Algorithmic Game Theory
Assignment 3

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1. How many MSNEs are there in the following normal form game?

   ▷ The set of players $(N) : \{1, 2\}$

   ▷ The set of strategies: $S_1 = \{A, B\}, S_2 = \{A, B, C\}$

   ▷ Payoff matrix:

		Player 2		
		A	B	C
Player 1	A	$(5, 0)$	$(5, 4)$	$(0, 3)$
	B	$(0, 4)$	$(0, 3)$	$(5, 2)$

   (a) 0
   (b) 1
   (c) 2
   (d) infinite

   The correct answer is (b).

2. How many MSNEs are there in the following normal form game?

   ▷ The set of players $(N) : \{1, 2\}$

   ▷ The set of strategies: $S_i = \{A, B, C\}$ for $i \in \{1, 2\}$

   ▷ Payoff matrix:

		Player 2		
		A	B	C
Player 1	A	$(1, 7)$	$(1, 1)$	$(7, 0)$
	B	$(5, 3)$	$(6, 4)$	$(5, 1)$
	C	$(3, 0)$	$(6, 5)$	$(6, 0)$

   (a) 0
   (b) 1
   (c) 2
   (d) infinite

   The correct answer is (d).
   Justification: Apply iterated elimination of strongly dominated strategy

3. In a normal form game, suppose the row and column players have m and n strategies respectively. What is the running time of the support enumeration algorithm on this game?

   (a) $\mathcal{O}(2^m + 2^n)$
   (b) $(2^m + 2^n)(m + n)^{\mathcal{O}(1)}$
   (c) $\mathcal{O}(2^{m+n})$
   (d) $2^{m+n}(m + n)^{\mathcal{O}(1)}$

   The correct answer is (d).

4. Yao's lemma is used to

   (a) upper bound the worst case expected running time of any algorithm for a problem
   (b) upper bound the average case running time of any algorithm for a problem
   (c) lower bound the worst case expected running time of any algorithm for a problem
   (d) lower bound the average case running time of any algorithm for a problem

   The correct answer is (c).
   Justification: Refer to week-3 Lecture-4

5. Which of the following values of x make the strategy profile (C, C) a strongly dominant strategy equilibrium of the game below?

   ▷ The set of players $(N) : \{1, 2\}$
   ▷ The set of strategies: $S_i = \{A, B\}$ for every $i \in [2]$
   ▷ Payoff matrix:

		Player 2	
		NC	C
Player 1	NC	$(-4, -4)$	$(-2, x)$
	C	$(x, -2)$	(x, x)

   (a) 0
   (b) -2
   (c) -4
   (d) -6

   The correct answer is (a).

6. Which of the following values of x make the strategy profile (C, C) a weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium of the game below?

   ▷ The set of players $(N) : \{1, 2\}$
   ▷ The set of strategies: $S_i = \{A, B\}$ for every $i \in [2]$
   ▷ Payoff matrix:

		Player 2	
		NC	C
Player 1	NC	$(-4, -4)$	$(-2, x)$
	C	$(x, -2)$	(x, x)

   (a) 0
   (b) -2
   (c) -4
   (d) -6

   The correct answer is (b).

7. Which of the following values of x make the strategy profile (C, C) not even a weakly dominant strategy equilibrium of the game below?

   ▷ The set of players $(N) : \{1, 2\}$
   ▷ The set of strategies: $S_i = \{A, B\}$ for every $i \in [2]$
   ▷ Payoff matrix:

		Player 2	
		NC	C
Player 1	NC	$(-4, -4)$	$(-2, x)$
	C	$(x, -2)$	(x, x)

   (a) 0

(b) -1

(c) -2

(d) -3

The correct answer is (d).

8. What type of equilibrium the game in Braess's paradox (with cost function and other details as discussed in the lecture) has after adding the high-capacity edge?

   (a) strongly dominant strategy equilibrium

   (b) weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium

   (c) very weakly dominant strategy equilibrium but not a weakly dominant strategy equilibrium

   (d) pure strategy Nash equilibrium but not a very weakly dominant strategy equilibrium

   The correct answer is (b).

9. There are n players. Each player announces a number in the set $\{1, 2, \ldots, 150\}$. A prize of 1 is split equally between all the people whose number is closest to $\frac{2}{5}$ of the average number. How many MSNEs this normal form game has?

   (a) 0

   (b) 1

   (c) 2

   (d) infinite

   The correct answer is (b).
   Justification: Apply iterated elimination of strongly dominated strategy.

10. Consider the following two player normal form game.

    ▷ $S_1 = \{x \in \mathbb{R} : 0 \leqslant x \leqslant 2\}, S_2 = \{x \in \mathbb{R} : 4 \leqslant x \leqslant 6\}$

    ▷ $u_1(x, y) = -u_2(x, y) = |x - y|$ for every $x \in S_1$ and $y \in S_2$

    Which one of the following is an MSNE of the above game?

    (a) $(0, 4)$

    (b) (uniform distribution over S_1, uniform distribution over S_2)

    (c) $(0, 6)$

    (d) (truncated normal distribution over S_1, truncated normal distribution over S_2)

    The correct answer is (a).