Lecture 5.3

Finding an MSNE

Qn: What is the computational complexity of finding an MSNE in a normal form game?

(i) do there exist at least two MSNEs?

(ii) does there exist an MSNE where player 1 receives at least some minimum utility?

(iii) does there exist an MSNE where both the players receive certain minimum utility?

(iv) does there exist any MSNE which involves some strategies of player 1 ?

(v) does there exist any MSNE which does not involve some strategies of player 1 ?

⋮

All the above problems are NP-complete.

Functional NP (FNP): This class consists of all problems in NP with the extra requirement that for Yes instances, the algorithm also needs to output a certificate.

Can we show MSNE problem for bimatrix games is FNP-complete? NO!

Theorem: If MSNE for bimatrix games is FNP-complete, then $NP \overset{\supseteq}{\underset{\subseteq}{=}} co\text{-}NP$.

Proof: Enough to prove $NP \subseteq \underline{co\text{-}NP}$.

Assumption: MSNE for bimatrix games is FNP-complete.

$\Rightarrow$ There exist a reduction from functional SAT to MSNE.

$\Rightarrow$ i) An algorithm A to construct an instance of MSNE A(f) from any Boolean formula f.

ii) An algorithm B to map every MSNE s^* of A(f) to either

a) yes and a satisfying assignment (certificate) for f

or b) no.

claim: SAT $\in$ co-NP.

Proof: To prove, there exist an efficiently verifiable certificate for the NO instances of SAT also. Let f be a NO instance.

We claim $(\underline{A(f)}, \underline{s}^*)$ a certificate, and the algorithm B is the poly-time verifier. ∎