Lecture 2.2

Two Person Zero Sum Game

Matching pennies:

	A	B
A	1, −1	−1, 1
B	−1, +1	1, −1

Rock-paper-Scissor:

	Rock	Paper	Scissor
Rock	0, 0	−1, +1	1, −1
Paper	1, −1	0, 0	−1, 1
Scissor	−1, 1	1, −1	0, 0

Strictly competitive games / Win-loss game / matrix game

Security of a player:

Unique PSNE : (B,B)

	A	B	max
A	2,2	2.5,1	2
B	-100,2	3,3	-100

maximum utility that a player can guarantee without assuming anything about other players.

Security level of the row player is 2.

"Reasoning" from security level of the row player, we can predict that players will play (A, A) which is not any equilibrium we have seen so far.

Security of a player in pure strategies.

Def$^{\underline{n}}$: $\Gamma = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$

security level of $\underbrace{\text{player } i \text{ in pure strategies}}$ or value

$$v_i = \max_{s_i \in S_i} \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$$

maxmin value of player i in pure strategies.

The value of both the players in pure strategies in the matching pennies game is -1.

Security level in mixed strategies

$\Gamma = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$

$$v_i = \sup_{\sigma_i \in \Delta(S_i)} \inf_{\sigma_{-i} \in \times_{j \in N, j \neq i} \Delta(S_j)} u_i(\sigma_i, \sigma_{-i})$$

maxmin value in mixed strategies.

Value of both the players in mixed strategies in matching pennies game is 0.

Obs: $$v_i = \sup_{\sigma_i \in \Delta(S_i)} \min_{s_{-i} \in S_{-i}} u_i(\sigma_i, s_{-i})$$

Proof: Follows from maximization of a convex combination

Theorem: $\Gamma = \langle N, (S_i)_{i\in N}, (u_i)_{i\in N} \rangle$. Let $(\sigma_i^*)_{i\in N}$ be an MSNE. Then,

$\forall i \in N, \quad u_i((\sigma_i^*)_{i\in N}) \geqslant v_i$

Proof: Follows immediately from the definitions. ∎