Other Forms of Games

Lecture 8.2

Normal form game models "simultaneous move" "complete information" setting.

Relaxing "simultaneous move" $\rightsquigarrow$ extensive form game

Relaxing "complete information" $\rightsquigarrow$ Bayesian game.

Bayesian Game

Definition: A Bayesian game $\Gamma = \langle N, (S_i)_{i \in N}, (\Theta_i)_{i \in N}, p, (u_i)_{i \in N} \rangle$

- N : set of players
- S_i : set of strategies for player $i \in N$
- Θ_i : set of types for player $i \in N$
- $p \in \Delta\left(\times_{i \in N} \Theta_i \right)$

- $u_i : \bigtimes_{i \in N} S_i \bigtimes_{i \in N} \Theta_i \longrightarrow \mathbb{R}$.

In a Bayesian game, Γ is common knowledge.

The prior distribution p is a common knowled and thus the posterior distribution $p(\cdot \mid \theta_i)$ is known to player i only.

Example : (Sealed Bid Selling Auction)

- $N = \{1, \ldots, n\}$ the set of players (buyers)
- $\Theta_i = [0, 1]$ the typeset (valuation) for player $i \in N$
- $S_i = [0, 1]$ the strategy set for player $i \in N$
- p is the product distribution $p = \bigtimes_{i=1}^{n} p_i$ where

  $p_i \sim \mathcal{U}([0,1])$

- Allocation function

$$a : \bigtimes_{i=1}^{n} S_i \longrightarrow \mathbb{R}^N \qquad a\left((s_i)_{i\in N}\right) = (a_i)_{i\in N}$$

$a_i = 1$ if and only if player i wins.

- payment function

$$q : \bigtimes_{i=1}^{n} S_i \longrightarrow \mathbb{R}^N \qquad q\left((s_i)_{i\in N}\right) = (q_i)_{i\in N}$$

where q_i is the money paid by player i.

- $u_i\left((\theta_i)_{i\in N}, (s_i)_{i\in N}\right) = a_i\left(\theta_i - q_i\right)$

A Bayesian game can be equivalently represented by a normal form game. Such a normal form game is called a Selten game.

Selten game: Let $\Gamma = \langle N, (\Theta_i)_{i \in N}, (S_i)_{i \in N}, p, (u_i)_{i \in N} \rangle$ be any Bayesian game. The corresponding normal form game $\Gamma^S = \langle N^S, (S_{\theta_i})_{\theta_i \in \Theta_i, i \in N}, (U_{\theta_i})_{\theta_i \in \Theta_i, i \in N} \rangle$

- $N^s = \bigcup_{i \in N} \Theta_i$

- $S_{\theta_i} = S_i \quad \forall \ \theta_i \in \Theta_i$

- $U_{\theta_i} : \bigtimes_{\substack{\theta_i \in \Theta_i \\ i \in N}} S_{\theta_i} \longrightarrow \mathbb{R}$

$$U_{\theta_i}\left(\underline{(s_{\theta_i})}_{\theta_i \in \Theta_i, i \in N} \right) = \mathbb{E}_{\theta_{-i} \sim p(\cdot | \theta_i)} \left[u_i\left((\theta_i, \theta_{-i}), (s_{\theta_i})_{i \in N} \right) \right]$$

$$= \sum_{\theta_{-i} \in \Theta_{-i}} p(\theta_{-i} | \theta_i) \ u_i\left((\theta_i, \theta_{-i}), (s_{\theta_i})_{i \in N} \right)$$