Seatwork (Permutations and Combinations)

Show your solutions

1. Given the 16 starting positions in a chessboard, how many ways can you arrange the following: (assuming the same pieces are identical)

   a. 8 black pawns

   b. 8 white pawns and 8 black pawns

   c. **8 black pawns 2 white knights 2 black knights**

   Complicated counting problems like these can be answered by breaking it down into multiple tasks:

   - **task 1** - **Selecting the 4 positions in the board that will be empty**. This is equal to $P(16, 4)$ or $\frac{16!}{(16-4)!}$. But since these 4 empty positions are identical, divide them by $4!$ giving us $\frac{16!}{(16-4)!4!}$. (You can summarize this whole task into $C(16, 4)$ and you'll get the same answer. I showed the division rule part to be clearer)

   - **task 2** - **Selecting the 8 pieces that will be black pawns.** Similar to the task above you can break this down to a permutation then division or combination. Either way this task will give us $C(12, 8)$ ways or $\frac{12!}{(12-8)!8!}$.

   - **task 3** - **Selecting 2 of the remaining 4 pieces to be white knights**. Similar to the task above you can break this down to a permutation and division or combination. Either way this task will give us $C(4, 2)$ ways or $\frac{4!}{(4-2)!2!}$.

   - (Note that we do not need to perform the task of selecting the black knights since the last two pieces will automatically be the black knights)

- Via multiplication rule the whole procedure is
  $C(16,4)C(12,8)C(4,2) = 5405400$.

d. 4 white pawns 4 black pawns and a white king

2. Prove the following identity (hint: look at where these coefficients lie in Pascals triangle)

$$\binom{n+1}{r+1} - \binom{n}{r+1} = \binom{n+1}{r} - \binom{n}{r-1}$$