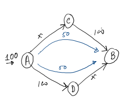
Lecture 3.3

Braess paradox:



Set of players: $\{1, \dots, 100\}$

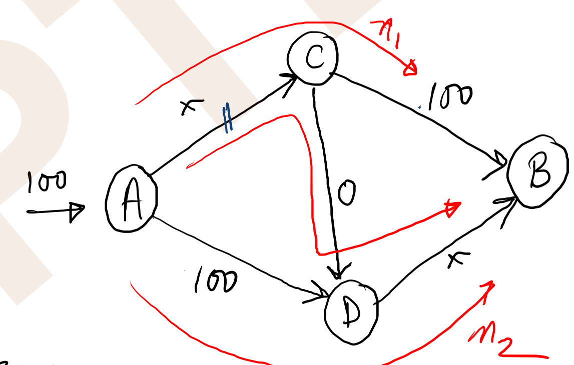
Set of strategies:

$\{A \to C \to B, A \to D \to B\}$

50 people using $A \to C \to B$ and the other 50 people using $A \to D \to B$ is a pure strategy Nash equilibrium.

Average delay of each player = 150

Strategy set = $\{A \to C \to B,\ A \to D \to B,\ A \to C \to D \to B\}$



<u>claim:</u> $A \to C \to D \to B$ is a weakly dominated strategy

→ The delay of a player i using $A \to C \to D \to B$ is $(n-n_2)+(n-n_1)$

$= 2n - (n_1+n_2) = 200 - (n_1+n_2)$

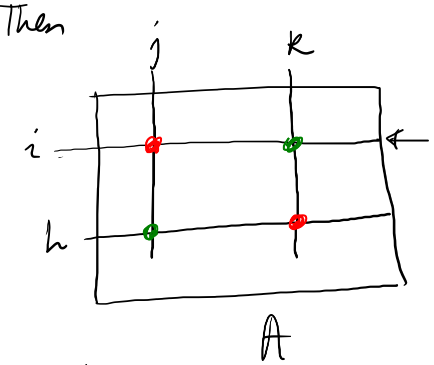
→ Suppose player i deviates to $A \to C \to D$. Then its utility is

$= (n-n_2) + 100 = 200 - n_2 \geq 200 - (n_1+n_2)$

All players playing $A \to C \to D \to B$ is a WDSE.

Average delay $= 200$

<u>Lemma:</u> Let $A \in \mathbb{R}^{m \times n}$ be a matrix. Let (i,j) and (h,k) be two PSNEs of the matrix game A. Then (h,j) and (i,k) are also PSNEs.



<u>Proof:</u> $\underline{A[h,j]} \leq A[i,j] \leq A[i,k] \leq A[h,k] \leq \underline{A[h,j]}$

$\blacksquare$