The Art of Computer Programming, Chapter #1
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1.1

1.[00] (b,c,d,a) ← (a,b,c,d):
Answer:
temp ← a, a ← b, b ← c, c ← d, d ← temp

2.[15] Proves $m > n$ in step E1.

Answer:
We know that r will always follow $0 \leq r < n$ because we defined it that way. From here in step E3, n is set to m ($m \leftarrow n$), and n is set to r ($n \leftarrow r$). From this we after the first iteration n will always be less than m.

3.[20] Write algorithm E such that it avoids trivial replacements of $m \leftarrow n$ as algorithm F.
Answer:
F1: [Assign m/n] $m \leftarrow \frac{m}{n}$, m is the remainder
F2: [Check if end] If m = 0 the algorithm terminates, with n as the answer.
F3: [Assign n/m] $n \leftarrow \frac{n}{m}$, n is the remainder
F4: [Check if end] If n = 0 the algorithm terminates, with m as the answer. If not go to step F1

4.[16] GCD of 2166 and 6099
Answer:
If we follow our answer from 3 we have,
F1: $m = 2166, n = 6099$ and $m \leftarrow \frac{2166}{6099}$ or 6099
F2: m != 0 therefore no termination
F3: $n \leftarrow \frac{6099}{2166}$ or 1767 (same as 6099%2166)
F4: n != 0 therefore no termination so back to step F1
F1(2): $m = 6099, n = 1767$ and $m \leftarrow \frac{6099}{1767}$ or 798
F2(2): m != 0 therefore no termination
F3(2): $n \leftarrow \frac{1767}{798}$ or 171
F4(2): Again start at F1
F1(3): $m =, n = 171$ and $m \leftarrow \frac{798}{171}$ or 114
F2(3): m != 0 therefore no termination
F3(3): $n \leftarrow \frac{171}{114}$ or 57
F4(3): Again start at F1
F1(4): $m = 114, n = 57$ and $m \leftarrow \frac{114}{57}$ or 0
F2(4): Termination answer is n, GCD = 57

5.[12] The flow chart on page xii does or doesn't meet what rules for classifying an algorithm?

Answer:
1. Finiteness: [No] Doesn't terminate if one doesn't get tired
2. Definiteness: [No] Doesn't meet because each step is not precisely defined
3. Input: [Yes] has zero or more inputs
4. Output: [No] Has no clear output
5. Effectiveness: [No] Each action is not guaranteed to be able to be done in a finite time. Steps can not be represented on paper.

6.[20] What is T_5, the average number of times step E1 is preformed when n = 5?

Answer:
Because we said $0 \leq r < n$ there are only the cases $m = 1, 2, 3, 4, 5$ and the times E1 will be called will be (2,3,4,3,1) respectively

7.[M21] Let m be known and n range over all positive integers. Let U_m be the average number of times E1 is called in Algorithm E. Show U_m is well defined.

Answer:
Similar to the answer for question 6 we know given m we know there are there are only limited possibilities for $n < m$ and a lot of cases where $n > m$. We also know that if $n > m$ after one run through they will swap into a $n < m$ case. From here we know it will follow the answer for T_m. So $U_m = T_m + 1$

8.[M25] Give a formal algorithm for calculating the GCD of positive integers m and n while specifying the variables found in Eq(3). Let the input be the string $a^m b^n$, that is m a's followed by n b's. **Hint:** Use Algorithm E but instead of division set $r \leftarrow |m - n|$ and $n \leftarrow \min(n, m)$

Answer:

9.[M30]

Answer:

1.2