# QUANTUM PHASE SHIFT AND GRAVITATIONAL TIME DILATION

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## 1. Introduction

In general relativity, clocks at different heights in a gravitational field run at different rates, with the higher one running faster. In quantum mechanics, particles at different energy levels rotate their quantum phase at different rates, with the higher-energy one rotating faster. We investigate whether these two phenomena can be viewed as essentially identical.

## 2. Gravitational Time Dilation

In general relativity, position in a gravitational field affects the speed of clocks, with higher ones running faster.

We consider the class of experiments where two clocks are initially synchronized, are separated to different heights and left there for a while, and are then reunited and the times compared. The higher clock will run faster than the lower one; labeling the clocks as $f$ and $s$, we have $t_f > t_s$. For the simplest case, of a uniform (non-curved) gravitational field with strength $g$, the time dilation $T_d$ between two observers at different heights is given by

$$T_d \equiv \frac{t_f}{t_s} = 1 + \frac{gh}{c^2}$$

where $h$ is the height difference and $c$ is the speed of light. (This is sufficient for our present purposes, so more complicated fields are left as an exercise for the interested reader.)

For a particle (or clock) of mass $m$, the energy difference between the positions of the two observers is $\Delta E = mgh$, the amount of work required to raise the mass, so we can rewrite the previous equation as

$$T_d = 1 + \frac{\Delta E}{mc^2} = 1 + \frac{\Delta E}{E} = \frac{E + \Delta E}{E} = \frac{E_f}{E_s}$$

where $E = mc^2$ is the energy equivalent of the mass $m$. Thus we have that

$$\frac{t_f}{t_s} = \frac{E_f}{E_s}$$

or, subtracting 1 from both sides and choosing one of the clocks arbitrarily as our reference, that

$$\frac{\Delta t}{t} = \frac{\Delta E}{E}$$

The time dilation, as a fraction of the total time elapsed, is exactly equal to the energy increase, as a fraction of the original energy.

## 3. Quantum Phase Shift

From the Schrödinger equation, we have that the phase of an eigenstate with energy $E$ oscillates as $e^{iEt/\hbar}$. Absolute phase appears impossible to measure and may in fact have no physical meaning whatsoever; however, relative phase can be easily observed through various interference experiments. For identical particles, or different trajectories of the same particle, higher energy will cause the wave function to oscillate more rapidly. Labeling the energy levels $f$ and $s$ as before and defining $\Delta E \equiv E_f - E_s$, we get a relative phase shift

$$\Delta\phi(t) = e^{i\Delta Et/\hbar}$$

For any wave at a constant energy level, and thus constant phase velocity, a shift in phase can be produced by a shift in time. If we set the phase shift due to $\Delta E$ to be equal to the phase shift due to a time delay $\Delta t$,

$$e^{i(E+\Delta E)t/\hbar} = e^{iE(t+\Delta t)/\hbar}$$

we get that

$$\Delta E \cdot t = E \cdot \Delta t$$

or

$$\frac{\Delta t}{t} = \frac{\Delta E}{E}$$

which is the same equation arrived at in the previous section. Thus it appears reasonable to view the phase shift as being due to time dilation, with the local clocks on different particle paths running at different rates. In terms of pure interference effects, this is experimentally indistinguishable from the standard interpretation. However, the standard interpretation assumes a single universal time at all points in space and is clearly not compatible with general relativity. Interpreting the phase shift as being due to time dilation, on the other hand, appears to be compatible with relativity, and indeed, if one chooses to view a particle's phase oscillation as its own local clock, may be inescapable.

A third possibility, that the actual speed and hence arrival time of the particle are affected, would be measurable by time-of-flight experiments. This effect is not predicted to occur in the standard or time dilation interpretations, and would be very surprising given the field-free (and hence force-free) phase shifts predicted and observed in various Ehrenberg-Siday-Aharonov-Bohm (ESAB) effects. It should, however, be relatively straightforward to test.

## 4. Interpretation and Experimental Verification

In this section, we assume the time-dilation interpretation is correct. Changes in the rate of time flow and changes in level of potential energy are therefore the same thing. The electrons in different orbitals in the same atom must be viewed as having different time dilations, even though they span the same space and have the same (non-moving) center of reference. Relativity shows that time is not a universal absolute, but rather depends on the position and velocity of the observer. To this, we must now also add the energy of the observer.

This is experimentally testable. Measurable predictions include

- A charged clock, placed inside a conductive cage, will run faster when the cage is given the same polarity charge, thereby raising the energy level of the clock, and slower when the cage is given the opposite charge. Radioactive ions or unstable particles will decay faster (or slower) under similar circumstances.
- Likewise, their decay rate will be different on different sides of the solenoid in a magnetic Aharonov-Bohm setup even though they never encounter any field. (The muon, with a half life of 2.2 $\mu$S, might be an attractive candidate for such an experiment.) Note that, although fairly weak fluxes are used in typical magnetic ESAB experiments because only $3.9 \cdot 10^{-7}$ gauss-cm$^2$ is required to rotate the phase by $2\pi$[1], much stronger fields could be used to test the time dilation effect. MRI machines with 10 tesla (= $10^5$ gauss) fields over areas greater than 100 cm$^2$ have been demonstrated, so fluxes of $10^7$ gauss-cm$^2$ are quite feasible.
- Alternately, geometric confinement could be used to raise the energy, along lines discussed in section 3 of [2]. This could be used on uncharged particles such as neutrons.

There are many other possibilities, but these few should suffice to demonstrate that the time dilation view makes different predictions than the phase shift view and that the differences are accessible to experimental test.

Of course, there remain questions about which object or particle should be considered when calculating the time dilation. Is it, for example, an entire atom (or ion) with its nucleus and electrons taken as a whole? Or should we, in the case of nuclear decay, only consider the nucleus? This is an empirical question, but there is evidence that the entire atom is the proper entity. The detection of interference fringes for large molecules such as $C_{70}$[3, 4] implies that such a bound collection can reasonably be viewed as having its own quantum phase as a single unified entity. Also, such questions may be moot; at least in GR, while $\Delta E$ is dependent on the particle's mass, $\Delta E/E$ and hence $T_d$ are not.

It should also be noted that, in special relativity, the time dilation for a particle or observer moving at velocity $v$ is $T_d = \sqrt{1 - v^2/c^2}$. The higher the velocity, the higher the kinetic energy, but the *slower* the clock is perceived to run. Thus kinetic energy does not appear to have the same relationship to time that potential energy does; even the sign is reversed.

## 5. Summary

We propose an alternate interpretation of the well known quantum phase shift as instead a time-dilation effect. The mathematics of this is essentially identical to that of gravitational time dilation in general relativity, indicating perhaps a deep connection between QM and GR. This interpretation is shown to have measurable consequences, and experiments are proposed that could test its validity.

## References

[1] W. Ehrenberg, R. E. Siday,. "The Refractive Index in Electron Optics and the Principles of Dynamics", *Proc. Phys. Soc.* B62: 821 (1949). doi:10.1088/0370-1301/62/1/303

[2] B.E. Allman, A. Cimmino, A.G. Klein, Reply to "Comment on Quantum Phase Shift Caused by Spatial Confinement" by Murray Peshkin, *Foundations of Physics* v.29 #3 325-332 (March 1999). doi:10.1023/A:1018858630047

[3] M. Arndt, O. Nairz, J. Petschinka, A. Zeilinger, "High Contrast Interference with $C_{60}$ and $C_{70}$", *C. R. Acad. Sci. Paris*, t.2 Srie IV, 581-585 (2001)

[4] O. Nairz, M. Arndt, A. Zeilinger, "Quantum Interference Experiments with Large Molecules", *American Journal of Physics* 71, 319 (2003)