APPLICATION OF DIFFERENTIAL EQUATIONS

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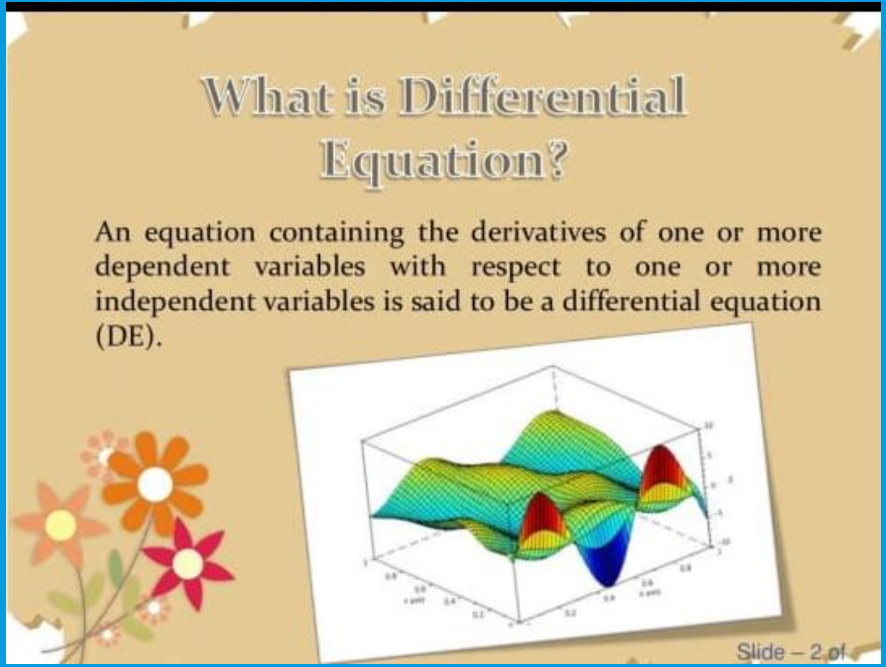
WHAT IS A DIFFERENTIAL EQUATION?

- A differential equation, in mathematics, can be referred to as an equation that relates to one or more functions and their derivatives.
- The said Functions usually represent physical quantities, and their derivatives represent their rates of change.
- The differential equation creates a relationship between these two.



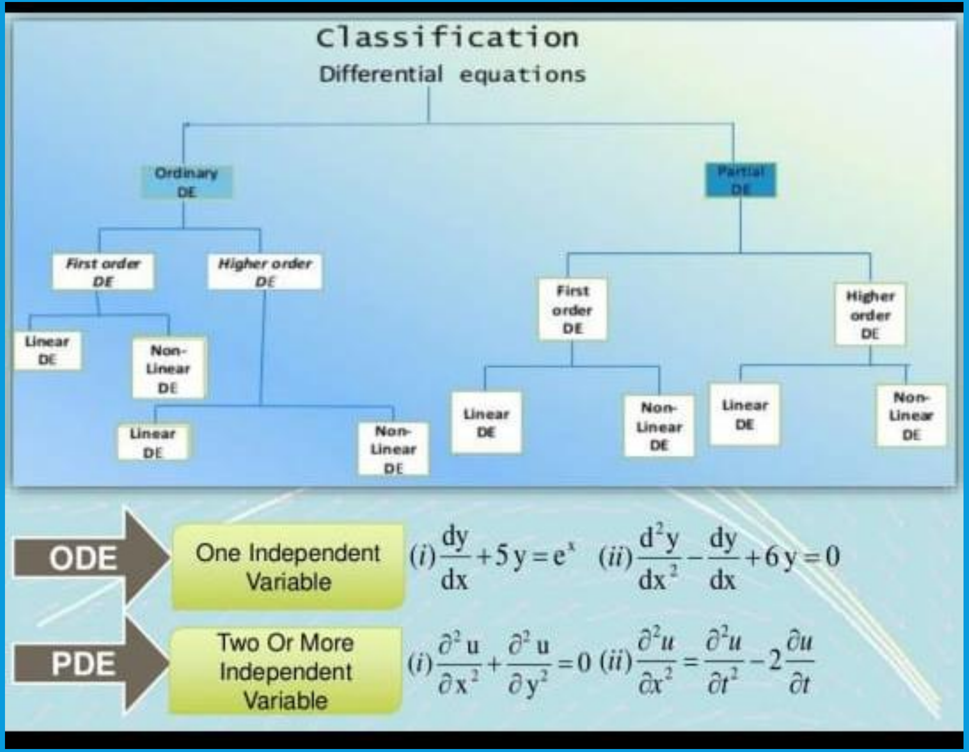
TYPES OF DIFFERENTIAL EQUATIONS

Differential equations can be categorized into three types:

Ordinary differential equations

Partial differential equations

Non-linear differential equations



EQUATION ORDER

Differential equations are described by their order, determined by the term with the highest derivatives. They are classified into:

1. First-order differential equation – equation containing only first derivatives
2. Second-order differential equation – equation containing the second Derivative and so on.

$\frac{dy}{dx}=5x+6$ has order 1 and is 1st degree

$\frac{d^2y}{dx^2}-\frac{dy}{dx}+9=0$ has order 2 and is 1st degree

$\frac{d^3y}{dx^3}=78xe^8$ has order 3 and is 1st degree

$\left(\frac{dy}{dx}\right)^2=5x+6$ has order 1 and is 2nd degree

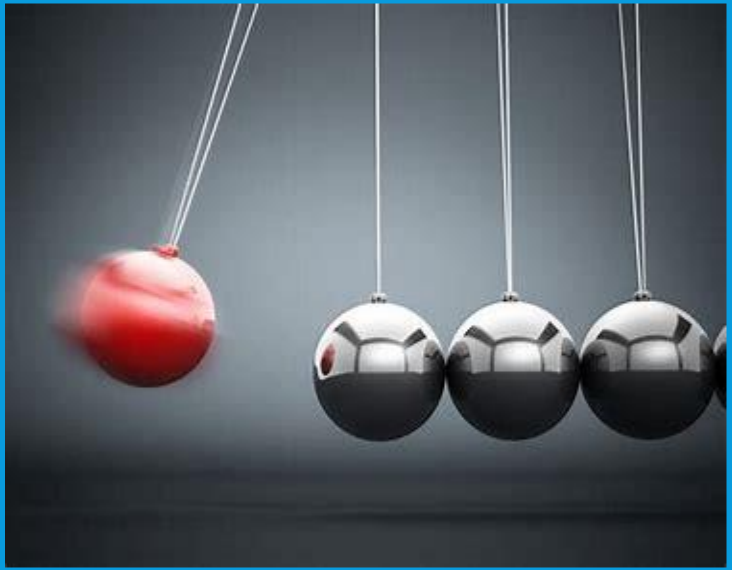
$\left(\frac{d^2y}{dx^2}\right)^7-\left(\frac{dy}{dx}\right)^{20}+9=0$ has order 2 and is 7th degree

$\left(\frac{d^3y}{dx^3}\right)^2=78xe^8$ has order 3 and is 2nd degree

APPLICATIONS OF DIFFERENTIAL EQUATIONS

- Ordinary differential equations are utilized in the real world :

To calculate the movement or flow of electricity,motion of an object to and from like a **pendulum** and to elucidate **thermodynamics** concepts.



- Moreover, they are used in the medical field

to check the **growth of diseases** in graphical representation.

APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

Partial differential equations can be used to describe a wide variety of phenomena in nature such as sound, heat, **electrostatics**, electrodynamics, fluid flow, elasticity, or quantum mechanics.

These physical phenomena which seem to be distinct can actually be formalized in terms of PDEs. While ordinary differential equations model one-dimensional dynamical systems, partial differential equations model multi-dimensional systems.

APPLICATIONS OF DIFFERENTIAL EQUATIONS IN REAL LIFE

The applications of differential equations in real life are as follows:

In Physics:

Study the movement of an object like a pendulum

Study the movement of electricity

To represent thermodynamics concepts

In Medicine:

Graphical representations of the development of diseases

In Mathematics:

Describe mathematical models such as:

Population explosion

Radioactive decay

APPLICATIONS OF DIFFERENTIAL EQUATIONS IN REAL LIFE

Geometrical applications:

To find the:

Slope of a tangent

Equation of tangent and normal

Length of tangent and normal

Length of sub-tangent and sub-normal

Physical application:

We can calculate

Velocity

Acceleration

EXPONENTIAL GROWTH - POPULATION

Let P(t) be a quantity that increases with time t and the rate of increase is proportional to the same quantity P as follows

$$dP / dt = kP$$

where d p / d t is the first derivative of P, $k > 0$ and t is the time.
The solution to the above first order differential equation is given by

$$P(t) = A e^{kt}$$

where A is a constant not equal to 0.
If $P = P_0$ at $t = 0$, then $P_0 = A e^0$ which gives $A = P_0$
The final form of the solution is given by

$$P(t) = P_o e^{kt}$$

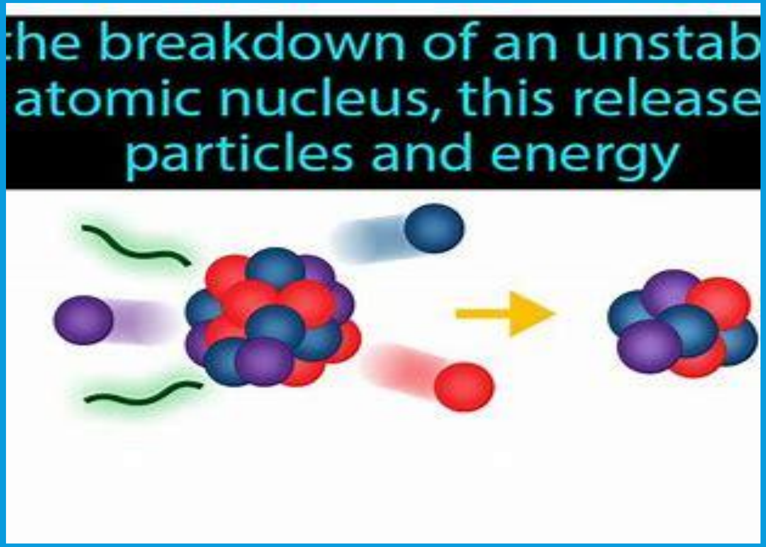
Assuming P_0 is positive and since k is positive, P(t) is an increasing exponential. $dP / dt = kP$ is also called an exponential growth model

EXPONENTIAL DECAY-RADIOACTIVE MATERIAL

- Let $M(t)$ be the amount of a product that decreases with time t and the rate of decrease is proportional to the amount M as follows

$$dM/dt = -kM$$

where dM/dt is the first derivative of M, $k > 0$ and t is the time.
Solve the above first order differential equation to obtain



$$M(t) = A e^{-kt}$$

where A is non zero constant.
It we assume that $M = M_0$ at $t = 0$, then
$M_0 = A e^0$ which gives $A = M_0$
The solution may be written as follows

$$M(t) = M_o e^{-kt}$$

Assuming M_0 is positive and since k is positive, $M(t)$ is an decreasing exponential. $dM/dt = -kM$ is also called an exponential decay model.

FALLING OBJECT

- An object is dropped from a height at time $t = 0$. If $h(t)$ is the height of the object at time t, $a(t)$ the acceleration and $v(t)$ the velocity. The relationships between a, v and h are as follows:
  $a(t) = dv / dt$, $v(t) = dh / dt$.
  For a falling object, $a(t)$ is constant and is equal to $g = -9.8$ m/s.

Combining the above differential equations, we can easily deduce the following equation:

$$d^2 h / dt^2 = g$$

Integrate both sides of the above equation to obtain
$dh / dt = g t + v_0$
Integrate one more time to obtain
$h(t) = (1/2) g t^2 + v_0 t + h_0$
The above equation describes the height of a falling object, from an initial height h_0 at an initial velocity v_0, as a function of time.

NEWTON'S LAW OF COOLING

- It is a model that describes, mathematically, the change in temperature of an object in a given environment. The law states that the rate of change (in time) of the temperature is proportional to the difference between the temperature T of the object and the temperature Te of the environment surrounding the object.

$$dT / dt = -k (T - Te)$$

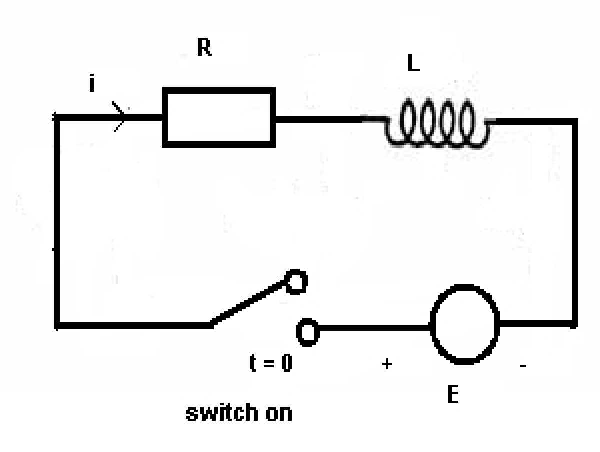
Let $x = T - Te$ so that $dx / dt = dT / dt$
Using the above change of variable, the above differential equation becomes

$$dx / dt = -k x$$

The solution to the above differential equation is given by $x = A e^{-kt}$ substitute x by T - Te
$T - Te = A e^{-kt}$
Assume that at $t = 0$ the temperature $T = To$
$To - Te = A e^{0}$
which gives $A = To - Te$
The final expression for T(t) i given by
$T(t) = Te + (To - Te)e^{-kt}$
This last expression shows how the temperature T of the object changes with time.

RL CIRCUIT

- Let us consider the RL (resistor R and inductor L) circuit shown above. At $t = 0$ the switch is closed and current passes through the circuit. Electricity laws state that the voltage across a resistor of resistance R is equal to R i and the voltage across an inductor L is given by L di/dt (i is the current). Another law gives an equation relating all voltages in the above circuit as follows:
  $L \, di/dt + Ri = E$, where E is a constant voltage.
  Let us solve the above differential equation which may be written as:
  $L [di / dt] / [E - R i] = 1$
  which may be written as
  $- (L / R) [- R \, d i] / [E - Ri] = dt$
  Integrate both sides
  $- (L / R) \ln(E - R i) = t + c$, c constant of integration.
  Find constant c by setting $i = 0$ at $t = 0$ (when switch is closed) which gives
  $c = (-L / R) \ln(E)$

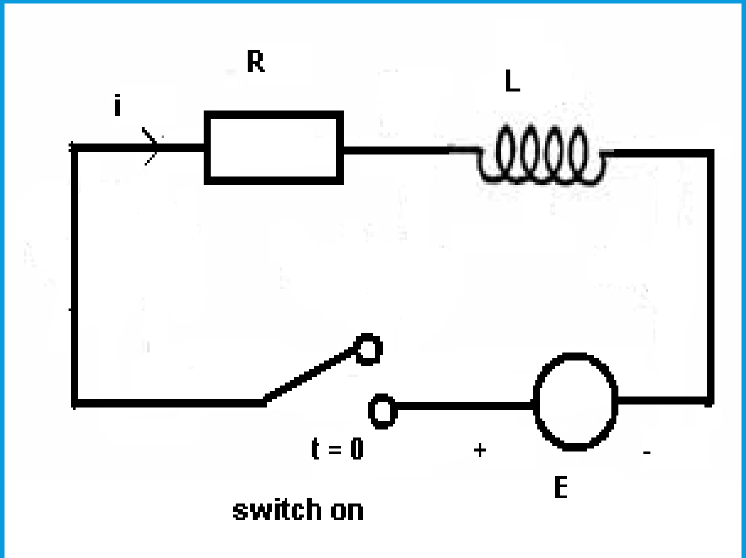


RL CIRCUIT

- Substitute c in the solution
  $- (L / R) \ln(E - R i) = t + (-L/R) \ln (E)$
  which may be written
  $(L/R) \ln (E) - (L / R) \ln(E - R i) = t$
  $\ln[E/(E - Ri)] = t(R/L)$
  Change into exponential form
  $[E/(E - Ri)] = e$
- t(R/L)
  Solve for i to obtain

$$i = (E/R) (1-e^{-Rt/L})$$

The starting model for the circuit is a differential equation which when solved, gives an expression of the current in the circuit as a function of time.



LOTKA-VOLTERRA EQUATIONS

- We let $R(t)$ be the number of prey (R for rabbits) and $W(t)$ be the number of predators (W for wolves) at time t.
- In the absence of predators, the ample food supply would support exponential growth of the prey, that is,

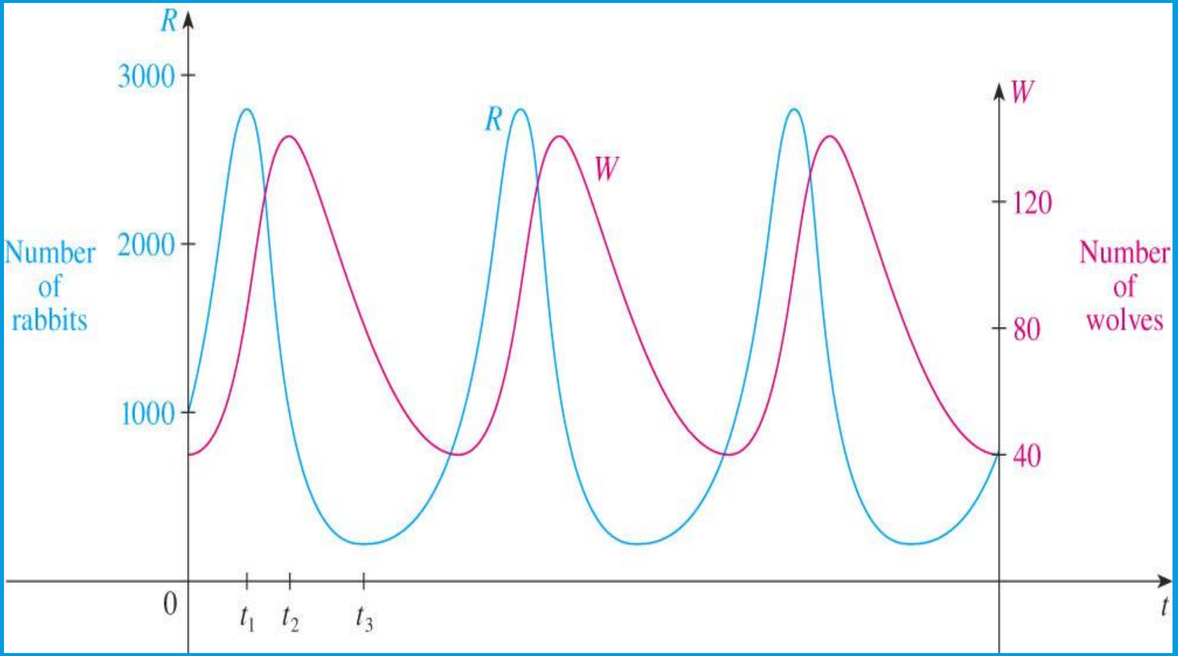
$$\frac{dR}{dt} = kR$$

where k is a positive constant.

- In the absence of prey, we assume that the predator population would decline at a rate proportional to itself, that is,

$$\frac{dW}{dt} = -rW$$

where r is a positive constant.

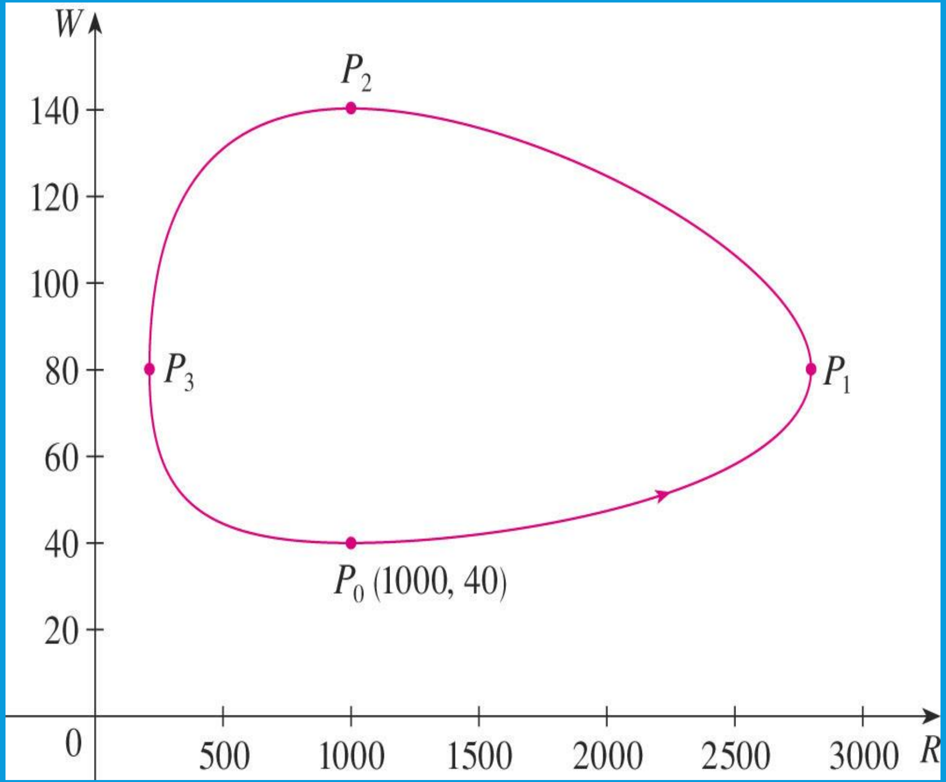


LOTKA-VOLTERRA EQUATIONS

They were proposed as a model to explain the variations in the shark and food-fish populations in the Adriatic Sea by the

Italian mathematician Vito Volterra (1860–1940).

$$\frac{dR}{dt} = kR - aRW \qquad \frac{dW}{dt} = -rW + bRW$$

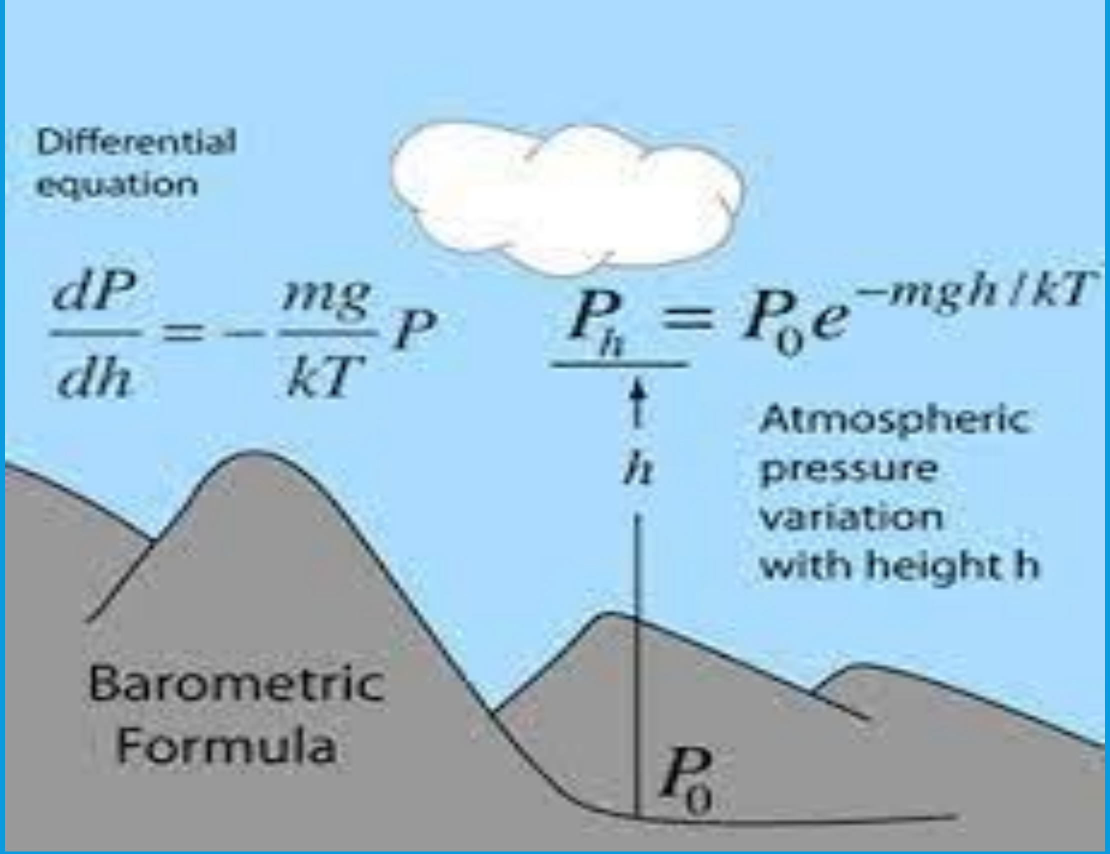


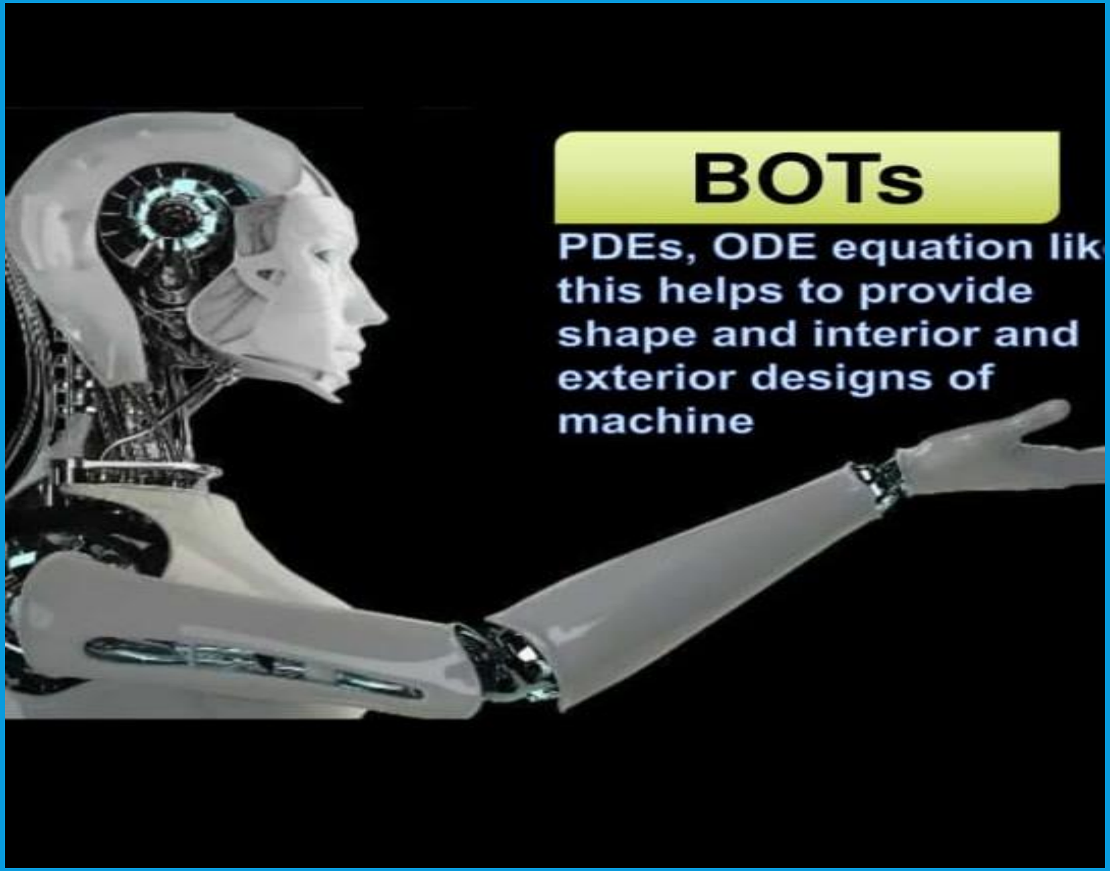
These equations are known as the

predator-prey equations,

or the Lotka-Volterra equations.

SOME OTHER APPLICATIONS





APPLICATION OF DIFFERENTIAL EQUATIONS

CODING PROGRAMS IN PYTHON PROGRAMMING LANGUAGE

CODE FOR SOLVING DIFFERENTIAL EQUATION

```
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt

# function that returns dy/dt
print("Solving and plotting graph for Differentia
l Equation in Python.")
def model(y,t):
    k = 0.3
    dydt = -k * y
    return dydt
```

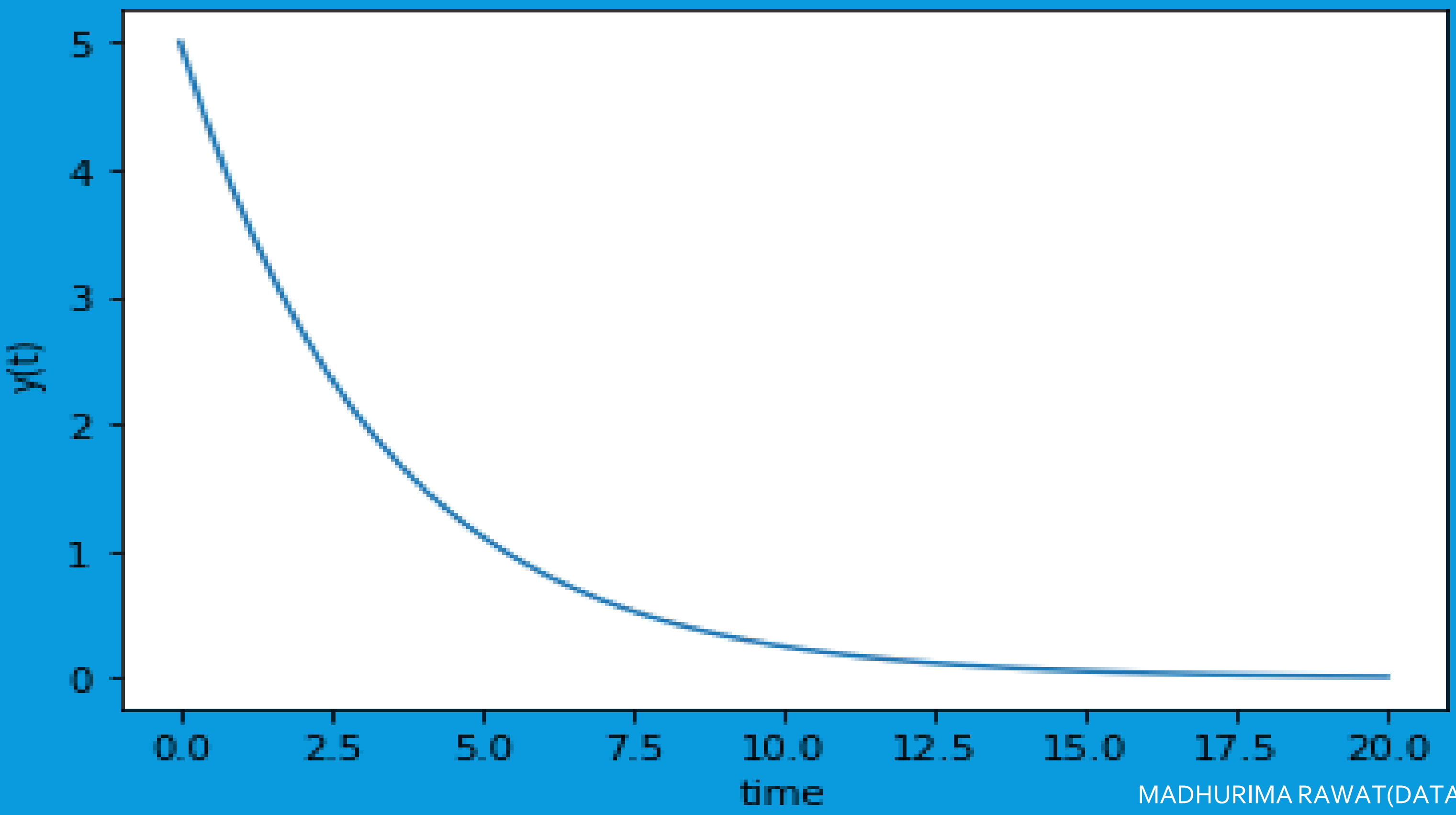
```
# initial condition
y0 = 5
# time points
t = np.linspace(0,20)

# solve ODE
y = odeint(model,y0,t)

# plot results
plt.plot(t,y)
plt.xlabel('time')
plt.ylabel('y(t)')
plt.show()
```

OUTPUT OF THE CODE

- Solving and plotting graph for Differential Equation in Python.



THANK YOU FOR LISTENING TO ME

ANY QUESTIONS