Algorithmic Game Theory
Assignment 12

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1. Let $\mathcal{D}$ be a domain contains single-parameter domain as its subset. Also, $\mathcal{D}$ itself is contained in the quasi-linear domain. Then, which of the following allocation rules must be implementable in $\mathcal{D}$?

   (a) affine maximizers
   (b) allocatively efficient
   (c) monotone
   (d) arbitrary

   The correct answer is (c).

2. What is the kind of domain of the type set of each player in the mechanism design problem of Knapsack allocation?

   (a) arbitrary
   (b) quasi-linear but not convex
   (c) convex but not single parameter
   (d) single parameter

   The correct answer is (d). Refer to Lecture 12.2.

3. In a stable matching instance, suppose we have n men and n women. Then which of the following is true in a stable matching of the instance?

   (a) A man may remain unmatched
   (b) A woman may remain unmatched
   (c) It can have a blocking pair
   (d) A man may not be matched with his most preferred woman

   The correct answer is (d). Refer to Lecture 12.3.

4. In a stable matching instance, suppose we have n men and n women. Then which of the following is true for the output of the men-proposing deferred acceptance algorithm?

   (a) Every man is matched with his most preferred woman among all stable matchings.
   (b) Every man is matched with his most preferred woman among all matchings.
   (c) Every woman is matched with his most preferred woman among all stable matchings.
   (d) Every woman is matched with his most preferred woman among all matchings.

   The correct answer is (a). Refer to Lecture 12.4.

5. Which of the following is not true for the output of men-proposing deferred acceptance algorithm?

   (a) Every man is matched with his most preferred woman among all stable matchings.
   (b) Every woman is matched with her least preferred man among all stable matchings.

(c) During the run of the algorithm, every man, once matched, never becomes unmatched.

(d) During the run of the algorithm, every woman, once matched, never becomes unmatched.

The correct answer is (a). Refer to Lecture 12.4.

6. What is the maximum total number of proposals men make in the deferred acceptance algorithm?

   (a) $\Theta(n)$

   (b) $\Theta(n \log n)$

   (c) $\Theta(n^2)$

   (d) $\Theta(n^2 \log n)$

   The correct answer is (c). Refer to Lecture 12.4.

7. Consider the following instance of the stable matching problem. The preference lists of men are:

$$m_1 : w_2 \succ w_1 \succ w_3 \succ w_4$$
$$m_2 : w_4 \succ w_1 \succ w_2 \succ w_3$$
$$m_3 : w_1 \succ w_3 \succ w_2 \succ w_4$$
$$m_4 : w_2 \succ w_3 \succ w_1 \succ w_4$$

   The preference lists of women are:

$$w_1 : m_1 \succ m_3 \succ m_2 \succ m_4$$
$$w_2 : m_3 \succ m_4 \succ m_1 \succ m_2$$
$$w_3 : m_4 \succ m_2 \succ m_3 \succ m_1$$
$$w_4 : m_3 \succ m_2 \succ m_1 \succ m_4$$

   In the men-optimal stable matching, who will be the partner of w_2?

   (a) m_1

   (b) m_2

   (c) m_3

   (d) m_4

   The correct answer is (d). Refer to Lecture 12.4.

8. Consider the following instance of the stable matching problem. The preference lists of men are:

$$m_1 : w_2 \succ w_1 \succ w_3 \succ w_4$$
$$m_2 : w_4 \succ w_1 \succ w_2 \succ w_3$$
$$m_3 : w_1 \succ w_3 \succ w_2 \succ w_4$$
$$m_4 : w_2 \succ w_3 \succ w_1 \succ w_4$$

   The preference lists of women are:

$$w_1 : m_1 \succ m_3 \succ m_2 \succ m_4$$
$$w_2 : m_3 \succ m_4 \succ m_1 \succ m_2$$
$$w_3 : m_4 \succ m_2 \succ m_3 \succ m_1$$
$$w_4 : m_3 \succ m_2 \succ m_1 \succ m_4$$

   In the men-pessimal stable matching, who will be the partner of w_2?

   (a) m_1

   (b) m_2

(c) m_3

(d) m_4

The correct answer is (c). Refer to Lecture 12.4.

9. Consider the following instance of the stable matching problem. The preference lists of men are:

$$m_1 : w_2 \succ w_1 \succ w_3 \succ w_4$$
$$m_2 : w_4 \succ w_1 \succ w_2 \succ w_3$$
$$m_3 : w_1 \succ w_3 \succ w_2 \succ w_4$$
$$m_4 : w_2 \succ w_3 \succ w_1 \succ w_4$$

The preference lists of women are:

$$w_1 : m_1 \succ m_3 \succ m_2 \succ m_4$$
$$w_2 : m_3 \succ m_4 \succ m_1 \succ m_2$$
$$w_3 : m_4 \succ m_2 \succ m_3 \succ m_1$$
$$w_4 : m_3 \succ m_2 \succ m_1 \succ m_4$$

In a run of men proposing deferred acceptance algorithm, how many times woman w_4 gets rejected?

(a) 1

(b) 2

(c) 3

(d) 0

The correct answer is (a). Refer to Lecture 12.4.

10. In the women-proposing deferred acceptance algorithm, which of the following is true?

(a) A woman, once matched, never becomes unmatched.

(b) A man, once matched, never becomes unmatched.

(c) A man never rejects any woman.

(d) No man or woman ever reject anyone.

The correct answer is (b). Refer to Lecture 12.4.