Homework 0

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1. Fill in the following table

p	q	$\neg p$	$\neg q$	$p \implies q$	$\neg p \implies \neg q$	$\neg q \implies \neg p$	$\neg(p \iff q)$
T	T	F	F	T	T	T	F
T	F	F	T	F	T	F	T
F	T	T	F	T	F	T	F
F	F	T	T	T	T	T	T

2. Which of the following expressions are tautologies?

- $(((p \implies q) \implies p) \implies p)$: Tautology
- $(((p \vee q) \wedge \neg q) \implies q)$: Tautology
- $(p \implies (\neg p \wedge q))$: Not a Tautology
- $(((p \vee \neg q) \implies r) \implies q)$: Not a Tautology
- $(((p \implies q) \vee (r \implies s)) \implies ((p \vee r) \implies (q \wedge s)))$: Tautology
- $(((p \implies q) \wedge (r \implies s)) \implies ((p \vee r) \implies (q \wedge s)))$: Tautology

3. State whether the given biconditional is true or false under the assumption that all variables are quantified over $\mathbb{R}$. Give a brief explanation for each.

(a) $x^2 = 9$ if and only if $x = 3$: False because $(-3)^2 = 9$
(b) x is a positive number if and only if $x > 0$: True because positive numbers are numbers that are greater than 0
(c) $|x|$ is a positive number if and only if $x \neq 0$: True because the absolute value of a number is always positive unless its 0
(d) A number x is rational if and only if it has a terminating decimal expansion : False because $\frac{1}{3}$ is rational but does not have a terminating decimal expansion
(e) Today is March 1 if and only if yesterday was February 28 : False because today is March 1st and yesterday was February 29th if its a leap year

LaTeX Exercises

1. Please type me! Sphinx of black quartz, judge my vow!
2. $e^{i\pi} + 1 = 0$
3. $e^{i\Theta} = \cos(\Theta) + i\sin(\Theta)$
4. $G_{\mu v} + \Lambda g_{\mu v} = \frac{8\pi G}{c^4} T_{\mu v}$
5. $x = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$
6. $\overrightarrow{L} = \overrightarrow{r} \times \overrightarrow{t}$

7. $(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}$
8. $\sqrt{\frac{a_1^2+\cdots+a_n^2}{n}} \geq \frac{a_1+\cdots+a_n}{n} \geq \sqrt[n]{a_1+\cdots+a_n} \geq \frac{n}{\frac{1}{a_1}+\cdots+\frac{1}{a_n}}$
9. $|\langle x,y\rangle| \leq \langle x,x\rangle * \langle y,y\rangle$
10.

A1: $\varphi \longrightarrow (\psi \rightarrow \varphi)$
A2: $(\varphi \longrightarrow (\psi \rightarrow \theta)) \longrightarrow ((\varphi \rightarrow \psi) \longrightarrow (\varphi \rightarrow \theta))$
A3: $(\neg\varphi \rightarrow \neg\psi) \longrightarrow (\psi \rightarrow \varphi)$