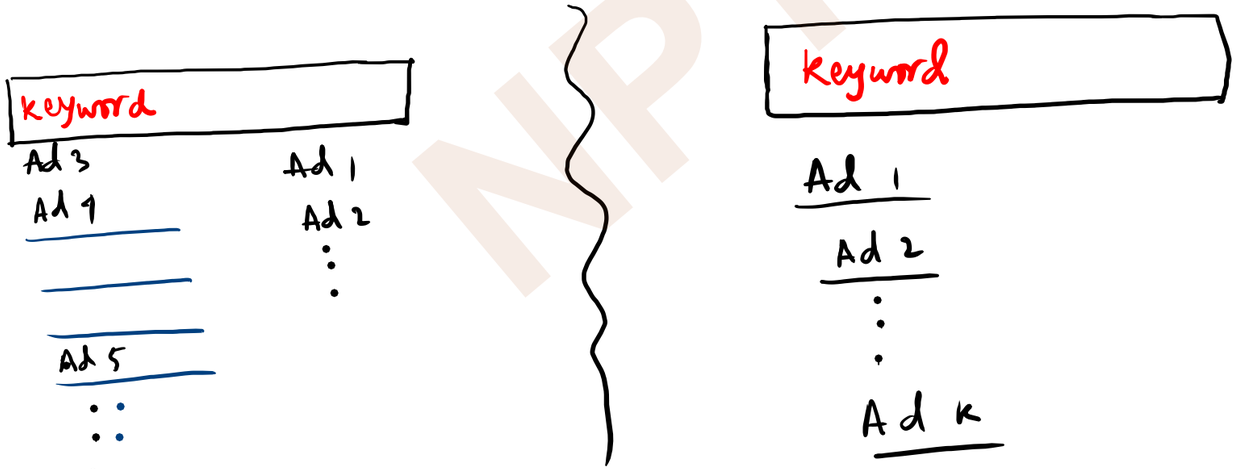
Lecture 11.5

Application of Mayerson Lemma:

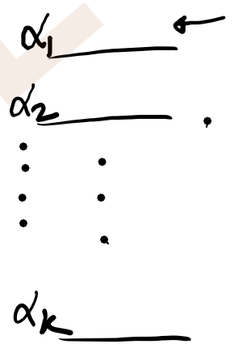
Sponsored Search Auction



Click-through rate (CTR)

$\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_k$

Each Ad j has a quality score β_j



Assumption: The probability that an user clicks an Ad j shown at position i is $(\alpha_i \cdot \beta_j)$.

Each advertiser i has valuation v_i for its ad, and if its Ad is shown in the j-th slot, then

the advertiser has valuation $v_i \alpha_j$.

Sponsored search auction belongs to single parameter domain.

Monotone allocation rule: CTRs: $\alpha_1, \dots, \alpha_k$

types: $v_1, \dots, v_n$ $\quad k \le n$

quality score: $\beta_1, \dots, \beta_n$.

$$\underset{\substack{k \in \mathcal{R} \\ \| \\ (k_1(\cdot), \dots, k_a(\cdot))}}{\arg\max} \left\{ \sum_{i=1}^{n} \mathbb{1}(k_i(\theta) \neq 0) \cdot \alpha_{k_i(\theta)} \cdot \beta_i \cdot v_i \right\}$$ is a monotone allocation rule.

The payment formula due to Myerson's lemma:

Fix player i: (v_i, v_{-i})

If player i's ad is not chosen in (v_i, v_{-i}) then $t_i(v_i, v_{-i}) = 0$.

If player i's ad is shown in the l-th slot for some $l \in \{1, \ldots, k\}$, then

$$t_i(v_i, v_{-i}) = \alpha_1 v_i \cdot (z_2 - z_1) + \alpha_2 v_i (z_3 - z_2) + \dots + (v_i - z_k) \cdot \alpha_k \cdot v_i$$