AN ELEMENTARY APPROACH TO QUANTUM TIME DILATION.

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Contents

1. Introduction

In General Relativity, clocks at different heights in a gravitational potential run at different rates, with the higher one running faster. In Quantum Mechanics, particles at different energy levels rotate their quantum phase at different rates, with the higher-energy one rotating faster. We propose here that these two phenomena are the same; that the phase oscillation of a particle can be viewed as its local clock, and that changes in phase frequency correspond to actual time dilations. We further demonstrate that the stationary time dilations predicted by QM and GR are identical. These considerations lead us directly to a theory which makes predictions very similar to those made by an earlier theory of Apsel, which we discuss in historical context. Both theories predict that the lifetime of a muon will be affected by an electrostatic potential, and are therefore easily testable with relatively inexpensive equipment.

2. Theory

In GR the time dilation is normally given as a function of position within a gravitational field, while in QM the phase shift is normally given as a function of the energy of a bound wave function. To compare them, we need to put them into common terms. We choose to do this by considering position in terms of energy, and phase shift in terms of time.

2.1. **Time Dilation in a Uniform Gravitational Field.**

2.1.1. *The Weak-Field Approximation.* It is sufficient for our present purposes to consider only the simplest case of a uniform (non-curved) gravitational field with strength g. The time dilation $T_d(h)$ between two observers at heights h and 0 is often given by the weak-field approximation[1]

$$T_d(h) \equiv \frac{t_h}{t_0} \approx 1 + \frac{gh}{c^2}$$

where t_h and t_0 are the times at the two observers. Since we only use the ratio of t_h to t_0, it does not matter much whether we consider them to be the instantaneous rate of time flow at each observer, or the total elapsed time measured on each clock for some experiment with synchronized start and end times.[2] The ratio is the same in either case.

For a particle of mass m, the energy difference between the positions of the two observers is $\Delta E = mgh$, the amount of work required to raise the mass from the lower observer to the upper one. Thus the energy of the particle at height 0 is $E_0 = E = mc^2$, the energy equivalent of the mass m, and at height h (as seen from height 0) is $E_h = E + \Delta E$. We can thus rewrite the previous equation as

$$T_d \approx 1 + \frac{mgh}{mc^2} = 1 + \frac{\Delta E}{E} = \frac{E + \Delta E}{E} = \frac{E_h}{E_0}$$

Thus we have that

$$\frac{t_h}{t_0} \approx \frac{E_h}{E_0}$$

The ratio of energies equals the ratio of times. This is the relation between time dilation and energy in GR. However, the derivation is only valid for $\Delta E \ll E$.

[1]For a derivation see e.g. [1], section 48. This approximation is only valid for $gh \ll c^2$, and has various problems that we will discuss in the next subsection.

[2]We assume that it is possible for stationary observers to synchronize their clocks and to measure time intervals with a synchronized beginning and end. This is obviously true if, for example, we accept the notion of simultaneity presented by Einstein in 1907 ([21], see also [22] and chapter 7 of [23]). This involves the assumption that the speed of light is isotropic. There is some question about the correctness of this assumption (see e.g. the discussion in [24] or [25]), but for the present purposes we can assume that all observers are arranged in a single vertical line and all light paths follow that geodesic. This eliminates the possibility of the Sagnac effect and other global topological anisotropies. For a general proof of the validity of synchronization in any stationary spacetime, see section 9.2 of [6].)

2.1.2. *The Exact Solution.* The linear approximation given above has severe problems in the limit of large h or strong g.

$$T_d(h) = e^{gh/c^2}$$

is the well-known unique exact solution for the uniform field[3]. The weak-field approximation is just the tangent line to this at $h = 0$.

At first it might appear that the simple relation given in the previous section needs to be modified. However, the formula $\Delta E = mgh$ depends on the assumption that the mass m does not change as we raise it. If we consider the mass to include the potential energy, so that a particle gets heavier as we raise it, and lighter as we lower it, then by an argument similar to that just given, we can conclude that the mass and energy depend exponentially on height

$$m_h = m_0 e^{gh/c^2}$$

$$E_h = m_0 c^2 e^{gh/c^2}$$

A further confirmation of this comes observing that, if we have 3 observers at heights a, b, and c, the time dilations among them must be logically consistent. By definition,

$$T_d(c - b) \equiv T_c/T_b$$

$$T_d(c - a) \equiv T_c/T_a$$

$$T_d(b - a) \equiv T_b/T_a$$

So that the relation

$$T_d(c - a) = T_d(c - b) \cdot T_d(b - a)$$

must hold for any exact solution. The exponential formula satisfies this, but the linear one does not.

The exact relationship then becomes

$$\frac{t_h}{t_0} \equiv T_d(h) = e^{gh/c^2} = \frac{m_h}{m_0} = \frac{E_h}{E_0}$$

so that it is still the case that

$$\frac{t_h}{t_0} = \frac{E_h}{E_0}$$

even using the exact equations. This relationship is thus shown to be valid everywhere.

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[3]It is e.g. equation 1.11 in [6]; see also chapter 9 of that work. The derivation there is more general and applies to any stationary potential, not just a uniform one.

2.2. **Quantum Phase Shift.** Standing wave solutions to the Schrödinger equation with energy E oscillate phase as $e^{-iEt/\hbar}$. Absolute phase appears impossible to measure and may in fact have no physical meaning whatsoever; however, relative phase can be easily observed through various interference experiments. For identical particles, or different trajectories of the same particle, higher energy will cause the wave function to rotate phase more rapidly. Labeling the energy levels h and 0 as before and defining $\Delta E \equiv E_h - E_0$, we get a relative phase shift

$$\Delta\phi(t) = -i\Delta Et/\hbar$$

A shift in phase can be produced by a shift in time. If we set the phase shift due to ΔE to be equal to the phase shift due to a time delay Δt,

$$-i(E + \Delta E)t/\hbar = -iE(t + \Delta t)/\hbar$$

we get that

$$\Delta E \cdot t = E \cdot \Delta t$$

or

$$\frac{\Delta t}{t} = \frac{\Delta E}{E}$$

so that, arbitrarily choosing E_0 as our reference level, we get

$$\frac{t_h}{t_0} = \frac{t_0 + \Delta t}{t_0} = 1 + \frac{\Delta t}{t_0} = 1 + \frac{\Delta E}{E_0} = \frac{E_0 + \Delta E}{E_0} = \frac{E_h}{E_0}$$

which is the same equation arrived at in the previous section.

An alternate path to the same result starts with de Broglie's original equation relating frequency to mass (Eq. 1.1.5 in [7])

$$h\nu_0 = m_0c^2$$

where $E_0 = m_0c^2$ is the rest mass energy as above. Then the energy ratio equals the frequency ratio which, by definition, equals the time dilation:

$$\frac{E_h}{E_0} = \frac{\nu_h}{\nu_0} = \frac{t_h}{t_0}$$

In summary, it appears entirely reasonable to view the phase shift as being due solely to time dilation, with the particle's phase oscillation *being* its local clock. Indeed, any other interpretation seems problematic.

2.3. **History.** As shown above, on the QM side it is not even required to have the Schrödinger equation; de Broglie's work of 1923-25 is sufficient. In fact, our time dilation could be considered to already be described by Schrödinger in late 1925 [8], when he gave the electron's frequencies in a hydrogen atom as

$$\nu_n = mc^2/h - R/n^2$$

except that Schrödinger did not interpret this as a time dilation.

Similarly, on the GR side it is only necessary to have the weak equivalence principle and $E = h\nu$ [6]; the full Einstein field equation is not necessary. Thus the present results *could* have been derived as early as 1926. Why they weren't seems rather mysterious.

A similar time dilation was proposed by Apsel in 1978-9 [9, 10]. His derivation starts from the Aharonov-Bohm effect, assuming that the variational principle

$$\delta \int_A^B d\tau = 0$$

of relativity applies to both gravitational and electromagnetic fields. He concludes, as we do, that "the physical time associated with the trajectory of a classical particle is related to the beats of the quasi-classical quantum mechanical wave function associated with the particle". Despite experimental confirmation two years later [11], it appears to have been largely ignored for three decades. Only a handful of other papers [12, 13, 14] refer to it. Ryff [13] rederives and generalizes Apsel's results starting from the equation

$$dx_4' = -\frac{i}{mc} p_\mu dx_\mu$$

Beil [14] gives a metric for which the Lorentz equation of motion is just the geodesic equation in a Finsler space where electromagnetism is a noncompact timelike fifth dimension. He notes "that not only does the electromagnetic energy tensor part ... appear in the curvature, but so does the matter term. Thus, one can say that everything in this theory is curvature." Although the theory's gauge is dependent on particle velocity, "The usual physically meaningful quantities all involve only the gauge-independent field $F_{\mu\nu}$. There may, though, be a way of using ideas such as those of Apsel (1979) to give a measurable significance to the gauge."

Time dilation due to strong external fields has been considered several times, for example by van Holten. In a 1991 paper[15] he derives the formula

$$dt = d\tau \frac{E - q\phi}{M}$$

where dt is the laboratory time and $d\tau$ is the proper time. But E and M in this formula must be the same, because in the absence of a potential ($\phi = 0$) when laboratory time *is* proper time ($dt = d\tau$), the formula gives

$$\frac{dt}{d\tau} = 1 = \frac{E}{M}$$

In our notation his equation thus reduces to

$$T + \Delta T = T \frac{E + \Delta E}{E}$$

which is equivalent to what we derived in section 1. Two years later[16] he gives the example of a muon spin down in an intense magnetic field and predicts a change $\delta\tau$ in the muon lifetime τ_μ given by

$$\frac{\delta\tau}{\tau_\mu} = -\frac{\vec{\mu}\cdot\vec{B}}{mc^2} = 0.28 \times 10^{-14} \times B$$

requiring a field strength on the order of 5 GT (producing an energy change of $|\vec{\mu}\cdot\vec{B}| \geq 1$ keV) to see a significant variation in lifetime[16]. His calculations for the muon appear to give identical results to what we would predict for the same energy shift, since his equation can be rewritten in our notation as

$$\frac{\Delta T}{T} = \frac{\Delta E}{E}$$

which is also the same as ours from section 1. However, Van Holten's theory differs notably from Apsel's in that it is EM gauge invariant, and thus predicts effects from fields but not from potentials. Despite commenting that "It is quite clear from this formula, that any quantity which contributes to the energy E in an observable way, also contributes to the time dilation"[16] (with which we heartily agree), he does not in either paper consider that an electric potential can have any time dilation effect, particularly in a field free region. This has significant implications for testability, because the same degree of dilation produced by the 5 GT field (which is many orders of magnitude beyond what can be generated in labs today) in his theory could, in the present theory and Apsel's theory, also be achieved by changing the potential of the muon by a kV or so (something that could be easily done by almost anyone). Essentially, he considers the effect to be a form of spin-orbit coupling.

The question of whether there might be some kind of time dilation associated with an electric potential was raised online in 2004 [17], but the discussion there was vague and inconclusive, and showed no awareness of Apsel's results.

More recent online discussions instigated by J. Duda (e.g. [18]) are quite aware of Apsel, and also of O. Heaviside's 1893 theory of Gravitomagnetism[19] and recent revivals and extensions of it by O. Jefimenko[20] and others, but appear so far to also be inconclusive. Duda, like van Holten, seems to think that strong fields (and not just potentials) are required, saying e.g. "Such experiment would need extremely strong electromagnetic field - like very near particles or near pulsars".([18] 8 Jan 2010)

2.4. **Limitations.** In special relativity, the time dilation for a particle or observer moving at velocity v is $T_d = \sqrt{1 - v^2/c^2}$. The higher the velocity, the higher the kinetic energy, but the *slower* the clock is perceived to run. Thus kinetic energy does not appear to have the same relationship to time that potential energy does; even the sign is reversed. A further difficulty, as noted by de Broglie [7], is that while all observers agree on stationary time dilation, the time dilation due to motion is reciprocal (I think your clock is slower than mine at the same time you think my clock is slower than yours). Working through these issues is beyond the scope of the current paper.

3. Experiment

How can quantum time dilation be most easily measured? If quantum time dilation is real, it should have occurred in many experiments that have already been carried out. In some cases, it would have been swamped by other effects such as velocity-based time dilation, but in others, it should have been noticeable.

For all charged particles, QTD predicts changes in lifetime at non-zero electric potential. Obviously this only has meaning for unstable particles. With stable particles, one appears to be limited to indirect tests based on interference effects

3.1. **Electrons and Positrons.**

Electrons and positrons, with the largest charge-to-mass ratio of any particle, would experience the greatest time dilation under the present theory. However, since they have infinite lifetimes, this would only be detectable as a phase shift which is identical to that predicted by standard theory.

One indirect test would be to set up an electron interference experiment and run it at various potentials. Near $\phi = +511\text{keV}$, $h\nu = E = m_0c^2 - e\phi$ predicts the electron phase frequency should go to zero, and its de Broglie wavelength go infinite; therefore the interference fringes should get wider as we approach that voltage and disappear when we hit it exactly.

3.2. **Muons.**

Radioactive ions or unstable charged particles will decay slower (or faster) when time-dilated. The muon, with a half life of 2.197 μS [32], is an attractive candidate for QTD experiments as it has the highest charge-to-mass ratio of any unstable particle. Surveys of methods of muon production can be found in [34] and [35]; see also section VI of [36]. Beam sources may be continuous or pulsed. Muon lifetime detectors adequate to measure time dilation effects can be simple and inexpensive enough to be used in an undergraduate physics lab [37].

The muon decay time can be very accurately calculated within the standard model given certain parameters [38]; some of these parameters can in turn be derived from muon lifetime data. Thus the precision of the standard model is dependent on the precision to which the muon lifetime is known.

In the following subsections we look at several ways of altering muon lifetimes. Much of what is said would apply equally well to other unstable particles such as pions.

3.2.1. *Aharonov-Bohm Effect On Muons.*

The Aharonov-Bohm effect[29] (actually first described by Ehrenberg and Siday [30]) is a phase shift induced by a magnetic vector potential in a region of space which has zero magnetic field. We interpret this phase shift as an actual time dilation, and so predict that the experienced time (and hence decay rate in the laboratory frame) will be different on different sides of the solenoid in an Aharonov-Bohm setup even though the particles never encounter any field. Although fairly weak fluxes are used in typical AB experiments (because only $3.9 \cdot 10^{-7}$ gauss-cm^2 is required to rotate the electron phase by 2π [30]), much stronger fields could be used to test the time dilation effect. MRI machines with 10 tesla (= 10^5 gauss) fields over areas greater than 100 cm^2 have been demonstrated, so total fluxes of 10^7 gauss-cm^2 and up are quite feasible.

Let's first analyze the situation for an electron. A phase shift of 2π happens when

$$\Delta E \cdot t = 2\pi\hbar = h = E \cdot \Delta t$$

so that for an electron

$$\Delta t = \frac{h}{E} = \frac{h}{m_e c^2} = \frac{6.626 \cdot 10^{-34}\mathrm{m}^2\mathrm{kg/S}}{(9.109 \cdot 10^{-31}\mathrm{kg}) \cdot (2.998 \cdot 10^8\mathrm{m/S})^2} = 8.093 \cdot 10^{-21}\mathrm{S}$$

is the time difference (as seen by the electrons, not by an external observer) between electron paths when the interference pattern has been shifted by one full fringe. This requires a total flux of $3.9 \cdot 10^{-7}$ gauss-cm^2 as noted above, but we should be able to use fluxes at least 10^{13}-10^{14} that large, leading to feasible Δts in the range of 10^{-7}-10^{-6} seconds. (Indeed the Brookhaven E821 experiment [33] applied a field of 1.45T over a ring with radius 7.11m; if the field had been uniform, the total contained flux would have been about $2.3 \cdot 10^{10}$ gauss-cm^2.)

For the muon the Δt for a single fringe shift is 207 times smaller (the ratio of the muon mass to the electron mass), or $3.91 \cdot 10^{-23}$S.

A 200 kV muon beam should travel roughly as fast as a 1 kV electron beam, or about 2% of the speed of light or $6 \cdot 10^6$ m/S. If we split this beam and send it around a solenoid with a radius of about 100 cm (and hence cross-sectional area 314 cm^2) we should be able to have each path be no longer than, say, 600 cm. With a 10T = 10^6 gauss field strength the total flux would be $3.14 \cdot 10^8$ gauss-cm^2 and the predicted $\Delta t = 3.91 \cdot 10^{-23} \frac{3.14 \cdot 10^8}{3.9 \cdot 10^{-7}}\mathrm{S} = 3.14 \cdot 10^{-8}\mathrm{S}$. The flight time of the muon would then be about 10^{-7} S. With a half life of 2.2 μS, and ignoring relativistic corrections, we would expect in the standard interpretation that a fraction $2^{-t/2.2\mu\mathrm{S}}$ of the muons on each path would remain undecayed

$$2^{-0.1/2.2} = 2^{-1/22} = 2^{-0.04545} = 0.969$$

so that about 3.1% of the muons would decay on each path. However, the time dilation predicted by the present theory would cause the fast-time path to experience a total time $(t + \Delta t)$ of $(0.1 + 0.03)\mu$S so that approximately

$$2^{-0.103/2.2} = 2^{-0.04682} = 0.968$$

would remain undecayed while on the slow-clock path

$$2^{-0.097/2.2} = 2^{-0.04409} = 0.970$$

would. Thus, for that flux, we would predict a roughly 3% increase in the decay rate on the fast-time path and a 3% decrease on the slow-time path. Larger fluxes would have larger effects. For large enough flux, the effect should be truly spectacular.

It is not necessary to actually interfere the two beams to measure this effect. One could, for example, just have a single beam of muons rotating in a cyclotron ring. A large confined and shielded flux through the ring should have a measurable effect on the decay rate of the muons; reversing the flux or the direction of rotation should reverse the effect.

It would also be possible to simply fire a beam of muons through the center hole of one or more shielded toroidal magnets, as was done in the elegant Hitachi experiment to demonstrate the AB effect with electrons [40], although much larger magnets with much larger fluxes would be desirable. There is no theoretical upper bound to the total encircling flux per length of muon path in this configuration, as there is no upper limit to the radial size of the core. If we assume a core with a contained field strength of 1T [4], and a toroidal shape with rectangular cross section (inside radius r_i, outside radius r_o, and thickness l), the cross-sectional area is given by $A = (r_o - r_i) \cdot l$ and the total flux would be $10^4 \cdot A$ gauss-cm^2 with the flux per length equal to $10^4(r_o - r_i)$ gauss-cm. The time shift per length (for a muon) is given roughly by

$$\frac{3.91 \cdot 10^{-23}\mathrm{S}}{3.9 \cdot 10^{-7}\mathrm{gauss} \cdot \mathrm{cm}^2} \cdot 10^4(r_o - r_i)\mathrm{gauss} \cdot \mathrm{cm} \approx 10^{-12}(r_o - r_i)\frac{\mathrm{S}}{\mathrm{cm}}$$

For a rather large core with $(r_o - r_i) = 1\mathrm{m}$ and $l = 1\mathrm{m}$ (weighing about 25 metric tons[5]) we would get a time shift of 10 nS. This has to be compared with the time of flight at fast but sub-relativistic speeds; at 10% of the speed of light a particle will only spend about 33 nS passing through the toroid, so a time shift of 10 nS represents a roughly 30% increase or decrease in the time experienced by the particle.. The special-relativistic time dilation at that speed is less than 1%.

3.3. **Neutrons.** Neutrons, being uncharged, are not subject to time dilation by electrostatic potentials. However they do have spin and hence a magnetic moment, and may be dilated by the Aharonov-Casher effect in the same way that charged particles may be dilated by the Aharonov-Bohm effect. Also, like everything, they are affected by gravitational time dilation.

3.4. **Other Forces.** The above discussion has been entirely about gravity and electromagnetism. However, other potentials should also cause similar time dilation.

In theory, geometric confinement could be used to raise the energy of a particle, along lines discussed in section 3 of [31]. This could be used on uncharged particles

[4]An electromagnet made of iron saturates at 1.6T, and NdFeB permanent magnets can have fields of 1.17 to 1.48T, so 1T is a little conservative.

[5]This is only moderately large by HEP standards; the storage ring magnet at the BNL muon g-2 experiment weighs 700 tons.[41]

such as neutrons, while most of the above approaches require charged particles. However, the effect for any realistic confinement may be too small to measure.

There are many other possibilities, including the strong and weak nuclear forces, but these few should suffice to demonstrate that quantum time dilation makes different predictions than the standard view of phase shift, and that the differences are accessible to experimental test.

4. CPT and C Invariance

It is often stated (e.g. in [27, 43, 44]) that the CPT theorem guarantees that particle and antiparticle masses and lifetimes are identical. However, this conclusion is only justified at zero potential (or with the further assumption of EM-gauge invariance, which renders potential irrelevant). A true CPT (or C) reflection must invert all charges in the universe, which necessarily inverts all electric potentials as well. Therefore, the CPT theorem only *really* proves that a particle's mass and lifetime at electric potential V must equal its antiparticle's mass and lifetime at potential $-V$. This holds true in QTD, since the dilations for those two cases are identical. Thus, the CPT theorem does not contradict the QTD claim that particles and their oppositely-charged antiparticles will be time-dilated oppositely at a non-zero potential and that their masses and lifetimes will differ there. QTD is completely compatible with the notion of CPT (or C) invariance.

5. Summary

We interpret the well known quantum phase shift at different energy levels as a time-dilation. The mathematics of this is essentially identical to that of gravitational time dilation in general relativity, indicating perhaps a deep and simple connection between QM and GR. This interpretation is shown to have measurable consequences which are supported by prior experimental data, and further experiments are proposed that could test its validity more directly.

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