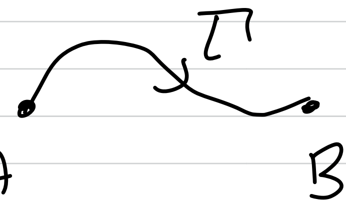
MMC

Dx Collagium.

Cauchy's Integral Theorem:-

Theorem:- Let Γ be a closed contour integral in $\mathbb{C}$, and suppose $f(z)$ is holomorphic on Γ and its interior. Then,

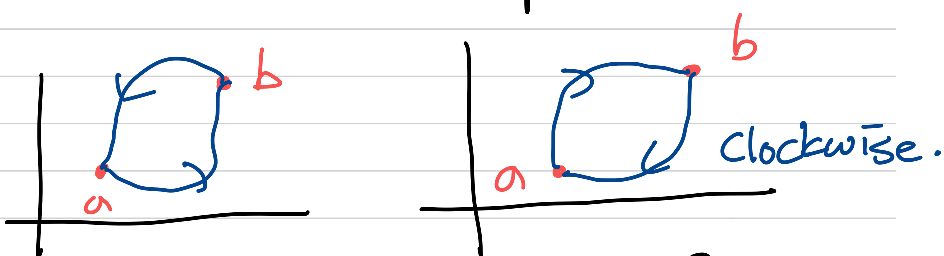
$$\int_{\Gamma} f(z)\, dz = 0.$$



Green's Theorem:-

Let $\gamma : [0,1] \to \mathbb{R}^2$ be a closed contour oriented so that the interior of the region is on the left of Γ. Suppose that P and Q are functions on an open set U containing Γ and interior D, so that P and Q are continuous partial derivatives on U:

Then,



$$\underbrace{\int_{\Gamma} (P\,dx + Q\,dy)}_{\text{counterclockwise}} = \iint_{D} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy$$

Proof. $f(z) = \underline{u(x,y) + i v(x,y)}$

$$dz = dx + i\,dy$$

$$\int_{T'} f(z)\,dz = \int_{T'} ((u(x,y) - v(x,y)\,dy)$$
$$+ i\,(u(x,y)\,dy + v(x,y)\,dx))$$

Thus, by Green's Theorem:-

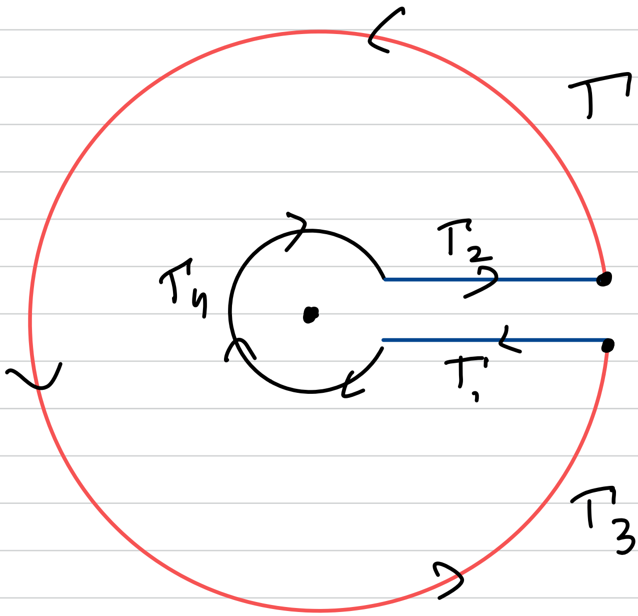
$$\int_{T'} f(z)\,dz = \iint_D \left[\left(-\frac{\partial v}{\partial x} - \underline{\frac{\partial u}{\partial y}}\right) + i\left(\underline{\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}}\right)\right]$$

By the <u>Cauchy-Rieman</u> equations,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

The integrant of the double integral is 0.

Exploring contour integrals



WTS:- $\int_{\Gamma} f(z)\,dz = 0.$

For the horizontal paths,

$\int_{\Gamma_1} f(z)\,dz = -\int_{\Gamma_2} f(z)\,dz$

Reversing the orientation.

$$\int_{\Gamma_3} f(z)\,dz + \int_{\Gamma_4} f(z)\,dz = 0.$$

$$\int_{T_3} f(z)\,dz = \int_{\tilde{T}_4} f(z)\,dz.$$

Integrating $\bar{z}$.

$\bar{z}$ is not holomorphic function. (antiholomorphic).

want to compute:- $\int_T \bar{z}\,dz$

Theorem:- Let T be a closed contour, enclosing a region D. Then

$$\int \bar{z}\,dz = 2i \times \text{Area}(D)$$

Proof.

$$\int_T \bar{z}\,dz = \int_T (x-iy)(dx+i\,dy)$$

$z = x+iy$

$$= \int_T [(x-iy)dx + (ix+y)\,dy]$$

Lastly, by Green's Theorem,

$$= \iint_D \left(\frac{\partial(ix+y)}{\partial x} - \frac{\partial(x-iy)}{\partial y} \right) dx\,dy$$

$$= \iint_D (\underline{i+i})\, dx\,dy$$

$$= 2i \times \text{Area}(D)$$

\# $$\iint_D dz\, d\bar{z} = 2i \times (D)$$