Lecture 4.5

Theorem: In an atomic network congestion game, suppose the following holds:

i) All players have the same source and destination.

ii) Cost functions have α-bounded jump.

iii) Max-gain version of ε-best response dynamic is used.

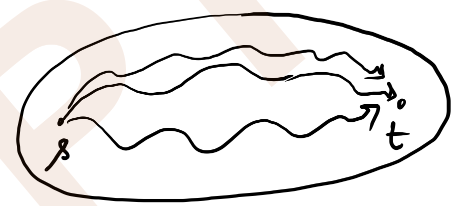
Then an ε-PSNE is reached in $O\left(\frac{n\alpha}{\varepsilon} \log \frac{\Phi(s^0)}{\Phi(s^{min})}\right)$ iterations.

Corollary: For an atomic network congestion game satisfying $(\star)$, an ε-PSNE can be computed in

$$O\left(\frac{n\alpha}{\varepsilon}\log\frac{\Phi(s^0)}{\Phi(s^{min})}\cdot \mathrm{poly}(N, m, n)\right)$$

Proof: Enough to show: each iterations of the max-gain version of ε-best response dynamics can be executed in $\mathrm{poly}(N, m, n)$ time.

- Let us fix a player i and a strategy profile s'.



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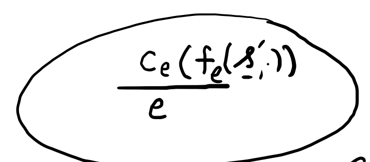
- Consider s'_{-i}, define weight of an edge e to be $c_e(f_e(s'_{-i}))$.



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Find the shortest $s-t$ path p.

- Let the costs of s_i and p be c_i' and c_p.
- If $c_p > (1-\varepsilon) c_i'$, then player i does not have

any ε-move.

- The absolute reduction in cost for player i is

  $c_i' - c_p$.

- Since single-source shortest weight path can be computed in polynomial time, the result follows. $\blacksquare$

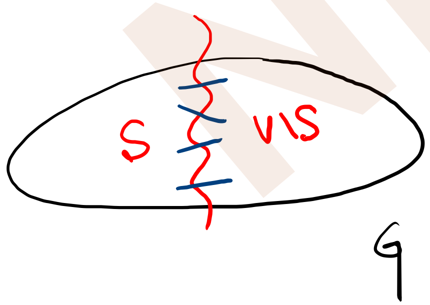
Qn: Can we relax the conditions in ☆?

Qn: Can the result be generalized to congestion games?

Local Search

A local search problem is to find a local minima for an optimization problem.

The canonical local search problem is weighted maximum-cut.

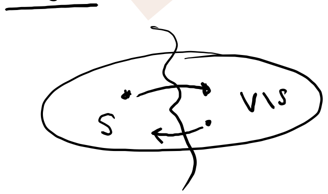


$S \neq \phi, S \neq V$

The weight of a cut is the sum of the weights of the cut edges (edges having one end point in S and another in V\S).

Local maximum cut

A cut whose size can not be increased further by moving any "single" vertex from its current set.



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i.e. no improvement possible by local moves.

Canonical NP problem is SAT.