Single Parameter Domain

Lecture 11.4

The allocation rules implementable in DSE in the single parameter domain are monotone allocation rule.

Recall: The allocation rules implementable in DSE in quasi-linear environment (Groves Theorem and Roberts Theorem) are the affine maximizers.

Example of affine maximizer:

$$k(\theta_1, \dots, \theta_n) = \underset{k \in \mathcal{R}}{\operatorname{argmax}} \sum_{i=1}^{n} a_i v_i(k, \theta) + c$$

$$a_i \geq 0, \quad c \in \mathbb{R}$$

Monotone

$\boxed{\text{Affine maximizer}}$

Examples of monotone allocation rules which are not affine maximizer:-

$$k(\theta_1, \dots, \theta_n) = \underset{k \in \mathcal{R}}{\operatorname{argmax}} \sum_{i=1}^{n} a_i v_i(k, \theta_i)^{\lambda_i} + c$$

$$a_i, \lambda_i \in \mathbb{R}_{\geq 0}, \quad c \in \mathbb{R}.$$

Recall, we have worked with a restricted version of single parameter domain.

Mayerson's Lemma: We have the following in any single-parameter domain.

(i) An allocation rule $k : \Theta \to \mathcal{R}$ is DSIC if and only if $k(\cdot)$ is monotone in each θ_i.

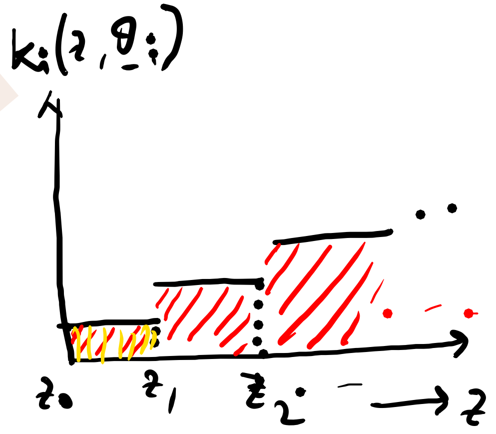
(ii) If $k(\cdot)$ is monotone, there there exist unique payment rules $t_1(\cdot), \ldots, t_n(\cdot)$ where players reporting 0 as their type do <u>not</u> pay anything such that the mechanism $f(\cdot) = (k(\cdot), t_1(\cdot), \ldots, t_n(\cdot))$ is DSIC.

(iii) The payment rule of part (ii) is given by the following explicit formula.

$$t_i(\theta_i, \underline{\theta}_{-i}) = -\int_0^{\theta_i} z \cdot \frac{d}{dz} k_i(z, \theta_{-i})\, dz.$$

where $k(\cdot) = (k_1(\cdot), \ldots, k_n(\cdot))$, $k_i(\cdot)$ is differentiable in its domain.

If $k_i(\cdot)$ is a step function having jumps at $z_0, z_1, z_2, \ldots$



$$t_i(\theta_i,\theta_{-i}) = k_i(z_0,\theta_{-i})\cdot(z_1-z_0) + k_i(z_1,\theta_{-i})\cdot(z_2-z_1) + \cdots$$

Uniqueness of VCG Payment Rule: Assume Θ_i is connected in some Euclidean space for every $i \in [n]$. Let $f(\cdot) = (k(\cdot), t_1(\cdot), \ldots, t_n(\cdot))$ be DSIC. If $f'(\cdot) = (k(\cdot), t_1'(\cdot), \ldots, t_n'(\cdot))$ is also DSIC, then

$$\forall \theta \in \Theta, \quad t_i'(\theta) = t_i(\theta) + h_i(\theta_{-i})$$

for some function $h_i : \Theta_{-i} \longrightarrow \mathbb{R}$.