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B11

Date 24, 11, 25
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Assignment

1)

Student	X (maths)	Y (stats)
1	10	15
2	15	12
3	20	18
4	25	24
5	30	30
6	35	36

$$\bar{X} = \frac{135}{6} = 22.5$$

$$\bar{Y} = \frac{135}{6} = 22.5$$

X	Y	(X-22.5)	(Y-22.5)	$(X-\bar{X})(Y-\bar{Y})$
10	15	-12.5	-7.5	93.75
15	12	-7.5	-10.5	78.75
20	18	-2.5	-4.5	11.25
25	24	2.5	1.5	3.75
30	30	7.5	7.5	56.25
35	36	12.5	13.5	168.75

Now find totals:

$$\Sigma(X-\bar{X})(Y-\bar{Y}) = 412.$$

$$\Sigma(X-\bar{X})^2 = 437.5$$

$$\Sigma(Y-\bar{Y})^2 = 427.5$$

$$r = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\sqrt{\Sigma(X-\bar{X})^2 \cdot \Sigma(Y-\bar{Y})^2}}$$

$$r = \frac{412.5}{\sqrt{437.5 \times 427.5}} = 0.956 \text{ Ans}$$

2) $X \sim N(\mu = 1200, \sigma = 100)$

a) $z = \frac{X-\mu}{\sigma} = \frac{1000-1200}{100} = \frac{-200}{100} = -2$

$P(X < 1000) = P(z < -2)$

$P(z < -2) = 0.0228$

$P(X < 1000) = 0.0228$ Ans

b) For 1100:

$z_1 = \frac{1100-1200}{100} = -1$

For 1300:

$z_2 = \frac{1300-1200}{100} = 1$

$P(1100 < X < 1300) = P(-1 < z < 1)$

$P(z < 1) = 0.8413$

$P(z < -1) = 0.1587$

So:

$P(-1 < z < 1) = 0.8413 - 0.1587 = 0.6826$

$P(1100 < X < 1300) = 0.6826$ Ans

3 a) Show W is a subspace of $M_{3\times3}(R)$

Recall $W = A \in M_{3\times3}(R) : A^T = -A$.

i) Nonempty: The zero matrix O satisfies $O^T = O = -O$. so $O \in W$. Hence $W \neq \phi$.

ii) Closed under addition: Take $A, B \in W$
Then
$$(A+B)^T = A^T + B^T = (-A) + (-B) = -(A+B).$$

iii) closed under scalar multiplication: For any Scalar $\lambda \in R$ and $A \in W$,

$$(\lambda A)^T = \lambda A^T = \lambda(-A) = -(\lambda A)$$

Since all three properties hold, W is a subspace of $M_{3\times3}(R)$

b) $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$, $a, b, c \in R$.

$E_1 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $E_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$, $E_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

$A = aE_1 + bE_2 + cE_3$

Linear independence: if
$\alpha E_1 + \beta E_2 + \gamma E_3 = 0$
Compare entries: the (1,2) entry gives
$\alpha = 0$;
(1,3) entry gives $\beta = 0$.
(2,3) entry gives $\gamma = 0$. so

$\alpha=\beta=\gamma=0$. Hence E_1, E_2, E_3 is linearly independent
Therefore E_1, E_2, E_3 is a basis of W. so

basis $=\{E_1, E_2, E_3\}$, $\dim W = 3$. Ans

4)

Person	X (Height cm)	Y (Weight kg)
1	150	50
2	160	55
3	165	58
4	170	60
5	175	63

$\overline{X} = \frac{820}{5} = 164$

$\overline{Y} = \frac{286}{5} = 57.2$

$S_{xy} = \sum (X-\overline{X})(Y-\overline{Y}) = 164.05$

$S_{xx} = \sum (X-\overline{X})^2 = 317.5$

Slope : $b = \frac{S_{xy}}{S_{xx}} = \frac{164.05}{317.5} \approx 0.5162$

Intercept :

$a = \overline{Y} - b\overline{X} = 57.2 - 0.5162(164) \approx -27.459$

a) $Y = -27.459 + 0.5162X$ Ans

b) $Y = -27.459 + 0.5162(168)$

$Y \approx 59.26$ kg Ans