

# Morphological Filtering

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- Morphological Filtering 개요
- Basic set theory
- Morphological operations for binary images
- Morphological operations for gray-scale images
- OpenCV를 사용한 Morphological operations 구현

# Morphological filtering 개요

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- *Morphology* relates to the structure or form of objects
- Useful for *smoothing out object outlines, filling small holes, eliminating small projection*, etc.
- Simplify a segmented image to facilitate the search for objects of interest
- Applicable to gray-level images as well as binary images

# Basic Set Theory

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Let  $A$  and  $B$  be sets in  $Z^2$

$a$  is an **element** of  $A$   $\Rightarrow a \in A$

$a$  is **not** an **element** of  $A$   $\Rightarrow a \notin A$

$A$  is a **subset** of  $B$   $\Rightarrow A \subseteq B$

The **union** of  $A$  and  $B$

$$\Rightarrow A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

The **intersection** of  $A$  and  $B$

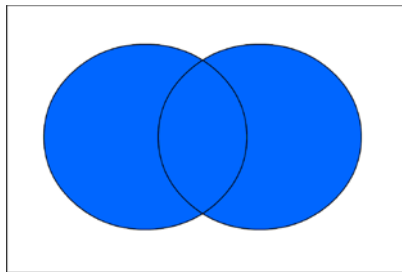
$$\Rightarrow A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

The **complement** of  $A$

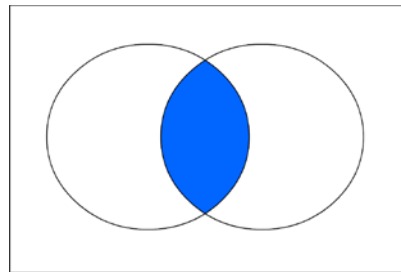
$$\Rightarrow A^c = \{x \mid x \notin A\}$$

The **difference** of  $A$  and  $B$

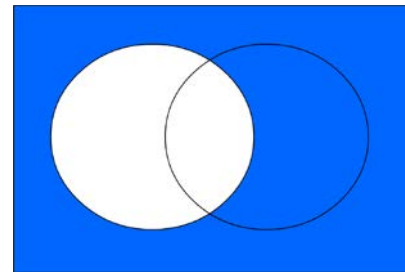
$$\Rightarrow A - B = \{x \mid x \in A \text{ and } x \notin B\}$$



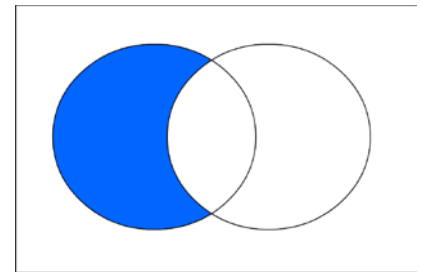
$A \cup B$



$A \cap B$



$A^c$



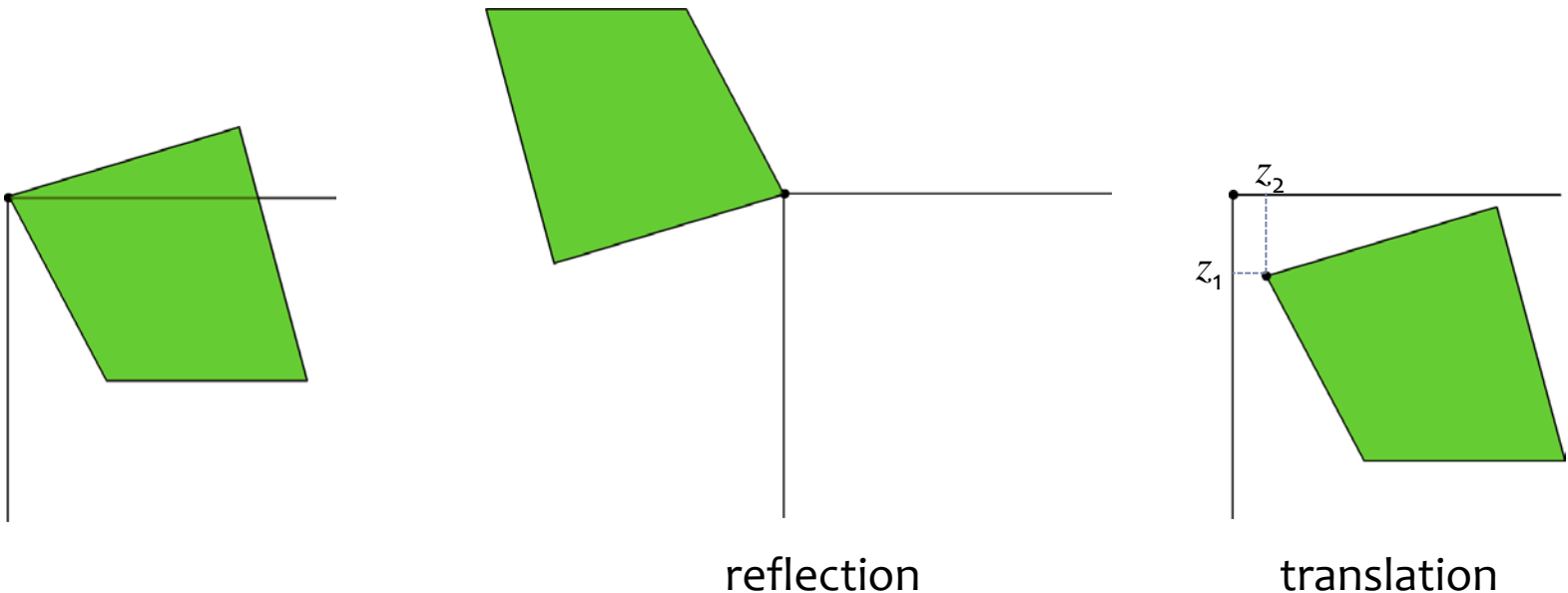
$A - B$

The **reflection** of  $A$

$$\rightarrow \hat{A} = \{x \mid x = -b, \text{ for } b \in A\}$$

The **translation** of  $A$

$$\rightarrow (A)_z = \{c \mid c = b + z, \text{ for } b \in A\}$$



# Morphological operations for binary images

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- Two principal morphological operations

## Dilation & Erosion

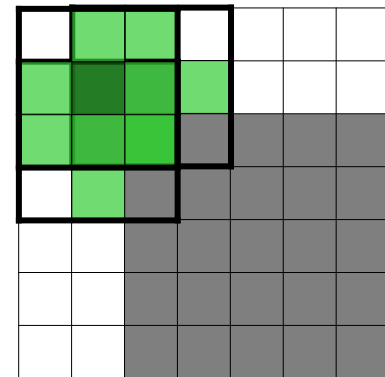
- Can be customized by the proper selection of the **structuring element**, which determines exactly how the object will be dilated or eroded
- Performed by laying the structuring element on the image and sliding it across the image in a manner similar to convolution

# Dilation

Expand objects, thus potentially filling in small holes and connecting disjoint objects

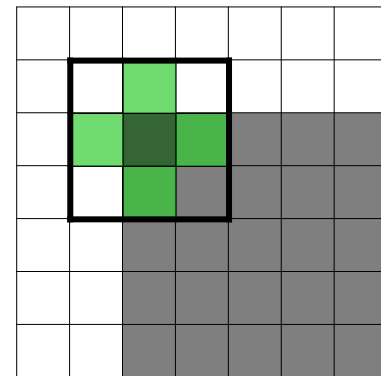
The dilation of  $A$  by the structuring element  $B$ ,

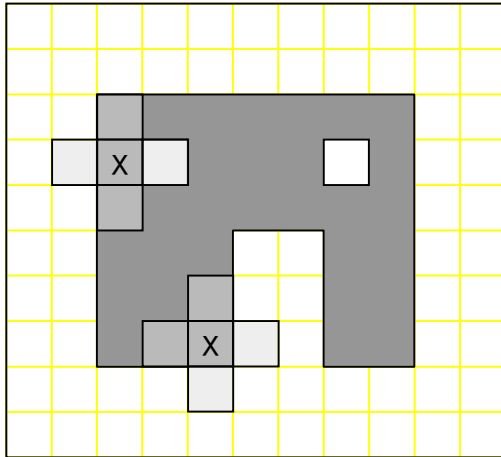
$$A \oplus B = \left\{ z / \left( \hat{B} \right)_z \cap A \neq \phi \right\}$$



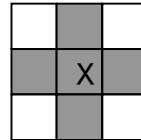


1. If the origin of the structuring element coincides with a '0' in the image, there is no change; move to the next pixel.
2. Otherwise (in case of '1'), perform the OR logic operation on all pixels within the structuring element



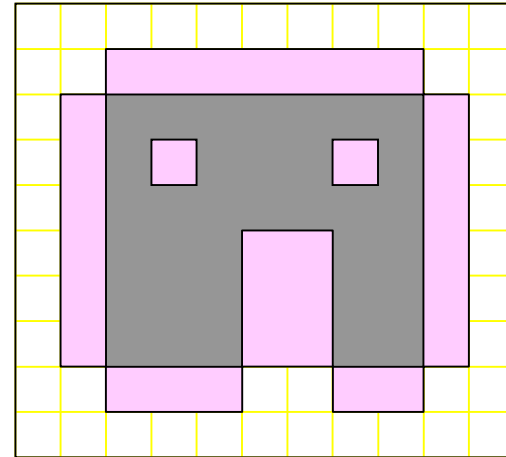


Original Image



Structuring  
Element

x : origin



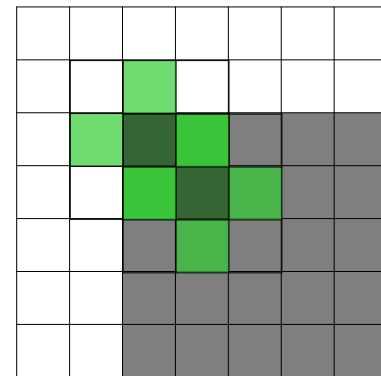
After Dilation

# Erosion

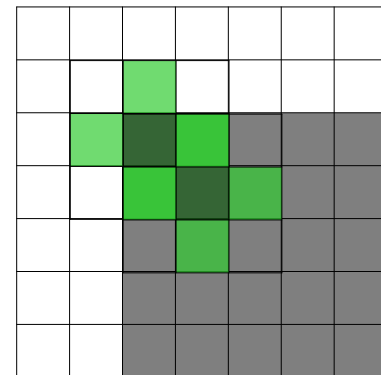
Shrinks objects by etching away(eroding)  
their boundaries

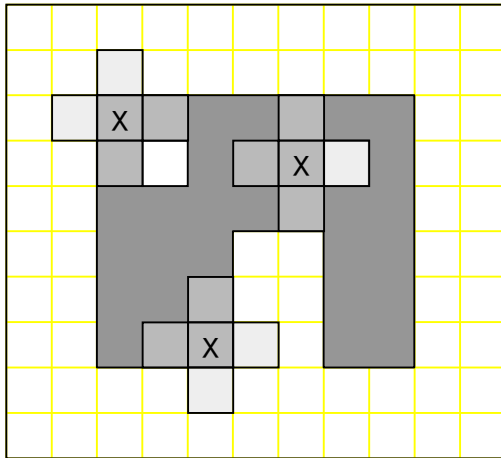
The erosion of  $A$  by  $B$ ,

$$A \ominus B = \{z / (B)_z \subseteq A\}$$

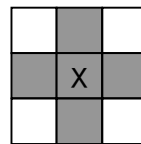


1. If the origin of the structuring element coincides with a '0' in the image, there is no change; move to the next pixel
2. If the origin of the structuring element coincides with a '1' in the image, and any of the '1' pixels in the structuring element extend beyond the object('1' pixels), then change the '1' pixel in the image to a '0'



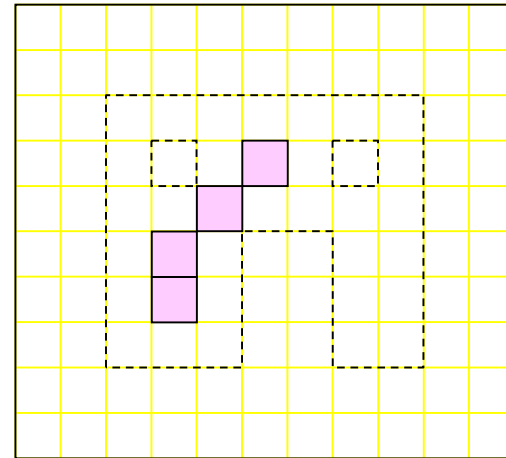


Original Image



Structuring  
Element

x : origin

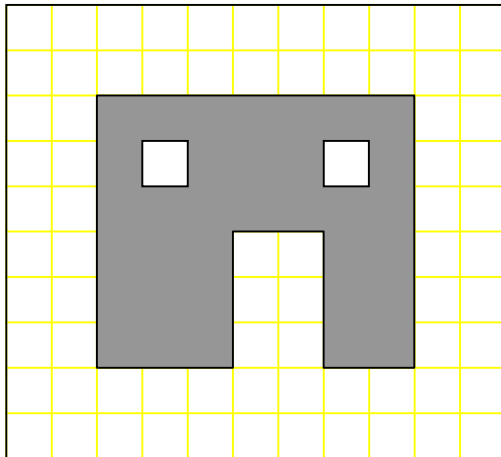


After Erosion

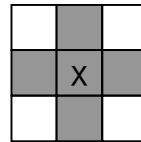
# Opening

$$A \circ B = (A \ominus B) \oplus B$$

- Erosion followed by a Dilation
- Eliminate all pixels in regions that are too small to contain the structuring element

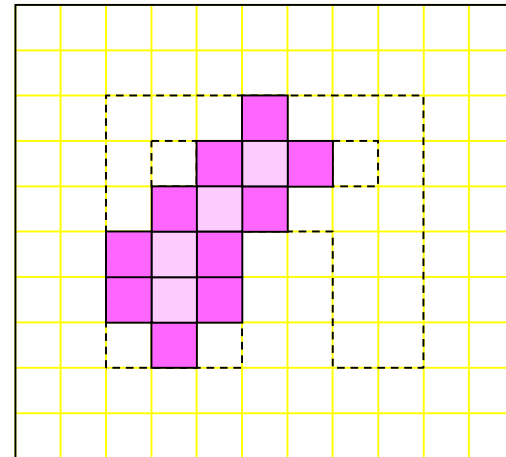


Original Image



Structuring  
Element

x : origin

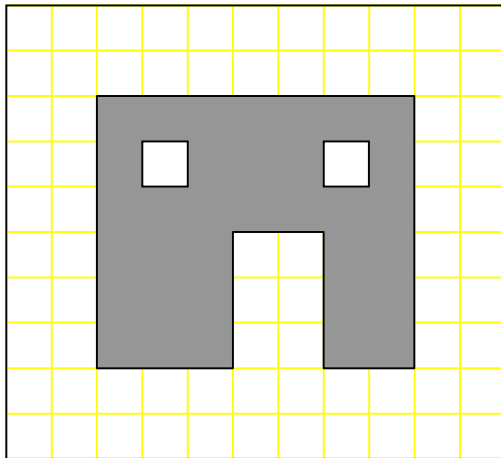


After Opening

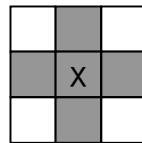
# Closing

$$A \bullet B = (A \oplus B) \ominus B$$

- Dilation followed by a Erosion
- Fill in hole and small gaps

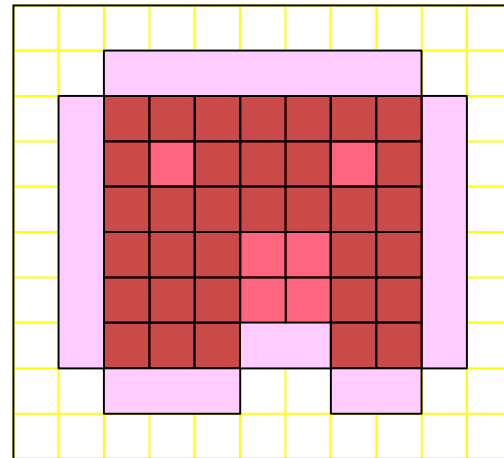


Original Image



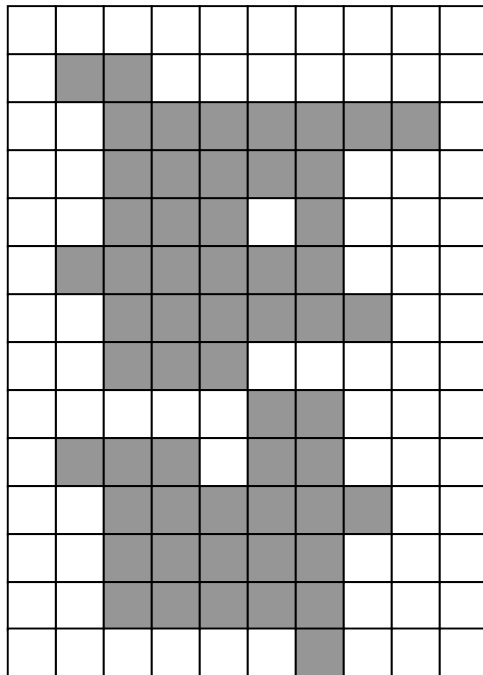
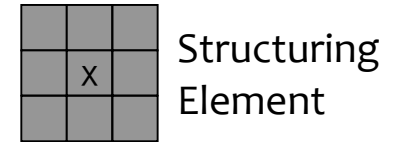
Structuring  
Element

x : origin

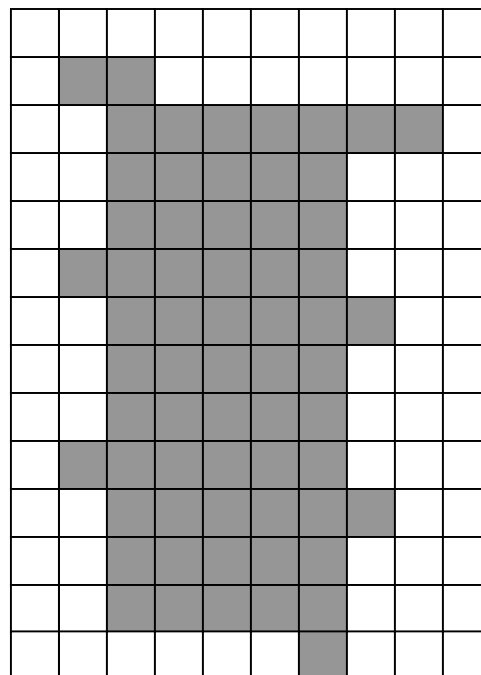


After Closing

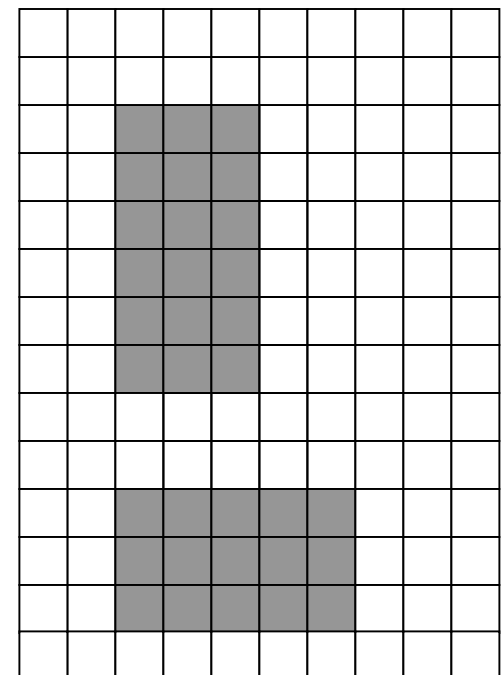
# example



Original Image



After Closing



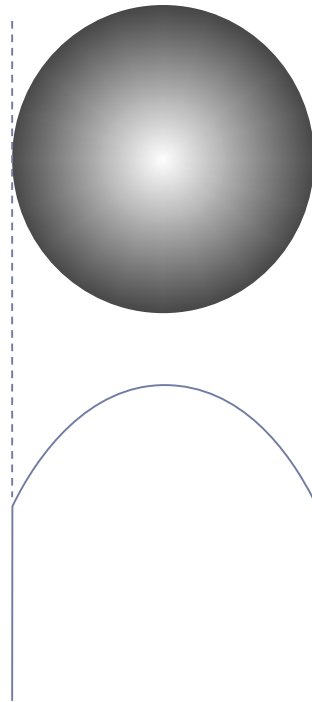
After Opening



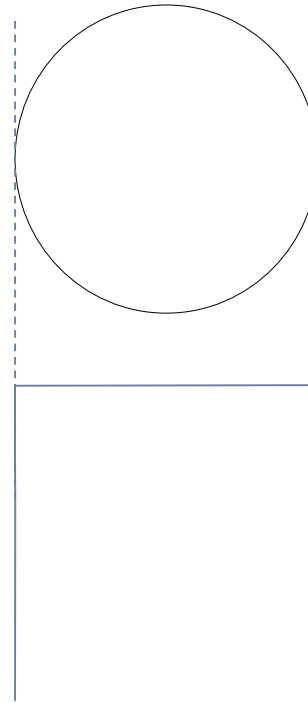
# Operations for gray-scale images

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Two types of a structuring element



nonflat



flat

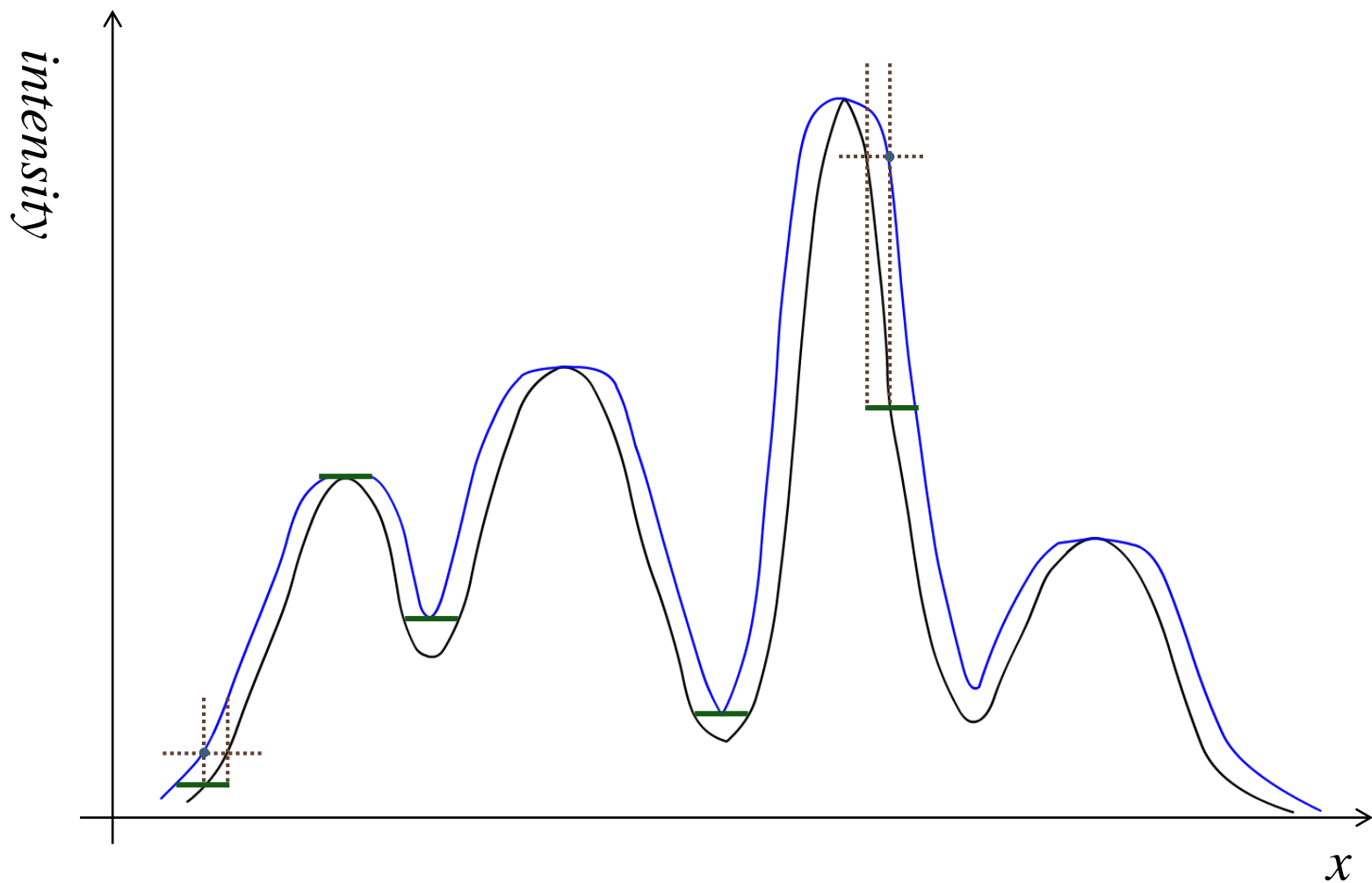
## Dilation of $f$ by $k$

$$(f \oplus k)(s, t) = \max_{(s, t) \in D_k} \{f(x - s, y - t) + k(s, t)\}$$

## Erosion of $f$ by $k$

$$(f \ominus k)(s, t) = \min_{(s, t) \in D_k} \{f(x + s, y + t) - k(s, t)\}$$

## Dilation with a flat structuring element



# OpenCV를 사용한 Morphological operations 구현

---

```
Mat    getStructuringElement( ... )  
void    dilate( ... )  
void    erode( ... )  
void    morphologyEx( ... )
```

```
Mat getStructuringElement(  
    int shape,  
    Size ksize,  
    Point anchor=Point(-1,-1)  
)
```

**shape**

- MORPH\_RECT: a rectangular structuring element
- MORPH\_ELLIPSE: an elliptic structuring element,  
Rect( 0, 0, ksize.width, ksize.height )
- MORPH\_CROSS: a cross-shaped structuring element
- CV\_SHAPE\_CUSTOM: custom structuring element

```

void dilate(
    InputArray src,
    OutputArray dst,
    InputArray element,
    Point anchor=Point(-1,-1),
    int iterations=1,
    int borderType=BORDER_CONSTANT,
    const Scalar& borderValue =
        morphologyDefaultBorderValue()
)

```

$$\text{dst}(x, y) = \max_{(x', y') : \text{element}(x', y') \neq 0} \text{src}(x + x', y + y')$$

```

void erode(
    InputArray src,
    OutputArray dst,
    InputArray element,
    Point anchor=Point(-1,-1),
    int iterations=1,
    int borderType=BORDER_CONSTANT,
    const Scalar& borderValue =
        morphologyDefaultBorderValue()
)

```

$$\text{dst}(x, y) = \min_{(x', y') : \text{element}(x', y') \neq 0} \text{src}(x + x', y + y')$$

```

void morphologyEx(
    InputArray src,
    OutputArray dst,
    int op,
    InputArray kernel,
    Point anchor=Point(-1,-1),
    int iterations=1,
    int borderType=BORDER_CONSTANT,
    const Scalar& borderValue =
        morphologyDefaultBorderValue()
)

```

op

- MORPH\_OPEN - an opening operation
- MORPH\_CLOSE - a closing operation
- MORPH\_GRADIENT - a morphological gradient  

$$(f \oplus b) - (f \ominus b)$$
- MORPH\_TOPHAT - “top hat”  $f - (f \circ b)$
- MORPH\_BLACKHAT - “black hat”  $(f \bullet b) - f$





dilation



erosion



gradient



dilation



erosion

```

void Dilation( const Mat &image, Mat &result, int type=0, int size=3 );
void Erosion( const Mat &image, Mat &result, int type=0, int size=3 );
void Morphology( const Mat &image, Mat &result, int op, int type=0, int size=3 );

int main(void){
    Mat image = imread( "text_m.bmp", -1 );
    if( image.data == NULL ) return -1;

    Mat dilation;
    Dilation( image, dilation );

    Mat erosion;
    Erosion( image, erosion );

    Mat morphology;
    Morphology( image, morphology, 2 );

    // Display the images
    namedWindow( "Image" );
    namedWindow( "Dilation" );
    namedWindow( "Erosion" );
    namedWindow( "Morphology" );
    imshow( "Image", image );
    imshow( "Dilation", dilation );
    imshow( "Erosion", erosion );
    imshow( "Morphology", morphology );

    waitKey();

    return 0;
}

```

```

void Dilation( const Mat &image, Mat &result, int type, int size )
{
    // allocate if necessary
    result.create( image.size(), image.type() );

    int dilation_type;
    if( type == 0 )
        dilation_type = MORPH_RECT;
    else if( type == 1 )
        dilation_type = MORPH_CROSS;
    else if( type == 2 )
        dilation_type = MORPH_ELLIPSE;

    Mat element = getStructuringElement(
        dilation_type, Size(size, size) );

    // Apply the dilation operation
    dilate( image, result, element );
}

```

```

void Erosion( const Mat &image, Mat &result, int type, int size )
{
    // allocate if necessary
    result.create( image.size(), image.type() );

    int erosion_type;
    if( type == 0 )
        erosion_type = MORPH_RECT;
    else if( type == 1 )
        erosion_type = MORPH_CROSS;
    else if( type == 2 )
        erosion_type = MORPH_ELLIPSE;

    Mat element = getStructuringElement(
        erosion_type, Size(size, size) );

    // Apply the dilation operation
    erode( image, result, element );
}

```

```
void Morphology( const Mat &image, Mat &result, int op, int type, int size )
{
    // allocate if necessary
    result.create( image.size(), image.type() );

    int operation;
    switch( op )
    {
    case 0:
        operation = MORPH_OPEN;
        break;
    case 1:
        operation = MORPH_CLOSE;
        break;
    case 2:
        operation = MORPH_GRADIENT;
        break;
    case 3:
        operation = MORPH_TOPHAT;
        break;
    case 4:
        operation = MORPH_BLACKHAT;
        break;
    }
}
```

```
int mor_type;
if( type == 0 )
    mor_type = MORPH_RECT;
else if( type == 1 )
    mor_type = MORPH_CROSS;
else if( type == 2)
    mor_type = MORPH_ELLIPSE;

Mat element = getStructuringElement( mor_type, Size(size, size) );

// Apply the specified morphology operation
morphologyEx( image, result, operation, element );
}
```

# Reference

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- R. Gonzalez, R. Woods, **Digital Image Processing (2nd Edition)**, Prentice Hall, 2002
- Scott E Umbaugh, **Computer Imaging**, CRC Press, 2005
- R. Laganière, **OpenCV2 Computer Vision: Application Programming Cookbook**, PACKT Publishing, 2011
- <http://docs.opencv.org>