Reading Summary Appendix A, B

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1 Appendix A Logic and Proof

Propositions

A **proposition** is a declarative statement that can be either true or false.
EX: 2 is a positive integer

Compound Statements

A **compound statement** is when two or more statements are combined using words like "or" and "and."
P and Q is true when both P and Q are true, otherwise it is false.
P or Q is true when one or both are true and false when both are false.

Negation

Negation($\neg$) changes the meaning of a proposition by inverting it. Changing its outcome from false to true and vice versa.

Quantifiers

Universal quantifiers ($\forall$) are used to state that a statement is true for all values of a variable. EX: For all x, x^2 is a positive integer.
Existential quantifiers ($\exists$) are used to state that a statement is true for at least one value of a variable. EX: There exists a positive integer x such that x^2 is a perfect square.
The negation of a statement with a universal quantifier is a statement with an existential quantifier.

Conditional Statements

A **conditional statement** is a statement is a *if-then* statement.
EX: If x is a positive integer, then x^2 is a positive integer.
Conditional statements are written symbolically as $Q \implies P$ $P \implies Q$ is a true statement when both P and Q are true and false when P is true and Q is false. The contrapositive of the conditional statement $P \implies Q$ is the statement $\neg Q \implies \neg P$.

Proofs

There are 3 main types of proofs:

- Direct Proof
- Contrapositive Proof
- Proof by Contradiction

Direct Proof

A **direct proof** is a proof that shows that a statement is true by showing that each of its premises is true. This method relies on the rule of logic called **modus ponens**: If R is a true statement and $R \implies S$ is a true conditional statement, then S is a true statement.
Example: (from the book) To prove the theorem $P \implies Q$ by the direct method, you find a series of statements P_1, P_2,..., P_n and the verify that each of the implications $P_n \implies P_{n+1}$ is true. Then the assumption that P is true and repeated use of **modus ponens** show that Q is true.

Contrapositive Proof

A **contrapositive proof** is a proof that shows that a statement is true by showing that the contrapositive of the statement is true.

p	q	$p \to q$	$q \to p$ (converse)	$\sim p \to \sim q$ (inverse)	$\sim q \to \sim p$ (contrapositive)
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Figure 1: Contrapositive Truth Table

Proof by Contradiction

A **proof by contradiction** is a proof that shows that a statement is true by showing that the negation of the statement is false.
To prove a multi conditional statement you must prove all parts of it.

2 Appendix B Sets and Functions

Sets

A **set** is a collection of objects. For Example $\mathbb{Z}$ is the set of all integers. $\emptyset$ is the empty set.

Subsets

A set B is said to be a subset of a set C (written $B \subseteq C$) provided that every element of B is also an element of C. Sets are also subsets of themselves

Set Operations

Set operations are used to combine sets.

- Union $\cup$ is the set of all elements that are in either set.
- Intersection $\cap$ is the set of all elements that are in both sets.
- Difference $\setminus$ is the set of all elements that are in the first set but not the second.
- Complement $\bar{A}$ is the set of all elements that are not in the set.

Functions

A **function** is a rule that assigns to each element of a set exactly one element of another set. **Example**: A set P of integers are mapped to a set Q of real numbers by the function $f(x) = x^2$. This can also be written as $P \to Q$.

Binary Operations

A **binary operation** is a function that takes two arguments and returns a single value.

Injective and Surjective Functions

A function is **injective** if no two elements of the domain map to the same element of the range. Or it is **one-to-one**.