TI DSP, MCU, Xilinx Zynq FPGA 기반의 프로그래밍 전문가 과정

<공학 수학>

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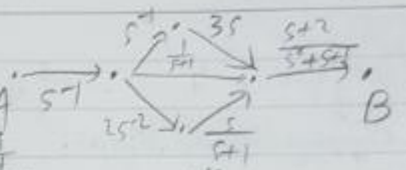
<5.23 수>

- Block diagram
- Signal Flow graph



$i = c\frac{dv}{dt}$

$V = L\frac{di}{dt}$

비율 $= \frac{L\frac{di}{dt}}{\frac{1}{c}\int i\,dt} \Rightarrow L\left\{\frac{L\frac{di}{dt}}{\frac{1}{c}\int i\,dt}\right\} =$ 상수

라플라스 변환 => 미분, 적분이 들어간 식의 비율을 알기 위해

$$\frac{L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt}{L\{f(t)\}(s)}$$

$0 \sim \infty$ 적분 => 미적분학 (이상적분)

어떻게 계산 하지? => 치환

$\lim_{k\to\infty}\int_0^k e^{-st} f(k)\,dk \Rightarrow$ 부분적분

$$f(x)g(x) = \int f'(x)g(x) + \int f(x)g'(x)$$

$$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)\,dx$$

$$\int e^{-sk} f(k) = -\frac{1}{s}e^{-sk} f(k) - \int\left(\frac{1}{s}e^{-sk}\right) f'(k)$$

다항식의 경우 차수만큼...

삼각함수는 두번하고 합성 혹은 e^{ix}

Ex) $L\{f(t)\} = ?\quad f(t) = e^{-at}$

$$\lim_{k\to\infty}\int_0^k e^{-st}e^{-at}\,dt = \int_0^\infty e^{-(s+a)t}\,dt = \frac{1}{-(s+a)}e^{-(s+a)t}\Big|_0^\infty$$

$$= \boxed{\frac{1}{s+a}}$$

Ex) $f(t) = \sin t$

$$L\{f(t)\} = \int_0^\infty e^{-st}\sin t\, dt = -\frac{1}{s}e^{-st}\sin t\Big|_0^\infty + \int_0^\infty \frac{1}{s}e^{-st}\cos t\, dt$$

$$\Rightarrow \int_0^\infty e^{-st}\sin t\, dt = -\frac{1}{s}e^{-st}\sin t\Big|_0^\infty - \frac{1}{s^2}e^{-st}\cos t\Big|_0^\infty - \int_0^\infty \frac{1}{s^2}e^{-st}\sin t\, dt$$

$$\Rightarrow \int_0^\infty e^{-st}\sin t\, dt = \frac{1}{s^2} - \int_0^\infty \frac{1}{s^2}e^{-st}\sin t\, dt$$

$$\Rightarrow \left(1+\frac{1}{s^2}\right)\int_0^\infty e^{-st}\sin t\, dt = \frac{1}{s^2}$$

$$\Rightarrow (s^2+1)\int_0^\infty e^{-st}\sin t\, dt = 1$$

$$L\{f(t)\} = \Rightarrow \boxed{\int_0^\infty e^{-st}\sin t\, dt = \frac{1}{s^2+1}}$$

$\frac{1}{2s}$

$\frac{1}{s_i}\int_0^\infty e^{-2st}\, dt$

$\frac{1}{-2s}e^{-2s}\Big|_0^\infty$

$f(t) = \sin t = \frac{e^{it}-e^{-it}}{2i}$ $\qquad \cos t = \frac{e^{it}+e^{-it}}{2}$

$$\int_0^\infty e^{-st}\sin t = \frac{1}{2i}\int_0^\infty e^{-st}(e^{it}-e^{-it}) = \frac{1}{2i}\int_0^\infty e^{-(s-i)t} - e^{-(s+i)t}\, dt$$

$$= \frac{1}{2i}\left[-\frac{1}{(s-i)}e^{-(s-i)t} + \frac{1}{(s+i)}e^{-(s+i)t}\right]_0^\infty = \frac{1}{2i}\left(\frac{1}{s-i} - \frac{1}{s+i}\right)$$

$\frac{1}{s^2+1}$

$$= \boxed{\frac{1}{s^2+1}}$$

Ex) $f(t) = \cos t$

$$L\{f(t)\} = \int_0^\infty e^{-st}\frac{e^{it}+e^{-it}}{2}\, dt = \frac{1}{2}\int_0^\infty e^{-(s-i)t} + e^{-(s+i)t}\, dt = \frac{1}{2}\left[\frac{1}{-(s-i)}e^{-(s-i)t} + \frac{1}{-(s+i)}e^{-(s+i)t}\right]_0^\infty$$

$$= \frac{1}{2}\left(\frac{1}{s-i} + \frac{1}{s+i}\right) = \frac{1}{2}\left(\frac{2s}{s^2+1}\right)$$

$$= \boxed{\frac{s}{s^2+1}}$$

<부분적분>

$$L\{f(t)\} = \int_0^\infty e^{-st}\cos t\, dt = -\frac{1}{s}e^{-st}\cos t\Big|_0^\infty - \int_0^\infty \frac{1}{s}e^{-st}\sin t\, dt$$

$$= -\frac{1}{s}e^{-st}\cos t\Big|_0^\infty - \left(-\frac{1}{s^2}e^{-st}\sin t\Big|_0^\infty + \int_0^\infty \frac{1}{s^2}e^{-st}\cos t\, dt\right)$$

$$= \frac{1}{s} - \int_0^\infty \frac{1}{s^2}e^{-st}\cos t\, dt$$

$$\Rightarrow \left(1+\frac{1}{s^2}\right)\int_0^\infty e^{-st}\cos t\, dt = \frac{1}{s}$$

$$\Rightarrow \boxed{\int_0^\infty e^{-st}\cos t\, dt = \frac{s}{s^2+1}}$$

$(s^2+1)\square = s$

$f(t) = \cos(\omega t)$

부분적분

$$L\{f(t)\} = \int_0^\infty e^{-st}\cos(\omega t)\, dt = -\frac{1}{s}e^{-st}\cos\omega t\Big|_0^\infty - \int_0^\infty \frac{1}{s}e^{-st}\omega\sin\omega t\, dt$$

$$= +\frac{1}{s} - \left[-\frac{1}{s^2}e^{-st}\omega\sin\omega t\Big|_0^\infty + \int_0^\infty \frac{1}{s^2}e^{-st}\omega^2\cos\omega t\, dt\right]$$

$$= \frac{1}{s} - \frac{\omega^2}{s^2}\int_0^\infty e^{-st}\cos\omega t\, dt$$

$$\Rightarrow \left(1+\frac{\omega^2}{s^2}\right)\int_0^\infty e^{-st}\cos\omega t\, dt = \frac{1}{s}$$

$$\int_0^\infty e^{-st}\cos\omega t\, dt = \frac{1}{s}\left(\frac{s^2}{s^2+\omega^2}\right) = \boxed{\frac{s}{s^2+\omega^2}}$$

Ex) $f(t) = t^2$

$$L\{f(t)\} = \int_0^\infty e^{-st}t^2\, dt = -\frac{1}{s}e^{-st}t^2\Big|_0^\infty + \int_0^\infty \frac{1}{s}e^{-st}(2t)\, dt$$

$$= -\frac{1}{s}e^{-st}t^2\Big|_0^\infty - \frac{1}{s^2}e^{-st}(2t)\Big|_0^\infty + \int_0^\infty \frac{1}{s^2}e^{-st}\cdot 2\, dt$$

$$= \int_0^\infty \frac{2}{s^2}e^{-st}\, dt = -\frac{2}{s^3}e^{-st}\Big|_0^\infty = \boxed{\frac{2}{s^3}} \Rightarrow \boxed{\frac{n!}{s^{n+1}}}$$

$$\int_0^\infty e^{-st}\left(\frac{e^{i\omega t}+e^{-i\omega t}}{2}\right)dt = \frac{1}{2}\int_0^\infty e^{-(s-i\omega)t} + e^{-(s+i\omega)t}\, dt$$

$$= \frac{1}{2}\left[\frac{1}{-(s-i\omega)}e^{-(s-i\omega)t} + \frac{1}{-(s+i\omega)}e^{-(s+i\omega)t}\right]_0^\infty$$

$$= \frac{1}{2}\left(\frac{1}{s-i\omega} + \frac{1}{s+i\omega}\right) = \boxed{\frac{s}{s^2+\omega^2}}$$

$f'(t)$

$$L\{f(t)\}(s) = \int_0^\infty e^{-st}f(t)\, dt = F(s)$$

$$L\{f'(t)\}(s) = \int_0^\infty e^{-st}f'(t)\, dt = [f(t)e^{-st}]_0^\infty - \int_0^\infty(-se^{-st})f(t)\, dt$$

$+s\int_0^\infty (e^{-st})f(t)\, dt$

$= -f(0) + sF(s)$

<1계 도함수의 라플라스 변환>

$sF(s) - f(0)$

〈2계도함수의 라플라스 변환〉

Date

$f''(t)$

$$L\{f''(t)\}(s) = \int_0^\infty \underbrace{f''(t)}_{f'}\underbrace{e^{-st}}_{g}\,dt = \underbrace{f'(t)}_{f}\underbrace{e^{-st}}_{g}\Big|_0^\infty - \left(\int_0^\infty \underbrace{f'(t)}_{f}\underbrace{(-se^{-st})}_{g'}\,dt\right)$$

$$= -f'(0) + s\boxed{\int_0^\infty f'(t)e^{-st}\,dt}$$

$$= -f'(0) + s(sF(s) - f(0)) = \boxed{s^2F(s) - sf(0) - f'(0)}$$

Ex) $y' + 4y + 1 = 0$

$$L\{y' + 4y + 1\} \Rightarrow sY(s) - y(0) + 4(Y(s)) + \frac{1}{s} = Y(s)(s+4) - y(0) + \frac{1}{s} = 0$$

$$Y(s) = \frac{1}{s+4}\left(y(0) - \frac{1}{s}\right)$$

부분 분수 전개

1. $\dfrac{A}{x-a_0} + \dfrac{B}{x-a_1} + \dfrac{C}{x-a_2} = \cancel{\dfrac{D}{ax^2+bx+c}} = \dfrac{D}{ax^3+bx^2+cx+d}$
2. $\dfrac{A}{x-a_0} + \dfrac{B}{x-a_1} + \dfrac{Cx+D}{x^2+a_2x+a_3} = \cancel{\dfrac{E}{ax^3+bx^2+cx+D}} = \dfrac{E}{ax^4+bx^3+cx^2+dx+e}$
3. $\dfrac{A}{x-a_0} + \dfrac{B}{(x-a_0)^2} + \dfrac{C}{x-a_1} = \cancel{\dfrac{D}{ax^2+bx+c}} = \dfrac{D}{ax^3+bx^2+cx+d}$

Ex) $$\frac{x^2+2x-3}{(x^2+x+5)(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+x+5}$$

$$x^2+2x-3 = A(x-2)(x^2+x+5) + B(x^2+x+5) + (Cx+D)(x^2-4x+4)$$

$$= A(x^3-x^2+3x-10) + B(x^2+x+5) + (Cx^3+(D-4C)x^2+4(C-D)x+4D)$$

$$= (A+C)x^3 + (-A+B-4C+D)x^2 + (3A+B+4C-4D)x$$

$A+C=0$

$-A+B+D-4C=1$

$3A+B+4C-4D=2$

$-10A+5B+4D=-3$

$A=-C$ $\quad +(-10A+5B+4D)=0$

$-3C+B+D=1$

$C+B-4D=2$

$10C+5B+4D=-3$

$5C+5B-20D=10$

$\frac{9}{4}$

$4C-5D=1$

$5C+24D=-13$

$5C-\frac{25}{4}D=\frac{5}{4}$

$\frac{121}{4}D=-\frac{57}{4}$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 4 & 1 & -4 & 1 & 1 \\ 3 & 1 & 4 & -4 & 2 \\ -10 & 5 & 0 & 4 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 1 & 1 \\ 0 & 1 & 1 & -4 & 2 \\ 0 & 5 & 10 & 4 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 1 & 1 \\ 0 & 0 & +4 & -5 & 1 \\ 0 & 0 & 25 & -1 & -8 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 1 & 1 \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{1}{4} \\ 0 & 0 & 25 & -1 & -8 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 1 & 1 \\ 0 & 0 & 1 & -\frac{5}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{121}{4} & -\frac{57}{4} \end{array}\right]$$

$$\frac{121}{4}D = -\frac{57}{4}$$

$$\boxed{D = \frac{-57}{121}}$$

$$C = \frac{121-285}{484} = \frac{-164}{484} = \frac{-41}{121} \qquad C + \frac{285}{484} = \frac{1}{4}$$

$$B + \frac{123}{121} - \frac{57}{121} = 1 \qquad B = \frac{57}{121}$$

$$A = \frac{41}{121}, \quad B = \frac{55}{121} \; \left(\frac{5}{11}\right), \quad C = -\frac{41}{121}, \quad D = -\frac{57}{121}$$

$$\frac{x^2+2x-3}{(x^2+x+5)(x-2)^2} = \frac{41}{121}\cdot\frac{1}{x-2} + \frac{5}{11}\frac{1}{(x-2)^2} - \frac{41}{121}\cdot\frac{x}{x^2+x+5} - \frac{57}{121}\cdot\frac{1}{x^2+x+5}$$

Ex) $y'' + 4y' + 3y = e^t$

$f(t) = \sin(\omega t)$

$F(s) = \frac{\omega}{s^2+\omega^2}$

$L\{e^{at}f(t)\} = \int_0^\infty e^{-st}e^{at}f(t)dt = \int_0^\infty e^{-(s-a)t}f(t)dt = F(s-a)$

$L^{-1}\left\{\frac{4}{s^2+4s+20}\right\} = \frac{4}{(s+2)^2+16} = \sin(4t)e^{-2t}$

ex) $\frac{s}{s^2+s+5} = \frac{s}{(s+\frac{1}{2})^2+\frac{19}{4}} = \cos\left(\frac{\sqrt{19}}{2}t\right)e^{-\frac{1}{2}t}$

ex) $L^{-1}\left\{\frac{3s-1}{s^2-6s+2}\right\} \Rightarrow \frac{3(s-3)+8}{(s-3)^2-7}$

$L(e^{at}\sinh(\omega t)) = \frac{\omega}{(a-s)^2-\omega^2}$

$L(e^{at}\cosh(\omega t)) = \frac{a-s}{(a-s)^2-\omega^2}$

라플라스 ; 제어기의 안정성 판별 (범용성↑)

푸리에 변환 ; 주파수 성분을 보기위해 (범용성↓)

Ex) $0 \le t < 2$, $t \ge 2$
$g(t) = 0$ $\quad g(t) = t^2+1$

$g(t) = H(t-2)(t^2+1)$

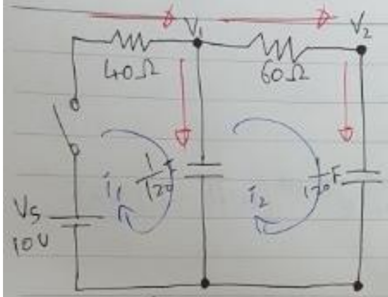
$\Rightarrow H(t-2)\{(t-2)^2+1\}$

오류 ; 원식은 t^2
헤비사이드 ; $t-2$
시간변화를 맞추기위해
$t-2$를 넣었더니 원식이 붕괴됨.
$\Rightarrow H(t-2)\{(t-2+2)^2+1\}$ 으로

$= H(t-2)\{(t-2)^2+4(t-2)+5\}$

$= e^{-2s}\left\{\frac{2}{s^3} + 4\frac{1}{s^2} + 5\cdot\frac{1}{s}\right\}$

<Laplace Transform Based RC-RC Filter>



(전하량) 스위치 떨어져 있으니까 초기에는 0
$q_1(0) = q_2(0) = 0$

망 전류법 $\quad 120\int(i_1-i_2)dt$

1. $40i_1 + 120\int i_1 dt = 10$
2. $60i_2 + 120\int i_2 dt + 120\int(i_2-i_1)dt = 0$

$i_c = C\frac{dv}{dt} \Rightarrow V = \frac{1}{C}\int i\,dt$

$\frac{dq}{dt} = i$

$\boxed{\text{★ 적분의 라플라스 변환} \quad L\{\int f(t)dt\}(s) = \frac{1}{s}F(s)}$ => 각각을 라플라스 변환한다.

1. $40I_1(s) + \frac{120}{s}(I_1(s) - I_2(s)) = \frac{10}{s}$
2. $60I_2(s) + \frac{120}{s}I_2(s) + \frac{120}{s}(I_2(s) - I_1(s)) = 0$

$\Rightarrow$ 1. $40I_1(s)s + 120I_1(s) - 120I_2(s) = 10 \Rightarrow (40s+120)I_1(s) - 120I_2(s) = 10$

$\Rightarrow$ 2. $60I_2(s)s + 120I_2(s) + 120I_2(s) - 120I_1(s) = 0 \Rightarrow (60s+240)I_2(s) - 120I_1(s) = 0$

교착상태

=
1) 그러므로 $I_1(s)$를 $I_2(s)$로 바꾸거나
2) $I_2(s)$를 $I_1(s)$로 바꾸도록 한다. => 상수값이 없는 2번 선택

$(60s+240)I_2(s) = 120I_1(s)$

$\boxed{I_2(s) = \frac{2}{s+4}I_1(s)}$

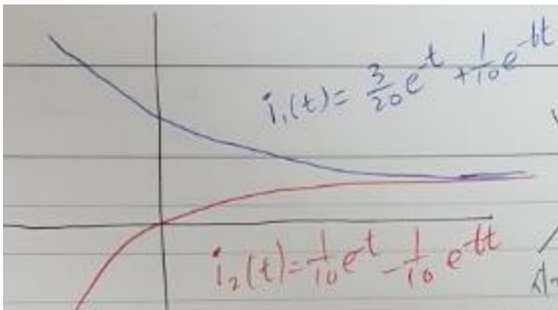
$\boxed{I_1(s) = \frac{s+4}{2}I_2(s)}$

$\boxed{I_2(s) = \frac{s+3}{3}I_1(s)}$

$6 = s^2+7s+12$ $\quad s = -1, -6$

$I_1(s) = \frac{3}{s+3}I_2(s)$ $\quad s^2+7s+6=0$

$s^2+7s+12-6=0$



$i_1(t)$와 $i_2(t)$는 서로를 방해함

시간이 지나면 합쳐지는 전류 $i_1(t), i_2(t)$ 2개가 서로 상쇄되서 안정화됨.

Z 변환 L변환을 컴퓨터에서 구현하자!

$I_1(s) = \frac{s+4}{4s^2+28s+24}$

$I_2(s) = \frac{1}{2s^2+14s+12}$

여기까지는 구하고 계산하죠?

1. 샘플링 기반 Z transform
2. 샘플링기반 회귀 분석
3. 회귀분석 결과와 아날로그 해석