Homework 3 problem 6, presentation

January 2023

1. Prove the following statements.

   (a) If $a \in \mathbb{Z}_n$ is a unit, then a is not a divisor of zero.
   Let a be a unit of $\mathbb{Z}_n$, and assume a is a divisor of zero.
   Because a is a unit there exists $b \in \mathbb{Z}_n$ such that $ab = 1$. And because a is a divisor of zero there exists $c \in \mathbb{Z}_n$ such that $ac = 0$ where $c \neq 0$.
   $[0] = [0 * b]$
   $0 = b(ac)$
   $0 = (ba)c$
   $0 = (1)c$
   $0 = c$
   Which is a contradiction. Therefore, a is not a divisor of zero.

   (b) If $a \in \mathbb{Z}_n$ is not a divisor of zero, then a is a unit.
   Contrapositive of the previous statement. If a is not a unit, then a is a divisor of zero.
   If a is not a unit of $\mathbb{Z}_n$, then $\gcd(a, n) = k \neq 1$.
   Then let $c = \frac{n}{k}$, $c \in \mathbb{Z}$, and so $0 < c < n$ and $b \in \mathbb{Z}_n$.
   $ac = a(\frac{n}{k}) = (\frac{a}{k})n$
   Since $k \mid a$, ac is equal to an integer times n Therefore $ac = 0$. Meaning a is a divisor of zero.
   Because the contrapositive is true, the original statement is true.