AN ELEMENTARY APPROACH TO QUANTUM TIME DILATION.

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Abstract. We present several arguments pointing towards a possible time dilation associated with the weak, strong, and electromagnetic potentials, similar to the well-known gravitational time dilation of General Relativity. This notion provides simple and direct links between GR and basic Quantum Mechanics. Although a fairly complete and detailed theory with these exact properties was published by Apsel three decades ago, it has been largely ignored and many objections to it have been made. We show that most of these objections are obviously wrong, and that the remainder are less certain than they might appear.

Regardless of the truth or falsity of the theory, we show that it makes definite predictions and is readily and inexpensively testable, and propose that such tests be performed.

Contents

1. The Unfinished Program of General Relativity

To set the context, it will be helpful to review the history of General Relativity and attempts at unified theories.

Before relativity, Heaviside had attempted to unify Newtonian gravity with EM in his theory of Gravitomagnetism[?]. While GM had problems, and was rendered obsolescent by relativity theory, it contained many interesting ideas and novel non-Newtonian predictions, such as measurable effects near rotating massive bodies. In this sense, it can be viewed as an early glimpse of the kind of torsional GR effects measured by Gravity Probe B. It is also worth noting that a revised and updated version of GM, with fewer obvious problems, has been developed by Jefimenko[?]; this line of thought has not completely died out.

Between 1915-1917, when GR was first published in more-or-less final form, and the early 1930s, many people tried to find theories uniting GR with classical EM. Analyzing this massive body of work is far beyond the scope of the present article; even a book-length review[?] was only able to cover some of the principal theories from this period. One of the more famous attempts[?], by H. Weyl, introduced the

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concept of gauge invariance, which was crucial to many later developments. Weyl wrote:

> ... as the fact of the propagation of action and of the existence of rigid bodies leads us to found the affine on the *metrical* character of the world which lies a grade lower, it immediately suggests itself to us, not only to identify the co-efficients of the quadratic groundform $g_{ik}dx_i dx_k$ with the potentials of the gravitational field, but also to identify **the co-efficients of the linear groundform** $\phi_i dx_i$ **with the electromagnetic potentials.** ... We thus arrive at the inference: **The world is a** $(3 + 1)$**-dimensional metrical manifold; all physical field-phenomena are expressions of the metrics of the world.**[**?**, p.283]

None of these Unified Geometrized Theories (UGTs) was entirely successful; the problem was harder than people thought. Both the triumphant development of Quantum Mechanics in 1925-27, and the discovery of new elementary particles and the weak and strong forces a few years later, made GR-EM unification seem increasingly irrelevant.

For most working physicists from then on, GR was considered to be a theory of gravitation only, with little or no relevance for other phenomena. Only a handful of researchers, including Einstein and Schrödinger, felt otherwise. In their view, the program of GR would not be complete until all forces had been geometrized:

> the conception Einstein put forward in 1915 embraced from the outset ... every kind of dynamical interaction, not just gravitation only. ... the very foundation of the theory, viz. the basic principle of equivalence and a gravitational field, clearly means that there is no room for any kind of 'force' to produce acceleration save gravitation, which however is not to be regarded as a force but resides on the geometry of space-time. Thus in fact, though not always in the wording, the mystic concept of force is wholly abandoned. ... we are in patent need of field-laws for the matter-tensor (e.g. for the electromagnetic field), laws that one would also like to conceive as purely geometrical restrictions on the structure of space-time.[**?**]

Our first argument for electromagnetic (and weak, and strong) time dilation, while admittedly vague and qualitative, follows from this viewpoint. We assume that such a unified geometrized theory can be formulated, with GR as a limiting case. Such a theory would presumably treat each force somewhat similarly. Since we know that there is a time dilation associated with the gravitational potential, it would be reasonable to expect a similar time dilation to be associated with each of the other potentials.

The mainstream view is that in GR only gravity needs to be geometrized; other forces do not. For example, S. Carroll writes

> in GR, gravity is represented by the curvature of space-time, not by a force. From the GR point of view, "particles move along geodesics, until forces knock them off." Gravity doesn't count as a force. If you consider the motion of particles under the influence of forces other than gravity, then they wont move along geodesics[**?**, p.14].

This is diametrically opposed to the Einstein-Schrödinger position.

2. Time dilation and gravitation in GR

It is easy to show that in general relativity, for slow-moving objects that are not close to a black hole, almost all of the acceleration of gravity is due to time dilation.[**?**, **?**, **?**, **?**] For example, the Schwarzchild metric is:

$$ds^2 = -(1 - 2GM/r)c^2dt^2 + (1 - 2GM/r)^{-1}dr^2 + r^2d\Omega^2$$

and if we take the weak-field limit ($2GM/r$ is small) and non-relativistic limit ($|dr/dt| << c$) we get

$$ds^2 \approx -c^2dt^2 + dr^2 + r^2d\Omega^2 + 2\phi(r)c^2dt^2$$

where $\phi(r) = GM/r$ is the Newtonian gravitational potential. The first three terms are just the flat-space metric of special relativity, so all of the curvature is in the time dimension; yet the geodesics in this metric reproduce Newtonian gravitational dynamics. So the traditional notion that gravity causes time dilation has it precisely backwards; to a very good approximation, time dilation causes gravity.

Thus we have a concrete example of a mass-induced time dilation gradient presenting itself as a $1/r^2$ "force". It therefore seems plausible that a similar charge-induced time dilation could present itself as the EM "force", which is also $1/r^2$. Note, however, that any extension of GR to cover EM requires substantially different geometry. Since positive and negative particles move in different directions under the influence of an electric field, such a theory cannot have GR-style geodesics that are the same for all particles.

3. Phase frequency as local clock: linking GR and QM

The argument above are of course very unsatisfactory by themselves. They make no quantitative predictions, and gives no hint of how to begin unifying GR with quantum theory. Fortunately, elementary considerations afford us some further insights into this problem. In GR, clocks deeper in a gravitational potential run slower. In QM, particles deeper in an electric potential have lower phase frequency. To make the analogy more precise is not difficult.

In GR the time dilation is normally given as a function of position within a gravitational potential, while in QM the phase shift is normally given as a function of the energy of a bound wave function. To compare them, we need to put them into common terms. We choose to do this by considering position in terms of energy, and phase shift in terms of time.

3.1. **Time Dilation in a Uniform Gravitational Field.**

3.1.1. *The Weak-Field Approximation.* It is sufficient for our present purposes to consider only the simplest case of a uniform gravitational field with strength g. The time dilation $T_d(h)$ between two observers at heights h and 0 is often given by the weak-field approximation[1]

$$T_d(h) \equiv \frac{t_h}{t_0} \approx 1 + \frac{gh}{c^2}$$

where t_h and t_0 are the times at the two observers. Since we only use the ratio of t_h to t_0, it does not matter much whether we consider them to be the instantaneous rate of time flow at each observer, or the total elapsed time measured on each clock for some experiment with synchronized start and end times.[2] The dimensionless ratio is the same in either case.

For a particle of mass m, the energy difference between the positions of the two observers is $\Delta E = mgh$, the amount of work required to raise the mass from the lower observer to the upper one. Thus the energy of the particle at height 0 is $E_0 = E = mc^2$, the energy equivalent of the mass m, and at height h (as seen from height 0) is $E_h = E + \Delta E$. We can thus rewrite the previous equation as

$$T_d \approx 1 + \frac{mgh}{mc^2} = 1 + \frac{\Delta E}{E} = \frac{E + \Delta E}{E} = \frac{E_h}{E_0}$$

Thus we have that

$$\frac{t_h}{t_0} \approx \frac{E_h}{E_0}$$

The ratio of energies equals the ratio of times. This is the relation between time dilation and energy in GR. However, the derivation is only valid for $\Delta E \ll E$.

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[1] For a derivation see e.g. [**?**, section 48]. This approximation is only valid for $gh \ll c^2$, and has various problems that we will discuss in the next subsection.

[2] We assume that it is possible for stationary observers to synchronize their clocks and to measure time intervals with a synchronized beginning and end. This is obviously true if, for example, we accept the notion of simultaneity presented by Einstein in 1907 ([**?**], see also [**?**] and chapter 7 of [**?**]). This involves the assumption that the speed of light is isotropic. There is some question about the correctness of this assumption (see e.g. the discussion in [**?**] or [**?**]), but for the present purposes we can assume that all observers are arranged in a single vertical line and all light paths follow that geodesic. This eliminates the possibility of the Sagnac effect and other global topological anisotropies. For a general proof of the validity of synchronization in any stationary spacetime, see [**?**, section 9.2].)

3.1.2. *The Exact Solution.* The linear approximation given above has severe problems in the limit of large h or strong g. First, two observers at different heights should predict the same relative time dilation between themselves

$$T_d(h) \cdot T_d(-h) = \frac{t_h}{t_0} \cdot \frac{t_{-h}}{t_0} = \frac{t_h}{t_0} \cdot \frac{t_0}{t_h} = 1$$

But for the linear formula, we have

$$T_d(h) = 1 + \frac{gh}{c^2}$$

and

$$T_d(-h) = 1 - \frac{gh}{c^2}$$

so that

$$T_d(h) \cdot T_d(-h) = \left(1 + \frac{gh}{c^2}\right)\left(1 - \frac{gh}{c^2}\right) = 1 - \frac{g^2h^2}{c^4} \neq 1$$

More generally, if we have three observers at heights 0, a, and $a+b$, each of them must correctly predict the time dilation between the other two. That is, we require that

$$T_d(a+b) = T_d(a) \cdot T_d(b)$$

for all a and b. But the linear formula fails this as well.

Finally, the formula gives absurd results for $h < -\frac{c^2}{g}$. The upper observer predicts that time should be flowing in opposite directions for the two observers, but the lower one does not. There is a kind of "event horizon" at $h = -\frac{c^2}{g}$ where time stops[3], but this is purely an artifact of the approximation and has no basis in reality.

The exponential equation

$$T_d(h) = e^{gh/c^2}$$

is the well-known unique exact solution for the uniform field[4]. The weak-field approximation is just the tangent line to this at $h = 0$.

At first it might appear that the simple relation given in the previous section needs to be modified. However, the formula $\Delta E = mgh$ depends on the assumption that the mass m does not change as we raise it. If we consider the mass to include the potential energy, so that a particle gets heavier as we raise it, and lighter as we lower it, then by an argument similar to that just given, we can conclude that the mass and energy depend exponentially on height

$$m_h = m_0 e^{gh/c^2}$$

[3]A uniform gravitational field is locally equivalent to an actual acceleration in flat spacetime. For this situation, in the SR literature the event horizon is known as the *Rindler Horizon* (see e.g. [**?**, **?**, **?**, **?**]).

[4]It is e.g. equation 1.11 in [**?**]; see also chapter 9 of that work. The derivation there is more general and applies to any stationary potential, not just a uniform one.

$$E_h = m_0 c^2 e^{gh/c^2}$$

A further confirmation of this comes observing that, if we have 3 observers at heights a, b, and c, the time dilations among them must be logically consistent. By definition,

$$T_d(c-b) \equiv t_c/t_b$$
$$T_d(c-a) \equiv t_c/t_a$$
$$T_d(b-a) \equiv t_b/t_a$$

So that the relation

$$T_d(c-a) = T_d(c-b) \cdot T_d(b-a)$$

must hold for any exact solution. The exponential formula satisfies this, but the linear one does not.

The exact relationship then becomes

$$\frac{t_h}{t_0} \equiv T_d(h) = e^{gh/c^2} = \frac{m_h}{m_0} = \frac{E_h}{E_0}$$

so that it is still the case that

$$\frac{t_h}{t_0} = \frac{E_h}{E_0}$$

even using the exact equations. This relationship is thus shown to be valid everywhere.

3.2. **Quantum Phase Shift.** Standing wave solutions to the Schrödinger equation with energy E oscillate phase as $e^{-iEt/\hbar}$. For different energy levels of identical particles, higher energy will cause the wave function to rotate phase more rapidly. Labeling the energy levels h and 0 as before and defining $\Delta E \equiv E_h - E_0$, we get a relative phase shift

$$\Delta\phi(t) = -i\Delta E t/\hbar$$

A shift in phase can be produced by a shift in time. If we set the phase shift due to ΔE to be equal to the phase shift due to a time delay Δt,

$$-i(E+\Delta E)t/\hbar = -iE(t+\Delta t)/\hbar$$

we get that

$$\Delta E \cdot t = E \cdot \Delta t$$

or

$$\frac{\Delta t}{t} = \frac{\Delta E}{E}$$

so that, arbitrarily choosing E_0 as our reference level, we get

$$\frac{t_h}{t_0} = \frac{t_0 + \Delta t}{t_0} = 1 + \frac{\Delta t}{t_0} = 1 + \frac{\Delta E}{E_0} = \frac{E_0 + \Delta E}{E_0} = \frac{E_h}{E_0}$$

which is the same equation arrived at in the previous section.

An alternate path to the same result starts with de Broglie's original equation relating frequency to mass (Eq. 1.1.5 in [**?**])

$$h\nu_0 = m_0 c^2$$

where $E_0 = m_0 c^2$ is the rest mass energy as above. Then the energy ratio equals the frequency ratio which, by definition, equals the time dilation:

$$\frac{E_h}{E_0} = \frac{\nu_h}{\nu_0} = \frac{t_h}{t_0}$$

In summary, it appears entirely reasonable to view the phase shift as being due solely to time dilation, with the particle's phase oscillation *being* its local clock. We call this viewpoint Quantum Time Dilation (QTD).

There is, of course, one other (non-deterministic, statistical) property of some elementary particles that could be used to measure their rate of time flow, and that is their rate of decay. Since the equations above indicate that any EM time dilation will be inversely proportional to the particle's rest mass, it should be easiest to detect this in the lightest possible unstable particle, namely, the muon. It is rather surprising that such elementary considerations as those given above can already make predictions about muon lifetimes under a unified theory whose mathematical form we do not yet know in detail. However, it seems unavoidable that any such theory must predict that the muon, with a mass of 105.7 MeV/c, will have its lifetime altered by about 1% in an electrostatic potential of 1.057 MV.

Of course, one would still like to have a detailed theory. Which brings us to the curious case of David Apsel.

3.3. **History.** As shown above, on the QM side it is not even required to have the Schrödinger equation; de Broglie's work of 1923-25 is sufficient. In fact, our time dilation could be considered to already be described by Schrödinger in late 1925 [**?**, **?**], when he gave the electron's frequencies in a hydrogen atom as

$$\nu_n = mc^2/h - R/n^2$$

except that Schrödinger did not interpret this as a time dilation.

Similarly, on the GR side it is only necessary to have the weak equivalence principle and $E = h\nu$ [**?**]; the full Einstein field equation is not necessary. Thus the results of the previous section *could* have been derived as early as 1926.

A similar time dilation was proposed by Apsel in 1978-9 [**?**, **?**]. His derivation starts from the Aharonov-Bohm effect, assuming that the variational principle

$$\delta \int_A^B d\tau = 0$$

of relativity applies to both gravitational and electromagnetic fields. He concluded, as we do, that "the physical time associated with the trajectory of a classical particle is related to the beats of the quasi-classical quantum mechanical wave function associated with the particle". This seems to be exactly the sort of theory anticipated by Schrödinger, but despite experimental support[?], it appears to have been largely ignored for three decades. Only a handful of other papers [?, ?, ?] refer to it. Ryff [?] rederives and generalizes Apsel's results starting from the equation

$$dx'_4 = -\frac{i}{mc} p_\mu dx_\mu$$

Beil [?] gives a metric for which the Lorentz equation of motion is just the geodesic equation in a Finsler space where electromagnetism is a noncompact timelike fifth dimension. He notes "that not only does the electromagnetic energy tensor part ... appear in the curvature, but so does the matter term. Thus, one can say that everything in this theory is curvature." Although the theory's gauge is dependent on particle velocity, "The usual physically meaningful quantities all involve only the gauge-independent field $F_{\mu\nu}$. There may, though, be a way of using ideas such as those of Apsel (1979) to give a measurable significance to the gauge."

The consequences for particle lifetimes were not lost on Apsel; he first proposed testing muon lifetimes in an electrostatic potential in his 1979 paper[?].

3.4. **Related theories.** Time dilation due to very strong external fields has been considered several times, for example by van Holten. In a 1991 paper[?] he derives the formula

$$dt = d\tau \frac{E - q\phi}{M}$$

where dt is the laboratory time and $d\tau$ is the proper time. But E and M in this formula must be the same, because in the absence of a potential ($\phi = 0$) when laboratory time *is* proper time ($dt = d\tau$), the formula gives

$$\frac{dt}{d\tau} = 1 = \frac{E}{M}$$

In our notation his equation thus reduces to

$$T + \Delta T = T \frac{E + \Delta E}{E}$$

which is equivalent to what we derived in section 1. Two years later[?] he gives the example of a muon spin down in an intense magnetic field and predicts a change $\delta\tau$ in the muon lifetime τ_μ given by

$$\frac{\delta\tau}{\tau_\mu} = -\frac{\vec{\mu} \cdot \vec{B}}{mc^2} = 0.28 \times 10^{-14} \times B$$

requiring a field strength on the order of 5 GT (producing an energy change of $|\vec{\mu} \cdot \vec{B}| \geq 1$ keV) to see a significant variation in lifetime[?]. His calculations for the muon appear to give identical results to what we would predict for the same energy shift, since his equation can be rewritten in our notation as

$$\frac{\Delta T}{T} = \frac{\Delta E}{E}$$

which is also the same as ours from section 1. However, Van Holten's theory differs notably from Apsel's in that it predicts effects from fields but not from potentials. Essentially, he considers the effect to be a form of spin-orbit coupling. Despite commenting that "It is quite clear from this formula, that any quantity which contributes to the energy E in an observable way, also contributes to the time dilation"[?] (with which we heartily agree), he does not in either paper consider that an electric potential can have any time dilation effect, particularly in a field free region. This has significant implications for testability, because the same degree of dilation produced by the 5 GT field (which is many orders of magnitude beyond what can be generated in labs today) in his theory could, in the present theory and Apsel's theory, also be achieved by changing the potential of the muon by a kV or so (something that could be easily done by almost anyone).

The question of whether there might be some kind of time dilation associated with an electric potential was raised online in 2004 [?], but the discussion there was vague and inconclusive, and showed no awareness of Apsel's results.

More recent online discussions instigated by J. Duda (e.g. [?]) are quite aware of Apsel, and also of the Heaviside & Jefimenko theories[?, ?] mentioned earlier, but appear so far to also be inconclusive. Duda, like van Holten, seems to think that strong fields (and not just potentials) are required, saying e.g. "Such experiment would need extremely strong electromagnetic field - like very near particles or near pulsars".[?, 8 Jan 2010]

In 2012 P. Ogonowski began publishing a theory of "Dilation As Field".[?, ?] It concludes "that gravitational field as well as electromagnetic field may be considered through one phenomena - time dilation."

4. Counterarguments

I am aware of five classes of counterarguments to Apsel-style theories. Some of them claim that no such effect can possibly exist; others, that even if it exists it would not constitute a time dilation.

4.1. Dismissal out of hand.

Some physicists reject the theory as so ridiculous that it does not even require an explanation of why it is wrong. As this is useless for advancing the debate, I mention it only in passing.

4.2. Naive Electromagnetic Gauge Invariance.

Many physical theories, such as classical EM and Van Holten's theory mentioned in the previous section, have a property that I will call Naive Electromagnetic Gauge Invariance. In NEGI theories, everything can be expressed in terms of fields acting locally; potentials can be viewed as having no physical reality but being merely aids to computation. NEGI would of course rule out any time dilation effects from an EM potential in a field free region, such as inside the sphere of a Van De Graaff generator. Many physicists seem to think that this is sufficient to disprove the theory.

The problem with this viewpoint is that it is flat-out wrong. The universe does *not* have the NEGI property; the Aharonov-Bohm effect suffices as a counterexample. The importance of this is often glossed over. For example, Jackson and Okun[**?**, p.24] write:

> ... gauge invariance is a manifestation of non-observability of A_μ. However integrals ... are observable when they are taken over a closed path, as in the Aharonov-Bohm effect ... The loop integral of the vector potential there can be converted by Stokes's theorem into the magnetic flux through the loop, showing that the result is expressible in terms of the magnetic field, albeit in a nonlocal manner. It is a matter of choice whether one wishes to stress the field or the potential, but the local vector potential is not an observable.

In any case, the NEGI idea (that fields acting locally on particles can explain everything) is admitted to be false.

4.3. **CPT and C Invariance.** It is often stated (e.g. in [**?**, **?**, **?**]) that the CPT theorem guarantees that particle and antiparticle masses and lifetimes are identical. However, this conclusion is only justified at zero potential, or with the further assumption of NEGI (which renders potential irrelevant). A true CPT reflection must invert all charges in the universe, which necessarily inverts all electric potentials as well. Therefore, the CPT theorem only *really* proves that a particle's mass and lifetime at 4-potential A must equal its antiparticle's mass and lifetime at 4-potential $-A$. This holds true in QTD, since the dilations for those two cases are identical. Thus, the CPT theorem does not contradict the QTD claim that particles and antiparticles will be time-dilated oppositely at a non-zero potential and that their lifetimes will differ there. QTD is completely compatible with the notion of CPT invariance.

4.4. **The S-Matrix and Accessible States.** An argument due to M. Gelfand[**?**] is based on the muon decay time being given by the S-matrix of the standard model. Since this matrix couples the muon initial state to possible final states, and placing the muon in a potential reduces or increases the number of accessible final states, Gelfand notes that one would expect an alteration in muon decay rate from this alone. So, while the previous arguments all claim that there can be no alteration in muon lifetime, this argument predicts that there *will* be an alteration, but explains it as being due to the mechanics of the S-matrix and not due to any time dilation. (Obviously, since this argument disagrees with the others about whether or not there will be an effect, at least one of them must be wrong.)

While this argument is subtle, I do not find it compelling. It is completely general and applies to any potential, including gravitational ones. Therefore, if we accept it, we must also accept that alteration of lifetime by a gravitational potential is entirely due to the S-matrix and not due to any time dilation. But we have substantial experimental evidence that gravitational time dilation exists, and our current best theory is that it is explained by GR alone.

This does not mean that the S-matrix approach is wrong; it merely means that the S-matrix is capable of expressing a time dilation. Thus, the ability to calculate accurate decay lifetimes entirely from S-matrix considerations does not preclude the existence of a potential-related time dilation.

One characteristic of a pure time dilation is that, all other things being equal, it must necessarily slow down (or speed up) all decay modes equally. Since muons have 3 known decay modes[**?**], this can be used as a test for whether lifetime alterations can reasonably be viewed as solely due to time dilation, or whether other factors must be invoked.

4.5. **Consequences of Linearity.** In the above theories, the energy of a charged particle is a linear function of potential, and the time flow is likewise linear and has many of the same problems as the weak-field gravitational approximation. In particular, for any particle there should be a potential at which the absolute phase frequency goes to zero; for example, the frequency of a μ^- should go to zero in a potential of $m_\mu c^2/q_\mu$ =+105.658 MV. The time flow at this potential must also be zero, so the predicted muon lifetime is infinite. Even worse, at higher potentials the time flow is predicted to be negative, and so is the lifetime. It is not clear what this could possibly mean.

One could conceive that the "real" theory, as in the case of gravitational time dilation, might be exponential rather than linear. That would solve all the problems of the previous paragraph; however, it would also appear to require abandoning the linear relation of frequency and energy ($E = h\nu$).

5. Experiment

How can quantum time dilation be most easily measured? If quantum time dilation is real, it should have occurred in many experiments that have already been carried out. In some cases, it would have been swamped by other effects such as velocity-based time dilation, but in others, it should have been noticeable.

For all charged particles, QTD predicts changes in lifetime at non-zero electric potential. Obviously this only has meaning for unstable particles. With stable particles, one appears to be limited to indirect tests based on interference effects

5.1. **Electrons and Positrons.** Electrons and positrons, with the largest charge-to-mass ratio of any particle, would experience the greatest time dilation under the present theory. However, since they have infinite lifetimes, this would only be detectable as a phase shift which is identical to that predicted by standard theory.

One indirect test would be to set up an electron interference experiment and run it at various potentials. Near $\phi = +511$keV, $h\nu = E = m_0c^2 - e\phi$ predicts the electron phase frequency should go to zero, and its de Broglie wavelength go infinite; therefore the interference fringes should get wider as we approach that voltage and disappear when we hit it exactly. If this fails to happen, then the linear version of the theory cannot be right.

This sort of experiment would be easy to do inside a large Van De Graaff generator, such as the one at Museum of Science in Boston, and not much more difficult in a moderately large enthusiast VDGG of say 0.5 meter diameter. The main difficulty would be getting the experiment to run off battery power and fit in the space allowed.

5.2. **Muons.** Radioactive ions or unstable charged particles will decay slower (or faster) when time-dilated. The muon, with a half life of 2.197 μS [**?**], is an attractive candidate for QTD experiments as it has the highest charge-to-mass ratio of any unstable particle. Surveys of methods of muon production can be found in [**?**] and [**?**]; see also section VI of [**?**]. Beam sources may be continuous or pulsed. Muon lifetime detectors adequate to measure time dilation effects can be simple and inexpensive enough to be used in an undergraduate physics lab [**?**].

The muon decay time can be very accurately calculated within the standard model given certain parameters [**?**]; some of these parameters can in turn be derived from muon lifetime data. Thus the precision of the standard model is dependent on the precision to which the muon lifetime is known.

In the following subsections we look at several ways of altering muon lifetimes. Much of what is said would apply equally well to other unstable particles such as pions.

5.2.1. *Electrostatic Potential Effect On Muons.* As mentioned above, QTD predicts that muon lifetime should be altered by an electrostatic potential. Since the muon mass-energy is 0.1134 AMU[**?**] = 105.6 MeV, a potential of 1.056 MV should alter the lifetime by 1%. Such a potential could be expected on a VDGG of about 70 cm diameter in air, which is within the reach of a serious hobbyist. A 1% effect is huge by HEP standards and should be easy to detect. Apsel (1979)[**?**] first proposed this kind of experiment; 37 years later, it still has never been performed.

Deep but small EM potentials exist around atomic nuclei. Thus, QTD (like Apsel[**?**]) predicts that decay times of bound μ^- in matter should be longer, and the lifetime should increase with atomic number. Such an effect does seem to be observable in light atoms, but is traditionally explained by other means, including special-relativistic time dilation from the orbital kinetic energy[5] and a reduction in accessible states for the decay products. Unfortunately, in heavy atoms, the muon orbital is so small that the muon spend much of its time *inside* the nucleus, so nuclear capture dominates and it is difficult to measure the muon decay lifetime. If a muon could be kept in a 2P orbital, or any other orbital which has a node

[5] 99% of bound muons end up in the 1S ground state, which has zero angular momentum. For atomic orbitals with zero angular momentum, the probability current is everywhere zero. Thus, explaining the change in decay time by SR time dilation requires assuming that the muon is "moving" even though there is zero current, which seems questionable.

through the nucleus and should thus have a greatly reduced rate of capture, the decay rate might be more easily observable for higher N.

5.2.2. *Aharonov-Bohm Effect On Muons.* The Aharonov-Bohm effect[**?**] (actually first described by Ehrenberg and Siday [**?**]) is a phase shift induced by a magnetic vector potential in a region of space which has zero magnetic field. We interpret this phase shift as an actual time dilation, and so predict that the experienced time (and hence decay rate in the laboratory frame) will be different on different sides of the solenoid in an Aharonov-Bohm setup even though the particles never encounter any field. Although fairly weak fluxes are used in typical AB experiments (because only $3.9 \cdot 10^{-7}$ gauss-cm^2 is required to rotate the electron phase by 2π [**?**]), much stronger fields could be used to test the time dilation effect. MRI machines with 10 tesla ($= 10^5$ gauss) fields over areas greater than 100 cm^2 have been demonstrated, so total fluxes of 10^7 gauss-cm^2 and up are quite feasible.

Let's first analyze the situation for an electron. A phase shift of 2π happens when

$$\Delta E \cdot t = 2\pi\hbar = h = E \cdot \Delta t$$

so that for an electron

$$\Delta t = \frac{h}{E} = \frac{h}{m_e c^2} = \frac{6.626 \cdot 10^{-34}\mathrm{m}^2\mathrm{kg/S}}{(9.109 \cdot 10^{-31}\mathrm{kg}) \cdot (2.998 \cdot 10^8 \mathrm{m/S})^2} = 8.093 \cdot 10^{-21}\mathrm{S}$$

is the time difference (as seen by the electrons, not by an external observer) between electron paths when the interference pattern has been shifted by one full fringe. This requires a total flux of $3.9 \cdot 10^{-7}$ gauss-cm^2 as noted above, but we should be able to use fluxes at least 10^{13}-10^{14} that large, leading to feasible Δts in the range of 10^{-7}-10^{-6} seconds. (Indeed the Brookhaven E821 experiment [**?**] applied a field of 1.45T over a ring with radius 7.11m; if the field had been uniform, the total contained flux would have been about $2.3 \cdot 10^{10}$ gauss-cm^2.)

For the muon the Δt for a single fringe shift is 207 times smaller (the ratio of the muon mass to the electron mass), or $3.91 \cdot 10^{-23}$S.

A 200 kV muon beam should travel roughly as fast as a 1 kV electron beam, or about 2% of the speed of light or $6 \cdot 10^6$ m/S. If we split this beam and send it around a solenoid with a radius of about 100 cm (and hence cross-sectional area 314 cm^2) we should be able to have each path be no longer than, say, 600 cm. With a $10\mathrm{T} = 10^6$ gauss field strength the total flux would be $3.14 \cdot 10^8$ gauss-cm^2 and the predicted $\Delta t = 3.91 \cdot 10^{-23} \frac{3.14 \cdot 10^8}{3.9 \cdot 10^{-7}}\mathrm{S} = 3.14 \cdot 10^{-8}\mathrm{S}$. The flight time of the muon would then be about 10^{-7} S. With a lifetime of 2.2 μS, and ignoring relativistic corrections, we would expect in the standard interpretation that a fraction $e^{-t/2.2\mu\mathrm{S}}$ of the muons on each path would remain undecayed

$$e^{-0.1/2.2} = e^{-0.04545} = 0.956$$

so that about 4.4% of the muons would decay on each path. However, the time dilation predicted by the present theory would cause the fast-time path to experience a total time $(t + \Delta t)$ of $(0.1 + 0.03)\mu$S so that approximately

$$e^{-0.103/2.2} = e^{-0.04682} = 0.954$$

would remain undecayed while on the slow-clock path

$$e^{-0.097/2.2} = e^{-0.04409} = 0.957$$

would. Thus, for that flux, we would predict a roughly 3% increase in the decay rate on the fast-time path and a 3% decrease on the slow-time path. Larger fluxes would have larger effects. For large enough flux, the effect should be truly spectacular.

It is not necessary to actually interfere the two beams to measure this effect. One could, for example, just have a single beam of muons rotating in a cyclotron ring. A large confined and shielded flux through the ring should have a measurable effect on the decay rate of the muons; reversing the flux or the direction of rotation should reverse the effect.

It would also be possible to simply fire a beam of muons through the center hole of one or more shielded toroidal magnets, as was done in the elegant Hitachi experiment to demonstrate the AB effect with electrons [**?**], although much larger magnets with much larger fluxes would be desirable. There is no theoretical upper bound to the total encircling flux per length of muon path in this configuration, as there is no upper limit to the radial size of the core. If we assume a core with a contained field strength of 1T [6], and a toroidal shape with rectangular cross section (inside radius r_i, outside radius r_o, and thickness l), the cross-sectional area is given by $A = (r_o - r_i) \cdot l$ and the total flux would be $10^4 \cdot A$ gauss-cm^2 with the flux per length equal to $10^4(r_o - r_i)$ gauss-cm. The time shift per length (for a muon) is given roughly by

$$\frac{3.91 \cdot 10^{-23}\mathrm{S}}{3.9 \cdot 10^{-7}\mathrm{gauss} \cdot \mathrm{cm}^2} \cdot 10^4(r_o - r_i)\mathrm{gauss} \cdot \mathrm{cm} \approx 10^{-12}(r_o - r_i)\frac{\mathrm{S}}{\mathrm{cm}}$$

For a rather large core with $(r_o - r_i) = 1$m and $l = 1$m (weighing about 25 metric tons[7]) we would get a time shift of 10 nS. This has to be compared with the time of flight at fast but sub-relativistic speeds; at 10% of the speed of light a particle will only spend about 33 nS passing through the toroid, so a time shift of 10 nS represents a roughly 30% increase or decrease in the time experienced by the particle.. The special-relativistic time dilation at that speed is less than 1%.

5.3. **Neutrons.** Neutrons, being uncharged, are not subject to time dilation by electrostatic potentials. However they do have spin and hence a magnetic moment, and may be dilated by the Aharonov-Casher effect in the same way that charged

[6] An electromagnet made of iron saturates at 1.6T, and NdFeB permanent magnets can have fields of 1.17 to 1.48T, so 1T is easily achievable.

[7] This is only moderately large by HEP standards; the storage ring magnet at the BNL muon g-2 experiment weighs 700 tons.[**?**]

particles may be dilated by the Aharonov-Bohm effect. Also, like everything, they are affected by gravitational time dilation.

5.4. **Other Forces.** The above discussion has been entirely about gravity and electromagnetism. However, other potentials should also cause similar time dilation.

In theory, geometric confinement could be used to raise the energy of a particle, along lines discussed in section 3 of [**?**]. This could be used on uncharged particles such as neutrons, while most of the above approaches require charged particles. However, the effect for any realistic confinement may be too small to measure.

There are many other possibilities, including the strong and weak nuclear forces, but these few should suffice to demonstrate that quantum time dilation makes different predictions than the standard view of phase shift, and that the differences are accessible to experimental test.

6. Summary

We interpret the well known quantum phase shift at different energy levels as a time-dilation. The mathematics of this is essentially identical to that of gravitational time dilation in general relativity, indicating perhaps a deep and simple connection between QM and GR. This interpretation is shown to have measurable consequences which have some support in prior experimental data, and further experiments are proposed that could test its validity more directly.

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