TI DSP, MCU 및 Xilinx Zynq FPGA 프로그래밍 전문가 과정

강사 – Innova Lee(이상훈)
gcccompil3r@gmail.com
학생 – GJ (박현우)
uc820@naver.com

목차

수학 – Fourier Series & Integral, Taylor Series

1) 푸리에 급수로 톱니파 만들기

2) 적분의 정의 (For Fourier Integral) & 푸리에 적분

3) 문제 – 비주기 함수 푸리에 적분으로 구하기 (레이더에 활용됨.)

4) 라플라스 적분

5) Fourier Transform & Taylor Series

1) 푸리에 급수로 톱니파 만들기

◎ 푸리에 급수로 톱니파 만들기

$$f(x) = \begin{cases} 0 & (-\pi < x < 0) \\ x & (0 < x < \pi) \end{cases}$$

※ 푸리에 급수를 이용하면 사각파 또는 sin파 등을 만들 수 있다.
추가공부: 파세발의 정리, 나이퀴스트의 정리

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{\pi} x\,dx = \left[\frac{1}{2}x^2\right]_0^{\pi} \times \frac{1}{\pi} = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos\left(\frac{n\pi}{T}x\right)dx = \frac{1}{\pi}\int_0^{\pi} \underbrace{x}_{g}\,\underbrace{\cos(nx)}_{f'}\,dx$$

$$\int f'(x)g(x) = f(x)g(x) - \int f(x)g'(x)$$

$$\int_0^{\pi}\cos(nx)x\,dx = \left[\frac{1}{n}\sin(nx)x\right]_0^{\pi} - \int_0^{\pi}\frac{1}{n}\sin(nx)\,dx$$

$$= \frac{\pi}{n}\sin(n\pi) + \frac{1}{n^2}\left[\cos(nx)\right]_0^{\pi}$$

$$= \frac{\pi}{n}\sin(n\pi) + \frac{1}{n^2}\left[\cos(n\pi) - 1\right]$$

if) n =정수

$$\therefore\ a_n = \frac{1}{\pi n^2}\left[\cos(n\pi) - 1\right]$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx) = \frac{1}{\pi}\int_0^{\pi} x\sin(nx)\,dx$$

$$\int_0^{\pi}\sin(nx)x\,dx = \left[-\frac{1}{n}\cos(nx)x\right]_0^{\pi} + \int_0^{\pi}\frac{1}{n}\cos(nx)\,dx$$

$$= -\frac{\pi}{n}\cos(n\pi)$$

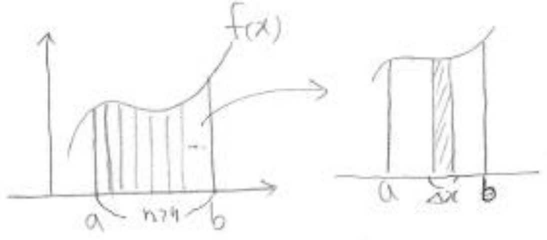
$$\therefore\ b_n = -\frac{1}{n}\cos(n\pi)$$

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left\{\frac{1}{n\pi^2}\left[\cos(n\pi) - 1\right]\cos(nx) - \frac{1}{n}\cos(n\pi)\cdot\sin(nx)\right\}$$

$\Longrightarrow$ $y = x$와 동치 $(x>0)$

2) 적분의 정의 (For Fourier Integral) & 푸리에 적분

ⓞ 적분의 정의 for Fourier integral



$$\boxed{\Delta x = \frac{b-a}{n}}$$

$$x_k = a + k\Delta x$$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\,\Delta x$$

* 각속도와 주기

$$f = \frac{1}{T} \Leftrightarrow T = \frac{1}{f} \xRightarrow{\text{원}}$$

원주: $2\pi r$

시간: $\dfrac{2\pi r}{v}$

$$\omega = \frac{2\pi}{T} \Rightarrow \omega = \frac{v}{r}$$

ⓞ 푸리에 적분

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left\{ a_n\cos\left(\frac{n\pi}{T}x\right) + b_n\sin\left(\frac{n\pi}{T}x\right)\right\}$$

대입

$$a_0 = \frac{1}{T}\int_{-T}^{T} f(x)\,dx$$

$$a_n = \frac{1}{T}\int_{-T}^{T} f(x)\cos\left(\frac{n\pi}{T}x\right)$$

$$b_n = \frac{1}{T}\int_{-T}^{T} f(x)\sin\left(\frac{n\pi}{T}x\right)$$

$$f(x) = \frac{1}{2T}\int_{-T}^{T} f(p)\,dp + \frac{1}{T}\sum_{n=1}^{\infty}\left\{\int_{-T}^{T} f(p)\cos\left(\frac{n\pi}{T}p\right)dp\cdot\cos\left(\frac{n\pi}{T}p\right) + \int_{-T}^{T} f(p)\sin\left(\frac{n\pi}{T}p\right)dp\cdot\sin\left(\frac{n\pi}{T}p\right)\right\}$$

전체원주기 n등분 $= \omega_n = \dfrac{n\pi}{T}$, $\Delta\omega = \dfrac{(n+1)\pi}{T} - \dfrac{n\pi}{T} = \dfrac{\pi}{T} \Rightarrow \dfrac{1}{T} = \dfrac{\Delta\omega}{\pi}$

$$f(x) = \underbrace{\frac{1}{2T}\int_{-T}^{T} f(p)\,dp}_{\to 0} + \frac{1}{\pi}\sum_{n=1}^{\infty}\left\{\cos(\omega_n p)\int_{-T}^{T} f(p)\cos(\omega_n p)\,dp + \sin(\omega_n p)\int_{-T}^{T} f(p)\sin(\omega_n p)\,dp\right\}\Delta\omega$$

$$\lim_{T\to\infty} f(x) = 0 + \boxed{\frac{1}{\pi}}\int_0^{\infty}\left\{\cos(\omega p)\boxed{\int_{-\infty}^{\infty} f(p)\cos(\omega p)\,dp} + \sin(\omega p)\boxed{\int_{-\infty}^{\infty} f(p)\sin(\omega p)\,dp}\right\}d\omega$$

$$A_\omega = \frac{1}{\pi}\int_{-\infty}^{\infty} f(p)\cos(\omega p)\,dp \quad , \quad B_\omega = \frac{1}{\pi}\int_{-\infty}^{\infty} f(p)\sin(\omega p)\,dp$$

$$\therefore \lim_{T\to\infty} f(x) = \int_0^{\infty}\left\{A_\omega\cos(\omega p) + B_\omega\sin(\omega p)\right\}d\omega \quad //$$

3) 문제 – 비주기 함수 푸리에 적분으로 구하기 (레이더에 활용됨.)

◎ 문제 – 비주기 함수 푸리에 적분으로 구하기 (레이더에 활용됨.)

$$A_w = \frac{1}{\pi}\int_{-\infty}^{\infty} f(p)\cos(wp)\,dp$$

$$= \frac{1}{\pi}\left\{\int_{-\infty}^{0} 0 + \int_{0}^{1} 1\cdot\cos(wp)\,dp + \int_{1}^{\infty} 0\right\}$$

$$= \frac{1}{\pi w}\left[\sin(wp)\right]_0^1$$

$$= \frac{\sin(w)}{\pi w}$$

$$B_w = \frac{1}{\pi}\int_{-\infty}^{\infty} f(p)\sin(wp)\,dp$$

$$= \frac{1}{\pi}\left\{\int_{-\infty}^{0} 0 + \int_{0}^{1} 1\cdot\sin(wp)\,dp + \int_{1}^{\infty} 0\right\}$$

$$= -\frac{1}{\pi w}\left[\cos(wp)\right]_0^1$$

$$= -\frac{\cos(w)}{\pi w} + \frac{1}{\pi w}$$

$$\therefore\ f(x) = \int_0^{\infty} \frac{\sin(w)}{\pi w}\cdot\cos(wp) - \frac{\cos(w)}{\pi w}\sin(wp) + \frac{\sin(wp)}{\pi w}\,dw$$

4) 라플라스 적분

◎ 라플라스 적분

Laplace Integral

$$f(x) = e^{-2x} \quad (x > 0) \qquad \int_0^\infty e^{-st}\cos(wt)\,dt = \frac{w}{s^2+w^2}$$

$$A_w = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos(wx)\,dx \qquad B_w = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin(wx)\,dx$$

$$= \frac{1}{\pi}\int_0^\infty e^{-2x}\cos(wx) \qquad = \frac{1}{\pi}\int_0^\infty e^{-2x}\sin(wx)\,dx$$

$$= \frac{1}{\pi}\frac{w}{2^2+w^2} \qquad = \frac{1}{\pi}\frac{2}{2^2+w^2}$$

$$\therefore f(x) = \int_0^\infty \frac{1}{\pi}\frac{w}{2^2+w^2}\cos(wx) + \frac{1}{\pi}\frac{2}{2^2+w^2}\sin(wx)\;dw$$

$$\circ\; e^{-2x} = \frac{2}{\pi}\int_0^\infty \frac{w}{2^2+w^2}\cos(wx)\,dw$$

$$\therefore \frac{\pi}{2}e^{-2x} = \int_0^\infty \frac{w}{2^2+w^2}\cos(wx)\,dw$$

$$\circ\; e^{-2x} = \frac{4}{\pi}\int_0^\infty \frac{1}{2^2+w^2}\sin(wx)\,dw$$

$$\therefore \frac{\pi}{4}e^{-2x} = \int_0^\infty \frac{1}{2^2+w^2}\sin(wx)\,dw$$

⇒ 반주기 이므로 2를 나눠야 함

5) Fourier Transform & Taylor Series

◎ Fourier Transform

$F(x) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$ ☆ ⇒ 컴퓨터가 이것을 계산하면 DFT 혹은 FFT

◎ Taylor Series (근사값 구하기, 조건: 무한번 미분 가능)

$\int_a^x f'(t)\,dt = f(x) - f(a)$

$\int_a^x \underset{f'}{1} \times \underset{g}{f'(t)}\,dt$, $\int f'g = fg - \int fg'$

$= [xf'(x)]_a^x - \int_a^x t f''(t)\,dt$ 대입 ☆ $x\int_a^x f''(t)\,dt = x(f'(x) - f'(a))$

$\Rightarrow xf'(x) = x\int_a^x f''(t)\,dt + xf'(a)$

$= xf'(x) - af'(a) - \int_a^x t f''(t)\,dt$

∘ $f(x) = f(a) + f'(a)(x-a) + \int_a^x (x-t) f''(t)\,dt$

$\int_a^x (x-t) f''(t)\,dt = \left[-\frac{1}{2}(x-t)^2 f''(t)\right]_a^x - \int_a^x -\frac{1}{2}(x-t)^2 f'''(t)\,dt$

$= -\frac{1}{2}\{0 - (x-a)^2 f''(a)\} + \frac{1}{2}\int_a^x (x-t)^2 f'''(t)\,dt$

∘ $f(x) = f(a) + f'(a)(x-a) + \frac{1}{2}(x-a)^2 f''(a) + \frac{1}{2}\int_a^x (x-2)^2 f'''(t)\,dt$

$\therefore$ $f(x) = f(a) + (x-a)f'(a) + \frac{1}{2}(x-a)^2 f''(a) + \cdots + \frac{(x-a)^n}{n!} f^{(n)}(a)$ 테일러 급수

$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \cdots + \frac{x^n}{n!} f^{(n)}(0)$ $(a=0)$ 맥클로린 급수