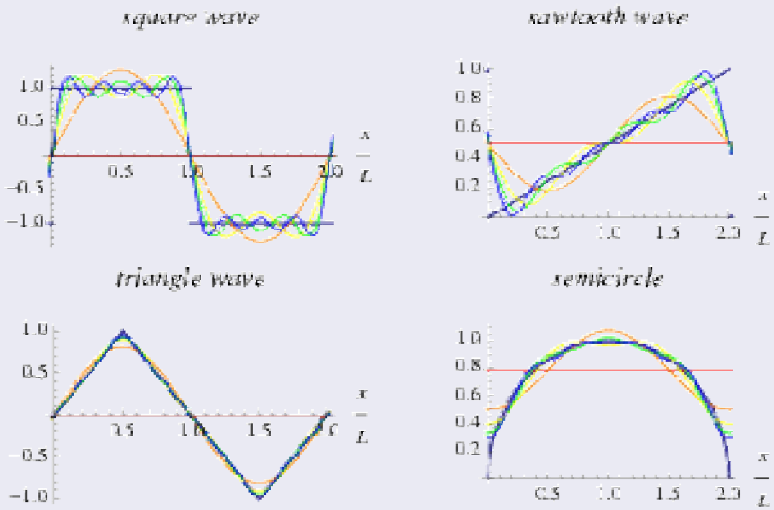
Part 4
Chapter 16

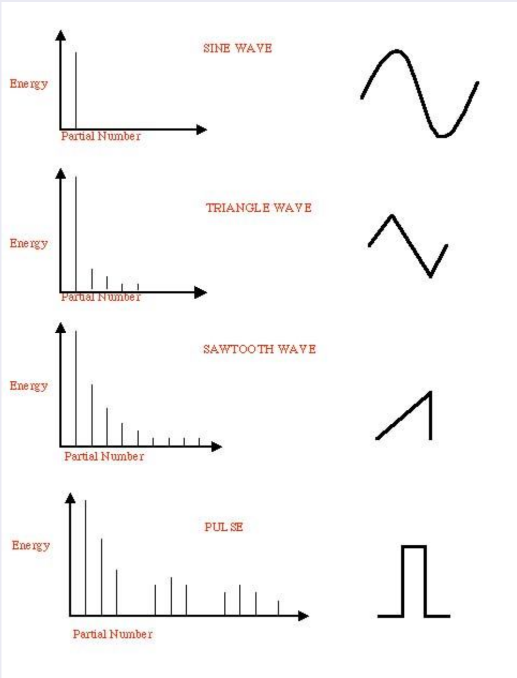
참고만 할 것
(시험범위 제외)

Fourier(푸리에) Analysis



Chapter Objectives

- Sinusoids for curve fitting.
- Fourier series
- Euler's formula.
- Fourier integral and transform
- Discrete Fourier transform (DFT)
- Fast Fourier transform (FFT)

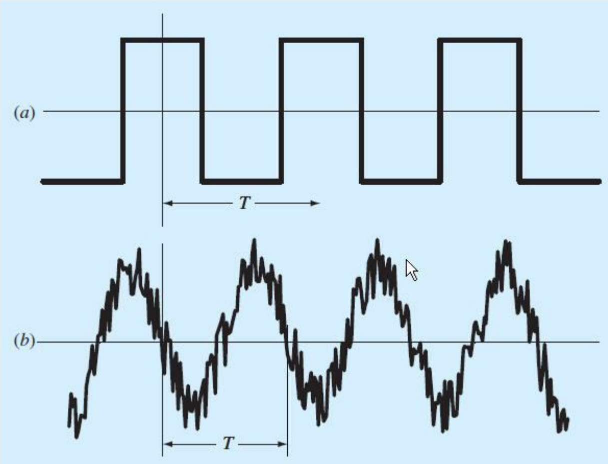


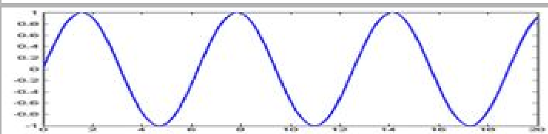
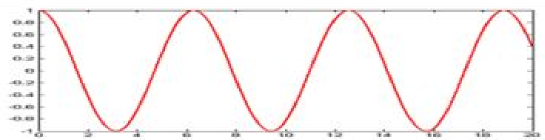
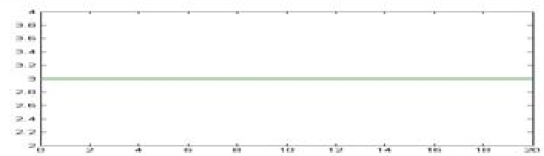
Fourier Analysis is based on the premise that more complicated functions(e.g., time series) can be represented by sum of simpler trigonometric functions.

Periodic Functions

- Any periodic function is one for which

$$f(t) = f(t + T) \quad \text{where } T = \text{the period}$$



function	graph	period
$\sin x$		2π
$\cos x$		2π
constant		ε

Examples

$$f(t) = \cos\frac{t}{3} + \cos\frac{t}{4}$$

Find its period (T)

$$f(t) = f(t+T) \Rightarrow \cos\frac{t}{3} + \cos\frac{t}{4} = \cos\frac{1}{3}(t+T) + \cos\frac{1}{4}(t+T)$$

$$\boxed{\cos\theta = \cos(\theta + 2m\pi)}$$

$$\frac{T}{3} = 2m\pi \qquad \Rightarrow \qquad T = 6m\pi$$

$$\frac{T}{4} = 2n\pi \qquad \Rightarrow \qquad T = 8n\pi$$

$$\Rightarrow \quad T = 24\pi$$ smallest T

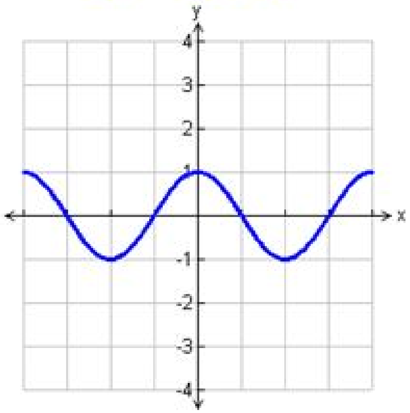
Vertical Stretch or Compression

(Amplitude change)

$$y = A\sin x \qquad y = A\cos x$$

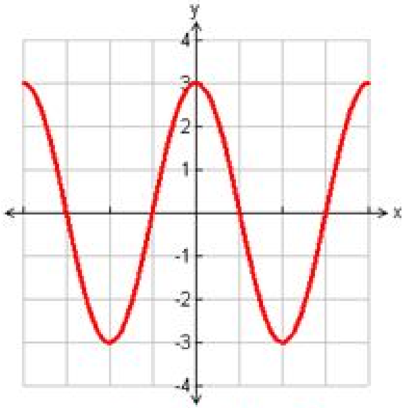
$$\boxed{\text{Amplitude} = |A|}$$

$y = \cos x$



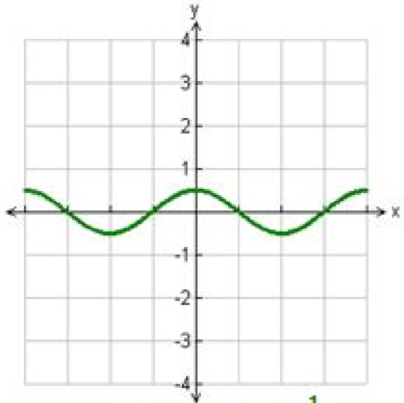
$Amplitude = 1$

$y = 3\cos x$



$Amplitude = 3$

$y = \frac{1}{2}\cos x$



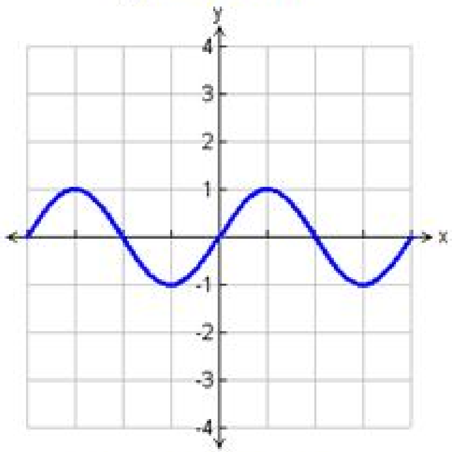
$Amplitude = \frac{1}{2}$

Horizontal Stretch or Compression (Period change)

$$y = A\sin \omega x \qquad y = A\cos \omega x$$

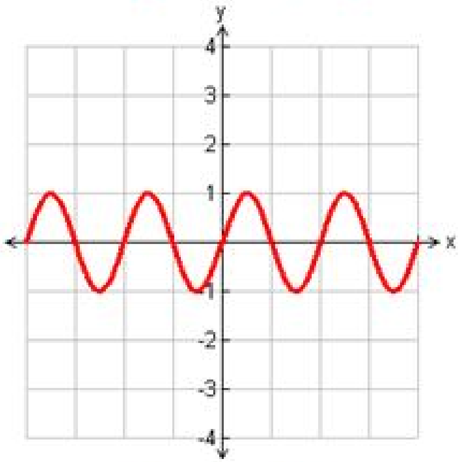
$$\boxed{\text{Period} = T = \frac{2\pi}{\omega}}$$

$y = \sin x$



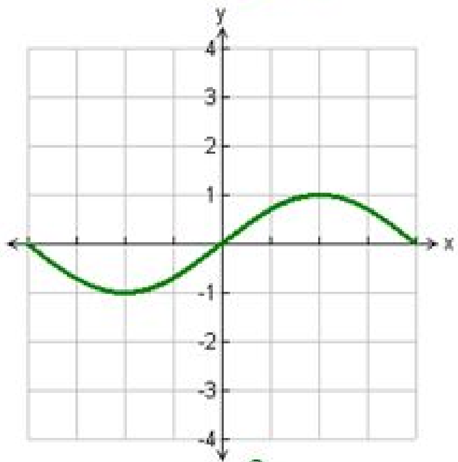
$period = T = 2\pi$

$y = \sin(2x)$



$period = T = \frac{2\pi}{2} = \pi$

$y = \sin\left(\frac{1}{2}x\right)$

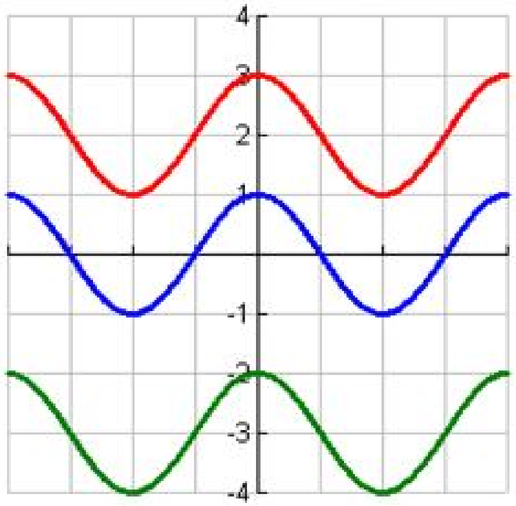


$period = T = \frac{2\pi}{\frac{1}{2}} = 4\pi$

Vertical Shifts

$$y = A\sin(\omega x) + B \qquad y = A\cos(\omega x) + B$$

Vertical Shift = B *units*



$y = \cos x + 2$ Shift 2 units upward

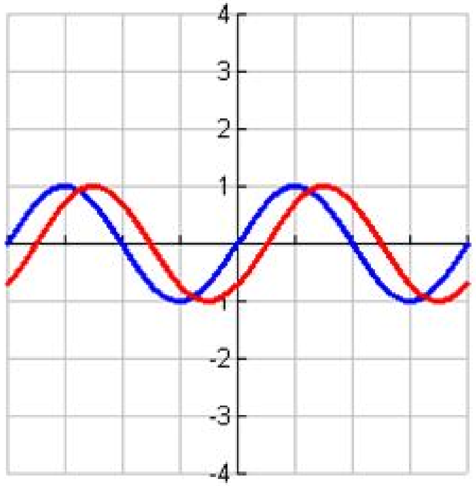
$y = \cos x$ *Parent Graph*

$y = \cos x - 3$ Shift 3 units downward

Phase Shifts

$$y = A\sin(\omega x - \phi) + B \qquad\qquad y = A\cos(\omega x - \phi) + B$$

$$\text{Phase shift} = \frac{\phi}{\omega}\ \textit{units}$$



$y = \sin x$

$y = \sin\left(x - \dfrac{\pi}{4}\right)$

Phase shift $\dfrac{\pi}{4}$ units to right.

$$y = A\sin(\omega x - \phi) = A\sin(\omega(x - \frac{\phi}{\omega})),\ \omega > 0$$

or

$$y = A\cos(\omega x - \phi) = A\cos(\omega(x - \frac{\phi}{\omega})), \omega > 0$$

Sinusoids (sine wave, 정현파(正弦波))

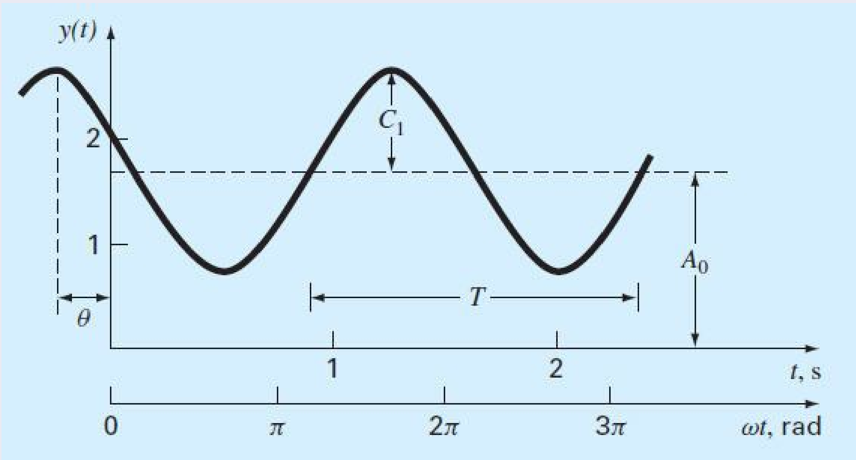
- Sine 또는 Cosine 함수로 표현된 함수
  - 규칙적인 반복 (주기, cycle)
  - 진폭(amplitude)

$$f(t) = A_0 + C_1 \cos(\omega_0 t + \theta)$$

Mean value | Amplitude | Angular frequency | Phase shift

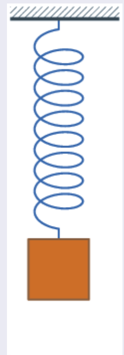
$$\omega_0 = 2\pi f = \frac{2\pi}{T}$$

T = period
f = Frequency (1/T)



A_0=1.7, C_1=1, T=1.5s
ω_0 = 2π/T = 2π/(1.5s),
θ = (π/3) radians = 1.0472 (= 0.25s),
f = 1/T = (1 cycle)/(1.5s) = 0.6667 Hz

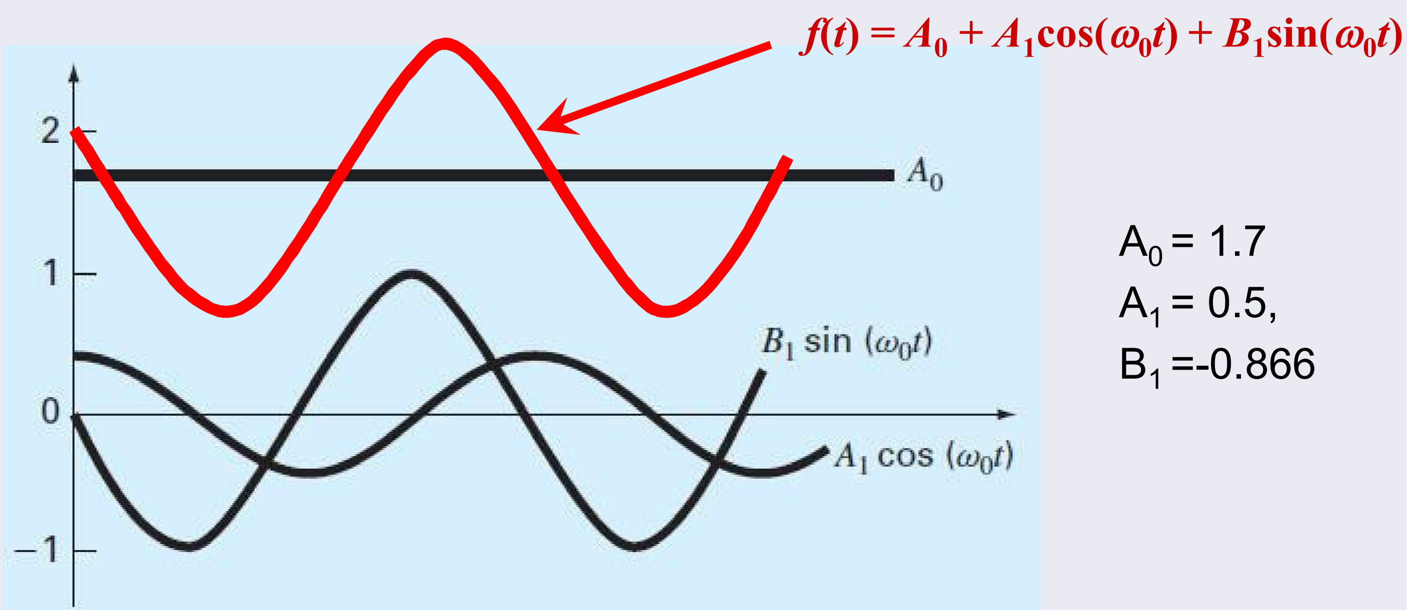
The oscillation of an spring-mass system around the equilibrium is a sine wave



Alternative Representation of Sinusoids

$$f(t) = A_0 + C_1\cos(\omega_0 t + \theta)$$

$$f(t) = A_0 + A_1\cos(\omega_0 t) + B_1\sin(\omega_0 t)$$



$A_0 = 1.7$
$A_1 = 0.5$,
$B_1 = -0.866$

- The two forms are related by

$$C_1 = \sqrt{A_1^2 + B_1^2} \qquad \theta = \arctan(-B_1/A_1)$$

Alternative Representation of Sinusoids

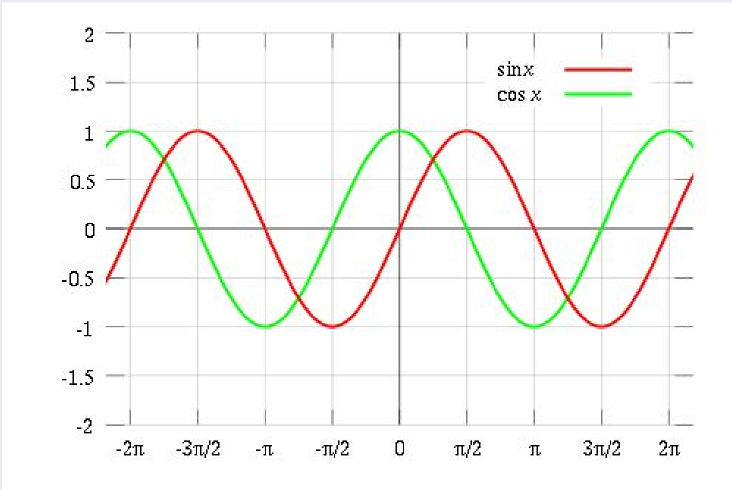
- $f(t) = A_0 + C_1\cos(\omega_0 t + \theta)$ ------------------------------------- (1)
  - $C_1\cos(\omega_0 t + \theta) = C_1[\cos(\omega_0 t)\cos(\theta) - \sin(\omega_0 t)\sin(\theta)]$ --------*(2)*
  - *(2)* → *(1)* : $f(t) = A_0 + A_1\cos(\omega_0 t) + B_1\sin(\omega_0 t)$
    - $A_1 = C_1\cos(\theta)$, $B_1 = -C_1\sin(\theta)$ →
    - $\theta = \arctan(-B_1/A_1)$, $C_1^2 = A_1^2 + B_1^2$

$$\boxed{f(t) = A_0 + A_1\cos(\omega_0 t) + B_1\sin(\omega_0 t) \text{ : Linear model}}$$

Alternative Representation of Sinusoids

- $f(t) = A_0 + C_1\cos(\omega_0 t + \theta)$ or
  $= A_0 + C_1\sin(\omega_0 t + \delta)$

  – $\sin(\omega_0 t + \delta) = \cos(\omega_0 t + \delta - \pi/2)$
  – $\theta = \delta - \pi/2$



Least-Square Fit of a Sinusoid

$$y = A_0 + A_1 \cos(\omega_0 t) + B_1 \sin(\omega_0 t) + e$$

$$y = a_0 z_0 + a_1 z_1 + a_2 z_2 + \cdots + a_m z_m + e$$

where $z_0 = 1$, $z_1 = \cos(\omega_0 t)$, $z_2 = \sin(\omega_0 t)$, and all other z's $= 0$.

$$S_r = \sum_{i=1}^{N} \left\{ y_i - [A_0 + A_1 \cos(\omega_0 t_i) + B_1 \sin(\omega_0 t_i)] \right\}^2$$

The normal equations

$$\begin{bmatrix} N & \Sigma\cos(\omega_0 t) & \Sigma\sin(\omega_0 t) \\ \Sigma\cos(\omega_0 t) & \Sigma\cos^2(\omega_0 t) & \Sigma\cos(\omega_0 t)\sin(\omega_0 t) \\ \Sigma\sin(\omega_0 t) & \Sigma\cos(\omega_0 t)\sin(\omega_0 t) & \Sigma\sin^2(\omega_0 t) \end{bmatrix} \begin{Bmatrix} A_0 \\ A_1 \\ B_1 \end{Bmatrix} = \begin{Bmatrix} \Sigma y \\ \Sigma y\cos(\omega_0 t) \\ \Sigma y\sin(\omega_0 t) \end{Bmatrix}$$

where there are N observations equi- spaced at intervals of Δt and with a total record length of $T = (N - 1)\ \Delta t$.

For this situation, the following average values can be determined

$$\frac{\Sigma\sin(\omega_0 t)}{N} = 0 \qquad \frac{\Sigma\cos(\omega_0 t)}{N} = 0$$

$$\frac{\Sigma\sin^2(\omega_0 t)}{N} = \frac{1}{2} \qquad \frac{\Sigma\cos^2(\omega_0 t)}{N} = \frac{1}{2}$$

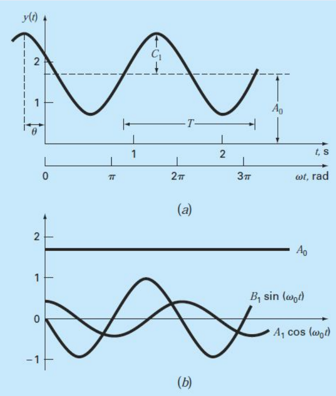
$$\frac{\Sigma\cos(\omega_0 t)\sin(\omega_0 t)}{N} = 0$$

for equispaced points the normal equations become

$$\begin{bmatrix} N & 0 & 0 \\ 0 & N/2 & 0 \\ 0 & 0 & N/2 \end{bmatrix} \begin{Bmatrix} A_0 \\ A_1 \\ B_1 \end{Bmatrix} = \begin{Bmatrix} \Sigma y \\ \Sigma y\cos(\omega_0 t) \\ \Sigma y\sin(\omega_0 t) \end{Bmatrix}$$

$$\begin{Bmatrix} A_0 \\ A_1 \\ B_1 \end{Bmatrix} = \begin{bmatrix} 1/N & 0 & 0 \\ 0 & 2/N & 0 \\ 0 & 0 & 2/N \end{bmatrix} \begin{Bmatrix} \Sigma y \\ \Sigma y\cos(\omega_0 t) \\ \Sigma y\sin(\omega_0 t) \end{Bmatrix}$$

$$\boxed{A_0 = \frac{\Sigma y}{N} \qquad A_1 = \frac{2}{N}\Sigma y\cos(\omega_0 t) \qquad B_1 = \frac{2}{N}\Sigma y\sin(\omega_0 t)}$$



$$y = 1.7 + \cos(4.189t + 1.0472)$$

Generate 10 discrete values for this curve at intervals of $\Delta t = 0.15$ for the range $t = 0$ to 1.35

The data required to evaluate the coefficients with $\omega = 4.189$ are

t	y	$y\cos(\omega_0 t)$	$y\sin(\omega_0 t)$
0	2.200	2.200	0.000
0.15	1.595	1.291	0.938
0.30	1.031	0.319	0.980
0.45	0.722	−0.223	0.687
0.60	0.786	−0.636	0.462
0.75	1.200	−1.200	0.000
0.90	1.805	−1.460	−1.061
1.05	2.369	−0.732	−2.253
1.20	2.678	0.829	−2.547
1.35	2.614	2.114	−1.536
$\sum =$	17.000	2.502	−4.330

$$A_0 = \frac{\Sigma y}{N} \qquad A_1 = \frac{2}{N}\Sigma y\cos(\omega_0 t) \qquad B_1 = \frac{2}{N}\Sigma y\sin(\omega_0 t)$$

$$A_0 = \frac{17.000}{10} = 1.7 \qquad A_1 = \frac{2}{10}2.502 = 0.500 \qquad B_1 = \frac{2}{10}(-4.330) = -0.866$$

$$y = A_0 + A_1 \cos(\omega_0 t) + B_1 \sin(\omega_0 t) + e$$

$$A_0 = \frac{17.000}{10} = 1.7 \qquad A_1 = \frac{2}{10} 2.502 = 0.500 \qquad B_1 = \frac{2}{10}(-4.330) = -0.866$$

Thus, the least-squares fit is $y = 1.7 + 0.500\cos(\omega_0 t) - 0.866\sin(\omega_0 t)$

$$\theta = \arctan\left(-\frac{-0.866}{0.500}\right) = 1.0472 \qquad C_1 = \sqrt{(0.5)^2 + (-0.866)^2} = 1.00$$

$$y = 1.7 + \cos(\omega_0 t + 1.0472)$$

$\delta = \theta + \pi/2$

or alternatively, $y = 1.7 + \sin(\omega_0 t + 2.618)$

General Model

- The foregoing analysis can be extended to the general model

$$f(t) = A_0 + A_1 \cos(\omega_0 t) + B_1 \sin(\omega_0 t) + A_2 \cos(2\omega_0 t) + B_2 \sin(2\omega_0 t) + \cdots + A_m \cos(m\omega_0 t) + B_m \sin(m\omega_0 t)$$

where, for equally spaced data, the coefficients can be evaluated by

$$A_0 = \frac{\Sigma y}{N}$$

$$\left.\begin{aligned} A_j &= \frac{2}{N} \Sigma y \cos(j\omega_0 t) \\ B_j &= \frac{2}{N} \Sigma y \sin(j\omega_0 t) \end{aligned}\right\} \qquad j = 1, 2, \ldots, m$$

- Although these relationships can be used to fit data in the regression sense, that is, *N > (2m + 1),*
  - an alternative application is to employ them for interpolation when N=(2m+1)
  - This is the approach used in the Continuous Fourier series.

Continuous Fourier Series

- In the course of studying heat-flow problems,
  - Fourier showed that an arbitrary periodic function can be represented by an infinite series of sinusoids of harmonically related frequencies.

- For a function with period *T, a continuous Fourier series can be written*

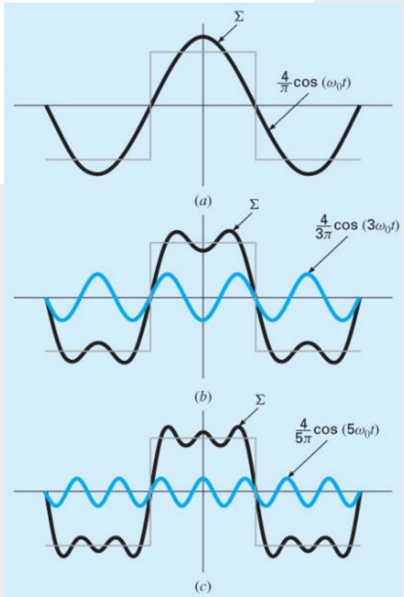
$$f(t) = a_0 + a_1\cos(\omega_0 t) + b_1\sin(\omega_0 t) + a_2\cos(2\omega_0 t) + b_2\sin(2\omega_0 t) + \cdots$$

or more concisely,

$$f(t) = a_0 + \sum_{k=1}^{\infty}[a_k\cos(k\omega_0 t) + b_k\sin(k\omega_0 t)]$$

where $\omega_0 = 2\pi/T$ is called the *fundamental frequency* and

its constant multiples $2\omega_0$, $3\omega_0$, etc., are called *harmonics.*



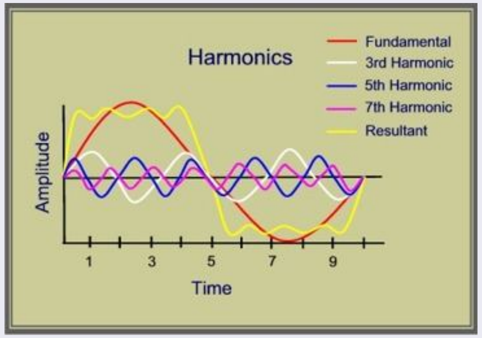
the coefficients can be computed via

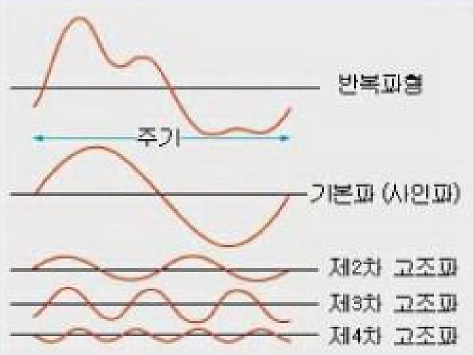
$$a_k = \frac{2}{T}\int_0^T f(t)\cos(k\omega_0 t)\,dt \quad \text{and} \quad b_k = \frac{2}{T}\int_0^T f(t)\sin(k\omega_0 t)\,dt \quad \text{for } k = 1, 2, \ldots$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

(Euler Formulas)

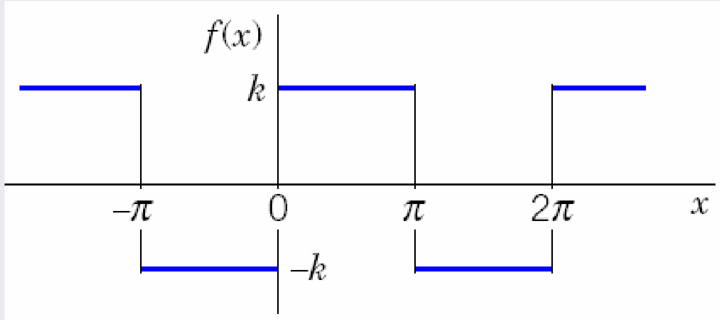
- Harmonics (高調波, 고조파)
  - 사인파가 아닌 주기적 반복파형은 기본주파수를 가지는 **사인파**와 사인파의 정수배의 주파수를 갖는 파동으로 분해되는데,
    - 이 때 반복파형을 구성하는 **기본파 이외의 파동들 : 고조파**
    - 주파수가 n배인 파동 : n차 고조파
    - 음 : 배음 (악기의 음색은 고조파를 포함하는 정도에 따라 달라짐)
    - 전자파 : 기본진동수에 대해 그 배수(倍數)에 따라 제2 또는 제3조파(調波)





Example : Periodic Rectangular Wave (1)

- Given function $f(x)$



$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$$
$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx,$$
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nxdx,$$
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nxdx$$

$$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } \quad 0 < x < \pi \end{cases},$$

$$a_0 = \frac{1}{2\pi}\left[-\int_{-\pi}^{0} kdx + \int_{0}^{\pi} kdx\right] = \frac{1}{2\pi}(-k\pi + k\pi) = 0$$

average height
(we can derive 0 from graph)

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-k)\cdot\cos nxdx + \int_{0}^{\pi} k\cdot\cos nxdx\right] = \frac{1}{\pi}\left[-\frac{k}{n}\sin nx\Big|_{-\pi}^{0} + \frac{k}{n}\sin nx\Big|_{-\pi}^{0}\right] = 0$$

$$(\sin(x))' = \cos(x)$$
$$(\cos(x))' = -\sin(x)$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax + C$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax + C$$

$$a_0 = a_n = 0$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-k)\cdot\sin nx dx + \int_0^{\pi} k\cdot\sin nx dx\right] = \frac{1}{\pi}\left[\frac{k}{n}\cos nx\Big|_{-\pi}^{0} - \frac{k}{n}\cos nx\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{k}{n} - \frac{k}{n}\cos n\pi - \frac{k}{n}\cos n\pi + \frac{k}{n}\right] = \frac{2k}{\pi n}(1-\cos n\pi)$$

$$b_n = \frac{2k}{\pi n}(1-\cos n\pi) = \begin{cases} \dfrac{4k}{n\pi} & \text{if } n = odd \\ 0 & \text{if } n = even \end{cases}$$

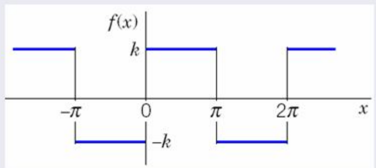
$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx,$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx,$$

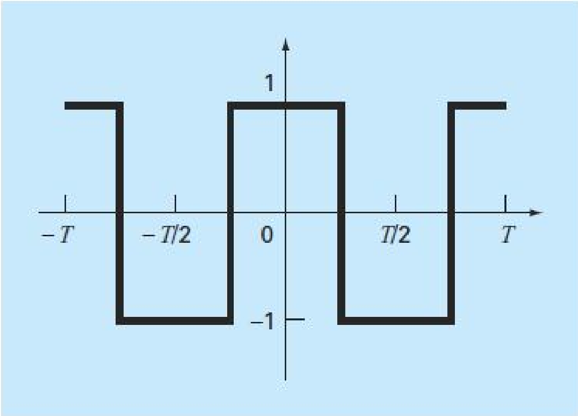
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

$$\cos n\pi = \begin{cases} -1 & if \ \ n = odd \\ 1 & if \ \ n = even \end{cases}$$



$$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$$

$$\boxed{\therefore f(x) = \frac{4k}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right)}$$



A square or rectangular waveform with a height of 2 and a period $T = 2\pi/\omega_0$.

Problem Statement. Use the continuous Fourier series to approximate the square or rectangular wave function

$$
f(t) = \begin{cases} -1 & -T/2 < t < -T/4 \\ 1 & -T/4 < t < T/4 \\ -1 & T/4 < t < T/2 \end{cases}
$$

Solution. Because the average height of the wave is zero, a value of $a_0 = 0$ can be obtained directly. The remaining coefficients can be evaluated as

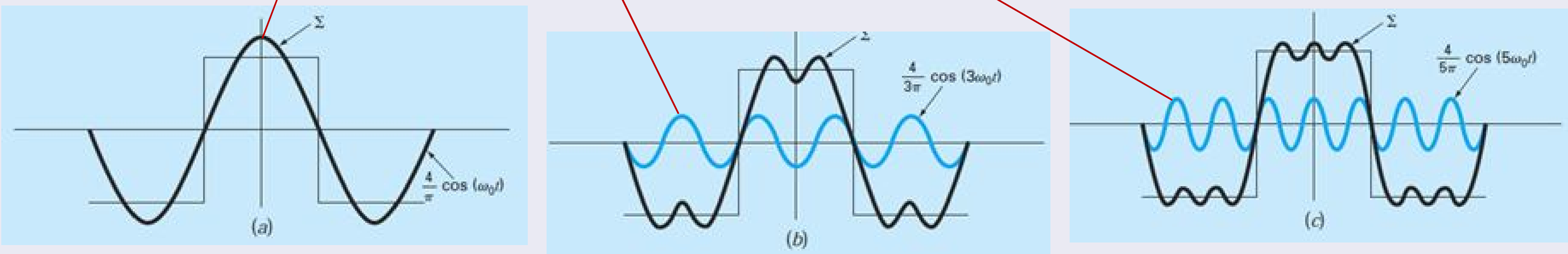
$$
\begin{aligned}
a_k &= \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(k\omega_0 t)\,dt \\
&= \frac{2}{T}\left[-\int_{-T/2}^{-T/4}\cos(k\omega_0 t)\,dt + \int_{-T/4}^{T/4}\cos(k\omega_0 t)\,dt - \int_{T/4}^{T/2}\cos(k\omega_0 t)\,dt\right]
\end{aligned}
$$

The integrals can be evaluated to give

$$a_k = \begin{cases} 4/(k\pi) & \text{for } k = 1, 5, 9, \ldots \\ -4/(k\pi) & \text{for } k = 3, 7, 11, \ldots \\ 0 & \text{for } k = \text{even integers} \end{cases}$$

Similarly, it can be determined that all the b's $= 0$. Therefore, the Fourier series approximation is

$$f(t) = \frac{4}{\pi}\cos(\omega_0 t) - \frac{4}{3\pi}\cos(3\omega_0 t) + \frac{4}{5\pi}\cos(5\omega_0 t) - \frac{4}{7\pi}\cos(7\omega_0 t) + \cdots$$



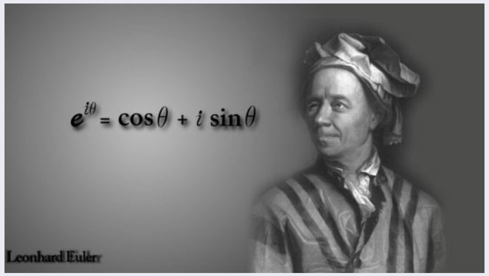
It should be mentioned that the square wave in Fig. is called an *even function* because $f(t) = f(-t)$. Another example of an even function is $\cos(t)$. It can be shown (Van Valkenburg, 1974) that the b's in the Fourier series always equal zero for even functions. Note also that *odd functions* are those for which $f(t) = -f(-t)$. The function $\sin(t)$ is an odd function. For this case, the a's will equal zero.

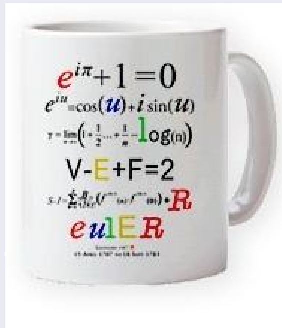
Euler's Formula

- Named after Leonhard Euler
  - the fundamental relationship between the trigonometric functions and the complex exponential function.
  - sometimes denoted **cis**(*x*) ("**c**osine plus ***i*** **s**ine")

  - Euler's formula states that, for any real number x in radians,

$$e^{ix} = \cos x + i \sin x$$



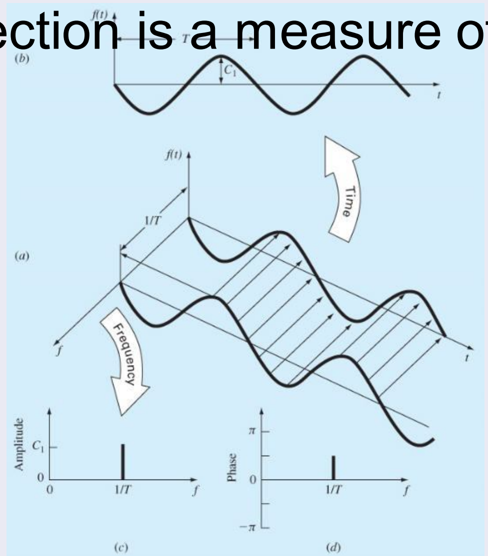


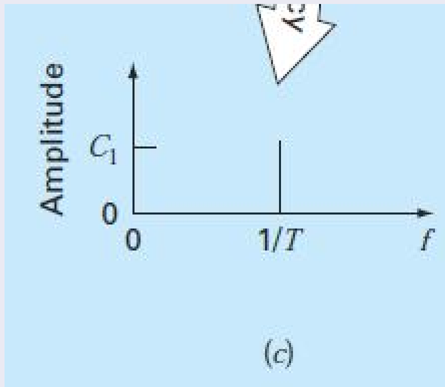
세상에서 가장 아름다운 공식

Time Versus Frequency Domains

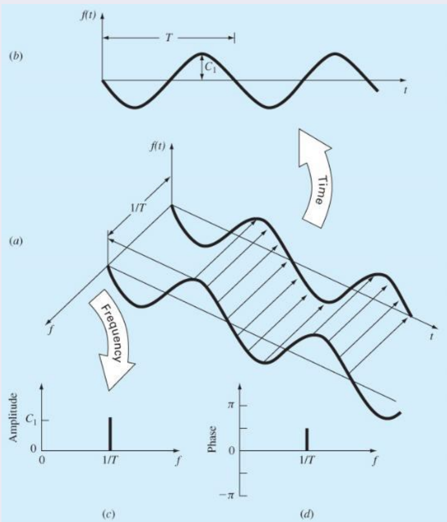
- To this point, our discussion of Fourier approximation has been limited to the *time domain.*
  - Although it is not as familiar, the *frequency domain provides an* alternative perspective for characterizing the behavior of oscillating functions.

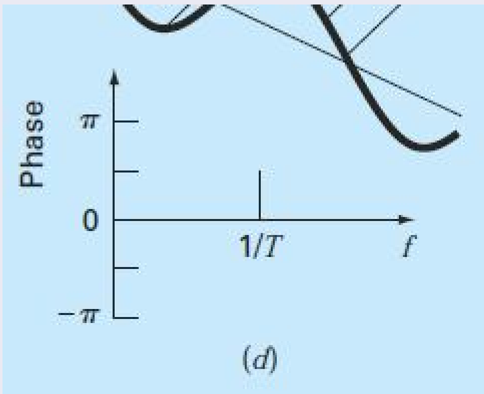
- As amplitude can be plotted versus time, so also can it be plotted versus frequency.
  - The sinusoid can be conceived of as existing a distance *1/T* out along the frequency axis and running parallel to the time axes.
  - As in (Fig. *c),* this projection is a measure of the sinusoid's maximum positive amplitude C1.



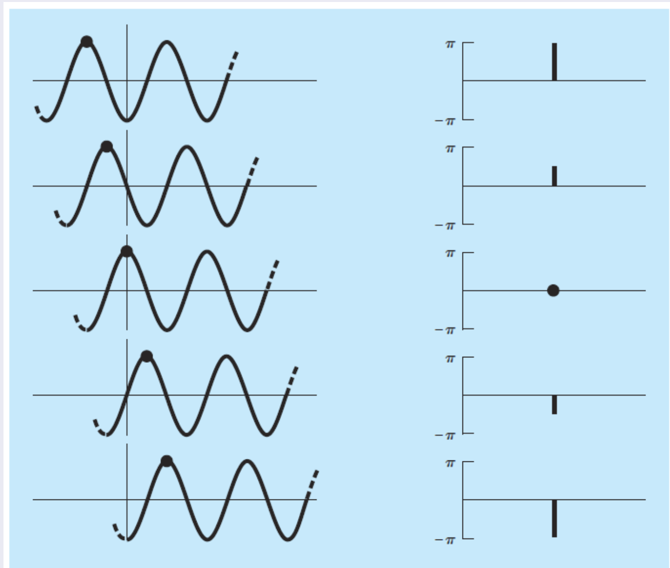


- One more parameter, namely, the phase angle, is required to position the curve relative to *t = 0.*
  - The phase angle is determined as the distance (in radians) from zero to the point at which the positive peak occurs.
  - If the peak occurs before zero, it is said to be advanced
    - the positive phase angle
  - If the peak occurs after zero, it is said to be delayed
    - the negative phase angle
  - For (Fig. d), the peak leads zero and the phase angle is plotted as $+\pi/2$.





line spectra



Fourier Integral and Transform

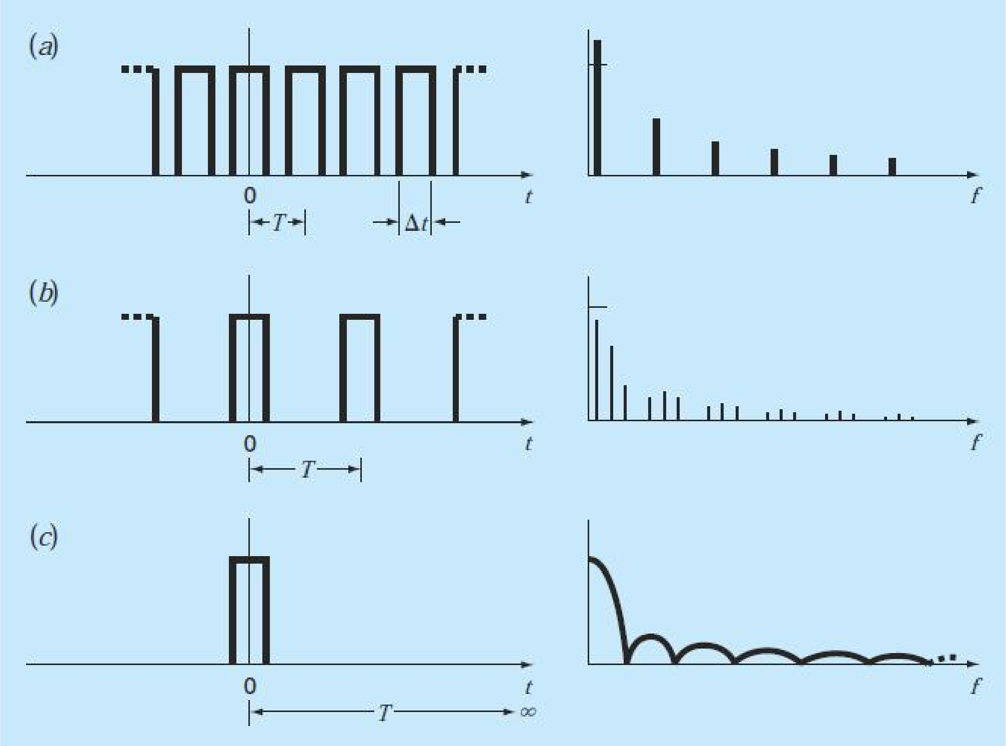
- Periodic functions can be approximated by Fourier series
  - Although the Fourier series is a useful tool for investigating the spectrum of a periodic function, there are many waveforms that do not repeat themselves regularly.
  - For example, a lightning bolt occurs only once (or at least it will be a long time until it occurs again), but it will cause interference with receivers operating on a broad range of frequencies (TVs, radios, and shortwave receivers).

- Such evidence suggests that a nonrecurring signal such as that produced by lightning exhibits a continuous frequency spectrum.

- Because such phenomena are of great interest to engineers, an alternative to the Fourier series would be valuable for analyzing these aperiodic waveforms.

- In (Fig. b), a doubling of the pulse train's period has two effects on the spectrum.
  - First, two additional frequency lines are added on either side of the original components.
  - Second, the amplitudes of the components are reduced.

  - As the period is allowed to approach infinity, these effects continue as more and more spectral lines are packed together until the spacing between lines goes to zero.
  - At the limit, the series converges on the continuous Fourier integral, depicted in (Fig. c).



- It can be derived from the exponential form of the Fourier series

$$f(t) = \sum_{k=-\infty}^{\infty} \tilde{c}_k e^{ik\omega_0 t} \qquad \text{where} \quad \tilde{c}_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-ik\omega_0 t}\, dt$$

$\omega_0 = 2\pi/T$ and $k = 0, 1, 2, \ldots$

- The transition from a periodic to a nonperiodic function can be effected by allowing the period to approach infinity.
  - as *T becomes infinite, the function never repeats* itself and thus becomes aperiodic.

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(i\omega_0) e^{i\omega_0 t}\, d\omega_0 \tag{19.25}$$

and the coefficients become a continuous function of the frequency variable ω, as in

$$F(i\omega_0) = \int_{-\infty}^{\infty} f(t) e^{-i\omega_0 t}\, dt \tag{19.26}$$

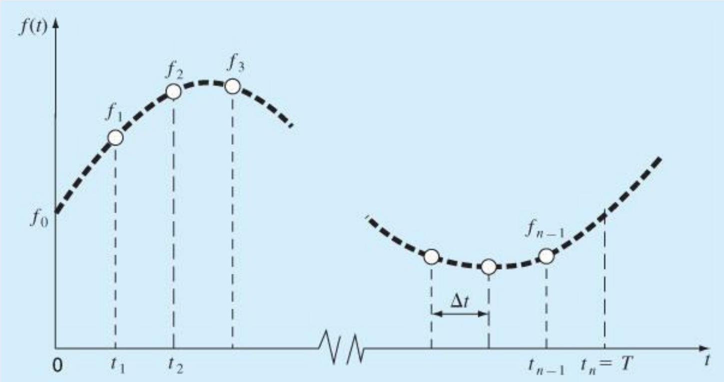
- The function *$F(i\omega_0)$, as defined by Eq. (19.26), is called the Fourier integral of f(t).*
  - Eqs. (19.25) and (19.26) are collectively referred to as the *Fourier transform pair.*
  - $F(i\omega_0)$ is also called the Fourier transform of f(t).

Discrete Fourier Transform (DFT)

- Spectral analysis is the process of identifying component frequencies in data.
  - For discrete data, the computational basis of spectral analysis is the discrete Fourier transform (DFT).
  - The DFT transforms time(or space)-based data into frequency-based data.

- In engineering, functions are often represented by finite sets of discrete values.
  - Data is often collected in or converted to such a discrete format.
    - N equi-spaced subintervals with widths of $\Delta t = T/N$.
    - f_n designates a value of the continuous function f(t) taken at t_n
  - Note that the data points are specified at $n = 0, 1, 2, \ldots, N - 1$.
    - *A value is not included* at $n = N$.

The sampling points of the discrete Fourier series.



- A discrete Fourier transform can be written as

$$F_k = \frac{1}{N}\sum_{n=0}^{N}\left[\, f_n \cos(k\omega_0 n) - i\, f_n \sin(k\omega_0 n)\right]$$

and

$$f_n = \sum_{k=0}^{N-1}\left[F_k \cos(k\omega_0 n) + iF_k \sin(k\omega_0 n)\right]$$

- (Eqs. 19.27 and 19.28) represent the discrete analogs of (Eqs. 19.26 and 19.25, *Fourier transform pair*), respectively.

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(i\omega_0)e^{i\omega_0 t}\, d\omega_0 \tag{19.25}$$

and the coefficients become a continuous function of the frequency variable ω, as in

$$F(i\omega_0) = \int_{-\infty}^{\infty} f(t)e^{-i\omega_0 t}\, dt \tag{19.26}$$

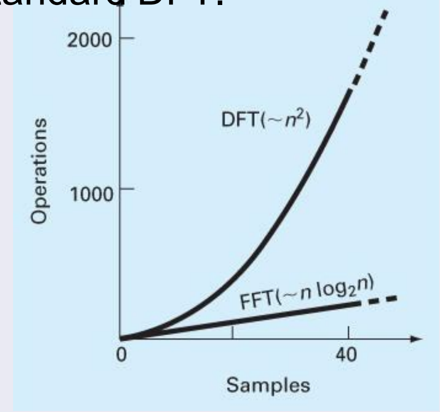
Fast Fourier Transform (FFT)

- It is computationally burdensome to calculates the Discrete Fourier Transform (DFT), because N^2 *operations are required.*
  - For data samples of even moderate size, the direct determination of the DFT can be extremely time-consuming.

- The Fast Fourier transform (FFT), is an algorithm that has been developed to compute the DFT in an extremely economical fashion.

  - Its speed stems from the fact that it utilizes the results of previous computations to reduce the number of operations.

  - In particular, it exploits the periodicity and symmetry of trigonometric functions to compute the transform with approximately $N \log_2 N$ *operations.*
    - *For* $N = 50$ samples, the FFT is about 10 times faster than the standard DFT.
    - For $N = 1000$, *it is about* 100 times faster.



THE END