Estimating spatial weights – a simple case

Presenter: Haoyang Li

15th May 2024

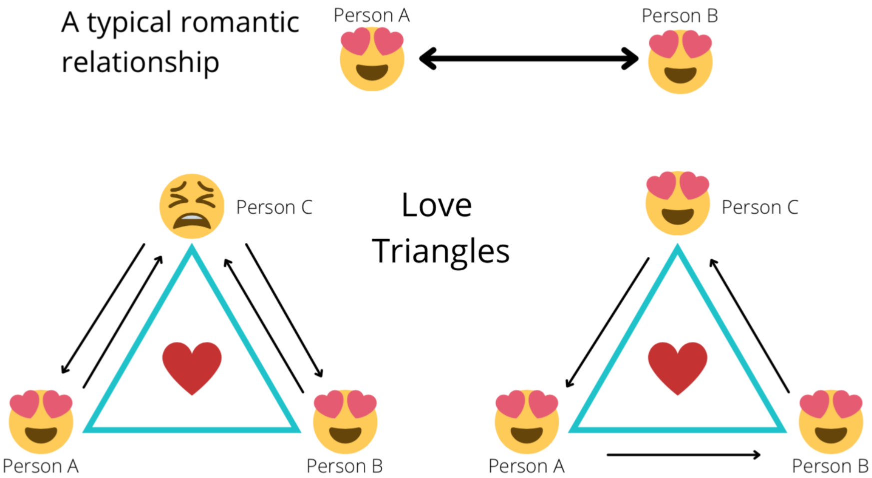


Figure: Love triangle

The setting

$$\boldsymbol{W} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \\ \boldsymbol{y_3} \end{pmatrix} = \rho \boldsymbol{W} \begin{pmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \\ \boldsymbol{y_3} \end{pmatrix} + \begin{pmatrix} \beta_1 \boldsymbol{V_1} \\ \beta_2 \boldsymbol{V_2} \\ \beta_3 \boldsymbol{V_3} \end{pmatrix} + \begin{pmatrix} \boldsymbol{e_1} \\ \boldsymbol{e_2} \\ \boldsymbol{e_3} \end{pmatrix}$$

Method: OCMT (Chudik et al., 2018)

Stage 1:

$\boldsymbol{y_1}$ on $\boldsymbol{y_2}$ (IV: $\boldsymbol{V_2}$).

$$\boldsymbol{y_1} = \phi_2 \boldsymbol{y_2} + \beta_1 \boldsymbol{V_1} + u_1$$

$\boldsymbol{y_1}$ on $\boldsymbol{y_3}$ (IV: $\boldsymbol{V_3}$).

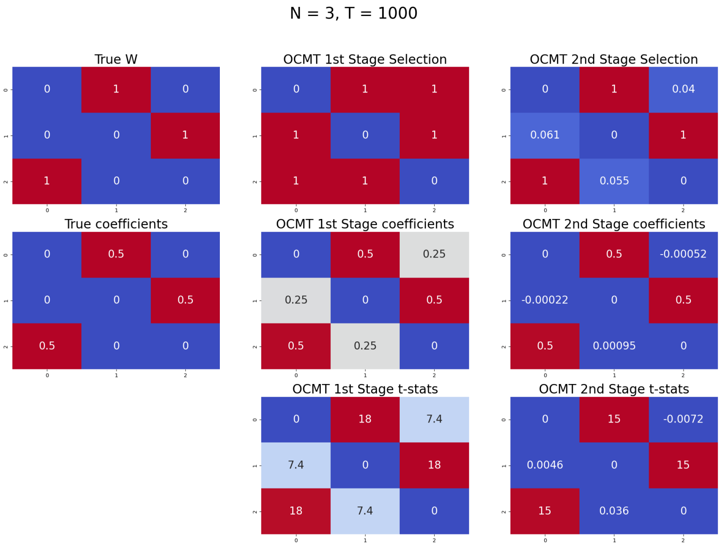
$$\boldsymbol{y_1} = \phi_3 \boldsymbol{y_3} + \beta_1 \boldsymbol{V_1} + u_1$$

Stage 2:

$$\tilde{\boldsymbol{y}}_{\boldsymbol{1}} = \gamma_2 \tilde{\boldsymbol{y}}_{\boldsymbol{2}} + \gamma_3 \tilde{\boldsymbol{y}}_{\boldsymbol{3}} + \boldsymbol{\epsilon_1}$$

$$\hat{\gamma}_2^{IV} = \rho + O(\frac{1}{T}), \quad \hat{\gamma}_3^{IV} = O(\frac{1}{T})$$

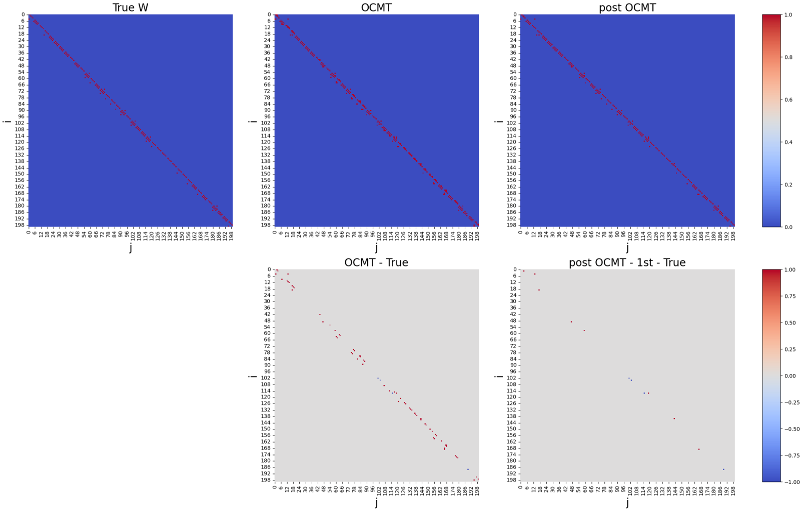
Monte Carlo



N = 3, T = 1000
True W
0 1 0
0 0 1
1 0 0
OCMT 1st Stage Selection
0 1 1
1 0 1
1 1 0
OCMT 2nd Stage Selection
0 1 0.04
0.061 0 1
1 0.055 0
True coefficients
0 0.5 0
0 0 0.5
0.5 0 0
OCMT 1st Stage coefficients
0 0.5 0.25
0.25 0 0.5
0.5 0.25 0
OCMT 2nd Stage coefficients
0 0.5 -0.00052
-0.00022 0 0.5
0.5 0.00095 0
OCMT 1st Stage t-stats
0 18 7.4
7.4 0 18
18 7.4 0
OCMT 2nd Stage t-stats
0 15 -0.0072
0.0046 0 15
15 0.036 0

what if N $\to \infty$?

N = 200, T = 200



Issues in the first step

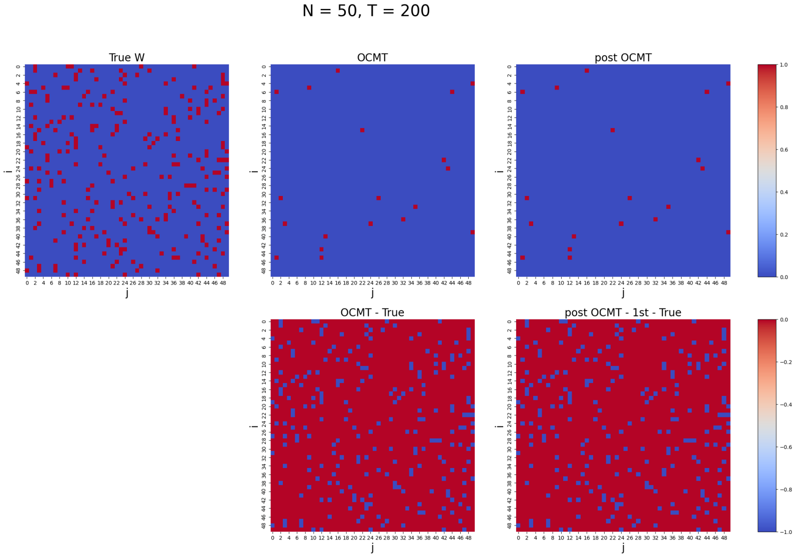
Selection standard:

$$t_{i,j} = \frac{\boldsymbol{\hat{y}_j' M_i y_i}}{\sqrt{(\sum_{k=1}^{N} p_{ik}\hat{\sigma}_k^2)\boldsymbol{\hat{y}_j' M_i \hat{y}_j}}}$$

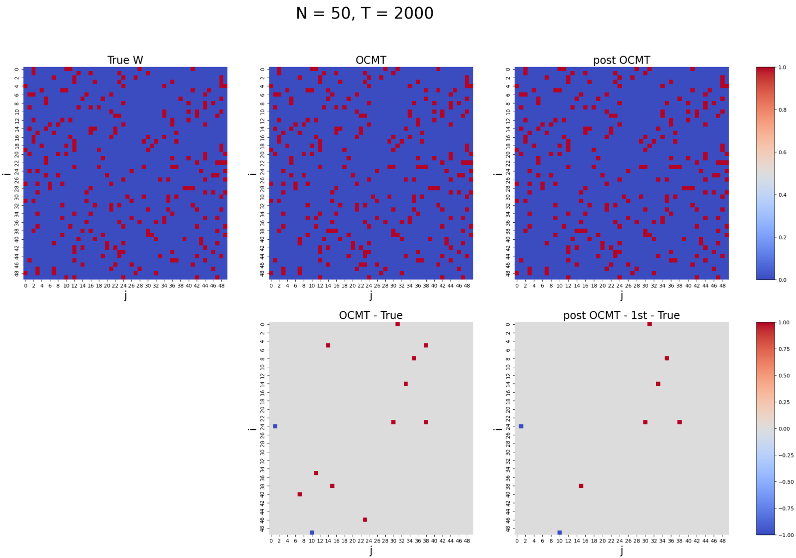
where $Var(e_{kt}) = \sigma_k^2$, $\boldsymbol{M_i = I - V_i(V_i' V_i)^{-1} V_i'}$, $\boldsymbol{\hat{y}_j} = \boldsymbol{V_j(V_j' V_j)^{-1} V_j' y_j}$

- Selecting too little true signals:
  Under assumption $|\rho \boldsymbol{w_i' \iota}| \leq 1$, further assume equal weights on k^* elements, then $|w_{ij}| \leq \frac{1}{\rho k^*}$. Larger $k^* \to$ smaller $t_{i,j}$.

Issues in the first step



Issues in the first step



Issues in the first step

Selection standard:

$$t_{i,j} = \frac{\hat{\boldsymbol{y}}_j' \boldsymbol{M}_i \boldsymbol{y}_i}{\sqrt{(\sum_{k=1}^{N} p_{ik}\hat{\sigma}_k^2)\hat{\boldsymbol{y}}_j' \boldsymbol{M}_i \hat{\boldsymbol{y}}_j}}$$

where $Var(e_{kt}) = \sigma_k^2$, $\boldsymbol{M_i} = \boldsymbol{I} - \boldsymbol{V_i}(\boldsymbol{V_i'}\boldsymbol{V_i})^{-1}\boldsymbol{V_i'}$, $\hat{\boldsymbol{y}}_j = \boldsymbol{V_j}(\boldsymbol{V_j'}\boldsymbol{V_j})^{-1}\boldsymbol{V_j'}\boldsymbol{y_j}$

- Selecting too little true signals:
  Under assumption $|\rho \boldsymbol{w_i}| \leq 1$, further assume equal weights on k^* elements, then $|w_{ij}| \leq \frac{1}{\rho k^*}$. Larger $k^* \to$ smaller $t_{i,j}$.

Solution 1: restrict $N = O(T^\alpha)$ with $0 < \alpha < 1$?
But how is α decided? Why not include all elements in regression at first?
Solution 2: Allow $N = O(T)$, impose stricter sparsity assumption.
How sparse is sparse?

Thank you!

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References I

Alexander Chudik, George Kapetanios, and M Hashem Pesaran. A one covariate at a time, multiple testing approach to variable selection in high-dimensional linear regression models. *Econometrica*, 86(4): 1479–1512, 2018.