(1973) (1977)

Gibbard-Satterwaite Theorem

Lecture 9.4

For any player $i \in [n]$ and any type $\theta_i \in \Theta_i$, we get a partial order R_i on the set $\mathcal{X}$ of outcomes.

$x, y \in \mathcal{X}$, $\quad x \, R_i^{\theta_i} \, y \overset{def^n}{\equiv} u_i(x, \theta_i) \geq u_i(y, \theta_i)$

$R_i^{\theta_i}$ is called the rational preference relation of player i when its type is θ_i.

We call a preference relation to be strict if that rational preference relation is a linear order/complete order. We denote the set of all possible strict rational preference relations on $\mathcal{X}$ by $\mathcal{L}(\mathcal{X})$.

<u>Recall:</u> $f : \underset{i \in [n]}{\times} \Theta_i \longrightarrow \mathcal{X}$

<u>Social choice function</u> (G.S Theorem) $f : \mathcal{L}(\mathcal{X})^n \longrightarrow \mathcal{X}$

Unanimity: A social choice function $f: \mathcal{L}(X)^n \to X$ is called unanimous if

$\forall\, P_1, \ldots, P_n \in \mathcal{L}(X)$ such that the best outcomes in all $P_1, \ldots, P_n$ are the same, say x.

we have $\quad f(P_1, \ldots, P_n) = x$

Ex-post efficiency $\Rightarrow$ unanimity.

Theorem: Let $f: \mathcal{L}(\mathcal{X})^n \longrightarrow \mathcal{X}$ be a social choice function such that

(i) We have at least 3 outcomes. That is $|\mathcal{X}| \geq 3$.

(ii) f is unanimous.

(iii) Every player has a strict rational preference relation.

Then f is dominant strategy incentive compatible if and only if f is a dictatorship.

Way-Outs from GS Impossibility

(1) Assume "more structure" on outcomes and "more structure" on the utility functions of the players.

— Quasilinear setting.

(2) Be satisfies with Bayesian incentive compatibility.

(3) "Computational barrier" can the manipulation problem be NP-hard?