

TI DSP, MCU 및 Xilinx Zynq FPGA 프로그래밍 전문가 과정

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1) 제어의 안정성

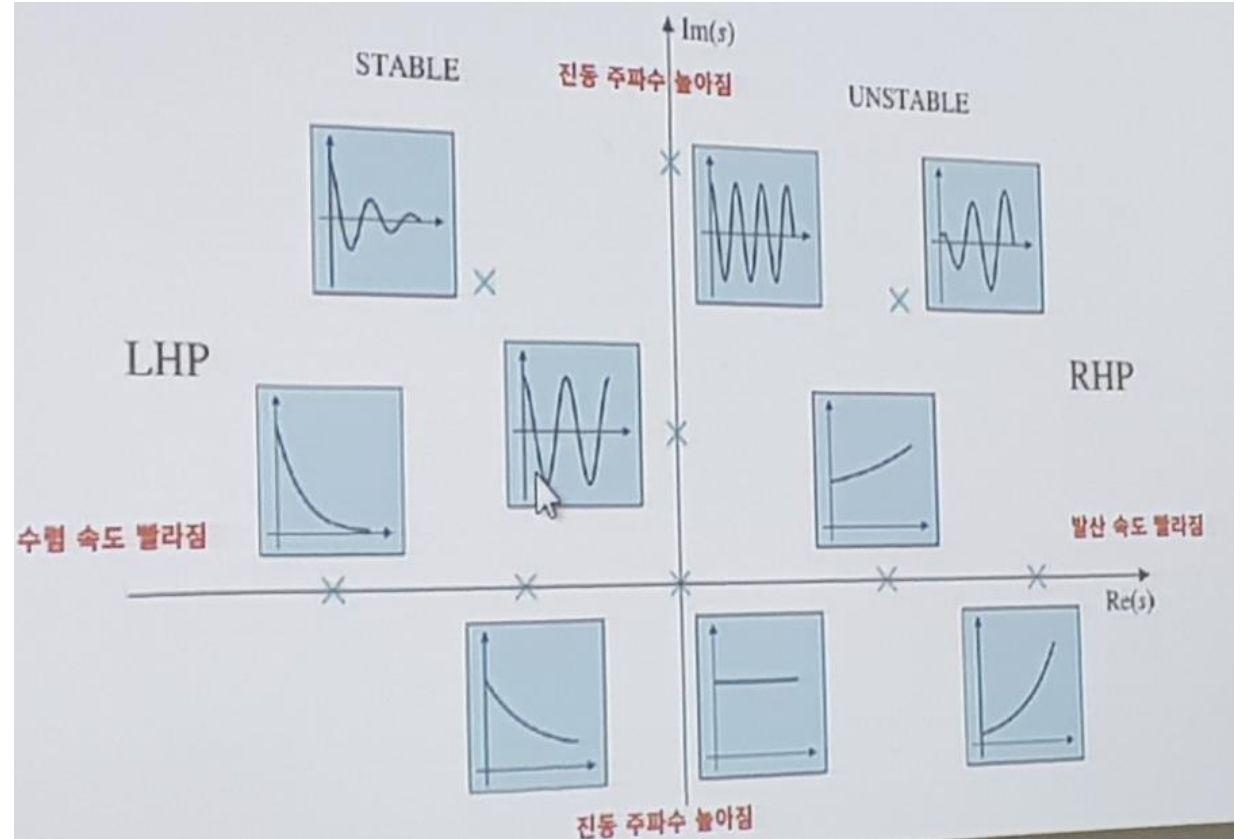
Downloads/Automatic_Control.pdf

$\alpha > 0$ 이면 발산 $\alpha < 0$ 이면 수렴한다.

$s = \alpha \pm j\omega$

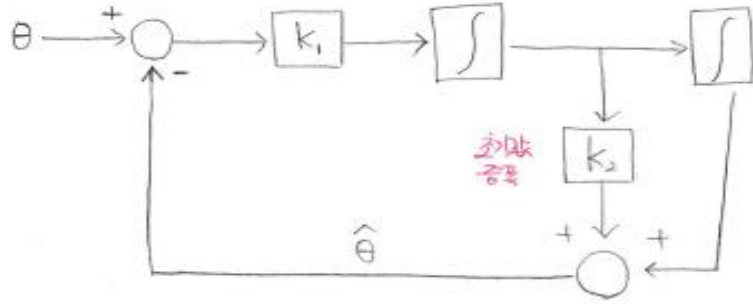
복소 평면에서 보자면 극점이 좌반평면에 있으면 수렴, 우반 평면에 있으면 발산한다.
극점이 좌반평면에 있으면서 허수축으로부터 멀수록 출력 신호의 수렴 속도가 빨라진다.
극점이 우반평면에 있으면서 허수축으로부터 멀수록 출력 신호의 발산 속도가 빨라진다.

극점의 허수 부분 ω 는 출력 신호의 진동 성분과 관계된다.
 ω 가 클수록, 즉 극점이 실수축으로부터 멀수록 진동 주파수는 높아진다.



2) Block Diagram

○ 제어공학 - Block Diagram



$$\hat{\theta} = k_2 \int_0^t k_1 (\theta(\tau) - \hat{\theta}(\tau)) d\tau + \int_0^t \int_0^{\tau} k_1 (\theta(\tau) - \hat{\theta}(\tau)) d\tau d\tau$$

$$\hat{\theta} = \frac{k_2 k_1}{s} (\theta(s) - \hat{\theta}(s)) + \frac{k_1}{s^2} (\theta(s) - \hat{\theta}(s))$$

$$\therefore \frac{\hat{\theta}}{\theta} = H(s) = \frac{k_1 (1 + k_2 s)}{s^2 + k_1 k_2 s + k_1}$$

○ 전달함수

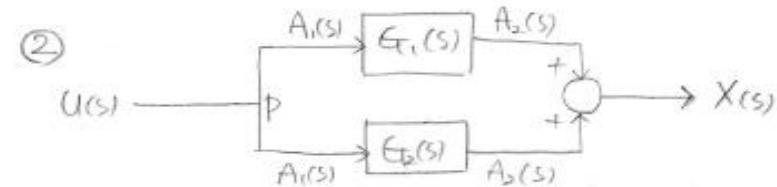


$$X(s) = A(s) \cdot G_2(s)$$

$$A(s) = U(s) G_1(s)$$

$$X(s) = G_1(s) G_2(s) U(s)$$

$$\therefore G(s) = \frac{X(s)}{U(s)} = G_1(s) G_2(s)$$



$$A_1(s) = U(s)$$

$$A_2(s) = A_1(s) G_1(s)$$

$$A_3(s) = A_1(s) G_2(s)$$

$$X(s) = A_2(s) + A_3(s)$$

$$X(s) = U(s) (G_1(s) + G_2(s))$$

$$\therefore G(s) = \frac{X(s)}{U(s)} = G_1(s) + G_2(s)$$

3) 2차 시스템의 Block Diagram

○ 2차 시스템

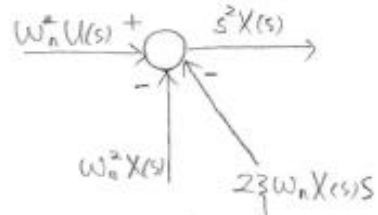
$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2x(t) = \omega_n^2u(t)$$

초기값: $x(0) = \dot{x}(0) = 0$

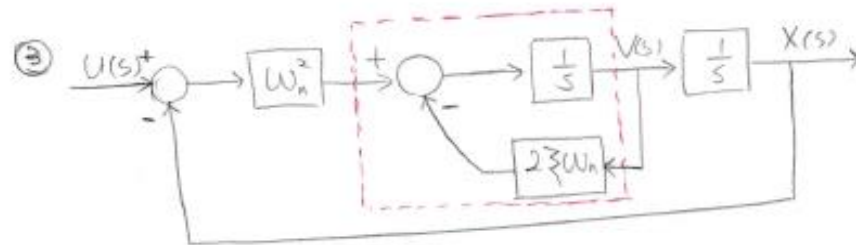
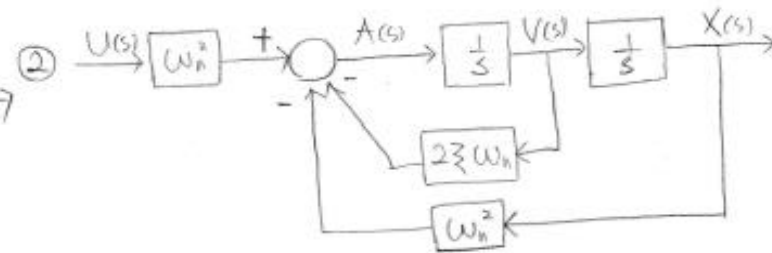
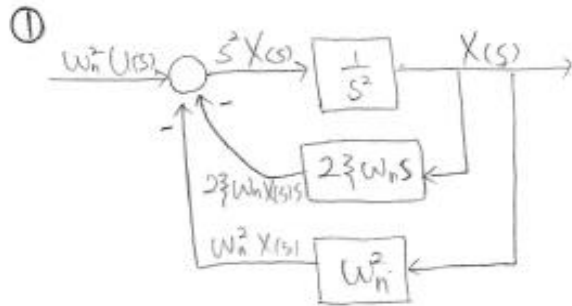
라플라스 변환

$$X(s)s^2 + \underbrace{2\zeta\omega_n}_{\text{저감비}} \underbrace{X(s)s}_{\text{출력}} + \underbrace{\omega_n^2}_{\text{공진주파수}} X(s) = \underbrace{\omega_n^2}_{\text{입력}} U(s)$$

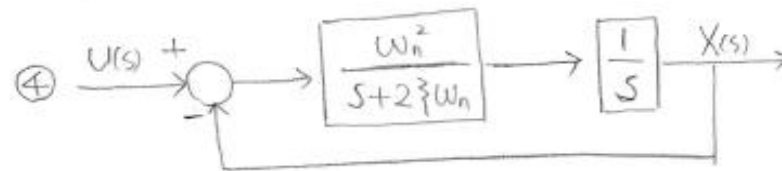
$$\omega_n^2 U(s) - 2\zeta\omega_n X(s)s - \omega_n^2 X(s) = X(s)s^2$$



블록다이어그램

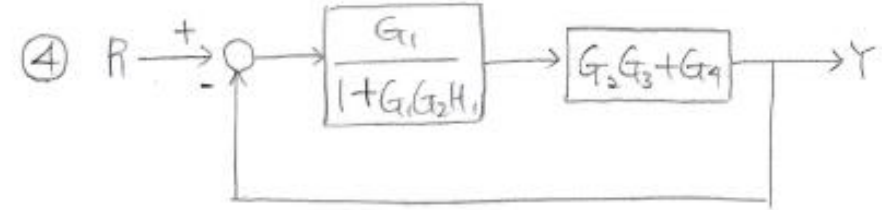
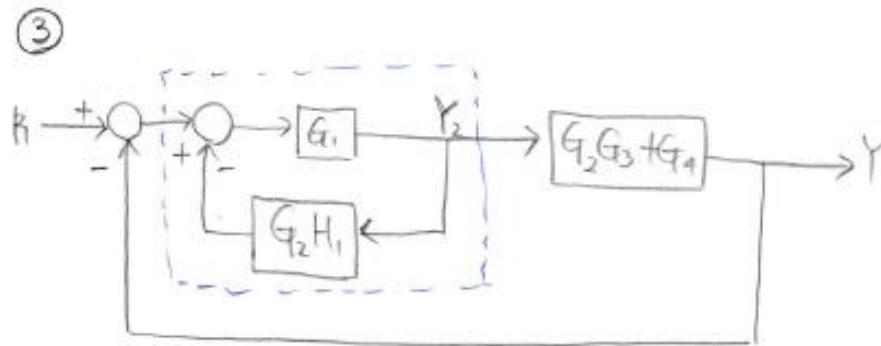
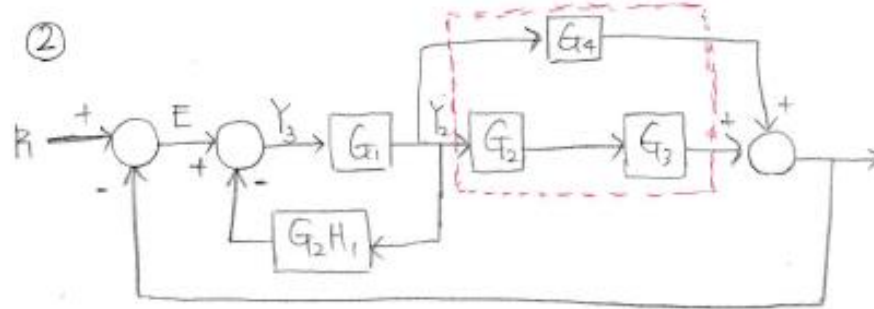
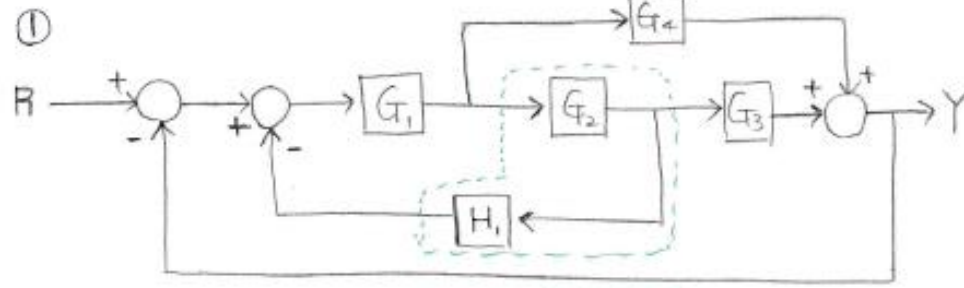


$$\frac{G(s)}{1+G(s)H(s)} = \frac{\frac{1}{s}}{1+\frac{2\zeta\omega_n}{s}} = \frac{1}{s+2\zeta\omega_n}$$



4) Block의 전달함수

o Block의 전달함수

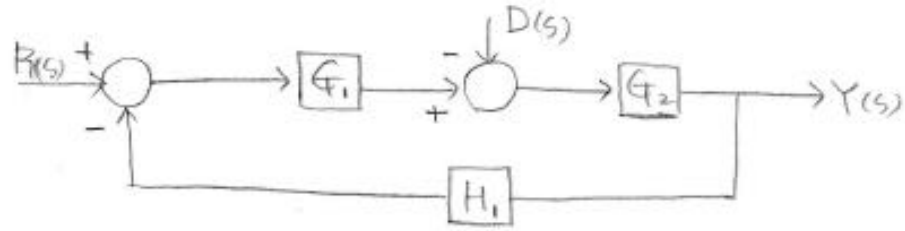


$$\therefore G(s) = \frac{\frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 H_1}}{1 + \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 H_1}}$$

$$= \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 H_1 + G_1 G_2 G_3 + G_1 G_4}$$

5) 외란(입력 다수)의 Block Diagram

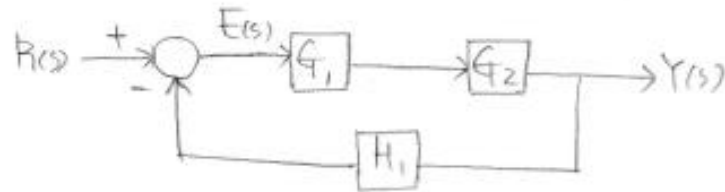
○ 외란(입력 다수)의 블록 다이어그램



$$Y_{\text{total}} = Y_R|_{D=0} + Y_D|_{R=0}$$

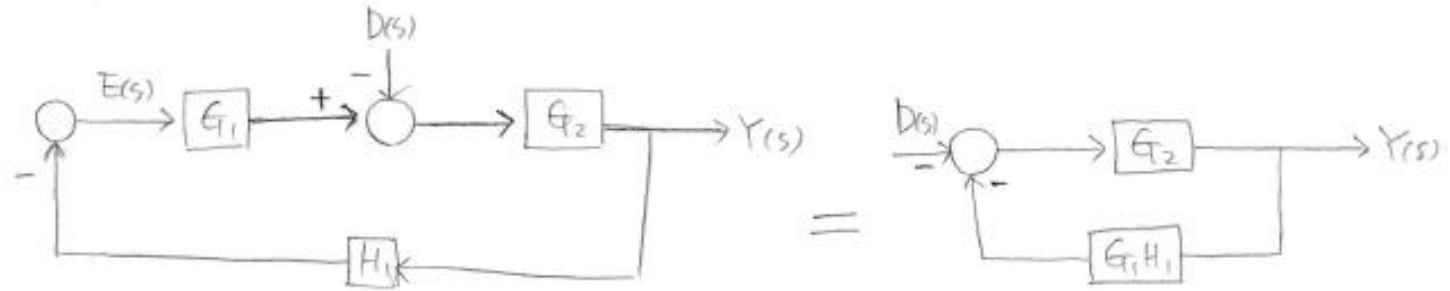
① $D(s) = 0$

$$\frac{Y(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H_1(s)}$$



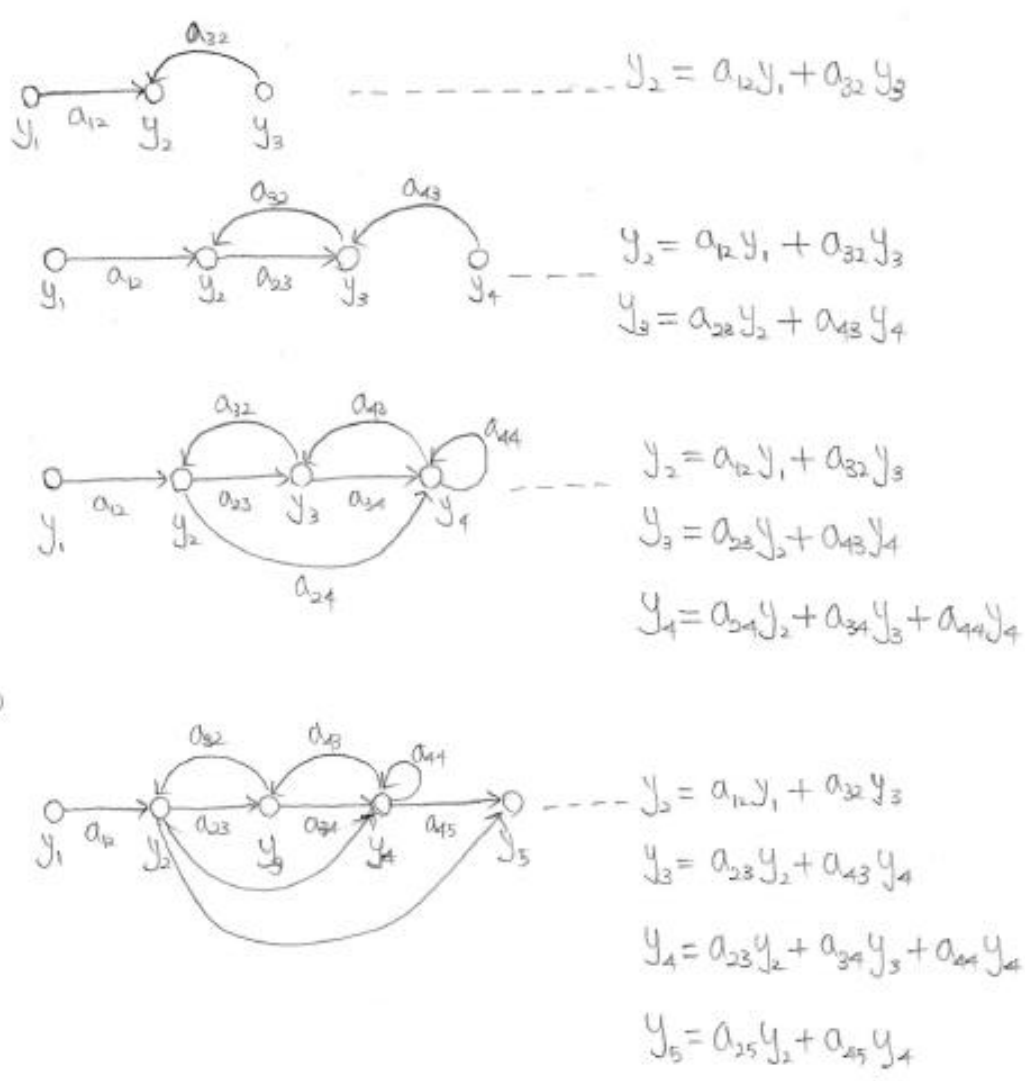
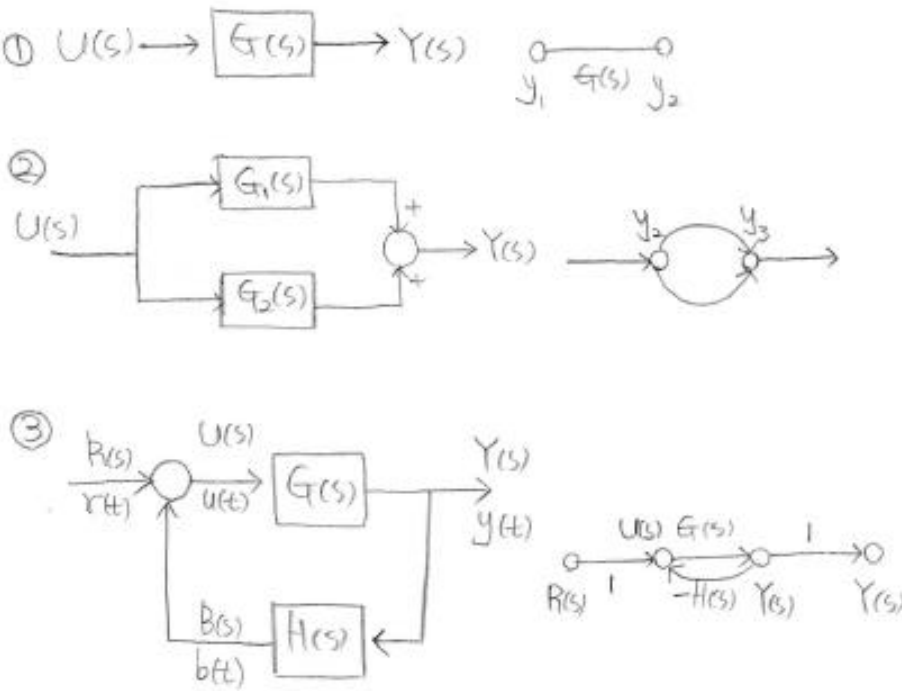
② $R(s) = 0$

$$\frac{Y(s)}{D(s)} = \frac{-G_2(s)}{1 + G_1(s)G_2(s)H_1(s)}$$



6) Signal Flow Graph

o Signal Flow Graph



7) SFG 기반 전달함수 구하기

○ SFG 기반 전달함수 구하기

$$M = \frac{y_{out}}{y_{in}} = \sum_{k=1}^N \frac{M_k \Delta_k}{\Delta}$$

y_{in} = 입력변수

y_{out} = 출력변수

M = 입력과 출력 변수 사이의 이득

N = 입력과 출력 사이의 전방경로 개수

M_k = 입력과 출력 변수 사이에 k 번째 전방경로의 이득

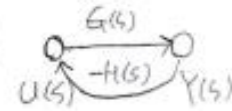
$$\Delta = 1 - \sum_i L_{i1} + \sum_j L_{j2} - \sum_k L_{k3} + \dots$$

Δ 는 $1 - (\text{각각의 모든 루프의 이득의 합}) + (\text{두 개의 비접촉루프의 가능한 모든 조합의 이득의 곱의 합}) - (\text{세 개의 비접촉루프의 가능한 모든 조합의 이득의 곱의 합}) + \dots$

L_m = m 개의 비접촉루프의 가능한 m 번째 ($m=1, 2, 3, \dots$) 조합의 이득

Δ_k = k 번째 전방경로와 접하지 않는 signal flow graph에 대한 Δ

○ 용어 정의

경로 :  같은 구간을 여러 번 거쳐도 무방

전방경로 :  $y_1 \rightarrow y_3$ 까지 가는 방법은
비접촉선과 접촉선 파악.

경로이득 : $y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4$ 의 경우 $a_{12} \times a_{23} \times a_{34}$

루프 : 시작과 끝이 같은 것.

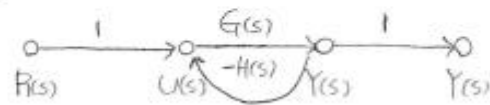


비접촉 루프 : Signal Flow Graph의 두 부분이 공통이 바디를 공유하지 않으면 비접촉이다.

8) SFG 예제

o SFG 예제

①



$$M_1 = G(s)$$

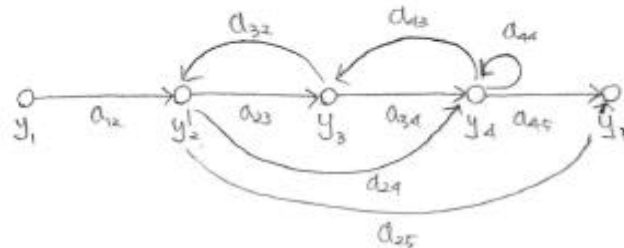
$$L_{11} = -G(s)H(s)$$

$$\Delta_1 = 1 \quad (\Delta_k = k\text{번째 전방경로와 접하지 않는 SFG에 대한 } \Delta)$$

$$\Delta = 1 - L_{11} \quad (\Delta = 1 - \sum_i L_{i1} + \sum_{j2} L_{j2} - \sum_k L_{k3} + \dots)$$

$$\therefore M = \frac{y_{out}}{y_{in}} = \sum_{k=1}^N \frac{M_k \Delta_k}{\Delta} = \frac{M_1 \Delta_1}{\Delta} = \frac{G(s)}{1 + G(s)H(s)}$$

②



• 전방경로 이득

$$y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow y_5 \quad M_1 = a_{12} a_{23} a_{34} a_{45}$$

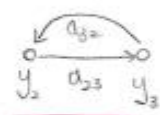
$$y_1 \rightarrow y_2 \rightarrow y_5$$

$$M_2 = a_{12} a_{25}$$

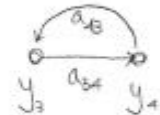
$$y_1 \rightarrow y_2 \rightarrow y_4 \rightarrow y_5$$

$$M_3 = a_{12} a_{24} a_{45}$$

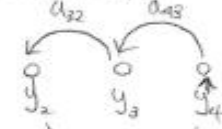
• Loop 이득 (L_{ij} : i 개의 비접촉 루프의 곱)



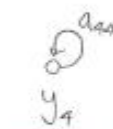
$$L_{11} = a_{23} a_{32}$$



$$L_{21} = a_{34} a_{43}$$



$$L_{31} = a_{23} a_{32} a_{43} a_{34}$$



$$L_{41} = a_{44}$$

$$L_{12} = a_{23} a_{32} a_{44}$$

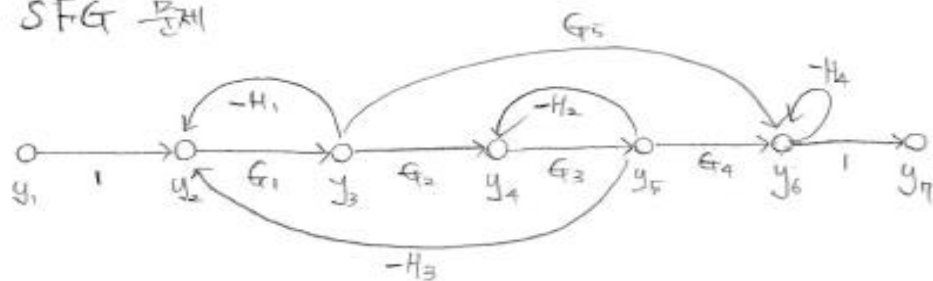
$$\Delta_1 = \Delta_3 = 1, \quad \Delta_2 = 1 - a_{34} a_{43} - a_{44} \quad (y_2 \rightarrow y_3 \rightarrow y_2, \quad y_4 \rightarrow y_4)$$

$$\Delta = 1 - (L_{11} + L_{21} + L_{31} + L_{41}) + L_{12} = 1 - (a_{23} a_{32} + a_{34} a_{43} + a_{23} a_{32} a_{43} + a_{44}) + a_{23} a_{32} a_{44}$$

$$\therefore M = \frac{y_2}{y_1} = \sum_{k=1}^N \frac{M_k \Delta_k}{\Delta} = \frac{M_1 \Delta_1}{\Delta} = \frac{a_{12} (1 - a_{34} a_{43} - a_{44})}{1 - (a_{23} a_{32} + a_{34} a_{43} + a_{23} a_{32} a_{43} + a_{44}) + a_{23} a_{32} a_{44}} \quad \therefore M = \frac{y_5}{y_1} = \sum_{k=1}^N \frac{M_k \Delta_k}{\Delta} = \frac{a_{12} a_{23} a_{34} a_{45} + a_{12} a_{25} (1 - a_{34} a_{43} - a_{44}) + a_{12} a_{24} a_{45}}{1 - (a_{23} a_{32} + a_{34} a_{43} + a_{23} a_{32} a_{43} + a_{44}) + a_{23} a_{32} a_{44}}$$

9) SFG 문제

○ SFG 문제

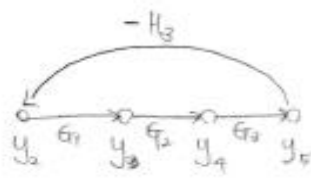
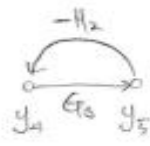
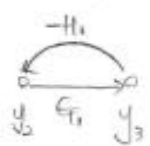


1) 전방경로이득

$$y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_6 \rightarrow y_7 \quad M_1 = G_1 G_2 G_3 G_4$$

$$y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_6 \rightarrow y_7 \quad M_2 = G_1 G_5$$

⇒ Loop 이득



$$L_{11} = -G_1 H_1, \quad L_{21} = -G_3 H_2, \quad L_{31} = -H_4, \quad L_{41} = -G_1 G_2 G_3 H_3$$

3) 비전환루프

$$L_{12} = (-G_1 H_1)(-G_3 H_2) = G_1 H_1 G_3 H_2, \quad L_{22} = G_1 H_1 H_4, \quad L_{32} = G_3 H_2 H_4, \quad L_{42} = G_1 G_2 G_3 H_3 H_4 \quad (2개 조합)$$

$$L_{31} = -G_1 H_1 G_3 H_2 H_4$$

$$\Delta = 1 - (L_{11} + L_{21} + L_{31} + L_{41}) + (L_{12} + L_{22} + L_{32} + L_{42}) - (L_{31})$$

$$= 1 + G_1 H_1 + G_3 H_2 + H_4 + G_1 G_2 G_3 H_3 + G_1 H_1 G_3 H_2 + G_1 H_1 H_4 + G_3 H_2 H_4 + G_1 H_1 G_3 H_2 H_4$$

4) 결과

$$\therefore M = \frac{y_6}{y_1} = \frac{M_1 \Delta_1}{\Delta} = \frac{1 - (L_{21} + L_{31} + L_{42})}{\Delta} = \frac{1 + G_3 H_2 + H_4 + G_3 H_2 H_4}{\Delta}$$

$$\therefore \frac{y_6}{y_1} = \frac{M_1 \Delta_1}{\Delta} = \frac{G_1 G_2 (1 - (L_{21}))}{\Delta} = \frac{G_1 G_2 (1 + H_4)}{\Delta}$$

$$\therefore \frac{y_6}{y_1} = \frac{M_1 \Delta_1}{\Delta} + \frac{M_2 \Delta_2}{\Delta}, \quad \Delta_1 = 1, \quad \Delta_2 = 1 + G_3 H_2$$

$$= \frac{G_1 G_2 G_3 G_4 + G_1 G_5 (1 + G_3 H_2)}{\Delta}$$