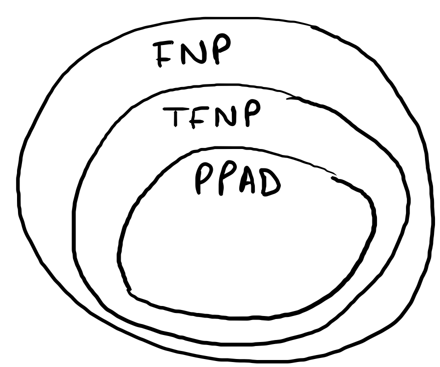
Lecture 5.5

Complexity of finding an MSNE in a bimatrix game.

Polynomial Parity Argument on Directed graphs (PPAD)

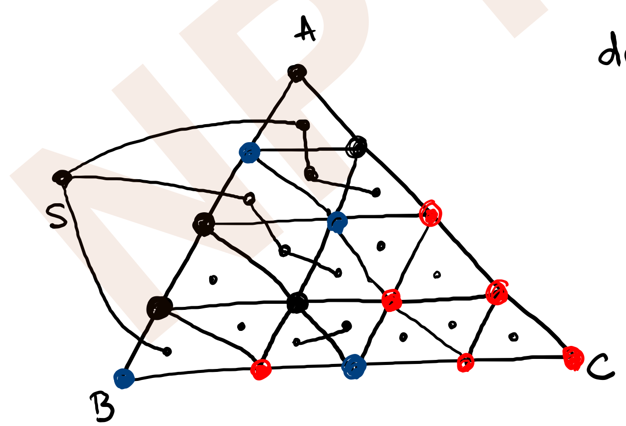


Theorem: MSNE problem for bimatrix games is PPAD-complete.

Sperner's Lemma

Lemma:

There exists at least one trichromatic triangle.



degree of any vertex
0/1/2

Proof: Construct graph with baby triangles as vertices.

We also have a special vertex

The degree of a trichromatic triangle must be 1.

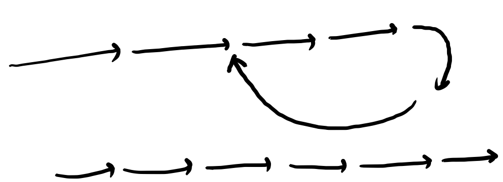
Fact: Any undirected graph has an even number of vertices of odd degree.

- The degree of S must be an odd number.

Hence, there exists at least one vertex other than S whose degree is odd. Equivalently, there exists a trichromatic baby triangle. ∎

Sperner's Problem: Given an oracle access to a sub-divided simplex $\left(= \{(x_1, \dots, x_n) \in \mathbb{R}^n : x_1, \dots, x_n \geq 0, \sum_{i=1}^{n} x_i = 1\} \right)$ in n-dimension, find a colorful baby simplex.

Sperner's problem $\in$ PPAD.



In any directed graph where the out-degree of every vertex is at most 1, if there exists a source node, then there exists a sink.

It turns out that the edges in the proof of Sperner's lemma can be directed appropriately such that any trichromatic baby triangle corresponds to sink nodes.

<u>Theorem</u>: Sperner's problem is PPAD-complete.