Xilinx Zynq FPGA, TI DSP, MCU 기반의 프로그래밍 및 회로 설계 전문가 과정

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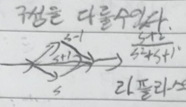
signal Flow Graph

↳ 그래프 이론 하면 네트워크 구성을 다룰 수 있다.



라플라스와 결합 가능

$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$

$0 \sim \infty$ 적분 = 미적분학 이상적분

$\lim_{k\to\infty}\int_0^k e^{-sk} f(k)\,dk \Rightarrow$ 부분적분

↳ $f(x)g(x) = \int f'(x)g(x) + \int f(x)g'(x)$

$\int f'(x)g(x) = f(x)g(x) - \int f(x)g'(x)$

$\mathcal{L}\{f(t)\}$ $\quad f(t) = e^{-at}$

ex) $\int_0^\infty e^{-st} e^{-at}\,dt$

$= \int_0^\infty e^{-(s+a)t}$

$= \lim_{k\to\infty}\int_0^k e^{-(s+a)k}\,dk$

$= \frac{1}{s+a}$

$f(t) = \sin(t)$ $\quad * \cos(t) = \frac{e^{it}+e^{-it}}{2}$

$\sin(t) = \frac{e^{it}-e^{-it}}{2i}$

$\int_0^\infty e^{-st}\left(\frac{e^{it}-e^{-it}}{2i}\right)dt$

$= \frac{1}{2i}\int_0^\infty e^{-(s-i)t} - e^{(s+i)t}\,dt$

$= \frac{1}{s^2+1}$

$s\int_0^\infty f'(t)e^{-st}\,dt \rightarrow sF(0) - f(0)$

2계 도함수 라플라스

$f''(t)$

$\mathcal{L}\{f''(t)\}(s) = \int_0^\infty f''(t)e^{-st}\,dt = \left[\underset{f}{f'(t)}\,\underset{g}{e^{-st}}\right]_0^\infty -$

$\int_0^\infty \underset{f}{f'(t)}\,\underset{g}{(-se^{-st})}\,dt$

$= f'(0) + \underline{s\int_0^\infty f'(t)e^{-st}\,dt}$

↳ $sF(s) - F(0)$

$= -f'(0) + s\{sF(s) - f(0)\}$

$= s^2F(s) - sf(0) - f'(0)$

2계 도함수 라플라스

* 부분분수 전개

1. $\frac{A}{x-a_0} + \frac{B}{x-a_1} + \frac{C}{x-a_2} = \frac{D}{ax^3+bx^2+cx+d}$

2. $\frac{A}{x-a_0} + \frac{B}{x-a_1} + \frac{Cx-D}{x^2-a_2x+a_1} = \frac{E}{ax^4+bx^3+cx^2+dx+e}$

3. $\frac{A}{x-a_0} + \frac{B}{(x-a_0)^2} + \frac{C}{(x-a_1)} = \frac{D}{ax^3+bx^2+cx+d}$

ex) $\frac{x^2+2x-3}{(x^2+x+5)(x-2)^2}$

$= \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+x+5}$

$x^2+2x-3 = (x-2)(x^2+x+5)A + (x^2+x+5)B + (x-2)^2(Cx+D)$

$= (x^3-x^2+3x-10)A + 6 + (x^3-4x+4)(Cx+D)$

$(A+C)x^3 + (A+B-4C+D)x^2 + (3A+B+4C-4D)x$

$+ (-10A+5B+4D)$

$A+C=0 \quad -A+B+4C+D=1,\ 3A+B+4C-4D=2$

$-10A+5B+4D=-3$

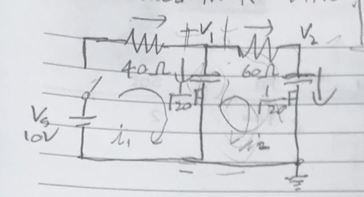
결론 $\frac{x^2+2x-3}{(x^2+x+5)(x-2)^2} = \frac{41}{121}\frac{1}{(x-2)} + \frac{5}{11}\frac{1}{(x-2)} -$

$\frac{41}{121}\frac{x}{x^2+x+5} - \frac{57}{121}\frac{1}{x^2+x+5}$

라플라스 역변환

Laplace Transform Based RC-RC Filter

RC-RC필터기반 라플라스 변환



$1. \frac{V_s - V_1}{40}$

$= \frac{10 - V_1}{40}$

$=$

$q_1(0) = q_2(0) = 0$

↳ 전하량 초기에 스위치가 떨어져있어 당연히 0이다.

$i_c = C\frac{dV}{dt} \Rightarrow V = \frac{1}{C}\int i\,dt.$

$\frac{dq}{dt} = i$

망전류법 (직선을 흐르는 전류동일)

$1.\ 40i + \frac{1}{C}$ $\quad \frac{1}{120} = C$

$1.\ 40i_1 + 120\int(i_1 - i_2)\,dt = 10$

$i_2 = i_1$ C의 충전을 방해

$2.\ 60i_2 + 120\int i_2\,dt + 120\int(i_2 - i_1)\,dt = 0$

*적분의 라플라스 변환

$L\{\int f(t)\,dt\} = \frac{1}{s}F(s)$

각각을 라플라스 변환

$1.\ 40I_1(s) + \frac{120}{s}\{I_1(s) - I_2(s)\} = \frac{10}{s}$

$2.\ 60I_2(s) + \frac{120}{s}I_2(s) + \frac{120}{s}\{I_2(s) - I_1(s)\} = 0$

$1.\ 40I_1(s)s + 120I_1(s) - 120I_2(s) = 10$

$2.\ 60I_2(s)s + 120I_2(s) + 120I_2(s) - 120I_1(s) = 0$

$\rightarrow (40s + 120)I_1(s) - 120I_2(s) = 10$
$(60s + 240)I_2(s) - 120I_1(s) = 0$ 교합상태

그러므로 $I_1(s)$를 $I_2(s)$로 바꾸거나 $I_2(s)$를 $I_1(s)$로 바꾸도록

$I_2(s) = \frac{120}{(60s + 240)}I_1(s)$

$1.\ (40s + 120)I_1(s) - 120\left(\frac{120}{(60s + 240)}\right)I_1(s)$ $\quad \frac{120}{60(s+4)} \quad \frac{2}{(s+4)}$

$1.\ (40s + 120)I_1(s) - \frac{240}{(s+4)}I_1(s)$

$(s+4)I_2(s) = 2I_1(s)$

$I_2(s) = \frac{2}{s+4}I_1(s)$

$1.\ 4I_1(s) + 12I_1(s) - 12I_2(s) = 1$

$4I_1(s)s + 12I_1(s) - 12\left(\frac{2}{s+4}\right)I_1(s) = 1$

$4s(s+4)I_1(s) + 12(s+4)I_1(s) - 24I_1(s) = s+4$

$I_1(s) = \frac{s+4}{4s^2 + 28s + 24}$ 라플라스 역변환

$I_2(s) = \frac{2}{4s^2 + 28s + 24}$

$4s^2 + 28s + 24 = 4(s^2 + 7s + 6) = 4(s+1)(s+6)$

$2s^2 + 14s + 12 = 2(s^2 + 7s + 6) = 2(s+1)(s+6)$

부분분수

$\frac{s+4}{4s^2 + 28s + 24} = \frac{1}{4}\left\{\frac{A}{(s+1)} + \frac{B}{(s+6)}\right\}$

$s + 4 = (s+6)A + (s+1)B$

$A + B = 1$
$6A + B = 4$
$-5A = -3$

$A = \frac{3}{5},\ B = \frac{2}{5}$ $\rightarrow i_1(t) = \frac{3}{20}e^{-t} + \frac{1}{10}e^{-6t}.$

$I_2(s)$

$\frac{1}{2s^2 + 14s + 12} = \frac{1}{2}\left\{\frac{A}{(s+1)} + \frac{B}{(s+6)}\right\}$

$A + B = 0$
$6A + B = 1$ $\quad A = \frac{1}{5} \quad B = -\frac{1}{5}$

$i_2(t) = \frac{1}{10}e^{-t} - \frac{1}{10}e^{-6t}$