Lecture 5.2

<u>Theorem</u>: PSNE problem for congestion games is PLS-complete. Moreover, the cuts are in one-to-one correspondence with strategy profiles.

<u>Fact</u>: There exist an instance of local weighted max cut where we need exponentially many local moves to find a local max-cut.

Corollary: The best-response dynamic can make exponentially many iterations to converge to a PSNE.

Theorem: PSNE problem is PLS-complete even for symmetric congestion games.

Proof: Membership in PLS follows from the fact that congestion games are potential games.

To prove PLS-hardness, we reduce from PSNE problem for congestion games.

Let $\Gamma = \langle N, E, (S_i)_{i \in N}, (c_e : \mathbb{N} \longrightarrow \mathbb{R})_{e \in E} \rangle$ be any congestion game. we now contruct a symmetric congestion game

$$\Gamma' = \langle N', E', (S_i')_{i \in N'}, (c_e' : \mathbb{N} \longrightarrow \mathbb{R})_{e \in E'} \rangle$$

Reduced instance

- $N' = N$
- $E' = E \cup \{ r_i : i \in N \}$
- $S_i' = \bigcup_{i \in N} \{ s_i \cup \{ r_i \} : s_i \in S_i \} \quad \forall\, i \in N'$
- $c_e' = c_e \quad \forall\, e \in E, \quad c_e'(\ell) = \begin{cases} 0 & \text{if } \ell \leq 1 \\ \infty & \text{o/w.} \end{cases} \quad \forall\, e \in \{ r_i : i \in N \}$

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Need an algorithm to construct a PSNE of the congestion game from a PSNE of the symmetric congestion game.

Let $s' = (s'_i)_{i \in N}$ be any PSNE of Γ'. Observe that each resource r_i is used by exactly one player in the strategy profile s'. Otherwise, there would exist another resource r_j which no player is using. Any of the multiple players using r_i can unilaterally deviate to any strategy containing r_j which reduces the cost of the deviating player.

However this contradicts our assumption that s' is a PSNE. We define a strategy profile s of Γ as follows: $s = (s_i)_{i \in N}$.

$$s_i = s'_i \text{ such that } \exists j \in N,\ s'_j = s_i \cup \{r_i\}$$

The values of the potential function are the same in both s and s'. ∎

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Corollary: The best response dynamic can take exponential number of iteration to find a PSNE even for symmetric congestion games.

Q. What is the complexity of finding an MSNE of a finite strategic form game?

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