Propositional Logic in Knowledge Representation and Reasoning

Introduction

❑Logic is a great knowledge representation language for many AI problems

❑**Propositional logic** is the simple foundation and fine for some AI problems

❑**First order logic** (FOL) is much more expressive as a KR language and more commonly used in AI

❑There are many variations: horn logic, higher order logic, three-valued logic, probabilistic logics, etc

Introduction

- Propositional logic (PL) is the simplest form of logic where all the statements are made by propositions.

- A proposition is a declarative statement which is either true or false. It is a technique of knowledge representation in logical and mathematical form.

Introduction

❑**Logical constants**: true, false

❑**Propositional symbols**: P, Q,... (**atomic sentences**)

❑Wrapping **parentheses**: (...)

❑Sentences are combined by **connectives**:

∧	and	[conjunction]
∨	or	[disjunction]
⇒	implies	[implication / conditional]
⇔	is equivalent	[biconditional]
¬	not	[negation]

❑**Literal**: atomic sentence or negated atomic sentence

P, ¬ P

Example:

a) It is Sunday.

b) The Sun rises from West (False proposition)

c) 3+3= 7(False proposition)

d) 5 is a prime number.

Facts About Propositional Logic:

❑ Propositional logic is also called Boolean logic as it works on 0 and 1.

❑ In propositional logic, we use symbolic variables to represent the logic, and we can use any symbol for a representing a proposition, such A, B, C, P, Q, R, etc.

❑ Propositions can be either true or false, but it cannot be both.

❑ Propositional logic consists of an object, relations or function, and **logical connectives**.

❑ These connectives are also called logical operators.

Examples of PL Sentences

❑
- $(P \wedge Q) \rightarrow R$
  "If it is hot and humid, then it is raining"
- $Q \rightarrow P$
  "If it is humid, then it is hot"
- Q
  "It is humid."
- We're free to choose better symbols, btw:
  Ho = "It is hot"
  Hu = "It is humid"
  R = "It is raining"

Propositional Logic (PL)

- Simple language for showing key ideas and definitions

- User defines set of propositional symbols, like P and Q

- User defines **semantics** of each propositional symbol:
  - P means "It is hot", Q means "It is humid", etc.

- A sentence (well formed formula) is defined as follows:
  - A symbol is a sentence
  - If S is a sentence, then $\neg$S is a sentence
  - If S is a sentence, then (S) is a sentence
  - If S and T are sentences, then (S $\vee$ T), (S $\wedge$ T), (S $\rightarrow$ T), and (S $\leftrightarrow$ T) are sentences
  - A sentence results from a finite number of applications of the rules

Some terms

- The meaning or **semantics** of a sentence determines its **interpretation**

- Given the truth values of all symbols in a sentence, it can be "evaluated" to determine its **truth value** (True or False)

- A **model** for a KB is a *possible world* – an assignment of truth values to propositional symbols that makes each sentence in the KB True

Facts About Propositional Logic:

❑ The propositions and connectives are the basic elements of the propositional logic.

❑ Connectives can be said as a logical operator which connects two sentences.

❑ A proposition formula which is always true is called **tautology**, and it is also called a valid sentence.

❑ A proposition formula which is always false is called **Contradiction**.

❑ Statements which are questions, commands, or opinions are not propositions such as "**Where is Rohini**", "**How are you**", "**What is your name**", are not propositions

Syntax Of Propositional Logic:

The syntax of propositional logic defines the allowable sentences for the knowledge representation. There are two types of Propositions:

a) **Atomic Propositions**
b) **Compound propositions**

Syntax Of Propositional Logic:

- **Atomic Proposition**: Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.

Example:

a) 2+2 is 4, it is an atomic proposition as it is a **true** fact.
b) "The Sun is cold" is also a proposition as it is a **false** fact.

Syntax Of Propositional Logic:

Compound proposition: Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives.

Example:
a) "It is raining today, and street is wet."
b) "Ankit is a doctor, and his clinic is in Mumbai."

Logical Connectives:

❑Logical connectives are used to connect two simpler propositions or representing a sentence logically.

❑ We can create compound propositions with the help of logical connectives.

❑There are mainly five connectives.

Logical Connectives:

1. **Negation:** A sentence such as ¬ P is called negation of P. A literal can be either Positive literal or negative literal.

2. **Conjunction:** A sentence which has ∧ connective such as, **P ∧ Q** is called a conjunction.
   Example: Rohan is intelligent and hardworking. It can be written as,
   P= Rohan is intelligent,
   Q= Rohan is hardworking. → P∧ Q.

Logical Connectives:

3. **Disjunction:** A sentence which has V connective, such as **P V Q**. is called disjunction, where P and Q are the propositions.
   Example: "Ritika is a doctor or Engineer",
   Here P= Ritika is Doctor. Q= Ritika is Doctor, so we can write it as **P V Q**.

4. **Implication:** A sentence such as P → Q, is called an implication. Implications are also known as if-then rules. It can be represented as:

Logical Connectives:

If it is raining, then the street is wet.

    Let P= It is raining, and Q= Street is wet, so it is represented as P → Q

5. **Biconditional:** A sentence such as **P⇔ Q is a Biconditional sentence, example If I am breathing, then I am alive**

    P= I am breathing, Q= I am alive, it can be represented as P ⇔ Q.

Propositional Logic Connectives:

Connective symbols	Word	Technical term	Example
∧	AND	Conjunction	A ∧ B
∨	OR	Disjunction	A ∨ B
→	Implies	Implication	A → B
⇔	If and only if	Biconditional	A ⇔ B
¬ or ~	Not	Negation	¬ A or ¬ B

Truth Table:

❏ In propositional logic, we need to know the truth values of propositions in all possible scenarios.

❏ We can combine all the possible combination with logical connectives, and the representation of these combinations in a tabular format is called **Truth table**.

Truth Table:

For Negation:

P	¬ P
True	**False**
False	**True**

For Conjunction:

P	Q	P∧ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**False**

Truth Table:

For disjunction:

P	Q	P ∨ Q.
True	True	**True**
False	True	**True**
True	False	**True**
False	False	**False**

For Implication:

P	Q	P→ Q
True	True	**True**
True	False	**False**
False	True	**True**
False	False	**True**

Truth Table:

For Biconditional:

P	Q	P⇔Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**True**

Truth Table With Three Propositions:

- We can build a proposition composing three propositions P, Q, and R. This truth table is made-up of 8n Tuples as we have taken three proposition symbols.

P	Q	R	¬R	Pv Q	PvQ→¬R
True	True	True	False	True	False
True	True	False	True	True	True
True	False	True	False	True	False
True	False	False	True	True	True
False	True	True	False	True	False
False	True	False	True	True	True
False	False	True	False	False	True
False	False	False	True	False	True

Precedence Of Connectives:

❑ Just like arithmetic operators, there is a precedence order for propositional connectors or logical operators.

❑ This order should be followed while evaluating a propositional problem .

Precedence Of Connectives:

Precedence	Operators
First Precedence	Parenthesis
Second Precedence	Negation
Third Precedence	Conjunction(AND)
Fourth Precedence	Disjunction(OR)
Fifth Precedence	Implication
Six Precedence	Biconditional

Logical Equivalence:

❑ Logical equivalence is one of the features of propositional logic.

❑ Two propositions are said to be logically equivalent if and only if the columns in the truth table are identical to each other.

❑ Let's take two propositions A and B, so for logical equivalence, we can write it as A⇔B.

❑ In below truth table we can see that column for ¬AV B and A→B, are identical hence A is Equivalent to B.

Logical Equivalence:

A	B	¬A	¬A∨B	A→B
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Properties of Operators:

❏ **Commutativity:**
P∧ Q= Q ∧ P, or
P ∨ Q = Q ∨ P.

❏ **Associativity:**
(P ∧ Q) ∧ R= P ∧ (Q ∧ R),
(P ∨ Q) ∨ R= P ∨ (Q ∨ R)

❏ **Identity element**:
P ∧ True = P,
P ∨ True= True.

Properties of Operators:

❑ **Distributive:**
P∧ (Q ∨ R) = (P ∧ Q) ∨ (P ∧ R).
P ∨ (Q ∧ R) = (P ∨ Q) ∧ (P ∨ R).

❑ **DE Morgan's Law:**
¬ (P ∧ Q) = (¬P) ∨ (¬Q)
¬ (P ∨ Q) = (¬ P) ∧ (¬Q).

❑ **Double-negation elimination:**
¬ (¬P) = P.

Limitations Of Propositional Logic:

❑We cannot represent relations like ALL, some, or none with propositional logic. Example:

All the girls are intelligent.
Some apples are sweet.

❑Propositional logic has limited expressive power.

❑In propositional logic, we cannot describe statements in terms of their properties or logical relationships.