Numerical Analysis

Mathematics of Scientific Computing

主讲人 朱亚菲
幻灯片制作 孙晓庆

中国海洋大学 信息科学与工程学院

2013 年 9 月 6 日

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Convergent Sequences

- In case of real numbers, a computer program may produce a sequence of real numbers x_1, x_2, x_3,... that are approaching the correct answer.
- if there corresponds to each positive ε a real numbers r such that $|x_n - l| < \varepsilon$ whenever $n > r$
- For example

$$\lim_{n\to\infty} \frac{n+1}{n} = 1$$

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Orders of Convergence

- Let $[x_n]$ be a sequence of real numbers tending to a limit x^*. We say that the rate of convergence is at least **linear** if there are a constant $c < 1$ and an integer N such that

$$|x_{n+1} - x^*| \le c|x_n - x^*|(n \ge N)$$

- We say that the rate of convergence is at least **superlinear** if there exists a sequence ε_n tending to 0 and an integer N such that

$$|x_{n+1} - x^*| \le \varepsilon_n|x_n - x^*|(n \ge N)$$

Orders of Convergence

- The convergence is at least **quadratic** if there are a constant C (not necessarily less than 1) and an integer N such that

$$|x_{n+1} - x^*| \leq C|x_n - x^*|^2 (n \geq N)$$

- In general, if there are positive constants C and α and an integer N such that

$$|x_{n+1} - x^*| \leq C|x_n - x^*|^\alpha (n \geq N)$$

  We say that the rate of convergence is of **order** α at least.

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Big O and Little o Notation

- Let $[x_n]$ and $[\alpha_n]$ be two different sequences, We write $x_n = O(\alpha_n)$ if there are constants C and n_0 such that $|x_n| \leq C|\alpha_n|$ when $n \geq n_0$. x_n is "big oh"of $[\alpha_n]$
- The equation $x_n = o(\alpha_n)$. For some ε_n have $\varepsilon_n \to 0$ and $|x_n| \leq \varepsilon_n|\alpha_n|$.

$$(\lim_{n\to\infty} \frac{x_n}{\alpha_n} = 0)$$

Big O and Little o Notation

- In general, we write $f(x) = O(g(x))$ $(x \to x^*)$ when there is a constant C and a neighborhood of x^* such that $|f(x)| \leq C|g(x)|$ in that neighborhood.
- Similarly, $f(x) = o(g(x))$ $(x \to x^*)$ means that

$$(\lim_{x \to x^*} \frac{f(x)}{g(x)} = 0)$$

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Theorem

- Let u and v be continuous real-valued functions on an interval [a,b], and suppose that $v \geq 0$. Then there exists a point ξ in $[a, b]$ such that

$$\int_a^b u(x)\,v(x)\,dx = u(\xi)\int_a^b v(x)\,dx$$

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Nested Multiplication

- A polynomials can be rewritten in a nested form that it requires only a few more than the minimum number of multiplications when evaluating it.
- The polynomial $p(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$
- can be written using standard mathematical notation involving the sum $\sum$ and product $\prod$ as follows:

$$p(x) = \sum_{k=0}^{n} a_k x^k$$

- To evaluate the polynomial efficiently, we can group the terms using nested multiplication:
  $p(x) = a_0 + x(a_1 + x(a_2 + ... + x(a_{n-1} + x a_n))...))$

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Least Upper Bound Axiom

Definition of supremum

The supremum of S is v $(v = supS = lubS)$ if and only if

1. v is an upper bound for S and
2. no real number smaller than v is an upper bound for S

Definition of infilmum

The infilmum of S is u $(u = supS = lubS)$ if and only if

1. u is a lower bound for S and
2. no real number greater than u is a lower bound for S

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Explicit Functions

- A explicit function is usually defind via an explicit formula, from which a value of function can be computed for each argument.
- for example $f(x) = \sqrt{7x^3 - 2x}$
- A function $y = f(x)$ is well defined by the following differential equation with an initial condition

$$\begin{aligned} y' &= 1 + \sin y \\ y(0) &= 0 \end{aligned}$$

Implicit Functions

- Let G be a function of two real variables defined and continuously differentiable in a neighborhood of (x_0, y_0),
  if $G(x_0, y_0) = 0$ and$\partial G/\partial y \neq 0$ at (x_0, y_0), then there is a positive δ and a continuously differentiable function f defined for $|x - x_0| < \delta$ such that $f(x_0) = y_0$ and $G(x, f(x)) = 0$ for $|x - x_0| < \delta$