TI DSP, MCU 및 Xilinx Zynq FPGA 프로그래밍 전문가 과정

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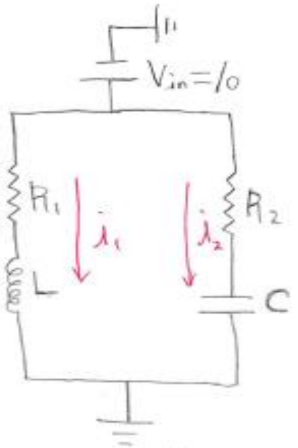
1) 라플라스 변환을 활용한 회로 해석

ㅇ 라플라스 변환을 활용한 회로 해석



1. $10 = i_1R_1 + Li_1'$

2. $10 = i_2R_2 + \frac{1}{C}\int i_2\,dt$

$$\Downarrow \mathcal{L}\{\ \}$$

1. $\frac{10}{s} = I_1(s)R_1 + L\{sI_1(s) - \overset{\nearrow 0}{i(0)}\}$

2. $\frac{10}{s} = I_2(s)R_2 + \frac{1}{C}\frac{1}{s}I_2(s)$

$$\Downarrow \text{정리}$$

$I_1(s) = \frac{10}{Ls^2 + R_1s}$, $I_2(s) = \frac{10C}{R_2Cs + 1}$

$V_L = L\frac{di}{dt}$

$i_c = C\cdot\frac{dV}{dt} \Rightarrow V_c = \frac{1}{C}\int i_c\,dt$

$$I_1(s) = \frac{10}{L\{(s+\frac{R_1}{2L})^2 - (\frac{R_1}{2L})^2\}} = \frac{20L}{R_1}\cdot\frac{\frac{R_1}{2L}}{(s+\frac{R_1}{2L})^2 - (\frac{R_1}{2L})^2}\cdot\frac{1}{L}$$

$\mathcal{L}^{-1}\{\frac{\omega}{s^2-\omega^2}\} = \cosh(\omega t)$

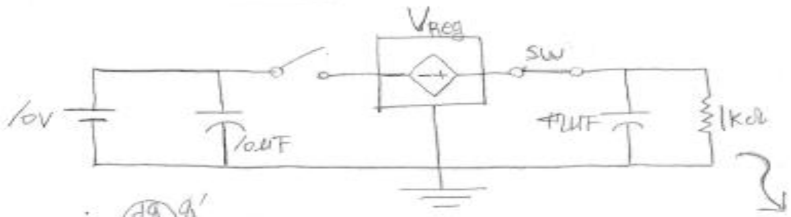
$\therefore\ \underline{i_1(t) = \frac{20}{R_1}\cosh(\frac{R_1}{2L}t)e^{-\frac{R_1}{2L}t}}$ $\underline{V_L(t) = \frac{R_1}{2L}\frac{20}{R_1}\sinh(\frac{R_1}{2L}t)e^{-\frac{R_1}{2L}t} - \frac{R_1}{2L}\frac{20}{R_1}\cosh(\frac{R_1}{2L}t)e^{\frac{R_1}{2L}t}}$

$I_2(s) = \frac{10}{R_2}\cdot\frac{1}{(s+\frac{1}{R_2C})} \xrightarrow{\mathcal{L}^{-1}} \underline{i_2(t) = \frac{10}{R_2}e^{-\frac{t}{R_2C}}}$, $V_c(t) = \frac{1}{C}\int i_2(t)\,dt$

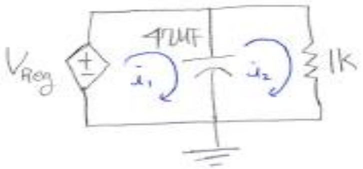
$= \underline{10(1 - e^{-\frac{t}{R_2C}})}$

2) 7805 레귤레이터를 Heaviside function과 laplace transform을 이용하여 해석하기

ㅇ 7805 레귤레이터를 헤비사이드 함수와 라플라스 변환을 이용하여 해석하기



$i = \frac{dq}{dt} \; q'$, $q = CV$



0 ~ 1μs 사이에 5V 스위치 off
1 ~ 2μs 사이에 4.95V 스위치 ON
2 ~ 3μs 사이에 5V 스위치 off
3 ~ 4μs 사이에 4.95V 스위치 ON

편의상 생략

$$V_{Reg} = 5[H(t)] - 0.05[H(t-10^{-6})] + \underline{0.05[H(t-2\times10^{-6})] - 0.05[H(t-3\times10^{-6})]}$$

$$1.\ V_{Reg} = \frac{q_1 - q_2}{c} \Rightarrow 5[H(t)] - 0.05[H(t-10^{-6})] = \frac{1}{c}(q_1 - q_2)$$

$$\mathcal{L}\{1번\} = \frac{5}{s} - 0.05\frac{1}{s}e^{-10^{-6}s} = \frac{1}{c}\{Q_1(s) - Q_2(s)\}$$

$$2.\ i_2 R + \frac{q_2 - q_1}{c} = 0 \Rightarrow q_2' R + \frac{q_2 - q_1}{c} = 0$$

$$\mathcal{L}\{2번\} = RsQ_2(s) - \cancel{q_2(0)}^{\,0} + \frac{1}{c}\{Q_2(s) - Q_1(s)\} = 0$$

$$Q_2(s) = \left(\frac{5}{s^2} - 0.05\frac{1}{s^2}e^{-10^{-6}s}\right)\frac{1}{R} = \frac{1}{s^2}(5 - 0.05e^{-10^{-6}s}) \times \frac{1}{10^3}$$

$$q_2(t) = \mathcal{L}^{-1}\{Q_2(s)\} = \{t\,5[H(t)] - 0.05(t-10^{-6})[H(t-10^{-6})]\} \times \frac{1}{10^3}$$

$$q_2'(t) = i_2(t) = \{5[H(t)] - 0.05[H(t-10^{-6})]\} \times \frac{1}{10^3} \,//$$

$$V_2(t) = i_2(t) \times 1k$$

$$= 5[H(t)] - 0.05[H(t-10^{-6})] \,//$$

3) 함수의 직교성 판단 (푸리에 급수 응용)

o 함수의 직교성 (푸리에 급수에 응용)

$\langle f(x), g(x) \rangle = \int_a^b f(x)g(x)\,dx = 0 \Rightarrow$ 두 함수가 서로 직교한다.

$\int_{-\pi}^{\pi} \sin(x)\sin(2x)\,dx \qquad \sin x = \frac{e^{ix}-e^{-ix}}{2i}, \quad \cos x = \frac{e^{ix}+e^{-ix}}{2}$

$= \int_{-\pi}^{\pi} \frac{e^{ix}-e^{-ix}}{2i} \cdot \frac{e^{2ix}-e^{-2ix}}{2i}\,dx$

$= -\frac{1}{4}\int_{-\pi}^{\pi} e^{3ix}+e^{-3ix}-(e^{ix}+e^{-ix})\,dx$

$= \frac{1}{2}\int_{-\pi}^{\pi} \cos(x)-\cos(3x)\,dx \Rightarrow \frac{1}{2}[\sin(\pi)-\sin(-\pi)] - \frac{1}{2}[\sin(3\pi)-\sin(-3\pi)]$ 0 0

※ 성질

1) $m \neq n \quad \langle f(x), g(x) \rangle = \int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,dx \Rightarrow \frac{1}{2}\left[\frac{\sin(m-n)x}{m-n} + \frac{\sin(m+n)x}{m+n}\right]_{-\pi}^{\pi}$

2) $m = n \quad \langle f(x), g(x) \rangle = \int_{-\pi}^{\pi} \sin(x)\sin(x)\,dx \Rightarrow -\frac{1}{2}\int \frac{e^{2ix}-e^{-2ix}}{2} - 1\,dx = \left[\frac{\cos 2x}{2} + \frac{1}{2}\right]_{-\pi}^{\pi}$

직교 X

※ $n \neq m$ 주기적분은 언제나 0 (cos, sin)

4,5) Fourier Series & 사각파 표현하기

○ Fourier Series

※ 주기적분의 특성

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{T}\right)x + b_n \sin\left(\frac{n\pi}{T}\right)x\right)$$ — 정의

$$\int_{-T}^{T} f(x) = \int_{-T}^{T} \frac{a_0}{2}\,dx \Rightarrow \underline{a_0 = \frac{1}{T}\int_{-T}^{T} f(x)\,dx}$$

$$\int_{-T}^{T} f(x)\cos\left(\frac{m\pi}{T}x\right)dx = \frac{a_0}{2}\int_{-T}^{T}\cancel{\cos\left(\frac{m\pi}{T}x\right)}^{0} + \sum_{n=1}^{\infty}\left\{\int_{-T}^{T} a_n \cos^2\left(\frac{m\pi}{T}x\right) + \underline{\int_{-T}^{T} b_n \cancel{\sin \cdot \cos}^{0}\,dx}\right\}$$

$$= \sum_{n=1}^{\infty}\left\{\underbrace{\int_{-T}^{T} a_n \cos^2\left(\frac{m\pi}{T}x\right)}_{a_n T}\right\} \Rightarrow \underline{a_n = \frac{1}{T}\int_{-T}^{T} f(x)\cos\left(\frac{m\pi}{T}x\right)dx}$$

$$\underline{b_n = \frac{1}{T}\int_{-T}^{T} f(x)\sin\left(\frac{m\pi}{T}x\right)dx}$$

○ 사각파 표현하기

$$f(x) = \begin{cases} 0 & (-\pi \le x < 0) \\ 1 & (0 \le x < \pi) \end{cases}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \left\{\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} 1\,dx\right\}\frac{1}{\pi} = 1$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos\left(\frac{n\pi}{T}x\right)dx$$

$$= \underline{\frac{1}{\pi}\left\{\int_{-\pi}^{\pi} 0\,dx + \int_{0}^{\pi}\cos\left(\frac{n\pi}{T}x\right)dx\right.} = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin\left(\frac{n\pi}{T}x\right)dx \Rightarrow \frac{1}{n\pi}\left(1-\cos(n\pi)\right)$$

$$\therefore f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\left\{-\frac{2}{n\pi}\sin\left(\frac{n\pi}{T}x\right)\right\}$$

$$\underline{= \frac{1}{2} + \sum_{n=1}^{\infty}\left\{\frac{1}{n\pi}\left[1-\cos(n\pi)\right]\sin(nx)\right\}}$$