Unit – II Probability & Statistics

Ans 1 ⇒ Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

(i) There should be finite number of trials.
(ii) The trials should be independent
(iii) each trial has exactly two outcomes: Success or failure
(iv) The probability of success remains the same in each trial.

E.g.:- Throwing a die 4 times is a case 4 Bernoulli trials. in which each trial is independent in which trial of getting a odd number has two outcomes either even (failure) or odd (success). The probability of success is same in each trial as all are independent events.

Ans 2 ⇒ The probability distribution of Bernoulli trials can be expressed as combination of probability of success failure and their product as :-

X	0	1	...	r	...	n
$P(X)$	$^nC_0 q^n$	$^nC_1 q^{n-1} p$	...	$^nC_r q^{n-r} p^r$	...	$^nC_n p^n$

$P(x)$ is called probability function of binomial distribution.

A binomial distribution with n-Bernoulli trials and probability of success in each trial as p, is denoted by $B(n,p)$.

Probability density function $r = x$:-

The probability of x success $P(X=x) = {}^nC_x q^{n-x} p^x$

In each trial; q = probability of failure, p = probability of success

n = no. of trial.

$$\left\{ \begin{array}{l} \text{Mean} = \mu = E(X) = np \\ \text{Variance} = Var(X) = \sigma_x^2 = npq = np(1-p) \end{array} \right\}$$

Ans 3 ⇒ • $P(X=6) = {}^{10}C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^4$

here $p = \frac{1}{2}$, $q = \frac{1}{2}$, $n = 10$, $r = 6$

$$P(X=6) = \frac{10!}{6!\,4!} \times \frac{1}{2^{10}} = \frac{10\times9\times8\times7}{4\times3\times2\times 2^{10}}$$

$$= \frac{210}{1024} = \frac{105}{512}$$

• $P(X \geq 6) = P(X=7) + P(X=8) + P(X=9) + P(X=10) + P(X=6)$

$$= \frac{10!}{2^{10}} \left\{ \frac{1}{7!\,3!} + \frac{1}{2!\,8!} + \frac{1}{1!\,9!} + \frac{1}{10!\,0!} + \frac{1}{6!\,4!} \right\}$$

$$= \frac{1}{2^{10}} \left\{ \frac{10\times9\times8}{3\times2} + \frac{10\times9}{2} + 10 + 1 + \frac{10\times9\times8\times7}{4\times3\times2} \right\}$$

$$= \frac{1}{2^{10}} \left\{ 120 + 45 + 11 + 210 \right\}$$

$$= \frac{386}{1024} = \frac{193}{512}$$

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$P(X \le 6) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6)$

$= \frac{10!}{2^{10}} \left\{ \frac{1}{0!\,10!} + \frac{1}{1!\,9!} + \frac{1}{2!\,8!} + \frac{1}{3!\,7!} + \frac{1}{4!\,6!} + \frac{1}{5!\,5!} + \frac{1}{6!\,4!} \right\}$

$= \frac{1}{2^{10}} \left\{ 1 + 10 + 45 + 120 + 420 + \frac{10\times9\times8\times7\times6}{5\times4\times3\times2} \right\}$

$= \frac{1}{2^{10}} \left\{ 596 + 252 \right\} = \frac{848}{1024} = \frac{53}{64}$

Ans 4 ⇒ $P(X \geq 1) = P(x=1) + \cdots P(x=10)$

Here $P(x=r) = {}^{n}C_r\, p^r q^{n-r}$

$n = 10,\ p = 10\% = 0.1,\ q = 1 - 0.1 = 0.9$

$\therefore P(x \geq 1) = \frac{10!}{1!\,9!} \times (0.9)^{10} + \frac{10!}{2!\,8!} \times (0.9)^9 (0.1) \cdots$

∵ probability success is 10% = 0.10 which is very low & number of trials is 10 which is large compared to success probability we will use poisson's distribution.

$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$

here λ = mean $= np = 10 \times 0.1 = 1$

$\therefore P(x \geq 1) = \sum_{i=1}^{10} \frac{(1)^{x_i} e^{-\lambda}}{x_i!} = e^{-1} \left\{ \frac{1}{1!} + \frac{1}{2!} + \cdots \frac{1}{10!} \right\}$

$= e^{-1} \left\{ \frac{9! + 8! + 7! + \cdots 1!}{10!} \right\}$ $\cancel{[\because 1! + 2! + 3! \cdots n!]}$

Ans

Ans 5 ⇒ $n = 5$ (∵ five cards are drawn successively with replacement)

(P.T.O.)

X = no of spades random discrete variable

- $P(X=5) = {}^5C_5\, q^0 p^5$

$p = \frac{13}{52} = \frac{1}{4}$ $\quad q = 1 - \frac{1}{4} = \frac{3}{4}$

$P(X=5) = 1 \times \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$

$P(X=3) = {}^5C_3\, q^2 p^3 = \frac{5!}{3!\,2!} \times \left(\frac{3}{4}\right)^2 \times \left(\frac{1}{4}\right)^3$

$= \frac{10 \times 9}{1024} = \frac{45}{512}$

$P(X=0) = {}^5C_0\, q^5 p^0 = \frac{5!}{5!\,0!} \times \left(\frac{3}{4}\right)^5$

$= \frac{243}{1024}$

Ans 6 ⇒ Condition to use poisson's distribution:-

- The probability of success is very low
- The number of trials is very large
- along with conditions satisfy for binomial distribution

$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$ (probability density function)

λ = mean = np

Variance = $\sigma_x^2 = npq$

Ans 7 ⇒ X = no of defective fuses found

$n = 200$ trials $\quad p = 2\%$ = probability of defective bulb in each trial.

$$P(X \leq 5) = \sum_{i=1}^{5} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

where, $\lambda = np = 200 \times \frac{2}{100} = 4$

$$\therefore P(X \leq 5) = e^{-4} \left\{ \frac{4^0}{0!} + \frac{4^1}{1!} + \frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} + \frac{4^5}{5!} \right\}$$

$$= e^{-4} \left\{ 1 + 4 + 8 + \frac{32}{3} + \frac{32}{3} + \frac{32 \times 4}{15} \right\}$$

$$= e^{-4} \left\{ 13 + \frac{64}{3} + \frac{128}{15} \right\} = \frac{e^{-4}}{15} \left\{ 195 + 320 + 128 \right\}$$

$$= \frac{e^{-4} \times 623}{15}$$

Ans 8 ⇒ X = no. of individual suffering from a bad reaction.

$p = 0.001$

$n = 2000$ individuals.

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad \lambda = np = 2000 \times 0.001 = 2$$

$$P(X = 3) = \frac{2^3 e^{-2}}{3!}$$

$$= \frac{8}{6} e^{-2} = \frac{4}{3} e^{-2}$$

$$P(X > 2) = 1 - P(X \leq 2)$$

$$= 1 - \left\{ P(X=1) + P(X=2) + P(X=0) \right\}$$

$$= 1 - \left\{ \frac{2e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!} + \frac{2^0 e^{-2}}{0!} \right\}$$

$$= 1 - e^{-2} \left\{ 2 + 2 + 1 \right\}$$

$$= 1 - 5e^{-2}$$

$$P(X=0) = \frac{2^0 e^{-2}}{0!} = e^{-2}$$

$$P(X>1) = 1 - P(X \leq 1)$$

$$= 1 - \{P(X=0) + P(X=1)\}$$

$$= 1 - \left\{\frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!}\right\}$$

$$= 1 - e^{-2}\{1+2\} = 1 - 3e^{-2}$$

Ans 9 ⇒ Suppose that in a sequence of trials we are interested in number of the trial on which first success occurs. The three assumptions for Bernoulli trials are satisfied but the extra assumption underlying the binomial distribution is not, i.e; n is not fixed.

For x^{th} trial to occur first success first $(x-1)$ failures will come.

Therefore :-

Probability distribution function

$$P(X=x) = \begin{cases} q^{(x-1)}p & x = 1, 2, 3, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{mean} = \frac{1}{p}$$

$$\text{Standard deviation} = \frac{\sqrt{q}}{p}$$

$$\text{variance} = \frac{q}{p^2}$$

Ans 10 ⇒ $p = 0.05$ $\quad x = 6.$

$P(X=6) = q^{(5)} p$ by geometric Distribution

$p = 0.05$ $q = 1 - 0.05 = 0.95$

$P(X=6) = (0.95)^5 \, 0.05 = 0.0386890469$

(calculator)

Ans 11 => $x = 5$ $p = \frac{1}{6}$ $q = \frac{5}{6}$

by Geometric Distribution

$P(X=5) = \left(\frac{5}{6}\right)^4 \frac{1}{6} = \frac{625}{6^5}$

Ans 12 => $p = 0.7$

$q = 1 - 0.7 = 0.3$

(i) $x = 3$

$P(X=3) = (q)^2 p = (0.3)^2 \, 0.7 = 0.09 \times 0.7$

$= 0.063$

(ii) Average number of shots needed to hit target is mean of distribution $= \frac{1}{p} = \frac{1}{0.7} = 1.4286$

Ans 13 => In Normal Distribution if $\mu = 0$ and $\sigma = 1$ it is called Standard normal Distribution. and probability density function is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

Convert x into a standard normal variate by formula $z = \frac{x - \mu}{\sigma}$

Find limit of z corresponding to limit of x by above formula. Use standard normal table for probability

$\therefore P(a < x < b) = P\left(\frac{a-\mu}{\sigma} < x < \frac{b-\mu}{\sigma}\right)$

Example:- If mean height is 151 cm of 500 students standard deviation is 15cm, probability & no of students between 120 and 155 cm.

$\Rightarrow \quad \mu = 151cm \qquad \sigma = 15cm$

$z = \frac{x-\mu}{\sigma}$

Now limits of $x = 120, 155$, i.e; $120 < x < 155$

limits of $z = \frac{120-151}{15}, \frac{155-151}{15}$

$= -\frac{31}{15}, \frac{4}{15}$

$\therefore \quad -2.067 < z < 0.267$

$\therefore P(-2.067 < z < 0.267) = P(-2.067 < z < 0) + P(0 < z < 0.267)$

~~from table we will get~~ zscore from the probability density function.

$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \qquad z \to [-2.067, 0]$
$\qquad \& \; z \to (0, 0.267)$

Ans 14 $\Rightarrow \quad \mu = 151 \qquad \sigma = 15 \qquad z = \frac{x-\mu}{\sigma}$

$x = 120 \qquad z = \frac{120-151}{15} = -2.067$

$x = 155 \Rightarrow z = 0.267$

$P(120 < x < 155) = P(-2.067 < z < 0.267)$
$= P(-2.067 < z < 0) + P(0 < z < 0.267)$

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$P(120 < x < 155) = 0.4808 + 0.1064 = 0.5872$

$\therefore$ No. of students $= 500 \times 0.5872 = 294$

Ans 15 ⇒ A random variable x is said to have an exponential distribution with parameter $\lambda > 0$, if its probability density function is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

The distribution is concerned where we are concerned with exponential growth or decay of organisms or matters

also $P(X \geq x) = \int_{-\infty}^{x} f(x)\,dx = \int_{0}^{x} \lambda e^{-\lambda x}\,dx$

$$= 1 - e^{-\lambda x}$$

$\therefore$

$$F(x) = P(X \geq x) \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \\ 1 & x = \infty \end{cases}$$

$$\text{Mean} = \mu = \int_{-\infty}^{\infty} x f(x)\,dx = \lambda\left[\frac{xe^{-\lambda x}}{-\lambda} - \frac{e^{-\lambda x}}{\lambda^2}\right]_0^{\infty}$$

$$= \frac{1}{\lambda}$$

$\therefore$ Variance $= \text{Var}(X) = \frac{1}{\lambda^2}$ &

Standard deviation $= \frac{1}{\lambda}$

Eg:- $f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$ (P.T.O.)

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$$P(x \geq 3) = \int_3^\infty f(x)\,dx = \int_3^\infty 2e^{-2x}\,dx$$

$$= 2\left[\frac{e^{-2x}}{-2}\right]_3^\infty = -(0 - e^{-6}) = e^{-6}$$

Mean $= \frac{1}{\lambda} = \frac{1}{2}$, Standard deviation $= \sigma = \frac{1}{\lambda} = \frac{1}{2}$

Ans 16 ⇒ $\because$ $f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$

$\lambda = 0.001$

$$P(x > 1200) = \int_{1200}^\infty f(x)\,dx = \int_{1200}^\infty \lambda e^{-\lambda x}\,dx$$

$$= \lambda\left[\frac{e^{-\lambda x}}{-\lambda}\right]_{1200}^\infty = e^{-0.001 * 1200}$$

$$= e^{-1.2} = 0.301$$