Inverse kinematic solution

$$\theta_1 = atan(\frac{Y}{X})$$

The distance between P1 and P3 is called d.
If we find d, the problem is almost solved because we would know the 3 sides of the triangle (P1, P2, P3). Let'

If P3proj is the projection of P3 on the (x, y) plan, and dproj is the distance between P0 and P3proj then :

$$d_{proj} = \sqrt{(X^2 + Y^2)}$$

If d13 is the distance between P1 and P3proj then :

$$d_{13} = d_{proj} - l_1$$

Hence :

$$d = \sqrt{(d_{13}^2 + Z^2)}$$

The sides of the triangle (P1, P2, P3) are known, Al-Kashi's theorem closes the deal.
Let's call b the angle (P1P2, P1P3) and a the angle (P1P3, P1P3proj) :

$$\theta_2 = a + b$$
$$a = atan(\frac{Z}{d_{13}})$$
$$b = AlKashi(l2, d, l3)$$
$$\theta_3 = AlKashi(l2, l3, d) + \pi$$

Where :

$$AlKashi(a, b, c) = \pm acos((a^2 + b^2 - c^2)/(2 * ab))$$