

# Reading Summary Appendix A, B

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## 1 Appendix A Logic and Proof

### Propositions

A **proposition** is a declarative statement that can be either true or false.

EX: 2 is a positive integer

### Compound Statements

A **compound statement** is when two or more statements are combined using words like "or" and "and."

$P$  and  $Q$  is true when both  $P$  and  $Q$  are true, otherwise it is false.

$P$  or  $Q$  is true when one or both are true and false when both are false.

### Negation

**Negation**( $\neg$ ) changes the meaning of a proposition by inverting it. Changing its outcome from false to true and vice versa.

### Quantifiers

Universal quantifiers ( $\forall$ ) are used to state that a statement is true for all values of a variable. EX: For all  $x$ ,  $x^2$  is a positive integer.

Existential quantifiers ( $\exists$ ) are used to state that a statement is true for at least one value of a variable.

EX: There exists a positive integer  $x$  such that  $x^2$  is a perfect square.

The negation of a statement with a universal quantifier is a statement with an existential quantifier.

### Conditional Statements

A **conditional statement** is a statement is a *if-then* statement.

EX: If  $x$  is a positive integer, then  $x^2$  is a positive integer.

Conditional statements are written symbolically as  $P \implies Q$ .  $P \implies Q$  is a true statement when both  $P$  and  $Q$  are true and false when  $P$  is true and  $Q$  is false. The contrapositive of the conditional statement  $P \implies Q$  is the statement  $\neg Q \implies \neg P$ .

### Proofs

There are 3 main types of proofs:

- Direct Proof
- Contrapositive Proof
- Proof by Contradiction

## Direct Proof

A **direct proof** is a proof that shows that a statement is true by showing that each of its premises is true. This method relies on the rule of logic called **modus ponens**: If  $R$  is a true statement and  $R \implies S$  is a true conditional statement, then  $S$  is a true statement.

**Example:** (from the book) To prove the theorem  $P \implies Q$  by the direct method, you find a series of statements  $P_1, P_2, \dots, P_n$  and verify that each of the implications  $P_n \implies P_{n+1}$  is true. Then the assumption that  $P$  is true and repeated use of **modus ponens** show that  $Q$  is true.

## Contrapositive Proof

A **contrapositive proof** is a proof that shows that a statement is true by showing that the contrapositive of the statement is true.

p	q	$p \rightarrow q$	$q \rightarrow p$ (converse)	$\sim p \rightarrow \sim q$ (inverse)	$\sim q \rightarrow \sim p$ (contrapositive)
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Figure 1: Contrapositive Truth Table

## Proof by Contradiction

A **proof by contradiction** is a proof that shows that a statement is true by showing that the negation of the statement is false.

To prove a multi conditional statement you must prove all parts of it.

## 2 Appendix B Sets and Functions

### Sets

A **set** is a collection of objects. For Example  $\mathbb{Z}$  is the set of all integers.  $\emptyset$  is the empty set.

### Subsets

A set  $B$  is said to be a subset of a set  $C$  (written  $B \subseteq C$ ) provided that every element of  $B$  is also an element of  $C$ . Sets are also subsets of themselves

### Set Operations

Set operations are used to combine sets.

- Union  $\cup$  is the set of all elements that are in either set.
- Intersection  $\cap$  is the set of all elements that are in both sets.
- Difference  $\setminus$  is the set of all elements that are in the first set but not the second.
- Complement  $\bar{A}$  is the set of all elements that are not in the set.

### Functions

A **function** is a rule that assigns to each element of a set exactly one element of another set.

**Example:** A set  $P$  of integers are mapped to a set  $Q$  of real numbers by the function  $f(x) = x^2$ . This can also be written as  $P \rightarrow Q$ .

## Binary Operations

A **binary operation** is a function that takes two arguments and returns a single value.

## Injective and Surjective Functions

A function is **injective** if no two elements of the domain map to the same element of the range. Or it is **one-to-one**.