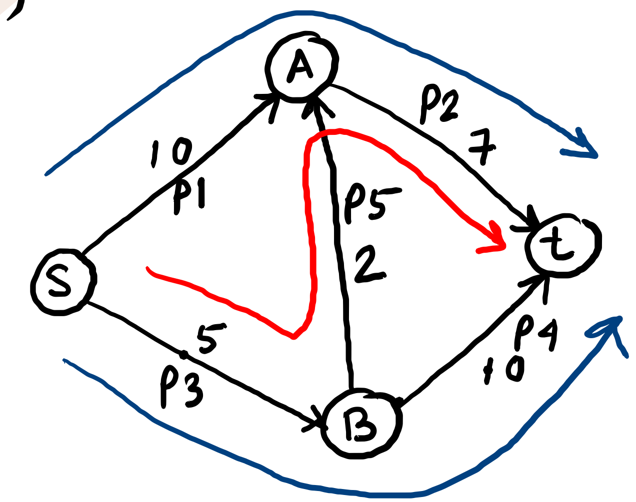
Examples of VCG Mechanism

Lecture 10.4

Example 3: (Strategic Network Formation)

Each link is owned by a strategic player. The delay/cost of each link is the private type of the player.



Set of allocation : {(1,1,0,0,0), (0,0,1,1,0), (0,1,1,0,1)}

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$-17 \qquad\qquad -15 \qquad\qquad -14$$

The allocatively efficient allocation : $S \to B \to A \to t$ i.e. $(0,1,1,0,1)$.

Payment received by agent 1 $= (-14) - (-14) = 0$

〃 2 $= (-5-2) - (-5-10) = 8$

〃 3 $= (-2-7) - (-10-7) = 8$

〃 4 $= (-14) - (-14) = 0$

〃 5 $= (-5-7) - (-5-10) = 3$.

Vickrey discount to player 1 is $0 - 0 = 0$

2 is $(-7) + 8 = 1$

3 is $(-5) + 8 = 3$

4 is $0 - 0 = 0$

5 is $(-2) + 3 = 1$

Weighted VCG

Affine Maximizer: An allocation rule $k : \times_{i=1}^{n} \Theta_i \to \mathcal{K}$ is called an affine maximizer if there exists $\mathcal{K}' \subseteq \mathcal{K}$ and $w_1, \ldots, w_n \in \mathbb{R}$ and $c_{k'} \in \mathbb{R}\ \forall k' \in \mathcal{K}'$ such that, for every $\theta \in \Theta$, we have the following.

$$k(\theta) \in \underset{k' \in \mathcal{K}'}{\operatorname{argmax}} \left[c_{k'} + \sum_{i=1}^{n} w_i v_i(k', \theta) \right]$$

An affine maximizer with $c_{k'} = 0 \ \forall k' \in \mathcal{K}'$ and $w_1, \ldots, w_n = 1$ is an allocatively efficient rule.

<u>Groves Payment for Affine Maximizer</u> Let $f(\cdot) = (k(\cdot), t_1(\cdot), \ldots, t_n(\cdot))$ be a social choice function where $k(\cdot)$ is an affine maximizer with parameters $w_1, \ldots, w_n, \mathcal{K}', \{c_{k'}\}_{k' \in \mathcal{K}'}$. Then $f(\cdot)$ is dominant strategy incentive compatible if the payment rules satisfy the following.

$\forall\, i \in [n]$,

$$t_i(\theta_i, \underline{\theta}_{-i}) = \sum_{\substack{j \in [n] \\ j \neq i}} \left[\frac{w_j}{w_i} v_j(k^*(\theta), \theta_j) + \frac{c\,k^*(\theta)}{w_i} \right] + h_i(\underline{\theta}_{-i}) .$$

for any $h_i : \Theta_{-i} \longrightarrow \mathbb{R}$.