Lecture 11.1

Brief recall of what we have learnt so far.

(1) Game Theory: studied various kinds of games, for example, normal form game, extensive form game, and Bayesian game. We studied various game theoretic tools, for examples, equilibrium concepts, learning dynamics to predict the outcome of a game. Moreover, we studied various algorithms

and corresponding complexity theoretic framework of PLS, PPAD etc. to formalize hardness results. Last but not the least, we studied cost/price of anarchy for PSNEs.

(2) Mechanism Design: "Reverse game theory". Given a social choice function $f : \times_{i=1}^{n} \Theta_i \longrightarrow \mathcal{X}$, can we design a game to "implement" f. This question

is settled by the Revelation Theorem. What are the social choice functions implementable? This is answered by Gibbard-Satterwaite Theorem. Assume quasi-linear environment —— (i) outcome has a special structure: (allocation, payment) (ii) utility function also has a special structure : utility is valuation + payment.

"Affine maximizers" are the social choice functions implementable in quasi-linear environment.

Characterization of DSIC Mechanisms:

(i) payment for player i depends on θ_i only through the allocation $k(\theta_i, \underline{\theta}_{-i})$

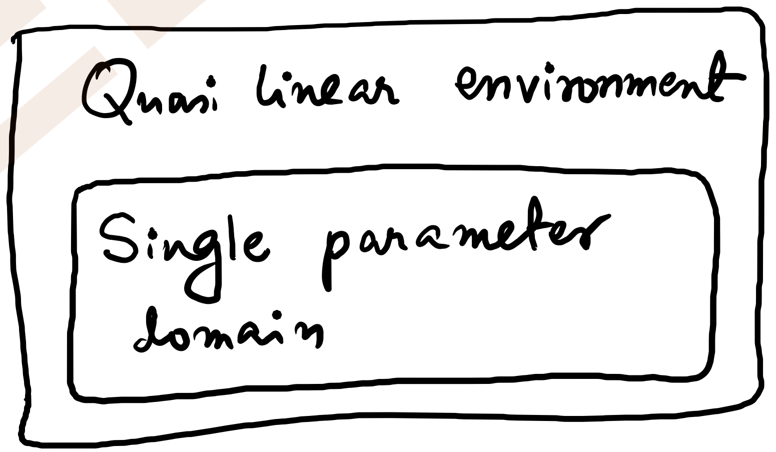
(ii) allocation function $k(\cdot)$ simultaneously optimizes for all the players.

$$k(\theta_i, \underline{\theta}_{-i}) \in \underset{K \in k(\cdot, \underline{\theta}_{-i})}{\arg\max} \left[v_i(k(\theta_i), \theta_i) + t_i(\theta_i, \underline{\theta}_{-i}) \right]$$

Single Parameter Domain

Quasi linear environment allows arbitrary valuation function

$$v_i : \mathcal{K} \times \Theta_i \longrightarrow \mathbb{R}$$



<u>Define:</u> A single parameter domain Θ_i is defined by $\mathcal{K}_i \subseteq \mathcal{K}$ and Θ_i is a real interval, $v_i(k, \theta_i) = \theta_i \ \forall k \in \mathcal{K}_i$, $v_i(k, \theta_i) = 0 \ \forall k \in \mathcal{K} \setminus \mathcal{K}_i$. Both $\mathcal{K}_i$ and

Θ_i are common knowledge.