Assignment 2
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1. To prove: $R S(a) R^T = S(Ra)$

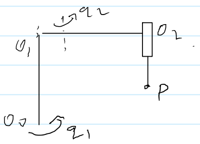
Consider $S(Ra).R.a$ — We will use the identity $S(a).p = a \times p$ to separate R and a in $S(Ra)$

$$S(Ra).(R.a) = (R.a) \times (R.a) = R.(a \times a) = R.S(a).a = R.S(a).I^3.a$$
$$= R.S(a).R^T.R.a$$

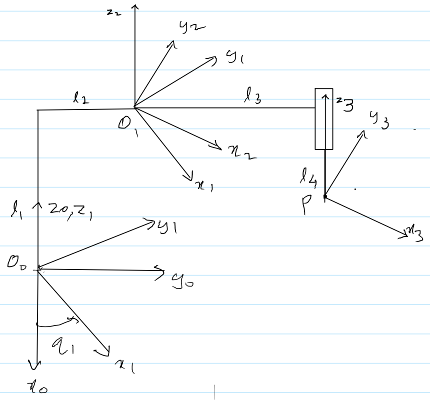
$$\Rightarrow S(Ra).(Ra) = (R.S(a)R^T)(Ra)$$

$$\Rightarrow \underline{S(Ra) = (R.S(a).R^T)}$$

2.



RRP SCARA schematic
initially, all links are in the y-z plane.



(indicated l_1 and l_2 correspond to initial setup)

l_4 is the fixed initial length before extension of the prismatic joint

$$R_0^1 = \begin{bmatrix} R_z q_1 \end{bmatrix} = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} R_z q_2 \end{bmatrix} = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ l_2 \\ l_1 \end{bmatrix}$$

$$R_2^3 = I^3 \qquad d_2^3 = \begin{bmatrix} 0 \\ l_3 \\ -l_4 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_1^2 = \begin{bmatrix} cq_2 & -sq_2 & 0 & 0 \\ sq_2 & cq_2 & 0 & l_2 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 1 & -l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ -d \end{bmatrix}$$ d is the extension of the prismatic joint

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & 0 \\ sq_2 & cq_2 & 0 & l_2 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 1 & -l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

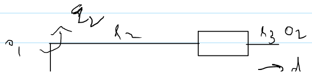
$$= \quad '' \quad \begin{bmatrix} cq_2 & -sq_2 & 0 & 0 \\ sq_2 & cq_2 & 0 & l_2 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_3 \\ -(d+l_4) \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -sq_2 l_3 \\ l_2 + cq_2 l_3 \\ l_1 - (d+l_4) \\ 1 \end{bmatrix} = \begin{bmatrix} -l_3 cq_1 sq_2 - l_3 sq_1 cq_2 - l_2 sq_1 \\ l_3 cq_1 cq_2 - l_3 sq_1 sq_2 + l_2 cq_1 \\ l_1 - (d+l_4) \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} -l_3 cq_1 sq_2 - l_3 sq_1 cq_2 - l_2 sq_1 \\ l_3 cq_1 cq_2 - l_3 sq_1 sq_2 + l_2 cq_1 \\ l_1 - (d+l_4) \end{bmatrix}$$

4.

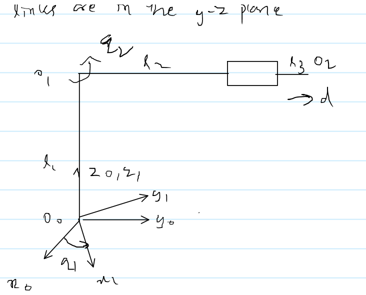
RRP Stanford Schematic
Initially all links are in the y-z plane



$$R_{01} = R_{z q_1} \qquad d_{01} = 0 \qquad H_{01} = \begin{bmatrix} R_{z q_1} & d_{01} \\ 0 & 1 \end{bmatrix}$$

$$R_{12} = R_{y,\pi/2} . R_{z, q_2} \qquad d_{12} = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} \qquad H_{12} = \begin{bmatrix} R_{y,\pi/2}.R_{z q_2} & d_{12} \\ 0 & 1 \end{bmatrix}$$

Initially all links are in the y-z plane



$R_{01} = R_{z q_1}$ $d_{01} = 0$ $H_{01} = \begin{bmatrix} R_{z q_1} & d_{01} \\ 0 & 1 \end{bmatrix}$

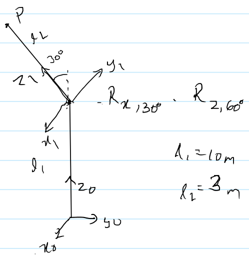
$R_{12} = R_{y,\pi/2} \cdot R_{z,q_2}$ $d_{12} = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$ $H_{12} = \begin{bmatrix} R_{y,\pi/2} \cdot R_{z q_2} & d_{12} \\ 0 & 1 \end{bmatrix}$

$R_{23} = I^3$ $d_{23} = \begin{bmatrix} 0 \\ l_2 + l_3 \\ 0 \end{bmatrix}$ $H_{23} = \begin{bmatrix} I^3 & d_{23} \\ 0 & 1 \end{bmatrix}$

$P_3 = \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix}$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_{01} \cdot H_{12} \cdot H_{23} \cdot \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

5.



$- R_{x,30°} \cdot R_{z,60°}$

$l_1 = 10m$

$l_2 = 3m$

At final orientation, obstacle is 3m directly above the drone

i.e w.r.t drone, position of obstacle is 0, 0, 3.

This means that the final rotation $R_{z,60°}$ does not affect relative position of obstacle.

$R_{01} = R_{x,30°}$ $d_{01} = [0, 0, 10]^T$

$$H_{01} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad P_1 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3/2 \\ 10 + 3\sqrt{3}/2 \\ 1 \end{bmatrix}$$

Position of obstacle in base frame is $\left(0\hat{\imath} - \frac{3}{2}\hat{\jmath} + \left(10 + \frac{3\sqrt{3}}{2}\right)\hat{k} \right) m$

6. Types of Gearboxes used for Motors in robotic applications:

<u>Planetary Gearbox</u>

Central sun gear rotates three to four planet gears that encircle it.

Pros- High torque produced in a small area, precise and durable

Cons- Complex design and construction, high power dissipation

<u>Cycloidal Drive</u>

Also called speed reducers, they reduce the speed of rotation of the input shaft. The ratio of reduction can be high with low play or backlash

Pros- Low backlash and hence lost energy, low noise, high impact resistance

Cons- Manufacturing is complex and requires high precision

<u>REFLEX Torque Amplifier</u>

Made by *Genesis Robotics*, lightweight and made of plastic. Meant to replace metal planetary gearboxes.

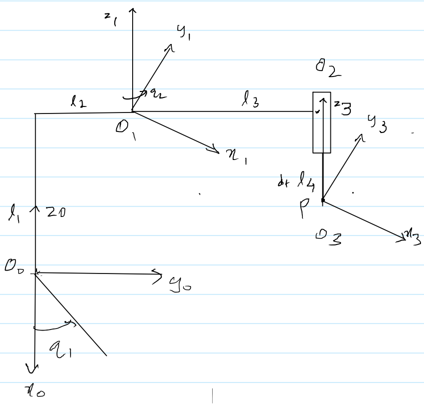
Pros- Because it's made of plastic, it's lighter and cheaper than conventional planetary gearboxes. Has no bearings, so gears can be hollow. Gears can be preloaded and with tapered teeth, zero backlash can be achieved.

Cons- Maintaining efficiency could be complex.

<u>Reference</u>

[1]
P. L. García, S. Crispel, E. Saerens, T. Verstraten, and D. Lefeber, "Compact Gearboxes for Modern Robotics: A Review," *Frontiers in Robotics and AI*, vol. 7, Aug. 2020, doi: https://doi.org/10.3389/frobt.2020.00103.

7.



$O_0 = [0\ 0\ 0]^T$ $O_1 = R_{z q_1} \cdot \begin{bmatrix} 0 \\ l_2 \\ l_1 \end{bmatrix}$ $z_0 = z_1 = z_2 = z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$O_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_2 \\ l_1 \end{bmatrix} = \begin{bmatrix} -l_2 sq_1 \\ l_2 cq_1 \\ l_1 \end{bmatrix}$$

Let $q_{12} = q_1 + q_2$

From Q2 we have

$$O_0^3 = \begin{bmatrix} -l_3 \cdot cq_1 sq_2 - l_3 sq_1 cq_2 - l_2 sq_1 \\ l_3 cq_1 cq_2 - l_3 sq_1 sq_2 + l_2 cq_1 \\ l_1 - (d + l_4) \end{bmatrix} = \begin{bmatrix} -l_3 sq_{12} - l_2 sq_1 \\ l_3 cq_{12} + l_2 cq_1 \\ l_1 - (d + l_4) \end{bmatrix}$$

$$O_0^2 = O_0^3 + \begin{bmatrix} 0 \\ 0 \\ (d + l_4) \end{bmatrix} = \begin{bmatrix} -l_3 sq_{12} - l_2 sq_1 \\ l_3 cq_{12} + l_2 cq_1 \\ l_1 \end{bmatrix}$$

$$\begin{matrix} a \\ b \\ c \end{matrix} \times \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} = \begin{matrix} -b \\ a \\ 0 \end{matrix} \qquad \begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & \\ a & b & \end{matrix}$$

$$J_1 = \begin{bmatrix} z_0 \times (O_0^3 - O_0^0) \\ z_0 \end{bmatrix} \quad J_2 = \begin{bmatrix} z_1 \times (O_0^3 - O_0^1) \\ z_1 \end{bmatrix} \quad J_3 \begin{bmatrix} z_2 \\ 0 \end{bmatrix}$$

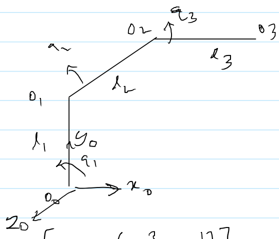
1

c

$$J = \begin{bmatrix} -l_3 cq_{12} - l_2 cq_1 & -l_3 cq_{12} & 0 \\ -l_3 sq_{12} - l_2 sq_1 & -l_3 sq_{12} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$ Manipulator Jacobian for RRP SCARA

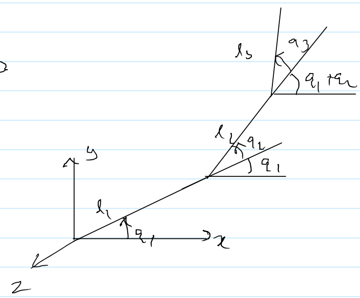
9. Schematic of the required RRR configuration :

All links are coplanar and are on the XY plane.



$q_{12} = q_1 + q_2$

$q_{123} = q_1 + q_2 + q_3$



$J_1 = \begin{bmatrix} z_0 \times (O_0^3 - O_0^0) \\ z_0 \end{bmatrix}$ $\quad J_2 = \begin{bmatrix} z_1 \times (O_0^3 - O_0^1) \\ z_1 \end{bmatrix}$ $\quad J_3 \begin{bmatrix} z_2 \times (O_0^3 - O_0^2) \\ z_2 \end{bmatrix}$

$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \underbrace{O_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} \quad O_2 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{bmatrix}}_{\text{From 2R manipulator}} \quad O_3 = O_2 + \begin{bmatrix} l_3 c_{123} \\ l_3 s_{123} \\ 0 \end{bmatrix} = \begin{bmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12} + l_3 s_{123}) & -(l_2 s_{12} + l_3 s_{123}) & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$ which is the required manipulator Jacobian