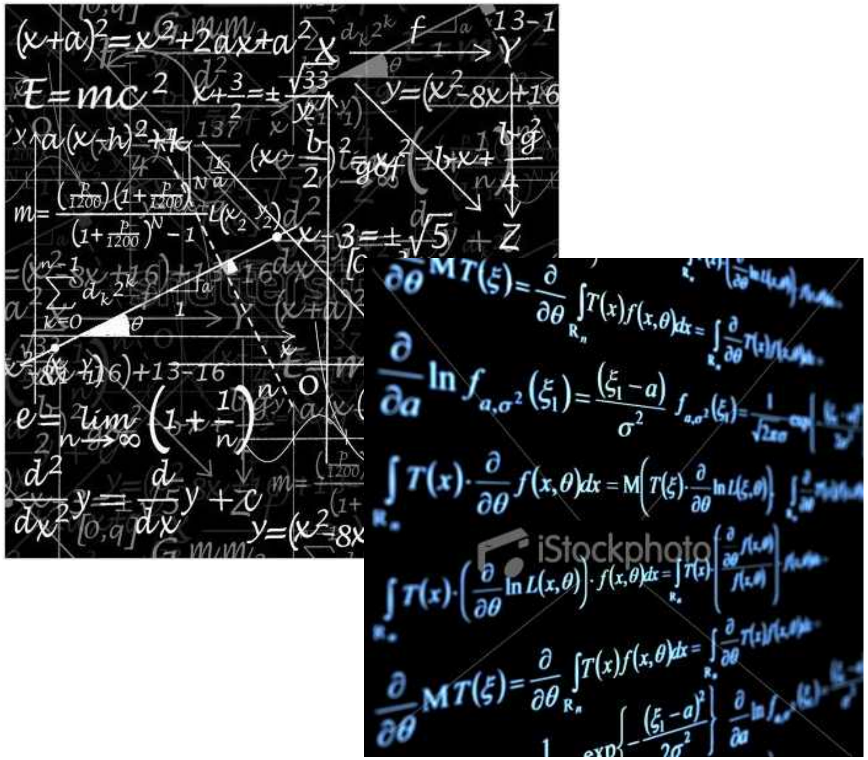
수치해석(Numerical Methods)



1

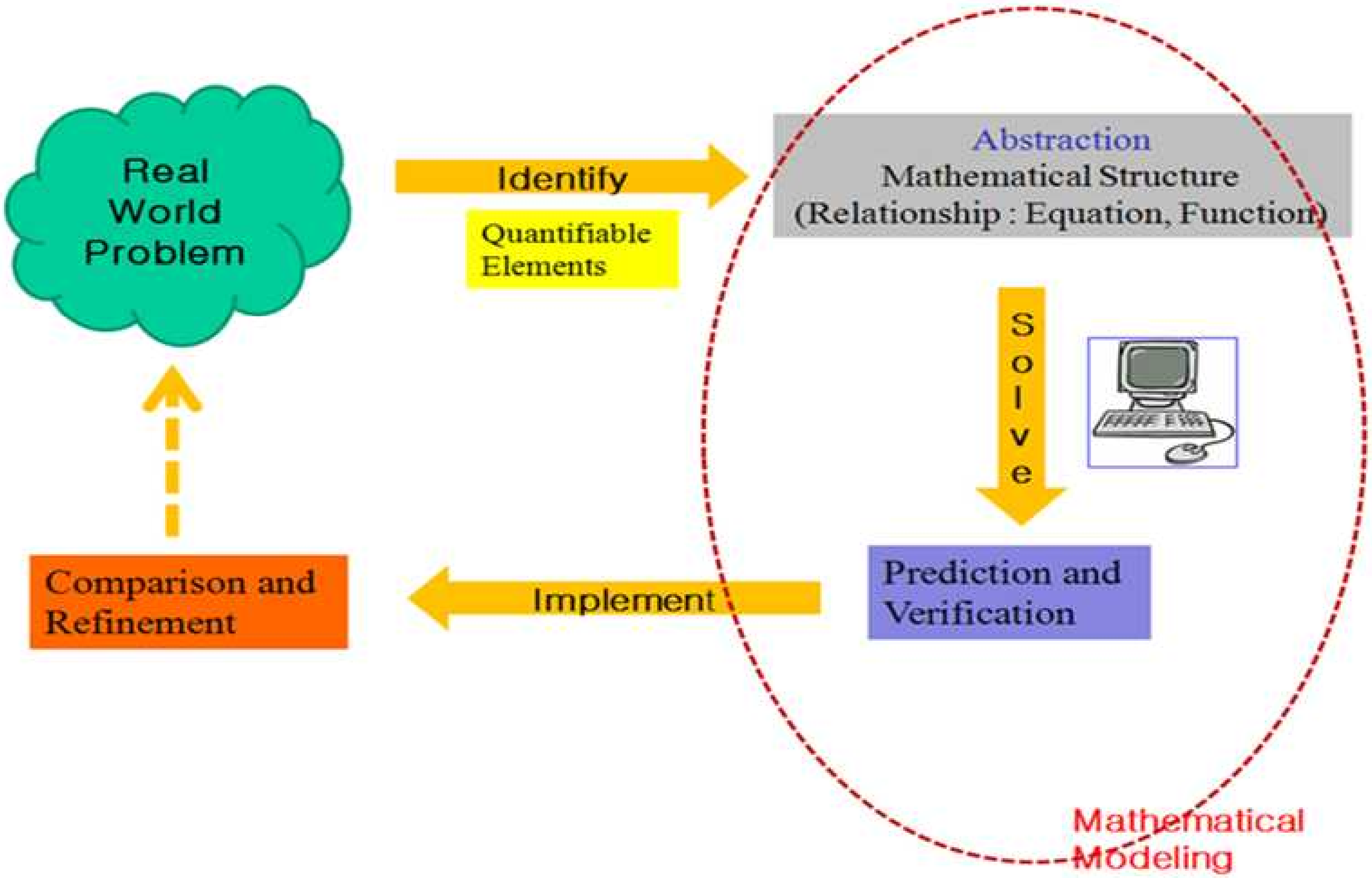
Mathematical Modeling, Numerical Methods, and Problem Solving

CHAPTER OBJECTIVES

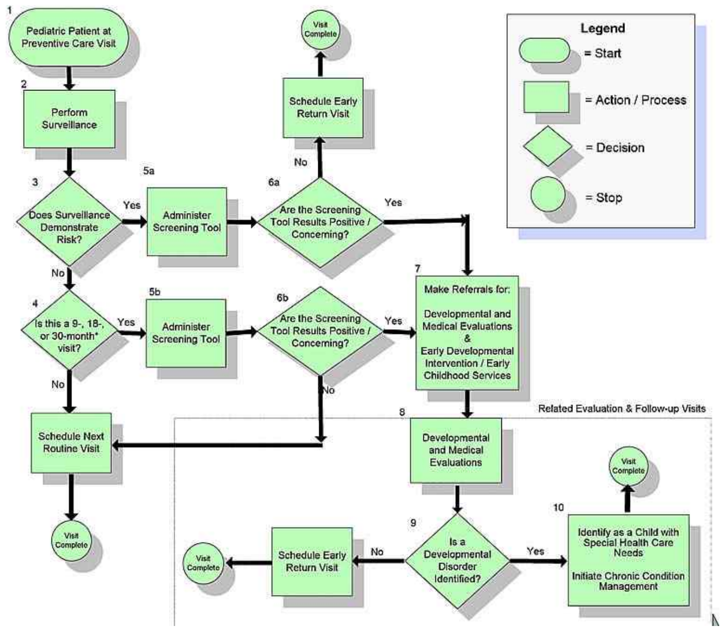
The primary objective of this chapter is to provide you with a concrete idea of what numerical methods are and how they relate to engineering and scientific problem solving. Specific objectives and topics covered are

- Learning how mathematical models can be formulated on the basis of scientific principles to simulate the behavior of a simple physical system.
- Understanding how numerical methods afford a means to generate solutions in a manner that can be implemented on a digital computer.
- Understanding the different types of conservation laws that lie beneath the models used in the various engineering disciplines and appreciating the difference between steady-state and dynamic solutions of these models.
- Learning about the different types of numerical methods we will be covering in this book.

Modeling

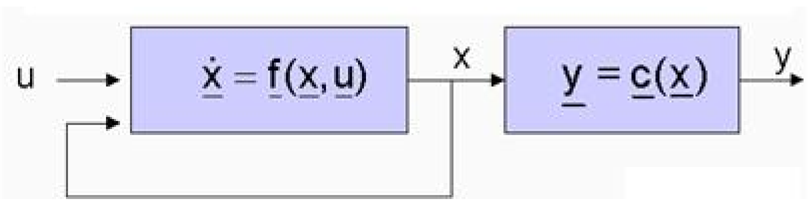

Real World Problem
Identify
Quantifiable Elements
Abstraction
Mathematical Structure
(Relationship : Equation, Function)
Solve
Prediction and Verification
Implement
Comparison and Refinement
Mathematical Modeling

Abstraction




```
*L
C       CALCULATE STATISTICS ON DATA FROM LOW SPEED READER
        SUM=0
        SUMSQ=0
        TYPE 100
100     FORMAT("ENTER THE NUMBER OF VALUES TO CACULATE STATISTICS ON",/)
        ACCEPT 10,N
10      FORMAT(I)
        DO 200 I=1,N
        READ 1,110,V
110     FORMAT(E)
        SUM=SUM + V
        SUMSQ=SUMSQ + V*V
        TYPE 120,I,V
120     FORMAT("VALUE",I,"IS",E,/)
200     CONTINUE
        SAMP=N
        AVRG=SUM/SAMP
        STD=SQTF(SUMSQ/SAMP - AVRG**2)
        TYPE 300,N,AVRG,STD
300     FORMAT("NUMBER OF VALUES",I,"MEAN",E,"STANDARD DEVIATION",E,/)
        END

*
.R FORT
?
```



$$a^x + b^y = c^z$$

Characteristics of mathematical models

- It describes a natural process (or system) in mathematical terms
- It represents an idealization and simplification of reality
- Finally, it yields reproducible results,
  - consequently, can be used for predictive purposes.

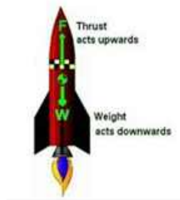
The time rate change of momentum of a body is equal to the resulting force acting on it.

F, m, a

$F = m * a$ Newton's second law of motion

where

F = net force acting on the body (N)
m = mass of the object (kg)
a = its acceleration (m/s^2)



Building Blocks of Mathematical Model

- Variables of the mathematical model
  - Decision (independent) variables
  - State (dependent) variables
    - represents system state and/or output variables
  - Exogenous variables
    - parameters or constants.

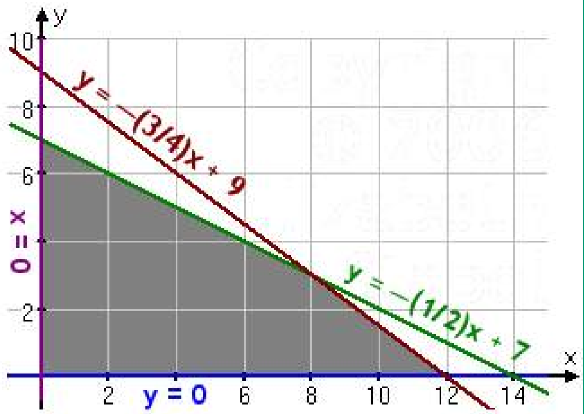
$$
\begin{aligned}
&\text{Maximize} && P = p_1x_1 + p_2x_2 + \cdots + p_kx_k \\
&\text{Subject to:} && a_{11}x_1 + a_{12}x_2 + \cdots + a_{1k}x_k \le q_1 \\
& && a_{21}x_1 + a_{22}x_2 + \cdots + a_{2k}x_k \le q_2 \\
& && \vdots \\
& && a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nk}x_k \le q_n \\
& && x_1, x_2, \cdots x_k \ge 0
\end{aligned}
$$

Real World Problem

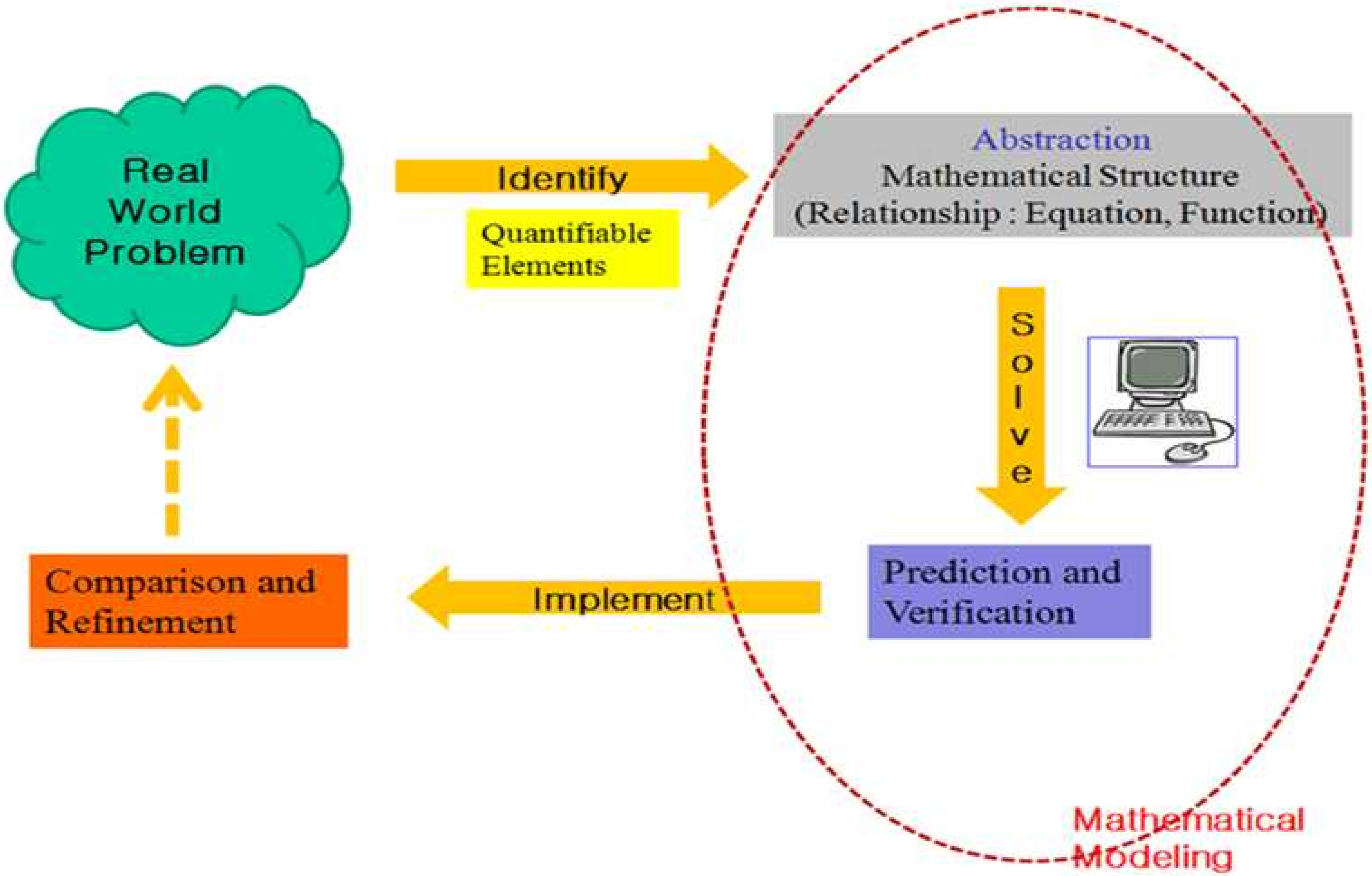
- You need to buy some filing cabinets.
  - Cabinet X costs $10 per unit, requires 6 square feet of floor space, and holds 8 cubic feet of files.
  - Cabinet Y costs $20 per unit, requires 8 square feet of floor space, and holds 12 cubic feet of files.
  - You have been given $140 for this purchase, though you don't have to spend that much.
  - The office has room for no more than 72 square feet of cabinets.
  - How many of which model should you buy, in order to maximize storage volume?

Mathematical Modeling

- Decision variables
  - x : number of model X cabinets purchased ($x \geq 0$)
  - y : number of model Y cabinets purchased ($y \geq 0$)
- State variable
  - Maximize Volume(V) = $8x + 12y$
- Parameters
  - Cost : $10x + 20y \leq 140$ → $y \leq -0.5x + 7$
  - Space : $6x + 8y \leq 72$ → $y \leq -0.75x + 9$
- Optimal solution
  - Test the corner points at (8, 3), (0, 7), and (12, 0),
  - Maximal volume of 100 cubic feet at (8,3)

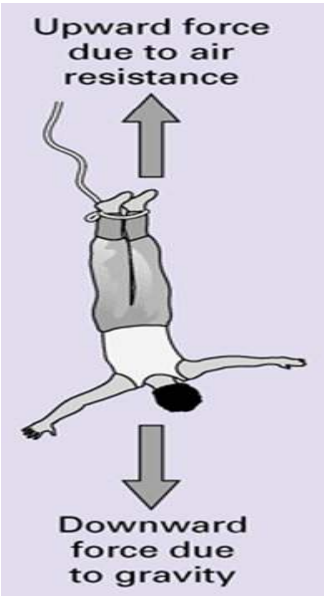


Mathematical Modeling



Bungee Jumper Problem

- Develop the mathematical model for the rate of change of velocity with respect to time



F_U = Force due to air resistance
(c=drag coefficient)

F_D = Force due to gravity

$$F = ma$$

$$a = \frac{F}{m}$$

$$\frac{dv}{dt} = \frac{F}{m}$$

$$F = F_D + F_U$$

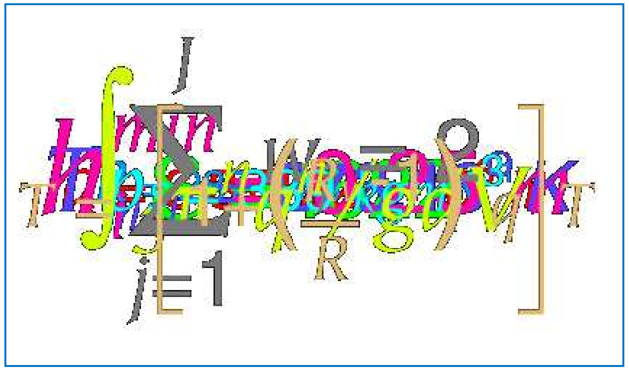
$$F_D = mg$$

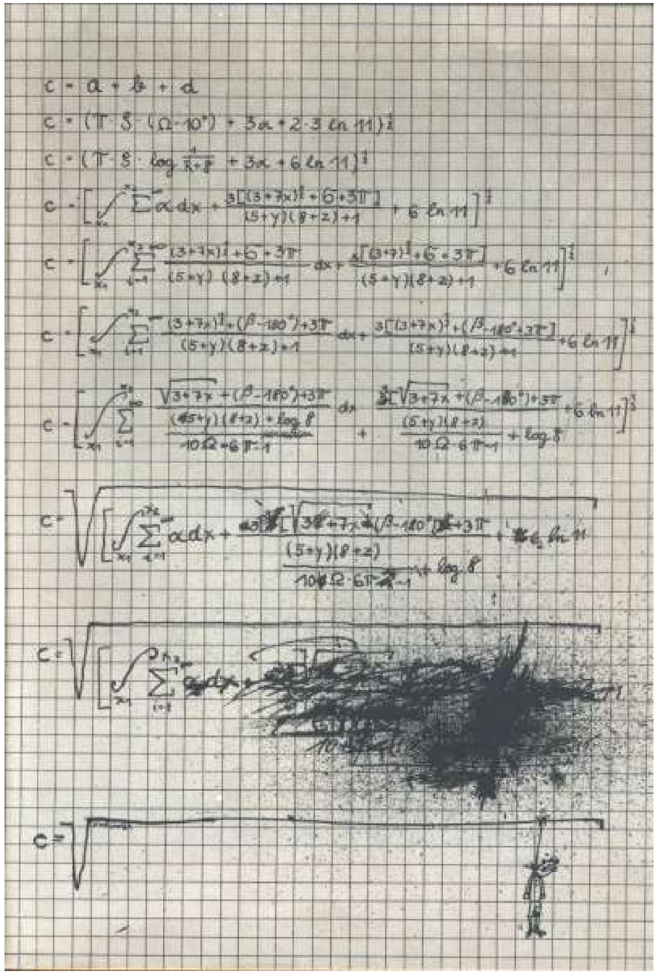
$$F_U = -cv^2$$

$$\frac{dv}{dt} = \frac{mg - cv^2}{m}$$

Analytical methods

$$v(t)=\sqrt{\frac{gm}{c_d}}\tanh\left(\sqrt{\frac{gc_d}{m}}\,t\right)$$





Numerical methods

- Numerical methods are those in which the mathematical problem is reformulated so it can be
- Solved by arithmetic operations.

$$\frac{dv}{dt} = \frac{mg - cv^2}{m} \qquad \frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

$$\frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$

Parachutist Problem

F_U = Force due to air resistance

$(c = drag\ coefficient)$

F_D = Force due to gravity

$$\frac{dv}{dt} = \frac{F}{m}$$

$$F = F_D + F_U$$

$$F_D = mg$$

$$F_U = -cv$$

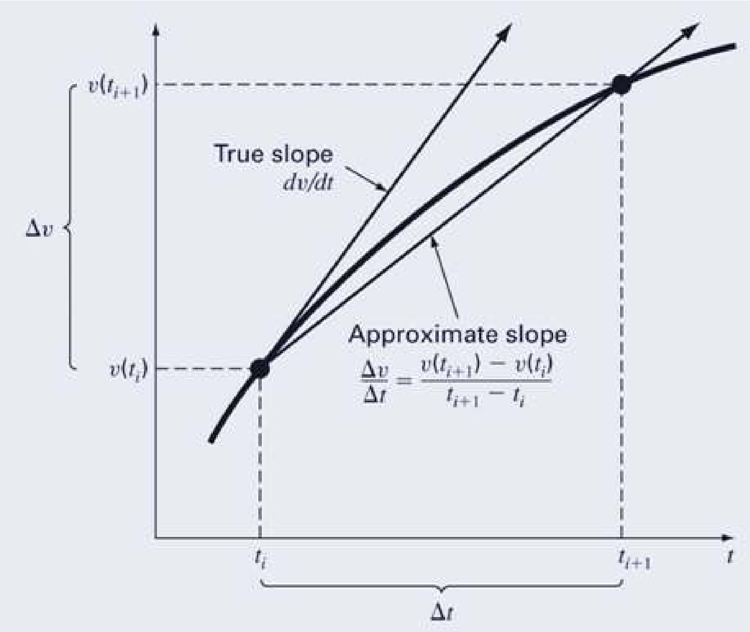
$$\frac{dv}{dt} = \frac{mg - cv}{m}$$

When Drag(F_U) is equal to Weight(F_D), acceleration is zero.
Velocity becomes constant (terminal velocity).

Numerical vs. Analytical solutions

$$v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$$

$$\frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$



Euler's Method

$$v(t_{i+1}) = v(t_i) + \left[g - \frac{c_d}{m} v(t_i)^2\right](t_{i+1} - t_i)$$

new = old + slope × step

Numerical vs. Analytical solutions

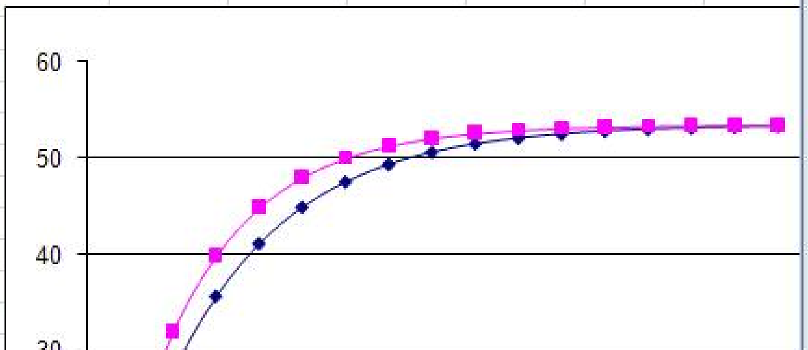
F4 f_x =($C$4*$C$2/$C$3)*(1-EXP(-$C$3*E4/$C$2))

$$v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$$

Parachute-Problem [호환 모드]

m=	68.1
c	12.5
g=	9.81
Δt	2

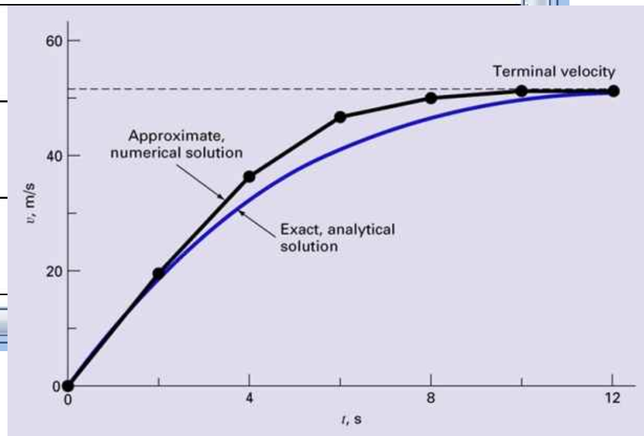
t	v	
0	0	0
2	16.42172	19.62
4	27.79763	32.03736
6	35.67812	39.89621
8	41.13722	44.87003
10	44.91893	48.01792
12	47.53865	50.01019
14	49.35343	51.27109
16	50.61058	52.06911
18	51.48146	52.57416
20	52.08475	52.89381
22	52.50267	53.09611
24	52.79218	53.22415
26	52.99273	53.30518
28	53.13166	53.35646
30	53.2279	53.38892
32	53.29457	53.40946



Sheet1 Sheet2

$$v(t_{i+1}) = v(t_i) + \left[g - \frac{c}{m}v(t_i)\right]\Delta t$$

G3 f_x =G2+$C$5*($C$2*$C$4-$C$3*G2)/$C$2



Analytical vs. Numerical solution

m=68.1 kg c=12.5 kg/s
g=9.8 m/s

t (sec.)	V (m/s)
0	0
2	16.40
4	27.77
8	41.10
10	44.87
12	47.49
∞	53.39

$\Delta t = 2$ sec

t (sec.)	V (m/s)
0	0
2	19.60
4	32.00
8	44.82
10	47.97
12	49.96
∞	53.39

$\Delta t = 0.5$ sec

t (sec.)	V (m/s)
0	0
2	17.06
4	28.67
8	41.94
10	45.60
12	48.09
∞	53.39

$\Delta t = 0.01$ sec

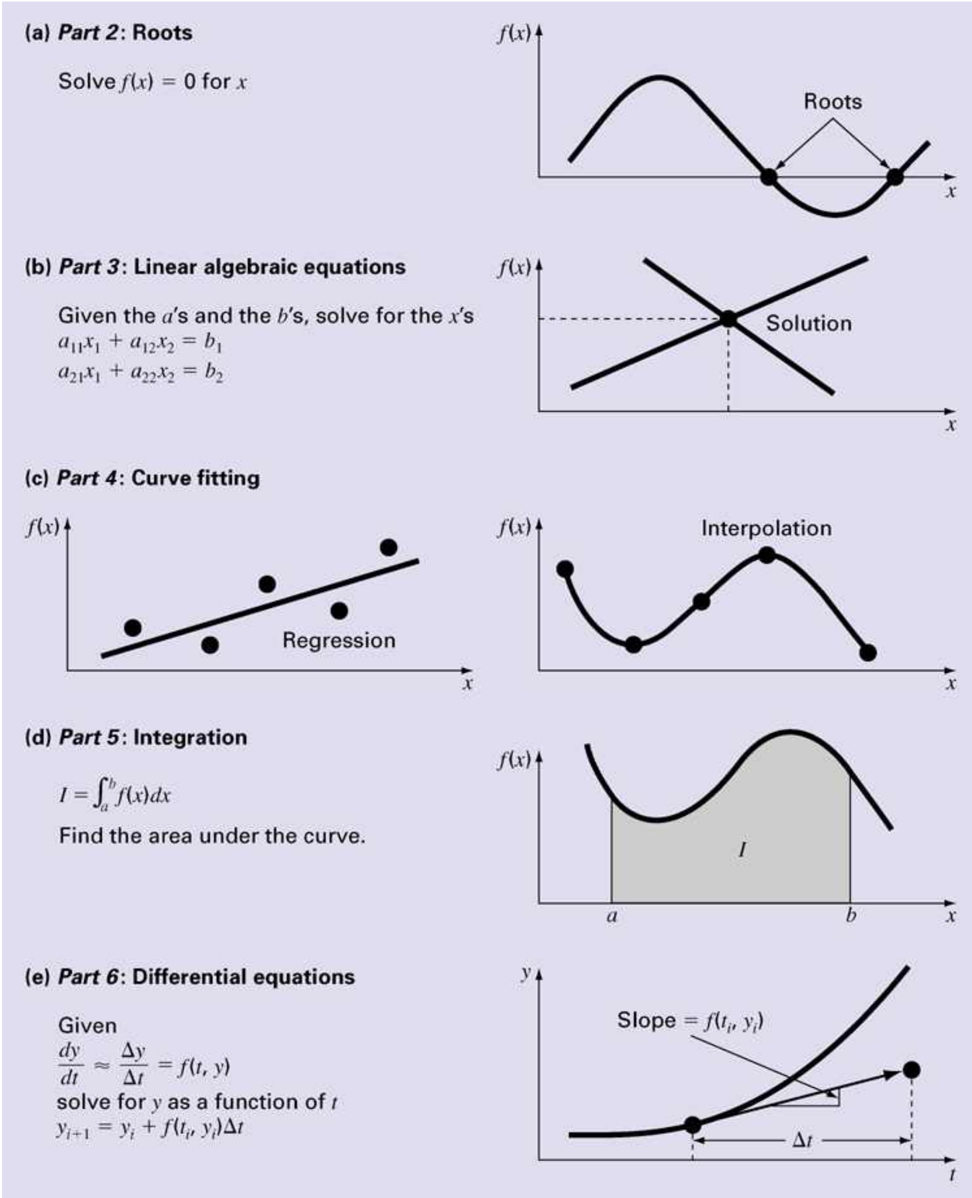
t (sec.)	V (m/s)
0	0
2	16.41
4	27.83
8	41.13
10	44.90
12	47.51
∞	53.39

$$v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$$

$$v(t_{i+1}) = v(t_i) + [g - \frac{c}{m} v(t_i)]\Delta t$$

CONCLUSION: If you want to minimize the error, use a smaller step size, Δt

Numerical Methods



THE END

Homework/Report