QUANTUM PHASE SHIFT CONSIDERED AS A GR-LIKE TIME DILATION

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Contents

1. Theory

1.1. **Introduction.** In general relativity, clocks at different heights in a gravitational field run at different rates, with the higher one running faster. In quantum mechanics, particles at different energy levels rotate their quantum phase at different rates, with the higher-energy one rotating faster. We propose here that these two phenomena can be viewed as essentially identical.

In GR the time dilation is normally given as a function of position within a gravitational field, while in QM the phase shift is normally given as a function of the energy of a wave function. To compare them, we need to put them into common terms. We choose to do this by considering position in terms of energy, and phase shift in terms of time.

1.2. **Time Dilation in a Uniform Gravitational Field: 1. The Weak-Field Approximation.** In general relativity, position in a gravitational field affects the speed of clocks, with higher ones running faster.

It is sufficient for our present purposes to consider only the simplest case of a uniform (non-curved) gravitational field with strength g. The time dilation T_d between two observers at different heights is normally given by the weak-field approximation

$$T_d \equiv \frac{t_f}{t_s} \approx 1 + \frac{gh}{c^2}$$

where h is the height difference and c is the speed of light. (For a derivation of this result see e.g. [1], section 48. This approximation is only valid for $gh \ll c^2$, and has various problems that we will discuss below.)

For a particle (or clock) of mass m, the energy difference between the positions of the two observers is $\Delta E = mgh$, the amount of work required to raise the mass from the lower observer to the upper, so we can rewrite the previous equation as

$$T_d = 1 + \frac{mgh}{mc^2} = 1 + \frac{\Delta E}{E} = \frac{E + \Delta E}{E} = \frac{E_f}{E_s}$$

where $E = mc^2$ is the energy equivalent of the mass m. Thus we have that

$$\frac{t_f}{t_s} = \frac{E_f}{E_s}$$

or, subtracting 1 from both sides and choosing one of the clocks arbitrarily as our reference, that

$$\frac{\Delta t}{t} = \frac{\Delta E}{E}$$

The time delta, as a fraction of the total reference time elapsed, is exactly equal to the energy delta, as a fraction of the original energy. This is the (approximate) relation between time dilation and energy in GR. The derivation is only valid for $\Delta E \ll E$.

1.3. **Time Dilation in a Uniform Gravitational Field: 2. The Exact Solution.** The linear approximation given above has severe problems in the limit of large h or strong g. First, two observers at different heights should predict the same relative time dilation between themselves

$$T_d(h) \cdot T_d(-h) = \frac{t_f}{t_s} \cdot \frac{t_s}{t_f} = 1$$

But for the linear formula, we have

$$T_d(h) = 1 + \frac{gh}{c^2}$$

and

$$T_d(-h) = 1 - \frac{gh}{c^2}$$

so that

$$T_d(h) \cdot T_d(-h) = (1 + \frac{gh}{c^2})(1 - \frac{gh}{c^2}) = 1 - \frac{g^2h^2}{c^4} \neq 1$$

More generally, if we have three observers at heights 0, a, and b, each of them must correctly predict the time dilation between the other two. That is, we require that

$$T_d(a + b) = T_d(a) \cdot T_d(b)$$

for all a and b. But the linear formula fails this as well.

Finally, the formula gives absurd results for $h < -\frac{c^2}{g}$. The upper observer predicts that time should be flowing in opposite directions for the two observers, but the lower one does not. There is a kind of "event horizon" at $h = -\frac{c^2}{g}$ where time stops, but (as we will show) this is purely an artifact of the approximation and has no basis in reality.

A uniform gravitational field is equivalent to an actual acceleration in flat spacetime. For this situation, in the GR literature the event horizon is known as the *Rindler Horizon* and is taken quite seriously (see e.g. [2] [3] [4] [5]). However the present theory views this as fundamentally incorrect.

An exact solution to the time dilation $T_d(h)$ at height h in a uniform field g must have the following properties:

(1) It must be continuous and differentiable.
(2) $T_d(h) \approx 1 + \frac{gh}{c^2}$ for small h; the approximate formula is valid near $h = 0$. In particular, $T_d(0) = 1$; two observers at the same height see the same time.
(3) $T_d(a + b) = T_d(a) \cdot T_d(b)$ Time dilation must be consistent across multiple observers at different heights. In particular, $T_d(h) \cdot T_d(-h) = T_d(h - h) = T_d(0) = 1$.

From the above we can readily see that for small δ

$$T_d(h + \delta) = T_d(h) \cdot T_d(\delta) = T_d(h) \cdot (1 + \frac{g\delta}{c^2})$$

and, taking a limit as $\delta \to 0$ in the usual way to find the derivative

$$\frac{dT_d}{dh}(h) = \lim_{\delta \to 0} \frac{T_d(h + \delta) - T_d(h)}{\delta} = \frac{g}{c^2} \cdot T_d(h)$$

we get a differential equation with the unique solution

$$T_d(h) = e^{gh/c^2}$$

It is easy to confirm that this function satisfies conditions (1)-(3) and that there is no event horizon or other discontinuity of any kind. It is the unique exact solution for a uniform field.

(In fact this equation was derived earlier. It is e.g. equation 1.11 in [6]. The derivation there is more general and applies to any stationary potential, not just a uniform one.)

Clearly the simple relation given in the previous section needs to be modified to take this into account. However, notice that the formula $\Delta E = mgh$ also depends on the assumption that the mass m does not change as we raise it. In reality, the mass includes the potential energy, so that a particle gets heavier as we raise it, and lighter as we lower it. By a derivation similar to that just given, we can conclude that the mass depends exponentially on height

$$m(h) = m_0 e^{gh/c^2}$$

The exact relationship is now

$$\frac{t_f}{t_s} \equiv T_d(h) = e^{gh/c^2} = \frac{m(h)}{m_0} = \frac{E_f}{E_s}$$

so that it is still the case that

$$\frac{t_f}{t_s} = \frac{E_f}{E_s}$$

and

$$\frac{\Delta t}{t} = \frac{\Delta E}{E}$$

even using the exact equations. This relationship is valid everywhere.

1.4. **Do Event Horizons Exist?** We noted above that, while the linear weak field approximation seems to indicate an event horizon in the uniform field, the exact solution has no such discontinuity. Here is a sketch of a simple proof that black holes do not actually have event horizons.

First, a non-rotating black hole can be represented by the Schwarzschild metric. This does not change over time, so it is a stationary potential. As noted above, the exponential formula for time dilation holds for all stationary potentials. Thus the time dilation at any point in the potential, relative to "cosmic time" at infinite distance, is given simply by $T_d = e^{\Phi/c^2}$ (where we take the convention that Φ is 0 at infinite distance and negative at all finite distances). The time dilation of observer B, as seen by observer A, is just $T_d = e^{(\Phi_B - \Phi_A)/c^2}$. There is no event horizon, where the time dilation goes to zero, for any possible stationary observer.

Of course, this is actually well-known (cf chapter 31 of MTW [7]).

We can perhaps generalize these results. Still considering only the case of a stationary potential with stationary observers,

Definition 1.1. $v(A, B) \iff B$ is visible to A. That is, light from B can reach A in finite time by A's clock.

Lemma 1.2. $v(A, B), v(B, C) \rightarrow v(A, C)$. *($v$ is transitive)*

1.5. **Quantum Phase Shift.** Standing wave solutions to the Schrödinger equation with energy E oscillate as $e^{-iEt/\hbar}$. Absolute phase appears impossible to measure and may in fact have no physical meaning whatsoever; however, relative phase can be easily observed through various interference experiments. For identical particles, or different trajectories of the same particle, higher energy will cause the wave function to rotate phase more rapidly. Labeling the energy levels f and s as before and defining $\Delta E \equiv E_f - E_s$, we get a relative phase shift

$$\Delta\phi(t) = -i\Delta Et/\hbar$$

For any wave at a constant energy level, and thus constant phase velocity, a shift in phase can be produced by a shift in time. If we set the phase shift due to ΔE to be equal to the phase shift due to a time delay Δt,

$$-i(E + \Delta E)t/\hbar = -iE(t + \Delta t)/\hbar$$

we get that

$$\Delta E \cdot t = E \cdot \Delta t$$

or

$$\frac{\Delta t}{t} = \frac{\Delta E}{E}$$

which is the same equation arrived at in the previous section.

An alternate path to the same result starts with de Broglie's original equation relating frequency to mass (Eq. 1.1.5 in [8])

$$h\nu_0 = m_0c^2$$

where $E_0 = m_0c^2$ is the rest mass energy as above. Then the energy ratio equals the frequency ratio which, by definition, equals the time dilation:

$$\frac{E_0 + m_0gh}{E_0} = \frac{E_f}{E_s} = \frac{\nu_f}{\nu_s} = \frac{t_f}{t_s}$$

In summary, it appears entirely reasonable to view the phase shift as being due solely to time dilation, with the local clocks on different particle paths running at different rates. Indeed, any other interpretation seems problematic.

1.6. **History.** As shown above, on the QM side it is not even required to have the Schrödinger equation; de Broglie's work of 1925 is sufficient. Similarly, on the GR side it is only necessary to have the weak equivalence principle and $E = h\nu$ ([6]); the full Einstein field equation is not necessary. Thus the present results *could* have been derived as early as 1926. Why they weren't seems rather mysterious.

A similar time dilation was proposed by Apsel [9]. His derivation starts from the Aharonov-Bohm effect and uses the full tensor machinery of standard GR. Despite experimental confirmation [10] it appears to have been ignored for three decades.

The idea of using quantum phase interference of entangled pairs to measure time dilation was proposed by Hwang et al. in 2002 [11], so they clearly understood that a potential-based time dilation would cause a phase shift; however, only gravitational time dilation was considered.

The question of whether there might be some kind of time dilation associated with e.g. an electric potential was raised in 2004 [12], but the discussion there was vague and inconclusive, and showed no awareness of Apsel's results.

1.7. **Thought Experiments.** In terms of pure interference effects, quantum time dilation appears to be experimentally indistinguishable from the standard interpretation of a phase shift without time dilation. Fortunately, there are many other forms of measurement which are not handicapped in this way.

1.7.1. *Experiment 1.* Two scientists, Fred and Sally, work in a lab at the foot of a mountain. They synchronize a pair of atomic clocks, and divide between them a pair of identical particles whose quantum phase has also been synchronized. As a control experiment, they wait until Fred's clock (and Sally's) shows exactly 1 hour and interfere the two particles. In the absence of decoherence, the phases should still precisely match. That is, $t_{1f} = t_{1s} = 1$ hour, $\Delta t_1 \equiv t_{1f} - t_{1s} = 0$, and $\Delta\phi_1 = 0$.

1.7.2. *Experiment 2.* As a second test, they synchronize as before. Fred now rides a computer-controlled cable car up the mountain, carrying his clock and particle with him, and immediately descends the same way. When he reaches the bottom, he and Sally note a slight time difference $\Delta t_2 \equiv t_{2f} - t_{2s}$ between their clocks. They wait one additional hour (on both clocks and then interfere their particles as before. They now measure a phase shift $\Delta\phi_2$ associated with ascending and descending the mountain; some of this might come from acceleration effects, and some from Fred being higher than Sally.

1.7.3. *Experiment 3.* Finally, having controlled for and measured all these effects, they synchronize once more. Fred ascends the mountain in exactly the same way as before, spends one hour by his clock at the summit, then descends. They interfere their particles and compare clocks. According to GR, their clocks should now be off, with Fred's clock showing a later time (t_{3f}) than Sally's (t_{3s}) due to gravitational time dilation. After adjusting for Δt_2 by computing $t_f \equiv t_{3f} - t_{2f}$ and $t_s \equiv t_{3s} - t_{2s}$, they would see a time dilation of

$$T_d \equiv \frac{t_f}{t_s} = 1 + \frac{gh}{c^2}$$

as in section 1. They also measure a phase shift $\Delta\phi_3$. After subtracting the phase shift from experiment 2 ($\Delta\phi_2$) to correct for any effects from ascending and descending, they would have a residual phase shift $\Delta\phi \equiv \Delta\phi_3 - \Delta\phi_2$ due entirely to Fred having been higher in the gravitational field than Sally.

The question is how to interpret this shift. In Fred's frame of reference at rest at the top of the mountain, the rate of phase rotation of his particle should have been perfectly normal, and hence the total rotation proportional to t_f. In Sally's frame of reference at rest in the lab, Fred's particle is at a higher energy level and should be rotating phase at a higher rate with the relative phase shift proportional to $t_s \Delta E$. Both their computations predict exactly the same phase shift. If Sally tries to adjust both for the higher energy and make a time-dilation adjustment at the same time, she will get the wrong answer. Either one works, but they cannot be used together. This is because they are the same thing viewed in two different ways.

(Note: We accept the notion of simultaneity presented by Einstein in 1907 ([13], see also [14] and chapter 7 of [15]). This involves the assumption that the speed of light is isotropic. There is some question about the correctness of this assumption (see e.g. the discussion in [16] or [17]), but for the present purposes we can assume that all observers are arranged in a single vertical line and all light paths follow that geodesic. This eliminates the possibility of the Sagnac effect and other global topological anisotropies. For a proof of the validity of synchronization in a stationary spacetime, see section 9.2 of [6]

2. Limitations

In special relativity, the time dilation for a particle or observer moving at velocity v is $T_d = \sqrt{1 - v^2/c^2}$. The higher the velocity, the higher the kinetic energy, but the *slower* the clock is perceived to run. Thus kinetic energy does not appear to have the same relationship to time that potential energy does; even the sign is reversed. A further difficulty, as noted by de Broglie [8], is that while all observers agree on gravitational time dilation, the time dilation due to motion is reciprocal (I think your clock is slower than mine at the same time you think my clock is slower than yours). Working through these issues is beyond the scope of the current paper.

3. Existing Experimental Results

Assuming that quantum time dilation is real, it should have occurred in many experiments that have already been carried out. In some cases, it would have been swamped by other effects such as velocity-based time dilation, but in others, it should have been noticeable. In this section, we try to compute the magnitude of the effect for various prior experiments to see whether the theory can be confirmed or disproved by them.

3.1. **Muonic Atoms.** Negative muons (μ^-) can be captured by normal atoms. In most cases the muon rapidly decays into the ground state $1S$ orbital. Because the muon orbit is much smaller than any of the electron orbits, to first order we can ignore the effect of the electron charge and treat the interaction between the nucleus and the muon as a hydrogen-like atom. The energy level is then given by the formula

$$-\frac{m_\mu}{m_e}\frac{Z^2}{1+m_\mu/M}\text{Ry}$$

where Z is the nuclear charge, M the nuclear mass, m_μ the muon mass, and $1\text{Ry} = 13.6\text{eV}$. An energy change of -1 Ry corresponds to a time dilation of

$$(1-1\text{Ry}/m_\mu c^2) = 1 - \frac{13.6\text{eV}}{105\cdot 10^6\text{eV}} = 0.999999871$$

which is not likely to be noticed. However, for heavy nuclei $m_\mu \ll M$ so we can ignore the correction term and the energy is roughly $-207Z^2\text{Ry}$. For the heavy non-radioactive nucleus $^{206}_{82}\text{Pb}$ we predict a potential-based time dilation of about 0.82; muons would decay about 18% slower in that situation due to this effect. However, other possible time-dilation effects such as those due to the velocity of the muon in the orbital must also be considered, as well as any effects from the nucleus that might accelerate or retard muon decay.

4. Proposed New Experiments

In this section, we assume the time-dilation interpretation is correct. Changes in the rate of time flow and changes in level of potential energy are therefore the same thing. The electrons in different orbitals in the same atom must be viewed as having different time dilations, even though they span the same space and have the same (non-moving) center of reference. Relativity shows that time is not a universal absolute, but rather depends on the position and velocity of the observer. To this, we must now also add the energy of the observer.

This is experimentally testable. Measurable predictions include

- A charged clock, placed inside a conductive cage, will run faster when the cage is given the same polarity charge, thereby raising the energy level of the clock, and slower when the cage is given the opposite charge. Radioactive ions or unstable charged particles will decay faster (or slower) under similar circumstances.
- Likewise, their decay rate will be different on different sides of the solenoid in a magnetic Ehrenberg-Siday-Aharonov-Bohm setup even though they never encounter any field. Note that, although fairly weak fluxes are used in typical ESAB experiments (because only $3.9\cdot 10^{-7}$ gauss-cm^2 is required to rotate the phase by 2π [18]), much stronger fields could be used to test the time dilation effect. MRI machines with 10 tesla ($= 10^5$ gauss) fields over areas greater than 100 cm^2 have been demonstrated, so total fluxes of 10^7 gauss-cm^2 are quite feasible.
- Alternately, geometric confinement could be used to raise the energy, along lines discussed in section 3 of [19]. This could be used on uncharged particles such as neutrons, while the above 2 approaches require charged particles.

There are many other possibilities, but these few should suffice to demonstrate that the time dilation view makes different predictions than the standard phase shift view and that the differences are accessible to experimental test.

4.1. **Muon-based Experiments.** The muon, with a half life of 2.2 μS, might be an attractive candidate for an experiment. While muons are difficult to produce by fission, fusion, or nuclear decay, beams of muons can be generated via pion decay, and have been used e.g. to test the standard model's prediction of their anomalous magnetic moment (see [20] and its references). A survey of methods of muon production can be found in [21]; see also section VI of [22]. Beams of muons produced by pion decay are inherently 100% polarized with spin opposite to the direction of emission ([21], p.16). Beam sources may be continuous or pulsed. Muon detectors adequate to measure time dilation effects can be simple and inexpensive enough to be used in an undergraduate physics lab[23].

The muon decay time can been very accurately calculated within the standard model given certain parameters [**?**]; one of these parameters can in turn be derived from muon lifetime data. Thus the precision of the standard model is dependent on the precision to which the muon lifetime is known.

Let's first analyze the situation for an electron. A phase shift of 2π happens when

$$\Delta E \cdot t = 2\pi\hbar = h = E \cdot \Delta t$$

so that for an electron

$$\Delta t = \frac{h}{E} = \frac{h}{m_e c^2} = \frac{6.626 \cdot 10^{-34}\mathrm{m}^2\mathrm{kg/S}}{(9.109 \cdot 10^{-31}\mathrm{kg}) \cdot (2.998 \cdot 10^8 \mathrm{m/S})^2} = 8.093 \cdot 10^{-21}\mathrm{S}$$

is the time difference (as seen by the electrons, not necessarily by an external observer) between electron paths when the interference pattern has been shifted by one full fringe. This requires a total flux of $3.9 \cdot 10^{-7}$ gauss-cm^2 as noted above, but we should be able to use fluxes at least 10^{13}-10^{14} that large, leading to feasible Δts in the range of 10^{-7}-10^{-6} seconds. (Indeed the Brookhaven E821 experiment[20] applied a field of 1.45T over a ring with radius 7.11m; if the field was uniform, the total contained flux was therefore about $2.3 \cdot 10^{10}$ gauss-cm^2.)

For the muon Δt is 207 times smaller (the ratio of the muon mass to the electron mass), or $3.91 \cdot 10^{-23}$S.

A 200 kV muon beam should travel roughly as fast as a 1 kV electron beam, or about 2% of the speed of light or $6 \cdot 10^6$ m/S. If we split this beam and send it around a solenoid with a radius of about 100 cm (and hence cross-sectional area 314 cm^2) we should be able to have each path be no longer than, say, 600 cm. With a 10T = 10^6 gauss field strength the total flux would be $3.14 \cdot 10^8$ gauss-cm^2 and the predicted $\Delta t = 3.91 \cdot 10^{-23} \frac{3.14 \cdot 10^8}{3.9 \cdot 10^{-7}}\mathrm{S} = 3.14 \cdot 10^{-8}\mathrm{S}$. The flight time of the muon would then be about 10^{-7} S. With a half life of 2.2 μS, and ignoring relativistic corrections, we would expect in the standard interpretation that a fraction $2^{-t/2.2\mu\mathrm{S}}$

of the muons on each path would remain undecayed

$$2^{-0.1/2.2} = 2^{-1/22} = 2^{-0.04545} = 0.969$$

so that about 3.1% of the muons would decay on each path. However, the time dilation predicted by the present theory would cause the fast-time path to experience a total time $(t + \Delta t)$ of $(0.1 + 0.03)\mu$S so that approximately

$$2^{-0.103/2.2} = 2^{-0.04682} = 0.968$$

would remain undecayed while on the slow-clock path

$$2^{-0.097/2.2} = 2^{-0.04409} = 0.970$$

would. Thus, for that flux, we would predict a roughly 3% increase in the decay rate on the fast-time path and a 3% decrease on the slow-time path. Larger fluxes would have larger effects. For large enough flux, the effect should be truly spectacular.

(Note that these calculations would give silly results for the slow-time path for high enough flux; it obviously cannot end up with negative experienced time-of-flight. This problem is due to using the linearized weak-field approximation. Exact calculation requires the exact formula.)

Note that it is not necessary to actually interfere the two beams to measure this effect. One could, for example, just have a single beam of muons rotating in a cyclotron ring. A large confined and shielded flux through the ring should have a measurable effect on the decay rate of the muons; reversing the flux or the direction of rotation should reverse the effect.

It would also be possible to simply fire a beam of muons through the center holes of one or more shielded toroidal inductors, as was done in the elegant Hitachi experiment to demonstrate the ESAB effect [25]. Much larger inductors with much larger fluxes would of course be desirable, but these are commercially available and relatively inexpensive. There is no theoretical upper bound to the total encircling flux per length of muon path in this configuration, as there is no upper limit to the radial size of the inductor cores. If we assume an iron core with a contained field strength of 1T (iron saturates at 1.6 T), and a toroidal shape with rectangular cross section (inside radius r_i, outside radius r_o, and thickness l), the cross-sectional area is given by $A = (r_o - r_i) \cdot l$ and the total flux would be $10^4 \cdot A$ gauss-cm^2 with the flux per length equal to $10^4(r_o - r_i)$ gauss-cm. The time shift per length (for a muon) is given roughly by

$$\frac{3.91 \cdot 10^{-23}\text{S}}{3.9 \cdot 10^{-7}\text{gauss} \cdot \text{cm}^2} \cdot 10^4(r_o - r_i)\text{gauss} \cdot \text{cm} \approx 10^{-12}(r_o - r_i)\frac{\text{S}}{\text{cm}}$$

If we assume a rather large core with $(r_o - r_i) = 1$m and $l = 1$m then we get a time shift of 10 nS. This has to be compared with the time of flight at fast but sub-relativistic speeds; at 10% of the speed of light a particle will only spend about 33 nS passing through the inductor, so a time shift of 10 nS represents a

30% increase or decrease in the time experienced by the particle.. (Again, these are based on the linear formula and thus are slightly off.) The special-relativistic time dilation at that speed is less than 1%.

4.2. **Determining Absolute Electric Potential.** Since a non-zero electric potential causes inverse time dilation for positive and negative particles, the lifetimes of (say) μ^+ and μ^- can be used to calculate the potential. This means that there is a global absolute zero potential which can be detected by experiment; it is that point at which positive and negative particles and antiparticles decay at the same rate.

Without measurement, there is no guarantee that the earth is at zero potential. Thus, we have no reason to expect that the lifetimes of charged particles will be exactly the same for positive and negative versions when measured in terrestrial laboratories. This has implications for those attempting to use e.g. the muon lifetime to determine parameters of the standard model. If they do not correct for time dilation due to the absolute potential of the earth, or equivalently take the geometric mean of μ^+ and μ^- lifetimes, their results will be slightly wrong.

Further corrections may also need to be made for the effective energy level of bound muons in solids. The time dilation for a μ^- in a 1S orbital in Pb is quite large, but even the relatively small time dilation for a μ^+ orbited by an e^- could be significant enough to perturb measurements intended to be accurate to 1 part per million.

5. Summary

We propose interpretating the well known quantum phase shift as instead a time-dilation effect. The mathematics of this is essentially identical to that of gravitational time dilation in general relativity, indicating perhaps a deep and simple connection between QM and GR. This interpretation is shown to have measurable consequences which are supported by prior experimental data, and further experiments are proposed that could test its validity more directly.

References

[1] D.F. Lawden, *An Introduction to Tensor Calculus, Relativity and Cosmology* 3rd Ed, J.Wiley (1982)

[2] "Rindler Coordinates" `http://en.wikipedia.org/wiki/Rindler_coordinates`

[3] G. Egan, "The Rindler Horizon", `http://www.gregegan.net/SCIENCE/Rindler/RindlerHorizon.html`

[4] L. Xiang, Z. Zheng, "Entropy of the Rindler Horizon", *Int. J. of Theoretical Physics* v.40 #10 1755-1760 (Oct 2001)

[5] H. Culetu, "Is the Rindler horizon energy nonvanishing?", *Int. J. Mod. Phys.* D15 2177-2180 (2006) DOI: 10.1142/S0218271806009601 arXiv:hep-th/0607049v2

[6] W. Rindler, *Relativity: Special, General, and Cosmological*, Oxford U. Press (2001)

[7] Misner, Thorne, Wheeler, *Gravitation*

[8] L. de Broglie, *Recherches sur la Théorie des Quanta* (1925), translated by A.F. Kracklauer as *On the Theory of Quanta* (2004) http://www.ensmp.fr/aflb/LDB-oeuvres/De_Broglie_Kracklauer.pdf
[9] D. Apsel, "Gravitation and electromagnetism", *General Relativity and Gravitation* v10 #4 297-306 (Mar 1979) DOI: 10.1007/BF00759487
[10] D. Apsel, "Time dilations in bound muon decay", *General Relativity and Gravitation* v13 #6 605-607 (Jun 1981) DOI: 10.1007/BF00757247
[11] W. Y. Hwang, D. Ahn, S. W. Hwang, Y. D. Han, "Entangled quantum clocks for measuring proper-time difference", *Eur. Phys. J. D,* v.19 #1, 129-132 (April 2002) doi:10.1140/epjd/e20020065
[12] "time dilation in an electromagnetic potential" (2004) http://www.physicsforums.com/archive/index.php/t-57510.html
[13] A. Einstein, "Über das Relativitätsprinzip und die aus demselben gezogenennFolgerungen", *Jahrbuch der Radioaktivität und Elektronik* 4, 411-462 (1907); English translation, "On the relativity principle and the conclusions drawn from it", *The Collected Papers*, v.2, 433-484 (1989)
[14] H.M. Schwartz, "Einstein's comprehensive 1907 essay on relativity, part 1", *Am. J. Physics* 45, 512-517 (1977)
[15] M. Jammer, *Concepts of Simultaneity: from Antiquity to Einstein and Beyond*, Johns Hopkins U. Press (2006)
[16] D. Soler. "Rigid Motions in Relativity: Applications", in L. Momas, J.Diaz Alonso (eds.), *A Century of Relativity Physics* 611-614, AIP (2006)
[17] R. Klauber, "New Perspectives on the Relativistically Rotating Disk and Non-time-orthogonal Reference Frames", *Found. Phys. Lett.* v.11 405-443 (1998) gr-qc/0103076
[18] W. Ehrenberg, R. E. Siday,. "The Refractive Index in Electron Optics and the Principles of Dynamics", *Proc. Phys. Soc.* B62: 821 (1949). doi:10.1088/0370-1301/62/1/303
[19] B.E. Allman, A. Cimmino, A.G. Klein, Reply to "Comment on Quantum Phase Shift Caused by Spatial Confinement" by Murray Peshkin, *Foundations of Physics* v.29 #3 325-332 (March 1999). doi:10.1023/A:1018858630047
[20] G.W. Bennett et al., "Measurement of the Negative Muon Anomalous Magnetic Moment to 0.7 ppm", *Physical Review Letters* 92; 1618102 (2004). arXiv:hep-ex/0401008 v3 (21 Feb 2004)
[21] G.H. Eaton, S.H. Kilcoyne, "Muon Production: Past, Present, and Future", in S.L. Lee et al. (eds), *Muon Science: Muons in Physics, Chemistry and Materials* 11-37, (1999)
[22] Y. Kuno, Y. Okada, "Muon decay and physics beyond the standard model", *Rev. Mod. Physics* 73 151-202 (Jan 2001)
[23] T. Coan, T. Liu, J. Ye, "A Compact Apparatus for Muon Lifetime Measurement and Time Dilation Demonstration in the Undergraduate Laboratory", *Am.J.Phys.* 74, 161-164 (2006) arXiv:physics/0502103v1
[24] T. van Ritbergen, R.G. Stuart, "Complete 2-Loop Quantum Electrodynamic Contributions to the Muon Lifetime in the Fermi Model", *Phys. Rev. Lett.* 82, 488-491 (1999) http://link.aps.org/doi/10.1103/PhysRevLett.82.488 DOI: 10.1103/PhysRevLett.82.488
[25] A. Tonomura et al., "Observation of Aharonov-Bohm Effect by Electron Holography," *Phys. Rev. Lett.* 48 1443-1446 (1982)
[26] H. Stephani et al., *Exact solutions of Einstein's field equations* 2nd Ed., Cambridge U. Press (2003)
[27] M.A.H. MacCallum, "Finding and using exact solutions of the Einstein equations", in L. Momas, J.Diaz Alonso (eds.), *A Century of Relativity Physics* 129-143, AIP (2006)