Lecture 9.2

The direct mechanism for the first price auction is <u>not</u>. even Bayesian Incentive compatible.

Revelation Principal for Dominant Strategy Incentive Compatibility:

Let $f: \times_{i\in[n]} \Theta_i \longrightarrow X$ be any social choice function.

Suppose an indirect mechanism $((S_i)_{i\in[n]}, g(\cdot))$ implements f in Dominant strategy equilibrium. Then the direct mechanism $((\Theta_i)_{i \in [n]}, f(\cdot))$ implements f in Dominant Strategy equilibrium.

<u>Proof:</u> Since $M = ((S_i)_{i \in [n]}, g(\cdot))$ implements f in DSE, there exists a very weakly dominant strategy equilibrium $(s_i^*(\cdot))_{i \in [n]}$ such that

$\forall\ (\theta_1, \dots, \theta_n) \in \bigtimes_{i=1}^{n} \Theta_i$

$$g(s_i^*(\theta_i))_{i \in [n]} = f((\theta_i)_{i \in [n]}) \quad \text{—— (1)}$$

$\forall i \in [n], \forall \theta_i \in \Theta_i, \forall a \in S_i, \forall a_{-i} \in S_{-i}, \forall \theta_{-i} \in \Theta_{-i}$

$$u_i\left(g\left(\left(s_i^*(\theta_i), a_{-i}\right)\right), (\theta_i)_{i \in [n]}\right) \geq u_i\left(g\left(\left(a, a_{-i}\right)\right), (\theta_i)_{i \in [n]}\right) \quad \text{—— (2)}$$

<u>To show:</u> $\left(f, (\Theta_i)_{i \in [n]}\right)$ is DSIC.

$\forall \theta_i' \in \Theta_i \; \forall \theta_{-i}' \in \Theta_{-i}$

$\forall i \in [n], \forall \theta_i \in \Theta_i, \forall \theta_{-i} \in \Theta_{-i}$

$$u_i\left(f(\theta_i, \theta_{-i}'), (\theta_j)_{j \in [n]}\right) \geq u_i\left(f(\theta_i', \theta_{-i}'), (\theta_j)_{j \in [n]}\right)$$

$$u_i\left(f(\theta_i, \underline{\theta}'_{-i}), (\theta_j)_{j\in[n]}\right)$$

$$= u_i\left(g\left(s_i^*(\theta_i), s_{-i}^*(\underline{\theta}'_{-i})\right), (\theta_j)_{j\in[n]}\right)$$ [from equation (1)]

$$\geq u_i\left(g\left(s_i^*(\theta'_i), s_{-i}^*(\underline{\theta}'_{-i})\right), (\theta_j)_{j\in[n]}\right)$$ [from equation (2), put $a = s_i^*(\theta'_i)$, $\underline{a}_{-i} = s_{-i}^*(\underline{\theta}'_{-i})$]

$$= u_i\left(f(\theta'_i, \underline{\theta}'_{-i}), (\theta_j)_{j\in[n]}\right)$$ [from equation (1)]

$\boxtimes$

Revelation Principle for BIC mechanisms

Let $f : \times_{i \in [n]} \Theta_i \longrightarrow \mathcal{X}$ be any social choice function. Suppose an indirect mechanism implements f in Bayesian Nash equilibrium. Then the direct mechanism $\left((\Theta_i)_{i \in [n]}, f(\cdot) \right)$ also implements $f(\cdot)$ in Bayesian Nash equilibrium.