Xilinx Zynq FPGA, TI DSP, MCU 기반의 프로그래밍 및 회로 설계 전문가 과정

강사 – Innova Lee(이상훈)

gcccompil3r@gmail.com

학생 – hoseong Lee(이호성)

hslee00001@naver.com

목차

Laplace transform

$\mathcal{L}\{f\}(s) = \int_0^{\infty} e^{-st} f(t)\,dt$

$\dfrac{\mathcal{L}\{f(t)\}}{\mathcal{L}\{f(t)\}(s)} = \int_0^{\infty} e^{-st} f(t)\,dt$

$0 \sim \infty$ 적분 ⇒ 미정적분(이상적분)

$\lim_{k\to\infty} \int_0^k e^{-sk} f(t)\,dk$ ⇒ 부분적분

$\cancel{f(x)g(x) = \int f'(x)g(x) - \int f(x)g'(x)}$

⇒ $\int f'(x)g(x) = f(x)g(x) - \int f(x)g'(x)$

⇒ $\int e^{-sk} f(k) = -\frac{1}{s} e^{-sk} f(k) - \int -\frac{1}{s} e^{-sk} f'(k)$

다항식의 경우 차수만큼 이짓을 해줘야한다.

삼각함수는 두번하면 합성 or e^{ix}

ex) $f(t) = e^{-at}$

$\lim_{k\to\infty} \int_0^k e^{-ak}\,dk = \lim_{k\to\infty} -\frac{1}{a}\left[e^{-ak}\right]_0^k = \lim_{k\to\infty} -\frac{1}{a} e^{-ak} + \frac{1}{a}$

$= \frac{1}{a} \Rightarrow \frac{1}{s+a}$

ex2) $f(t) = \sin(t)$

$\lim_{k\to\infty} \int_0^k e^{-sk} \cdot \sin(k)\,dk = -e^{-sk}\cdot\cos k + \frac{1}{s}\int e^{-sk}\cdot\cos k\,dk$

$f(x) \cdot g(x)$

$\frac{1}{2i}\left(\frac{1}{s+i} - \frac{1}{s-i}\right) \quad \frac{s-i-s-i}{(s+i)(s-i)} \quad \frac{-2i}{2i} \quad \frac{-1}{(s+i)(s-i)} = \frac{1}{s^2+1}$

$\int f(x) g'(x) = f(x)g(x) - \int f'(x)g(x)$

ex2) $f(t) = \sin(t)$

$$\int_0^\infty e^{-st} \cdot \sin(t) dt = \lim_{x\to\infty} \int_0^x e^{-sx} \cdot \sin x \, dx$$

$$\lim_{x\to\infty} A = -e^{-sx} \cdot \cos x - s \int_0^x e^{-sx} \cos x$$

$$= -e^{-sx} \cdot \cos x - s \left(e^{-sx} \sin x + s \int_0^x e^{-sx} \sin x \, dx \right)$$

$$= -e^{-sx} \cdot \cos x - s \cdot e^{-sx} \cdot \sin x - s^2 \int_0^x e^{-sx} \sin x \, dx$$

$$\lim_{x\to\infty} (1+s^2) A \quad \lim_{x\to\infty} \frac{-e^{-sx} \cdot \cos x - s e^{-sx} \sin x}{1+s^2} = \lim_{x\to\infty} \frac{-e^{-sx}(\cos x + s \sin x)}{1+s^2}$$

$$= \frac{1}{1+s^2}$$

⇒ ~~이런풀이법~~ $\cos(t) = \frac{e^{it} + e^{-it}}{2}$, $\sin(t) = \frac{e^{it} - e^{-it}}{2i}$

ex3) $f(t) = \cos(\omega t)$ $\quad \int_0^\infty e^{-st} \cos(\omega t) dt = \frac{1}{2} \int_0^\infty e^{-st} (e^{i\omega t} + e^{-i\omega t})$

$$= \frac{1}{2} \int_0^\infty e^{-(s-i\omega)t} + e^{-(s+i\omega)t} dt$$

$$= \frac{1}{2} \cdot \frac{1}{-(s-i\omega)} \left[e^{-(s-i\omega)t} \right]_0^\infty + \frac{1}{2} \frac{1}{-(s+i\omega)} \left[e^{-(s+i\omega)t} \right]_0^\infty$$

$$= \frac{1}{2(s-i\omega)} + \frac{1}{2(s+i\omega)} = \frac{1}{2} \frac{2s}{(s-i\omega)(s+i\omega)} = \frac{s}{s^2+\omega^2}$$

$$\int_0^\infty e^{-st} \sin \omega t = \frac{1}{2i} \int e^{-st} (e^{i\omega t} - e^{-i\omega t}) = \frac{1}{2i} \int_0^\infty e^{-(s t - i\omega t)} - e^{-(s t + i\omega t)}$$

$$\left[\quad \frac{1}{\quad} e^{-(s+i\omega)t} = \frac{1}{2i(s+i\omega)} \cdot e^{-(s+i\omega)t} \right]_0^\infty$$

ex4) $f(t) = t^2$

$-\frac{1}{s} \cdot e^{-st} t^2 - \frac{2}{s} \int_0^\infty e^{-st} t \, dt$

$\Rightarrow \int_0^\infty e^{-st} \cdot t^2 \, dt = \cancel{-\frac{1}{s} \int e^{-st}}$

$f'(x) g(x)$ $\quad f(x)g(x) - \int f(x) g'(x)$

$\left(-\frac{1}{s} e^{-st} t - \frac{1}{s} \int_0^\infty e \ldots\right)$

$= \left[-\frac{1}{s} \cdot e^{-st} \cdot t^2 - \frac{2}{s^2} e^{-st} t + \frac{2}{s^3} (e^{-st}) \right]_0^\infty$

$= \frac{2}{s^2} + \frac{2}{s^3} = \frac{2}{s^2}\left(1 + \frac{1}{s}\right) = \frac{2}{s^3}$

$f(x) = x \ln x \quad f'(x) = \ln x + x \cdot \frac{1}{x} \Rightarrow \frac{\lambda!}{s^{\lambda+1}}$

도함수의 라플라스 트랜스폼.

• 1계도함수 라플라스 변환

$sF(s) - f(0)$

$\Rightarrow \boxed{\mathcal{L}\{f(t)\}(s) = \int_0^\infty e^{-st} f(t) \, dt} = F(s)$

$\mathcal{L}\{f'(t)\}(s) = \int_0^\infty e^{-st} f'(t) \, dt = [f(t) e^{-st}]_0^\infty - \int_0^\infty -s e^{-st} f(t)$

• 2계도함수 라플라스 변환

$f''(t)$

$\mathcal{L}\{f''(t)\}(s) = \int_0^\infty f''(t) e^{-st} \, dt = [f'(t) e^{-st}]_0^\infty - \int_0^\infty f'(t) \cdot -s e^{-st} \, dt$

$= -f'(0) + s \int_0^\infty f'(t) e^{-st} \, dt$

$= -f'(0) + s(sF(s) - f(0))$

$= s^2 F(s) - s f(0) - f'(0)$

ex1) $y' + 4y + 1$

$\mathcal{L}\{y'\} = s$

$\mathcal{L}\{y\} =$

$\mathcal{L}\{1\} = \int_0$

$sY(s) - y(0)$

$Y(s)$

ex2) $\frac{x^2+2x \ldots}{(x^2+x+5)}$

역라플라스

특별한 함수가

↳ 보여.

이를 사용하

Laplace Transfo...

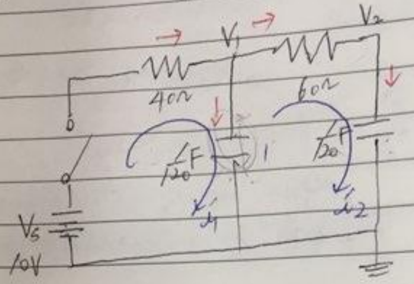
RCRC

$\cancel{f(t) = \sin(\omega t)}$

$\cancel{F(s) = \frac{\omega}{s^2+\omega^2}}$

$g_1(0) = g_2(0) = 0$

$\cancel{\frac{10 - V_1}{40} =}$



망전류법

$$120\int(i_1 - i_2)\,dt$$

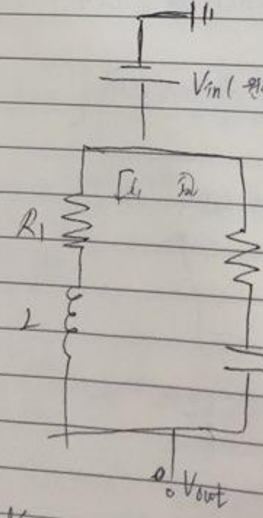
1. $40 i_1 + \cancel{120\int i_1\,dt} = 10$

2. $60 i_2 + 120\int i_2\,dt$ ~~$+ 120\int(i_2 - i_1)$~~

Ⓥ $i_C = C\frac{dV}{dt} \Rightarrow V = \frac{1}{C}\int i\,dt$ > 들은 전략에서 씀.

전하량. $\frac{dQ}{dt} = i$, $Q = CV$

$V = IR$



V_{in} (원래는 RF 신호가 들어와야함) $= 10$

$$\cancel{V_{in} = R_1 i + L\frac{di}{dt} + R_2 i_2 + \frac{1}{C}\int}$$

1. $10 = i_1 R_1 + L i_1'$

2. $10 = i_2 R_2 + \frac{1}{C}\int i_2\,dt$

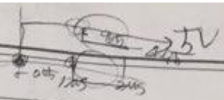
$\Downarrow$ 라플라스 트랜스폼

$$\frac{10}{s} = I_1(s)R_1 + L\{sI_1(s) - i_1(0)\}$$

$$\frac{10}{s} = I_2(s)R_2 + \frac{1}{C}\cdot\frac{1}{s}I_2(s)$$

$V_L = L\frac{di}{dt}$

$i_C = C\frac{dV}{dt}$

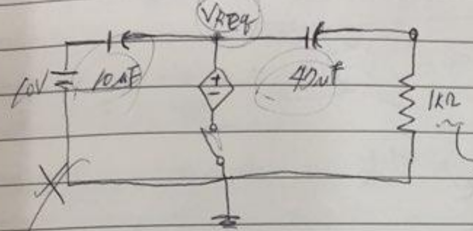


$$\int_0^\infty e^{-st}\cdot\cancel{(sI_1(s)-i_1(0))}$$

$5H(t)-0.05$

⇒ ~~종속전원이기가 없으므로 그냥 바로 풀면된다.~~

$$I_1(s)=\frac{10}{Ls^2+R_1s} \qquad I_2(s)=\frac{10}{\frac{1}{c}+R_2s}$$



커패시터안에서 전압이 실제로 변한다.
↳ 음리모형값을 미리 해준는다.

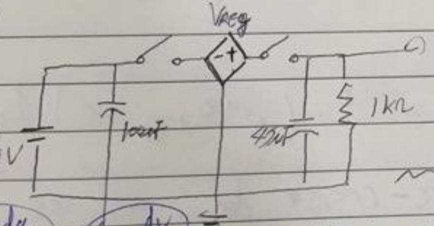
5V
4.95V

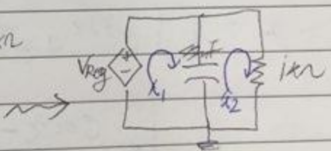
~~이면 V에 걸린것이다.~~

$$V_{Req}=5H(t)-0.05[H(t-10^{-6})]+0.05((H(t-2\times10^{-6})-0.05[H(t-3\times10^{-6})]$$

$q_2(0)$

q_2 $\frac{dq}{dt}$ ⇒ $I=\frac{dq}{dt}$ q_2





$i=\frac{dq}{dt}$ $i_c=C\frac{dv}{dt}$ ⇒ 둘중 하나로 해도 결과에는 변함이 없음.

1. $V_{Req}=\frac{q_1-q_2}{c}=\frac{q_1-q_2}{49}10^6$ $q=CV$ $\therefore V=\frac{1}{c}q$

2. $i_2R+\frac{q_2-q_1}{c}=0 \Rightarrow 1000\dot{q}_2+\frac{q_2-q_1}{49}10^{6}=0$

값없이 풀자. 그냥!

$$5H(t)-0.05H(t-10^{-6})+0.05H(t-2\times10^{-6})-0.05H(t-3\times10^{-6})=\frac{10^6}{49}(q_1-q_2)$$

~~2주1번3주~~

⇒ q_1, q_2를 시간에 대한 함수로 바꾼
이를 시간에 대해 적분하면 I가 나옴.
q_2에 $\frac{1}{c}$을 곱하면 전압을 구할 수 있다.

ex1) $y' + 4y + 1 = 0$

$\mathcal{L}\{y'\} = sY_s - y(0)$

$\mathcal{L}\{y\} = Y(s)$

보통, 주파수

$\mathcal{L}\{1\} = \int_0^\infty e^{-st}\,dt = \left[-\frac{1}{3}e^{-st}\right]_0^\infty = \frac{1}{3}$

$sY(s) - y(0) + 4Y(s) + \frac{1}{3} = 0$

$Y(s) = \left(y(0) - \frac{1}{3}\right)\frac{1}{(s+4)}$

오차를 제일 줄인

1. 샘플링기반 Z Transform

2. 샘플링 기반 차분방정식

3. 차분방정식 결과와 아날로그 해석

└ 데이터 자체 결과는 제일 좋음.

→ 범용적으로 쓰임 컴퓨터로 이용해 구현해냄

안정성 → Z transform

• 라플라스 제어기 안정 → 단일미분방정식이 없음

• 퓨리에 이건 그냥 주파수측면에서 보려고

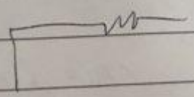
ex2) $\frac{x^2+2x-3}{(x^2+x+5)(x-2)^2} = \frac{(x+3)(x-1)}{(x^2+x+5)(x-2)(x-2)} = \left\{\frac{(x^2-2)^2}{(x^2+x+5)} + \frac{x^2+x+5}{(x-2)^2}\right\}$

역라플라스 → 회로 · 그전에 Heaviside Function을 알아보자.

특별한 함수가 특정시간때에 함수를 도출할 수 있음.

↳ 왜, 시간에 대한 정보가 없기 때문에 시간에 대한 함수를 만들어줌.

이를 사용하는 목적.



최초의 정의식에 $\cos\left(\frac{m\pi}{T}x\right)$를 곱하고 적분해보자.

$$\int_{-T}^{T} f(x)\cdot\cos\left(\frac{m\pi}{T}x\right)dx = \int_{-T}^{T}\frac{a_0}{2}\cos\left(\frac{m\pi}{T}x\right)dx + \int_{-T}^{T}\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi}{T}x\right)\cdot\cos\left(\frac{m\pi}{T}x\right)$$

$$+\, b_n\cos\left(\frac{m\pi}{T}x\right)\sin\left(\frac{n\pi}{T}x\right)$$

$\cos n\pi$ $(-1)^n$

$$=\sum_{n=1}^{\infty}\int_{-T}^{T}a_n\cos\frac{m\pi}{T}x\cos\frac{n\pi}{T}x\ (m=n)$$

$$a_nT$$

$$\Rightarrow a_n=\frac{1}{T}\int_{-T}^{T}f(x)\cdot\cos\left(\frac{n\pi}{T}x\right)dx \quad \begin{cases}0 & (m\neq n)\\ T & (m=n)\end{cases}$$

$$\int_{-T}^{T}f(x)\sin\left(\frac{m\pi}{T}x\right)dx=\frac{a_0}{2}\int_{-T}^{T}\sin\left(\frac{m\pi}{T}x\right)+\sum_{n=1}^{\infty}\int_{-T}^{T}a_n\cos Ax\sin mx$$

$$+\int_{-T}^{T}b_n\sin Ax\sin Ax\,dx\}$$

$$\sum_{n=1}^{\infty}b_nT$$

$$b_nT$$

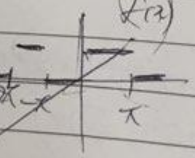
$$\Rightarrow b_n=\frac{1}{T}\int_{-T}^{T}f(x)\sin\left(\frac{n\pi}{T}x\right)dx$$

$$\sin^2 x=\frac{1}{4}\left(e^{2ix}+e^{-2ix}-2\right)$$

$$=\frac{1}{2}\left(\frac{e^{2ix}+e^{-2ix}}{2}-1\right)$$

$$(\cos 2x)^0-1$$

$$f(x)\begin{cases}0 & (-\pi\le x<0)\\ 1 & (0\le x<\pi)\end{cases}$$



$$\frac{1}{2\pi}\int_{-2\pi}^{2\pi}f(x)\sin\frac{n\pi}{T}x\,dx$$

$$0\ \frac{1}{2\pi}\int_{-2\pi}^{-\pi}\sin\frac{n\pi}{2}x\,dx$$

$$\left[\frac{1}{2\pi}\frac{2}{n}\cos\frac{n\pi}{2}x\right]_{-2\pi}^{-\pi}$$

$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{1}{\pi n^2}\{\cos(n\pi) - 1\} \cdot \cos(n\pi) - \frac{1}{n}\cos(n\pi) \cdot \sin n\pi x\}$

$\Rightarrow y = x$ 와 동치 $(x > 0)$ $\longrightarrow$ 일반적인 RC 회로로는 직선을 만들지 못한다

$\longrightarrow$ 정현~~[illegible]~~ 합성으로 만들 수가 있다.

푸리에 적분. $\rightarrow$ 비주기함수도 푸리에 시리즈로 풀 수가 있다.

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi}{T} x) + b_n \sin(\frac{n\pi}{T} x)$ $\Rightarrow$ $a_0 = \frac{1}{T}\int_{-T}^{T} f(x)dx$

$a_n = \frac{1}{T}\int_{-T}^{T} f(x)\cos(\frac{n\pi}{T}x)$

$b_n = \frac{1}{T}\int_{-T}^{T} f(x)\sin(\frac{n\pi}{T}x)$

$-\pi, 0$ $\quad \pi, 2\pi$ $\quad \cos\frac{n x}{2}$

$\sin x$ $\quad \sin -x$ $\quad \cancel{\sin x}$

$$a_n = \frac{1}{2\pi}\int_{-2\pi}^{2\pi} f(x)\cdot \cos\frac{n\pi}{2\pi}x\,dx$$

$$= \frac{1}{2\pi}\left(\int_{-2\pi}^{-\pi} 1\cdot\cos\frac{n}{2}x + \int_0^{\pi} 1\cdot\cos\frac{n}{2}x\,dx\right)$$

$$= \frac{1}{2\pi}\left(\left[\frac{2}{n}\sin\frac{n}{2}x\right]_{-2\pi}^{-\pi} + \left[\frac{2}{n}\sin\frac{n}{2}x\right]_{0}^{\pi}\right)$$

$$= \frac{1}{2\pi}\left(\frac{2}{n}\left(\cancel{\sin\left(-\frac{n}{2}\pi\right)} - \sin(-n\pi)\right) + \frac{2}{n}\left(\cancel{\sin\frac{n}{2}\pi}\right)\right) = \frac{1}{2\pi}\cdot\left(\frac{2}{n}\cdot\sin n\pi\right)$$

$$= \frac{1}{n\pi}\sin n\pi \rightarrow 0$$

$$b_n = \frac{1}{2\pi}\int_{-2\pi}^{2\pi} f(x)\cdot\sin\left(\frac{n\pi}{2\pi}x\right)dx$$

$$= \frac{1}{2\pi}\left(\int_{-2\pi}^{-\pi}\sin\left(\frac{n}{2}x\right)dx + \int_0^{\pi}\sin\frac{n}{2}x\,dx\right)$$

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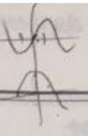
$$= -\frac{1}{2\pi}\left\{\left[\frac{2}{n}\cos\frac{n}{2}x\right]_{-2\pi}^{-\pi} + \left[\frac{2}{n}\cos\frac{n}{2}x\right]_0^{\pi}\right\} = -\frac{1}{2\pi}\left\{\frac{2}{n}\left(\cos\left(-\frac{n}{2}\pi\right) - \cos(-n\pi)\right) + \frac{2}{n}\left(\cos\frac{n}{2}\pi - 1\right)\right\}$$

$$= -\frac{1}{2\pi}\left(\frac{2}{n}\left(\cos\frac{n\pi}{2} - \cos n\pi + \cos\frac{n\pi}{2} - 1\right)\right)$$

$$\boxed{= -\frac{1}{n\pi}\left(2\cos\frac{n}{2}\pi - \cos n\pi - 1\right)}$$

(0

$-\pi, 0$ $\quad \pi, 2\pi$ $\quad \cos\frac{n\pi}{2}$



$\sin x$ $\quad \sin -x$ $\quad \cancel{\sin x}$

$$a_n = \frac{1}{2\pi}\int_{-2\pi}^{2\pi} f(x)\cdot\cos\frac{n\cancel{\pi}}{2\cancel{\pi}}x\,dx$$

$$= \frac{1}{2\pi}\left(\int_{-2\pi}^{-\pi}1\cdot\cos\frac{n}{2}x + \int_{0}^{\pi}1\cdot\cos\frac{n}{2}x\,dx\right)$$

$$= \frac{1}{2\pi}\left(\left[\frac{2}{n}\sin\frac{n}{2}x\right]_{-2\pi}^{-\pi} + \left[\frac{2}{n}\sin\frac{n}{2}x\right]_{0}^{\pi}\right)$$

$$= \frac{1}{2\pi}\left(\frac{2}{n}\left(\cancel{\sin\left(-\frac{n}{2}\pi\right)} - \sin(-n\pi)\right) + \frac{2}{n}\left(\cancel{\sin\frac{n}{2}\pi}\right)\right) = \frac{1}{\cancel{2}\pi}\cdot\left(\frac{\cancel{2}}{n}\cdot\sin n\pi\right)$$

$$= \frac{1}{n\pi}\sin n\pi \to 0$$

$$= 0$$

$$b_n = \frac{1}{2\pi}\int_{-2\pi}^{2\pi} f(x)\cdot\sin\left(\frac{n\cancel{\pi}}{2\cancel{\pi}}x\right)dx$$

$$= \frac{1}{2\pi}\left(\int_{-2\pi}^{-\pi}\sin\left(\frac{n}{2}x\right)dx + \int_{0}^{\pi}\sin\frac{n}{2}x\,dx\right)$$

n

$$= -\frac{1}{2\pi}\left\{\left[\frac{2}{n}\cos\frac{n}{2}x\right]_{-2\pi}^{-\pi} + \left[\frac{2}{n}\cos\frac{n}{2}x\right]_{0}^{\pi}\right\} = -\frac{1}{2\pi}\left\{\left(\frac{2}{n}\left(\cos\left(-\frac{n}{2}\pi\right) - \cos(-n\pi)\right)\right.\right.$$

$$\left.\left. + \frac{2}{n}\left(\cos\frac{n}{2}\pi - 1\right)\right)\right.$$

$$= -\frac{1}{\cancel{2}\pi}\left(\frac{\cancel{2}}{n}\left(\cos\frac{n\pi}{2} - \cos n\pi + \cos\frac{n\pi}{2} - 1\right)\right)$$

$$\boxed{= -\frac{1}{n\pi}\left(2\cos\frac{n}{2}\pi - \cos n\pi - 1\right)}$$

co