ENGINEERING MATHEMATICS-I

SYLLABUS

- **UNIT-1** univariate calculus.
- Unit -2 multivariate calculus.
- Unit-3 vector calculus.
- Unit-4 complex number.
- Unit-5 fourier series and fourier transform.

$$\frac{d x^n}{dx}$$

$$\frac{d x^2}{dx} = 2x$$

$$\frac{d 2x}{dx} = 2$$

$$\frac{d \log x}{dx}$$

n^{th}

❖ Leibnitz's theorem:-

= the n'th differential coefficient of the product of two functions is conventially . Evaluated by the use of this theorem, its statement is:-

Theorem:- if u and v are two functions of x, then

$$D^n(u.v) = D^n(u).v + {}^nC_1 D^{n-1}u.D^1v + {}^nC_2 D^{n-2}u.D^2v + \cdots + {}^nC_n D^{n-n}u.D^nv + \cdots$$

$$= {}^nC_0 D^n u . D^0 v$$

$$D^n(u.v) = D^n(u).v + {}^nC_1 D^{n-1}u.D^1v + {}^nC_2 D^{n-2}u.D^2v + \cdots$$

$$\frac{d(u.v)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$x^3 \cos x$

$3x^2$

$6x$

6

Q. $x^3\cos x$, find n'th differential coefficient ?

$$D^n \cos x = \cos\left(x + \frac{n\pi}{2}\right)$$

$$D^n \sin x = \sin\left(x + \frac{n\pi}{2}\right)$$

$$\boxed{D^n e^x = e^x}$$

$$^nC_3 = \frac{\underline{n-3}\,(n-2)(n-1)n}{6 \times \underline{n-3}}$$

$$\frac{\underline{n}}{\underline{2} \times \underline{n-2}} = \frac{\cancel{\underline{n-2}}(n-1)(n)}{\cancel{\underline{n-2}} \times 2} = \frac{(n-1)n}{2}$$

$$x^3 \cos x$$

$$D^n(x^3 \cos x) = D^n \cos x \cdot (x^3) + {}^nC_1 D^{n-1}\cos x \cdot Dx^3 + {}^nC_2 D^{n-2}\cos x \cdot D^2x^3$$

$$+ {}^nC_3 D^{n-3}\cos x \cdot D^3 x^3 + \ldots$$

$$= x^3\cos\left(x + \frac{n\pi}{2}\right) + n\cos\left\{x + (n-1)\frac{\pi}{2}\right\} \times 3x^2 + \frac{n(n-1)}{2}\cos\left(x + (n-2)\frac{\pi}{2}\right) \cdot 6x + \frac{n(n-1)(n-2)}{6}\cos\left(x + (n-3)\frac{\pi}{2}\right) \cdot 6$$

$$^nC_r = \frac{\underline{n}}{\underline{r}\,.\,\underline{n-r}}$$

or

$$\frac{n!}{r!\,(n-r)!}$$

$$\frac{\underline{n}}{\underline{1} \times \underline{n-1}} = \frac{\cancel{\underline{n-1}} \cdot n}{\cancel{\underline{n-1}}} = n$$

$$\underline{4} = 4 \times 3 \times 2 \times 1$$

$$\frac{dx^n}{dx} = \boxed{nx^{n-1}}$$

Q. $f(x) = x^3 \log x$, then find the n'th differential coefficient ?

H.W.

$$f(x) = x^3 \log x$$

$$\underset{V}{x^3} \quad \underset{U}{\log x}$$

$$\boxed{D^n \log x = \frac{(-1)^{n-1}\,\lfloor n-1}{x^n}}$$

$$\boxed{D^n\left(\frac{1}{x}\right) = \frac{(-1)^n\,\lfloor n}{x^{n+1}}}$$

Q. $y = x^2e^x$, prove that $y_n = \frac{1}{2}n(n-1)\frac{d^2y}{dx^2} - n(n-2)\frac{dy}{dx} + \frac{1}{2}(n-1)(n-2)y.$

$$y = x^2 e^x$$

$$\frac{dy}{dx} = \frac{d x^2 e^x}{dx} = x^2 \frac{de^x}{dx} + e^x \frac{dx^2}{dx}$$

$$= x^2 \cdot e^x + e^x \cdot 2x$$

$$\frac{dy}{dx} = x^2 e^x + 2x \cdot e^x \quad \text{—— (i)}$$

$$\frac{dy}{dx} = y + 2x \cdot e^x \quad \text{—— (ii)}$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + 2\frac{d\,xe^x}{dx}$$

$$= \frac{dy}{dx} + 2\{xe^x + e^x\}$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + 2xe^x + 2e^x$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + \frac{dy}{dx} - y + 2e^x$$

$$\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - y + 2e^x \quad \text{—— (iii)}$$

$$y = x^2 e^x$$

$$D^n y = D^n x^2 e^x$$

$$\left(\begin{aligned} D^n y &= y_n \\ D^{n+1} y &= y_{n+1} \end{aligned} \right)$$

$$y_n = D^n e^x \cdot x^2 + {}^nC_1 D^{n-1} e^x \, D x^2 + {}^nC_2 D^{n-2} e^x D^2 x^2$$

$$y_n = e^x \cdot x^2 + n \left(e^x \cdot 2x \right) + \frac{n(n-1)}{2} e^x \cancel{2}$$

$$y_n = y + n\left(\frac{dy}{dx} - y\right) + \frac{n(n-1)}{2}\left(\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y\right)$$

$$y_n = y + n\frac{dy}{dx} - ny + \frac{n(n-1)}{2}\frac{d^2y}{dx^2} - n(n-1)\frac{dy}{dx} + \frac{n(n-1)}{2} y$$

$$y_n = \frac{n(n-1)}{2}\frac{d^2y}{dx^2} + (n - n^2 + n)\frac{dy}{dx} + y\left(\frac{n^2}{2} - \frac{n}{2} - n + 1\right)$$

$$y_n = \frac{n(n-1)}{2}\frac{d^2y}{dx^2} - n(n-2)\frac{dy}{dx} + y\left\{ n\left(\frac{n}{2} - 1\right) - 1\left(\frac{n}{2} - 1\right)\right\}$$

$$y_n = \frac{n(n-1)}{2}\frac{d^2y}{dx^2} + n(n-2)\frac{dz}{dx} + z\left\{\left(\frac{n}{2}-1\right)(n-1)\right\}$$

$$y_n = \frac{n(n-1)}{2}\frac{d^2z}{dx^2} - n(n-2)\frac{dz}{dx} + \frac{1}{2}(n-1)(n-1)\, z$$

Proved

❖ maclaurin's theorem:-

= if f(x) is a function of x such that it can be expanded in accending power of x. and this expansion be differentiable any no. of times, then

$$F(x) = f(0)+x/1!f'(0) + x^2/2!f''(0)+\ldots\ldots\ldots\ldots$$

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \ldots$$

Q. $f(x) = e^x$

If $x = 0$

$f(x) = e^x$

$f'(x) = e^x$

$f''(x) = e^x$

$f'''(x) = e^x$

$f(0) = e^0 = 1$

$f'(0) = e^0 = 1$

$f''(x) = e^0 = 1$

$f'''(x) = e^0 = 1$

$$f(x) = f(0) + \frac{x^1}{\lfloor 1} f'(0) + \frac{x^2}{\lfloor 2} f''(0) + \text{---}$$

$$= 1 + \frac{x}{1} \times 1 + \frac{x^2}{2} \times 1 + \text{---}$$

$$= 1 + x + \frac{x^2}{2} + \text{----}$$

Q. $f(x) = \cos x,$

$f(x) = \cos x$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f'''(x) = \sin x$

$f(0) = \cos 0 = 1$

$f'(0) = 0$

$f''(0) = -1$

$f'''(0) = 0$

$$f(x) = f(0) + \frac{x}{\underline{|1}} f'(0) + \frac{x^2}{\underline{|2}} f''(0) + \cdots$$

$$= 1 + x \times 0 + \frac{x^2}{2} \times -1 + \cdots$$

$$= 1 - \frac{x^2}{2} + \cdots$$

❖ Taylor's theorem :-

$\frac{d \log \sin x}{d \sin x} \times \frac{d \sin x}{dx} = \frac{1}{\sin x} \times \cos x = \cot x$

= if f(a+h) [where a is independent of h] be a function of this variable h. such that it can be expanded in accending power of h and this expansion be differentiable of any no. of times , then

$F(a+h) = f(a)+f'(a).h^1/1! + f''(a)\,h^2/2!+f'''(a)\,h^3/3!+\ldots\ldots$

Q. f(x)=log(sinx), in power of (x-2)

$f(x) = \log \sin x$

$f(2 + x-2) =$

$f'(x) = \cot x$

$f''(x) = -\operatorname{cosec}^2 x$

$f'''(x) = 2\operatorname{cosec} x \cdot \operatorname{cosec} x \cdot \cot x$

$= 2\operatorname{cosec}^2 x \cdot \cot x$

$f(2)$

$$f(x) = \log \sin x$$

$$f(2) = \log \sin 2$$

$$f'(x) = \cot x$$

$$f'(2) = \cot 2$$

$$f''(x) = -\operatorname{cosec}^2 x$$

$$f''(2) = -\operatorname{cosec}^2 2$$

$$f'''(x) = 2\operatorname{cosec}^2 x \cdot \cot x$$

$$f'''(2) = 2\operatorname{cosec}^2 2 \cdot \cot 2$$

$$f(x) = f(a) + \frac{f'(x) \cdot h^1}{\lfloor 1} + \frac{f''(x) \cdot h^2}{\lfloor 2} + \text{—}$$

$$f(2 + x - 2) = \log \sin 2 + \frac{\cot 2 \, (x-2)^1}{\lfloor 1} + \frac{\operatorname{cosec}^2 2 \, (x-2)^2}{\lfloor 2}$$

$$+ \frac{\quad}{\quad}$$

THANK YOU