Lecture 5.1

Polynomial Local Search (PLS)

An abstract problem in PLS complexity class is defined by the following three algorithms.

i) An algorithm to pick an initial solution.

ii) An algorithm to compute the value of a solution.

iii) An algorithm to determine if a solution is a local optimal or executes a "local move" which improves the value of the solution.

PLS = the set of all abstract search problems.

<u>Observation:</u> The problem finding a PSNE in a congestion game belongs to PLS. More generally, the PSNE problem for finite potential games is in PLS.

Reduction in PLS class: A PLS problem P_1 reduces to another PLS problem P_2 in polynomial time if we have the following two algorithms.

$P_1 \leq P_2$

(i) An algorithm A to construct from every instance x of P_1 to an instance $A(x)$ of P_2

(ii) Another algorithm B to construct a solution of x from a solution of $A(x)$.

Fact: Local weighted maximum cut is PLS-complete.

Theorem: PSNE problem for congestion games is PLS-complete.

Proof: Need to show:

i) membership in PLS ←

ii) PLS-hardness.

Since congestion games are finite potential games, PSNE for ______ belongs to PLS.

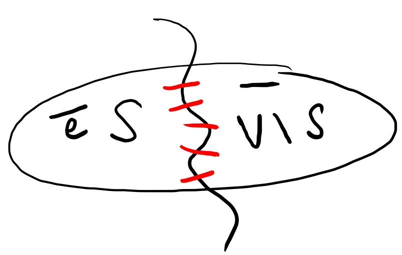
To show PLS-hardness, we reduce from local weighted max-cut.

Let $(G=(V,E), w)$ be any instance of local weighted max-cut.

- Set of players (N) : V.

- Set of resources : $\{ r_e , \bar{r}_e : e \in E \}$
- Strategy set of player v : $\{ \{ r_e : e \in \delta(v) \} , \{ \bar{r}_e : e \in \delta(v) \} \}$
- Cost of r_e or $\bar{r}_e$: 0 if at most one player uses it ; $w(e)$ otherwise.

Recall, $\Phi(s) = \sum_{e \in E} \sum_{i=1}^{f(e)} c_e(i)$.



$\Rightarrow$ Players in S play $\{ r_e : e \in \delta(v) \}$

" " $V \setminus S$ " $\{ \bar{r}_e : e \in \delta(v) \}$

A cut of weight $w(S,\bar{S})$ corresponds to a strategy profile of potential $\sum_{e\in E} w(e) - w(S, V\setminus S)$.

Locally maximum cut $\longleftrightarrow$ local minimum of potential function.

Given a PSNE, if player v plays $\{\bar{r_e} : e \in E\}$, then put v in $V\setminus S$, otherwise put v in S. $\blacksquare$