PCA

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2022-10-02

INSTALL AND LOAD PACKAGES

```r
# Install pacman ("package manager") if needed
if (!require("pacman")) install.packages("pacman")
```

```
## Loading required package: pacman
```

```r
# pacman must already be installed; then load contributed
# packages (including pacman) with pacman
pacman::p_load(GPArotation, magrittr, pacman, psych, rio,
  tidyverse, tinytex)
# GPArotation: Gradient Projection Algorithm Rotation for FA
# magrittr: for pipes
# pacman: for loading/unloading packages
# psych: for psychometric functions
# rio: for importing data
# tidyverse: for so many reasons
# install.packages('tinytex') for rmarkdown of pdf
# tinytex::install_tinytex()
```

LOAD AND PREPARE DATA

```r
# Import data from CSV, save as "df"
df <- import("../data/b5.csv")
```

```r
# Get column names
df %>% colnames()
```

```
##  [1] "E1"  "E2"  "E3"  "E4"  "E5"  "E6"  "E7"  "E8"  "E9"  "E10" "N1"  "N2"
## [13] "N3"  "N4"  "N5"  "N6"  "N7"  "N8"  "N9"  "N10" "A1"  "A2"  "A3"  "A4"
## [25] "A5"  "A6"  "A7"  "A8"  "A9"  "A10" "C1"  "C2"  "C3"  "C4"  "C5"  "C6"
## [37] "C7"  "C8"  "C9"  "C10" "O1"  "O2"  "O3"  "O4"  "O5"  "O6"  "O7"  "O8"
## [49] "O9"  "O10"
```

```r
# PRINCIPAL COMPONENT ANALYSIS ##############################

# Three methods in R
```

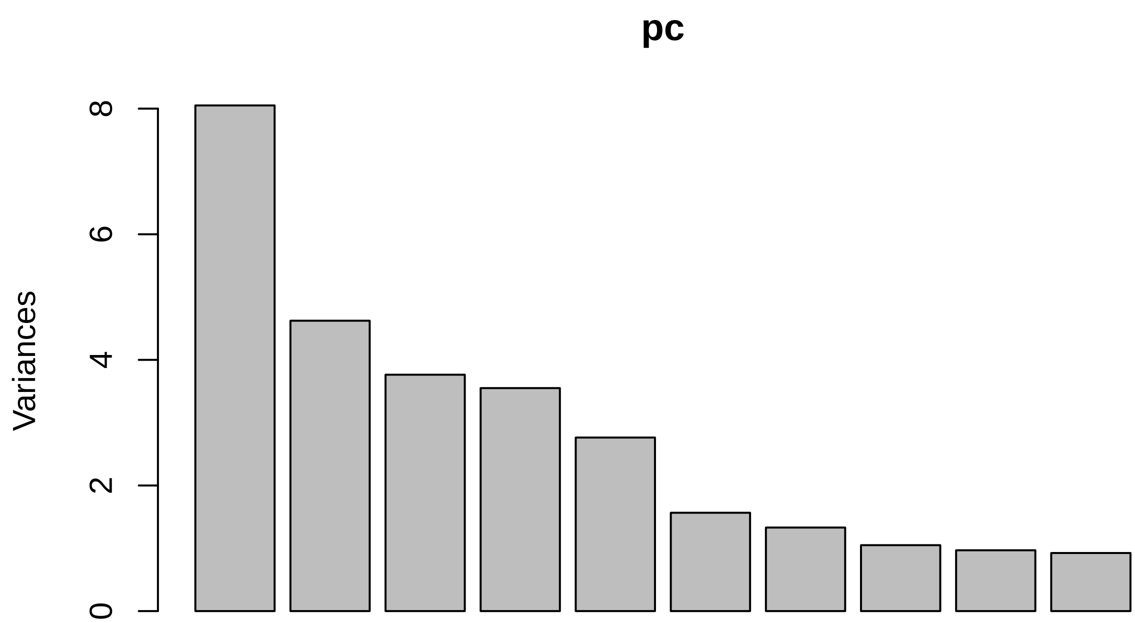
```r
#?prcomp     # Most common method
#?princomp   # Slightly different method
#?principal  # Method from psych package (my favorite)
```

```r
# Principal components model using default method
pc <- df %>%
  prcomp(
    center = TRUE,  # Centers means to 0 (optional)
    scale = TRUE    # Sets unit variance (helpful)
  )
```

```r
# Get summary stats
summary(pc)
```

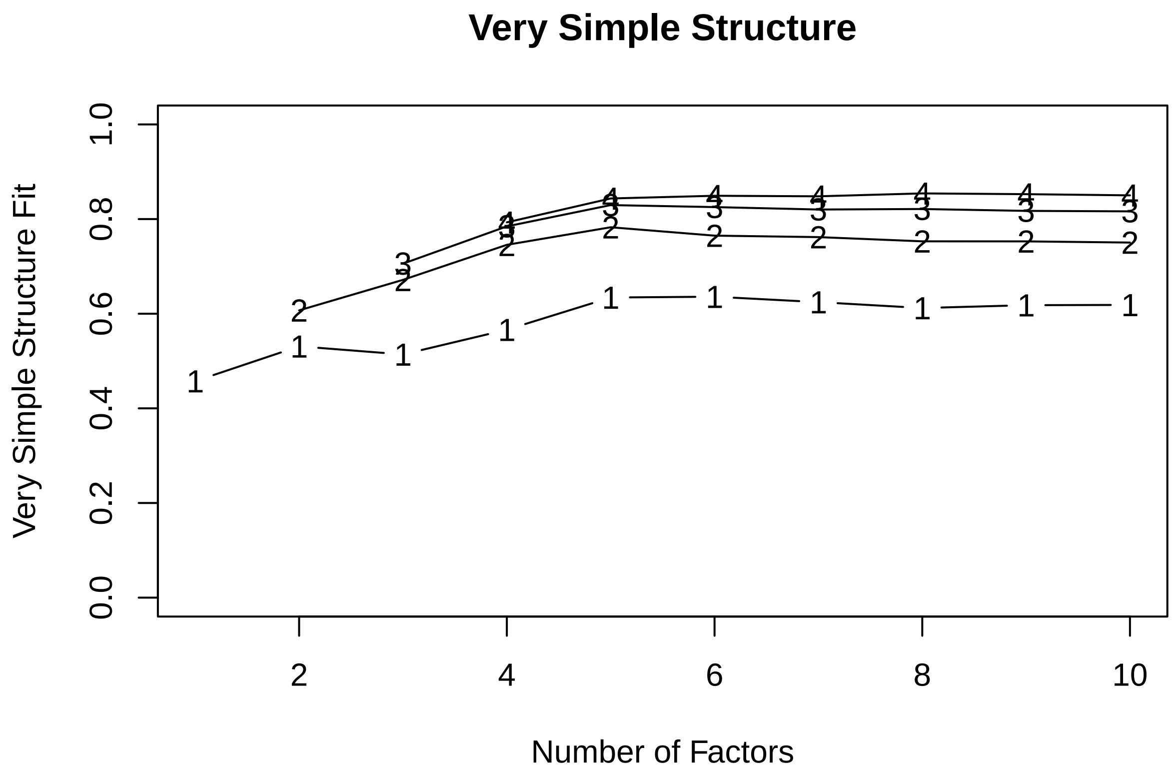
```
## Importance of components:
##                          PC1     PC2     PC3    PC4     PC5    PC6     PC7
## Standard deviation     2.837 2.15015 1.94001 1.8842 1.66227 1.2511 1.15342
## Proportion of Variance 0.161 0.09246 0.07527 0.0710 0.05526 0.0313 0.02661
## Cumulative Proportion  0.161 0.25348 0.32875 0.3998 0.45502 0.4863 0.51293
##                            PC8     PC9   PC10    PC11    PC12    PC13    PC14
## Standard deviation     1.02448 0.98413 0.9617 0.94714 0.93146 0.91905 0.89562
## Proportion of Variance 0.02099 0.01937 0.0185 0.01794 0.01735 0.01689 0.01604
## Cumulative Proportion  0.53392 0.55329 0.5718 0.58973 0.60708 0.62398 0.64002
##                           PC15   PC16   PC17    PC18    PC19    PC20    PC21
## Standard deviation     0.88770 0.8574 0.8544 0.84801 0.82540 0.81318 0.81020
## Proportion of Variance 0.01576 0.0147 0.0146 0.01438 0.01363 0.01323 0.01313
## Cumulative Proportion  0.65578 0.6705 0.6851 0.69946 0.71309 0.72631 0.73944
##                           PC22    PC23   PC24    PC25    PC26    PC27    PC28
## Standard deviation     0.79644 0.78210 0.7681 0.76291 0.75388 0.74457 0.72978
## Proportion of Variance 0.01269 0.01223 0.0118 0.01164 0.01137 0.01109 0.01065
## Cumulative Proportion  0.75213 0.76436 0.7762 0.78780 0.79917 0.81026 0.82091
##                           PC29    PC30    PC31    PC32    PC33    PC34    PC35
## Standard deviation     0.72285 0.71456 0.70840 0.70125 0.69817 0.69424 0.66920
## Proportion of Variance 0.01045 0.01021 0.01004 0.00984 0.00975 0.00964 0.00896
## Cumulative Proportion  0.83136 0.84157 0.85161 0.86144 0.87119 0.88083 0.88979
##                           PC36    PC37    PC38    PC39    PC40    PC41   PC42
## Standard deviation     0.66858 0.65893 0.64839 0.64463 0.63571 0.63006 0.6165
## Proportion of Variance 0.00894 0.00868 0.00841 0.00831 0.00808 0.00794 0.0076
## Cumulative Proportion  0.89873 0.90741 0.91582 0.92413 0.93221 0.94015 0.9477
##                           PC43    PC44    PC45   PC46   PC47    PC48    PC49
## Standard deviation     0.61161 0.60308 0.58938 0.5873 0.5702 0.56901 0.55859
## Proportion of Variance 0.00748 0.00727 0.00695 0.0069 0.0065 0.00648 0.00624
## Cumulative Proportion  0.95523 0.96251 0.96946 0.9764 0.9829 0.98933 0.99557
##                           PC50
## Standard deviation     0.47050
## Proportion of Variance 0.00443
## Cumulative Proportion  1.00000
```

```r
# Screeplot of eigenvalues
plot(pc)
```



VERY SIMPLE STRUCTURE

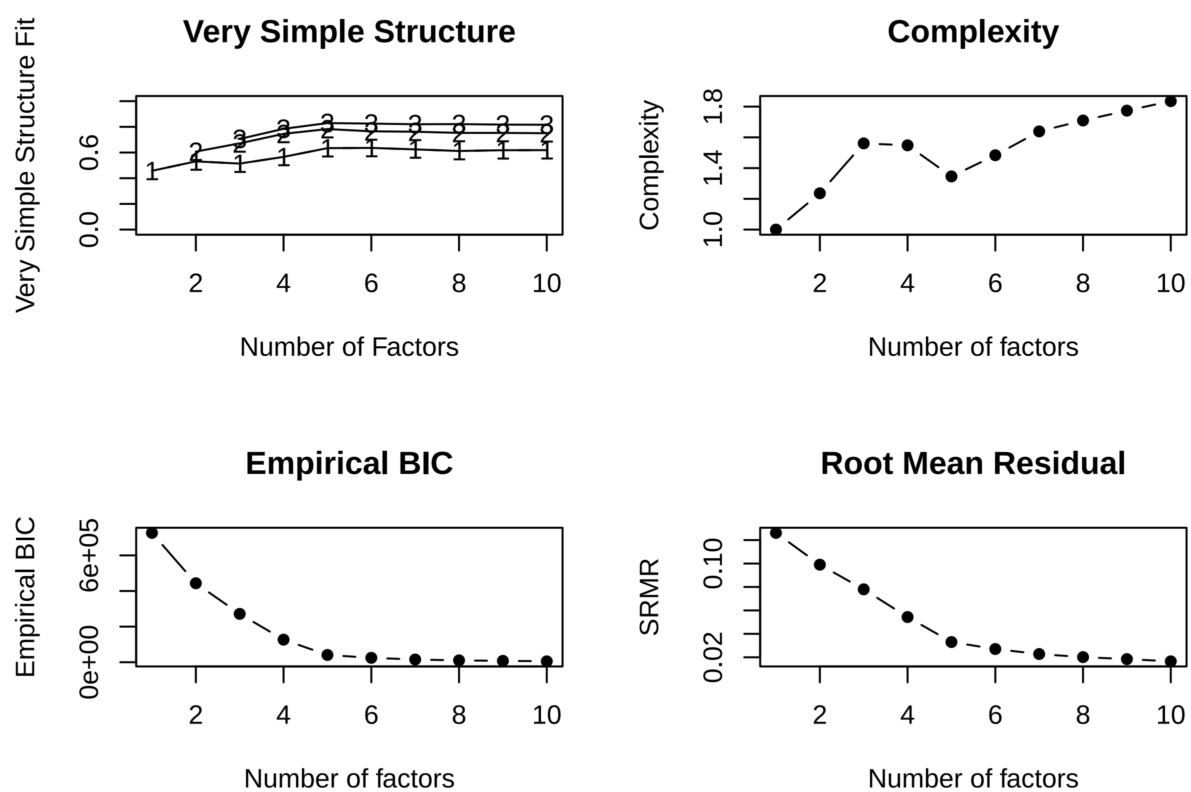
```
# Use "Very Simple Structure" to suggest number of factors
# Note: MAP = Minimum Absolute Partial correlation;
# n is the proposed maximum number of factors
df %>%
  select(1:50) %>%
  vss(n = 10)
```




```
##
## Very Simple Structure
## Call: vss(x = ., n = 10)
## Although the VSS complexity 1 shows  6  factors, it is probably more reasonable to think about  2  fa
## VSS complexity 2 achieves a maximimum of 0.78  with  5  factors
##
## The Velicer MAP achieves a minimum of 0.01  with  6  factors
## BIC achieves a minimum of  13459.82  with  10  factors
## Sample Size adjusted BIC achieves a minimum of  15906.84  with  10  factors
##
## Statistics by number of factors
##    vss1 vss2    map  dof  chisq prob sqresid  fit RMSEA    BIC  SABIC complex
## 1  0.46 0.00 0.0234 1175 247942    0      76 0.46 0.105 236370 240104     1.0
## 2  0.53 0.61 0.0175 1126 181584    0      55 0.61 0.092 170494 174073     1.2
## 3  0.51 0.67 0.0133 1078 133756    0      41 0.71 0.081 123139 126565     1.6
## 4  0.57 0.75 0.0103 1031  94542    0      29 0.79 0.069  84388  87664     1.5
## 5  0.63 0.78 0.0060  985  56723    0      22 0.85 0.055  47022  50152     1.3
## 6  0.64 0.76 0.0057  940  45066    0      19 0.86 0.050  35809  38796     1.5
## 7  0.62 0.76 0.0059  896  36826    0      18 0.87 0.046  28002  30849     1.6
## 8  0.61 0.75 0.0062  853  29070    0      17 0.88 0.042  20670  23380     1.7
## 9  0.62 0.75 0.0067  811  24933    0      16 0.88 0.040  16946  19523     1.8
## 10 0.62 0.75 0.0071  770  21043    0      15 0.89 0.037  13460  15907     1.8
##    eChisq  SRMR eCRMS   eBIC
## 1  737929 0.126 0.129 726357
## 2  454648 0.099 0.103 443558
## 3  282088 0.078 0.083 271472
```

```
## 4  136940 0.054 0.059 126786
## 5   50472 0.033 0.037  40771
## 6   33927 0.027 0.031  24670
## 7   24042 0.023 0.027  15218
## 8   18801 0.020 0.024  10400
## 9   15790 0.018 0.023   7803
## 10  12722 0.017 0.021   5139
```

```
# Or use "nfactors" to do the same; includes VSS
df %>%
  select(1:50) %>%
  nfactors(n = 10)
```



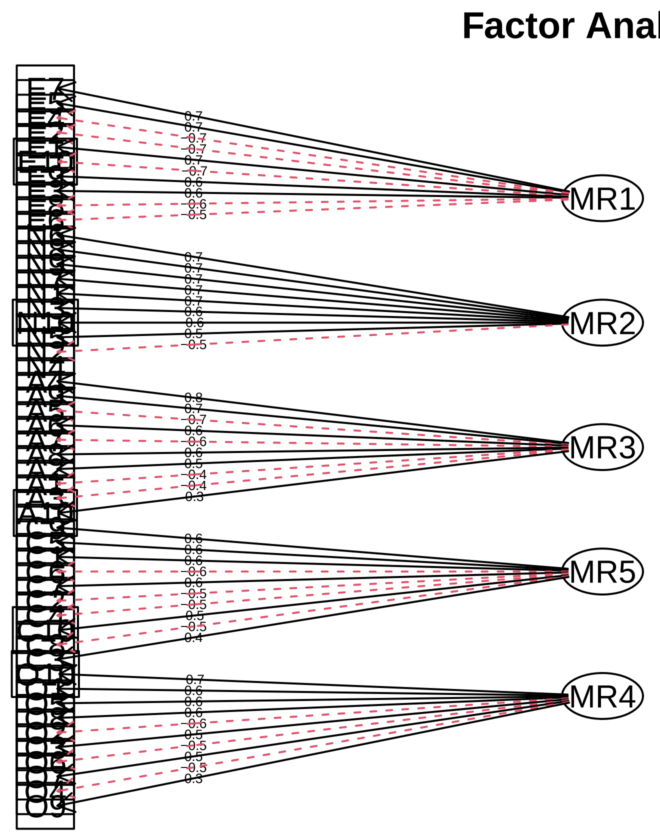

```
##
## Number of factors
## Call: vss(x = x, n = n, rotate = rotate, diagonal = diagonal, fm = fm,
##     n.obs = n.obs, plot = FALSE, title = title, use = use, cor = cor)
## VSS complexity 1 achieves a maximimum of Although the vss.max shows  6  factors, it is probably more
## VSS complexity 2 achieves a maximimum of 0.78  with  5  factors
## The Velicer MAP achieves a minimum of 0.01  with  6  factors
## Empirical BIC achieves a minimum of  5139.05  with  10  factors
## Sample Size adjusted BIC achieves a minimum of  15906.84  with  10  factors
##
## Statistics by number of factors
##    vss1 vss2   map  dof  chisq prob sqresid  fit RMSEA   BIC  SABIC complex
```

```
## 1  0.46 0.00 0.0234 1175 247942    0       76 0.46 0.105 236370 240104      1.0
## 2  0.53 0.61 0.0175 1126 181584    0       55 0.61 0.092 170494 174073      1.2
## 3  0.51 0.67 0.0133 1078 133756    0       41 0.71 0.081 123139 126565      1.6
## 4  0.57 0.75 0.0103 1031  94542    0       29 0.79 0.069  84388  87664      1.5
## 5  0.63 0.78 0.0060  985  56723    0       22 0.85 0.055  47022  50152      1.3
## 6  0.64 0.76 0.0057  940  45066    0       19 0.86 0.050  35809  38796      1.5
## 7  0.62 0.76 0.0059  896  36826    0       18 0.87 0.046  28002  30849      1.6
## 8  0.61 0.75 0.0062  853  29070    0       17 0.88 0.042  20670  23380      1.7
## 9  0.62 0.75 0.0067  811  24933    0       16 0.88 0.040  16946  19523      1.8
## 10 0.62 0.75 0.0071  770  21043    0       15 0.89 0.037  13460  15907      1.8
##    eChisq  SRMR eCRMS   eBIC
## 1  737929 0.126 0.129 726357
## 2  454648 0.099 0.103 443558
## 3  282088 0.078 0.083 271472
## 4  136940 0.054 0.059 126786
## 5   50472 0.033 0.037  40771
## 6   33927 0.027 0.031  24670
## 7   24042 0.023 0.027  15218
## 8   18801 0.020 0.024  10400
## 9   15790 0.018 0.023   7803
## 10  12722 0.017 0.021   5139
```

FACTOR ANALYSIS

Factor analysis using minimum residual (minres) method and oblimin rotation, which is useful for simple structure. Need to enter desired number of factors (from VSS or nfactors above).

```
# Calculate and plot factors with fa()
df %>%
  select(1:50) %>%       # Select variables
  fa(                    # Use fa() function
    nfactors = 5,        # Use five factors
    rotate = "oblimin"   # Oblimin oblique rotation
  ) %T>%                 # T-pipe
  fa.diagram() %>%       # Diagram of factors and variables
  print()                # Print results
```



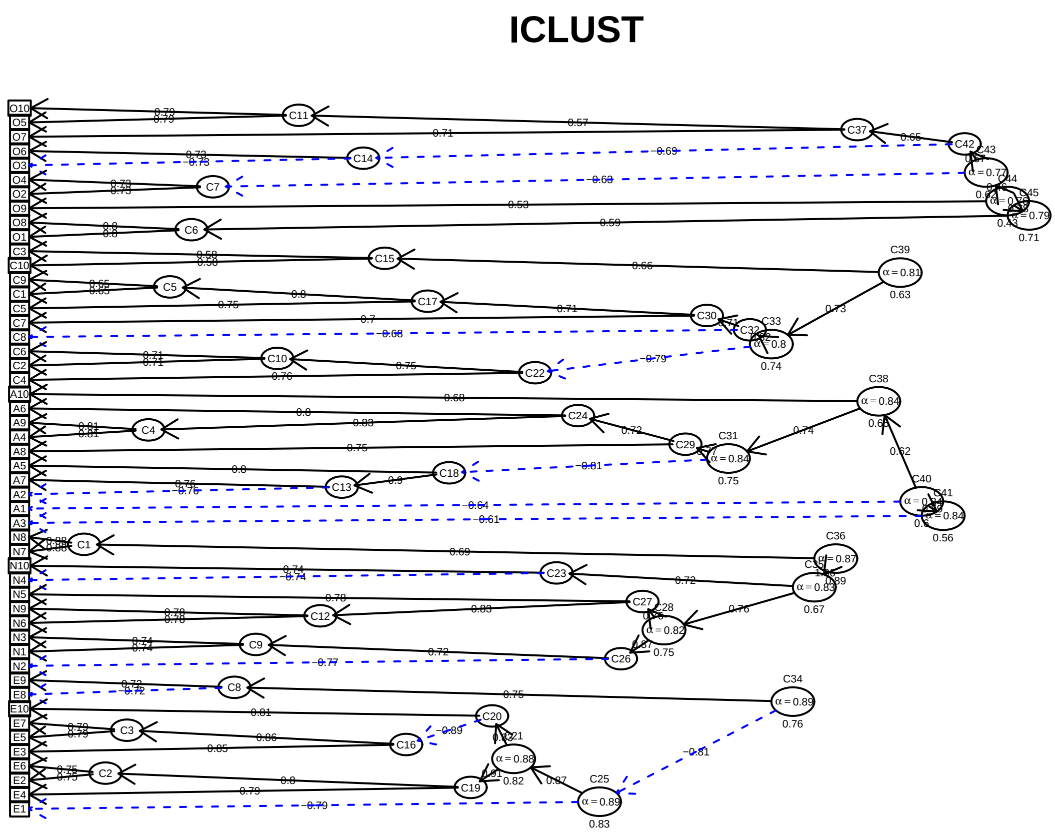

```
## Factor Analysis using method =  minres
## Call: fa(r = ., nfactors = 5, rotate = "oblimin")
## Standardized loadings (pattern matrix) based upon correlation matrix
##       MR1   MR2   MR3   MR5   MR4   h2   u2 com
## E1   0.69  0.04 -0.03 -0.01  0.01 0.46 0.54 1.0
## E2  -0.70 -0.08 -0.04  0.04  0.00 0.48 0.52 1.0
## E3   0.63 -0.17  0.16  0.09 -0.06 0.58 0.42 1.3
## E4  -0.72  0.05  0.04  0.00  0.03 0.52 0.48 1.0
## E5   0.72  0.03  0.12  0.07  0.03 0.60 0.40 1.1
## E6  -0.54  0.01 -0.08  0.01 -0.19 0.40 0.60 1.3
## E7   0.74  0.00  0.06  0.02 -0.01 0.57 0.43 1.0
## E8  -0.60 -0.04  0.11  0.07 -0.01 0.33 0.67 1.1
## E9   0.64  0.04 -0.09 -0.02  0.09 0.40 0.60 1.1
## E10 -0.65  0.10  0.03  0.00  0.01 0.45 0.55 1.0
## N1  -0.06  0.69  0.10  0.05 -0.05 0.49 0.51 1.1
## N2   0.07 -0.51 -0.01 -0.09  0.05 0.27 0.73 1.1
## N3  -0.11  0.61  0.20  0.10  0.01 0.43 0.57 1.4
## N4   0.14 -0.30 -0.07  0.07 -0.07 0.14 0.86 1.8
## N5   0.01  0.53  0.01 -0.05 -0.11 0.32 0.68 1.1
## N6   0.00  0.75  0.06 -0.01 -0.07 0.57 0.43 1.0
## N7   0.06  0.71 -0.05 -0.08  0.02 0.52 0.48 1.1
## N8   0.05  0.74 -0.06 -0.08  0.01 0.58 0.42 1.0
## N9   0.04  0.73 -0.15  0.04  0.00 0.54 0.46 1.1
## N10 -0.21  0.58  0.03 -0.11  0.08 0.47 0.53 1.4
## A1   0.05  0.09 -0.44  0.02 -0.07 0.20 0.80 1.2
## A2   0.28 -0.04  0.50 -0.04  0.05 0.41 0.59 1.6
```

```
## A3   0.17  0.27 -0.41 -0.15  0.09 0.27 0.73 2.6
## A4  -0.06  0.03  0.80 -0.01 -0.01 0.62 0.38 1.0
## A5  -0.06  0.04 -0.66  0.05  0.00 0.45 0.55 1.0
## A6  -0.05  0.12  0.61  0.02 -0.08 0.37 0.63 1.1
## A7  -0.23  0.09 -0.61  0.05 -0.01 0.51 0.49 1.4
## A8   0.05 -0.02  0.58  0.05  0.02 0.36 0.64 1.0
## A9   0.04  0.10  0.70  0.03  0.04 0.51 0.49 1.1
## A10  0.28 -0.08  0.34  0.11  0.06 0.31 0.69 2.4
## C1   0.01 -0.02 -0.04  0.60  0.12 0.39 0.61 1.1
## C2   0.05  0.04  0.08 -0.54  0.12 0.31 0.69 1.2
## C3  -0.08  0.05  0.07  0.40  0.27 0.24 0.76 2.0
## C4  -0.03  0.30  0.01 -0.53  0.02 0.45 0.55 1.6
## C5   0.08  0.01  0.00  0.63 -0.08 0.41 0.59 1.1
## C6   0.02  0.10  0.05 -0.59  0.06 0.38 0.62 1.1
## C7  -0.06  0.14  0.00  0.56  0.05 0.30 0.70 1.2
## C8  -0.02  0.16 -0.11 -0.45 -0.03 0.30 0.70 1.4
## C9   0.05  0.11  0.04  0.64 -0.03 0.41 0.59 1.1
## C10  0.00  0.05  0.03  0.47  0.24 0.28 0.72 1.5
## O1  -0.03 -0.03 -0.04  0.02  0.60 0.36 0.64 1.0
## O2   0.06  0.21 -0.02  0.05 -0.56 0.36 0.64 1.3
## O3  -0.02  0.10  0.07 -0.10  0.53 0.30 0.70 1.2
## O4   0.09  0.13 -0.13  0.11 -0.47 0.26 0.74 1.5
## O5   0.15 -0.01 -0.06  0.14  0.58 0.41 0.59 1.3
## O6  -0.04  0.04 -0.08  0.06 -0.49 0.27 0.73 1.1
## O7   0.02 -0.10 -0.03  0.17  0.49 0.30 0.70 1.3
## O8  -0.04  0.08 -0.11 -0.05  0.56 0.32 0.68 1.1
## O9  -0.19  0.15  0.21  0.04  0.35 0.20 0.80 2.7
## O10  0.13  0.02  0.00  0.02  0.66 0.48 0.52 1.1
##
##                        MR1  MR2  MR3  MR5  MR4
## SS loadings           5.07 4.55 3.74 3.27 3.20
## Proportion Var        0.10 0.09 0.07 0.07 0.06
## Cumulative Var        0.10 0.19 0.27 0.33 0.40
## Proportion Explained  0.26 0.23 0.19 0.16 0.16
## Cumulative Proportion 0.26 0.49 0.67 0.84 1.00
##
##  With factor correlations of
##       MR1   MR2   MR3   MR5   MR4
## MR1  1.00 -0.24  0.25  0.09  0.16
## MR2 -0.24  1.00 -0.03 -0.24 -0.08
## MR3  0.25 -0.03  1.00  0.14  0.07
## MR5  0.09 -0.24  0.14  1.00  0.05
## MR4  0.16 -0.08  0.07  0.05  1.00
##
## Mean item complexity =  1.3
## Test of the hypothesis that 5 factors are sufficient.
##
## The degrees of freedom for the null model are  1225  and the objective function was  19.15 with Chi S
## The degrees of freedom for the model are 985  and the objective function was  3
##
## The root mean square of the residuals (RMSR) is  0.03
## The df corrected root mean square of the residuals is  0.04
##
## The harmonic number of observations is  18930 with the empirical chi square  50472.27  with prob <  (
```

```
## The total number of observations was  18930  with Likelihood Chi Square =  56722.95  with prob <  0
##
## Tucker Lewis Index of factoring reliability =  0.808
## RMSEA index =  0.055  and the 90 % confidence intervals are  0.054 0.055
## BIC =  47022.17
## Fit based upon off diagonal values = 0.97
## Measures of factor score adequacy
##                                                    MR1  MR2  MR3  MR5  MR4
## Correlation of (regression) scores with factors   0.95 0.95 0.94 0.91 0.91
## Multiple R square of scores with factors          0.91 0.90 0.87 0.84 0.83
## Minimum correlation of possible factor scores     0.81 0.79 0.75 0.67 0.66
```

HIERARCHICAL CLUSTERING

```r
# Hierarchical clustering of items with iclust()
df %>%
  select(1:50) %>%
  iclust()
```




```
## ICLUST (Item Cluster Analysis)
## Call: iclust(r.mat = .)
##
## Purified Alpha:
##  C34  C36  C41  C39  C45
```

```
## 0.89 0.87 0.84 0.81 0.79
##
## G6* reliability:
##  C34  C36  C41  C39  C45
## 0.80 0.91 0.80 0.78 0.83
##
## Original Beta:
##  C34  C36  C41  C39  C45
## 0.76 0.89 0.56 0.63 0.71
##
## Cluster size:
## C34 C36 C41 C39 C45
##  10  10  10  10  10
##
## Item by Cluster Structure matrix:
##       O   P   C34   C36   C41   C39   C45
## E1  C34 C34  0.67 -0.17  0.22  0.05  0.11
## E2  C34 C34 -0.69  0.10 -0.27 -0.02 -0.10
## E3  C34 C34  0.70 -0.38  0.43  0.22  0.08
## E4  C34 C34 -0.73  0.24 -0.23 -0.09 -0.09
## E5  C34 C34  0.76 -0.20  0.39  0.16  0.16
## E6  C34 C34 -0.62  0.18 -0.30 -0.10 -0.29
## E7  C34 C34  0.75 -0.22  0.33  0.11  0.12
## E8  C34 C34 -0.56  0.11 -0.09  0.03 -0.08
## E9  C34 C34  0.62 -0.15  0.14  0.03  0.18
## E10 C34 C34 -0.68  0.27 -0.22 -0.09 -0.11
## N1  C36 C36 -0.20  0.70 -0.01 -0.15 -0.11
## N2  C36 C36  0.17 -0.54  0.07  0.06  0.10
## N3  C36 C36 -0.19  0.61  0.09 -0.06 -0.05
## N4  C36 C36  0.17 -0.38  0.03  0.14 -0.01
## N5  C36 C36 -0.15  0.54 -0.08 -0.23 -0.15
## N6  C36 C36 -0.18  0.74 -0.04 -0.23 -0.13
## N7  C36 C36 -0.14  0.72 -0.12 -0.27 -0.05
## N8  C36 C36 -0.16  0.76 -0.13 -0.29 -0.07
## N9  C36 C36 -0.19  0.69 -0.23 -0.19 -0.09
## N10 C36 C36 -0.33  0.67 -0.14 -0.28 -0.02
## A1  C41 C41 -0.12  0.07 -0.43 -0.09 -0.12
## A2  C41 C41  0.42 -0.12  0.60  0.09  0.15
## A3  C41 C41 -0.01  0.24 -0.40 -0.26  0.03
## A4  C41 C41  0.16  0.02  0.76  0.10  0.07
## A5  C41 C41 -0.24  0.04 -0.67 -0.08 -0.08
## A6  C41 C41  0.07  0.12  0.57  0.05 -0.03
## A7  C41 C41 -0.42  0.15 -0.69 -0.11 -0.12
## A8  C41 C41  0.22 -0.07  0.60  0.15  0.09
## A9  C41 C41  0.21  0.05  0.70  0.12  0.12
## A10 C41 C41  0.42 -0.22  0.48  0.21  0.16
## C1  C39 C39  0.09 -0.20  0.10  0.61  0.15
## C2  C39 C39  0.03  0.13  0.00 -0.53  0.08
## C3  C39 C39  0.01 -0.06  0.13  0.42  0.27
## C4  C39 C39 -0.15  0.41 -0.14 -0.61 -0.06
## C5  C39 C39  0.12 -0.17  0.15  0.62 -0.02
## C6  C39 C39 -0.05  0.21 -0.07 -0.62  0.01
## C7  C39 C39 -0.03  0.00  0.07  0.51  0.05
## C8  C39 C39 -0.15  0.27 -0.24 -0.52 -0.09
```

```
## C9  C39 C39  0.09 -0.08  0.16  0.61  0.01
## C10 C39 C39  0.08 -0.10  0.14  0.48  0.24
## O1  C45 C45  0.10 -0.08  0.03  0.09  0.60
## O2  C45 C45 -0.12  0.21 -0.07 -0.09 -0.59
## O3  C45 C45  0.06  0.07  0.08 -0.05  0.54
## O4  C45 C45 -0.08  0.10 -0.13 -0.01 -0.50
## O5  C45 C45  0.25 -0.16  0.09  0.21  0.61
## O6  C45 C45 -0.16  0.08 -0.13 -0.03 -0.53
## O7  C45 C45  0.14 -0.20  0.08  0.25  0.49
## O8  C45 C45  0.02  0.06 -0.08 -0.04  0.54
## O9  C45 C45 -0.10  0.15  0.15  0.05  0.32
## O10 C45 C45  0.24 -0.10  0.11  0.10  0.68
##
## With eigenvalues of:
## C34 C36 C41 C39 C45
## 4.9 4.5 3.9 3.3 3.2
##
## Purified scale intercorrelations
##  reliabilities on diagonal
##  correlations corrected for attenuation above diagonal:
##       C34   C36   C41   C39   C45
## C34  0.89 -0.30  0.39  0.13  0.20
## C36 -0.26  0.87 -0.12 -0.30 -0.12
## C41  0.34 -0.11  0.84  0.22  0.15
## C39  0.11 -0.25  0.18  0.81  0.14
## C45  0.17 -0.10  0.12  0.11  0.79
##
## Cluster fit =  0.75   Pattern fit =  0.98  RMSR =  0.04
```

PC WITH K FACTORS

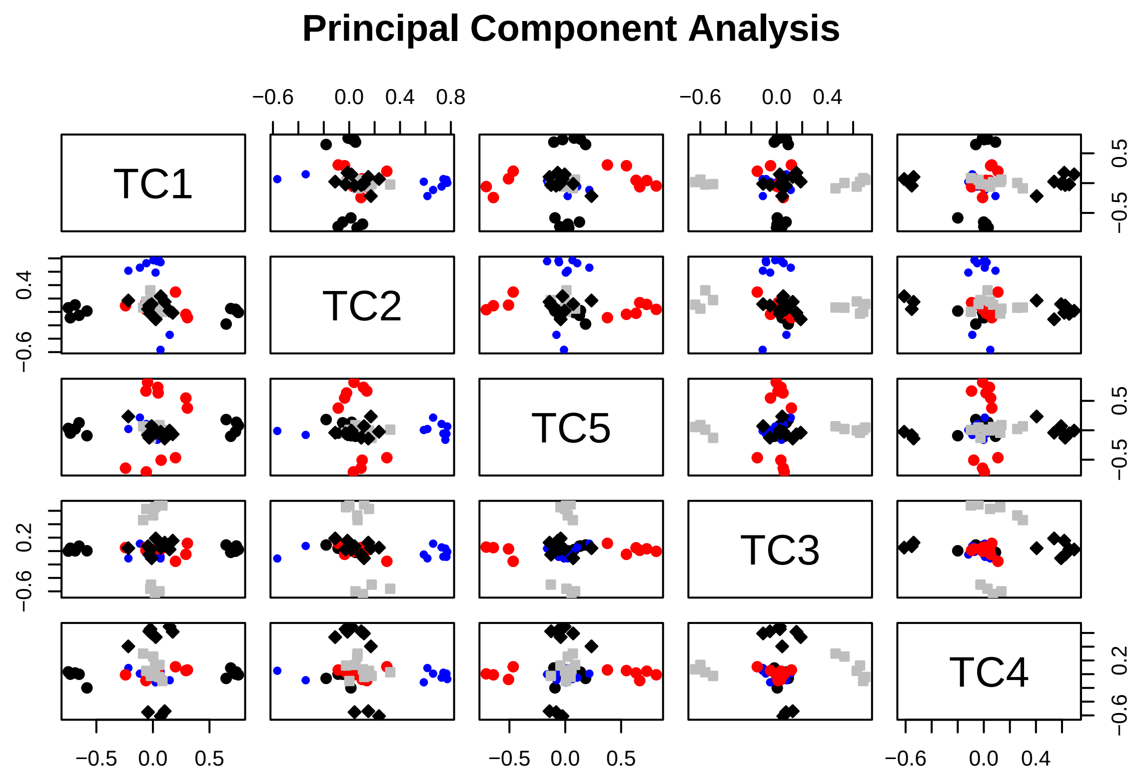
```r
# First PCA with no rotation, specify 5 factors
df %>% principal(nfactors = 5)
```

```
## Principal Components Analysis
## Call: principal(r = ., nfactors = 5)
## Standardized loadings (pattern matrix) based upon correlation matrix
##       RC1   RC2   RC5   RC3   RC4   h2   u2 com
## E1   0.71 -0.05  0.05  0.01  0.03 0.52 0.48 1.0
## E2  -0.72  0.00 -0.12  0.03 -0.03 0.53 0.47 1.1
## E3   0.67 -0.26  0.26  0.13 -0.02 0.60 0.40 1.7
## E4  -0.74  0.15 -0.05 -0.02  0.00 0.57 0.43 1.1
## E5   0.74 -0.08  0.22  0.10  0.07 0.62 0.38 1.3
## E6  -0.60  0.08 -0.16 -0.02 -0.23 0.45 0.55 1.5
## E7   0.75 -0.10  0.16  0.05  0.03 0.61 0.39 1.1
## E8  -0.62  0.02  0.06  0.07 -0.03 0.40 0.60 1.1
## E9   0.67 -0.04 -0.03 -0.01  0.12 0.46 0.54 1.1
## E10 -0.68  0.19 -0.06 -0.02 -0.02 0.50 0.50 1.2
## N1  -0.11  0.73  0.07 -0.01 -0.07 0.55 0.45 1.1
## N2   0.11 -0.56  0.01 -0.05  0.07 0.33 0.67 1.1
## N3  -0.14  0.66  0.18  0.06 -0.01 0.49 0.51 1.3
## N4   0.16 -0.37 -0.04  0.10 -0.07 0.18 0.82 1.7
```

```
## N5  -0.04  0.59 -0.02 -0.11 -0.14 0.38 0.62 1.2
## N6  -0.06  0.77  0.03 -0.08 -0.09 0.61 0.39 1.1
## N7   0.00  0.73 -0.09 -0.15  0.00 0.57 0.43 1.1
## N8  -0.01  0.76 -0.09 -0.16 -0.02 0.61 0.39 1.1
## N9  -0.04  0.74 -0.19 -0.04 -0.03 0.58 0.42 1.1
## N10 -0.26  0.65 -0.04 -0.17  0.06 0.52 0.48 1.5
## A1   0.00  0.08 -0.50 -0.01 -0.09 0.26 0.74 1.1
## A2   0.35 -0.05  0.57  0.00  0.09 0.46 0.54 1.7
## A3   0.13  0.27 -0.46 -0.20  0.09 0.35 0.65 2.3
## A4   0.04  0.06  0.80  0.04  0.02 0.65 0.35 1.0
## A5  -0.13  0.02 -0.71  0.01 -0.02 0.52 0.48 1.1
## A6  -0.01  0.16  0.65  0.03 -0.08 0.45 0.55 1.2
## A7  -0.31  0.10 -0.67  0.00 -0.05 0.56 0.44 1.5
## A8   0.12 -0.02  0.64  0.09  0.04 0.43 0.57 1.1
## A9   0.12  0.12  0.73  0.07  0.07 0.56 0.44 1.1
## A10  0.36 -0.13  0.42  0.15  0.09 0.35 0.65 2.6
## C1   0.05 -0.10  0.01  0.65  0.12 0.45 0.55 1.1
## C2   0.06  0.10  0.05 -0.59  0.14 0.38 0.62 1.2
## C3  -0.04  0.02  0.09  0.47  0.29 0.31 0.69 1.8
## C4  -0.06  0.38 -0.04 -0.59  0.02 0.49 0.51 1.8
## C5   0.09 -0.08  0.06  0.67 -0.10 0.48 0.52 1.1
## C6   0.00  0.17  0.01 -0.64  0.07 0.44 0.56 1.2
## C7  -0.05  0.09  0.03  0.61  0.04 0.38 0.62 1.1
## C8  -0.06  0.23 -0.17 -0.52 -0.03 0.36 0.64 1.7
## C9   0.07  0.04  0.09  0.68 -0.04 0.48 0.52 1.1
## C10  0.04  0.00  0.06  0.53  0.25 0.35 0.65 1.5
## O1   0.03 -0.04 -0.04  0.05  0.65 0.43 0.57 1.0
## O2   0.00  0.23 -0.03  0.01 -0.61 0.43 0.57 1.3
## O3   0.04  0.12  0.07 -0.09  0.59 0.37 0.63 1.2
## O4   0.03  0.13 -0.13  0.08 -0.54 0.34 0.66 1.3
## O5   0.22 -0.06 -0.03  0.18  0.62 0.47 0.53 1.5
## O6  -0.10  0.05 -0.09  0.04 -0.56 0.33 0.67 1.1
## O7   0.08 -0.14 -0.01  0.22  0.54 0.36 0.64 1.5
## O8   0.00  0.09 -0.13 -0.05  0.61 0.40 0.60 1.1
## O9  -0.17  0.19  0.21  0.05  0.40 0.27 0.73 2.5
## O10  0.20 -0.02  0.02  0.05  0.70 0.53 0.47 1.2
##
##                        RC1  RC2  RC5  RC3  RC4
## SS loadings           5.52 5.15 4.35 3.91 3.82
## Proportion Var        0.11 0.10 0.09 0.08 0.08
## Cumulative Var        0.11 0.21 0.30 0.38 0.46
## Proportion Explained  0.24 0.23 0.19 0.17 0.17
## Cumulative Proportion 0.24 0.47 0.66 0.83 1.00
##
## Mean item complexity =  1.3
## Test of the hypothesis that 5 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.04
##  with the empirical chi square  81657.81  with prob <  0
##
## Fit based upon off diagonal values = 0.95
```

```r
# Second PCA with oblimin (oblique) rotation
df %>%
```

```
principal(
  nfactors = 5,
  rotate = "oblimin"
) %>%
plot()  # Plot position of variables on components
```



The End!