

Задача 1.15

Для каких n получим сумму ряда аналитическим способом:

$$\begin{aligned} S_N &= \sum_{n=0}^N \frac{48}{5(n^2+6n+5)} = \frac{48}{5} \sum_{n=0}^N \frac{1}{(n+1)(n+5)} = \left\{ \frac{1}{(n+1)(n+5)} = \frac{A}{n+1} + \frac{B}{n+5} = \frac{1}{4} \frac{1}{n+1} - \frac{1}{4} \frac{1}{n+5} \right\} = \frac{12}{5} \sum_{n=0}^N \left(\frac{1}{n+1} - \frac{1}{n+5} \right) = \\ &= \left\{ \sum_{n=0}^N \frac{1}{n+1} - \frac{1}{n+5} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cancel{\frac{1}{6}} + \cancel{\frac{1}{7}} + \cancel{\frac{1}{8}} + \cancel{\frac{1}{9}} + \cancel{\frac{1}{10}} + \dots = 1 + \frac{6+4+3}{12} = 2\frac{1}{12} = \frac{25}{12} \right\} = \frac{12}{5} \cdot \frac{25}{12} = \boxed{5} \end{aligned}$$

$$\text{Ответ: } \sum_{n=0}^{\infty} \frac{48}{5(n^2+6n+5)} = 5$$