Minimum Spanning Tree

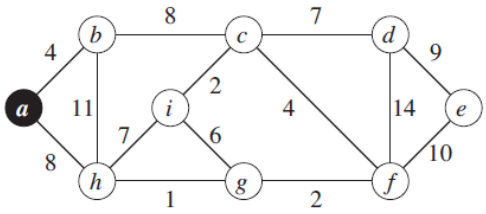
A minimum spanning tree (MST) is a subset of the edges of a connected, weighted graph that connects all the vertices without any cycles and has the minimum possible total edge weight.

Prim's Algorithm:

MST-Prim(G, w, r)

```
1  for each u ∈ V [G]
2        do key[u] ← ∞
3           π [u] ← NIL
4  key[r] ← 0
5  Q ← V [G]
6  while Q #= ∅
7        do u ← EXTRACT-MIN (Q)
8           for each v ∈ Adj[u]
9                 do if v ∈ Q and w(u, v) < key[v]
10                      then π [v] ← u
11                           key[v] ← w(u, v)
```

(a)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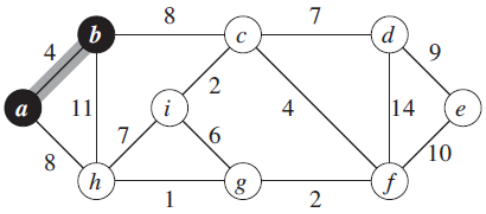
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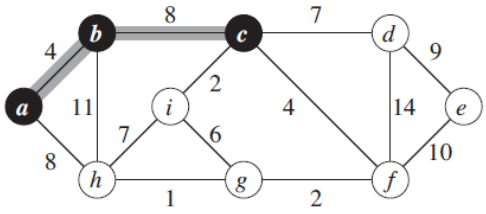
(c)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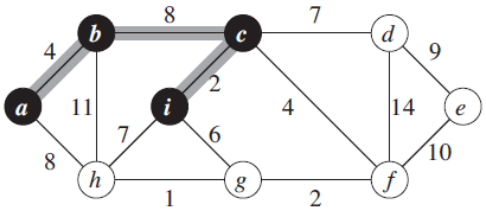
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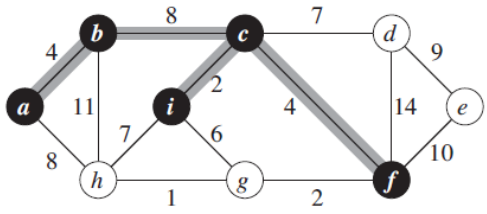
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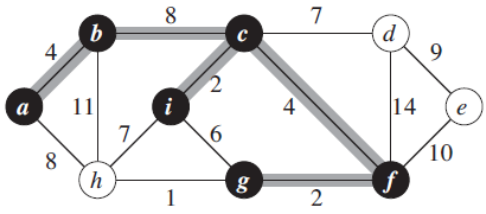
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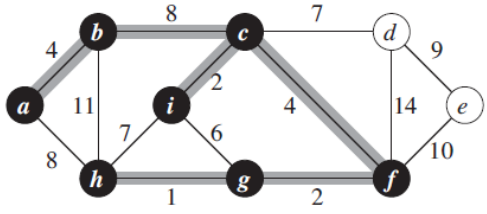
(g)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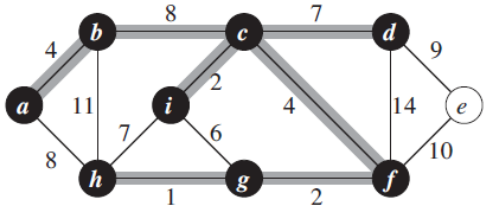
(h)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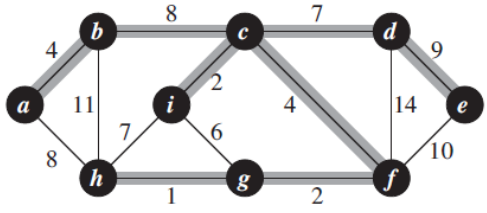
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Time Complexity

- Using an adjacency matrix: O(V^2) time complexity for each iteration to find the minimum weight edge and update distances.

- Using an adjacency list and a binary heap: O((V + E) log V) time complexity, where V is the number of vertices and E is the number of edges.