Lecture 5.4

Computational Complexity of Finding an MSNE

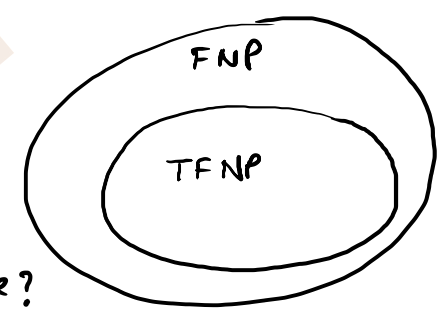
- Functional NP.

Theorem: If MSNE is FNP-complete, then NP = co-NP.

Total FNP (TFNP): The set of all problems in FNP which have only yes instances.

Examples: MSNE, factoring.

Theorem: If any problem in TFNP is FNP-complete, then we have NP = co-NP.



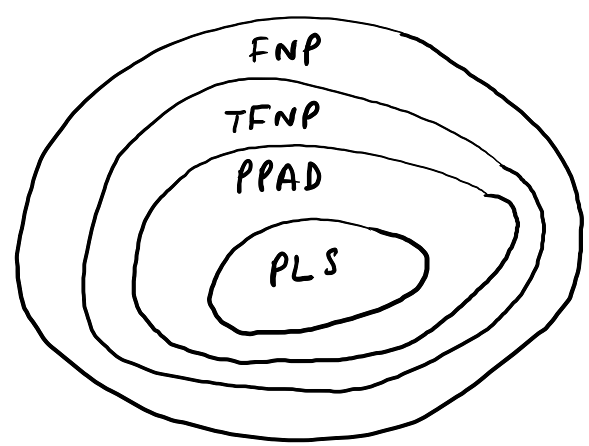
Q: Can we show MSNE is TFNP-complete?

No!

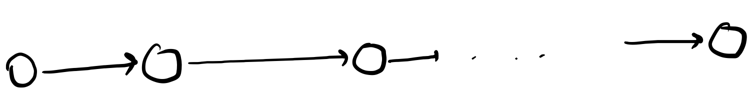
Because TFNP is a "semantic" complexity class whereas most other complexity classes, e.g. NP, XP, PSPACE, FNP are "syntactic" complexity class.

For TFNP, we do not have any complete problem.

Way-out: Define appropriate "syntactic" subclass of TFNP which contain MSNE.



PPAD (Polynomial Parity Argument on Directed graphs)



PLS: Finding a sink in an acyclic graph.

PPAD: search for sink or a source node in a directed graph

Definition: PPAD consists of search problems which has the following two algorithms:

(i) An algorithm to pick an initial solution.

(ii) Given an intermediate solution, an algorithm to find the "next intermediate solution" or output that the current solution is a local optimum.

Although not immediately clear, there exists algorithms for computing an MSNE (e.g. Lemke-Howson's algorithm) that traverses certain directed graph on the strategy profiles. Due to this, MSNE problem for bimatrix games belongs to PPAD.

Can we show that MSNE is PPAD-complete?

<u>YES!</u>

BROWER'S PROBLEM: Finding a stationary (fixed) point of a function $f : [0,1] \to [0,1]$, continuous.

Such functions has a fixed point. A point $x \in [0,1]$ is called a fixed point if $f(x) = x$.

SPERNER'S PROBLEM:

PAPADMITRIOU AND DASKALAKIS. "ACM DESSERTATION AWARD"