Iterative Elimination of Dominated Strategies

Lecture 3.2

Example 1:

	A	B	C
→ A	2, 3	3, 0	0, 1
→ B	0, 0	1, 6	4, 2

$u_2(A, \sigma) = 1.5 > 1 = u_2(A, C)$

$u_2(B, \sigma) = 3 > 2 = u_2(B, C)$

Claim: $\sigma = (A : \frac{1}{2}, B : \frac{1}{2})$ strongly dominates the pure strategy C for the column player. ✓

Reduced game:

	A	B
A	2 , 3	3 , 0
~~B~~	~~0 , 0~~	~~1 , 6~~

$u_1(A,A) = 2 > 0 = u_1(B,A)$

$u_1(A,B) = 3 > 1 = u_1(B,B)$

Claim: The strategy B is strongly dominated by the strategy A for the row player.

Reduced game:

	A	~~B~~
A	2 , 3	~~3 , 0~~

Claim: The strategy B is strongly dominated by the strategy A for the column player.

<u>Reduced game:</u>

	A
A	2,3

(A,A) is the unique MSNE for the given game. ∎

<u>Lemma:</u> Given a game $\Gamma = \langle N, (S_i)_{i\in N}, (u_i)_{i\in N} \rangle$, if a pure strategy $s_i \in S_i$ is weakly dominated by some mixed strategy, then there exists an MSNE $(\sigma_i^*)_{i\in N}$ such that $\sigma_i^*(s_i) = 0$.

<u>Proof:</u> Analogous to the lemma for strongly dominated strategy. ∎

Example:

	A	B
A	2, 3	3, 3

A is a weakly dominated strategy for the column player.

$\sigma \in \Delta(\{A, B\})$, (A, σ) is an MSNE.

Example: Suppose there are 50 students in a class. Each student writes a number in $\{0, 1, 2, \ldots, 100\}$. Let $s_1, \ldots, s_{50}$ be the numbers written.

$$l = \frac{2}{3} \cdot \frac{s_1 + \cdots + s_{50}}{50}$$

The winner is the student whose number is closest to l.

Find an MSNE of the game.

Iterative elimination of strongly dominated strategies.

The strategies $68, 69, \ldots, 100$ are weakly dominated by the strategy 67.

<u>Reduced game:</u> $S_i = \{0, 1, \ldots, 67\}$

$\ell \leq \frac{2}{3} \cdot 67 \approx 44.\ldots$

<u>Reduced game:</u> $S_i = \{0, 1, \ldots, 45\}$

$\vdots$

$S_i = \{0\}$

$(0, 0, \ldots, 0)$ is an MSNE.