Mechanism Design

Lecture 8.5

There is a social planner who wants to compute a social choice function $f : \underset{i \in N}{\times} \Theta_i \rightarrow X$.

But the inputs of the function f are held by n-strategic players. Mechanism designer designs a game with these n-players in such a way which enables the mechanism designer to implement the social choice function f.

We have n players. Each player $i \in [n]$ has a utility function $u_i : \mathcal{X} \times \Theta \longrightarrow \mathbb{R}$; $\Theta = \bigtimes_{i \in [n]} \Theta_i$.

We also assume that there is a prior distribution $p \in \Delta(\Theta)$.

Mechanism designer decide the set S_i of strategies of player i, and a function

$$g : \bigtimes_{i=1}^{n} S_i \longrightarrow \mathcal{X}.$$

$$u_i((s_i)_{i \in [n]}) = u_i(g((s_i)_{i \in [n]}), \theta). \quad \forall \theta \in \Theta$$

Indirect Mechanism. An indirect mechanism is a tuple $M = \langle N = [n], \mathcal{X}, (\Theta_i)_{i \in N}, (S_i)_{i \in N}, \mathbb{P} \in \Delta(\Theta),$

$g : \bigtimes_{i \in N} S_i \longrightarrow \mathcal{X}, \quad u_i : \mathcal{X} \times \Theta \longrightarrow \mathbb{R} \rangle$

- N : set of players
- $\mathcal{X}$: set of outcome

- Θ_i : type set of player i
- S_i : strategy set of player i
- $\mathbb{P}$: common prior distribution over Θ
- g : maps strategy profiles to outcomes
- u_i : utility function of player i.

If $N, \mathcal{X}, (\Theta_i)_{i \in N}, \mathbb{P}, u_i$ is clear from the context, we denote an indirect mechanism simply as

$$\left((S_i)_{i \in N}, g(\cdot) \right).$$

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A mechanism is called *direct* if $S_i = \Theta_i$ for all $i \in [n]$, and $g = f$. Clearly direct mechanism belongs to the set of indirect mechanisms. Typically $N, \mathcal{X}, (\Theta_i)_{i \in N}, \mathbb{P}, u_i$ are clear from the context and we denote a direct mechanism simply as $\left((\Theta_i)_{i \in N}, f(\cdot) \right)$.

Example (Buying Auction): one buyer and n potential sellers. The mechanism is

- $N : \{0, 1, \dots, n\}$ player 0 is the buyer and players $1, \dots, n$ are the sellers.

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- $\mathcal{X} = \Big\{ (a_0, a_1, \ldots, a_n, p_0, p_1, \ldots, p_n) \in \mathbb{R}^{2n+2} : a_i \in \{0, \pm 1\}, i \in \{0, \ldots, n\},$

$$\sum_{i=0}^{n} a_i = 0, \quad \sum_{i=0}^{n} p_i = 0 \Big\}$$

$a_i = 1$ if player i receives the item

$= -1$ ——"—— i gives ——"——

$= 0$ o/w.

p_i is the payment made by player i.

- Θ_i : set of all possible valuations of the item to player i

- S_i : the set of all possible bids.
- g : decides the outcome
- $u_i((a_0, \ldots, a_n, p_0, \ldots, p_n)) = a_i \theta_i - p_i$

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