Algorithmic Game Theory
Assignment 10

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1. Suppose we have at least 2 players and the setting is quasi-linear. Let $\mathcal{X}$ be the set of pareto efficient social choice functions and $\mathcal{Y}$ the set of allocatively efficient social choice functions. Then, which one of the following is true?

   (a) $\mathcal{X} \subsetneq \mathcal{Y}$
   (b) $\mathcal{Y} \subsetneq \mathcal{X}$
   (c) $\mathcal{X} = \mathcal{Y}$
   (d) $\mathcal{X} \cap \mathcal{Y} = \emptyset$

   The correct answer is (a). Refer to Lecture 10.1.

2. Suppose we have at least 2 players and the setting is quasi-linear. Let $\mathcal{X}$ be the set of pareto efficient social choice functions and $\mathcal{Y}$ the set of weakly budget balanced social choice functions. Then, which one of the following is true?

   (a) $\mathcal{X} \subsetneq \mathcal{Y}$
   (b) $\mathcal{Y} \subsetneq \mathcal{X}$
   (c) $\mathcal{X} = \mathcal{Y}$
   (d) $\mathcal{X} \cap \mathcal{Y} = \emptyset$

   The correct answer is (a). Refer to Lecture 10.1.

3. Suppose we have at least 2 players and the setting is quasi-linear. Let $\mathcal{X}$ be the set of pareto efficient social choice functions and $\mathcal{Y}$ the set of social choice functions which are allocatively efficient as well as strictly budget balanced. Then, which one of the following is true?

   (a) $\mathcal{X} \subsetneq \mathcal{Y}$
   (b) $\mathcal{Y} \subsetneq \mathcal{X}$
   (c) $\mathcal{X} = \mathcal{Y}$
   (d) $\mathcal{X} \cap \mathcal{Y} = \emptyset$

   The correct answer is (c). Refer to Lecture 10.1.

4. Suppose we have at least 2 players and the setting is quasi-linear. Let $\mathcal{X}$ and $\mathcal{Y}$ be the set of respectively Groves and Clarke mechanisms. Then, which one of the following is true?

   (a) $\mathcal{X} \subsetneq \mathcal{Y}$
   (b) $\mathcal{Y} \subsetneq \mathcal{X}$
   (c) $\mathcal{X} = \mathcal{Y}$
   (d) $\mathcal{X} \cap \mathcal{Y} = \emptyset$

   The correct answer is (b). Refer to Lecture 10.2.

5. Which one of the following does not always hold for every Clarke mechanisms?

   (a) It is allocatively efficient.
   (b) It is strictly budget balanced.

(c) It is dominant strategy incentive compatible.

(d) It is Bayesian incentive compatible.

The correct answer is (b). Refer to Lecture 10.2.

6. Consider a scenario with a set 5 sellers selling one copy of an identical item with valuations respectively $v_1 = 23, v_2 = 15, v_3 = 11, v_4 = 8, v_5 = 2$ and one buyer. The buyer wishes to buy 3 items and each seller can supply at most one items. Compute the amount of money received by the second seller under the VCG payment rule.

Answer range: 6 to 10.

7. Consider a scenario with a set 5 sellers selling one copy of an identical item with valuations respectively $v_1 = 23, v_2 = 15, v_3 = 11, v_4 = 8, v_5 = 2$ and one buyer. The buyer wishes to buy 3 items and each seller can supply at most one items. Compute the amount of money received by the third seller under the VCG payment rule.

Answer range: 6 to 10.

8. Consider a scenario with a set 5 sellers selling one copy of an identical item with valuations respectively $v_1 = 23, v_2 = 15, v_3 = 11, v_4 = 8, v_5 = 2$ and one buyer. The buyer wishes to buy 3 items and each seller can supply at most one items. Compute the amount of money received by the fourth seller under the VCG payment rule.
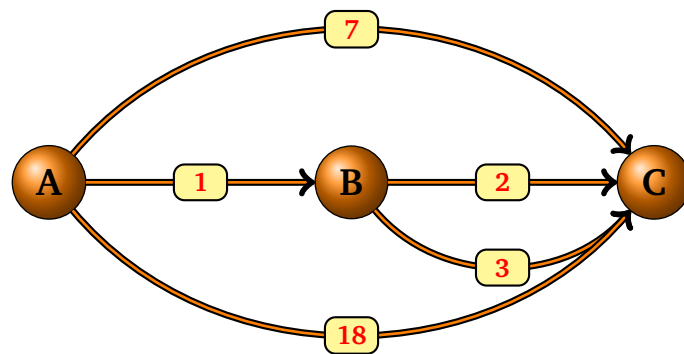
Answer range: -2 to 2.

Figure 1: Traffic network

9. Consider the traffic network in Figure 1 where the number on each edge is the cost to traverse that edge. Also each edge is owned by a player and the cost of that edge is private information known only to that player. Suppose a buyer want to buy an A to C path. The goal of the buyer is to minimize the cost of the path. If VCG mechanism is used, compute the payment received by the owner of the edge (b, c).

Answer range: 2 to 4

10. Consider the traffic network in Figure 1 where the number on each edge is the cost to traverse that edge. Also each edge is owned by a player and the cost of that edge is private information known only to that player. Suppose a buyer want to buy an A to C path. The goal of the buyer is to minimize the cost of the path. If VCG mechanism is used, compute the payment received by the owner of the edge (a, b).

Answer range: 5 to 9