Operations Research Case 2 Assignment

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1. Please formulate a Linear Program that may find an optimal allocation of CSRs. You may have noticed that as your Linear Program can only have fractional variables, which seems to be weird for CSR allocation and the calculation of lack amounts. In this Cast Assignment, let's ignore this issue. When one needs to produce a real schedule, she/he will do some rounding and manual adjustment. In short, please formulate a "Linear" Program and do not have non-fractional variables. For this problem, please write down a compact mathematical formulation. Do not submit a computer program.

Step 1: Define Variables and Parameters

【Define Variables】

$$N_{ds} = numbers\ of\ CSRs\ working\ in\ shift\ s\ in\ date\ d$$
$$N = N_{31\times 14}$$

【Define Parameters】

Shift arrangement:

$$A_{sp} = Working\ status\ at\ period\ p\ in\ shift\ s$$
$$A = A_{14\times 24}$$

Shift Category	Shift	09:00 09:30	09:30 10:00	10:00 10:30	10:30 11:00	11:00 11:30	11:30 12:00	12:00 12:30	12:30 13:00	13:00 13:30	13:30 14:00	14:00 14:30	14:30 15:00	15:00 15:30	15:30 16:00	16:00 16:30	16:30 17:00	17:00 17:30	17:30 18:00	18:00 18:30	18:30 19:00	19:00 19:30	19:30 20:00	20:00 20:30	20:30 21:00
Morning	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
	2	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
	3	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0
	4	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0
	5	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
Afternoon	7	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	8	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	9	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	10	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0
Night	11	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1
	12	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1
	13	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1
Leave	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 1 Shift working in every period

Demands estimated:

$$D_{dp} = demand\ of\ CSRs\ at\ period\ p\ at\ date\ d$$
$$D = D_{31\times 24}$$

Time	1	2	3	4	5	6	7	8	9	10
09:00~09:30	29	20	21	29	20	21	17	29	20	21
09:30~10:00	24	20	18	24	20	18	18	24	20	18
10:00~10:30	25	21	20	25	21	20	17	25	21	20
10:30~11:00	24	22	20	24	22	20	18	24	22	20
11:00~11:30	24	19	22	24	19	22	20	24	19	22
11:30~12:00	24	18	19	24	18	19	17	24	18	19
12:00~12:30	17	12	13	17	12	13	12	17	12	13
12:30~13:00	13	14	11	13	14	11	12	13	14	11
13:00~13:30	17	14	14	17	14	14	11	17	14	14
13:30~14:00	23	18	17	23	18	17	15	23	18	17
14:00~14:30	21	20	20	21	20	20	18	21	20	20
14:30~15:00	23	20	21	23	20	21	17	23	20	21
15:00~15:30	21	18	18	21	18	18	16	21	18	18
15:30~16:00	18	20	18	18	20	18	17	18	20	18
16:00~16:30	20	17	17	20	17	17	14	20	17	17
16:30~17:00	20	17	15	20	17	15	15	20	17	15
17:00~17:30	16	11	15	16	11	15	13	16	11	15
17:30~18:00	11	11	13	11	11	13	11	11	11	13
18:00~18:30	10	8	8	10	8	8	9	10	8	8
18:30~19:00	8	8	6	8	8	6	7	8	8	6
19:00~19:30	8	9	6	8	9	6	8	8	9	6
19:30~20:00	8	8	8	8	8	8	7	8	8	8
20:00~20:30	9	8	7	9	8	7	7	9	8	7
20:30~21:00	8	7	6	8	7	6	6	8	7	6

Figure 2 Demand in day 1 to day 7 in the following month

Step 2: Define Objective Function

The main idea of this task is to minimize the "Lack" of CSRs' supply, which "lack" means we care only about the insufficient of supplements instead of over supplements. Therefore, objective function has to detect those "lack" periods that lack numbers' summation. So that objective function may be designed as follows.

⇨ $Assume\ S_{dp}\ as\ lack\ of\ supplements\ in\ dated\ at\ period\ p$

$$S_{dp} = D_{dp} - (NA)_{dp}$$

However, we only care the "lack" of supplements, so we only need positive S_{dp}.

$$Positive(x) = \{x\ , if\ x \geq 0\ 0\ , if\ x < 0$$

$$z = \sum_{d=1}^{31} \sum_{p=1}^{24} Positive(S_{dp})$$

To Linearize the problem, we have to add additional constraints.

⇨ $Assume\ \omega_{dp} = Positive(S_{dp})$

⇨ $\omega_{dp} \geq 0, \omega_{dp} \geq S_{dp}$

Therefore, the objective function becomes as followed

$$z = \sum_{d=1}^{31} \sum_{p=1}^{24} \omega_{dp}$$

$$\omega_{dp} \geq 0$$

$$\omega_{dp} \geq S_{dp}$$

$$\forall d = 1,2, \dots ,31\ \ \forall p = 1,2, \dots ,24$$

Step 3: Define CSRs allocation's constraint

(1) Day off principle

It is known that each CSR has at least 8 day off in a month. So the company is going to make at least $8\ \times\ CSRs'\ number$ shift 0 in the entire month. So this constraint may to represent as followed:

$$\sum_{d=1}^{31} N_{ds}\ \geq\ 8 \times 40 = 320\ , s = 14\ (s = 14\ is\ shift\ 0)$$

(2) Shift and Leave request

If a CSR requests a shift at a certain date, we can reserve 1 in the certain date.

$$e.g. If\ there\ are\ x\ CSR\ x\ request\ shift\ s_x\ in\ date\ d_x$$

$$N_{d_x s_x} \geq x$$

If CSR requests a shift at a certain date, we can reserve 1 on the certain date.

(3) Senior limits and manager limits

As most of the CSRs have at least two years of experience, and the manager limit seems not a big deal, we will simply ignore them.

(4) At most one night shift and two afternoon shifts per week.

For night shift

$$\sum_{d=i}^{i+6} \sum_{s=11}^{13} N_{ds} \leq 1 \times 40, \forall i = 1,2,\dots,25$$

$$\sum_{d=i}^{i+6} \sum_{s=7}^{10} N_{ds} \leq 2 \times 40, \forall i = 1,2,\dots,25$$

(5) At least one day off in seven consecutive days.

$$\sum_{d=i}^{i+6} N_{ds} \leq 1 \times 40, \forall i = 1,2,\dots,25$$

2. Submit a (set of) computer program that may solve the Linear Program you formulated in Problem 1. You may submit an AMPL model file and an AMPL data file. Alternatively, you may submit a (set of) Python or C++ programs that invoke gurobi to solve this problem.

The program is solution.py. Before Running the program, please make sure that the excel data “OR108-2_case02_data.xlsx” is in the same folder with the program. While running the program, please do not open “OR108-2_case02_data.xlsx”; otherwise our program would fail.

To ease the process of reading the file in python, before starting programming, we have preprocessed the data. We change the sheet “shifts” to the following form. Therefore, if you want to execute our program, please make sure your data are also process in the same way, or use the file contained in the folder we submit.

Shift Category	Shift	09:00 09:30	09:30 10:00	10:00 10:30	10:30 11:00	11:00 11:30	11:30 12:00	12:00 12:30	12:30 13:00	13:00 13:30	13:30 14:00	14:00 14:30	14:30 15:00	15:00 15:30	15:30 16:00	16:00 16:30	16:30 17:00	17:00 17:30	17:30 18:00	18:00 18:30	18:30 19:00	19:00 19:30	19:30 20:00	20:00 20:30	20:30 21:00
Morning	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
	2	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
	3	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0
	4	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0
	5	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
Afternoon	7	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	8	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	9	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	10	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0
Night	11	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1
	12	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1
	13	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1
Leave	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 3 The sheet "shifts" after procession

After running the program, you can see three sheets are added.

- Output Data:

  It shows the allocation of CSRs each day in the after month.
- Omega:

  It shows the lack of supply every period in each day. If the demand is accomplished, the number will be 0. Otherwise, if the demand is not accomplished, the number will be a positive number.

- Output Demand:

  It shows the demand minus supply in every period each day in the following month.

3. Please summarize the optimal solution you obtain with your computer programs submitted in Problem 2. You should summarize the allocation of CSRs and the total lack amount returned by your computer programs.

The matrix in the following picture means how many CRSs work in the specific date and shift. There are three extra columns from excel arithmetic in the right hand side of the picture.

Date	1	2	3	4	5	6	7	8	9	10	11	12	13	0		Week_Day_off	night	afternoon
1	2	13	0	0	0	0	0	0	0	7	5	2	1	10	40	79.5	40	72.5
2	0	9	2	0	0	0	1	0	0	5	4	1	3	15	40	72.5	40	70.5
3	0	0	0	0	0	0	4.5	6.5	3	3.5	4.5	1.5	0	16.5	40	72	40	76.5
4	11	0	0	0	0	0	0	8	7	2	1	0	3	8	40	71.5	40	70
5	0	9	2	3	0	0	2	0	0	2	4	0	3	15	40	68.5	40	61
6	0	0	6	2	0	0	9	5	0	0	0	0	3	15	40	53.5	37	66
7	0	0	0	1	15	13	0	0	1	6	4	0	0	0	40	51.5	40	63
8	9	3	0	0	9	3	0	0	5	0	8	0	0	3	40	67.5	40	62
9	0	0	0	4.5	1	0	5.5	4.5	2	0	7.5	0.5	0	14.5	40	65.5	40	60
10	0	6	0	0	1	0	5	0	3	3	5	1	0	16	40	57.5	40	54
11	0	0	0	0	9	14	0	0	0	8	4	0	0	5	40	41.5	40	50
12	0	0	0	0	16	11	2	0	0	7	0	0	4	0	40	44.5	40	51
13	0	0	0	3	0	7	2	0	3	6	0	6	0	13	40	55.5	40	58
14	8	0	6	0	0	0	0	5	0	1	0	0	4	16	40	48.5	39	61
15	9	4	2	0	11	2	0	0	3	0	0	0	8	1	40	41	40	64
16	0	0	6.5	0	9	4	6	0	0	0	0	0.5	7.5	6.5	40	40	40	64
17	8	0	0	0	16	3	0	0	7	0	0	0	6	0	40	48.5	40	62
18	0	0	0	0	7	12	0	0	2	7	3	1	0	8	40	63.5	40	62
19	0	0	0	0	5	4	9	0	7	0	4	0	0	11	40	72.5	40	60
20	4	0	0	8	3	0	0	14	0	0	0	0	5	6	40	77.5	40	63
21	0	0	9	0	4.5	4	9	0	0	0	0	0	5	8.5	40	89.5	40	55
22	7	0	0	2	12	8	0	0	3	0	8	0	0	0	40	97	39	53
23	5	3	0	5	0	0	0	2	2	0	7	1	0	15	40	107	39	57
24	0	5	2	2	3	0	6	0	1	0	6	0	0	15	40	107	38	70
25	0	9	3	0	0	0	2	0	0	5	1	0	3	17	40	102	40	68
26	0	0	0	1	0	0	8	5	6	0	4	0	0	16	40			
27	0	7	0	4	0	0	3	0	3	0	3	0	2	18	40			
28	2	11	0	0	0	0	0	0	0	7	4	0	0	16	40			
29	5	10	0	0	0	0	0	0	3	4	6	2	0	10	40			
30	0	0	0	1	0	0	6	5	6	0	4	0	3	15	40			
31	5	9	3	0	0	0	0	1	0	4	5	2	1	10	40			

Figure 4 The optimal solution obtained by above-mentioned computer programs

First, the "Week_Day_off" column means how many CSRs get day offs every seven consecutive days. All the numbers bigger than 40, so that constraint(5) is true.

Secondly, the night column means how many CSRs get night shifts every seven consecutive days. Most of them reach the upper bound (40), also binding to the constraints, so that constraint (4)'s night constraint is true.

Last, the afternoon column means how many CSRs get afternoon shifts in seven consecutive days. As we can see there doesn't exist a week that is bigger than the upper bound (80), so that constraint (4)'s afternoon constraint is true.

By our omega definition, we know that zero number is good, meaning supply is bigger or equal to demand. On the contrary, the nonzero number means the lack of CSRs in the shifts. As we can see, the night shifts, 20 to 24 shifts are lacking CSRs. And the total lack of demands is 222.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	5	4
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	1	1	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	5	4	3
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	4	3	3	2
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	4	3	3	2
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	1	2	2
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	1	1	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	1	2	2
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	4	3	3	2
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	1	2	2
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	4	3
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	4	3	3	2
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	4	3	3	2
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	1	2	2
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	3	2	2	1
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	1	2	2
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	1	1	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

Figure 5 Omega matrix

The output demand matrix shows the number of demands minus supply. The supply in the morning and afternoon is sufficient. However, the company needs more CSRs in night shifts to fulfill its demand.

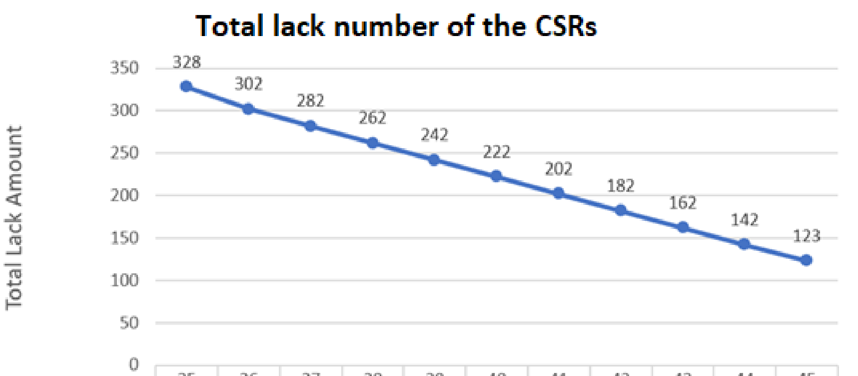
Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	-5	-4	-5	-5	0	-3	0	0	0	-2	-7	-9	-12	-10	-4	-6	-2	-5	0	0	0	1	0
2	-2	-2	-1	0	-3	0	-4	0	0	0	0	-5	-7	-5	-8	-1	-6	-2	-6	0	1	0	0	-1
3	-2.5	-5.5	-3.5	-3.5	-1.5	0	0	0	0	0	0	-2.5	-5.5	-5.5	-6.5	-4	-2.5	-9	-15.5	0	0	2	1	0
4	0	-5	-4	-5	-5	-4	0	0	0	0	-9	-9	-11	-14	-12	-8	-12	-10	-11	4	4	4	5	4
5	-2	-2	-1	0	-3	0	-4	0	0	0	0	-5	-7	-5	-8	-1	-7	0	-3	1	2	1	1	0
6	-1	-4	-2	-2	0	-3	0	0	0	0	-3	-4	-7	-7	-8	-7	-7	-4	-9	3	3	5	4	3
7	-10	-9	-10	-9	-7	-6	-11	0	0	-2	0	-8	-9	-8	-11	-6	-8	0	-2	3	4	3	3	2
8	-5	-10	-9	-10	-10	-2	0	0	0	0	-7	-5	-7	-10	-8	0	-4	-2	-3	0	0	0	1	0
9	-5.5	-5.5	-4.5	-3.5	-6.5	0	0	0	-4	0	0	-4.5	-6.5	-4.5	-7.5	0	-5.5	-8.5	-12	0	1	0	0	-1
10	-3	-6	-4	-4	-2	0	0	0	0	0	0	-2	-5	-5	-6	-3	-2	-3	-9	0	0	2	1	0
11	-4	-3	-4	-3	-1	0	-5	0	-1	-3	0	-9	-10	-9	-12	-7	-9	-1	-3	3	4	3	3	2
12	-7	-10	-5	-7	-9	-5	-11	0	0	0	0	-6	-8	-6	-7	-7	-6	-3	-7	2	2	1	2	2
13	-3	-2	-3	-2	0	-3	0	0	-6	0	0	-10	-11	-10	-13	-12	-8	0	-8	1	2	1	1	0
14	-2	-5	0	-2	-4	0	0	0	0	0	-6	-6	-8	-6	-7	-7	-6	0	-4	2	2	1	2	2
15	0	-5	-4	-5	-5	-5	-3	0	0	0	-7	-5	-7	-10	-8	0	-4	0	-1	0	0	0	1	0
16	-2	-2	-1	0	-3	-4	-3.5	0	0	0	-4.5	-4.5	-6.5	-4.5	-7.5	0	-5.5	-2.5	-6	0	1	0	0	-1
17	-10	-13	-11	-11	-9	-12	-10	-2	0	0	-4	-3	-6	-6	-7	-3	-3	0	-5	0	0	2	1	0
18	-3	-2	-3	-2	0	0	-4	0	0	-1	0	-8	-9	-8	-11	-7	-8	-1	-4	3	4	3	3	2
19	-7	-10	-5	-7	-9	-1	0	0	0	0	-7	-6	-8	-6	-7	-7	-6	-10	-14	2	2	1	2	2
20	0	-5	-4	-5	-5	-5	-8	0	0	0	-2	-8	-10	-13	-11	-6	-10	-8	-9	3	3	3	4	3
21	-2.5	-2.5	-1.5	-0.5	-3.5	-4.5	-1.5	0	0	0	-7	-7	-9	-7	-10	-5	-11	-3	-6	3	4	3	3	2
22	-3	-8	-7	-8	-8	0	0	0	0	0	-5	-5	-7	-10	-8	0	-4	0	-1	0	0	0	1	0
23	-5	-5	-4	-3	-6	0	0	0	-4	0	0	-5	-7	-5	-8	-1	-6	0	-4	0	1	0	0	-1
24	-4	-7	-5	-5	-3	0	0	0	0	0	0	-1	-4	-4	-5	-1	-1	0	-5	0	0	2	1	0
25	-3	-2	-3	-2	0	-2	-5	0	0	0	0	-6	-7	-6	-9	-4	-6	0	-2	3	4	3	3	2
26	-6	-9	-4	-6	-8	0	0	0	0	0	-6	-6	-8	-6	-7	-7	-6	-13	-17	2	2	1	2	2
27	-3	-2	-3	-2	0	0	-2	0	-1	0	0	-5	-6	-5	-8	-2	-4	0	-2	2	3	2	2	1
28	-6	-9	-4	-6	-8	0	-6	0	0	0	0	-6	-8	-6	-7	-7	-6	-1	-5	2	2	1	2	2
29	-1	-6	-5	-6	-6	0	0	0	0	0	-5	-7	-9	-12	-10	-4	-6	-2	-5	0	0	0	1	0
30	-2	-2	-1	0	-3	0	0	0	0	0	-4	-5	-7	-5	-8	-1	-7	-13	-16	1	2	1	1	0
31	0	-5	-4	-5	-5	0	0	0	0	0	-5	-7	-9	-12	-10	-4	-6	0	-3	0	0	0	1	0

Figure 6 Output demand matrix

4. Please rerun your programs several times, each time with a different number of CSRs, to see how the number of CSRs affect the total lack amount

To see how the number of the CSRs affects the company, we can start from CSR = 35, and add CSR by 1 every time we run the program. After that, we observe that the change of objective values(total lack amount) remains the same to 20 when CSR = 36, 37,...,44. To find the relationship between "20" and the change of CSR, we trace back to the chart "Omega matrix", and we find that if CSR = 36, 37,...,44, only the night shifts, 20-24, does not have sufficient CSRs. Therefore, every time we add one another CSR, the total lack amount will decrease by (24 - 20 + 1) * 4 = 20, where 4 means the number of weeks in a month.

If CSR = 45, the decremental amount will be 19 < 20, which means the action of adding CSR is not as efficient as before. To sum up, the objective value keeps improve when we increase the CSR number in the {35, 36, …, 44}, but the effect becomes worse along the increment of it.



	35	36	37	38	39	40	41	42	43	44	45
CSR number	328	302	282	262	242	222	202	182	162	142	123

Figure 7 the trend of total lack number of the CSRs