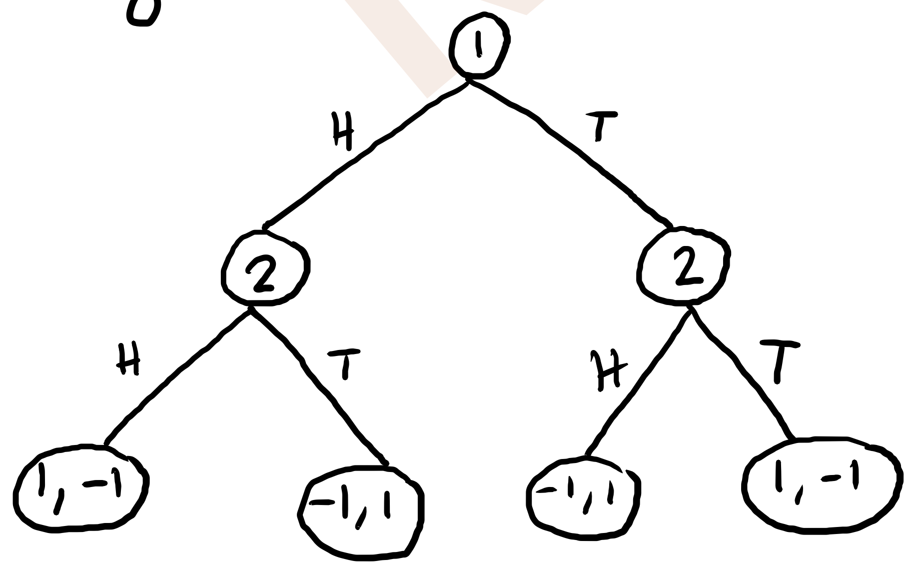
Lecture 8.4

Extensive Form Game

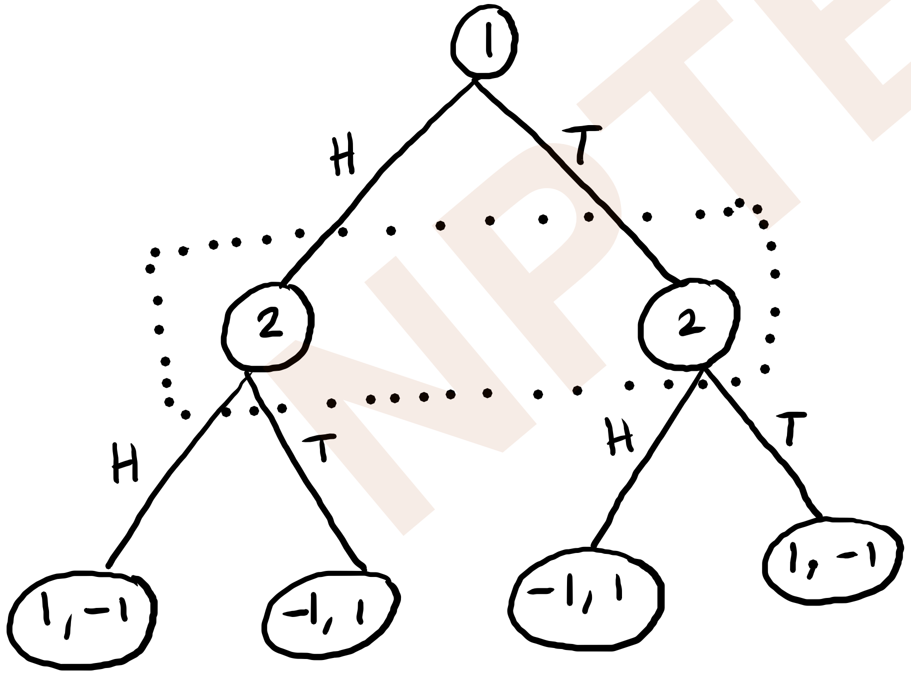
Relaxes "simultaneous move" condition of normal form game.

"Game tree"

Example (Matching Pennies with Observation):



Example 2 (Matching Pennies without Observation)



Information set.

Definition (Information set): An information set of a player is a subset of the player's decision nodes which are indistinguishable to him.

- In the matching pennies game with observation, player 1 has only one information set $\{\epsilon\}$ whereas player 2 has two information sets $\{H\}$ and $\{T\}$.
- In the matching pennies game without observation, player 2 has one information set $\{H, T\}$.

<u>Definition</u> (Extensive form game).

$\Gamma = \langle N, (S_i)_{i\in N}, H, P, (\mathbb{I}_i)_{i\in N}, C, (u_i)_{i\in N} \rangle$ where

- N = set of players.
- S_i = set of strategies for player i.
- H : set of all paths from root to leaf nodes. S_H is the set of all proper sub-histories.
- $P : S_H \longrightarrow N$ maps nodes to player.
- $\mathbb{I}_i$: set of all information sets of player i.

- $C : \bigcup_{i \in N} \mathbb{I}_i \longrightarrow \bigcup_{i \in N} S_i$, $C(J) \subseteq S_i \ \forall J \in \mathbb{I}_i$

- $u_i : \mathbb{H} \longrightarrow \mathbb{R}$ maps histories (leaf nodes) to the utility of player $i \in N$

- An extensive form game is called a perfect information game if all its information sets are singleton. Otherwise, the game is called an imperfect information game.

Representing Extensive form games as strategic form games.

Given an extensive form game $\Gamma = \langle N, (S_i)_{i\in N}, H, P, (\mathbb{I}_i)_{i\in N}, C, (u_i)_{i\in N} \rangle$, the corresponding normal form game

$\Gamma^s = \langle N^s, (S_i')_{i\in N}, (u_i')_{i\in N} \rangle$ is given by

- $N^s = N$
- $S_i' = \{ s_i : \mathbb{I}_i \to S_i \mid s_i(J) \in C(J) \ \forall J \in \mathbb{I}_i \}$
- $u_i'(s_1, \ldots, s_n)$ is the utility that player i

receives if all the players play according to $(s_1, \ldots, s_n)$.