Algorithmic Game Theory
Assignment 2

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1. Which one of the following is an MSNE of the rock-paper-scissor game?

   (a) $\left(\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right)$
   (b) $\left(\left(\frac{1}{2},\frac{1}{2},0\right),\left(0,\frac{1}{2},\frac{1}{2}\right)\right)$
   (c) $\left(\left(0,\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{2},\frac{1}{2},0\right)\right)$
   (d) $\left(\left(\frac{1}{2},0,\frac{1}{2}\right),\left(\frac{1}{2},0,\frac{1}{2}\right)\right)$

   The correct answer is (a).

2. Consider the following battle of sexes game.

   ▷ The set of players $(N):\{1,2\}$
   ▷ The set of strategies: $S_i=\{A,B\}$ for every $i\in[2]$
   ▷ Payoff matrix:

		Player 2	
		A	B
Player 1	A	$(1,2)$	$(0,0)$
	B	$(0,0)$	$(2,1)$

   Which one of the following is an MSNE of the above normal form game?

   (a) $\left(\{A:\frac{1}{2},B:\frac{1}{2}\},\{A:\frac{1}{2},B:\frac{1}{2}\}\right)$
   (b) $\left(\{A:\frac{1}{3},B:\frac{2}{3}\},\{A:\frac{2}{3},B:\frac{1}{3}\}\right)$
   (c) $\left(\{A:\frac{2}{3},B:\frac{1}{3}\},\{A:\frac{1}{3},B:\frac{2}{3}\}\right)$
   (d) there is no MSNE

   The correct answer is (b).

3. Consider the following coordination game.

   ▷ The set of players $(N):\{1,2\}$
   ▷ The set of strategies: $S_i=\{A,B\}$ for every $i\in[2]$
   ▷ Payoff matrix:

		Player 2	
		A	B
Player 1	A	$(10,10)$	$(0,0)$
	B	$(0,0)$	$(1,1)$

   (a) $\left(\{A:\frac{1}{2},B:\frac{1}{2}\},\{A:\frac{1}{2},B:\frac{1}{2}\}\right)$
   (b) $\left(\{A:\frac{10}{11},B:\frac{1}{11}\},\{A:\frac{10}{11},B:\frac{1}{11}\}\right)$
   (c) $\left(\{A:\frac{1}{11},B:\frac{10}{11}\},\{A:\frac{1}{11},B:\frac{10}{11}\}\right)$
   (d) there is no MSNE

   The correct answer is (c).

4. Consider the following battle of sexes game.

- ▷ The set of players $(N) : \{1, 2\}$
- ▷ The set of strategies: $S_i = \{A, B\}$ for every $i \in [2]$
- ▷ Payoff matrix:

		Player 2	
		A	B
Player 1	A	(1, 2)	(0, 0)
	B	(0, 0)	(2, 1)

What is the security of the row player in pure strategies?

(a) 0
(b) 1
(c) 2
(d) since the game is not a zero-sum game, the concept of security does not make any sense.

The correct answer is (a).

5. Consider the following battle of sexes game.

- ▷ The set of players $(N) : \{1, 2\}$
- ▷ The set of strategies: $S_i = \{A, B\}$ for every $i \in [2]$
- ▷ Payoff matrix:

		Player 2	
		A	B
Player 1	A	(1, 2)	(0, 0)
	B	(0, 0)	(2, 1)

What is the security of the row player in mixed strategies?

(a) 0
(b) 1
(c) 1.5
(d) since the game is not a zero-sum game, the concept of security does not make any sense.

The correct answer is (c).
Justification: Refer to week-2 lecture-2

6. In a two-player normal form game Γ, let (σ_1^*, σ_2^*) be a MSNE. If we multiply utilities of every player in every strategy profile by 10, then (σ_1^*, σ_2^*) continues to be a MSNE if

(a) the strategy set of each player is finite
(b) (σ_1^*, σ_2^*) is never an MSNE of the modified game
(c) (σ_1^*, σ_2^*) is always an MSNE of the modified game; no condition is required
(d) data insufficient

The correct answer is (c).

7. Let $\mathcal{A}$ be a 10×10 matrix of a matrix game. If $\mathcal{A}$ is anti-symmetric, then what is the value of the row player in mixed strategies?

(a) 0
(b) 10
(c) -10
(d) data insufficient

The correct answer is (a).

8. Let $\mathcal{A}$ be a 10×10 matrix of a matrix game. If $\mathcal{A}$ is symmetric, then what is the value of the row player in mixed strategies?

(a) 0

(b) 10

(c) -10

(d) data insufficient

The correct answer is (d).

9. Let $\mathcal{A}$ be a $n \times n$ matrix of a matrix game. Assume that (i, j) and (h, k) are two PSNEs of the matrix game. Then (i, k) is also a PSNE when

   (a) A has full rank

   (b) A is symmetric

   (c) A is anti-symmetric

   (d) always, no extra condition is required.

   The correct answer is (d).
   Justification: If (i, j) and (h, k) are two PSNEs of the matrix game, then (i, k) and (h, j) are also other two PSNEs of the game

10. Suppose in a matrix game, the players have 3 strategies each. Which numbers among $\{0, 1, 2, \ldots, 9\}$ cannot be the total number PSNEs in the matrix game?

   (a) 2

   (b) 3

   (c) 4

   (d) 5

   The correct answer is (d).