The magnitude of electromagnetic time dilation.

Howard A. Landman[1]*

[1]*Fort Collins, Colorado, USA howard@riverrock.org*

Since there is a time dilation associated with the gravitational potential, theories unifying gravity and electromagnetism naturally give rise to the question of whether there might be a time dilation associated with the electromagnetic 4-potential. While this notion has been widely rejected, a handful of theories explicitly predict such an effect. At least to first order, all of them agree on its magnitude, but it has not been clear why. Here we show that the magnitudes of both gravitational and EM time dilations can be computed from elementary considerations ($E = h\nu$ and $E = mc^2$) that are independent of specific unified theories. This demonstrates that EM time dilation must be a feature of any unified theory that is compatible with both Special Relativity and Quantum Mechanics; and more generally, that there must be a time dilation associated with all potentials, including the weak and strong nuclear ones. This constitutes physics beyond the Standard Model, since the SM excludes all such effects. The ubiquity of time dilation may allow it to be used as a central guiding principle of future unified theories, and provide a fresh approach to the problem of quantum gravity.

1 Introduction

From the first publication of General Relativity in 1915 to about 1930, hundreds of classical theories were proposed attempting to unify gravity and electromagnetism[1]. While none of these was completely successful, some of them were very influential. For example, Weyl's Space-Time-Matter theory[2] introduced the notion of gauge invariance, while Kaluza-Klein theory[3,4] used a compact 5th dimension and was an important precursor to string theory.

Given that there is a time dilation associated with the gravitational potential in GR, it seems reasonable to wonder whether there might be a similar time dilation associated with the EM potential in such unified theories. Sadly, this question has rarely been asked, let alone answered. Even after nearly a century, we don't know whether Kaluza-Klein theory has this feature or not. David Apsel in 1978-1981 gave probably the first unified theory to explicitly predict such a time dilation[5–7], and only a handful of subsequent papers[8–15] mention anything similar. At least to first order, all of these theories agree on the magnitude of EM time dilation.

In this paper we show why they must. We derive the magnitudes of both gravitational and electromagnetic time dilations from elementary considerations that do not depend on the machinery of GR or any specific unified theory, and thereby demonstrate that they must be features of any unified theory that is compatible with both Special

Relativity and Quantum Mechanics.

2 History

Einstein first derived gravitational time dilation in his 1907 paper on the Relativity Principle[16]. He began in §18 by using Special Relativity to show that clocks at different X-positions in a reference frame accelerated in the X-direction cannot run at the same rate; then in §19 used the Equivalence Principle to infer that the same thing must be true for clocks at different values of a gravitational potential Φ. He carried out all the arguments to first order to give the linear form $T_d = 1 + \Phi/c^2$, which has since become called the weak-field approximation, although he did note in passing that the actual formula must be $T_d = e^{\Phi/c^2}$. The linear approximation cannot be exactly correct because it has two problems: it is logically inconsistent since $(1+\Phi/c^2)(1-\Phi/c^2) \neq 1$, and there is an event horizon at $\Phi = -c^2$. The exponential form solves both of those problems.

The conclusion of the 1907 argument is that acceleration causes the rate of time flow to be a function of position in the direction of the acceleration. It did not matter to Einstein whether the acceleration was caused by a rocket, or by standing on the ground in a gravitational field. Although he didn't mention it, it is worth noting that the acceleration of a charged particle by an electric field is not immune to this argument.

Neither are accelerations due to the weak and strong forces; *all* accelerations of a given magnitude *must* cause exactly the same time dilation.

The philosophical question here is whether EM acceleration is ”gravity-like”, i.e. whether the Equivalence Principle applies to EM. This is a yes/no question with only two possible answers. If it does, then application of the Einstein 1907 argument forces EM time dilation, and gives a magnitude identical to that computed below. If it doesn’t, then there can be no EM time dilation. Weyl explicitly assumed that it doesn’t[2, pp. 304-305]; most other researchers have implicitly assumed the same without even discussing it.

After General Relativity in 1915 and the Schwarzschild solution in 1916, another view became possible, although it is still not widely appreciated. Taking the weak field ($r_s \ll r$) and low speed ($\frac{dr}{dt} \ll c$) limit of the Schwarzschild metric leaves us with the Newtonian metric

$$ds^2 = (dx^2 + dy^2 + dz^2 - c^2dt^2) + (-2\frac{GM}{r})dt^2$$

which is just flat Minkowski spacetime plus the time dilation field. In this metric, space is completely flat and only time is curved, and the curved time gives geodesics that match Newtonian gravity. This pure time dilation field appears as a $1/r^2$ “force”. So in the Newtonian limit of GR, matter causes a time dilation field and the time dilation gradient causes gravitational acceleration.[1] The direction of cause and effect is completely

[1]This line of thought was anticipated by several early unified theories, although they tended to describe

reversed from the 1907 argument.

If we accept both of these arguments, then we cannot have any acceleration without an associated time dilation gradient, and we cannot have any time dilation gradient without an associated acceleration. The two are inextricably linked.

3 Gravitational time dilation from $E = h\nu$ and $E = mc^2$

In this section we use a new method to derive gravitational time dilation without directly invoking relativity theory. We assume only that particles have a rest energy associated with their mass, given by $E = mc^2$, and a frequency associated with their energy, given by $E = h\nu$.

In a uniform gravitational field of strength g, raising a particle by a height z requires work mgz. Thus, to an observer at height 0, the total energy of the particle at height z is given by $E(z) = mc^2 + mgz$ and its frequency by $\nu(z) = E(z)/h$.

However, an observer already at height z would perceive the particle to have merely frequency $\nu(0) = mc^2/h$. This can only be true if the two observers have clocks running at different rates, in the ratio

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it as a speed-of-light field rather than a time dilation field. For example, Ishiwara wrote in 1912 that ”if the speed of light varies in space and in time, then these variations lead to the appearance precisely there of a gravitational field.”[17]

$$T_d = \frac{\nu(z)}{\nu(0)} = \frac{E(z)}{E(0)} = \frac{mc^2 + mgz}{mc^2} = 1 + \frac{gz}{c^2}$$

which is the weak-field approximation to GR's gravitational time dilation (with $\Phi = gz$). As above, the linear form can't be exactly correct but the exponential form $T_d = e^{gz/c^2}$ is.

This derivation appears in some sense to be quantum, since it utilizes $E = h\nu$. But because time dilation is a dimensionless ratio, h cancels out and its precise value doesn't matter. This means that the classical ($h \to 0$) limit is exactly the same as the "quantum" result.

Both this derivation and Einstein's 1907 one avoid almost all the assumptions of GR, so each of them implies that any other theory that predicts a gravitational time dilation must have the same relation of dilation to potential as GR. From this viewpoint, the existence and magnitude of gravitational time dilation cannot be viewed as a confirmation of GR specifically, but only of a class of theories of which GR is the best known example.

4 Electromagnetic time dilation by the same method

We now consider the case of a particle with mass m and charge q in an electrostatic potential V. The potential energy is qV, so the corresponding time dilation (to first order) must be

$$T_d = \frac{mc^2 + qV}{mc^2} = 1 + \frac{qV}{mc^2}$$

As in the gravitational case, the linear form cannot be completely right, and the exponential form $T_d = e^{qV/mc^2}$ is the most obvious candidate to replace it. But unlike in the gravitational case, here both charge and mass matter, or more precisely the dilation is a function of the charge/mass ratio q/m. This means that a simple Riemannian manifold is inadequate, and the geometry of any unified theory has to be something more complicated, like a Finsler space. Uncharged particles should be completely unaffected. For a given non-zero q, lighter particles will be dilated more strongly than heavier particles. The electron, being the lightest charged particle and having the highest charge/mass ratio, should be affected the most. But since electrons have infinite lifetime, the only observable effect on them is the shift in phase frequency. Although this is universally observed, most physicists would not consider it proof of or even evidence for time dilation.

Thus, for experimental testing, we are lead to the muon. With a mass-energy of

$m_\mu c^2 = 105.7$ MeV, it is still light enough to have its mean lifetime of 2.2 μS affected by a modest potential. For example, a potential of 1.057 MV should alter its lifetime by about 1%; such a potential could be achieved by a Van de Graaff generator with a sphere of about 76 cm diameter in air, which is well within reach of a serious hobbyist. Apsel first proposed this kind of experiment in 1979[6]; 40 years later it still has never been performed.

Negative muons (μ^-) bound to low-Z nuclei are also known to have lengthened lifetimes. The normal explanation for this is that the muon has a kinetic energy given by the quantum virial theorem, and an average velocity corresponding to that kinetic energy, and a special-relativistic time dilation corresponding to that velocity. However, Apsel has argued that this calculation does not match the experimental data very well, and that adding a (smaller) electromagnetic time dilation term gives a better fit[7]. If so, we may have already been seeing evidence for decades. The effect should be more obvious for higher Z. Unfortunately, as Z increases, nuclear capture by a proton begins to dominate, and we don't have good data on non-capture decay rates for most elements.

For magnetic interactions, the potential energy is $-\vec{\mu} \cdot \vec{B}$, where $\vec{\mu}$ is the magnetic moment and $\vec{B}$ is the magnetic field, and so to first order we get

$$T_d = 1 + \frac{-\vec{\mu} \cdot \vec{B}}{mc^2}$$

The muon's measured magnetic moment is $\mu = -4.49 \times 10^{-26}$ J/T. To get the same

1% level of time dilation, say between spin-up and spin-down muons, we would need to place them in a field of

$$1.057\,\mathrm{MeV} \times \frac{1\,\mathrm{J}}{6.24 \times 10^{12}\,\mathrm{MeV}} \times \frac{1\,\mathrm{T}}{2 \times (4.49 \times 10^{-26}\,\mathrm{J})} \approx 1.89 \times 10^{12}\,\mathrm{T}$$

Van Holten thought that 5×10^9 T might suffice for detection, and could be found in the vicinity of a magnetar[11,12]. But given that the world record magnetic fields are in the range of 45–330 T, this seems far beyond the reach of current experiment. Only the electrostatic part of the effect is amenable to testing. Thus, ignoring magnetic (and gravitomagnetic) terms, we get a unified time dilation equation

$$T_d \approx e^{(m\Phi + qV)/mc^2}$$

One characteristic of a pure time dilation is that, all other things being equal, it must necessarily slow down (or speed up) all decay modes equally. Since muons have 3 known decay modes[25], this can be used as a test for whether lifetime alterations can reasonably be viewed as solely due to time dilation, or whether other factors must be invoked.

Charged pions (π^+, π^-) have a charge-mass ratio 0.757 as large as a muon's, and would also be reasonable for such experiments, but would require about $0.757^{-2} = 1.745$ times as many data points to get the same statistical significance.

5 Counterarguments

In this section we point out flaws in two of the main counterarguments to EM time dilation theories.

Naive Gauge Invariance In many physical theories, such as classical EM and Van Holten's theory mentioned in the previous section, everything can be expressed in terms of fields acting locally, and potentials can be viewed as having no physical reality but being merely aids to computation. This would of course rule out any time dilation effects from an EM potential in a field free region, such as inside the sphere of a Van De Graaff generator. Many physicists seem to think that this is sufficient to disprove the theory.

The problem with this viewpoint is that it is flat-out wrong. The universe does *not* have that property; the Aharonov-Bohm effect[18, 19] suffices as a counterexample. The importance of this is often glossed over. For example, Jackson and Okun[20, p.24] write:

> … gauge invariance is a manifestation of non-observability of A_μ. However integrals … are observable when they are taken over a closed path, as in the Aharonov-Bohm effect … The loop integral of the vector potential there can be converted by Stokes's theorem into the magnetic flux through the loop, showing that the result is expressible in terms of the magnetic

> field, albeit in a nonlocal manner.

Contrast this with the discussion in Feynman Vol. II[21] lecture 15-5, where the central importance of the potential is emphasised:

> The fact that the vector potential appears in the wave equation of quantum mechanics (called the Schrödinger equation) was obvious from the day it was written. That it cannot be replaced by the magnetic field in any easy way was observed by one man after the other who tried to do so. This is also clear from our example of electrons moving in a region where there is no field and being affected nevertheless. But because in classical mechanics $\boldsymbol{A}$ did not appear to have any direct importance and, furthermore, because it could be changed by adding a gradient, people repeatedly said that the vector potential had no direct physical significance — that only the magnetic and electric fields are "right" even in quantum mechanics. It seems strange in retrospect that no one thought of discussing this experiment until 1956 . . . The implication was there all the time, but no one paid attention to it. . . . It is interesting that something like this can be around for thirty years but, because of certain prejudices of what is and is not significant, continues to be ignored.

In either case, the idea that fields acting locally can explain everything is admitted to be false.

It is also worth noting that gravitational time dilation itself is locally non-observable. There is no contradiction in claiming that a locally non-observable potential can have a locally non-observable effect; this is precisely how gravitational time dilation works. However the situation is somewhat different for EM time dilation. Since the effect is a function of the charge/mass ratio, the time dilation experienced by (say) a muon and a human observer is predicted to be different at the same potential. This makes EM time dilation locally observable, except to the muon itself.

CPT Invariance It is often stated (e.g. in [22–24]) that the CPT theorem guarantees that particle and antiparticle masses and lifetimes are identical. However, this conclusion is only justified at zero potential, or with the further assumption of naive gauge invariance (which renders potential irrelevant). A true CPT reflection must invert all charges and magnetic moments in the universe, which necessarily inverts all EM potentials as well. Therefore, the CPT theorem only *really* proves that a particle's mass and lifetime at 4-potential $\boldsymbol{A}$ must equal its antiparticle's mass and lifetime at 4-potential -$\boldsymbol{A}$. This holds true under EM time dilation, since the dilations for those two cases are identical. Thus, the CPT theorem does not contradict the claim that particles and antiparticles will be time-dilated oppositely at a non-zero potential and that their lifetimes will differ there.

EM time dilation is completely compatible with the notion of CPT invariance.

6 Summary

We reviewed two early derivations of gravitational time dilation and gave a new elementary derivation of it. Both the 1907 Einstein derivation and this new method can be trivially modified to give derivations of electromagnetic time dilation as well, which agree in magnitude with the handful of prior theories predicting such an effect. That EM time dilation seems so inescapably implied, and is yet so widely rejected, points perhaps to a deep paradox in current physical thought. Since testing for the first-order electrostatic effect would be quite easy and cheap, it seems worthwhile to actually perform that experiment.

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