Numerical Analysis

Mathematics of Scientific Computing

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To compute the dominant eigenvalue of A,
A has the following two properties:

- There is a single eigenvalue of maximum modulus.
- There is a linearly independent set of n eigenvectors.

Two assumpting:

- The eigenvalues λ_1, λ_2,..., λ_n can be labed $|\lambda_1| > |\lambda_2| \geq ... \geq |\lambda_n|$.
- There is a basic $u^{(1)}, u^{(2)}, ..., u^{(n)}$, for that $Au^{(j)} = \lambda_j u^{(j)}$

$x^{(0)} = a_1 u^{(1)} + a_2 u^{(2)} + ... + a_n u^{(n)} \qquad (a_1 \neq 0)$
$x^{(1)} = Ax^{(0)} \qquad x^{(2)} = Ax^{(1)} \ ... \qquad x^{(k)} = Ax^{(k-1)}$
$x^{(k)} = A^k x^{(0)}$
$x^{(0)} = u^{(1)} + u^{(2)} + ... + u^{(n)}$
$x^{(k)} = A^k u^{(1)} + A^k u^{(2)} + ... + A^k u^{(n)}$
$x^{(k)} = \lambda_1^k u^{(1)} + \lambda_2^k u^{(2)} + ... + \lambda_k^k u^{(n)}$
$x^{(k)} = \lambda_1^k [u^{(1)} + (\lambda_2/\lambda_1)^k u^{(2)} + ... + (\lambda_n/\lambda_1)^k u^{(n)}]$
Since $|\lambda_1| > |\lambda_j|$, so $k \to \infty$, $(\lambda_j/\lambda_1)^k \to 0$
$x^{(k)} = \lambda_1^k [u^{(1)} + \varepsilon^{(k)}]$
$\varphi(x^{(k)}) = \lambda_1^k [\varphi(u^{(1)}) + \varphi(\varepsilon^{(k)})]$
When$k \to \infty$, $r_k = \frac{\varphi(x^{(k+1)})}{\varphi(x^{(k)})} = \lambda_1 \frac{[\varphi(u^{(1)}) + \varphi(\varepsilon^{(k+1)})]}{[\varphi(u^{(1)}) + \varphi(\varepsilon^{(k)})]} \to \lambda_1$

Algorithm of Power Method

input A, x, M

for $k = 1$ to M do

$y \leftarrow Ax$

$r \leftarrow \varphi(y)/\varphi(x)$

$x \leftarrow y$

output k, x, r

end do

```
function [x,r]=powerMethod(A,x,M)
for k=1:M
    y=A*x
    r=linearFun(y)/linearFun(x)
    x=y/max(abs(y))
end
```

```
function f=linearFun(x)
f=x(2);
```

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The inverse power method comput the smallest engenvalue of A.
Eigenvalues of A can be arranged as follows:
$|\lambda_1| > |\lambda_2| \geq ... \geq |\lambda_n| > 0$.
Eigenvalues of A^{-1} can be arranged as follows:
$|\lambda_n^{-1}| > |\lambda_{n-1}^{-1}| \geq ... \geq |\lambda_1^{-1}| > 0$.
$x^{(k+1)} = A^{-1}x^{(k)}$

```
function [x,r]=inversepower(A,x,M)
for k=1:M
    y=A\x
    r=linearFun(y)/linearFun(x)
    x=y/max(abs(y))
end
```

```
function f=linearFun(x)
f=x(2);
```

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Summary

Method	Equation	Computes
power	$x^{(k+1)} = Ax^{(k)}$	lagest eigenvalue
inverse power	$Ax^{(k+1)} = x^{(k)}$	smallest eigenvalue
shifted power	$x^{(k+1)} = (A - uI)x^{(k)}$	eigenvalue farthest from u
shifted inverse power	$(A - uI)x^{(k+1)} = x^{(k)}$	eigenvalue closest to u

The code of shifted power

```
function [x,r]=shiftpower(A,x,M,u, I)
for k=1:M
    y=(A-u*I)*x
    r=linearFun(y)/linearFun(x)
    x=y/max(abs(y))
end
```

```
function f=linearFun(x)
f=x(2);
```

The code of shifted inverse power

```
function [x,r]=invshiftpower(A,x,M,u, I)
for k=1:M
    y=(A-u*I)\x
    r=linearFun(y)/linearFun(x)
    x=y/max(abs(y))
end
```

```
function f=linearFun(x)
f=x(2);
```

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QR-Factorization

$A = QR$

A is an $m \times n$ matrix, Q is an $m \times m$ unitary matrix and R is an $m \times n$ upper triangular martrix.

$A_1 = A$
$A_1 = Q_1 R_1$
$A_2 = R_1 Q_1,\ A_2 \sim A_1 = A$
$A_2 = Q_2 R_2$
$A_3 = R_2 Q_2,\ A_3 \sim A_2 \sim A_1 = A$
we have iterative process as follows:
$A_k = Q_k R_k\ (A_1 = A)$
$A_{k+1} = R_k Q_k,\ (k = 1, 2 \cdots\cdots)$
$A_{k+1} \sim A$
A_k converges to the upper triangular matrix whose diagonal elements are $\lambda_1, \lambda_2, \cdots\cdots \lambda_n$.