Lecture 4.2

Succinct Games

Graphical games, sparse games, symmetric games, anonymous games, network congestion games.

6. Congestion game: Generalization of network congestion game. We have a set $\mathcal{E}$ of resources. A strategy is a subset of $\mathcal{E}$. A strategy set of a player is a subset of power set of $\mathcal{E}$.

Load of a resource e is the number of players in a strategy profile using that resource.

Each resource has a non-decreasing cost function.

The utility of a player in a strategy profile is the sum of the cost of the resources it is using.

7. <u>Multi-matrix game:</u> For each $(i,j) \in [n] \times [n]$, we have
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad i \neq j$

a $m \times m$ matrix A^{ij}

The utility of player i in a strategy profile $(s_j)_{j \in [n]}$,

is $\displaystyle\sum_{\substack{j=1 \\ j \neq i}}^{n} A^{ij}_{(s_i, s_j)}$

Potential Game

A game is called a potential game if it has a "potential function".

Theorem: (Rosenthal) Every network congestion game has at least one PSNE.

Proof: Let f be a flow in G.

$$f : E(G) \longrightarrow \mathbb{N}$$ associated with strategy profiles.

"potential function maps strategy profiles to real numbers."

$$\Phi(f) = \sum_{e \in E[G]} \sum_{i=1}^{f(e)} c_e(i)$$

If there is a player i who can reduce its cost by changing its path from P_i to P_i', then the reduction in cost of player i is the same as the reduction in potential value.

f be the flow corresponding to (P_i, P_{-i})

$\hat{f}$ —————————— (P_i', P_{-i})

$$\Phi(\hat{f}) - \Phi(f) = \sum_{e \in P_i' \setminus P_i} c_e(\hat{f}_e) - \sum_{e \in P_i \setminus P_i'} c_e(f_e)$$

$$= \underbrace{\sum_{e \in P_i'} c_e(\hat{f}_e) - \sum_{e \in P_i} c_e(f_e)}$$

decrease in cost for player i by deviating unilaterally from P_i to P_i'.

$\cancel{\notin P_i, P_i'}$ $\quad \overline{\in P_i, \notin P_i'}$

$\overline{\in P_i', \notin P_i}$ $\quad \overline{\in P_i, \in P_i'}$

G

The domain of Φ is a finite set. So it attains a minimum value. Cleary, the pure strategy profile corresponding to the minimum value is a PSNE. $\blacksquare$

Potential Game: A game $\Gamma = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$ is called a potential game if there exists a function Φ s.t.

$$\forall i \in N \ \forall s_i, s_i' \in S_i, s_{-i} \in S_{-i}, \quad \Phi(s_i, s_{-i}) - \Phi(s_i', s_{-i}) = u_i(s_i, s_{-i}) - u_i(s_i', s_{-i}).$$

$\blacksquare$