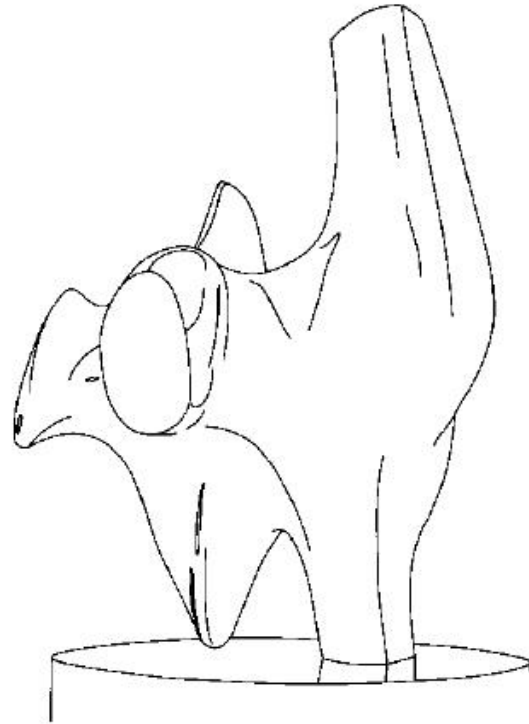
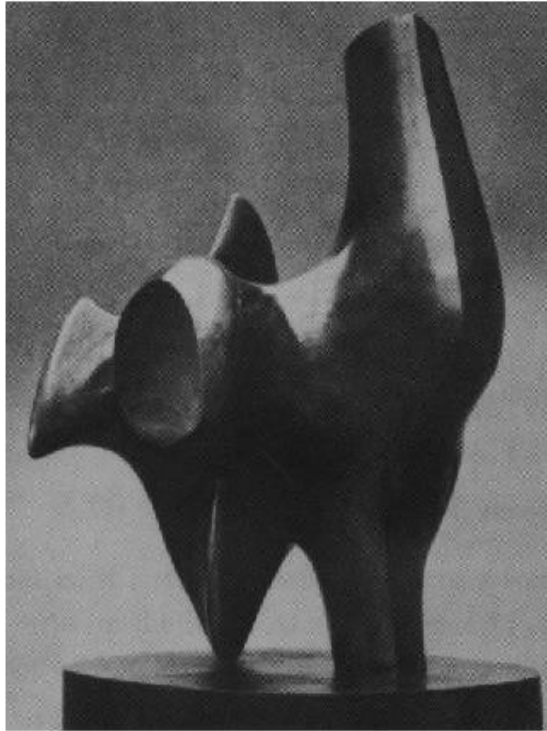


# Edge Detection

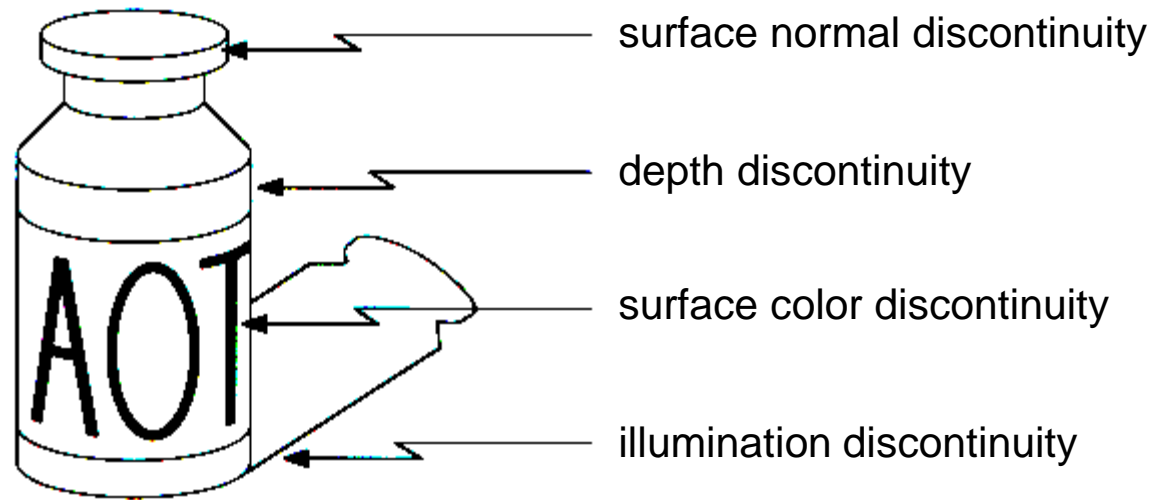
# Edge detection



Convert a 2D image into a set of curves

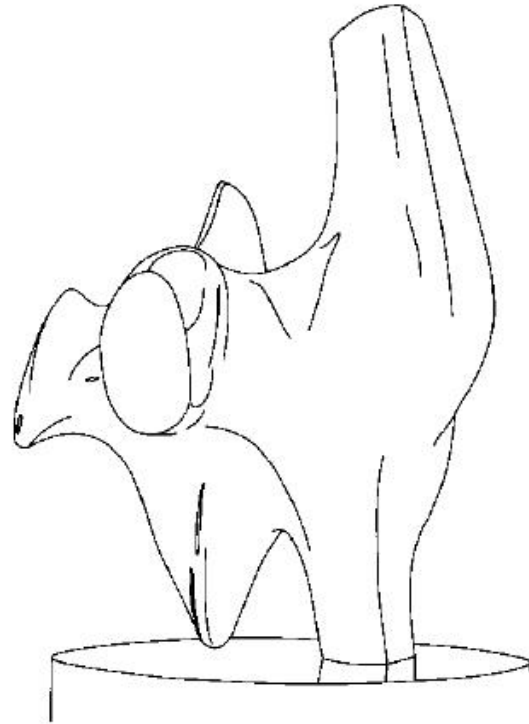
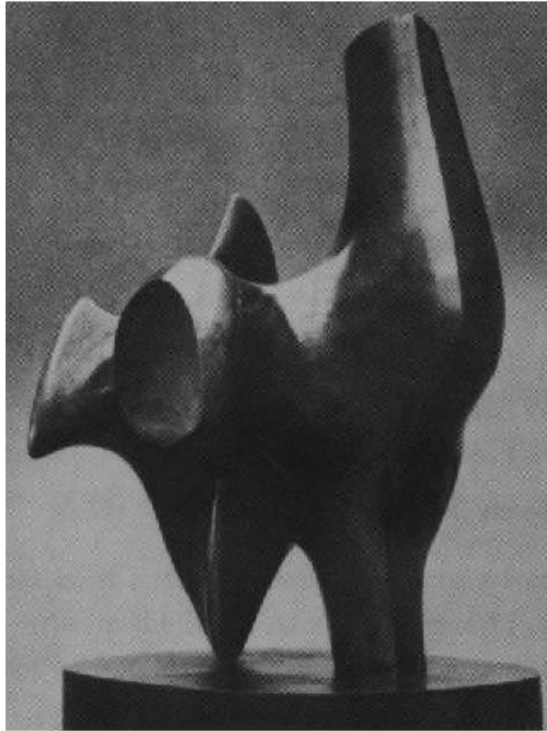
- Extracts salient features of the scene
- More compact than pixels

# Origin of Edges



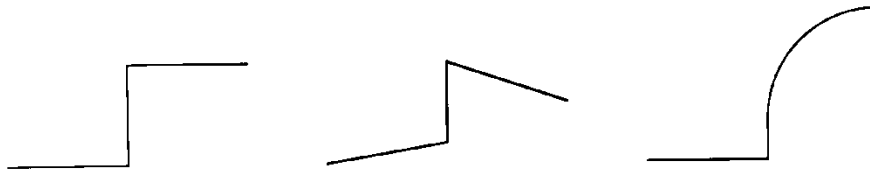
Edges are caused by a variety of factors

# Edge detection



How can you tell that a pixel is on an edge?

# Profiles of image intensity edges



Step Edges



Roof Edge

Line Edges



# Edge detection

1. Detection of short linear edge segments (edgels)
2. Aggregation of edgels into extended edges  
(maybe parametric description)

# Edgel detection

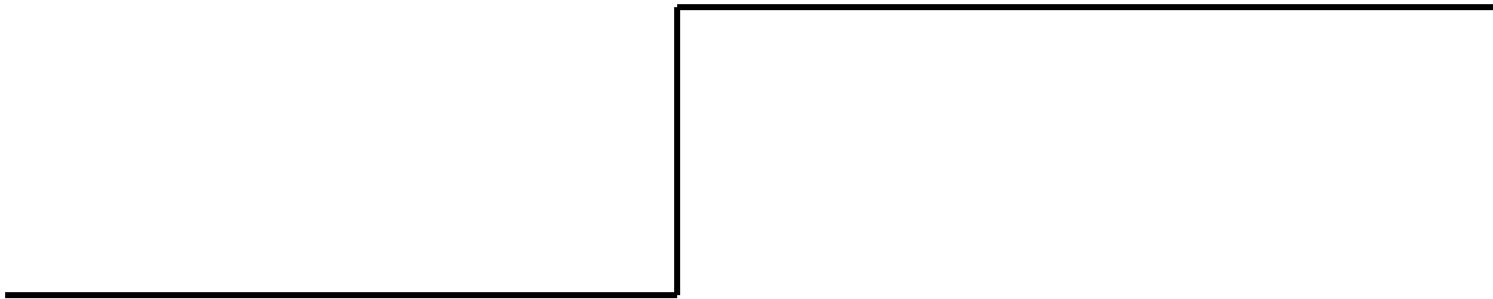
- Difference operators
- Parametric-model matchers

# Edge is Where Change Occurs

Change is measured by derivative in 1D

Biggest change, derivative has maximum magnitude

Or 2<sup>nd</sup> derivative is zero.



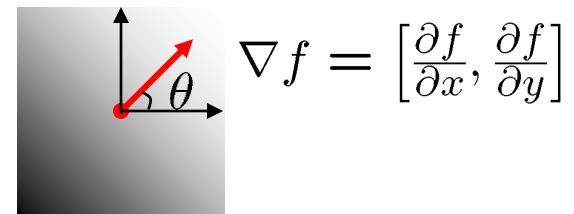
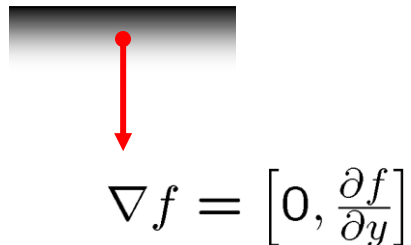
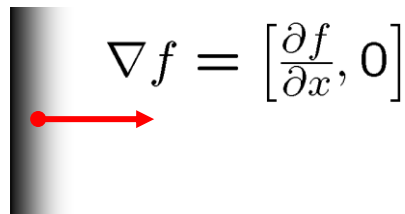


# Image gradient

The gradient of an image:

$$\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

The gradient points in the direction of most rapid change in intensity



The gradient direction is given by:

$$\theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

- how does this relate to the direction of the edge?

The *edge strength* is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

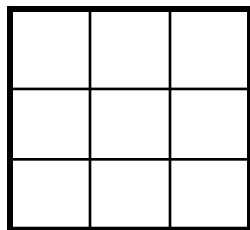
# The discrete gradient

How can we differentiate a *digital* image  $f[x,y]$ ?

- Option 1: reconstruct a continuous image, then take gradient
- Option 2: take discrete derivative (finite difference)

$$\frac{\partial f}{\partial x}[x, y] \approx f[x + 1, y] - f[x, y]$$

How would you implement this as a cross-correlation?



$H$

# The Sobel operator

Better approximations of the derivatives exist

- The *Sobel* operators below are very commonly used

$$\frac{1}{8} \begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$$

$s_x$

$$\frac{1}{8} \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

$s_y$

- The standard defn. of the Sobel operator omits the  $1/8$  term
  - doesn't make a difference for edge detection
  - the  $1/8$  term **is** needed to get the right gradient value, however

# Gradient operators

 $\Delta_1$ 
 $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ 
 $\Delta_2$ 
 $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ 

(a)

 $\Delta_1$ 
 $\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ 
 $\Delta_2$ 
 $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$ 

(b)

 $\Delta_1$ 
 $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ 
 $\Delta_2$ 
 $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$ 

(c)

 $\Delta_1$ 
 $\begin{bmatrix} -3 & -1 & 1 & 3 \\ -3 & -1 & 1 & 3 \\ -3 & -1 & 1 & 3 \\ -3 & -1 & 1 & 3 \end{bmatrix}$ 
 $\Delta_2$ 
 $\begin{bmatrix} 3 & 3 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ -3 & -3 & -3 & -3 \end{bmatrix}$ 

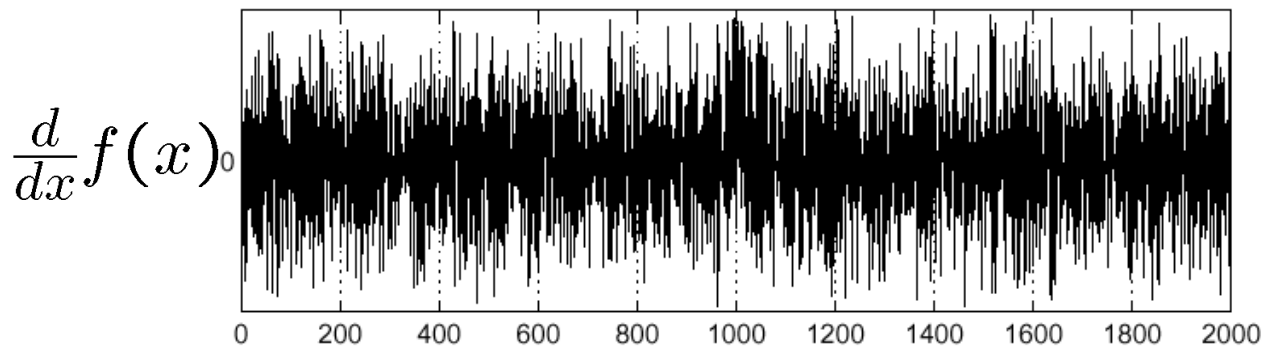
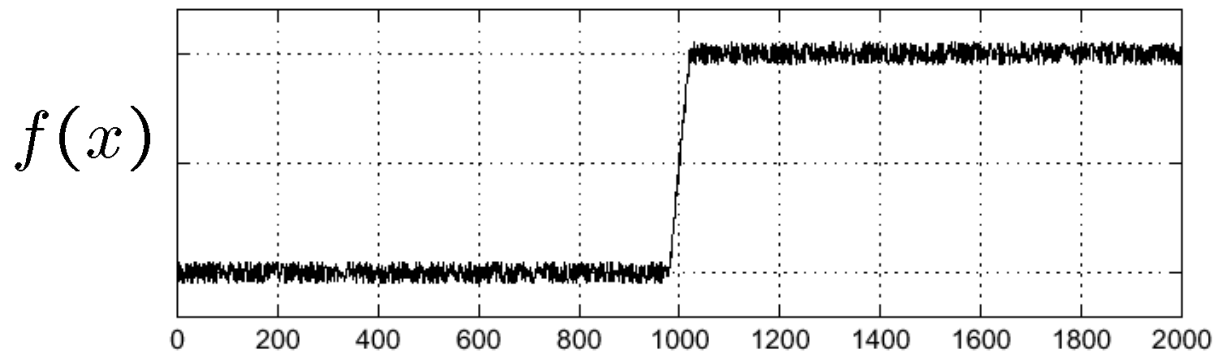
(d)

(a): Roberts' cross operator (b): 3x3 Prewitt operator  
(c): Sobel operator (d) 4x4 Prewitt operator

# Effects of noise

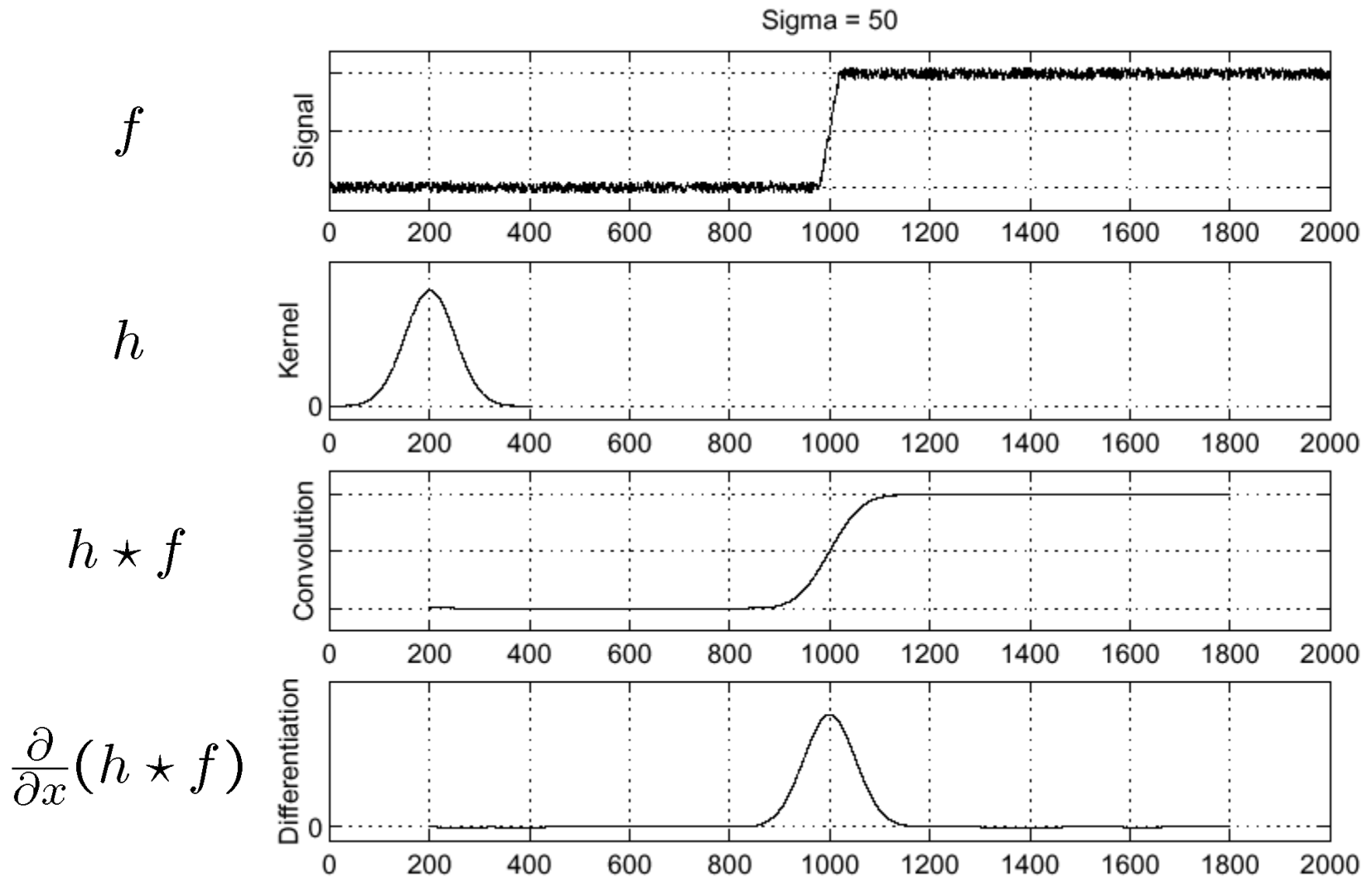
Consider a single row or column of the image

- Plotting intensity as a function of position gives a signal



Where is the edge?

# Solution: smooth first

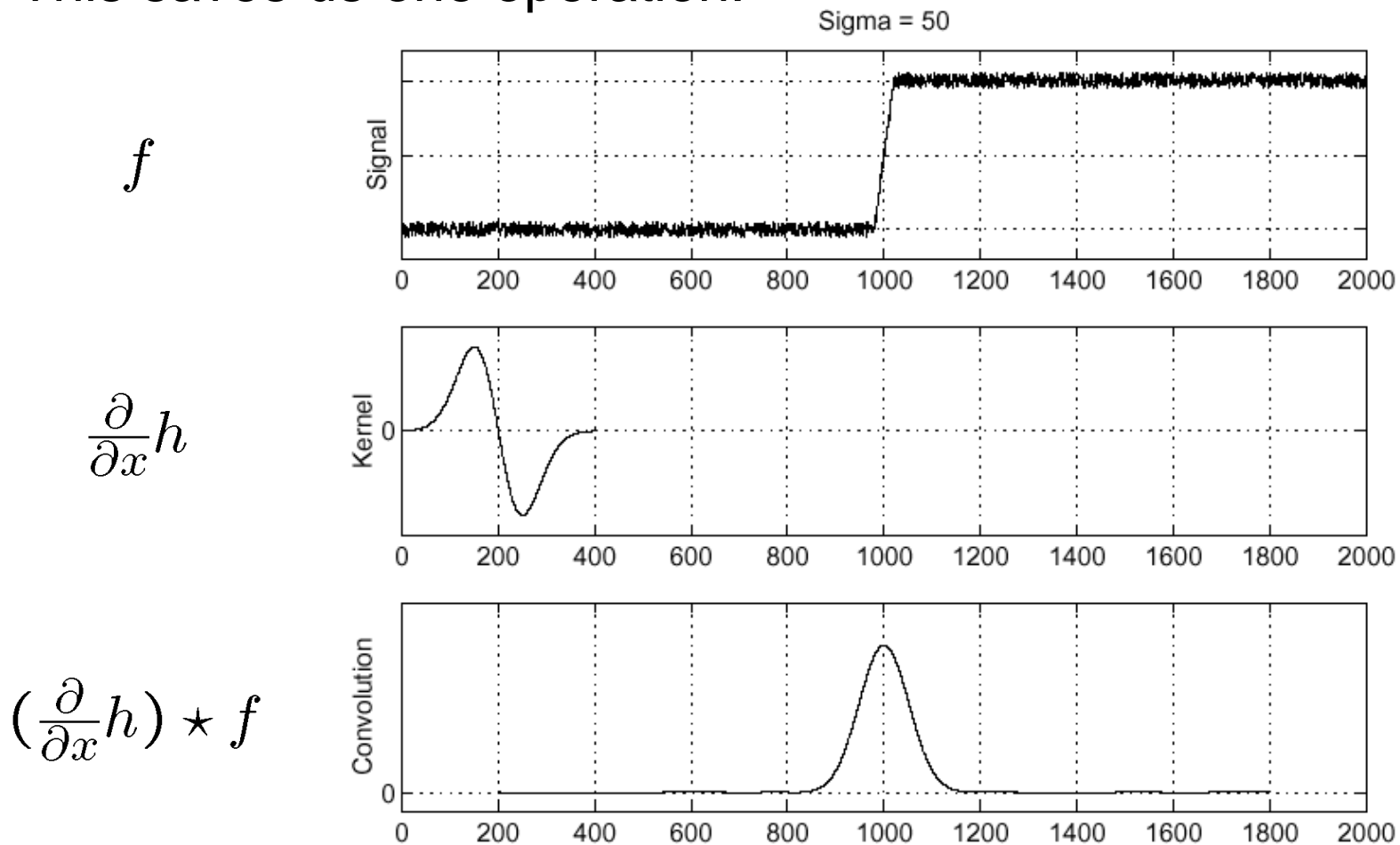


Where is the edge? Look for peaks in  $\frac{\partial}{\partial x}(h \star f)$

# Derivative theorem of convolution

$$\frac{\partial}{\partial x}(h \star f) = \left(\frac{\partial}{\partial x}h\right) \star f$$

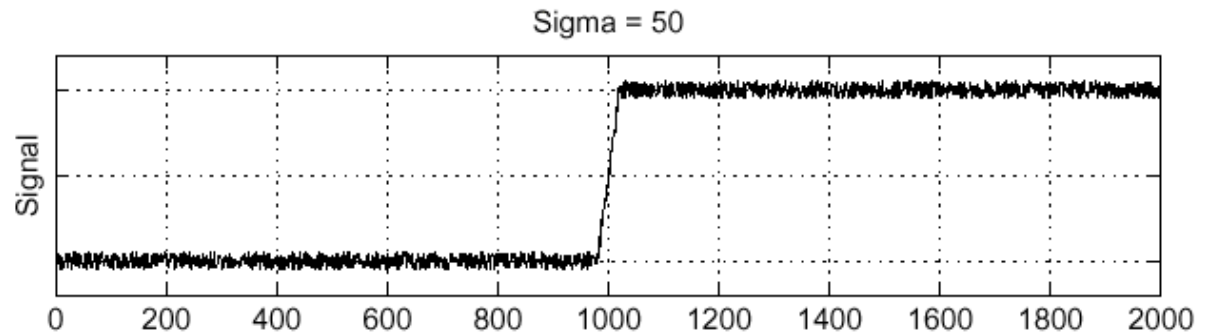
This saves us one operation:



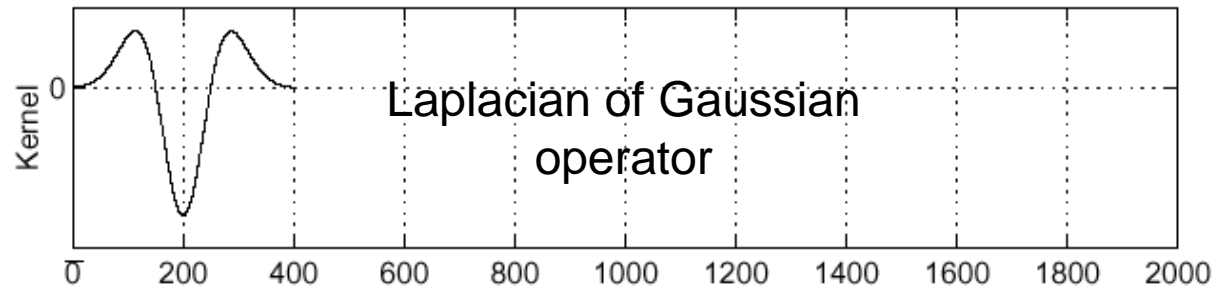
# Laplacian of Gaussian

Consider  $\frac{\partial^2}{\partial x^2}(h \star f)$

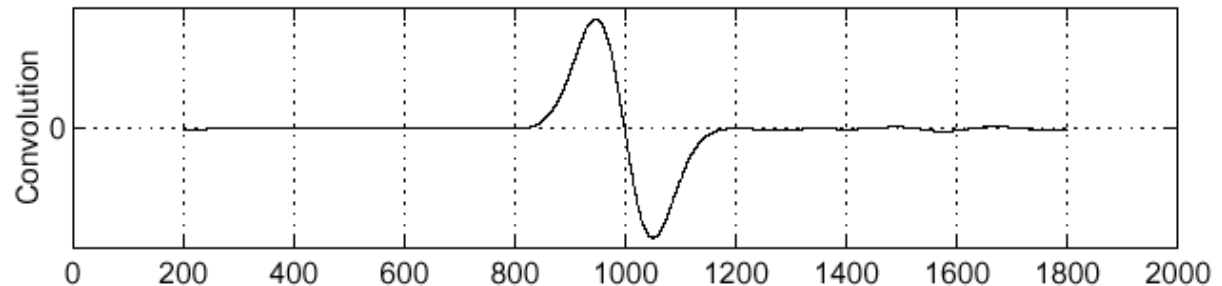
$f$



$\frac{\partial^2}{\partial x^2}h$



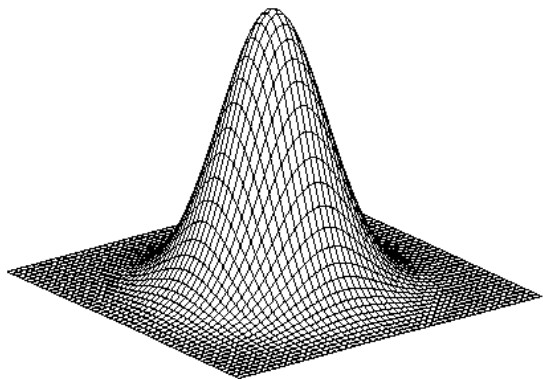
$(\frac{\partial^2}{\partial x^2}h) \star f$



Where is the edge?      Zero-crossings of bottom graph

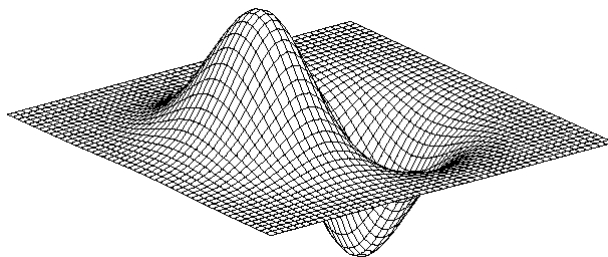


# 2D edge detection filters



Gaussian

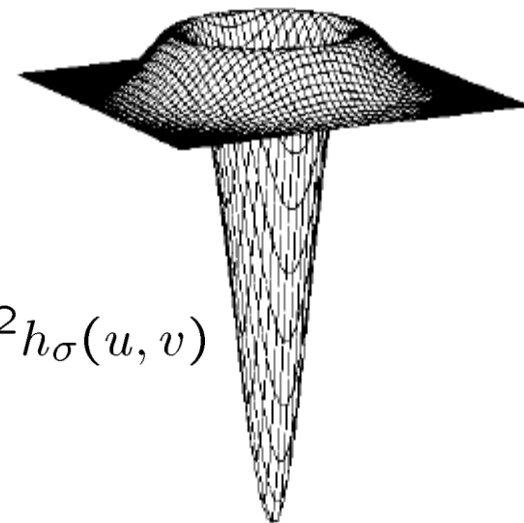
$$h_{\sigma}(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_{\sigma}(u, v)$$

Laplacian of Gaussian



$$\nabla^2 h_{\sigma}(u, v)$$

$\nabla^2$  is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

# Optimal Edge Detection: Canny

Assume:

- Linear filtering
- Additive iid Gaussian noise

Edge detector should have:

- Good Detection. Filter responds to edge, not noise.
- Good Localization: detected edge near true edge.
- Single Response: one per edge.

# Optimal Edge Detection: Canny (continued)

Optimal Detector is approximately Derivative of Gaussian.

Detection/Localization trade-off

- More smoothing improves detection
- And hurts localization.

This is what you might guess from (detect change) + (remove noise)

# The Canny edge detector



original image (Lena)

# The Canny edge detector



norm of the gradient

# The Canny edge detector



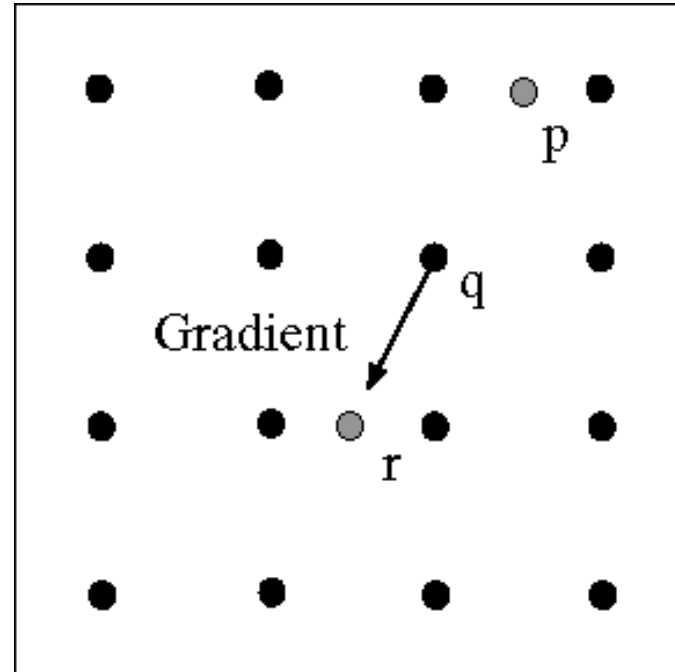
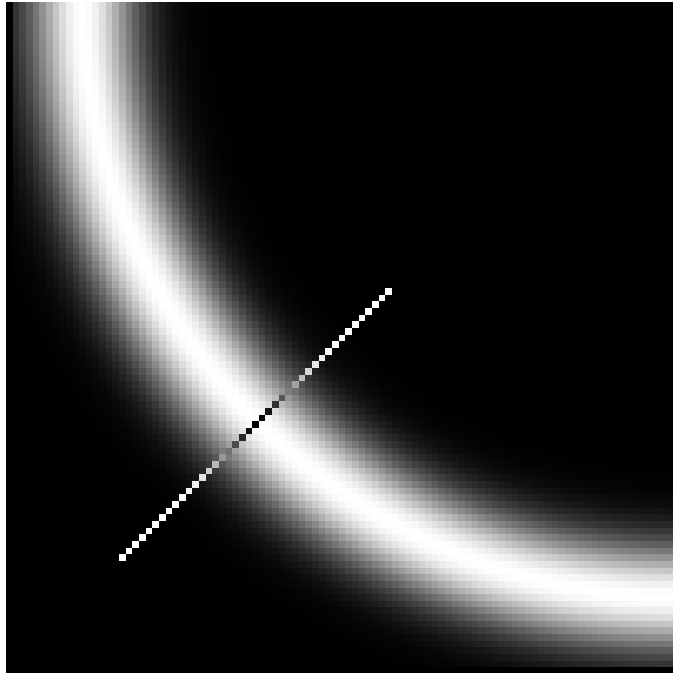
thresholding

# The Canny edge detector



thinning  
(non-maximum suppression)

# Non-maximum suppression



Check if pixel is local maximum along gradient direction

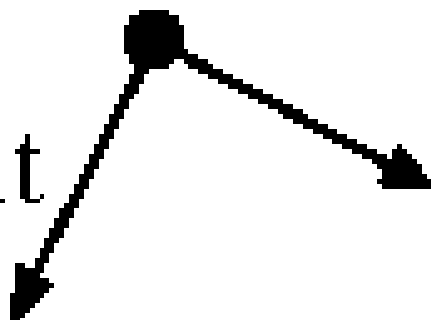
- requires checking interpolated pixels  $p$  and  $r$



Predicting  
the next  
edge point

Assume the  
marked point is an  
edge point. Then  
we construct the  
tangent to the edge  
curve (which is  
normal to the  
gradient at that  
point) and use this  
to predict the next  
points (here either  
r or s).

Gradient



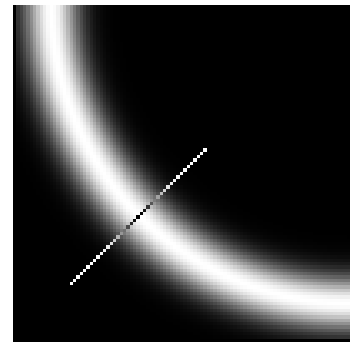
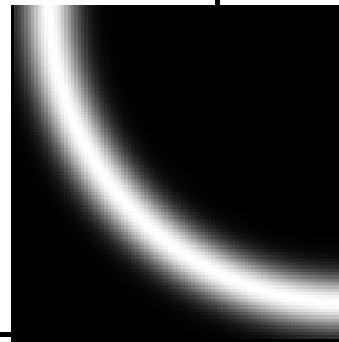
r



s



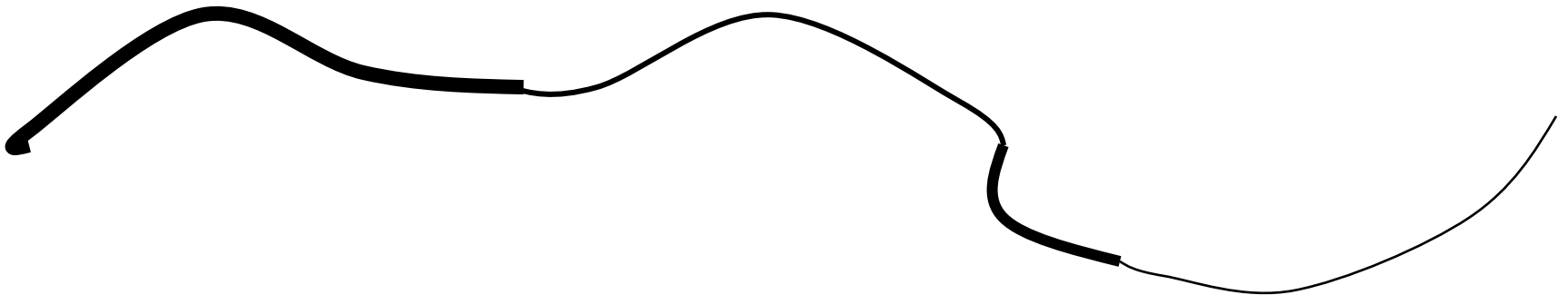
(Forsyth & Ponce)



# Hysteresis

Check that maximum value of gradient value is sufficiently large

- drop-outs? use **hysteresis**
  - use a high threshold to start edge curves and a low threshold to continue them.



# Effect of $\sigma$ (Gaussian kernel size)



original



Canny with  $\sigma = 1$



Canny with  $\sigma = 2$

The choice of  $\sigma$  depends on desired behavior

- large  $\sigma$  detects large scale edges
- small  $\sigma$  detects fine features



# Scale

Smoothing

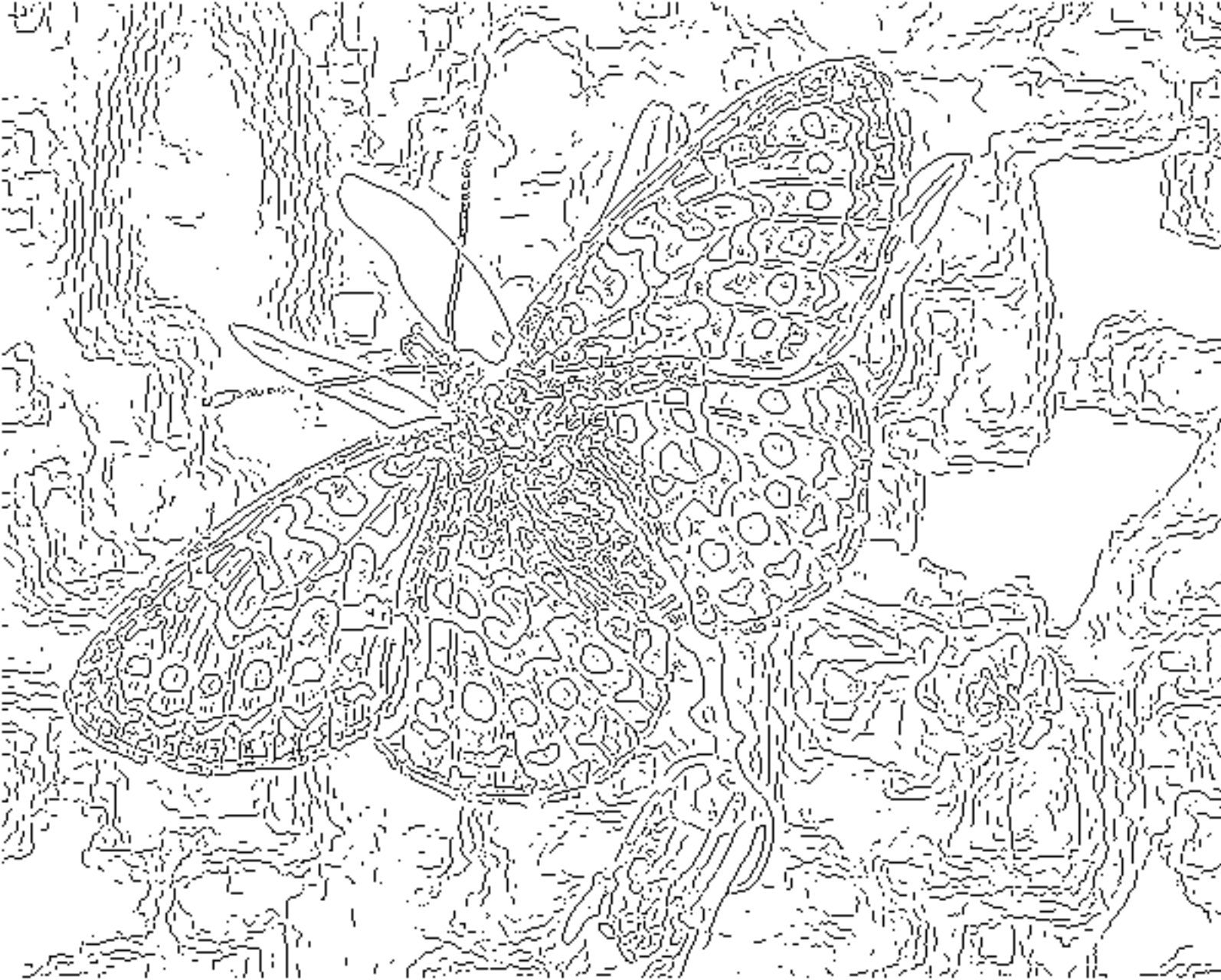
Eliminates noise edges.

Makes edges smoother.

Removes fine detail.

(Forsyth & Ponce)





fine scale  
high  
threshold



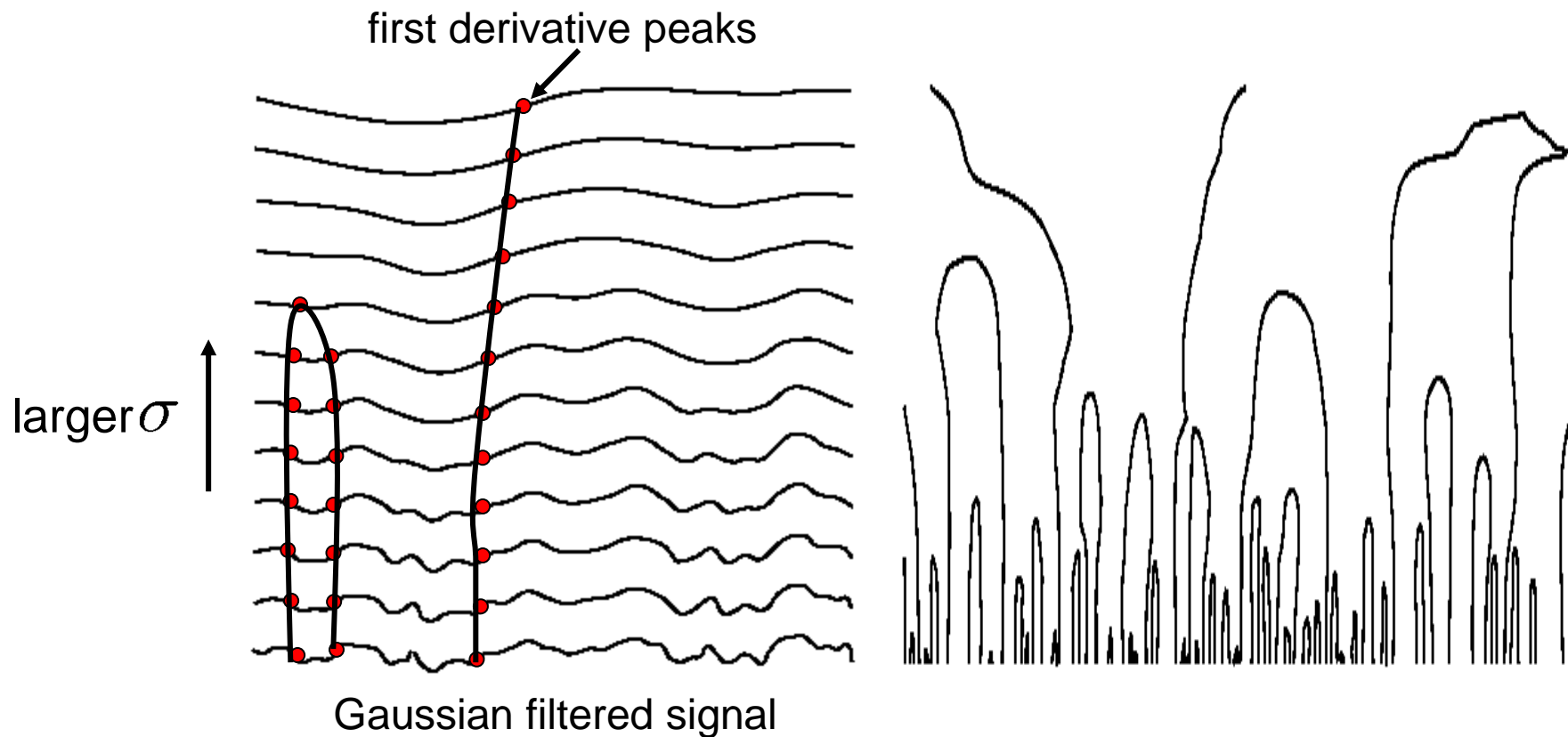
coarse  
scale,  
high  
threshold



coarse  
scale  
low  
threshold



# Scale space (Witkin 83)



## Properties of scale space (w/ Gaussian smoothing)

- edge position may shift with increasing scale ( $\sigma$ )
- two edges may merge with increasing scale
- an edge may **not** split into two with increasing scale

# Edge detection by subtraction



original

# Edge detection by subtraction



smoothed (5x5 Gaussian)

# Edge detection by subtraction

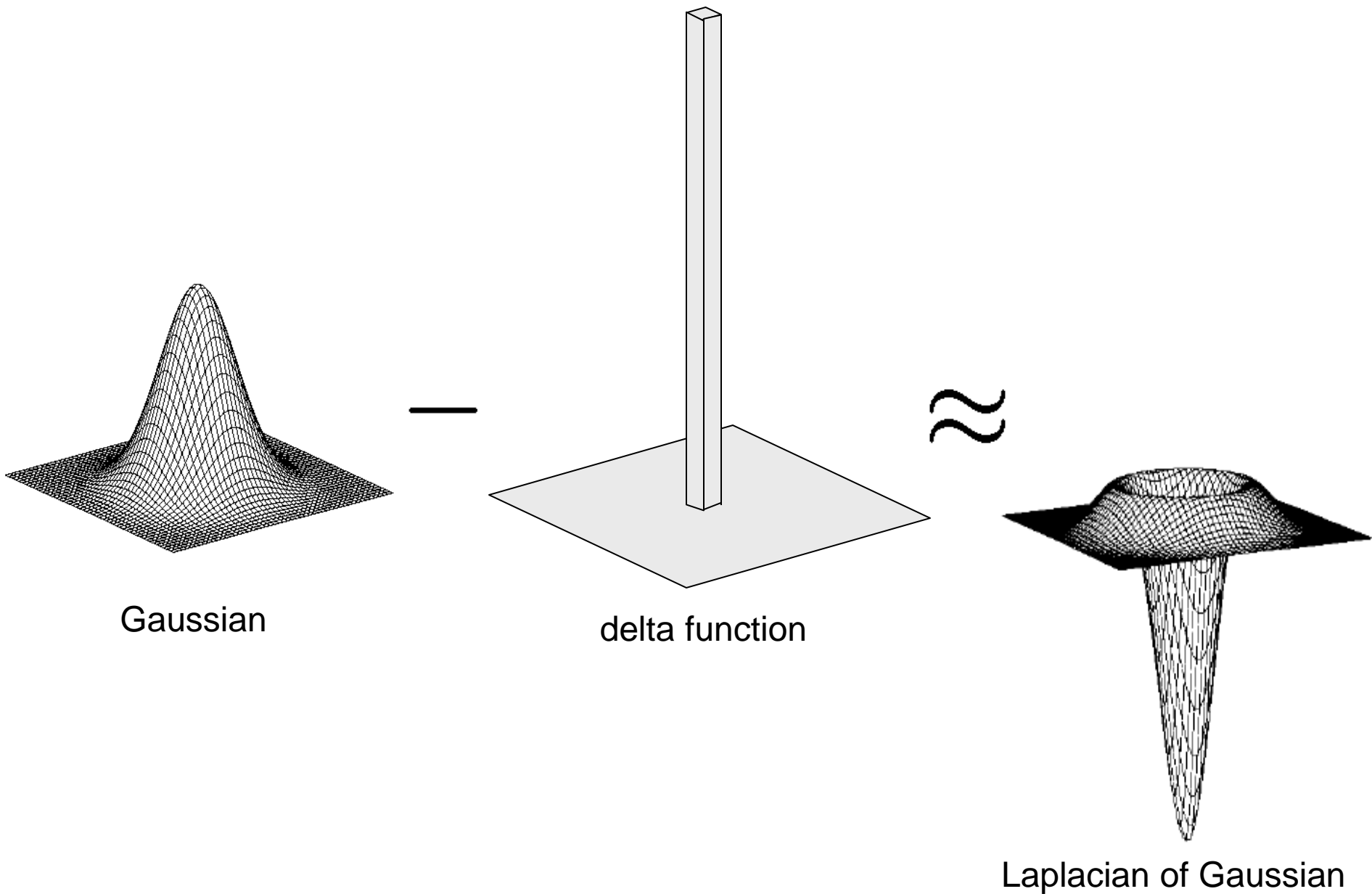


smoothed – original  
(scaled by 4, offset +128)

Why does  
this work?

filter demo

# Gaussian - image filter



An edge is not a line...



How can we detect *lines* ?

# Finding lines in an image

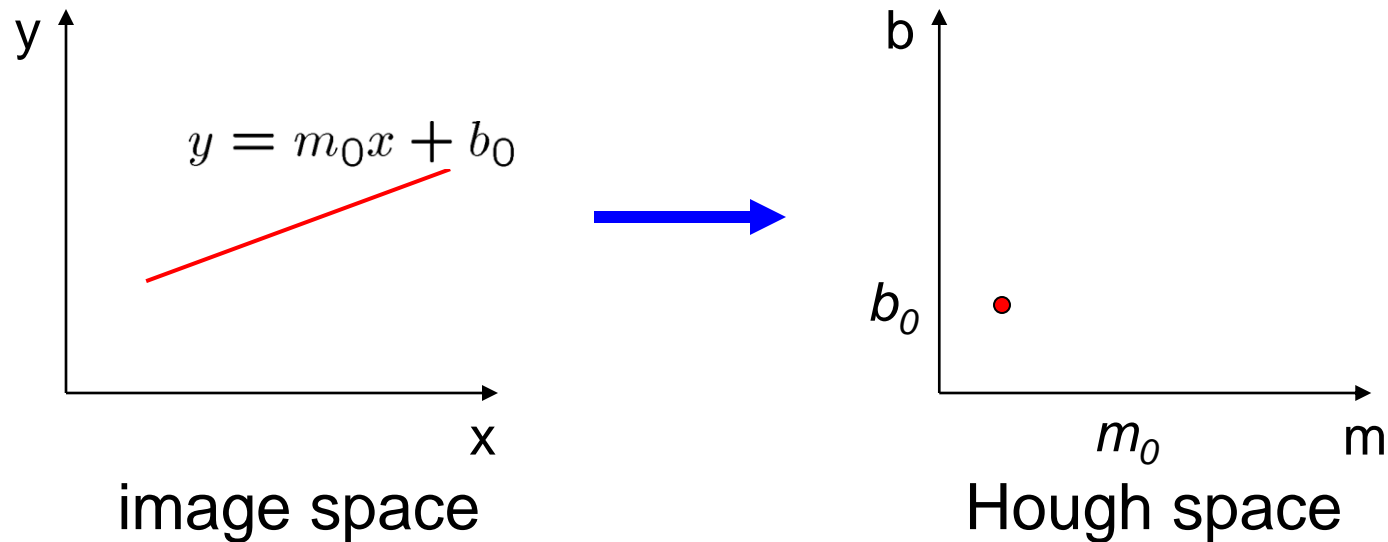
## Option 1:

- Search for the line at every possible position/orientation
- What is the cost of this operation?

## Option 2:

- Use a voting scheme: Hough transform

# Finding lines in an image

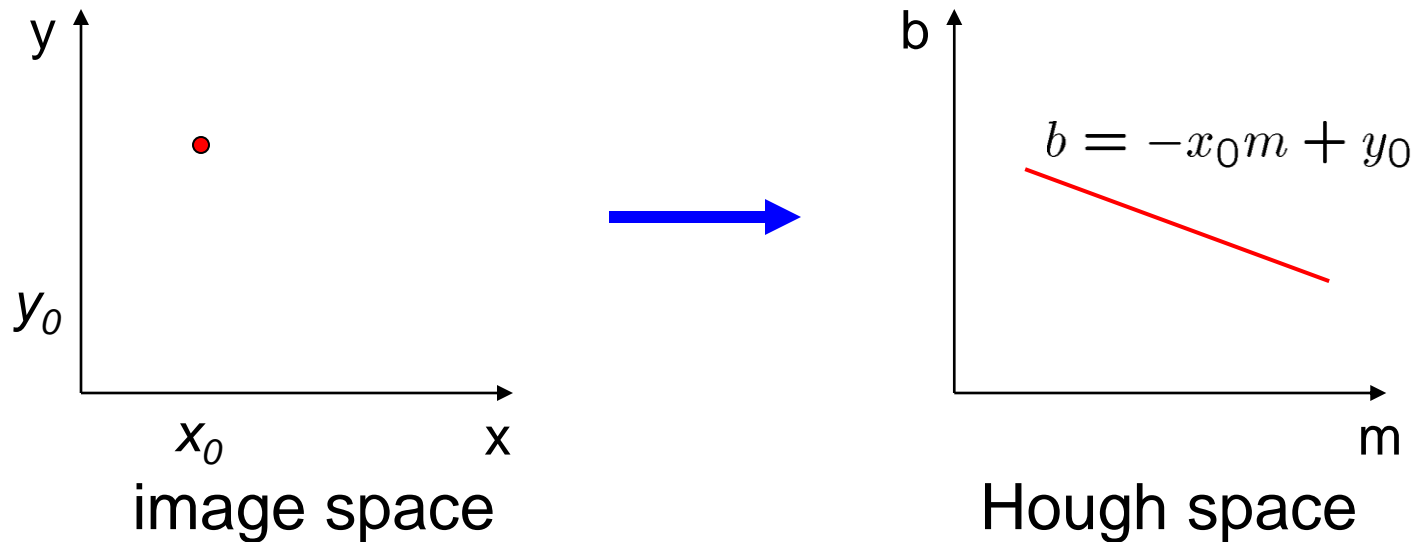


## Connection between image $(x,y)$ and Hough $(m,b)$ spaces

- A line in the image corresponds to a point in Hough space
- To go from image space to Hough space:
  - given a set of points  $(x,y)$ , find all  $(m,b)$  such that  $y = mx + b$



# Finding lines in an image



## Connection between image $(x,y)$ and Hough $(m,b)$ spaces

- A line in the image corresponds to a point in Hough space
- To go from image space to Hough space:
  - given a set of points  $(x,y)$ , find all  $(m,b)$  such that  $y = mx + b$
- What does a point  $(x_0, y_0)$  in the image space map to?
  - A: the solutions of  $b = -x_0m + y_0$
  - this is a line in Hough space

# Hough transform algorithm

Typically use a different parameterization

$$d = x\cos\theta + y\sin\theta$$

- $d$  is the perpendicular distance from the line to the origin
- $\theta$  is the angle this perpendicular makes with the x axis
- Why?

# Hough transform algorithm

Typically use a different parameterization

$$d = x \cos \theta + y \sin \theta$$

- $d$  is the perpendicular distance from the line to the origin
- $\theta$  is the angle this perpendicular makes with the x axis
- Why?

## Basic Hough transform algorithm

1. Initialize  $H[d, \theta] = 0$
2. for each edge point  $I[x, y]$  in the image  
    for  $\theta = 0$  to  $180$   
         $d = x \cos \theta + y \sin \theta$   
         $H[d, \theta] += 1$
3. Find the value(s) of  $(d, \theta)$  where  $H[d, \theta]$  is maximum
4. The detected line in the image is given by  $d = x \cos \theta + y \sin \theta$

What's the running time (measured in # votes)?

# Extensions

Extension 1: Use the image gradient

1. same
2. for each edge point  $I[x,y]$  in the image  
    compute unique  $(d, \theta)$  based on image gradient at  $(x,y)$   
     $H[d, \theta] += 1$
3. same
4. same

What's the running time measured in votes?

# Extensions

## Extension 1: Use the image gradient

1. same
2. for each edge point  $I[x,y]$  in the image  
compute unique  $(d, \theta)$  based on image gradient at  $(x,y)$   
 $H[d, \theta] += 1$
3. same
4. same

What's the running time measured in votes?

## Extension 2

- give more votes for stronger edges

## Extension 3

- change the sampling of  $(d, \theta)$  to give more/less resolution

## Extension 4

- The same procedure can be used with circles, squares, or any other shape

# Hough demos

Line : <http://www.dai.ed.ac.uk/HIPR2/houghdemo.html>

<http://www.dis.uniroma1.it/~iocchi/slides/icra2001/java/hough.html>

Circle : <http://www.markschulze.net/java/hough/>

# Hough Transform for Curves

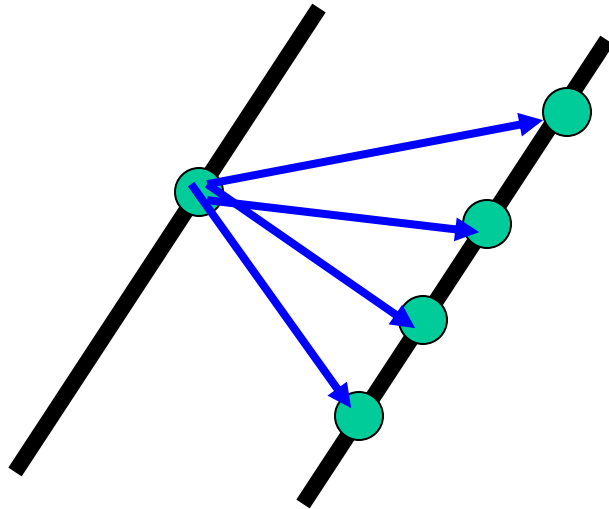
The H.T. can be generalized to detect any curve that can be expressed in parametric form:

- $Y = f(x, a_1, a_2, \dots, a_p)$
- $a_1, a_2, \dots, a_p$  are the parameters
- The parameter space is  $p$ -dimensional
- The accumulating array is LARGE!

# Corner detection

Corners contain more edges than lines.

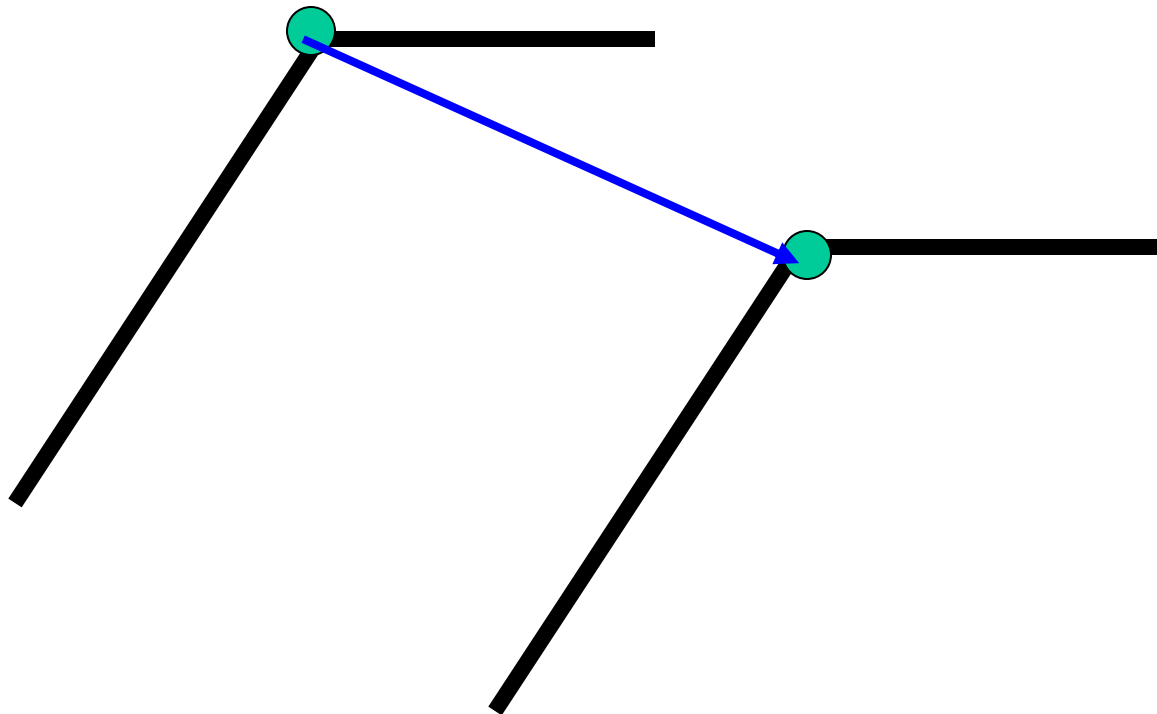
A point on a line is hard to match.



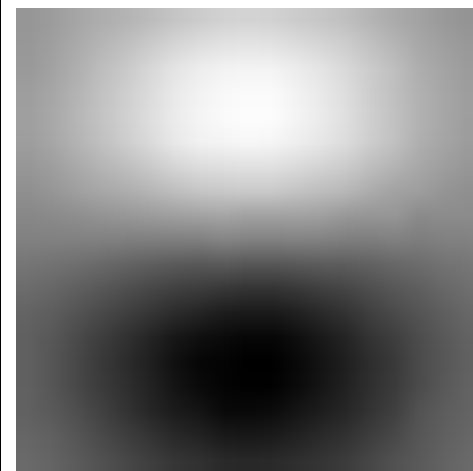
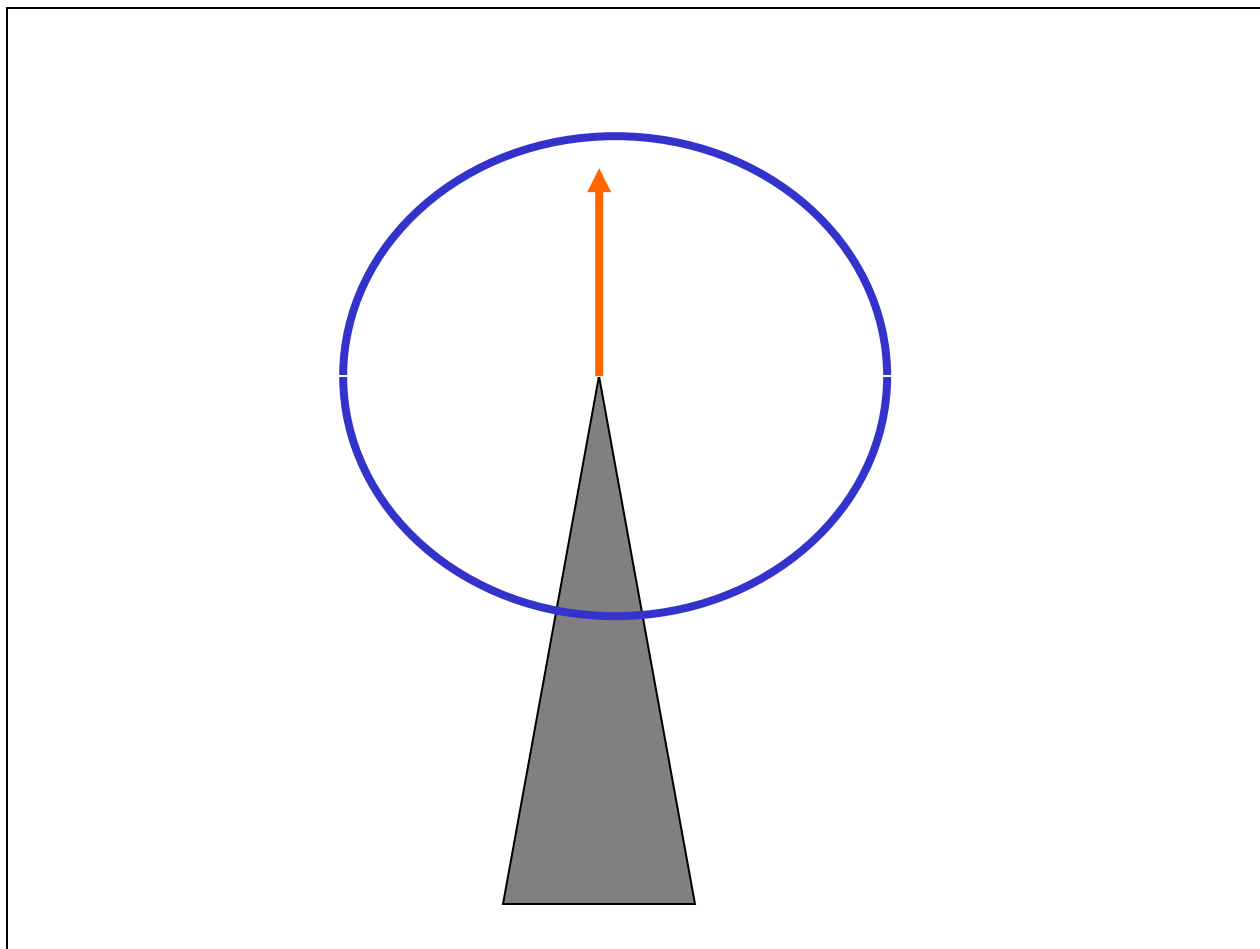


# Corners contain more edges than lines.

A corner is easier



# Edge Detectors Tend to Fail at Corners



# Finding Corners

Intuition:

- Right at corner, gradient is ill defined.
- Near corner, gradient has two different values.

# Formula for Finding Corners

We look at matrix:

Sum over a small region,  
the hypothetical corner

$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

Matrix is symmetric

Gradient with respect to x,  
times gradient with respect to y

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

***WHY THIS?***

First, consider case where:

$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

This means all gradients in neighborhood are:

(k,0) or (0, c) or (0, 0) (or off-diagonals cancel).

What is region like if:

1.  $\lambda_1 = 0$ ?
2.  $\lambda_2 = 0$ ?
3.  $\lambda_1 = 0$  and  $\lambda_2 = 0$ ?
4.  $\lambda_1 > 0$  and  $\lambda_2 > 0$ ?

# General Case:

From Linear Algebra, it follows that because  $C$  is symmetric:

$$C = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

With  $R$  a rotation matrix.

So every case is like one on last slide.

# So, to detect corners

Filter image.

Compute magnitude of the gradient everywhere.

We construct  $C$  in a window.

Use Linear Algebra to find  $\lambda_1$  and  $\lambda_2$ .

If they are both big, we have a corner.