Before Starting

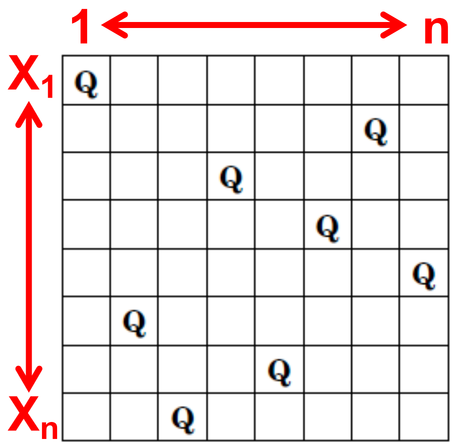
- Form a group of two students.
- Download the latest release of MiniZinc (20/06/23).
- Use a separate folder for each problem.
- Configure the solver to obtain the solution statistics, to search for one or all solutions, and to set a time limit when needed.
- Use commas when reporting big numbers. E.g.,
  - 976474 instead of 976,474

N-Queens

- Place n queens in an nxn board so that no two queens can attack each other.

Q							
						Q	
			Q				
					Q		
							Q
	Q						
				Q			
		Q					

Row Model



- Variables and Domains
  - A variable for each row $[X_1, X_2, \ldots, X_n]$ → no row attack
  - Domain values [1..n] represent the columns:
    - $X_i = j$ means that the queen in row i is in column j
- Constraints
  - alldifferent($[X_1, X_2, \ldots, X_n]$) → no column attack
  - for all $i<j$ $|X_i - X_j| \neq |i - j|$ → no diagonal attack

RC Combined Model 1

- Variables
  - $[X_1, X_2, \ldots, X_n], [Y_1, Y_2, \ldots, Y_n] \in [1..n]$
- Constraints
  - alldifferent($[X_1, X_2, \ldots, X_n]$)
  - alldifferent($[Y_1, Y_2, \ldots, Y_n]$)
  - for all $i<j$ $|X_i - X_j| \neq |i - j|$
  - for all $i<j$ $|Y_i - Y_j| \neq |i - j|$
- Channeling Constraints
  - for all i,j $X_i = j \leftrightarrow Y_j = i$

RC Combined Model 2

- Variables
  - $[X_1, X_2, \ldots, X_n], [Y_1, Y_2, \ldots, Y_n] \in [1..n]$
- Constraints
  - ~~alldifferent~~ $\cancel{([X_1, X_2, \ldots, X_n])}$
  - ~~alldifferent~~ $\cancel{([Y_1, Y_2, \ldots, Y_n])}$
  - for all i<j $|X_i - X_j| \neq |i - j|$
  - for all i<j $|Y_i - Y_j| \neq |i - j|$
- Channeling Constraints
  - for all i,j $X_i = j \leftrightarrow Y_j = i$

RC Combined Model 3

- Variables
  - $[X_1, X_2, \ldots, X_n], [Y_1, Y_2, \ldots, Y_n] \in [1..n]$
- Constraints
  - ~~alldifferent~~$\cancel{([X_1, X_2, \ldots, X_n])}$
  - ~~alldifferent~~$\cancel{([Y_1, Y_2, \ldots, Y_n])}$
  - for all $i<j$ $|X_i - X_j| \neq |i - j|$
  - ~~for all~~ $\cancel{i<j \quad |Y_i - Y_j| \neq |i - j|}$
- Channeling Constraints
  - for all i,j $X_i = j \leftrightarrow Y_j = i$

Alldifferent Model

- Variables
  - $[X_1, X_2, \ldots, X_n] \in [1..n]$
- Constraints
  - alldifferent($[X_1, X_2, \ldots, X_n]$)
  - alldifferent($[X_1 + 1, X_2 + 2, \ldots, X_n + n]$)
  - alldifferent($[X_1 - 1, X_2 - 2, \ldots, X_n - n]$)

Combined Alldifferent and Symmetry Breaking Model

- Variables
  - for all i, $X_i \in [1..n]$, for all i, j $B_{ij} \in [0..1]$
- Constraints
  - alldifferent($[X_1, X_2, \ldots, X_n]$)
  - alldifferent($[X_1 + 1, X_2 + 2, \ldots, X_n + n]$)
  - alldifferent($[X_1 - 1, X_2 - 2, \ldots, X_n - n]$)
  - lex≤(B , π(B)) for all π
    - Study Section 2.6.6 of the MiniZinc Tutorial.
- Channeling Constraints
  - for all i,j $X_i = j \leftrightarrow B_{ij} = 1$

N-Queens

- Download the zip file containing the templates for the:
  - the row model;
  - the rc combined models;
  - the alldifferent model;
  - the combined alldifferent and sym. breaking combined model.
- Complete the implementation of the 6 models.
- Search for **all solutions** for N = 8, 9, 10, 12 using the default search of Gecode.

N-Queens

- Report the number of solutions and failures in two tables.

n	#sols	r	rc1	rc2	rc3	alldiff
8						
9						
10						
12						

n	#sols	alldiffsym
8		
9		
10		
12		

- Answer briefly the following questions on the solver performance.
  1. What is happening when going r → rc1 → alldiff ? Why?
  2. What is happening when going rc1 → rc2 → rc3 ? Why?
  3. What is happening when going alldiff → alldiffsym? Why?

A Sequence Puzzle

- Find the code of my safe composed of 10 digits, where the first digit gives the number of 0s in the code, the second the number of 1s, the third the number of 2s, and so on with the 10th digit giving the number of 9s in the code.
- Solve the puzzle with a more general version: find a sequence of n integers $X_0,\ldots, X_{n-1}$ that contains values between 0 and $n-1$, in a way that any value i appears X_i times in the sequence.

A Sequence Puzzle

- E.g., with n = 5, a solution is [2, 1, 2, 0, 0]:
  - $X_0 = 2 \rightarrow$ 0 appears 2 times in the sequence
  - $X_1 = 1 \rightarrow$ 1 appears once in the sequence
  - $X_2 = 2 \rightarrow$ 2 appears 2 times in the sequence
  - $X_3 = 0 \rightarrow$ 3 appears 0 times in the sequence
  - $X_4 = 0 \rightarrow$ 4 appears 0 times in the sequence

A Sequence Puzzle

- Model the puzzle as follows.
  - Variables and Domains
    - $X_0, \ldots, X_{n-1} \in \{0, \ldots, n-1\}$
  - Constraints
    - for all i, $X_i = \sum_j (X_j = i)$
  - Implied constraints
    - $\sum_i X_i = n$
    - $\sum_i X_i * i = n$

A Sequence Puzzle

- Search for **one solution** for N = 500 and N=1000, using the default search of Gecode.
- Consider the two models with and without implied constraints and report the number failures and total time in a table.

n	Base		Base + Implied	
	Fails	Time	Fails	Time
500				
1000				

- Answer briefly the following question on the solver performance.
  - What is happening when going base → base+implied ? Why?