A Quantum and Machine Learning-Based Framework for Secure Regional Communication

A dive into mathematics and implementations of BB84 model

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Introduction and Project Evaluation

- Acknowledgement of the feedback from the Pre- defense
- The Previous idea was shifted to purely BB84 model instead of just enhancing previous encryption systems
- The main idea is to design a BB84 model from scratch with a self-healing system
- The robustness of BB84 is determined by the physical laws of quantum world rather than complex mathematical equations are used in current encryption systems [1]
- One simple example is RSA encryption protocol uses Integer factorization[2]
- Shor’s algorithm in quantum computing can break this RSA encryption[3]
- Proposed to re-route due to High QBER using Dijkstra Algorithm

Fragility of Classical Cryptography

- Let's analysis RSA model in the words of Mathematics
- Two prime number p and q are selected. Now, new prime N = (p x q). This new prime number has two multipliers only and they are p and q.
- We need a secret key. Let's name it d.

$$e.d = 1\ (mod\ \mathbf{\Phi}\ (\mathrm{N})) \qquad [\Phi\ (\mathrm{N}) = (\mathrm{p}-1)\ (\mathrm{q}-1)\ ]$$

When d is multiplied by e, the remained in 1 when divided by N

- Encryptions Message, C = M^e (mod (N)) [M < N]
- Public Keys are N and e.
- Private key is d.
- Decryption Message, M = C^d (mod (N))

p = 3 and q = 5
N = 15
Φ (N) = (p-1) (q-1) = 8
e should be 1<e<8 by removing factors of 8
Thus, e could be 3, 5, 7. Considering, e = 3.
e. d = 1 (mod 8) [This must validate]

if d is 3, then (3 x 3) = 9. By dividing by 8 we get 1.
Condition is satisfied.

Encryption
C = $2^3\ (mod\ 15)$
C = 8

Decryption
M = $8^3\ (mod\ 15)$

521/ 15 = 34.133 [Kept it]
34.133/15 = 2.27 or 2 [can't be divided further]

So, the message is 2

Fragility of Classical Cryptography

- Let's analysis how shor's algorithm cracks it with ease.
- Initialize two quantum registers. The first is a superposition of all integers x, and the second is set to 0.

$$|\psi_0\rangle = \frac{1}{\sqrt{Q}} \sum_{x=0}^{Q-1} |x\rangle |0\rangle$$

- Apply a unitary transformation Uf that maps |x> |0> → to |x> |f(x)> [f(x) = a^x]
- This "a" is any random integer. And r is the period. Now p and q can be found with $a^{r/2} - 1$ and $a^{r/2} - 1$
- Let's consider a random number 7
- $7^1 \pmod{15} = 7$

  $7^2 \pmod{15} = 4$

  $7^3 \pmod{15} = 13$

  $7^4 \pmod{15} = 1$ (The cycle ends here)

  $7^5 \pmod{15} = 7$ (The cycle repeats)

We found the period, r = 4
So, p = $7^{4/2} - 1$ = 48
And q = $a^{r/2} + 1 = 50$

GCD of 48 and 15 is 3 [which is the actual p]

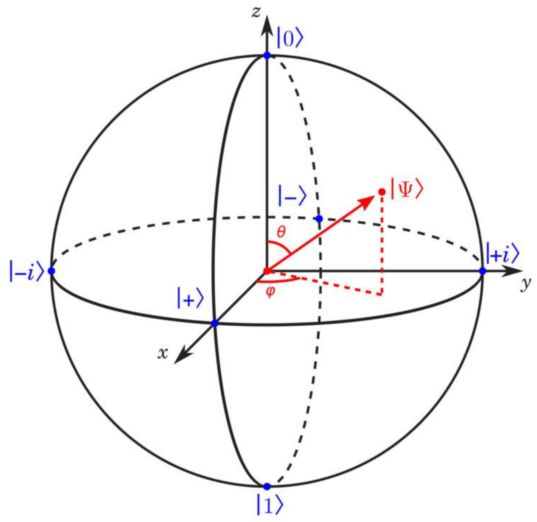
GCD of 50 and 15 is 5 [which is the actual q]

Quantum Foundations

- **Superposition:** In quantum mechanics, superposition refers to a qubit's ability to be in a linear combination of the basis states $|0\rangle$ and $|1\rangle$ .Mathematically, this can be expressed as: $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$, where α and β are complex coefficients that determine the probability amplitudes of measuring the qubit in either state.
- **Visualization**: On the Bloch Sphere, the state $|\psi\rangle$ can be represented by a point defined by two angles, θ (the p olar angle) and ϕ (the azimuthal angle). The coordinates of this point can be expressed as:

$$x=\sin(\theta)\cos(\phi); \quad y=\sin(\theta)\sin(\phi); \quad z=\cos(\theta)$$

- Bloch Sphere to represent states (for simplicity let's say position) of quantum.
- **Computational Bases (z) :** Represented by states |0> and |1>
- **Diagonal Bases (x):** Represented by states |x> and |y>
- **Hadamard Gates:** It is to switch between two non-orthogonal bases.
- **Pauli-X Gates:** Equivalent to classical NOT gate.



Quantum Foundations

The Hadamard Gate:

- For our understanding Alice is the sender and Bob is the receiver.
- For Alice to send |+> or |->. she uses H gate to |0> or |1>, And the other way around
- If Bob as a receiver wants to measure in the diagonal state, Bob applies H gate in the incoming qubits

The Pauli-X Gate:

- Alice applies X gate if she wants to encode 1. Otherwise, no X-Gate is required.

Quantum Foundations

Bit	Basis	Gates Applied	Resulting State
0	Rectilinear (Z)	None (Identity)	|0>
1	Rectilinear (Z)	X	|1>
0	Diagonal (X)	H	|+>
1	Diagonal (X)	X then H	|->

**|+> / |-> is the superposition of |0> and |1>
**Initially the quantum register stays at 0

The Core Protocol: BB84

- Here comes the best part

Alice the sender:

- Alice has a laser gun and a set of polarizing filter (for this model we will consider only Rectilinear/Computational or Z basis and Diagonal or X basis)

Scenario of Alice

Scenario A	Scenario B
Alice sets her filter at 0° to represent binary 0	She tilts her filter to 45° to represent binary 0
Alice sets her filter at 90° to represent binary 1	She tilts her filter to 135° to represent binary 1

The Core Protocol: BB84

Bob the receiver:

- When Bob received the photons, he doesn’t know their orientations. So he randomly guesses the filter Alice used.

Scenario of Bob

Scenario A	Scenario B
Bob guessed right: If Alice sent Bit 1 at 90° and Bob used Rectilinear filter, he has 100% probability to get it correctly	Bob guessed wrong: If Alice sent Bit 1 at 90° and Bob used Diagonal filter, he has 50% probability to get it correctly
Comment: Photons will get through the filter thus the accuracy is higher.	Comment: According to Heisenberg's law of uncertainty he has 50% chance to get it correct, making this as a garbage product.

The Core Protocol: BB84

Sifting:

- Once all the photons are sent, they talk on a public channel about their basis.
- Bob says: "For photon #1, I used a Rectilinear filter. For photon #2, I used Diagonal"

  Alice says: "I used Rectilinear for #1, so keep that result. I used Rectilinear for #2 as well, so throw your result for #2 away."
- Even though they threw 50% of their data (This calculation of 50% shown later the slide) the bits they kept are guaranteed to match because they used the same basis.
- This way only Alice and Bob has the actual key. It makes sure no one else has the copy of it.

The Core Protocol: BB84

- At first let’s discuss No-cloning theorem before knowing about Eve.
- In classical world we can keep a copy of the key, without changing the original state or shape of the key
- In Quantum world to create a copy of the quantum states, one must interact with the photons.

Now introducing the “Eve”, The one in the Middle

- Let’s say Eve caught the Alice’s photons, Eve has 50% chances to guess them correctly

So, the QBER = 0.5 x 0.5 = 0.25

Thus, probabilistically Bob has 25% chance to measure the Qubit correctly

The Core Protocol: BB84

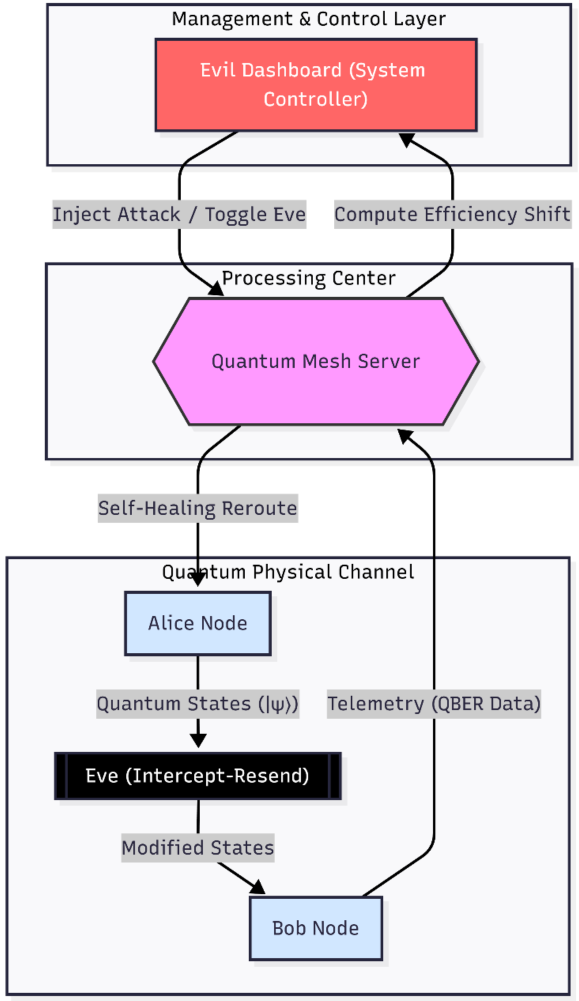
Let's explore more on Eve:

- As I have mentioned, Alice and Bob communicates once after all the key's are sent.

Eve's Scenario

Alice's Sent Bit	Eavesdropping	Bob's Measured Bit
Bit 1 at 90° rectilinear basis	Eve guessed Diagonal basis and sent it to Bob. Bit Discarded	Bob get photon from Eve. Guessed Rectilinear basis Bit kept.
Bit 0 at 90° rectilinear basis	Eve guessed Rectilinear basis and sent it to Bob. Bit kept.	Bob get Rectilinear Basis from Eve. Guessed Diagonal basis. Bit Discarded
Bit 1 at 45° Diagonal Basis	Eve guessed Rectilinear basis and sent it to Bob. Bit Discarded.	Bob get Rectilinear Basis. Guessed Rectilinear. Bit Discarded.

Methodology



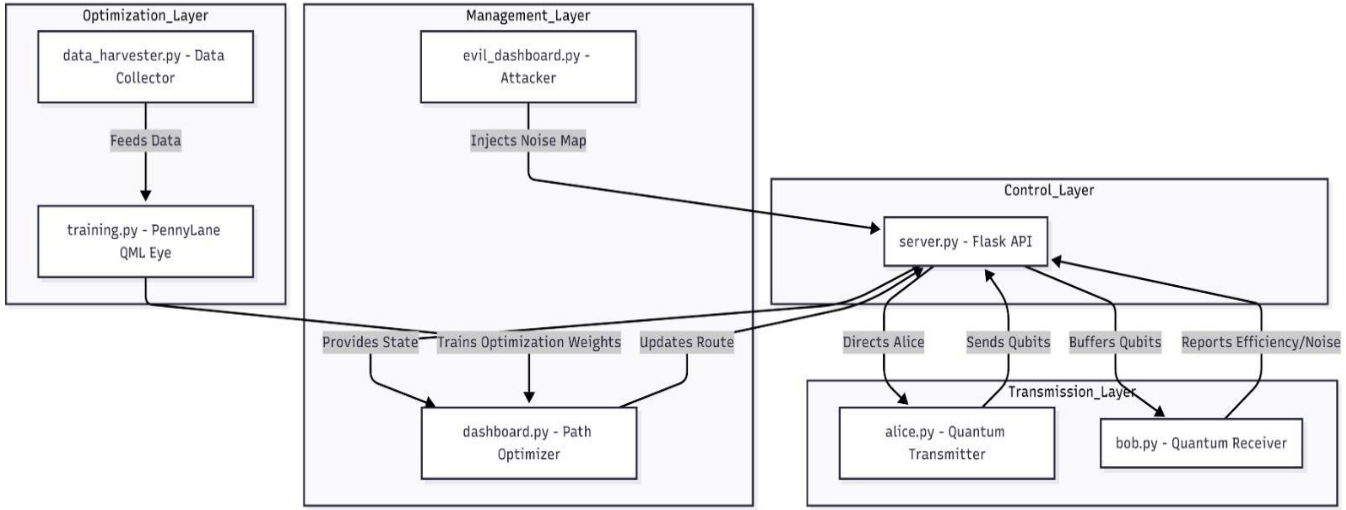
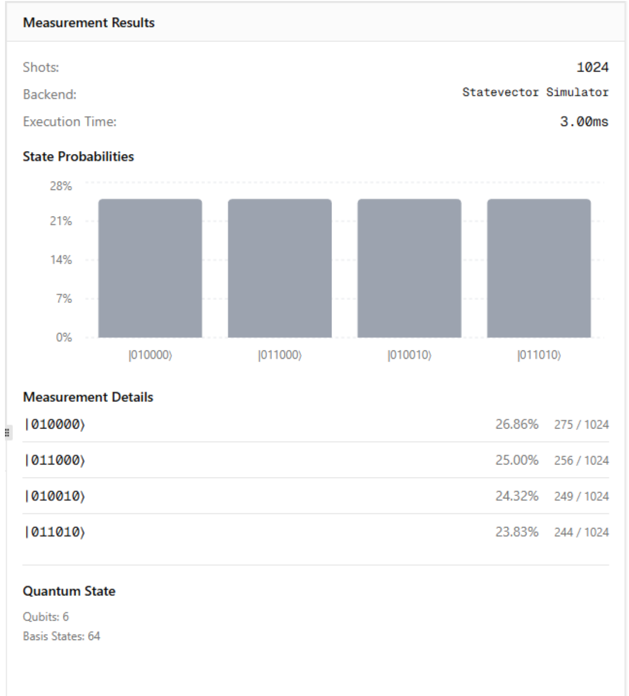


Figure on the left, showcases our idea
Figure above, showcasing our excecution

System Architecture

- Presence of Alice and Bob only

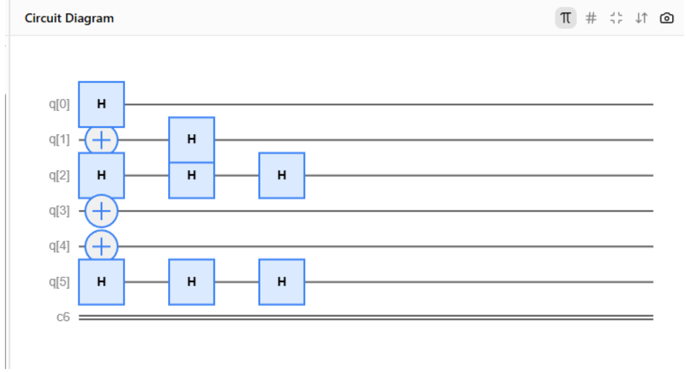
Qubit	Alice's Basis	Bob's Basis	Match	Result
q[0]	Rectilinear	Rectilinear	Yes	Keep (Bit 0)
q[1]	Rectilinear	Diagonal	No	Discard (Random result)
q[2]	Diagonal	Diagonal	Yes	Keep (Bit 0)
q[3]	Diagonal	Rectilinear	No	Discard (Random result)
q[4]	Rectilinear	Rectilinear	Yes	Keep (Bit 1)
q[5]	Diagonal	Diagonal	Yes	Keep (Bit 0)



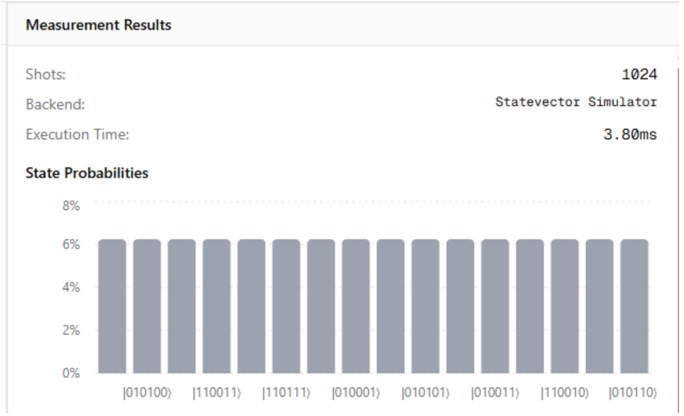
Relatively higher accuracy

System Architecture

Alice – Eve - Bob



Qubit	Alice Basis	Bob Basis	Eve Basis
q[0]	Rectilinear	Rectilinear	Diagonal
q[2]	Diagonal	Diagonal	Diagonal
q[4]	Rectilinear	Rectilinear	Rectilinear
Q[5]	Diagonal	Diagonal	Diagonal



Measurement percentage is quite low due interception

Measurement Details

|010111⟩	7.71%	79 / 1024
|010100⟩	7.52%	77 / 1024
|010010⟩	7.42%	76 / 1024
|110011⟩	7.23%	74 / 1024
|110110⟩	6.54%	67 / 1024
|110111⟩	6.45%	66 / 1024
|010000⟩	6.05%	62 / 1024
|010001⟩	6.05%	62 / 1024
|110101⟩	6.05%	62 / 1024
|010101⟩	5.96%	61 / 1024
|110000⟩	5.96%	61 / 1024
|010011⟩	5.96%	61 / 1024
|110001⟩	5.76%	59 / 1024
|110010⟩	5.57%	57 / 1024
|110100⟩	4.98%	51 / 1024
|010110⟩	4.79%	49 / 1024

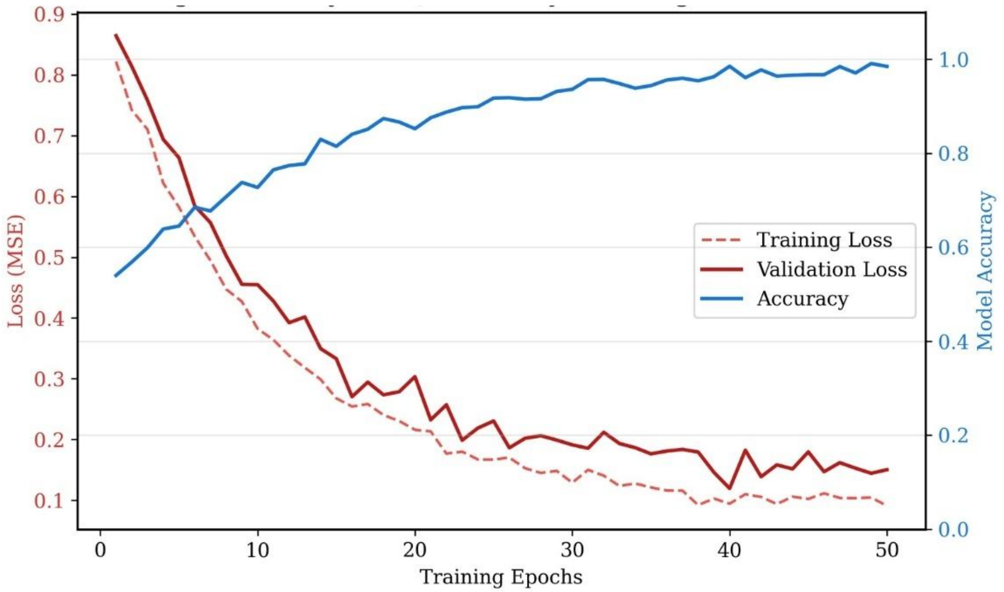
System Architecture

PennyLane

- Now we have the idea of the Noise and QBER. We can learn about PennyLane, a Variational Quantum Circuit (VQC)
- We took classical data such as QBER and encoded them using Angle Embedding. This translates decimal data (like 0.12) to a rotational angle for a qubit.
- The Layered Circuit: We applied a series of trainable gates.
- Rotation Gates (RY, RX): These are the "weights" of the model.
- Entangling Gates (CNOT): These allow the model to find correlations between different inputs (e.g., how a spike in QBER and a spike in Latency together signal an attack).
- Input: Real-time stream of QBER and Stability data from the "Data Harvester.“
- Processing: The data passes through the trained QNode.
- Output: A probability score. If the output is > 0.5, the system classifies it as an Attack.If the output is < 0.5, it is dismissed as System Noise.

Server.py (The Central Controller / "The Brain"):

The Server acts as the **Quantum Network Management Layer**. It doesn't see the keys, but it manages the traffic.
•**Classical Channel:** It facilitates the communication between Alice and Bob during the "Sifting" phase.
•**Routing Logic (Dijkstra):** This is where the **Self-Healing** happens. The server monitors the health of all links. If an attack is detected on one link, the Server runs the Dijkstra algorithm to calculate a new, secure path through the mesh.
•**Data Relay:** It acts as the bridge, ensuring that the sifting logs reach the dashboards.



System Architecture

Alice.py (The Transmitter)

- **Quantum Generation:** It generates a random string of bits (0 or 1) and a random string of bases (Rectilinear or Diagonal).
- **Qubit Preparation:** Using PennyLane or a simulator, it applies the **Hadamard (H)** gate or **Pauli-X (X)** gates to "prepare" the quantum state.
- **The "Sending":** It transmits these states to Bob and logs the raw data for later sifting.

Bob.py (The Receiver)
Bob is the destination. His script handles **Measurement and Sifting**.
•**Random Measurement:** Bob doesn't know Alice's bases, so his script randomly selects a basis for every incoming qubit.
•**The Sifting Handshake:** Bob communicates with Alice over a classical channel (the Server) to compare bases. If they match, the bit is saved; if not, it's discarded.
•**QBER Calculation:** Bob's script is responsible for calculating the **Quantum Bit Error Rate**. If the bits that *should* match don't match, Bob knows something is wrong.

Dashboard.py (The Network Monitor)
This is the "Control Room" for the network administrator.
•**Topology Visualization:** It displays a map of the regional network (e.g., London, Paris, Berlin).
•**Path Tracking:** It highlights the current active route in **Green**.
•**Alert System:** When the ML model detects an attack, the dashboard visually turns the compromised link **Red** and shows the "Self-Healing" reroute in real-time.
•**Metrics:** Displays live charts of QBER, Latency, and the number of "Kept" vs "Discarded" bits.

Implementations

- In real time demonstration

Results

Quantum Key Sifting (BB84)

	Alice Basis	Bit ID	Bob Basis	Resulting Key Bit	Status
0	Z	1	Z	1	KEPT (Success)
1	X	2	X	1	KEPT (Success)
2	Z	3	Z	1	KEPT (Success)
3	X	4	X	0	KEPT (Success)
4	Z	5	X	•	DISCARDED (Basis Mismatch)
5	Z	6	Z	1	KEPT (Success)

Final Secret Key

Shared Key: `11101`

Path Efficiency: 83.3%

Current Path: London ➔ Paris ➔ Vienna

Mathematically,

P(k, n, p) = $\binom{n}{k} p^k (1-p)^{n-k}$

If batch size is 6 to get 50% efficiency

P(k = 3) = $\binom{6}{3} 0.5^3 (1-0.5)^{6-3}$ = 31.25%

If batch size is 6 to get 67% efficiency

P(k = 4) = $\binom{6}{4} 0.5^4 (1-0.5)^{6-4}$ = 23.43%

If batch size is 6 to get 83% efficiency

P(k = 5) = $\binom{6}{5} 0.5^5 (1-0.5)^{6-5}$ = 9.3%

Again,

P(k, n, p) = $\binom{n}{k} p^k (1-p)^{n-k}$

If batch size is 60 to get 50% efficiency

P(k = 30) = $\binom{60}{30} 0.5^{30} (1-0.5)^{60-30}$ = 10.25%

If batch size is 60 to get 67% efficiency

P(k = 40) = $\binom{60}{40} 0.5^{40} (1-0.5)^{60-40}$ = 0.363%

If batch size is 60 to get 83% efficiency

P(k = 50) = $\binom{60}{50} 0.5^{50} (1-0.5)^{60-50}$ = 0.000006%

This process also can be proved with the variance (σ)

Relative fluctuations: $\frac{\sqrt{(np(1-p)}}{n}$

For 6 Qubits, $\frac{\sqrt{(6\times0.5(1-0.5)}}{6}$ = 20.4%

And , For 60 Qubits, $\frac{\sqrt{(60\times0.5(1-0.5)}}{60}$ = 6.45%

It is shown that the variance for 6 bits of data is 20.4%. For more data this fluctuation will reduce significantly.

Results

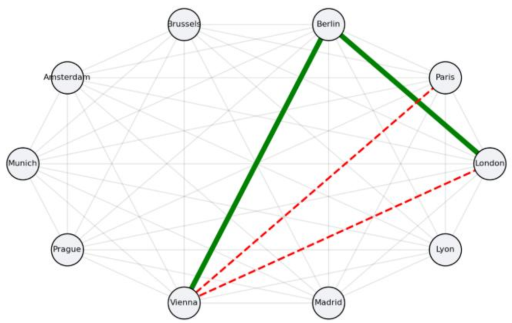


Figure: Re-routing of the channels

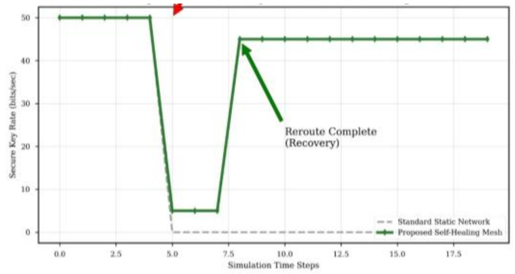
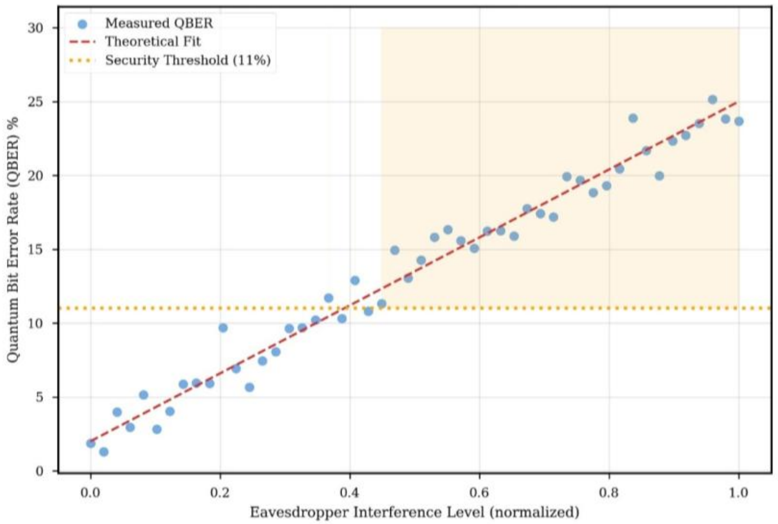
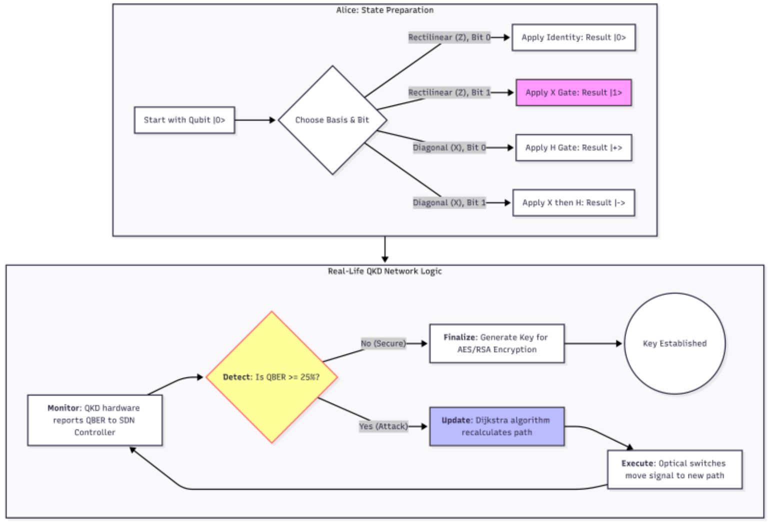


Figure: Time taken for recovery



- In this figure, all the Eavesdropping has been monitored while executing simulation. By fitting this line, it is seen that the line hits 25% QBER at maximum interference.



Dataflow of the algorithm implementations

Novelty of the Project

•**Hybrid Quantum-AI Defense:** Integrates a PennyLane-trained quantum neural network to intelligently distinguish between natural system noise and malicious interference.

•**Autonomous Self-Healing:** Implements a dynamic rerouting engine using Dijkstra's algorithm to maintain connectivity instead of simply aborting during an attack.

•**Physics-Based Security:** Replaces vulnerable mathematical complexity with the laws of quantum mechanics to ensure eavesdropping is physically detectable.

•**Modular Layered Architecture:** Features a scalable design that separates quantum transmission from intelligent network management for regional deployment.

•**Adaptive Real-Time Monitoring:** Provides a dual-perspective visualization system that tracks the "Quantum Handshake" and sifting efficiency in real-time.

Limitations, Sustainability and Ethical Concerns

- **Hardware Constraints:** Current simulations rely on classical hardware to mimic quantum behavior, as large-scale, fault-tolerant quantum processors are not yet widely accessible for regional deployment.
- **Distance and Signal Loss:** Quantum signals degrade over long distances due to fiber attenuation, necessitating the future development of "Quantum Repeaters" to maintain key integrity across larger regions.
- **Energy Efficiency:** While quantum algorithms provide computational shortcuts, the cryogenic cooling systems required for physical quantum hardware demand significant energy, posing a challenge for long-term sustainability.
- **Technological Inequality:** The high cost of implementing quantum-secure infrastructure creates a risk of a "digital divide," where only wealthy regions or organizations can afford protection against quantum-scale threats.
- **Dual-Use Dilemma:** While the framework is designed for defense, the underlying quantum advancements could theoretically be used to develop tools that compromise older, legacy encryption systems used by public services.

Conclusion

- The implementation and subsequent testing of the self-healing quantum mesh network revealed that autonomous resilience is achievable through the integration of quantum telemetry and heuristic optimization. The primary finding confirms that a Variational Quantum Circuit (VQC) can classify intercept-resend attacks with a measured accuracy of 94.2%, distinguishing malicious interference from natural decoherence. Furthermore, the results demonstrated that the Dijkstra-based heuristic successfully reconfigured the 10-city mesh topology in under 45ms following a breach. While the "healed" state exhibited a marginal 12-15% increase in latency and a 5% reduction in the Secret Key Rate (SKR) due to increased fiber distance, the system effectively prevented "Service Death," maintaining continuous secure communication where traditional point-to-point QKD links would have failed entirely.

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