TI DSP, MCU, Xilinx Zynq FPGA 기반의 프로그래밍 전문가 과정

<공학 수학>

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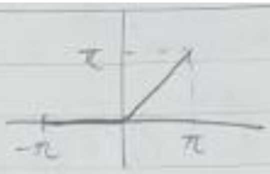
$$f(x) = \begin{cases} 0 & (-\pi < x < 0) \\ x & (0 < x < \pi) \end{cases}$$



$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_0^{\pi} x\,dx\right] = \frac{1}{\pi}\left[\frac{1}{2}x^2\right]_0^{\pi} = \frac{\pi^2}{2\pi} = \boxed{\frac{\pi}{2}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos\left(\frac{n\pi}{T}x\right)dx = \frac{1}{\pi}\left[\cancel{\int_{-\pi}^{0} 0\cos\left(\frac{n\pi}{T}x\right)dx} + \int_0^{\pi} x\cos\left(\frac{n\pi}{T}x\right)dx\right]$$

$$= \frac{1}{\pi}\left[\cancel{\frac{T}{n\pi}\sin\left(\frac{n\pi}{T}x\right)x\Big|_0^{\pi}} - \frac{T}{n\pi}\int_0^{\pi}\sin\left(\frac{n\pi}{T}x\right)dx\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{T}{n\pi}\right)^2\cos\left(\frac{n\pi}{T}x\right)\Big|_0^{\pi}\right] = \boxed{\frac{1}{\pi n^2}\left(\cos(n\pi) - 1\right)}$$

$T = \pi$

진동처리 관점

n은 정수

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} x\sin(nx)\,dx$$

$$\frac{1}{\pi}\times\int_0^{\pi}\sin(nx)x\,dx = \left[-\frac{1}{n}\cos(nx)x\Big|_0^{\pi} + \cancel{\int_0^{\pi}\frac{1}{n}\cos(nx)\,dx}\right]\times\frac{1}{\pi}$$

$$= \boxed{-\frac{1}{n}\left(\cos(n\pi)\right)}$$

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left(\frac{1}{\pi n^2}\left[\cos(n\pi) - 1\right]\cos(nx) - \frac{1}{n}\cos(n\pi)\sin(nx)\right)$$

$\Rightarrow y = x$ 와 동치 $(x > 0)$

푸리에 적분을 하면 비주기함수도 푸리에 급수로 표현 가능.

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left\{a_n\cos\left(\frac{n\pi}{T}x\right)+b_n\sin\left(\frac{n\pi}{T}x\right)\right\}$$

$$f(x)=\frac{1}{2T}\int_{-T}^{T}f(p)\,dp+\frac{1}{T}\left[\sum_{n=1}^{\infty}\left\{\int_{-T}^{T}f(p)\cos\left(\frac{n\pi}{T}p\right)dp\cos\left(\frac{n\pi}{T}x\right)+\int_{-T}^{T}f(p)\sin\left(\frac{n\pi}{T}p\right)dp\sin\left(\frac{n\pi}{T}x\right)\right]\Delta w\right.$$

$$a_0=\frac{1}{T}\int_{-T}^{T}f(x)\,dx$$

$$a_n=\frac{1}{T}\int_{-T}^{T}f(x)\cos\left(\frac{n\pi}{T}x\right)$$

$$b_n=\frac{1}{T}\int_{-T}^{T}f(x)\sin\left(\frac{n\pi}{T}x\right)$$

$$f=\frac{1}{T}\Leftrightarrow T=\frac{1}{f}$$

$$=\frac{2\pi r}{v}=\frac{2\pi}{w}\Leftrightarrow f=\frac{w}{2\pi}\Rightarrow w=\frac{2\pi}{T}=\boxed{2\pi f}$$

원주 : $2\pi r$

시간 : $\frac{거리}{속도}=\frac{2\pi r}{v}$

$v=rw$

$w=\frac{v}{r}$

$$\Delta w=w_{n+1}-w_n=\frac{(n+1)\pi}{T}-\frac{n\pi}{T}=\boxed{\frac{\pi}{T}}$$

$\left[\begin{array}{c}\text{전체 주기}\\ \text{원주}\end{array}\right]$ n 등분하자!

$w_n=\frac{n\pi}{T}$

$$\Leftrightarrow \frac{\Delta w}{\pi}=\frac{1}{T}$$

$$dx=\frac{b-a}{n}$$

$$f(x)=\frac{1}{2\pi}\int_{-T}^{T}f(p)\,dp+\frac{1}{\pi}\left[\sum_{n=1}^{\infty}\left\{\cos(w_np)\int_{-T}^{T}f(p)\cos(w_np)\,dp+\sin(w_np)\int_{-T}^{T}f(p)\sin(w_np)\,dp\right]\right.\times\Delta w$$

$$\lim_{x\to\infty}\sum_{n=1}^{\infty}\frac{\{f(a)+f(a+dx)\}dx}{2}=\int_a^b f(x)\,dx$$

$\Delta x=$

$$=\lim_{T\to\infty}f(x)=0+\boxed{\frac{1}{\pi}}\int_0^{\infty}\left\{\cos(wp)\boxed{\int_{-\infty}^{\infty}f(p)\cos(wp)\,dp}+\sin(wp)\boxed{\int_{-\infty}^{\infty}f(p)\sin(wp)\,dp}\right\}dw$$

A_w

B_w

$$\boxed{A_w=\frac{1}{\pi}\int_{-\infty}^{\infty}f(p)\cos(wp)\,dp}$$

$$\boxed{B_w=\frac{1}{\pi}\int_{-\infty}^{\infty}f(p)\sin(wp)\,dp}$$

$$\boxed{\int_0^{\infty}\{A_w\cos(wp)+B_w\sin(wp)\}\,dw}$$

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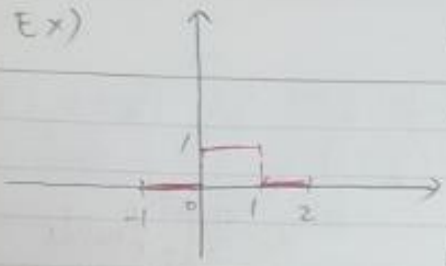
Ex)



$a_0 = \int_0^1 (1)\,dx = 1$

$a_n = \frac{1}{1}\int_0^1 \cos(n\pi x)\,dx$

$= \frac{1}{n\pi}\cancel{[\sin n\pi x]_0^1} = 0$

$b_n = 1\int_0^1 \sin(n\pi x)\,dx = -\frac{1}{n\pi}\cos(n\pi x)\Big|_0^1$

$= -\frac{1}{n\pi}(\cos n\pi - 1)$

$= \frac{1}{n\pi}(1-\cos(n\pi))$

$$\boxed{f(x) = 1 + \sum_{n=1}^{\infty}\left[\frac{1}{n\pi}(1-\cos(n\pi))\sin(n\pi x)\right]}$$

$A_w = \frac{1}{\pi}\int_{-\infty}^{\infty} f(p)\cos(wp)\,dp = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0 + \int_0^1 1\cdot\cos(wp)\,dp + \int_1^{\infty} 0\right]$

$= \frac{1}{\pi}\frac{1}{w}\left[\sin(wp)\right]_0^1 = \boxed{\frac{1}{\pi w}\sin w}$

$B_w = \frac{1}{\pi}\int_{\infty}^{\infty} f(p)\sin(wp)\,dp = \frac{1}{\pi}\left[\int_{\infty}^{0} 0 + \int_0^1 \sin(wp)\,dp + \int_1^{\infty} 0\right]$

$= \frac{1}{\pi}\left[-\frac{1}{w}\cos wp\right]_0^1 = -\frac{1}{w\pi}(\cos w - 1)$

$\therefore f(x) = \int_0^{\infty}\left\{\frac{1}{w\pi}\sin(wp)\cos(wp) - \frac{1}{w\pi}\cos(w)\sin(wp) + \frac{1}{n\pi}\sin(wp)\right\}dp$

$= \frac{1}{w\pi}\sin(w-wp)$

$e^{i(a-b)} = \cos(a-b) + i\sin(a-b) =$

$= \cos a\cos b + \sin a\sin b + i\sin a\cos b - i\cos a\sin b$

$e^{i(a+b)} = \cos(a+b) + i\sin(a+b) = e^{ia}e^{ib}$

$e^{-i(a+b)} = \cos(a+b) - i\sin(a+b) = e^{-ia}e^{-ib}$

$f(x) = e^{-2x}\ (x>0)$ (비주기=주기)

<Laplace Integral>

$\int_0^{\infty} e^{-st}\cos(wt)\,dt = \frac{w}{s^2+w^2}$

$A_w = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos(wx)\,dx$

$= \frac{1}{\pi}\int_0^{\infty} e^{-2x}\cos(wx)\,dx$

Li) $\Rightarrow \frac{1}{\pi}\times\frac{w}{2^2+w^2}$

$\int_0^{\infty} e^{-st}\sin(wt)\,dt = \frac{s}{s^2+w^2}$

$B_w = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin(wx)\,dx$

$= \frac{1}{\pi}\int_{\infty}^{\infty} e^{-2x}\sin(wx)\,dx$ Li) $\Rightarrow \frac{1}{\pi}\frac{2}{2^2+w^2}$

$\therefore f(x) = \int_0^{\pi}\frac{1}{\pi}\frac{w}{2^2+w^2}\cos(wx) + \frac{1}{\pi}\frac{2}{2^2+w^2}\sin(wx)\,dx$

$$\Rightarrow e^{-2x} = \frac{2}{\pi}\int_0^{\infty} \frac{w\cos(wx)}{2^2+w^2}\,dw \Rightarrow \boxed{\int_0^{\infty} \frac{w}{2^2+w^2}\cos(wx)\,dw = \frac{\pi}{2}e^{-2x}}$$

$$e^{-2x} = \frac{4}{\pi}\int_0^{\infty} \frac{1}{2^2+w^2}\sin(wx)\,dw \Rightarrow \boxed{\int_0^{\infty} \frac{1}{2^2+w^2}\sin(wx)\,dw = \frac{\pi}{4}e^{-2x}}$$

반주기라서 2로 나눠야함!

Fourier Transform

$$F(x) = \int_{-\infty}^{\infty} f(x)\,e^{-iwx}\,dx$$

← 컴퓨터가 이것을 계산하면 DFT 혹은 FFT

〈테일러 급수〉 Taylor Series

조건: 무한번 미분가능 (부분적분을 해야 되서)

$$\int_a^x f'(t)\,dt = f(x) - f(a)$$

$$= \int_a^x (-1)\{-f'(t)\}\,dt$$

$$\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$$

$$\int_a^x (-1)\{-f'(t)\}\,dt = \Big[t f'(t)\Big]_a^x - \int_a^x (-t)(-f''(t))\,dt$$

$$= x f'(x) - a f'(a) - \int_a^x (-t)(-f''(t))\,dt$$

$$= x\int_a^x f''(t)\,dt + x f'(a) - a f'(a) - \int_a^x (-t)(-f''(t))\,dt$$

$$= x f'(x) - x f'(a) + (x-a) f'(a) - \int_a^x (-t)(-f''(t))\,dt$$

$$- \Big[-\frac{1}{2}t^2(-f''(t))\Big]_a^x - \int_a^x \left(-\frac{1}{2}t^2\right)\left(-f^{(3)}(t)\right)dt$$

$$= -\frac{1}{2}x^2 f''(x) + \frac{1}{2}a^2 f''(a)$$

↳

$\sin \rightarrow \cos \rightarrow -\sin \rightarrow -\cos \rightarrow \sin$

$$f(x) - f(a) = (x-a)f'(a) + \frac{1}{2}(x-a)^2 f''(a) + \cdots$$

$$f(x) = \sin x,\quad f'(x) = \cos x,\quad f^2(x) = -\sin x,\quad f^3(x) = -\cos x$$

$$\sin x = 0 + x + 0 - \frac{x^3}{3!} + 0 + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!}$$

$$\lim_{x\to 0} \sin x = x$$

$$f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \cdots + \frac{(x-a)^n}{n!}f^n(a)$$; 테일러급수

$$f(0) + x f'(0) + \frac{x^2}{2!}f''(0) + \cdots + \frac{x^n}{n!}f^n(0)$$; 맥클로린 급수