Image Smoothing

김성영교수
금오공과대학교
컴퓨터공학과

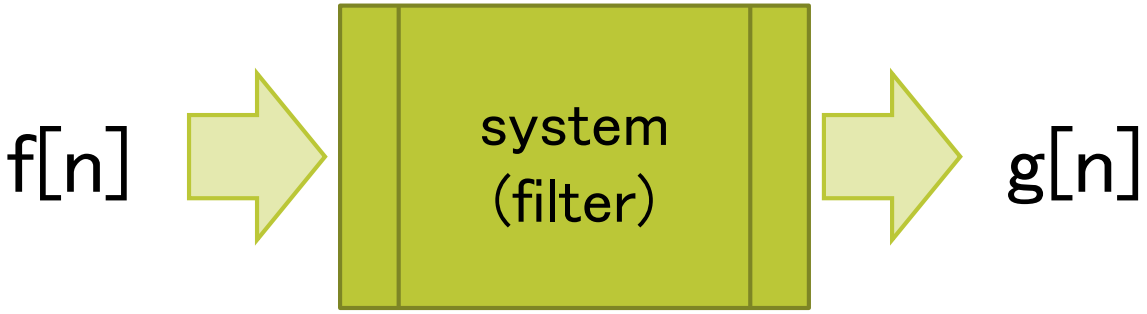
학습 □ □

- Image filtering
- Image smoothing

Image filtering

Filtering has its roots in the use of the Fourier transform for **signal processing**

spatial filtering



System

일련의 입력 신호를 처리하여 또 다른 일련의 출력 신호를 만들어 내는 실체

선형시스템 vs. 비선형시스템

$$f(ax_1[n]+bx_2[n]) = af(x_1[n])+bf(x_2[n])$$

시불변시스템 vs. 시변시스템

$$f(x[n-n_0]) \rightarrow y[n-n_0]$$

Filter

신호의 일부 성분을 제거하거나 일부 특성을 변경하기 위해 설계된 시스템의 한 종류

유한 임펄스 응답 (FIR) vs. 무한 임펄스 응답 (IIR)

$$y[n] = x[n] * h[n]$$
$$= \sum_{k=-\infty}^{\infty} x[n-k]h[k] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

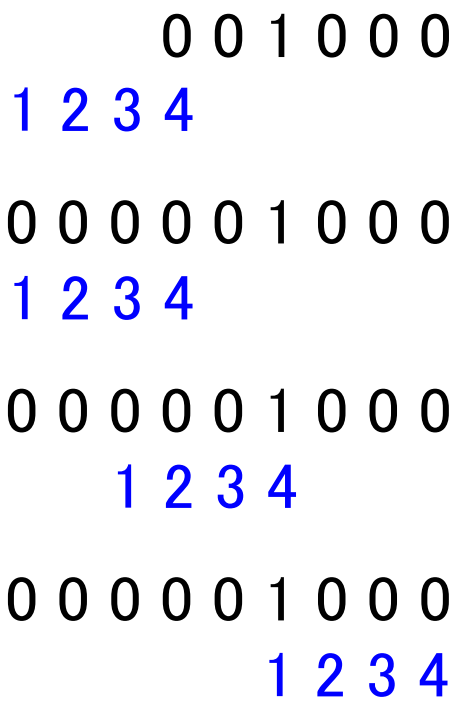
예: 3점의 계산을 위한 FIR 필터

$$h[n] = \left[\frac{1}{2} \;\; \frac{1}{3} \;\; \frac{1}{6} \right]$$

$$y[0] = x[0-0]h[0] + x[0-1]h[1] + x[0-2]h[2]$$
$$= 3 \times \frac{1}{2} + 0 \times \frac{1}{3} + 0 \times \frac{1}{6}$$

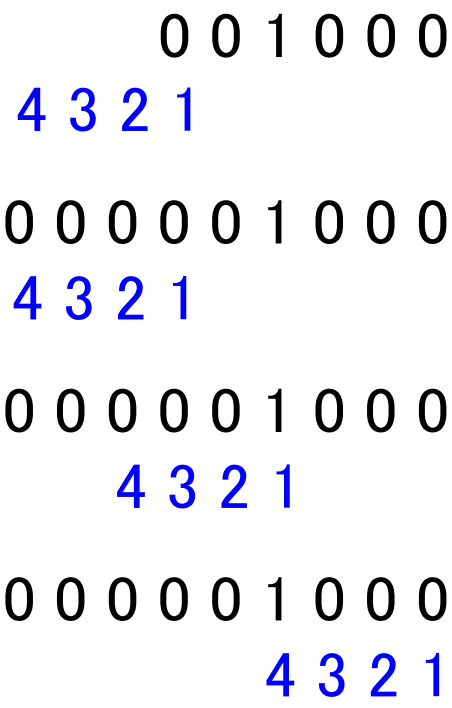
n	…	−2	−1	0	1	2	3	4	5	6	7	…
x[n]	0	0	0	3	6	9	6	3	0	0	0	0
h[n]		1/6	1/3	1/2								
y[n]	0	0	0	1/2	4	7	7	7/2	2	1/2	0	0

$h[n] = [1\,2\,3\,4] \quad x[n] = [0\,0\,1\,0\,0\,0]$



0 0 4 3 2 1

Correlation



0 0 1 2 3 4

Convolution

Convolution

마스크(mask), 필터(filter), 템플릿(template), 커널(kernel)

$$g(x,y) = h(x,y) * f(x,y) = \sum_{s=-a}^{a} \sum_{t=-a}^{b} h(s,t) f(x-s, y-t)$$

단,

$a = (m\text{-}1) / 2$

$b = (n\text{-}1) / 2$

입력 영상(f)의 크기: M x N

마스크(h)의 크기: m x n

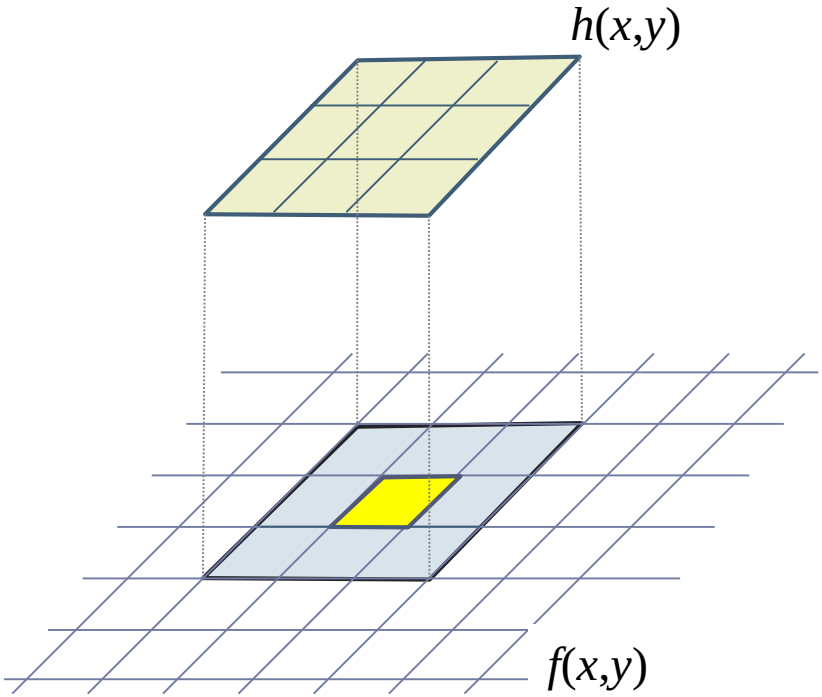
a	b	c
d	e	f
g	h	i

$h(x,y)$

$*$

r	s	t
u	v	w
x	y	z

$f(x,y)$



$$g = a \cdot z + b \cdot y + c \cdot x +$$
$$d \cdot w + e \cdot v + f \cdot u +$$
$$g \cdot t + h \cdot s + i \cdot r$$

Correlation

$$g(x,y) = h(x,y) \times f(x,y) = \sum_{s=-a}^{a} \sum_{t=-a}^{b} h(s,t)\, f(x+s, y+t)$$

a	b	c
d	e	f
g	h	i

$h(x,y)$

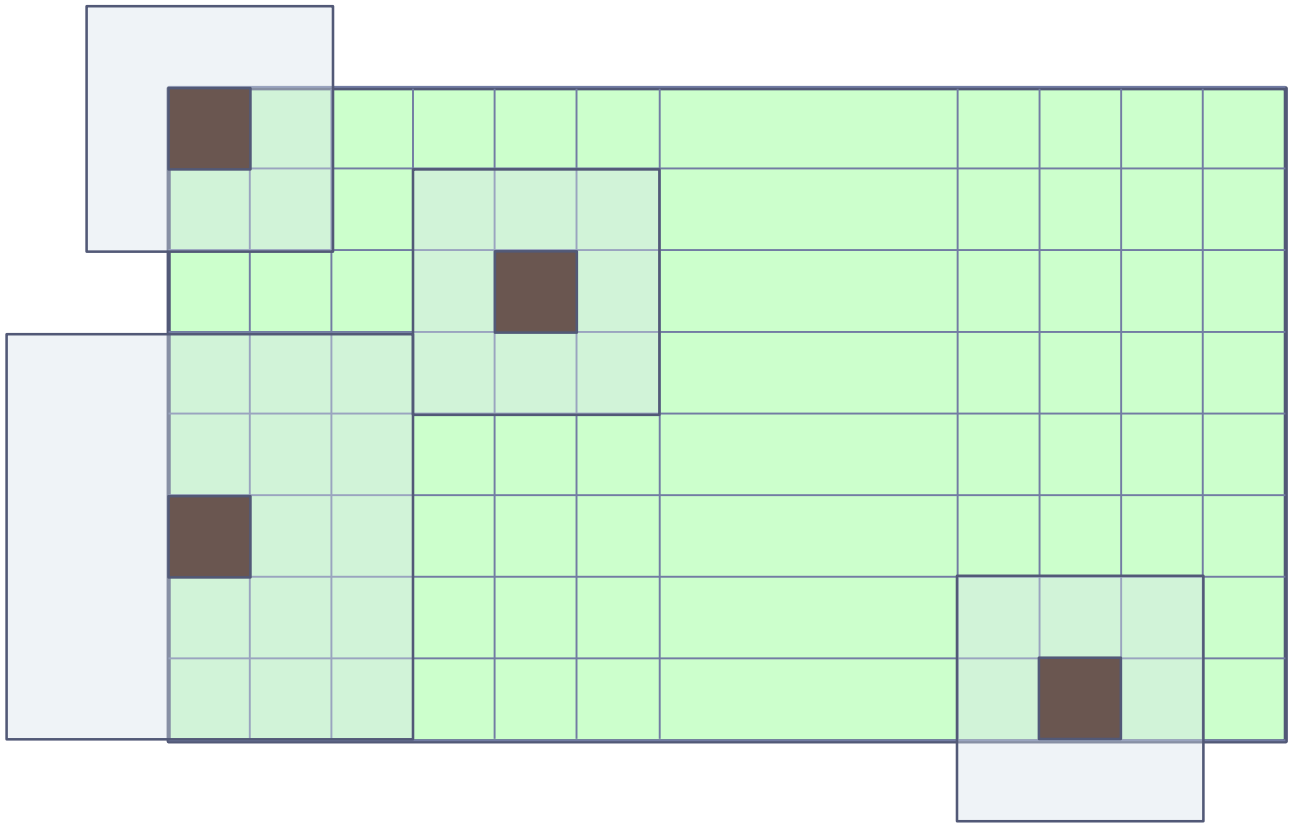
x

r	s	t
u	v	w
x	y	z

$f(x,y)$

$$g = a \cdot r + b \cdot s + c \cdot t +$$
$$d \cdot u + e \cdot v + f \cdot w +$$
$$g \cdot x + h \cdot y + i \cdot z$$

경계 처리



① 상수 값(예를 들어, 0)을 덧붙임

② 경계에 있는 픽셀 값을 복사

③ 영상을 주기적인 신호로 해석하여 맞은 편 픽셀 값을 복사 (Wrap-around)

④ 모든 이웃 픽셀이 정의되는 위치에서 convolution 연산을 시작하고 출력 영상의 경계 영역의 값은 입력 영상의 값으로 복사

Image smoothing

Used to give an image a softer or special effect or to eliminate noise

done by various types of mean, Gaussian and median filters

Mean filtering

$$\frac{1}{9}\left(v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 + v_8 + v_9\right)$$

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} \quad \frac{1}{10}\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 1\end{bmatrix} \quad \frac{1}{16}\begin{bmatrix}2 & 1 & 2\\ 1 & 4 & 1\\ 2 & 1 & 2\end{bmatrix} \quad \frac{1}{16}\begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix}$$

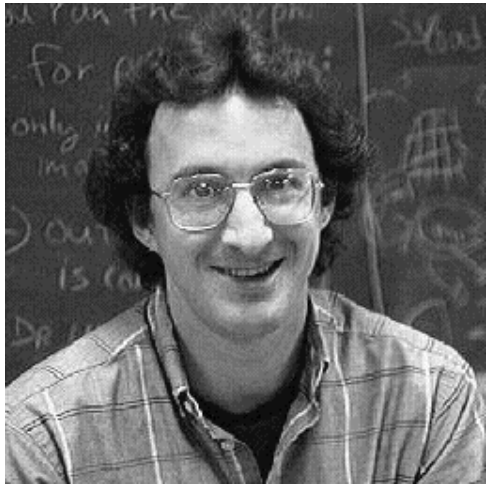
Box filtering

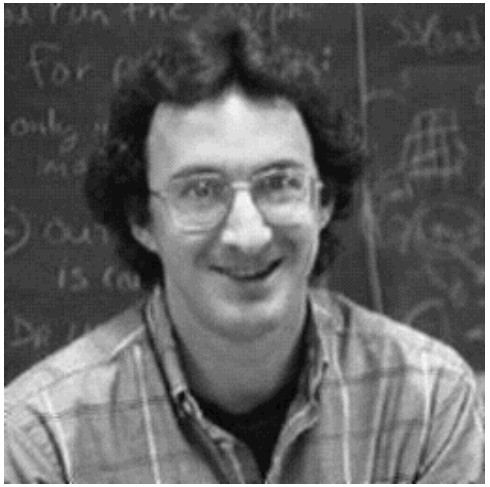
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	100	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10

Mean filtering

10	10	10	10	10	10	10	10
10	20	20	20	10	10	10	10
10	20	20	20	10	10	10	10
10	20	20	20	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10

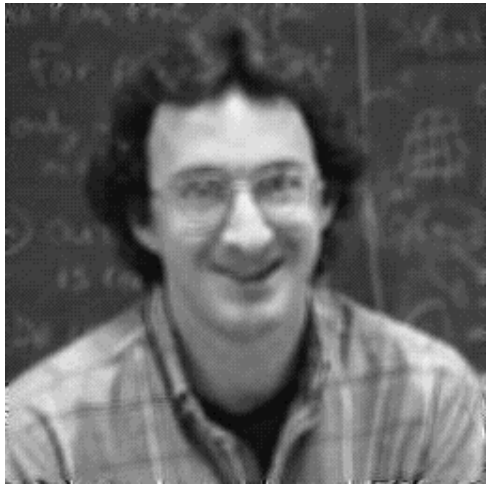
Original image





3*3
Mean filtering

5*5



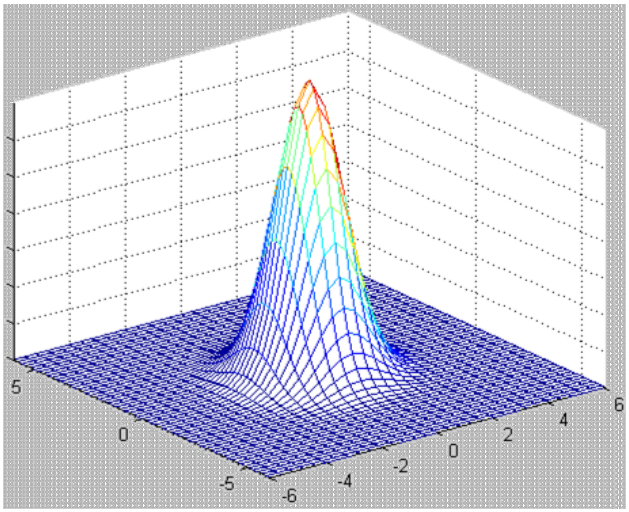
7*7

Gaussian filtering

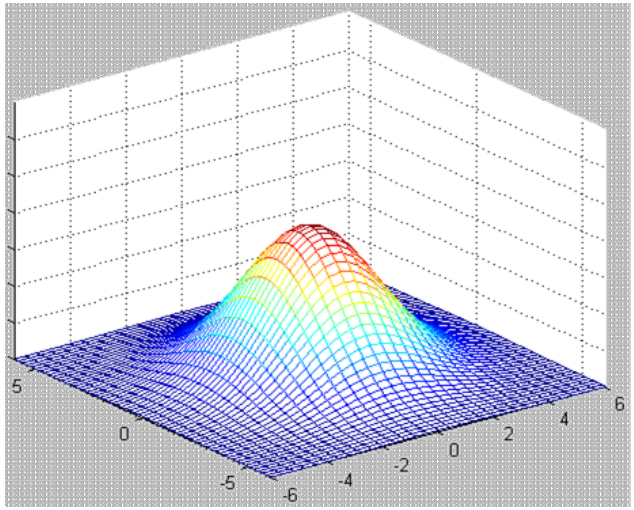
$$I'(x,y) = \sum_{s=-a}^{a} \sum_{t=-a}^{a} G(s,t) I(x+s, y+t)$$

$$G(s,t) = \frac{1}{2\pi\sigma^2} e^{-\frac{s^2+t^2}{2\sigma^2}}$$

±2.5σ: 98.76%
±3.0σ: >99%



σ=1



σ=2

Gaussian theoretically has infinite support, but we need a filter of finite size

$$\frac{1}{273}$$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

Discrete approximation to Gaussian function with $\sigma=1.0$

σ=1

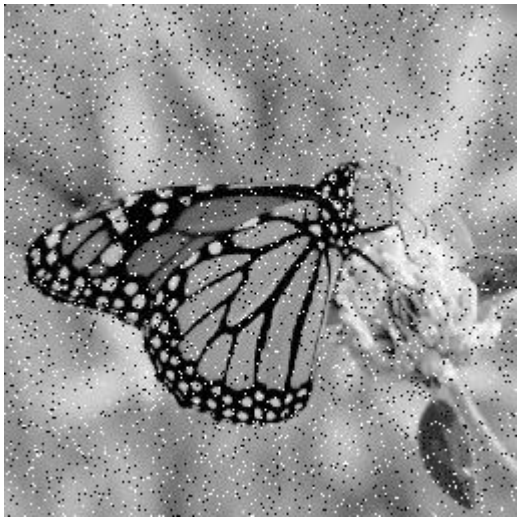
σ=2

Median filtering

Nonlinear filter

Useful for removing salt-, pepper-, or salt-and-paper noise

$$\begin{bmatrix} 5 & 5 & 6 \\ 3 & 4 & 5 \\ 3 & 4 & 7 \end{bmatrix} \quad (3,3,4,4,\mathbf{5},5,5,6,7) \quad \begin{bmatrix} 5 & 5 & 6 \\ 3 & \boxed{5} & 5 \\ 3 & 4 & 7 \end{bmatrix}$$

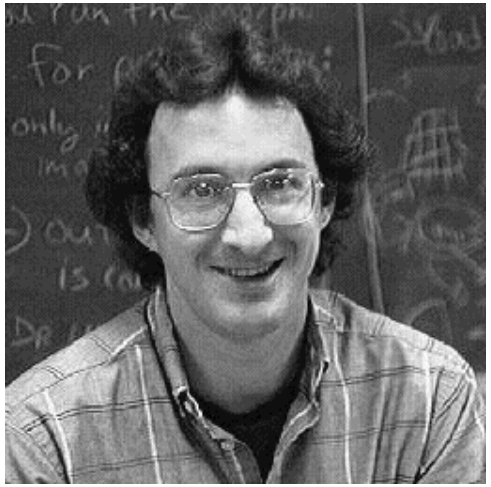


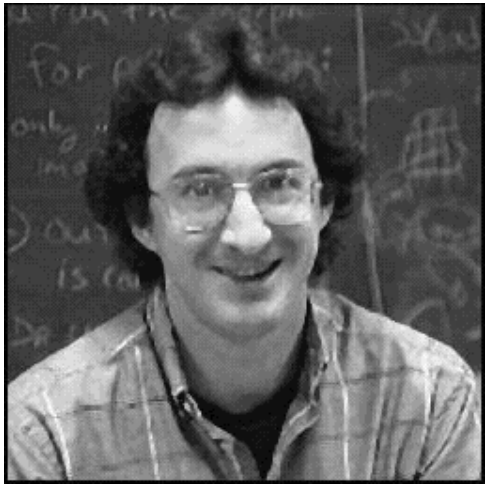
Original image

Mean Filtering

Median Filtering

Original image





3*3 Median filter

5*5

7*7

요약

- Image filtering
  - spatial filtering
  - with convolution, correlation
- Image smoothing
  - 영상을 부드럽게 변경하거나 노이즈를 제거
  - mean, Gaussian and media filters

Reference

- R. Gonzalez, R. Woods, **Digital Image Processing (2nd Edition),** Prentice Hall, 2002
- Scott E Umbaugh, **Computer Imaging**, CRC Press, 2005
- 김우생, **영상처리 및 패턴인식 배움터**, 생능, 2007