Algorithmic Game Theory
Assignment 9

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1. Suppose we have at least 2 players. Let $\mathcal{X}$ be the set of social choice functions implementable by a direct mechanism in dominant strategy equilibrium and $\mathcal{Y}$ the set of social choice functions implementable by a indirect mechanism in dominant strategy equilibrium. Then, which one of the following is true?

   (a) $\mathcal{X} \subsetneq \mathcal{Y}$
   (b) $\mathcal{Y} \subsetneq \mathcal{X}$
   (c) $\mathcal{X} = \mathcal{Y}$
   (d) $\mathcal{X} \cap \mathcal{Y} = \emptyset$

   The correct answer is (c). Refer to Lecture 9.2.

2. Suppose we have at least 2 players. Let $\mathcal{X}$ be the set of social choice functions implementable by a direct mechanism in Bayesian Nash equilibrium and $\mathcal{Y}$ the set of social choice functions implementable by a indirect mechanism in Bayesian Nash equilibrium. Then, which one of the following is true?

   (a) $\mathcal{X} \subsetneq \mathcal{Y}$
   (b) $\mathcal{Y} \subsetneq \mathcal{X}$
   (c) $\mathcal{X} = \mathcal{Y}$
   (d) $\mathcal{X} \cap \mathcal{Y} = \emptyset$

   The correct answer is (c). Refer to Lecture 9.2.

3. Suppose we have at least 2 players. Let $\mathcal{X}$ be the set of social choice functions implementable by an indirect mechanism in dominant strategy equilibrium and $\mathcal{Y}$ the set of social choice functions implementable by a indirect mechanism in Bayesian Nash equilibrium. Then, which one of the following is true?

   (a) $\mathcal{X} \subsetneq \mathcal{Y}$
   (b) $\mathcal{Y} \subsetneq \mathcal{X}$
   (c) $\mathcal{X} = \mathcal{Y}$
   (d) $\mathcal{X} \cap \mathcal{Y} = \emptyset$

   The correct answer is (a). Refer to Lecture 9.1.

4. Suppose we have at least 2 players. Let $\mathcal{X}$ be the set of social choice functions implementable by an indirect mechanism in dominant strategy equilibrium and $\mathcal{Y}$ the set of social choice functions implementable by a direct mechanism in Bayesian Nash equilibrium. Then, which one of the following is true?

   (a) $\mathcal{X} \subsetneq \mathcal{Y}$
   (b) $\mathcal{Y} \subsetneq \mathcal{X}$
   (c) $\mathcal{X} = \mathcal{Y}$
   (d) $\mathcal{X} \cap \mathcal{Y} = \emptyset$

The correct answer is (a). Refer to Lecture 9.1 and 9.2.

5. Suppose we have at least 2 players. Let $\mathcal{X}$, $\mathcal{Y}$, and $\mathcal{Z}$ be the set of all social choice functions which are respectively ex-post individually rational, interim individually rational, and ex-ante individually rational. Then, which one of the following is true?

   (a) $\mathcal{X} \subsetneq \mathcal{Y} \subsetneq \mathcal{Z}$
   (b) $\mathcal{X} \supsetneq \mathcal{Y} \supsetneq \mathcal{Z}$
   (c) $\mathcal{X} = \mathcal{Y} = \mathcal{Z}$
   (d) $\mathcal{X} \cap \mathcal{Z} = \mathcal{Y}$

   The correct answer is (a). Refer to Lecture 9.3.

6. Suppose we have at least 2 players. Let $\mathcal{X}$ and $\mathcal{Y}$ be the set of all respectively uninanimous and onto social choice functions. Then, which one of the following is true?

   (a) $\mathcal{X} \supsetneq \mathcal{Y}$
   (b) $\mathcal{X} = \mathcal{Y}$
   (c) $\mathcal{X} \subsetneq \mathcal{Y}$
   (d) $\mathcal{X} \cap \mathcal{Y} = \emptyset$

   The correct answer is (c). Refer to Lecture 9.4.

7. Suppose we have at least 2 players. Let $\mathcal{X}$ and $\mathcal{Y}$ be the set of all respectively uninanimous and ex-post efficient social choice functions. Then, which one of the following is true?

   (a) $\mathcal{X} \supsetneq \mathcal{Y}$
   (b) $\mathcal{X} = \mathcal{Y}$
   (c) $\mathcal{X} \subsetneq \mathcal{Y}$
   (d) $\mathcal{X} \cap \mathcal{Y} = \emptyset$

   The correct answer is (c). Refer to Lecture 9.4.

8. Suppose we have at least 2 players. Let $\mathcal{X}$ and $\mathcal{Y}$ be the set of all respectively dictatorship and ex-post efficient social choice functions. Then, which one of the following is true?

   (a) $\mathcal{X} \supsetneq \mathcal{Y}$
   (b) $\mathcal{X} = \mathcal{Y}$
   (c) $\mathcal{X} \subsetneq \mathcal{Y}$
   (d) $\mathcal{X} \cap \mathcal{Y} = \emptyset$

   The correct answer is (d). Refer to Lecture 9.4.

9. Suppose we have at least 2 players. Let $\mathcal{X}$ and $\mathcal{Y}$ be the set of all respectively dictatorship and unanimous social choice functions. Then, which one of the following is true?

   (a) $\mathcal{X} \supsetneq \mathcal{Y}$
   (b) $\mathcal{X} = \mathcal{Y}$
   (c) $\mathcal{X} \subsetneq \mathcal{Y}$
   (d) $\mathcal{X} \cap \mathcal{Y} = \emptyset$

   The correct answer is (d). Refer to Lecture 9.4.

10. Suppose we have only 1 player. Let $\mathcal{X}$ and $\mathcal{Y}$ be the set of all respectively dictatorship and ex-post efficient social choice functions. Then, which one of the following is true?

    (a) $\mathcal{X} \supsetneq \mathcal{Y}$
    (b) $\mathcal{X} = \mathcal{Y}$
    (c) $\mathcal{X} \subsetneq \mathcal{Y}$
    (d) $\mathcal{X} \cap \mathcal{Y} = \emptyset$

    The correct answer is (b). Refer to Lecture 9.4.