Making Dumplings

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# Background

I think the assumptions in our text (see [appendix](#_Appendix)) are a little inaccurate & unpractical, especially the shapes. After searching the Internet, I found some images (see below) of dumplings in real life, labeling them with upper-left, upper-right, lower-left, lower-right (***ul, ur, ll, lr***), respectively.



*Dumplings in real life*

As we shall see, the real dumplings are not **sphere-like** actually, but more in a way to be **ellipsoid-like**[[[1]](#footnote-1)](#ref_ellipse_like)(or some other particular shape) & with a considerable area of **fin** on their body (which is kinda like dinosaurs with fins on their backs but without any legs😂), and their bottom bases like **oval** shapes. Besides, we may consider the thickness of the flour as well. Thus, we can rebuild the model by taking above parameters into consideration and classify them into two cases as follows:

* None-thickness & with fin
* Thickness & with fin

Before proceeding, I’d like to discuss **Tangyuan-like** dumplings, of which the thickness must be taken into consideration, and the stuffing and flour are virtually concentric balls. Let inner radius be , and outer radius be . Hence, it’s very easy to see that the consumption of stuffing and flour depends on the ratio of their volumes, and it should be

Instantiate it with and we can get , (almost ) and it only depends on the ratio . Let’s find the critical point that is exactly, all we have to is to let , and we can therefore obtain critical point . If **flour is insufficient**, we just need to adjust the such that , otherwise let .

Note that even if (relatively very thin layer of flour), the volume ratio of flour to stuffing isn’t that low, it’s 371:729, and if we slightly diminish , say into , the ratio becomes , flour even accounts more!

*Tangyuan-like dumpling diagram*

flour

stuffing

# None-thickness & With Fin

Now, look back at dumpling [ur](#fin), one with fin on its back (~cute).

OK, let us obstruct the cutaway view (see[*dumpling cutaway view diagram*](#cutaway_view)). It seems we need to add more assumptions to move on.

fin

body

*dumpling cutaway view diagram*

*b*

*y*

*a*

*x*

*O*

*Bottom base elliptic curve diagram*

**

*rugby football*

## Assumptions

There are several reasonable assumptions that will make us proceed easier.

* The body of a dumpling is virtually a hemisphere elliptic ball (see above [*rugby football*](#rugby_football), but the dumpling only have the upper half), satisfying . Hence, , and the bottom base curve: .
* Associated dependencies of radii of these two curves & might have following two possible idealized cases:
  + Ratio  is invariable everywhere, i.e. .
  + Or alternatively,

## Model Establishing

### By first case of assumption 2:

#### Building

Let , where is a constant (here subscript *e* means ellipse-like).

Then via the similarity principles (we can divide the random curves into many small segments or partitions, then each of them can be processed by elementary mathematics – triangle similarity theorems), we can determine some useful results.

*Concentric random shape of curves*

Since is the upper curve of an ellipse, we can directly get

the area of the body by those known formulas, generating

,

And therefore,

The surface area of body (including bottom base) is

So, the total area of each dumpling will be

As for its volume, it simply is

Thus we obtain

From the expression, we can see that if **scale with the same proportion**[[[2]](#footnote-2)](#ref_same_proportion), say , then , where the coefficient is a **positive** constant depending on initial values .

#### Analysis

* Qualitatively, as grows, ratio of stuffing to flour of each dumpling () goes up as well, which indicates **unit area of flour can contain more stuffing as we increase the size of the dumplings**. And hence, with the same amount of flour, if we enlarge the size of each dumpling, then we will consume more stuffing ($ as well😂, but may bring better flavor).
* Quantitatively, we need to delve deeper into details.

Let , and , once parameters are determined, will be fixed. In addition, if scale with the **same proportion** , the coefs remain unchanged. In other words, if , then

This is reasonable, we can imagine the as a virtual, equivalent radius of a round-like 3D object, which is also the exact case when the ellipsoid is specialized as a sphere.

Hence we have

Well, this looks so like the expressions in our text if we substitute the stuff before with , and replace the long scary coef before by . Then it can yield exactly the same results as our text. But, uh, wait a minute, what the hell is the **fin** doing then, isn’t it supposed to have some effect on the results?

Actually, replacing the coefs before with constants respectively is reasonable under the big **precondition** that **scale with the same proportion**, for as we have claimed before – the coefs remain unchanged under this context.

fin

body

*Concentric circles that with ratio of fin to body*

Let’s verify it with an extreme example – supposing , then naturally , and let ratio of fin to body of fin be . Without any difficulties, we can get

which totally agree with results of equation if we initialize with .

We can see that, by the special case, , the results are consistent with our text, even though we have additionally considered the **fin** on the dumpling’s back. So, processing in a duplicate way as our text does, we can get the final answer times.

Note that if each guy has their tempers, I mean, they refuse to scale in tune, then can be variable, and have their own particular relationships with , depending on . So, we cannot regard they as constants that has nothing to do with in such case. See also [prior footnote](#ref_same_proportion).

(Actually, I made a mistake at first, and I wanna note it down here.)

**[** Since the flour and stuffing are used in a specific proportion, the number of dumplings of flour and stuffing should be the same, i.e.,

By maintaining the amount of flour, and varying , we have

So, the answer we get is times, i.e. the proportion we scale. If we double the dumpling’s virtual radius (or ), we will consume twice stuffing as much as before under the invariable amount of flour.

Of course, the answer is flawed. The issue is that if we change the practical volume into , both parts of equation () will also be changed from to , thus rendering equation () nonsense.**]**

### By **second case of assumption 2:**

fin

body

*Fin with identical width everywhere*

In this section, we will discuss the situation where the width of the fin is virtually identical everywhere, denoted by (see diagram above). The width , however, is determined by .

#### Building

From the discuss in previous section, we have known that

Now let’s think about how to calculate the area of fin.

Let the radius of be , then the area under curve is

Note that the area under is

Hence, the area of fin is

We’d better figure out a way of determining .

Let be a point on , then its coordinate is . Inserting point into the ellipse equation , we have

Therefore,

Let

which is known as the **Complete Elliptic Integral of the First Kind.**

Then we have,

.

For simplicity of writing, we denote by (which is a constant, and can be numerically calculated by some mathematical software, for example, provided by is a good tool for numeric computation of elliptic integral).

Thus,

Similar to the approaches we adopted before, let

Note that is a constant depending on .

Let .

#### Analysis

* Qualitatively

Let’s think about the problem from a new perspective – unit. We know that are units of length, so does , it could be something like, , or whatever. All they have to follow is that guaranteeing their final result be measurable by length & in the process, each term must be manipulative (can do operations, e.g. In second one, term & term are both based on unit of area, so they two can do subtraction operation, after square root, it becomes unit of length, then it can be added by operand with the same unit of type, i.e. ).

By observing these examples, we can see that the manipulativeness insures the homogeneity[[[3]](#footnote-4)](#ref_homogeneous) of function . Further, is of first order, for width is one-dimensional. Therefore, we have

So, if we try to scale the size with proportion of , then

In fact, by equation ([6](#eq_6)) & ([7](#eq_7)), we have

Hence, the conclusion is identical to that of in [previous qualitative analysis](#case_1_qualitative_analysis), provided, of course, that **scale with the same proportion.**

* Quantitatively

The result varies depending on the width , and there are a couple of possible situations as follows (Again, we only discuss the situations where scale with the same proportion).

1. is a linear function of .

This case may be the simplest & most intuitive, for example, it can be , which suggests width must be larger than one eighth of , and larger than a quarter of .

To be complemented …

1. is a function of , i.e.,

This case is also kinda possible.

## Summery

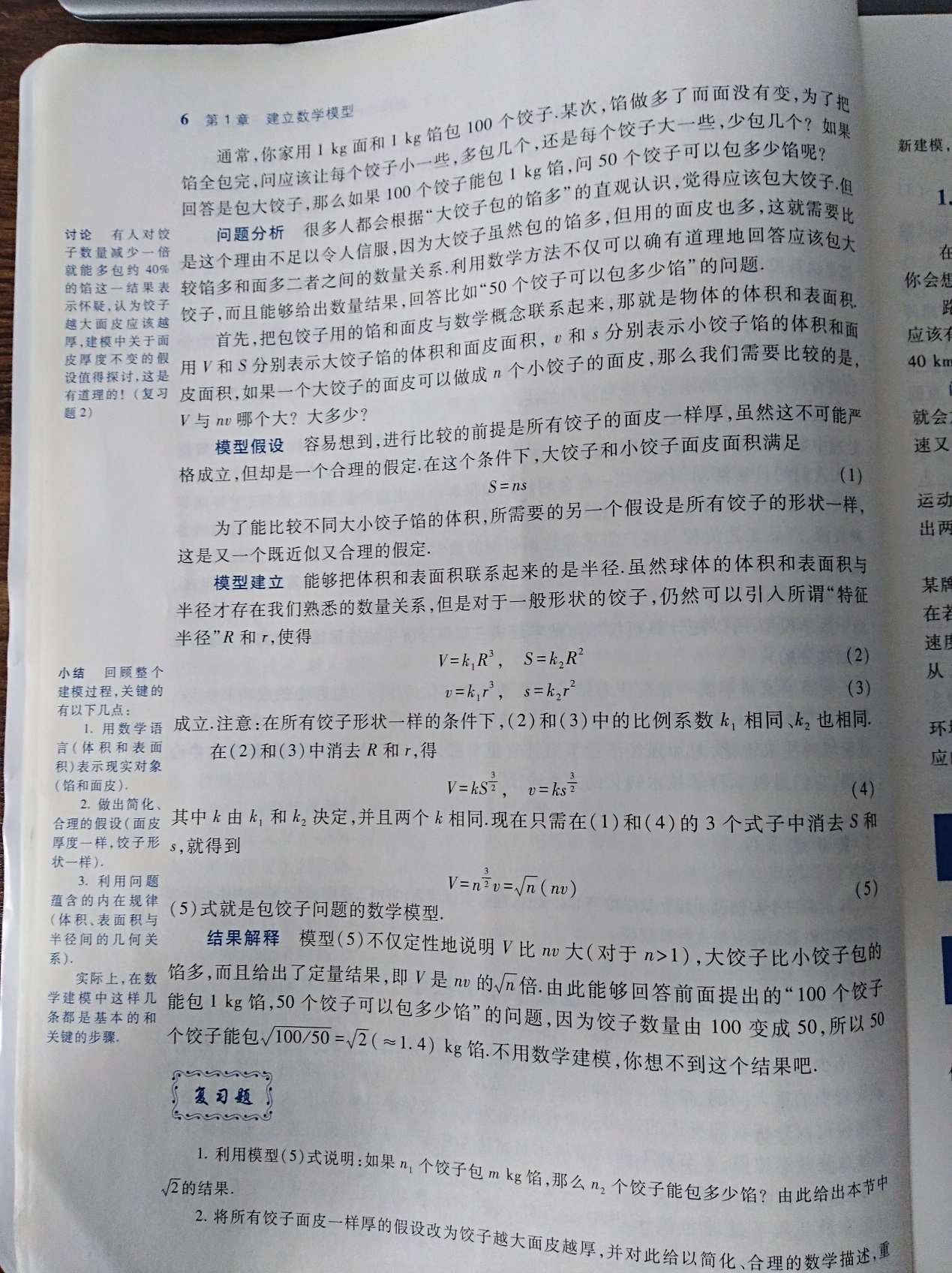
See analysis part of each section:

* [Case 1](#_Analysis),
* [Case 2](#_Analysis_1),

# Thickness & With Fin

To be complemented …

# Appendix



*Model in our text*

1. Though in reality they both can be seen as ball-like, and a sphere is a specialization of an ellipsoid (). Here we wanna generalize it and see how results look. [↑](#footnote-ref-1)
2. Well, you might ask what if they are out of tune with each other, say expanding with proportions . Yeah, it happens, and it **will influence the results quantitatively**. In the terms of qualitativeness, however, it remains unchanged as that of with the same proportion. It can be explained like this:

   where represents denominator, represents the th item of numerator. In general, we just need to substitute *3.6* with *the total product of each parameter*, but yielding identical result – if we enlarge the size, goes up; and if we decrease the size, falls off.

   Oops, wait a minute, some might ask what if it’s a compound, I mean, the scaling coefs are not all larger than 1 or not all smaller than 1 (e.g., ). Well, it’s sounds possible, and it requires further discussion (usually complicated). But, in such cases, where the dumplings attempt to get slimmer and taller so that they girls can become more good-looking and thus attracting boys, I dunno what the hell the dumpling maker is thinking about, maybe he’s an ingenious artist. (Goddamn it), why must we this trouble ourselves, huh? [↑](#footnote-ref-2)
3. I looked up in the dictionary for the word homogeneous, and surprisingly found that the meaning in the dict agrees so much well with the example I’d given. Here is the meaning in the dict: consisting of things or people that are all the same or all of the same type. [↑](#footnote-ref-4)