

### Succinct Games

| Lecture 4.2

Graphical games, sparse games, symmetric games,  
anonymous games, network congestion games.

6. Congestion game: Generalization of network congestion game. We have a set  $\mathcal{R}$  of resources. A strategy is a subset of  $\mathcal{R}$ . A strategy set of a player is a subset of power set of  $\mathcal{R}$ .

Load of a resource  $e$  is the number of players in a strategy profile using that resource. Each resource has a non-decreasing cost function. The utility of a player in a strategy profile is the sum of the cost of the resources it is using.

7. Multi-matrix game: For each  $(i, j) \in [n] \times [n]$ ,  $i \neq j$ , we have  
a  $m \times m$  matrix  $A^{ij}$

The utility of player  $i$  in a strategy profile  $(s_j)_{j \in [n]}$ ,

$$is \sum_{\substack{j=1 \\ j \neq i}}^n A^{ij}(s_i, s_j)$$

### Potential Game

A game is called a potential game if it has a "potential function".

Theorem: (Rosenthal) Every network congestion game has at least one PSNE.

Proof: Let  $f$  be a flow in  $G$ .  
 $f: E(G) \rightarrow \mathbb{N}$  associated with strategy profiles.

"potential function maps strategy profiles to real numbers."

$$\Phi(f) = \sum_{e \in E(G)} \sum_{i=1}^{f(e)} c_e(i)$$

If there is a player  $i$  who can reduce its cost by changing its path from  $p_i$  to  $p_i'$ , then the reduction in cost of player  $i$  is the same as the reduction in potential value.

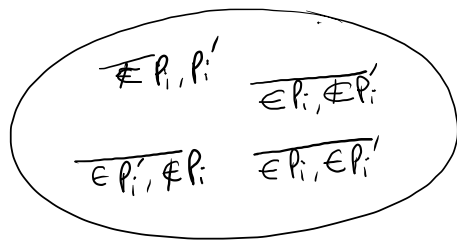
$f$  be the flow corresponding to  $(P_i, P_{-i})$

$\hat{f}$

$(P'_i, P_{-i})$

$$\Phi(\hat{f}) - \Phi(f) = \sum_{e \in P'_i \setminus P_i} c_e(\hat{f}_e) - \sum_{e \in P_i \setminus P'_i} c_e(f_e)$$

$$= \sum_{e \in P'_i} c_e(\hat{f}_e) - \sum_{e \in P_i} c_e(f_e)$$



$G$

decrease in cost for player  $i$  by deviating unilaterally from  $P_i$  to  $P'_i$ .

The domain of  $\Phi$  is a finite set. So it attains a minimum value. Clearly, the pure strategy profile corresponding to the minimum value is a PSNE.  $\square$

Potential Game: A game  $\Gamma = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$  is called a potential game if there exists a function  $\Phi$  s.t.

$$\forall i \in N \forall s_i, s'_i \in S_i, s_{-i} \in S_{-i}, \Phi(s_i, s_{-i}) - \Phi(s'_i, s_{-i}) = u_i(s_i, s_{-i}) - u_i(s'_i, s_{-i}).$$