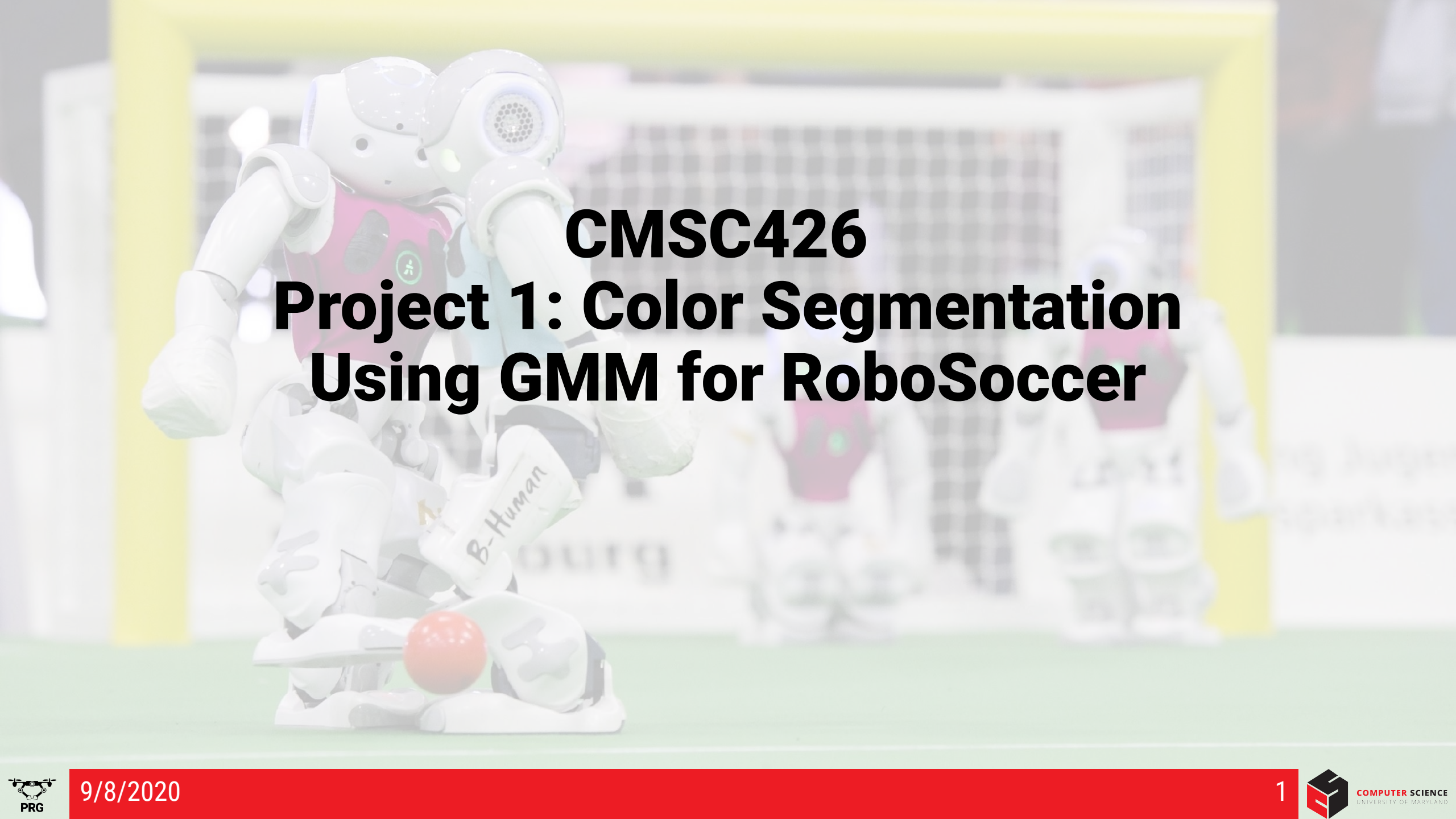
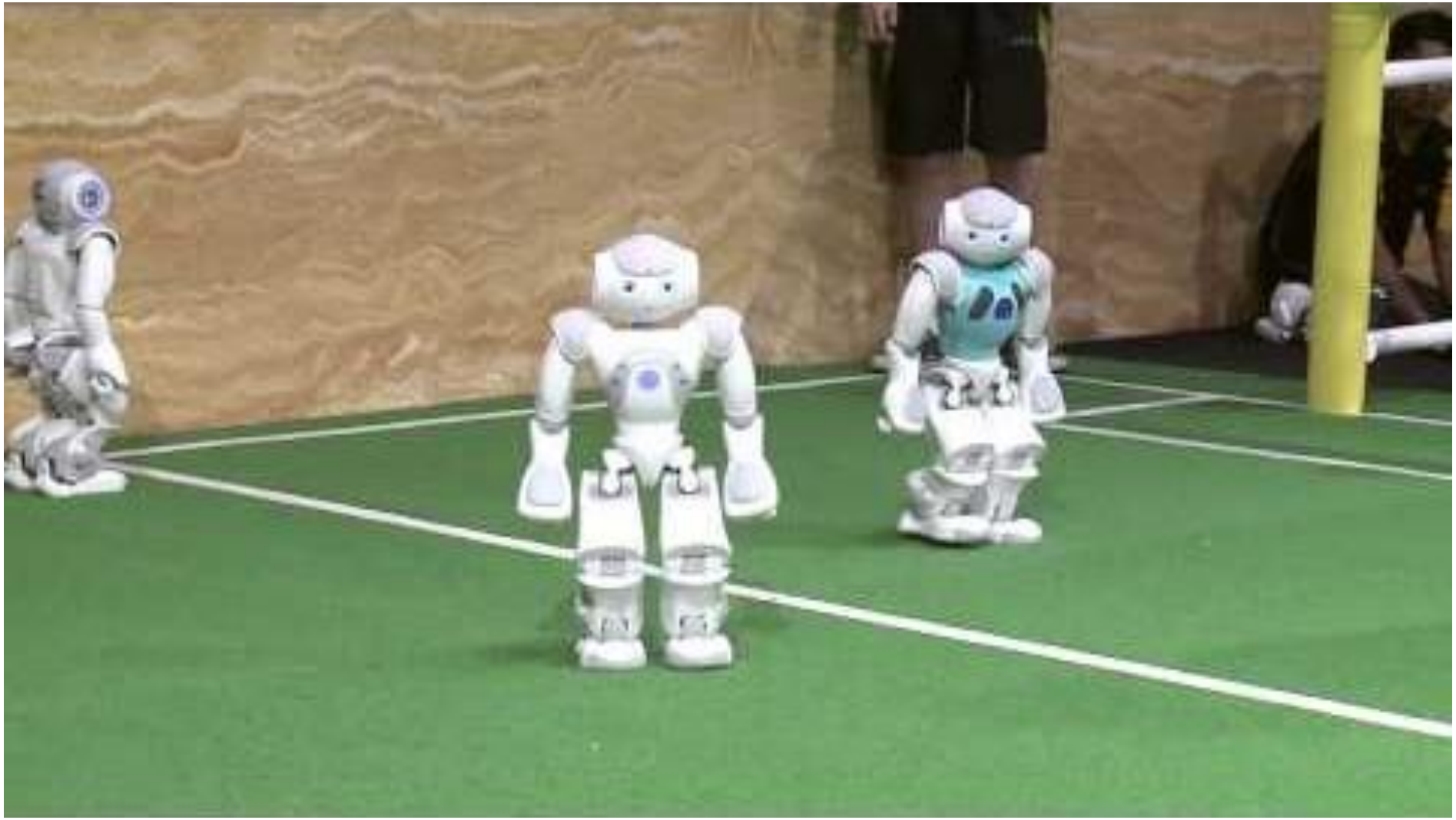


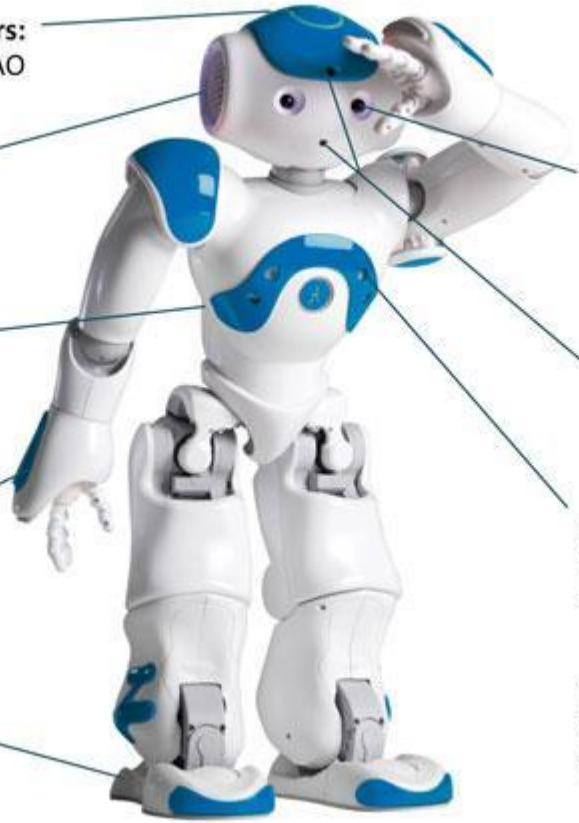
CMSC426
Project 1: Color Segmentation Using GMM for RoboSoccer

RoboSoccer

RoboSoccer



Data: From the Nao Robot



Tactile sensors:
Menu to interact non-verbally with NAO

Speakers (x2):
NAO talks, prompts, shares his story, plays music…

Battery:
NAO is free to navigate without being connected to a power source.

Prehensile hands with sensors:
To grasp small items and to work on object exchange and turn-taking

Foot bumpers:
Another way to interact with NAO.

Microphones (x4):
NAO detects the origin of sounds and understands what you say.

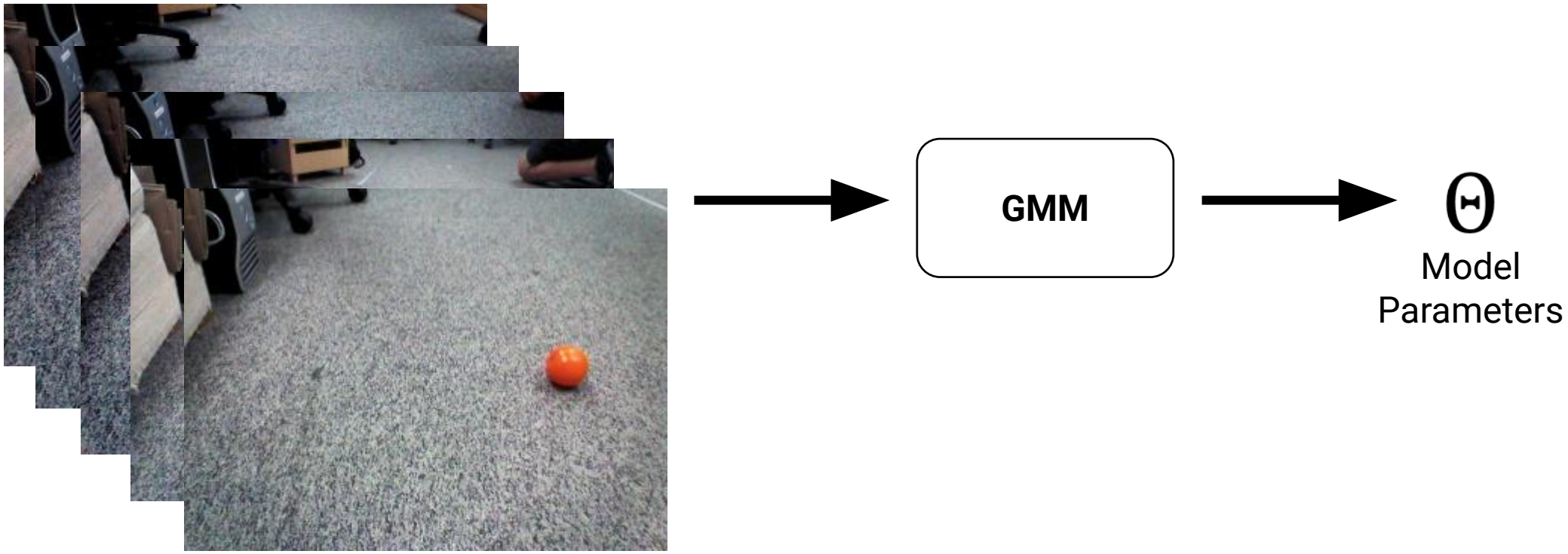
Eyeleds:
NAO uses color code to express emotions and even play edutaining color games with your children!

Cameras (x2):
NAO recognizes pre-recorded faces, pictures , reads books, imitates.

Sonars (x4):
NAO detects whether something stands closely in front of him.

Wifi Connection:
NAO can use information from the web

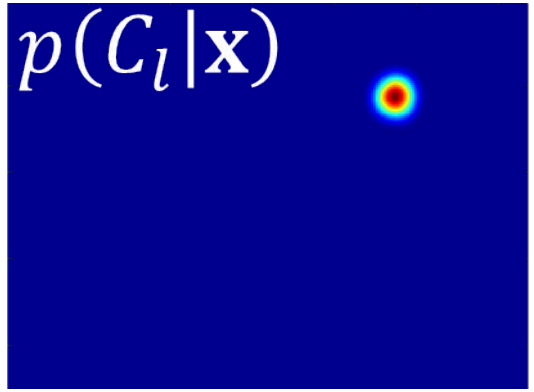
Color Segmentation: IO for Training



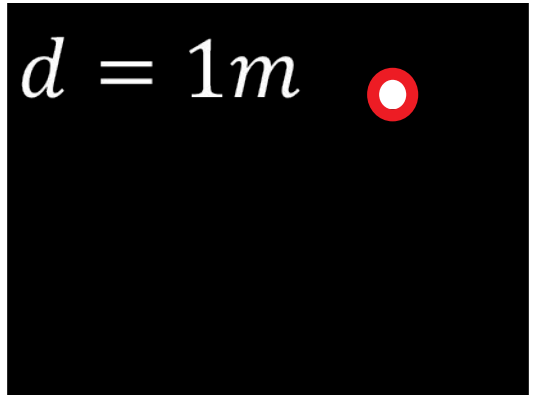
Color Segmentation: IO for Testing

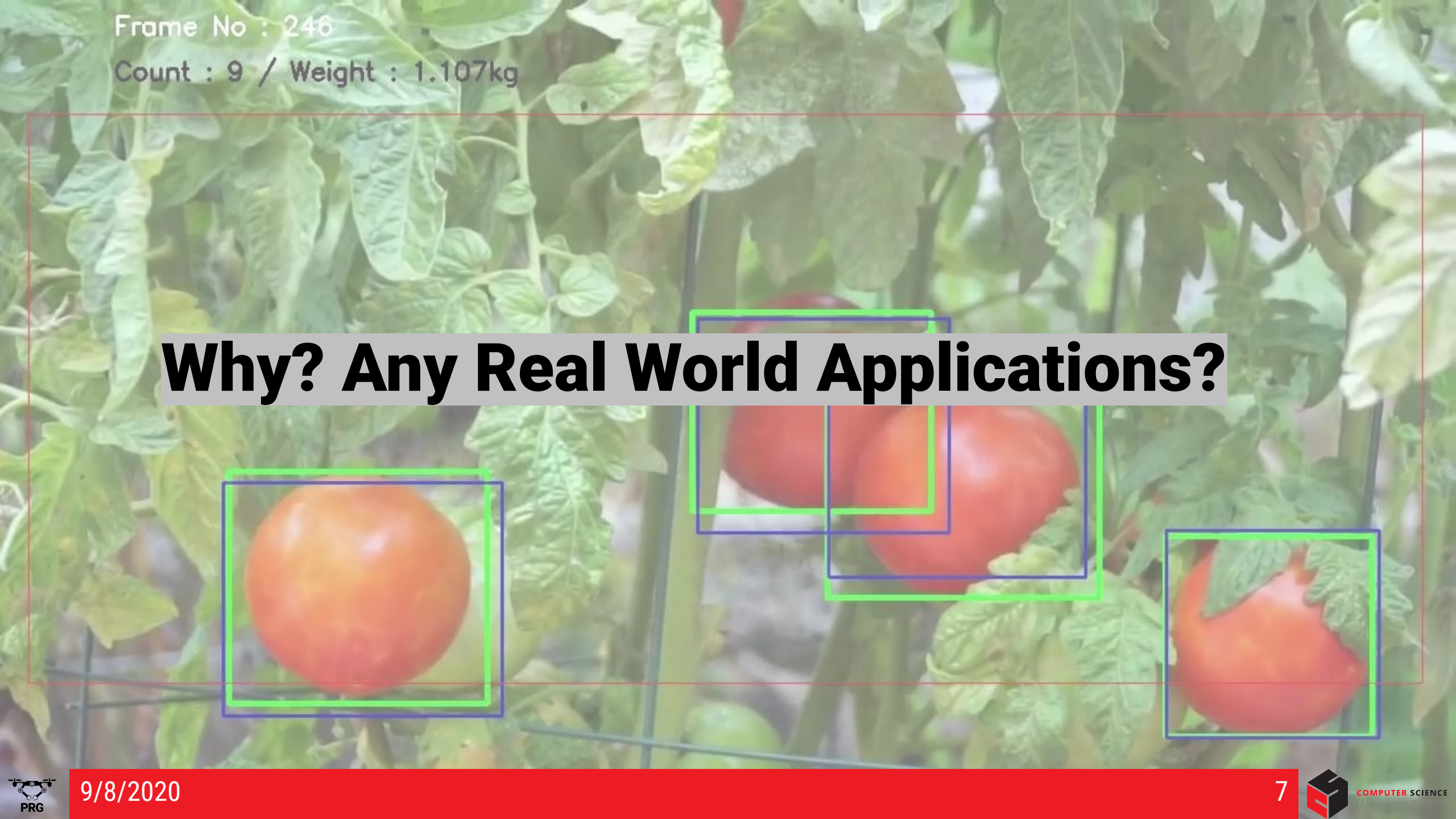
GMM

Θ



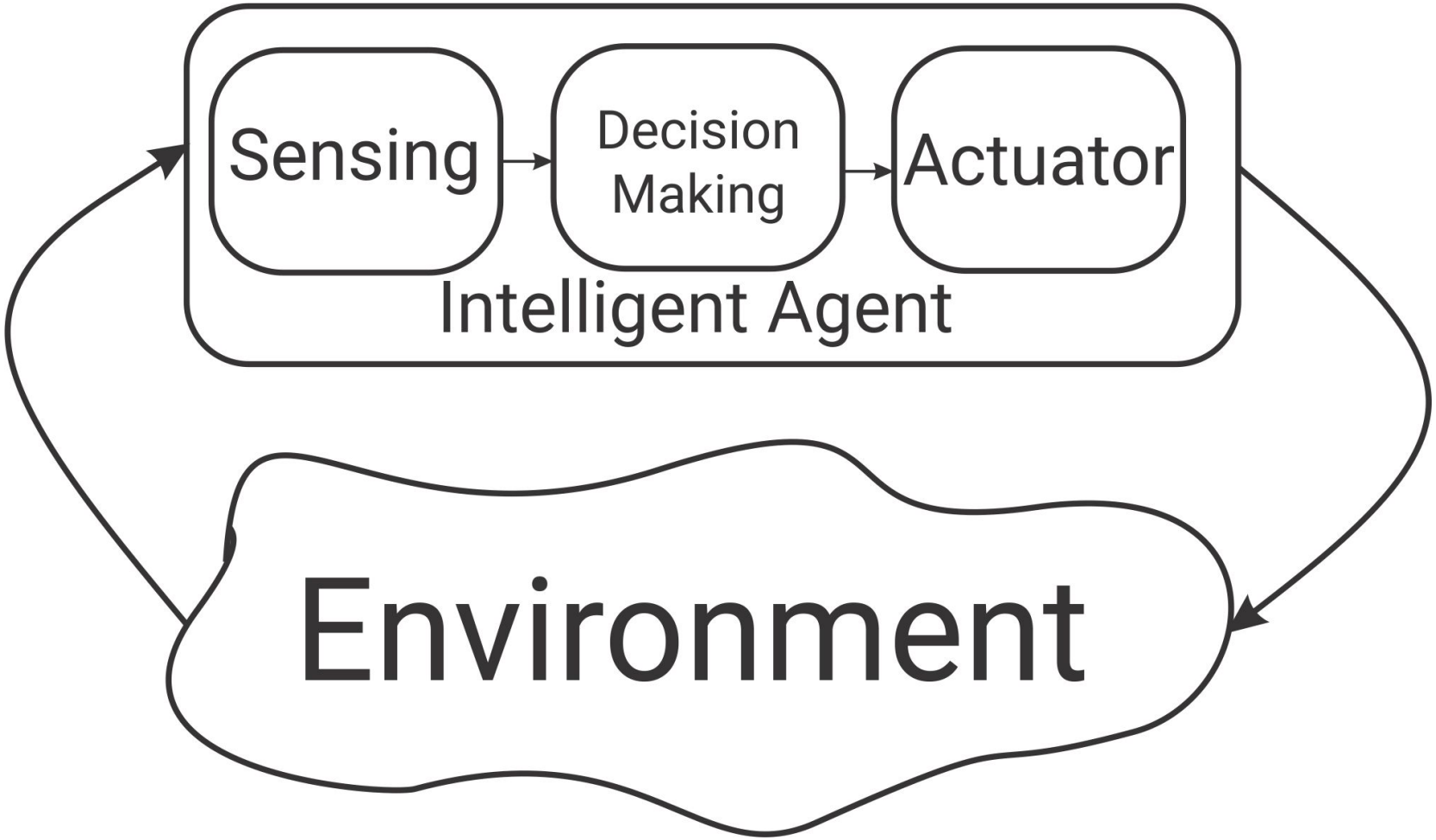
Ball localizer and
Distance predictor





Why? Any Real World Applications?

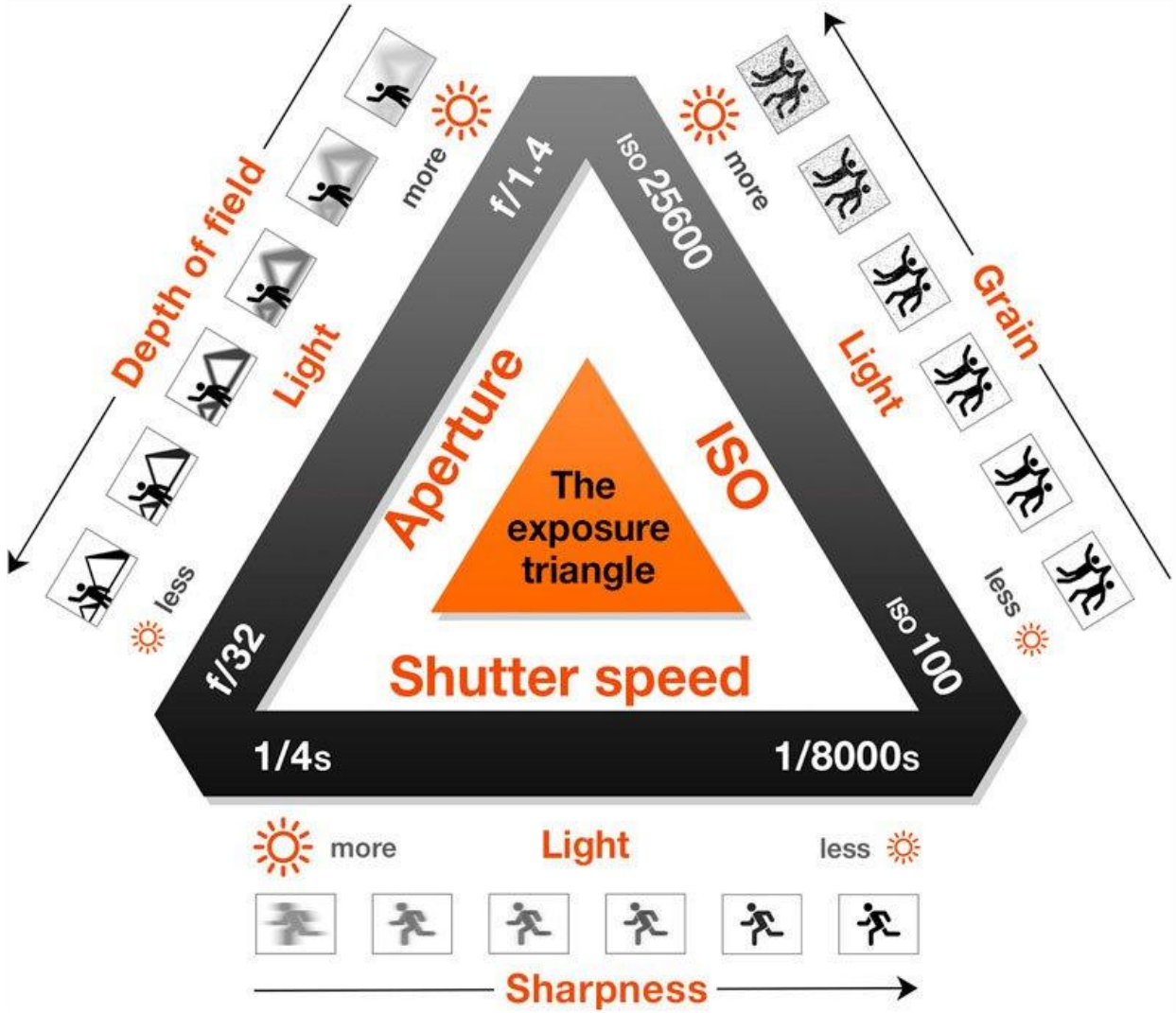
Intelligent Systems



Imaging

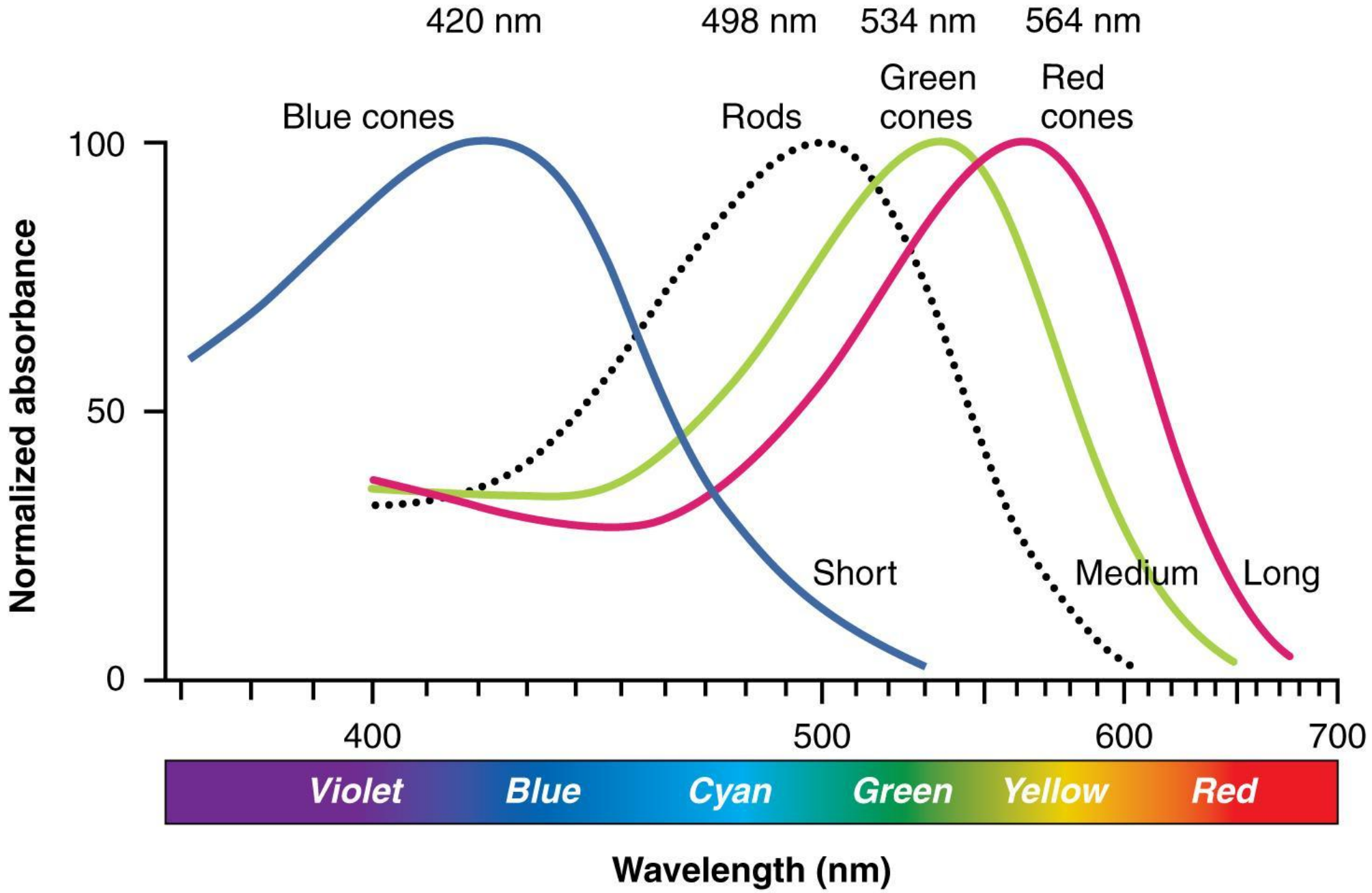
Measures: $\frac{\#\text{Photons}}{\text{Sampling Time}}$

Exposure Triangle

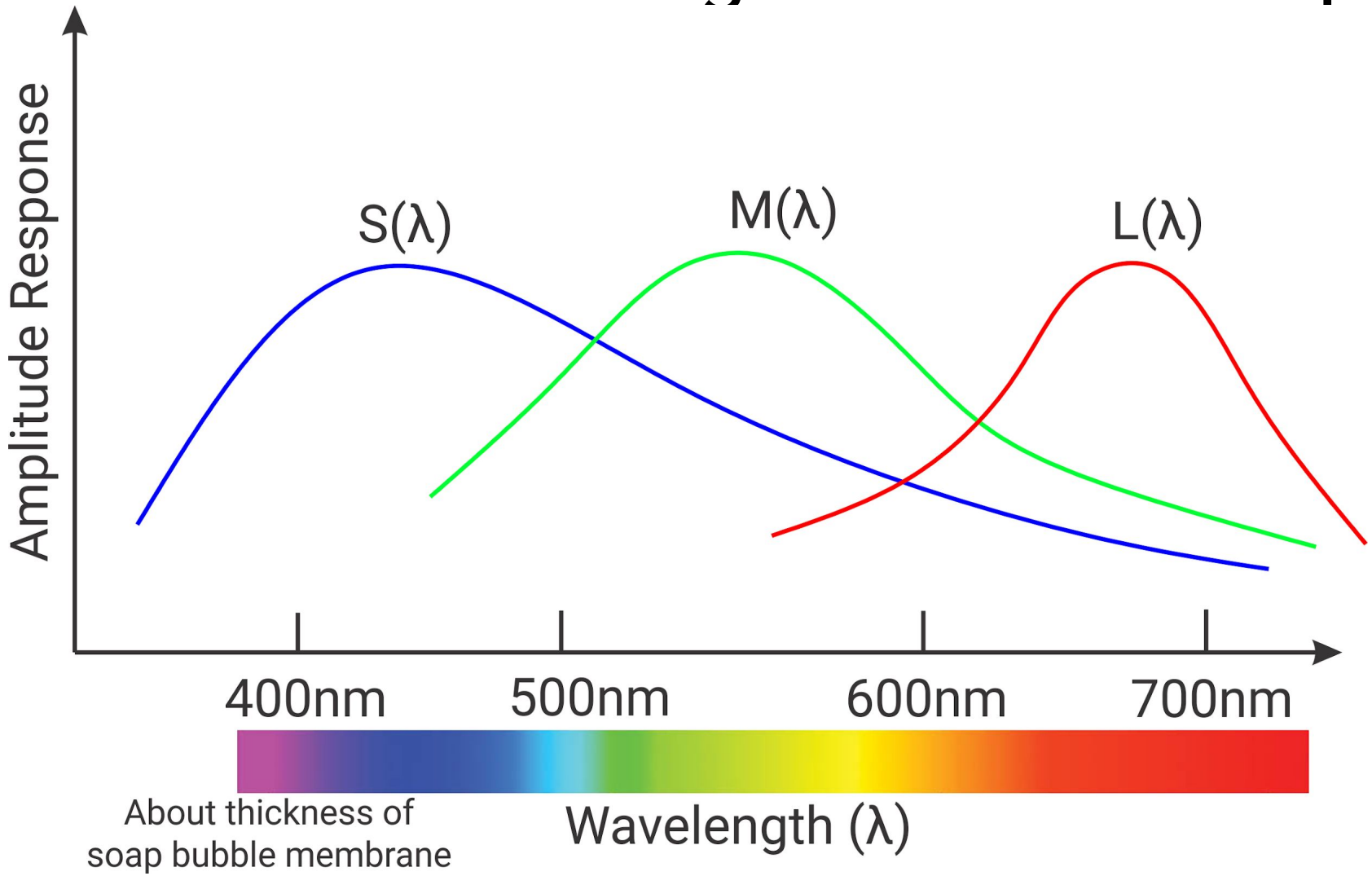


RGB Color Imaging

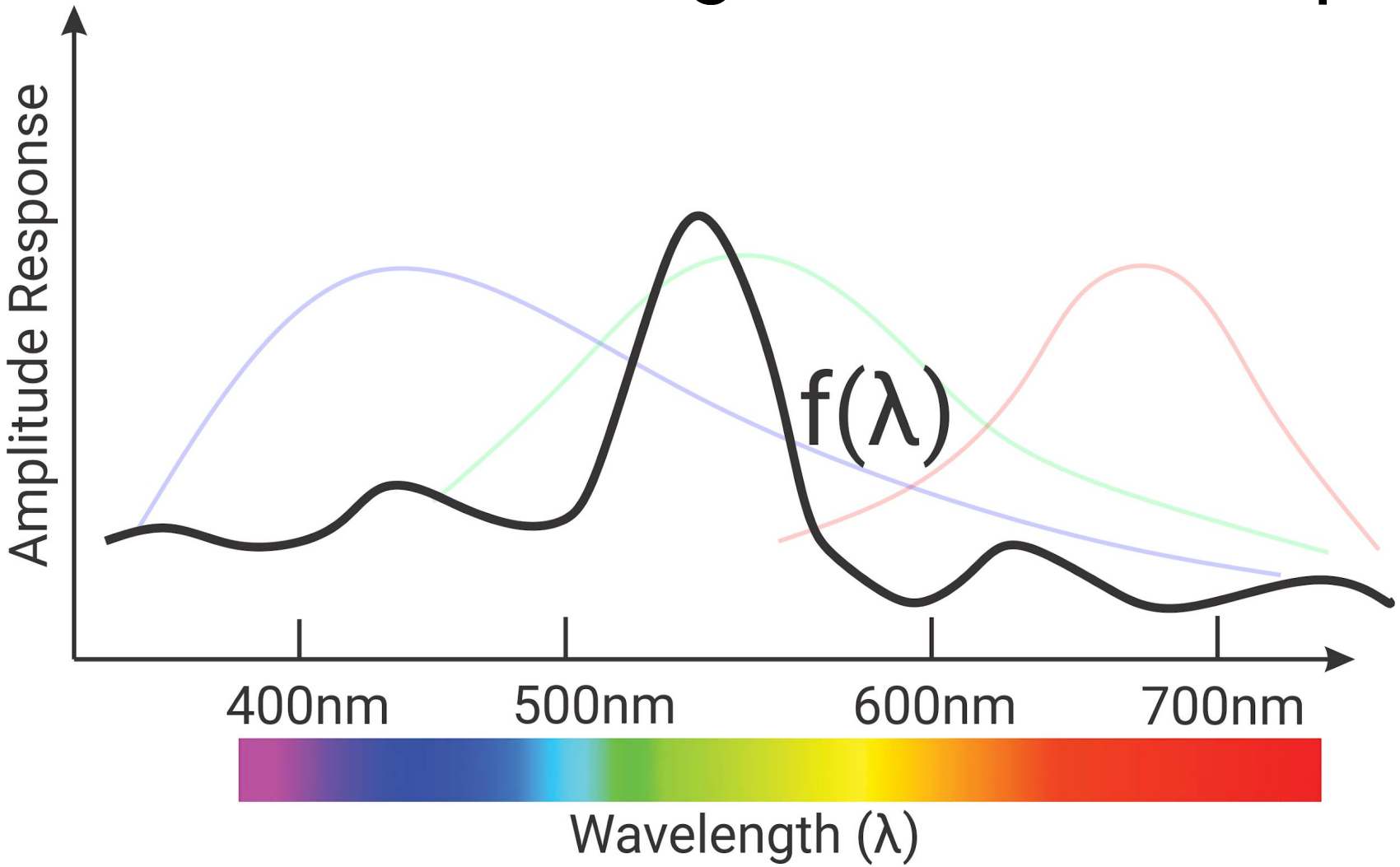
Why RGB?


420 nm
498 nm
534 nm
564 nm
Blue cones
Rods
Green cones
Red cones
100
50
0
Normalized absorbance
Short
Medium
Long
400
500
600
700
Violet
Blue
Cyan
Green
Yellow
Red
Wavelength (nm)

Mathematical Modelling of Color Perception



Mathematical Modelling of Color Perception



Mathematical Modelling of Color Perception

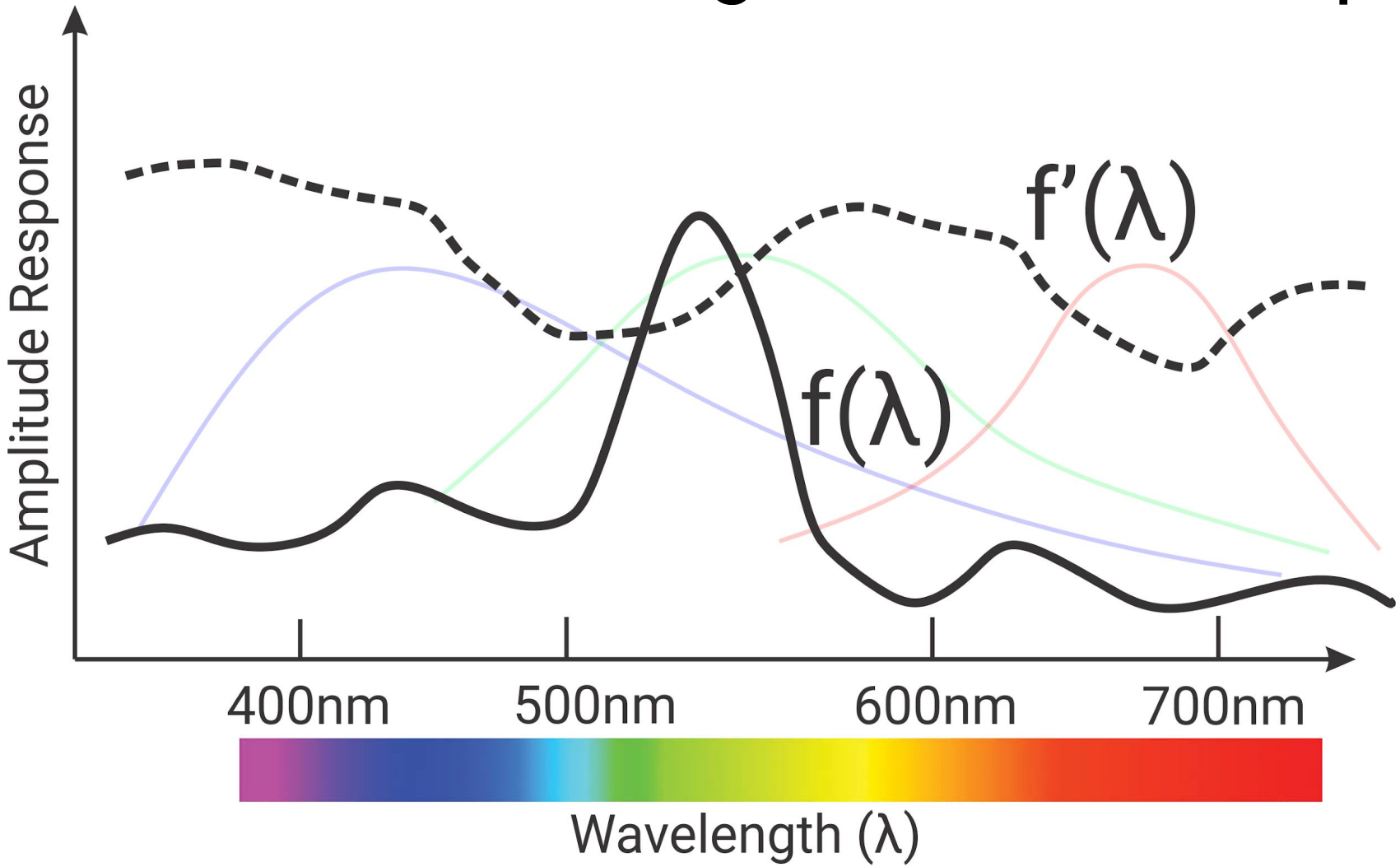
$$S_{res} = \int S(\lambda) f(\lambda) d\lambda$$

$$M_{res} = \int M(\lambda) f(\lambda) d\lambda$$

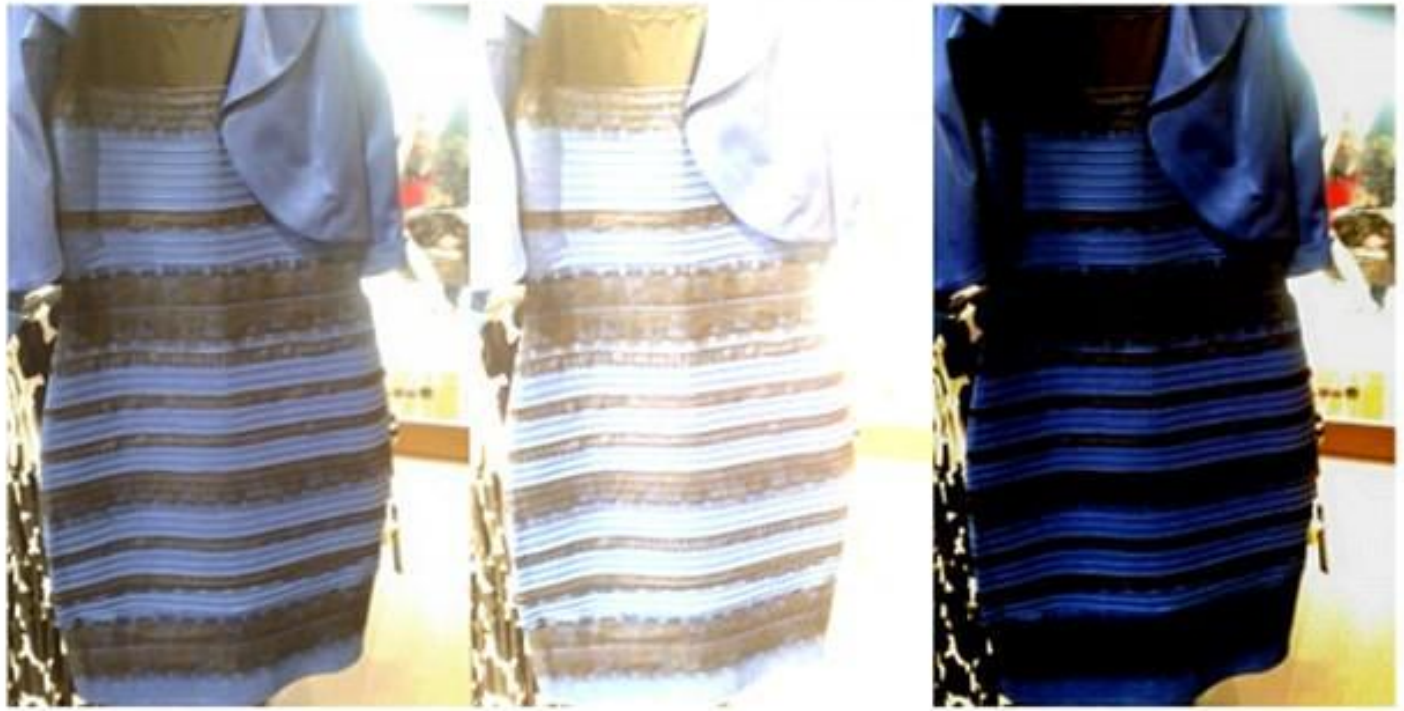
$$L_{res} = \int L(\lambda) f(\lambda) d\lambda$$

$$\mathbb{R}^{\infty} \rightarrow \mathbb{R}^{3}$$

Mathematical Modelling of Color Perception



Color Illusion

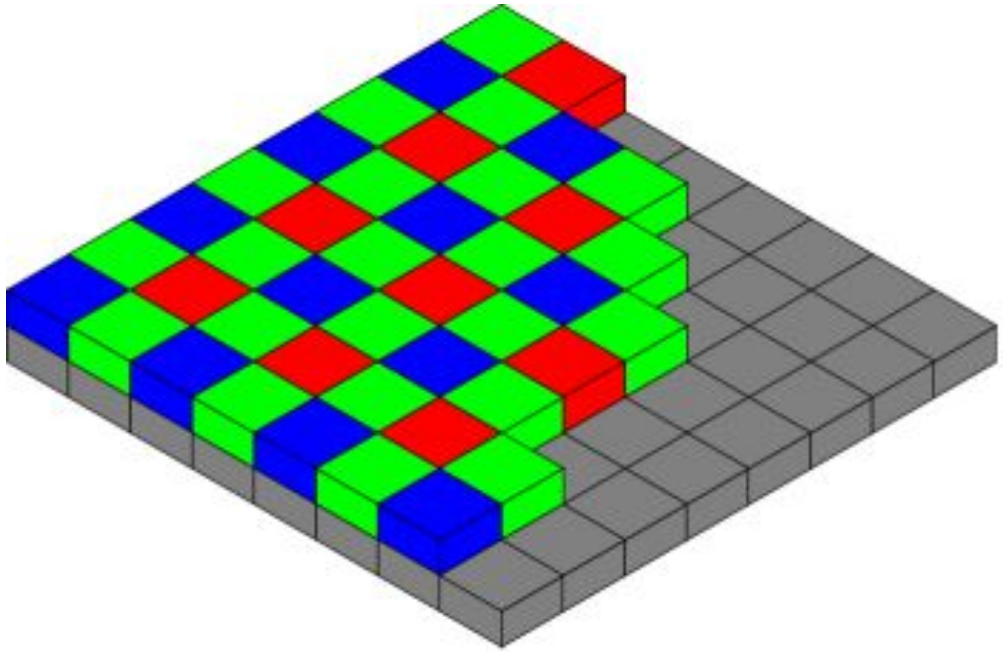


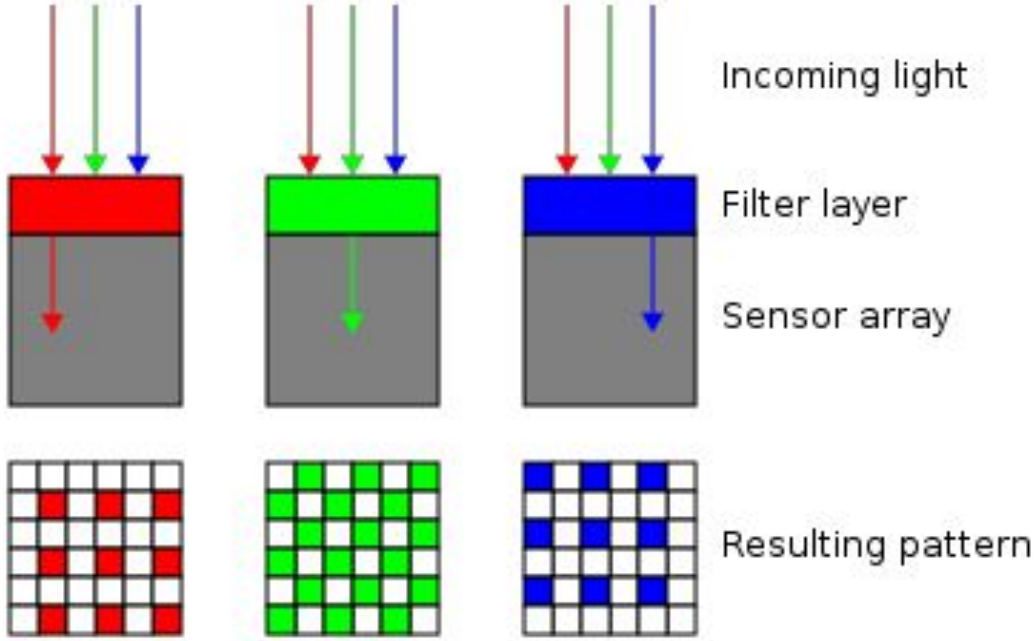
ORIGINAL
(Blue and Gold)
+0% brightness, +0% contrast

BRIGHTER
(White and Gold)
+40% brightness, +40% contrast

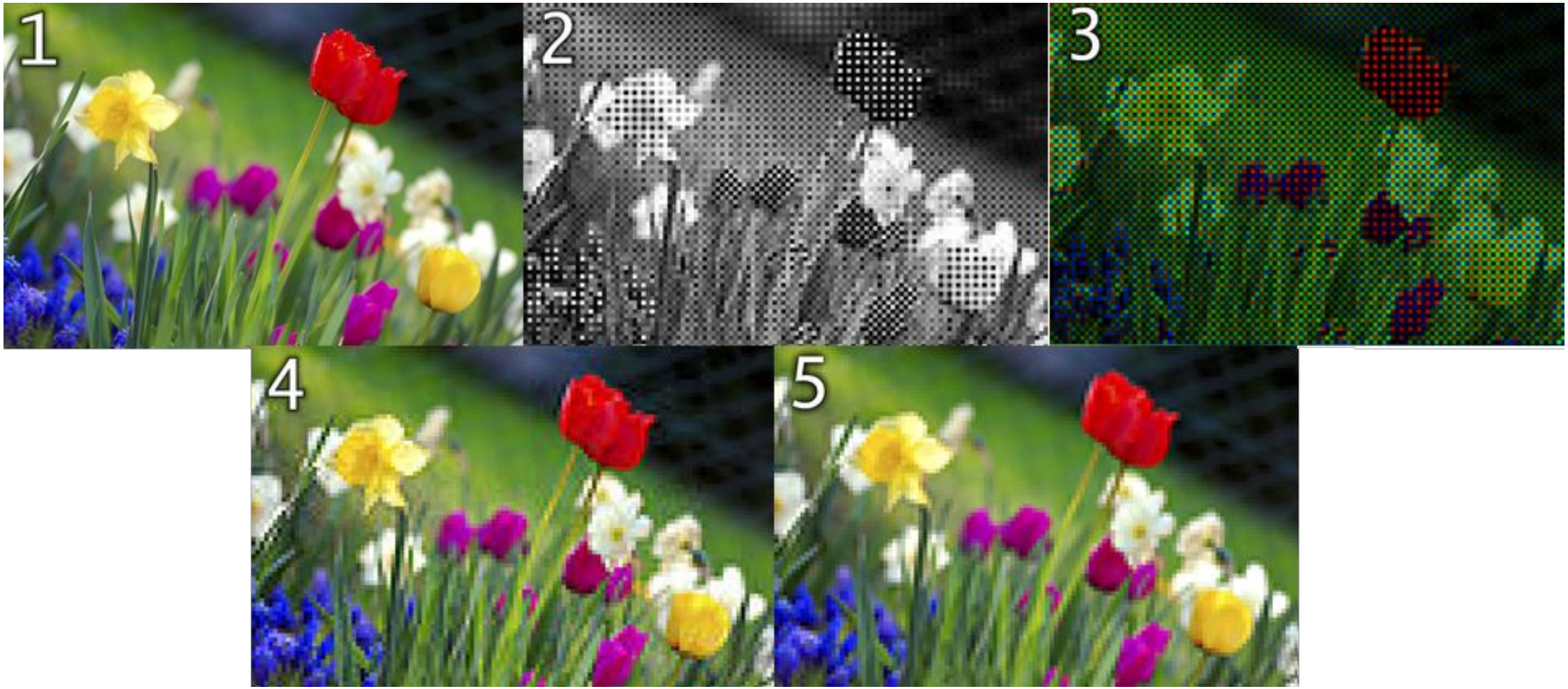
DARKER
(Blue and Black)
-30% brightness, +40% contrast

How is RGB captured?

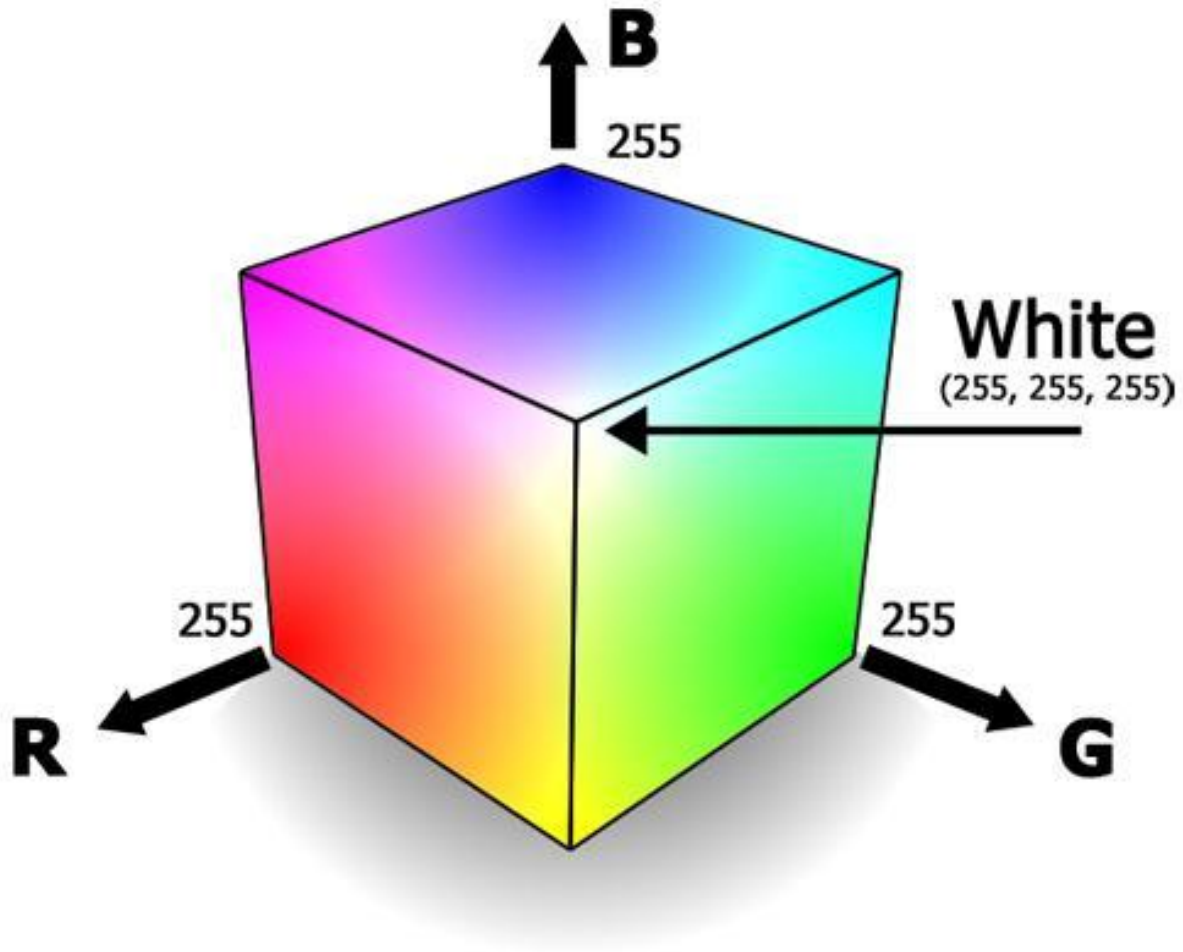




Bayer Pattern



RGB Color Cube



Different Color Spaces: HSV

$$R' = \frac{R}{255}$$

$$G' = \frac{G}{255}$$

$$B' = \frac{B}{255}$$

$$C_{max} = \max(R', G', B')$$

$$C_{min} = \min(R', G', B')$$

$$\Delta = C_{max} - C_{min}$$

Hue is calculated as follows:

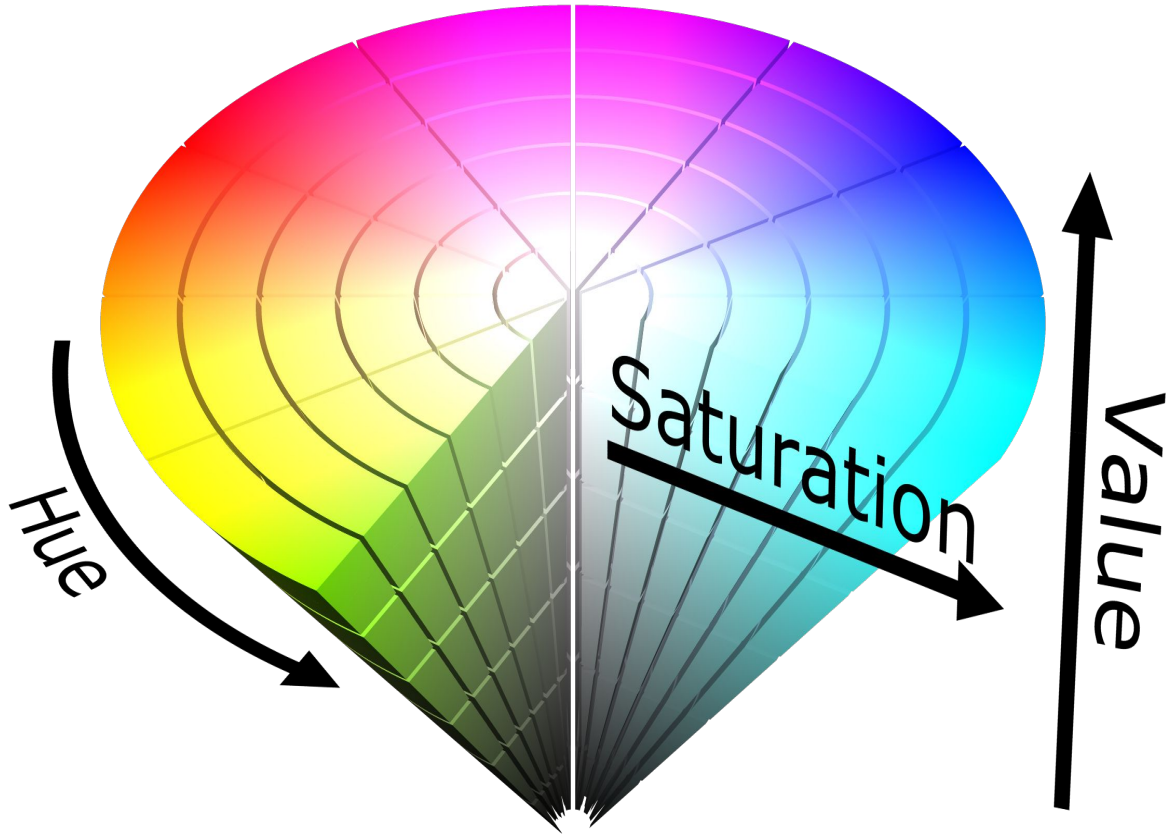
$$\begin{cases} 0^\circ & \text{if } \Delta = 0 \\ 60^\circ \times \frac{G'-B'}{\Delta} 60^\circ \times \frac{B'-R'}{\Delta} + 2 & \text{if } C_{max} = G' \\ 60^\circ \times \frac{R'-G'}{\Delta} + 4 & \text{if } C_{max} = B' \end{cases}$$

Saturation is calculated as follows:

$$\begin{cases} 0 & \text{if } C_{max} = 0 \\ \frac{\Delta}{C_{max}} & \text{if } C_{max} \neq 0 \end{cases}$$

Value is calulated as follows:

$$V = C_{max}$$

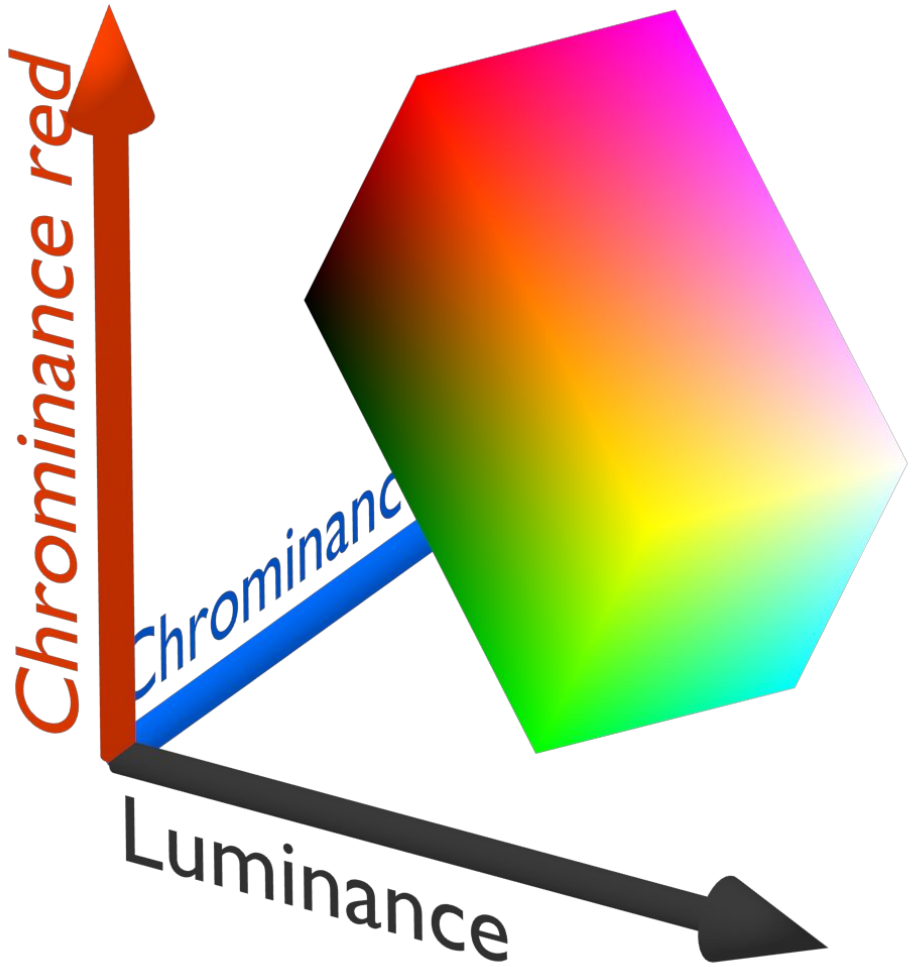


Different Color Spaces: YCbCr

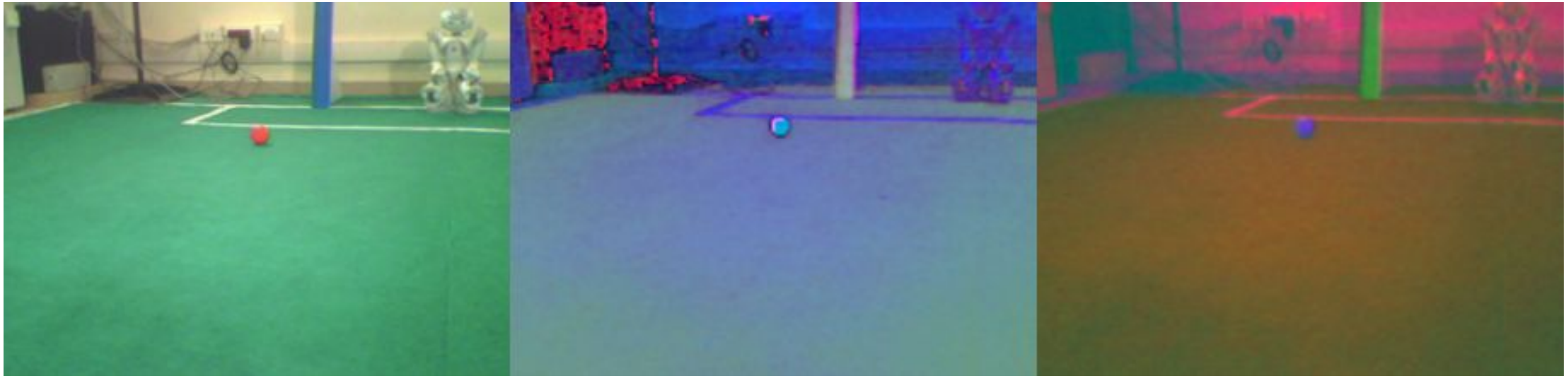
$$Y = 0.299R + 0.587G + 0.114B$$

$$Cb = -0.169R + -0.331G + 0.500B + 128$$

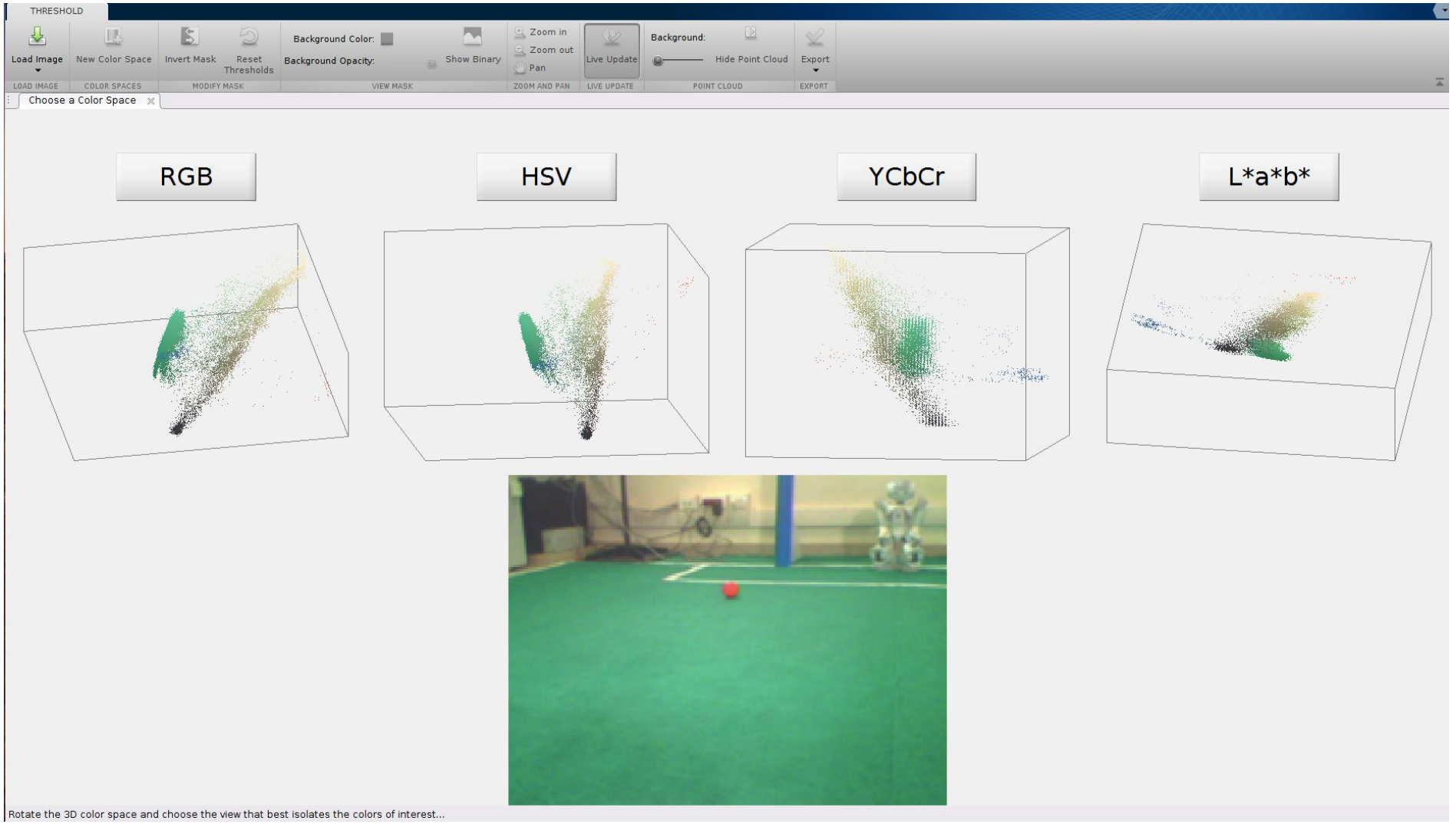
$$Cr = 0.500R + -0.419G + -0.081B + 128$$



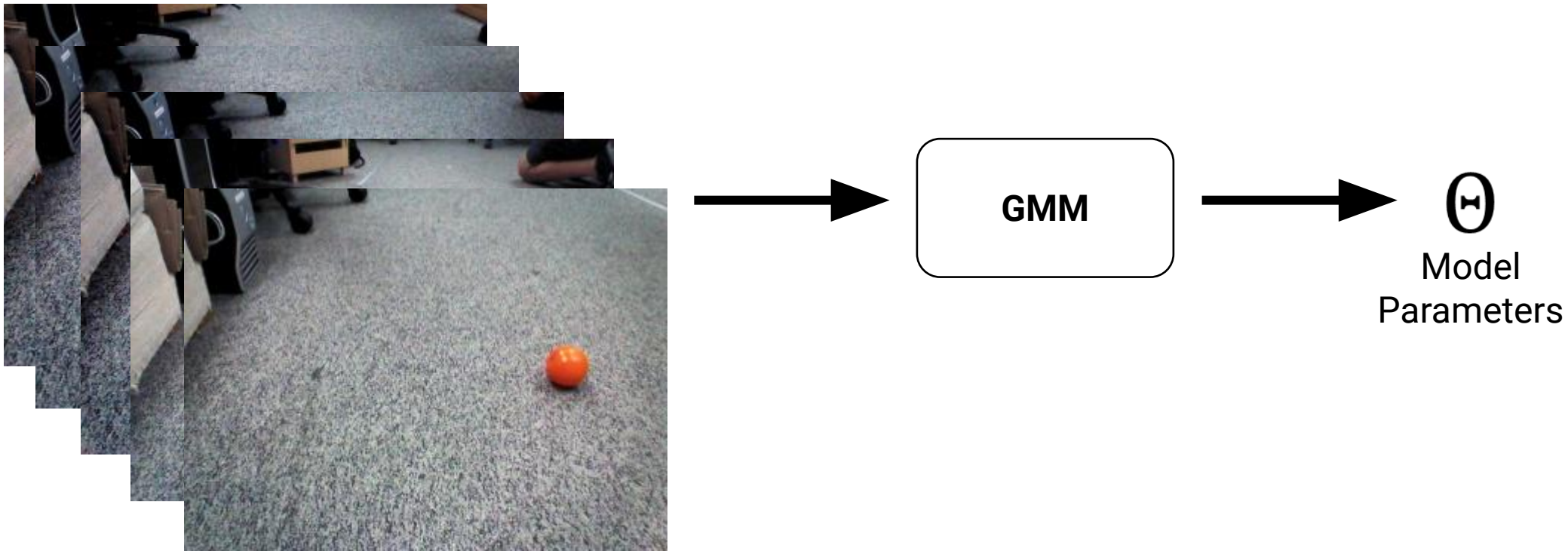
Robosoccer in RGB, HSV and YCbCr

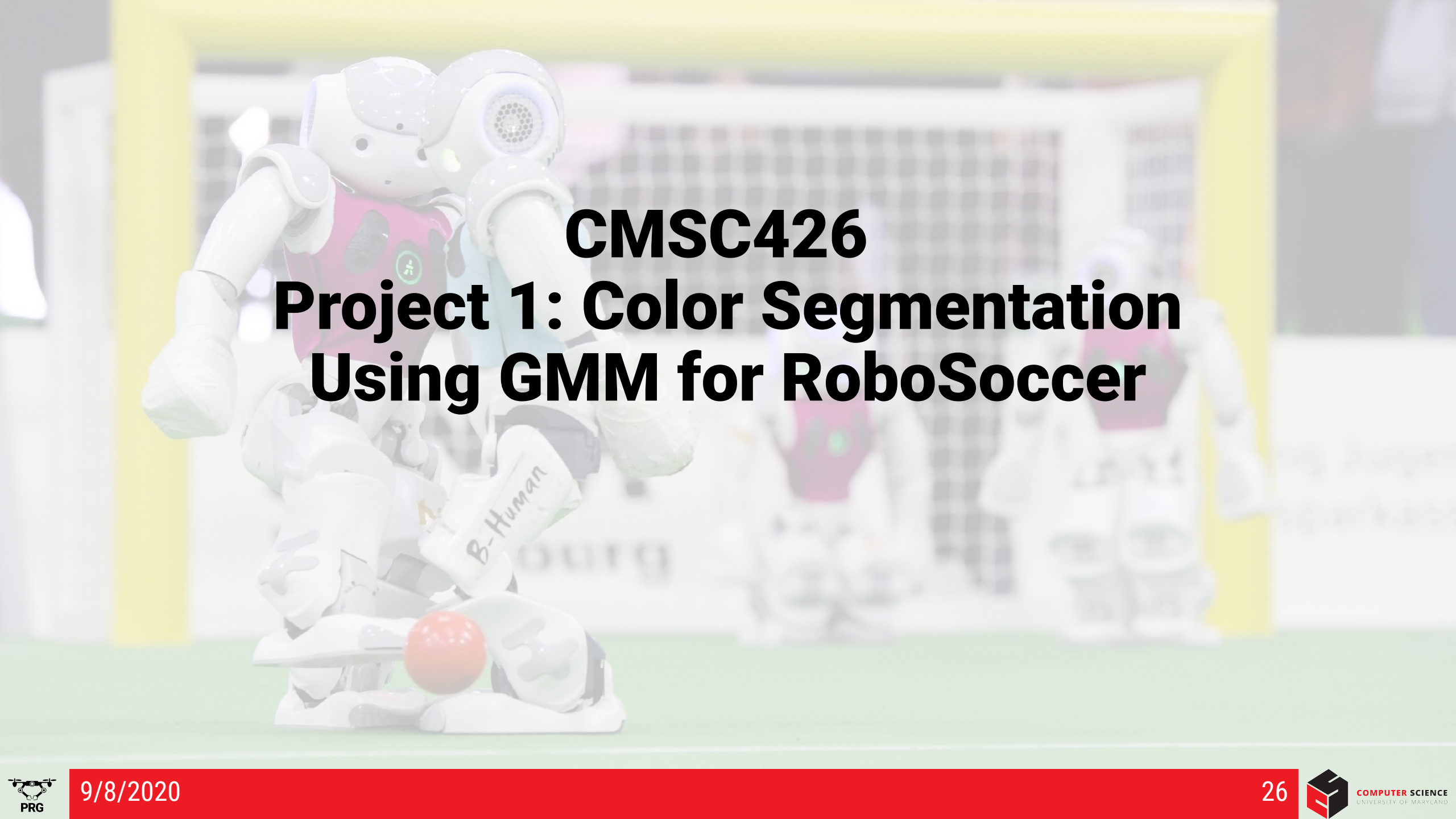


Mathematical View of Color Thresholding



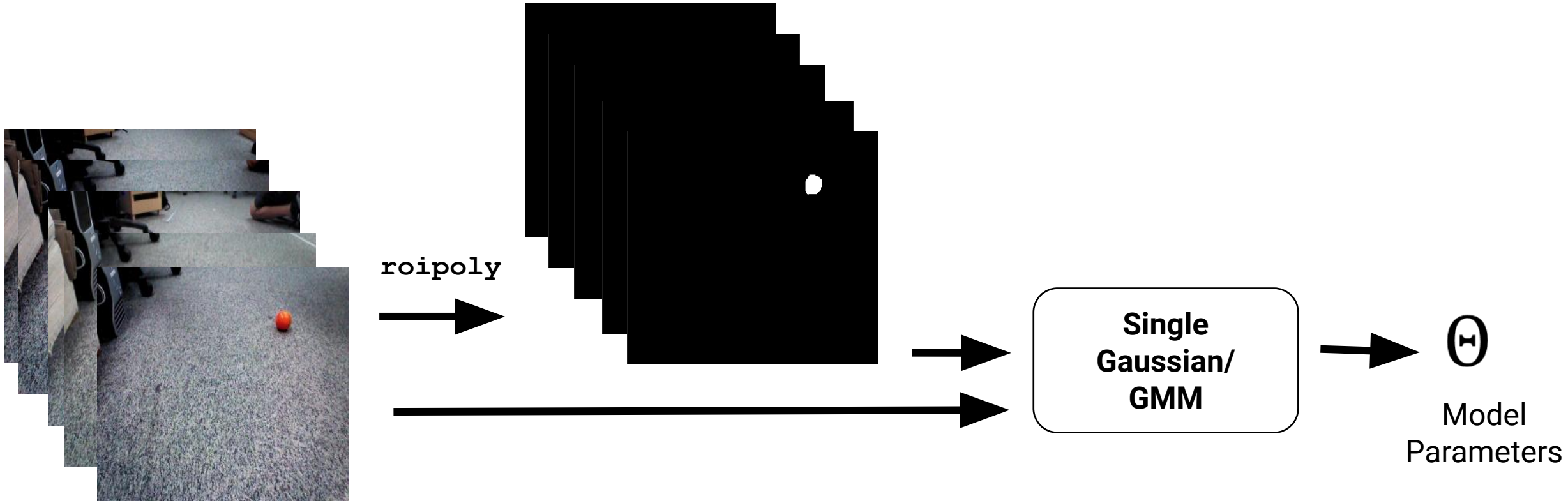
Color Segmentation: IO for Training





CMSC426
Project 1: Color Segmentation Using GMM for RoboSoccer

Project Pipeline



Mathematical View of Color Thresholding

$\mathbf{x}$ is a pixel's RGB value

$p(\mathbf{x}|C_l)$: Likelihood

$p(C_l|\mathbf{x})$: Posterior

How do we get $p(C_l|\mathbf{x})$?

Bayes' Rule!

Prior

$$p(C_l|\mathbf{x}) = \frac{p(\mathbf{x}|C_l)p(C_l)}{\sum_{i=1}^{L} p(\mathbf{x}|C_i)p(C_i)}$$

Color Thresholding Using Single Gaussian

What is function we should use for $p(C_l)$?

Any function of your choice!

Generally Uniform, i.e., $p(C_l) = \frac{1}{L}$

One could model more common colors with higher $p(C_l)$!

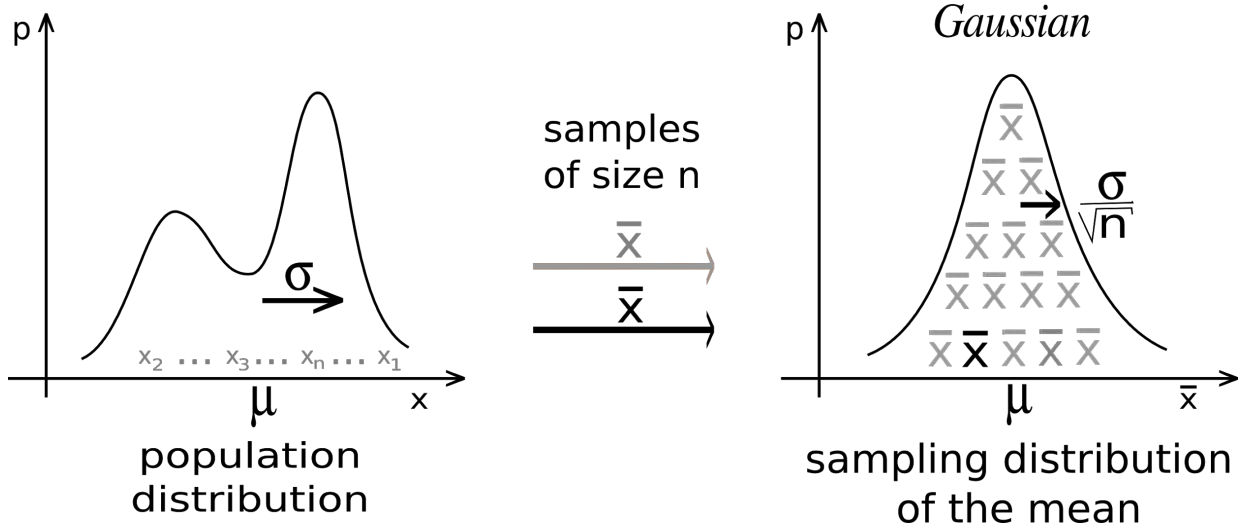
Color Thresholding Using Single Gaussian

What is function we should use for $p(\mathbf{x}|C_l)$?

$$p(\mathbf{x}|C_l) = \frac{1}{\sqrt{(2\pi)^3|\Sigma|}}\exp\left(\frac{-1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right) = \mathcal{N}(x|\mu,\Sigma)$$

Why Gaussian?

Central Limit Theorem



Color Thresholding Using Single Gaussian

What do we need to obtain $\Theta = [\boldsymbol{\mu}, \Sigma]$?

$\mathbf{x}$: Data samples

$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$ Empirical Mean

$\boldsymbol{\Sigma} = \frac{1}{N}(\mathbf{x} - \boldsymbol{\mu})(\mathbf{x} - \boldsymbol{\mu})^{\mathrm{T}}$ Empirical Co-variance

Programming Tips:

$$p(C_l|\mathbf{x}) \propto p(\mathbf{x}|C_l)p(C_l)$$
$$p(C_l) = \frac{1}{L}$$

$$p(\mathbf{x}|C_l) = \frac{1}{\sqrt{(2\pi)^3|\Sigma|}}\exp\left(\frac{-1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right) = \mathcal{N}(x|\mu,\Sigma)$$

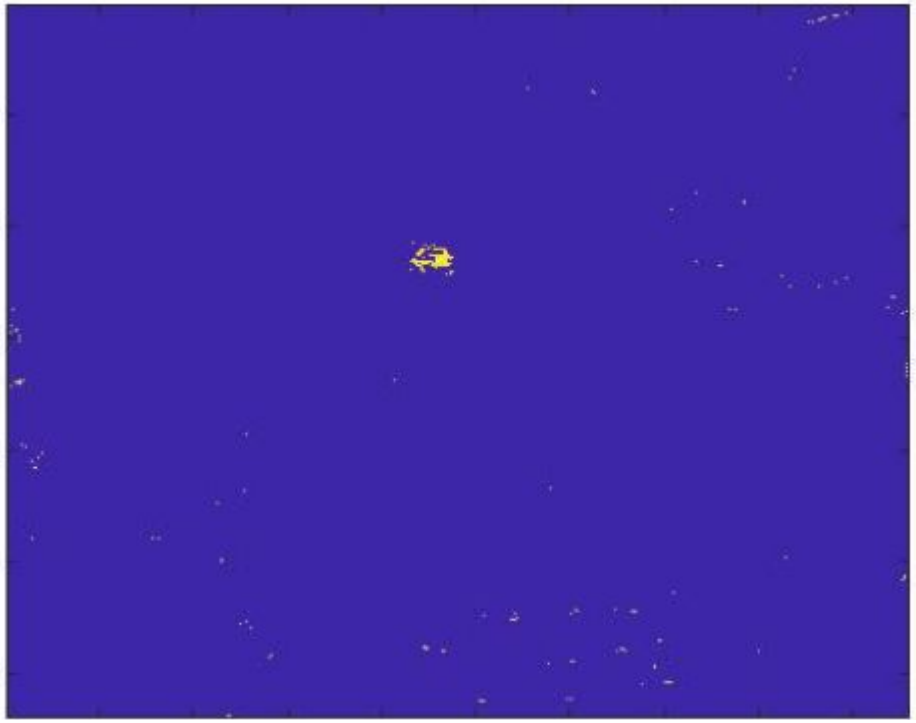
For Orange Pixel's only

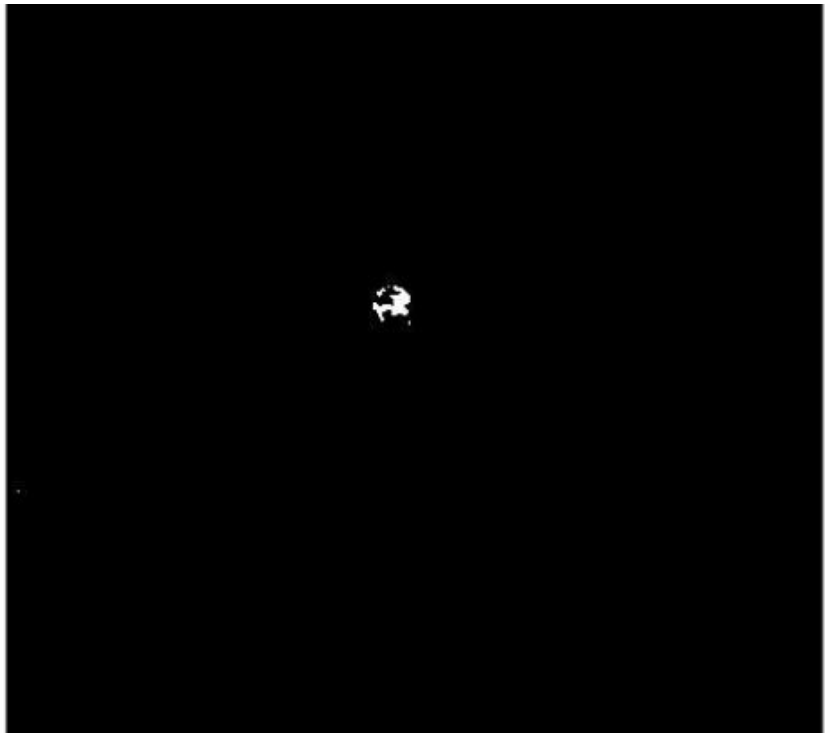
$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\boldsymbol{\Sigma} = \frac{1}{N}(\mathbf{x}-\boldsymbol{\mu})(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}$$ Empirical Co-variance

x is a pixel's RGB value

Sample Output



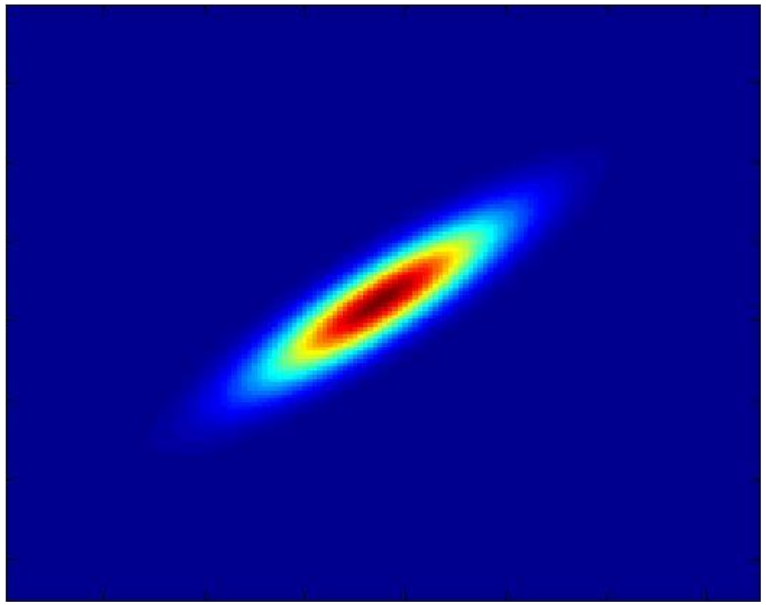


$$p(C_l|x) \geq \tau$$

N-D Gaussian

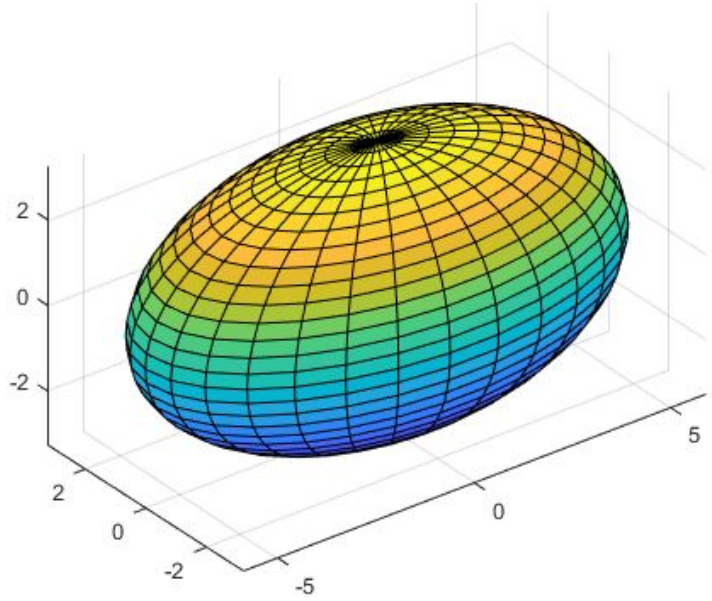
What is the shape of a 2D Gaussian?

Ellipse

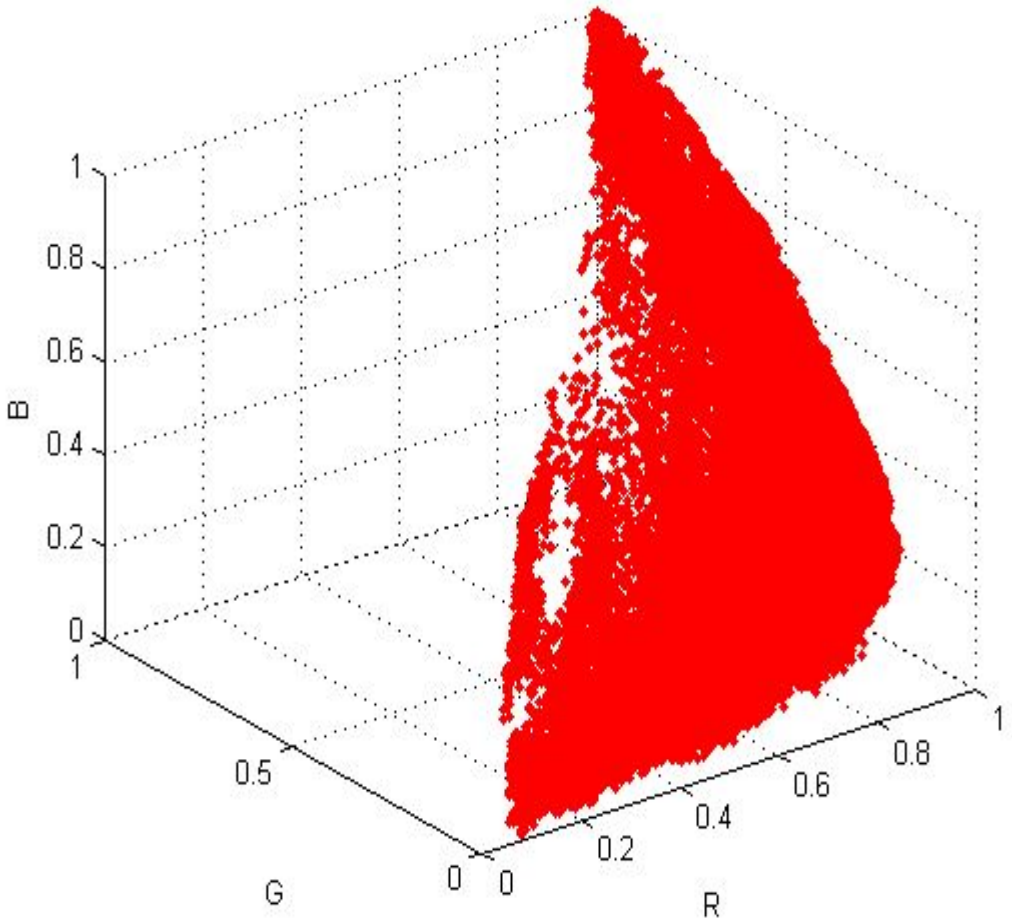


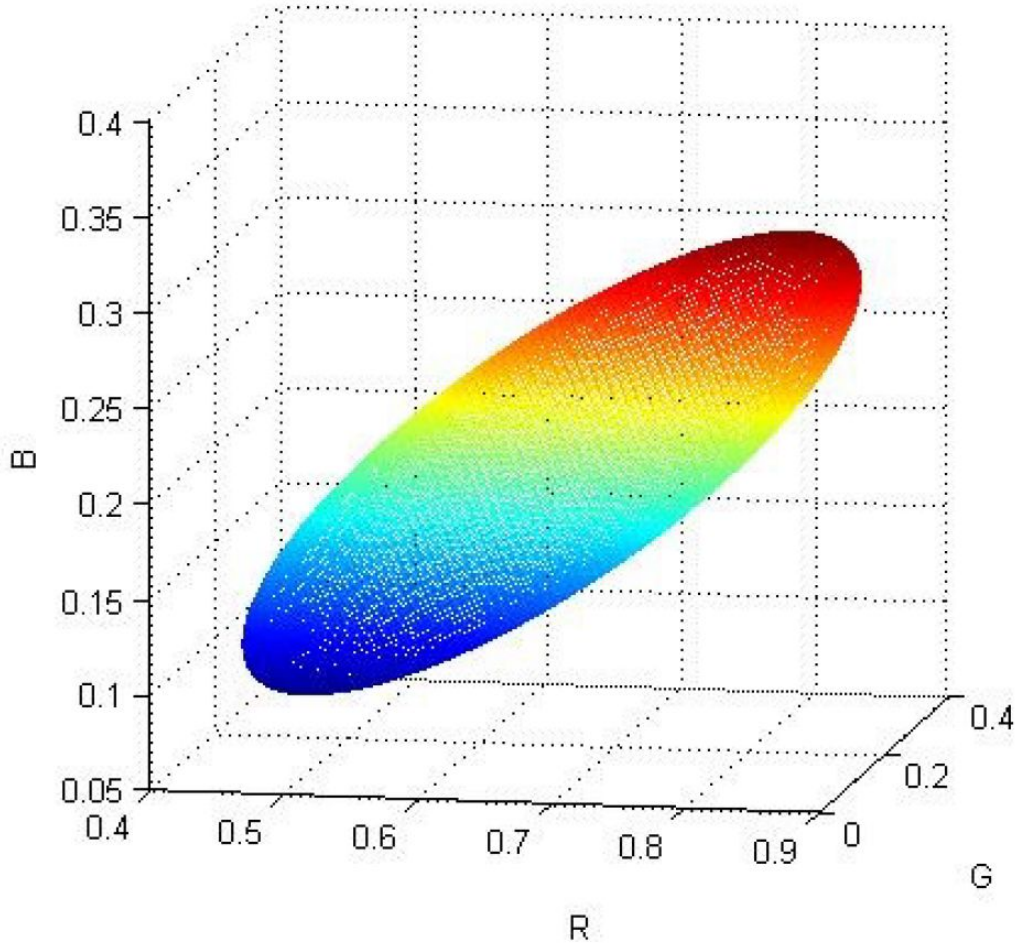
What is the shape of a 3D Gaussian?

Ellipsoid



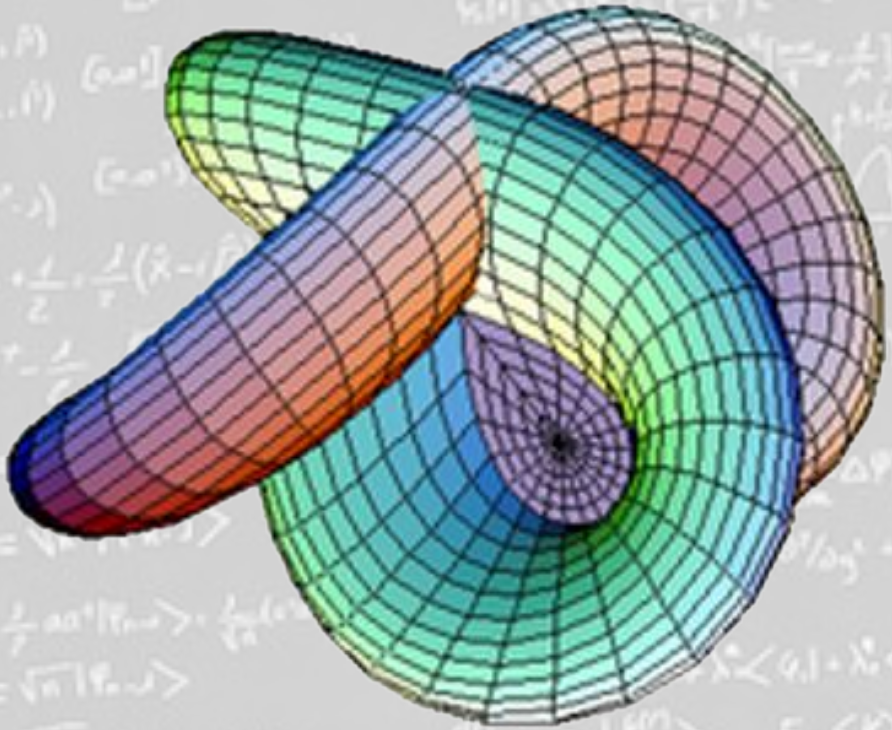
Problems with Single Gaussian





Solution?

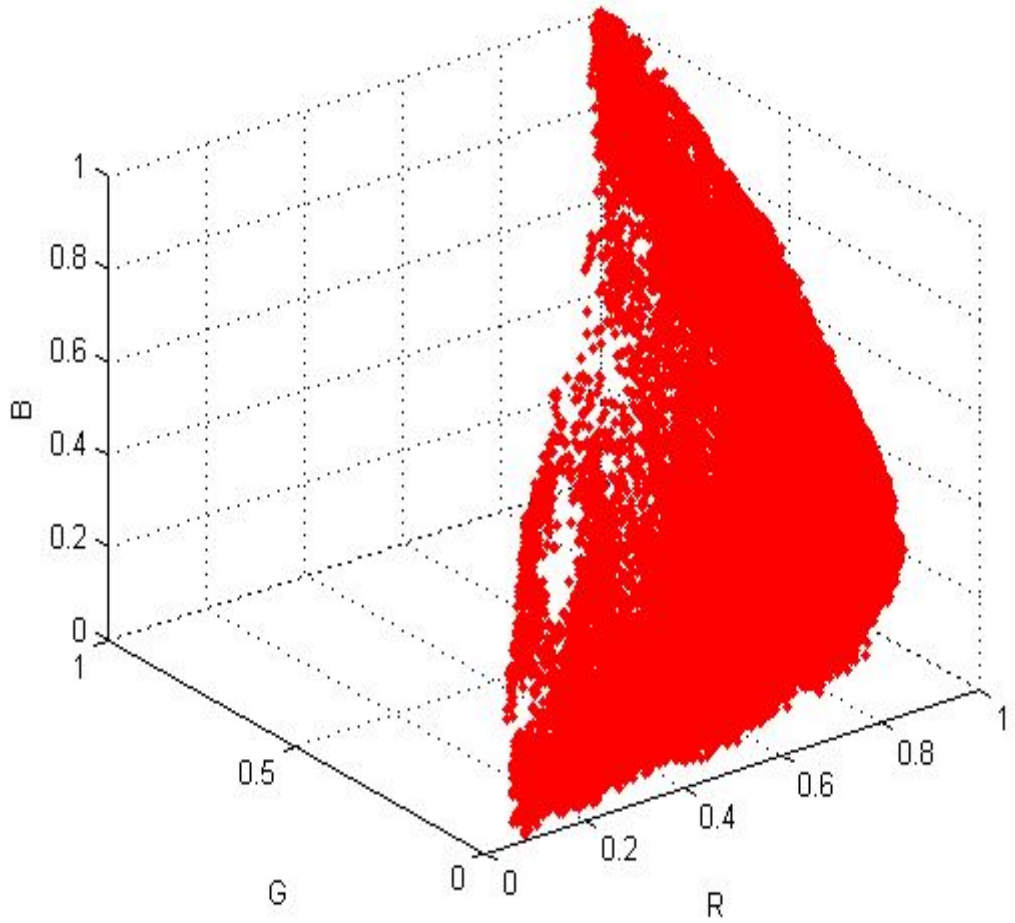
Fancy function/Manifold

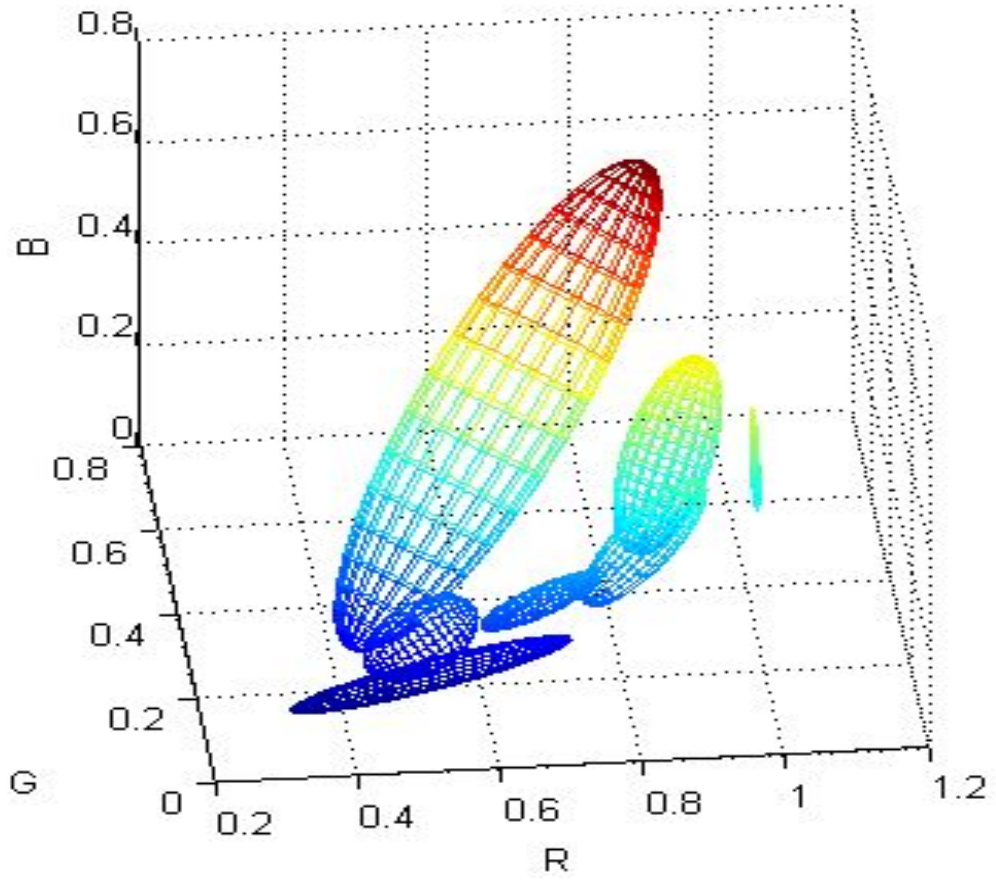


Too Complicated!

Use sum of known function as an efficient and easy approximation!

Gaussian Mixture Models to the rescue!





Math Behind Awesome GMMs

$$p(\mathbf{x}|C_l) \sim \sum_{i=1}^{K} \pi_i \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_i, \Sigma_i)$$

Given $\mathbf{x}$, we want to find $\Theta = [\pi_i, \boldsymbol{\mu}_i, \Sigma_i] \; \forall \; i \in [1, K]$

More formally, we want to maximize the correctness of $p(C_l|\mathbf{x})$

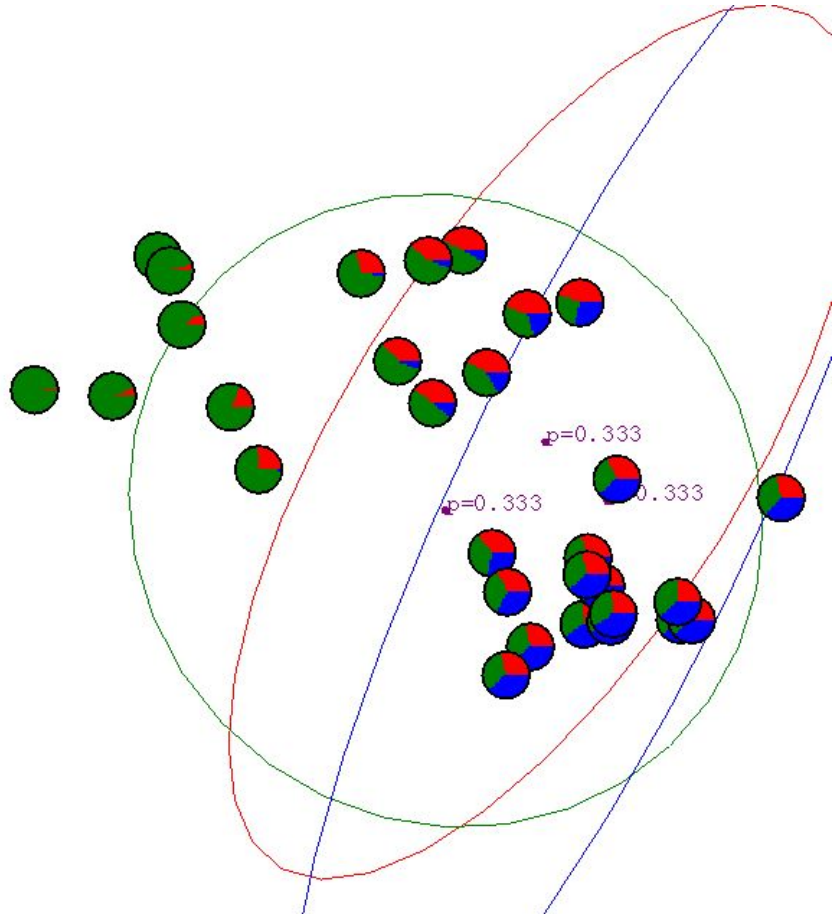
$$\underset{\Theta}{\operatorname{argmax}} \sum_{i=1}^{N} \log \, p(C_l|\mathbf{x})$$

Maximum Log Likelihood

Expectation Maximization for GMM

Initialization:

Randomly choose $\pi_i, \boldsymbol{\mu}_i, \Sigma_i \; \forall \; i \in [1, K]$



E-Step:

Evaluate Model/ Assign points to model

Cluster Weight $\alpha_{i,j} = \frac{\pi_j p(\mathbf{x}_i|C_j)}{\sum_{j=1}^{K} \pi_j p(\mathbf{x}_i|C_j)}$

i is the data point index, j is the cluster index

Expectation Maximization for GMM

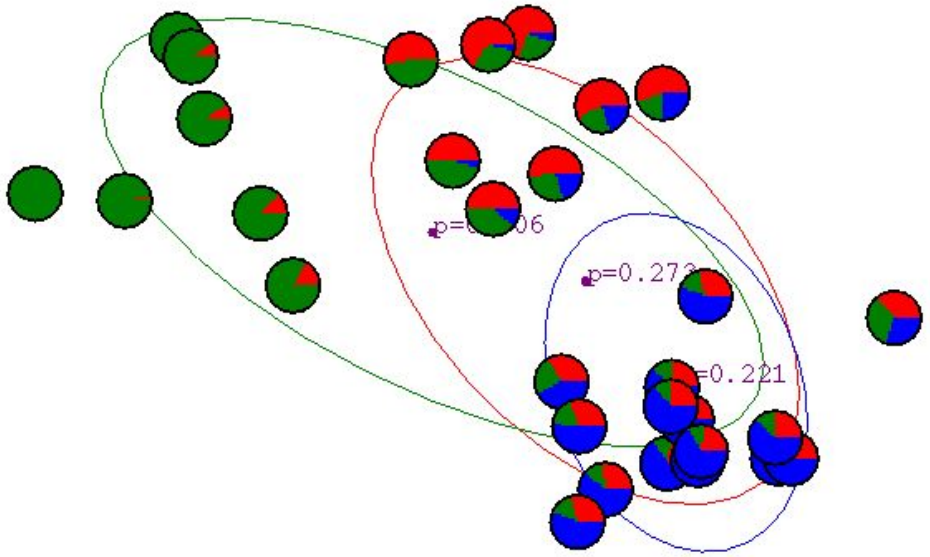
M-Step:

Evaluate best model to fit points

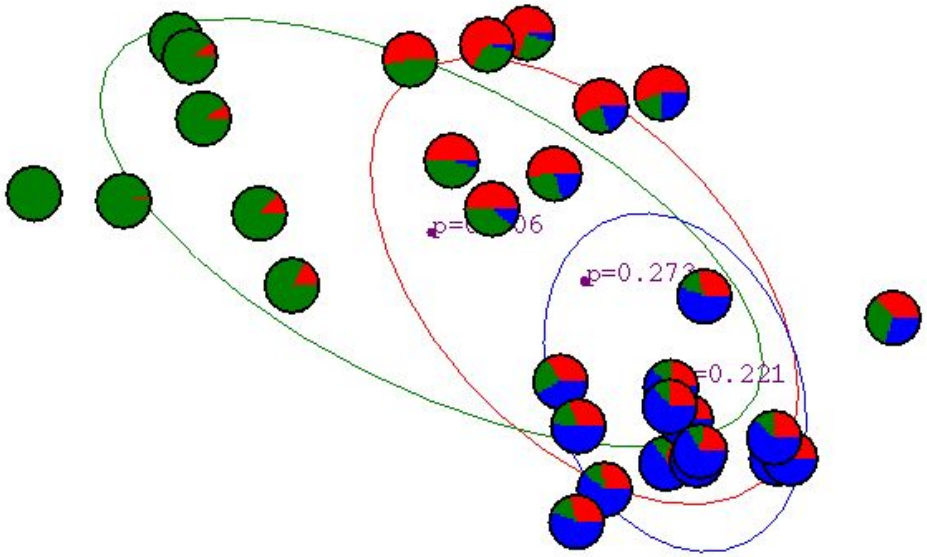
$$\mu_j = \frac{\sum_{i=1}^{N} \alpha_{i,j}\, \mathbf{x}_i}{\sum_{i=1}^{N} \alpha_{i,j}}$$

$$\Sigma_j = \frac{\sum_{i=1}^{N} \alpha_{i,j}\, (\mathbf{x}_i - \boldsymbol{\mu}_j)(\mathbf{x}_i - \boldsymbol{\mu}_j)^{\mathrm{T}}}{\sum_{i=1}^{N} \alpha_{i,j}}$$

$$\pi_j = \frac{1}{N} \sum_{i=1}^{N} \alpha_{i,j}$$



Expectation Maximization for GMM



E-Step:

Evaluate Model/ Assign points to model

Cluster Weight $\alpha_{i,j} = \frac{\pi_j p(\mathbf{x}_i|C_j)}{\sum_{j=1}^{K} \pi_j p(\mathbf{x}_i|C_j)}$

i is the data point index, j is the cluster index

Expectation Maximization for GMM

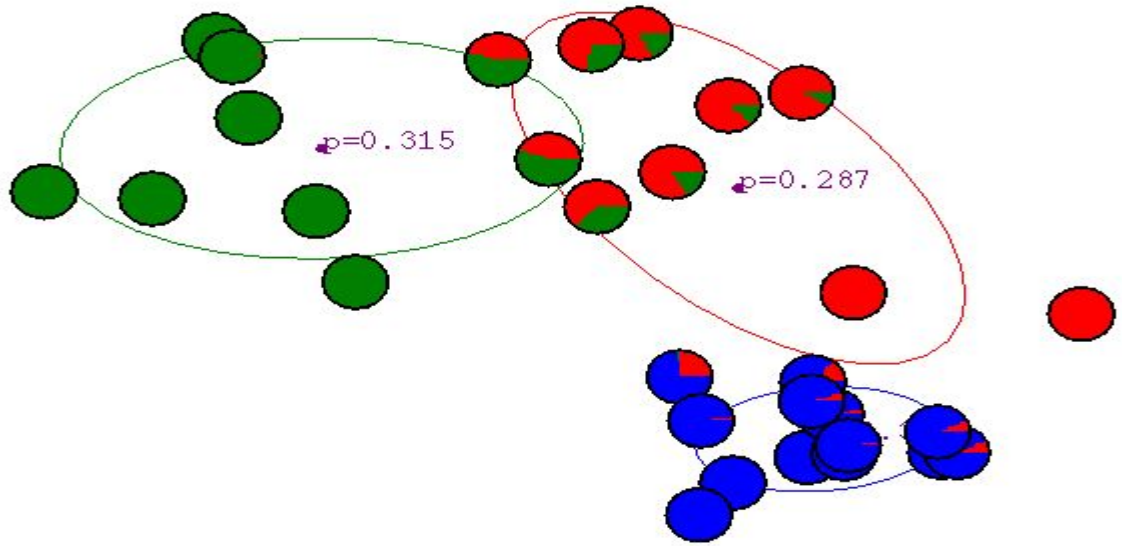
M-Step:

Evaluate best model to fit points

$$\mu_j = \frac{\sum_{i=1}^{N} \alpha_{i,j}\, \mathbf{x}_i}{\sum_{i=1}^{N} \alpha_{i,j}}$$

$$\Sigma_j = \frac{\sum_{i=1}^{N} \alpha_{i,j}\, (\mathbf{x}_i - \boldsymbol{\mu}_j)(\mathbf{x}_i - \boldsymbol{\mu}_j)^{\mathrm{T}}}{\sum_{i=1}^{N} \alpha_{i,j}}$$

$$\pi_j = \frac{1}{N} \sum_{i=1}^{N} \alpha_{i,j}$$

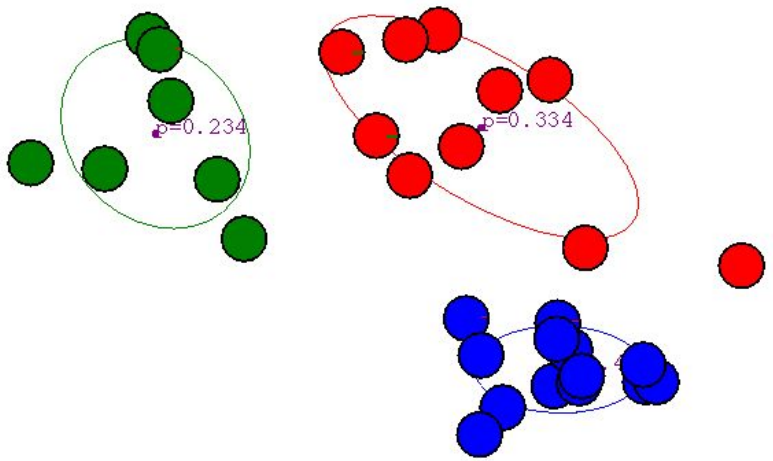


Expectation Maximization for GMM

Repeat E and M steps until convergence

Convergence measure:

$$\|\mu^t - \mu^{t-1}\|$$



Programming tips(Pseudocode)

Algorithm 1 GMM

```
1:  procedure GMM
2:      set τ                                   ▷ threshold for clustering.
3:      set K                                   ▷ number of gaussians
4:      if training then
5:          load train_images
6:          trainGMM(K)
7:      else if  then
8:          load test_images
9:          cluster = testGMM(Model, τ)
10:     end if
11:     d = measureDepth(cluster)
12:     plotGMM(Model)                          ▷ visualize GMM ellipsoids
13:     return cluster, d                       ▷ cluster and cluster depth
14:
15: end procedure
```

Training GMM (Only for Orange Pixels)

Randomly choose $\pi_i, \boldsymbol{\mu}_i, \Sigma_i \; \forall \; i \in [1, K]$

Evaluate Model/ Assign points to model

Cluster Weight $\alpha_{i,j} = \frac{\pi_j p(\mathbf{x}_i|C_j)}{\sum_{j=1}^{K} \pi_j p(\mathbf{x}_i|C_j)}$

i is the data point index, j is the cluster index

Algorithm 2 trainGMM

```
1: procedure TRAINGMM(K)
2:     set ϵ                                   ▷ convergence criteria
3:     Initialize Model        ▷ initialize scaling factor, gaussian mean and covariance
4:     while i ≤ max_iters and abs(Mean - prevMean) > ϵ do
5:         expectation_step
6:         maximization_step
7:         increment i;
8:     end while
9:     save Model              ▷ save scaling factor, gaussian mean and covariance
10: end procedure
```

$$\mu_j = \frac{\sum_{i=1}^{N} \alpha_{i,j} \mathbf{x}_i}{\sum_{i=1}^{N} \alpha_{i,j}}$$

$$\Sigma_j = \frac{\sum_{i=1}^{N} \alpha_{i,j} (\mathbf{x}_i - \boldsymbol{\mu}_j)(\mathbf{x}_i - \boldsymbol{\mu}_j)^{\mathrm{T}}}{\sum_{i=1}^{N} \alpha_{i,j}}$$

$$\pi_j = \frac{1}{N} \sum_{i=1}^{N} \alpha_{i,j}$$

Algorithm 3 testGMM

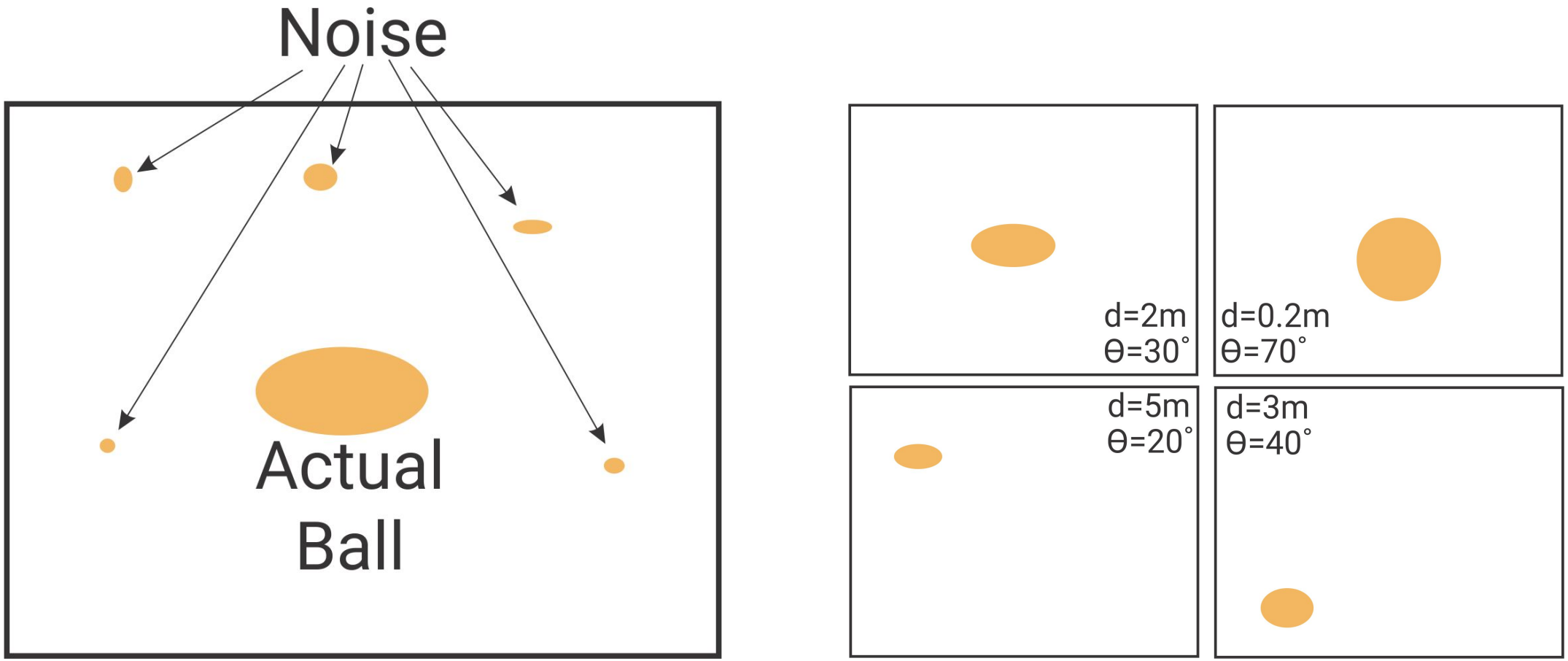
```
1: procedure TESTGMM(Model, τ)
2:     load Model                          ▷ load scaling factor, gaussian mean and covariance
3:     computePosterior(Model)
4:     getCluster(τ)                       ▷ use thresholding to get the orange ball
5: end procedure
```

$$p(C_l|\mathbf{x}) \propto p(\mathbf{x}|C_l)p(C_l)$$
$$p(C_l) = \frac{1}{L}$$

$$\forall \mathbf{x} \quad p(\mathbf{x}|C_l) = \sum_{i=1}^{K} \pi_i \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_i, \Sigma_i)$$

x is a pixel's RGB value

Estimating Distance To The Ball



`bwmorph, regionprops, fit`

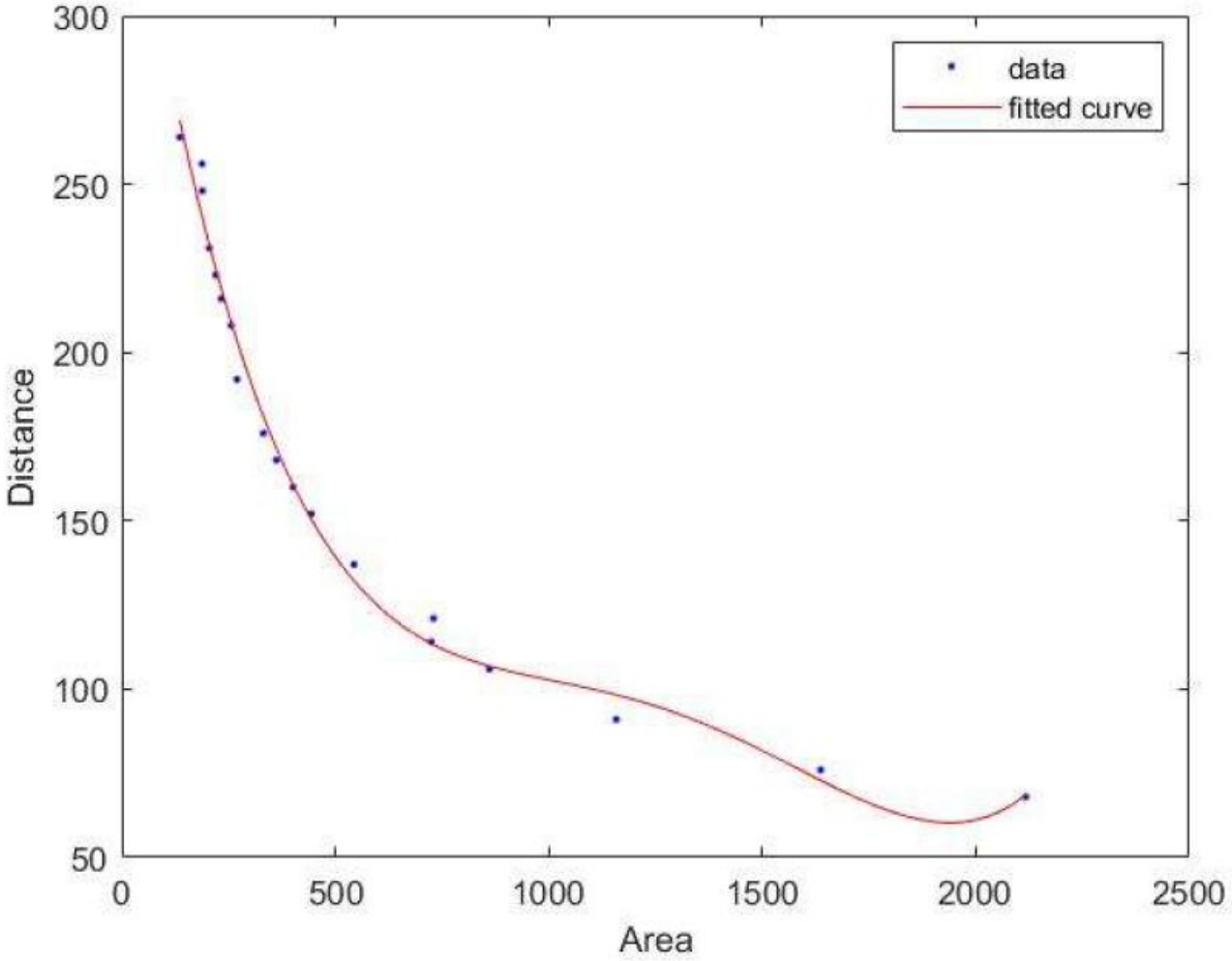


Figure: Distance vs Area Curve

Train and Test Set

- Few images released with project assignment: Train set.
  Use these images to train and validate your algorithm.

- Since the testing data hasn't been released to you yet, you should do a 80/20 split of your training data itself, and leave the 20% for testing/validating. It's not a good idea to use the same data for training and testing, since that generally leads to overfitting.

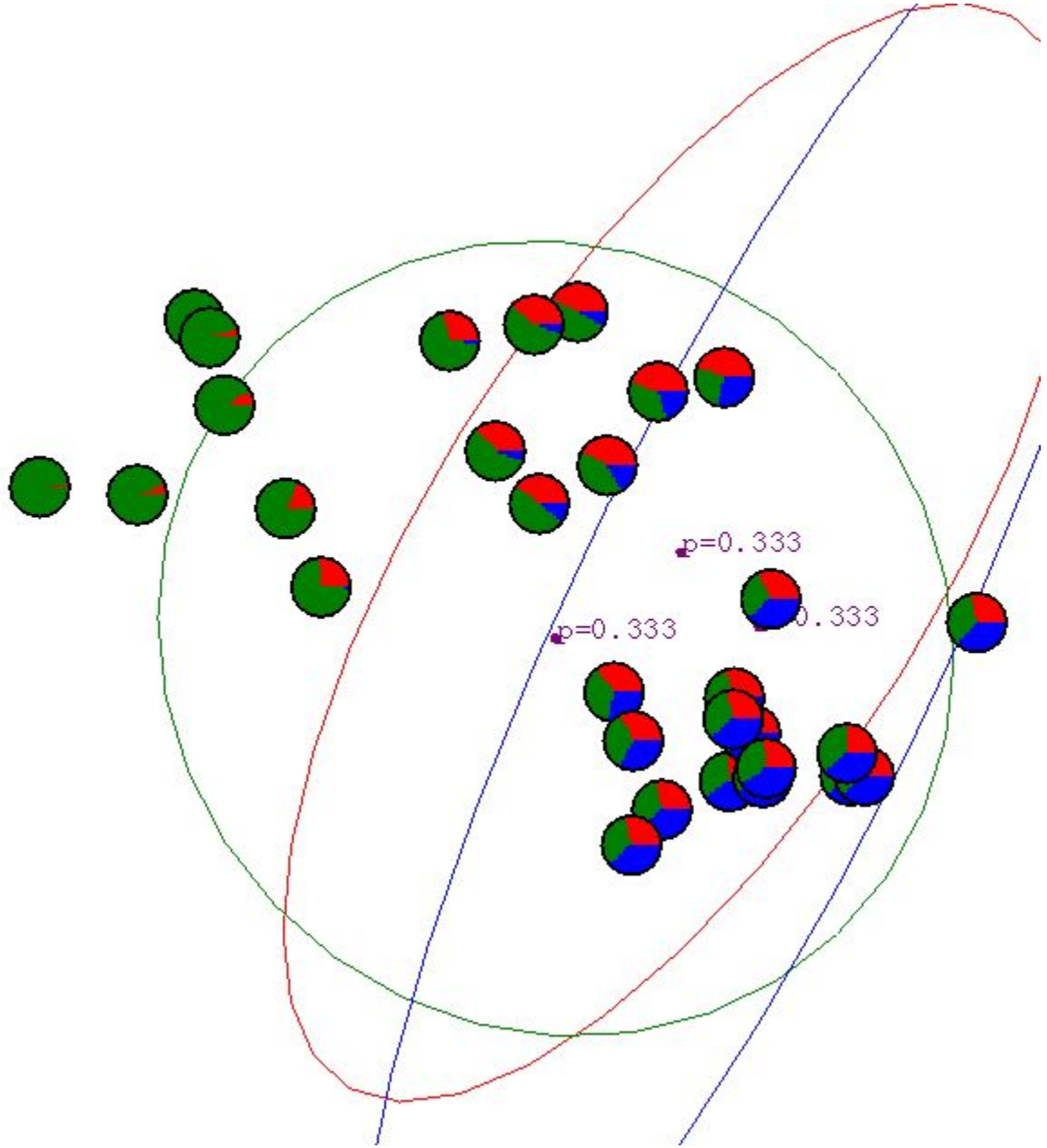
- 24hrs before the deadline, new images will be released.
  Only predict on these images, **DO NOT** train/ fit model on these images!

Improving Robustness

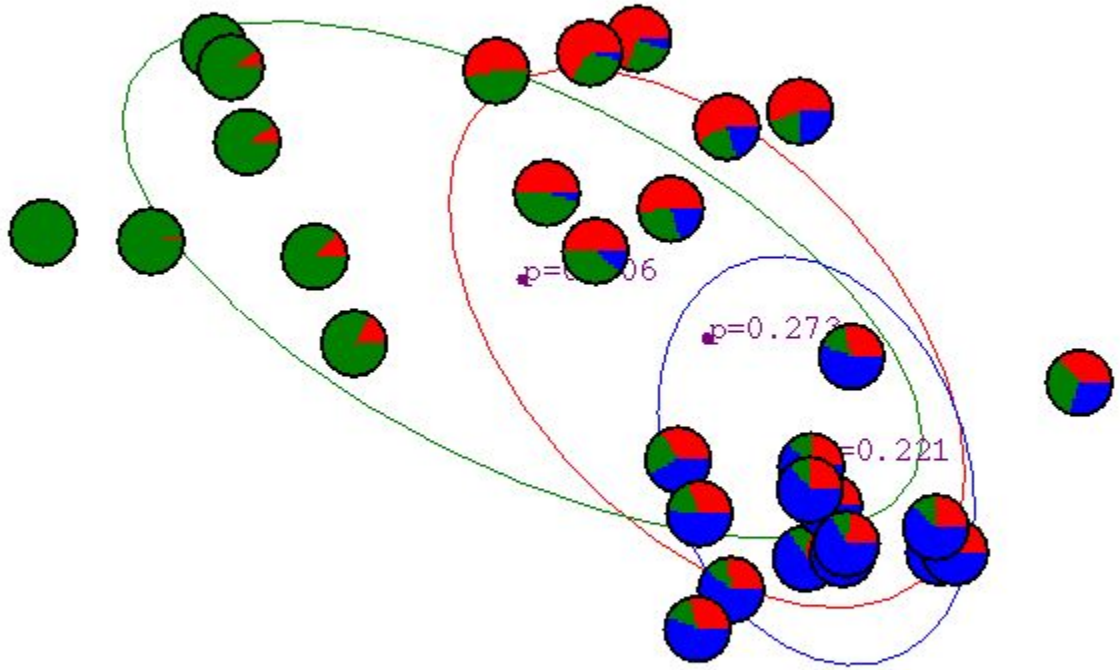
- Think of alternative colorspaces and custom colorspaces
- Hough circles to find circular blobs
- Think of methods to combine stray pixels, i.e., ball split into smaller parts/blobs
- Augment Training set to account for noise and illumination changes
- Using 3D geometry to estimate distance

Good Luck!

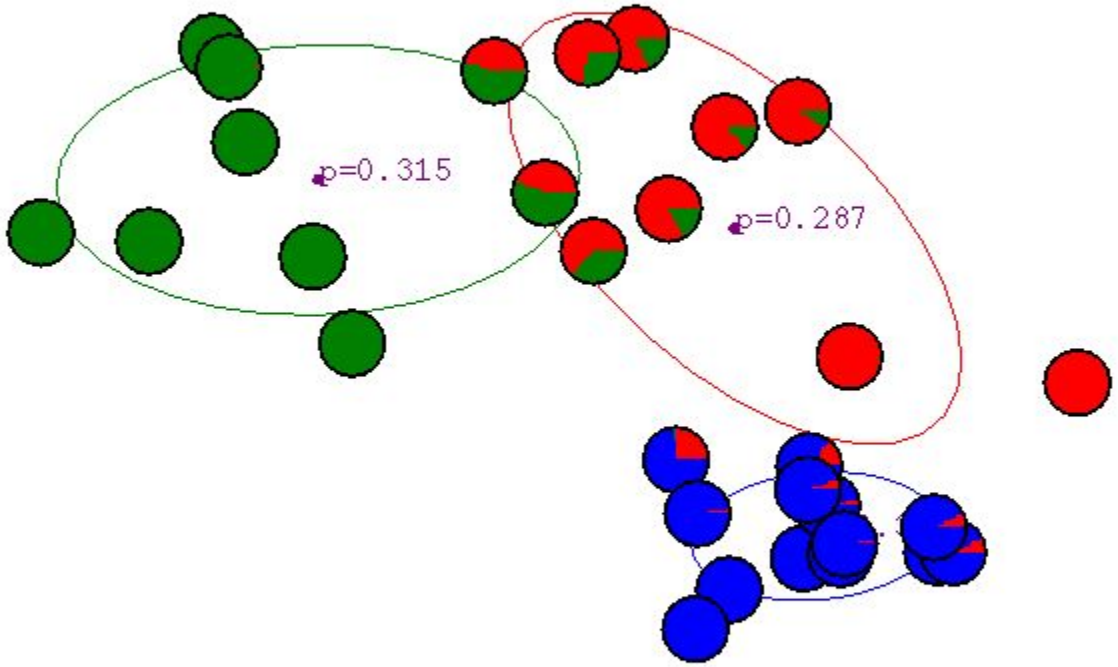
GMM Intuition



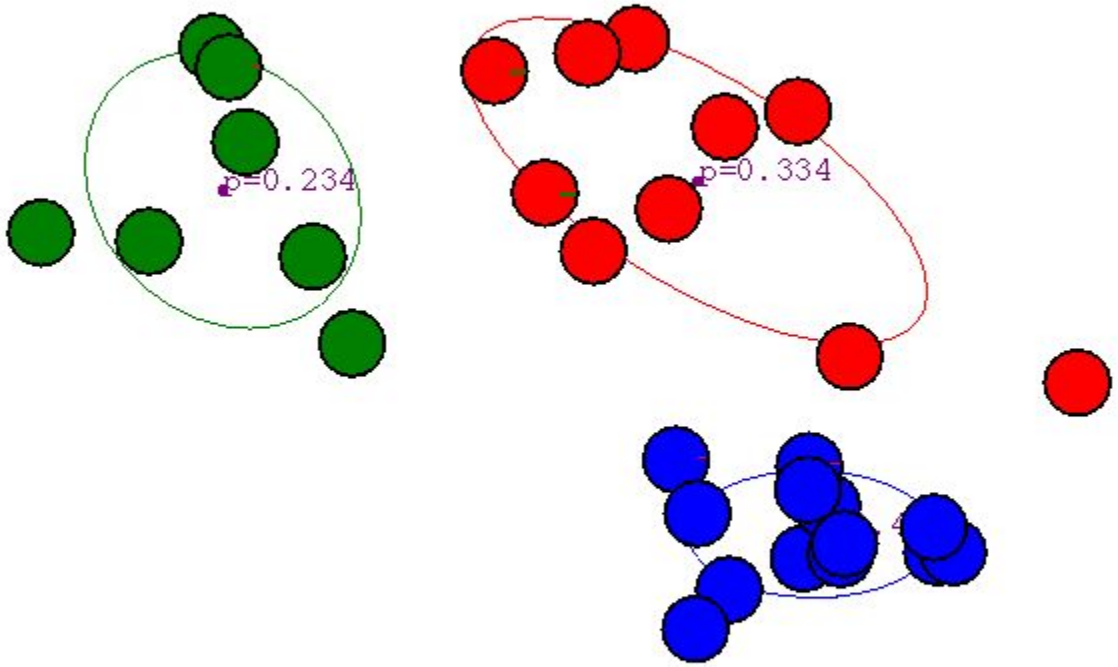
GMM Intuition



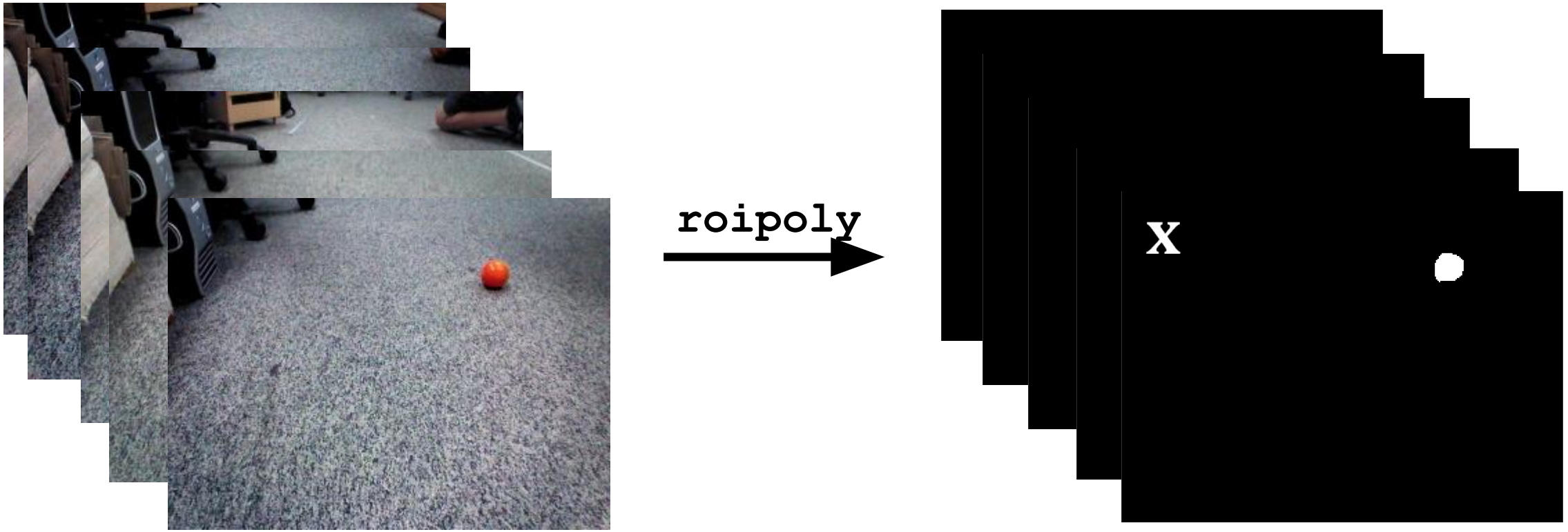
GMM Intuition



GMM Intuition



Color Segmentation using GMM



Color Segmentation using GMM

For all training images:

$p(C_l|\mathbf{x}) \propto p(\mathbf{x}|C_l)p(C_l)$ Posterior

$p(C_l) = \frac{1}{L}$ Prior

$p(\mathbf{x}|C_l) = \sum_{i=1}^{K} \pi_i \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_i, \Sigma_i)$ Likelihood

$\Theta = [\pi_i, \boldsymbol{\mu}_i, \Sigma_i] \; \forall \; i \in [1, K]$ Estimated using EM

For any/each test image:

$\forall \mathbf{x} \quad p(\mathbf{x}|C_l) = \sum_{i=1}^{K} \pi_i \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_i, \Sigma_i)$ Predict using GMM

$\mathbf{x}$ is a pixel's RGB value