Lecture 1.3

Ex: Tragedy of Commons:

$$(s_1, \dots, s_i, \dots, s_n) \equiv (s_i, s_{-i})$$

- $N = \{1, 2, \dots, n\}$
- $S_i = \{0, 1\}, \ \forall i \in N$
- Utility: $u_i(s_1, \dots, s_i, \dots, s_n) = u_i(s_i, s_{-i})$

$$= s_i - \left[\frac{5(s_1 + s_2 + \dots + s_n)}{n}\right]$$

Ex: (Auction)

- Players: $N = \{1, 2, \dots, n\}$ (n sellers)
- Strategy set: $S_i = \mathbb{R}_{\geq 0}$, $\forall i \in N$
- Valuation of the item to player i: $v_i \in \mathbb{R}_{\geq 0}$, $\forall i \in N$
- Allocation function. $a : \bigtimes_{i \in N} S_i \longrightarrow \mathbb{R}^N$

  $(s_1, \dots, s_n) \longmapsto (a_1, \dots, a_n)$ $\quad a_i = \begin{cases} 1 & \text{if } i \text{ "wins"} \\ 0 & \text{o/w.} \end{cases}$

- Payment $p : \bigtimes_{i \in N} S_i \longrightarrow \mathbb{R}^N$

  $(s_1, \dots, s_n) \longmapsto (p_1, \dots, p_n)$ $\quad p_i$ = money received by player i.

First price payment rule:

$$p_i = \begin{cases} b_i & \text{if } a_i = 1 \\ 0 & \text{o/w} \end{cases}$$

Second price payment rule:

$$p_i = \begin{cases} \min\limits_{\substack{j \neq i \\ j \in N}} b_j & \text{if } a_i = 1 \\ 0 & \text{o/w.} \end{cases}$$

Utility function:

$$u_i(b_i, b_{-i}) = a_i(p_i - v_i)$$

Dominant Strategy Equilibrium

	c	nc
c	-5, -5	-1, -10
nc	-10, -1	-2, -2

irrespective of what other player players, playing "c" is strictly always better. "c" is called a strongly dominant strategy.

Def^n. In a normal form game $\Gamma = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$, a strategy $s_i^* \in S_i$ is called a strongly dominant strategy for player i if

$$u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_{-i} \in S_{-i} \left(= \bigtimes_{\substack{j \in N \\ j \neq i}} S_j\right), \forall s_i \in S_i, s_i \neq s_i^*$$

Strongly dominant strategy equilibrium (SDSE)

Defn. Given $\Gamma = \langle N, (S_i)_{i\in N}, (u_i)_{i\in N} \rangle$ a strategy profile $(s_i^*)_{i\in N} \in S \left(:= \bigtimes_{i\in N} S_i \right)$ is called a strongly dominant strategy equilibrium if each s_i^* is a strongly dominant strategy for player i.

Ex: (C, C) is a SDSE for the prisoner's dilemma game.