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Tutorial-3

Q.1) if $\{f(k)\} = 4^k$ for $k<0$
3^k for $k \geq 0$
find $Z\{f(k)\}$

Ans:-

$f(k) = 4^k \quad k<0$

$$Z\{f(k)\} = \sum_{k=-\infty}^{-1} f(k). z^{-k}$$

$$= \sum_{k=-\infty}^{-1} 4^k . z^{-k}$$

$$= \sum_{k=1}^{\infty} 4^{-k}. z^k$$

$$= 4^{-1}. z + 4^{-2}. z^2 + 4^{-3} z^3 + 4^{-4}. z^4 + ---$$

$$= \frac{z}{4} + \left(\frac{z}{4}\right)^2 + \left(\frac{z}{4}\right)^3 + \left(\frac{z}{4}\right)^4 + ---$$

$$= \frac{z}{4}\left[1 + \frac{z}{4} + \left(\frac{z}{4}\right)^2 + ----\right]$$

$$= \frac{z}{4} \frac{1}{1-z/4}$$

$$= \frac{z}{4} . \frac{4}{4-z}$$

$$Z\{f(k)\} = \frac{z}{4-z} \quad k<0 \quad |z| < |4|$$

$f(k) = 3^k \quad k \geq 0$

$$Z\{f(k)\} = \sum_{k=0}^{\infty} f(k). z^{-k}$$

$$= \sum_{k=0}^{\infty} 3^k . z^{-k}$$

$$= 3^0 . z^{-0} + 3^1 z^{-1} + 3^2 z^{-2} + 3^3 z^{-3} + ---$$

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$= 1 + \frac{3}{z} + \left(\frac{3}{z}\right)^2 + \left(\frac{3}{z}\right)^3 + \cdots$

$= \frac{1}{1 - 3/z} = \frac{z}{z-3}$ $\quad k \geq 0 \quad |3| < |z|$

$z\{f(k)\} = \frac{z}{4-z} + \frac{z}{z-3}$

$= \frac{z^2 - 3z + 4z - z^2}{(4-z)(z-3)}$

$= \frac{z}{(4-z)(z-3)} = \frac{-z}{(z-4)(z-3)}$

if $|z| > 4$ and $|3| > |z|$

i.e $3 < |z| < 4$

Q.2) Find the z-transform of $f(k) = \frac{a^k}{k!}$, $k \geq 0$

→ $z\{f(k)\} = \sum_{k=0}^{\infty} f(k) \cdot z^{-k}$

$= \sum_{k=0}^{\infty} \frac{a^k}{k!} z^{-k}$

$= \sum_{k=0}^{\infty} \left(\frac{a}{z}\right)^k \cdot \frac{1}{k!}$

$= \frac{\left(\frac{a}{z}\right)^0}{0!} + \left(\frac{a}{z}\right)^1 \cdot \frac{1}{1!} + \left(\frac{a}{z}\right)^2 \cdot \frac{1}{2!} + \left(\frac{a}{z}\right)^3 \cdot \frac{1}{3!} + \cdots$

$= 1 + \frac{a}{z} + \left(\frac{a}{z}\right)^2 \cdot \frac{1}{2!} + \left(\frac{a}{z}\right)^3 \cdot \frac{1}{3!} + \cdots$

Q.3) Find the z-transform of $\{3^{|k|}\}$

→ $\cancel{Z\{f(k)\} = \sum_{k=-\infty}^{\infty} 3^{|k|}}$

$$3^{|k|} = 3^{k} \quad k \geq 0$$
$$3^{-k} \quad k < 0$$

$$Z\{f(k)\} = \sum_{k=-\infty}^{\infty} 3^{|k|} \cdot z^{-k}$$
$$= \sum_{k=-\infty}^{-1} 3^{-k} \cdot z^{-k} + \sum_{k=0}^{\infty} 3^{k} \cdot z^{-k}$$
$$= \sum_{k=1}^{\infty} 3^{k} \cdot z^{k} + \sum_{k=0}^{\infty} 3^{k} \cdot z^{-k}$$
$$= \sum_{k=1}^{\infty} \left(3z\right)^{k} + \sum_{k=0}^{\infty} \left(\frac{3}{z}\right)^{k}$$
$$= \left[(3z)^{1} + (3z)^{2} + (3z)^{3} + (3z)^{4} + \cdots\right]$$
$$+ \left[\left(\frac{3}{z}\right)^{0} + \left(\frac{3}{z}\right) + \left(\frac{3}{z}\right)^{1} + \left(\frac{3}{z}\right)^{2} + \cdots\right]$$
$$= 3z\left[1 + 3z + 3z^{2} + 3z^{3} + \cdots\right] +$$
$$\left[1 + \left(\frac{3}{z}\right) + \left(\frac{3}{z}\right)^{2} + \left(\frac{3}{z}\right)^{3} + \cdots\right]$$
$$= 3z \cdot \frac{1}{1-3z} + \frac{1}{1-3/2}$$
$$= \frac{3z}{1-3z} + \frac{z}{z-3}$$

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$$= \frac{3z^2 - 9z + z - 3z^2}{(1-3z)(z-3)}$$

$$= \frac{-8z}{(1-3z)(z-3)}$$

Q.4) Find the z-transform $\sin\left(\frac{k\pi}{4} + a\right)$

Ans:- $z\{f(k)\} = z\left\{\sin\left(\frac{k\pi}{4} + a\right)\right\}$

$$= z\left\{\sin\frac{k\pi}{4} \cdot \cos a + \cos\frac{k\pi}{4} \cdot \sin a\right\}$$

$$= \cos a \cdot z\left\{\sin\frac{k\pi}{4}\right\} + \sin a \, z\left\{\cos\frac{k\pi}{4}\right\}$$

$$= \cos a \cdot \frac{z \sin \pi/4}{z^2 - 2z\cos\pi/4 + 1} + \sin a \cdot \frac{(z^2 - z\cos\pi/4)}{z^2 - 2z\cos\pi/4 + 1}$$

$$\cancel{= \frac{\cos a \cdot \frac{z}{\sqrt{2}}}{z^2 - \frac{2z}{\sqrt{2}} + 1} + \frac{\sin a\left(z^2 - \frac{z}{\sqrt{2}}\right)}{z^2 - \frac{2z}{\sqrt{2}} + 1}}$$

$$= \frac{\cos a \cdot z \cdot \sin\pi/4 + \sin a \cdot (z^2 - z\cos\pi/4)}{z^2 - 2z\cos\pi/4 + 1}$$

$$= \frac{z\left(\cos a \sin\pi/4 + \sin a \cdot z - \sin a \cos\pi/4\right)}{z^2 - 2z\cos\pi/4 + 1}$$

$$= \frac{z\left(\sin\pi/4 \cos a - \cos a \sin\pi/4\right) + z^2 \sin a}{z^2 - 2z\cos\pi/4 + 1}$$

$$\frac{z\left(\sin\left(\frac{\pi}{4}-a\right)\right)+z^2\sin a}{z^2-\sqrt{2}z+1}$$

Q.5) Find z - transform $k^2 e^{-2k}$

Ans:
$$z\{e^{-2k}\} = z\{(e^{-2})^k\}$$

$$= \frac{z}{z-e^{-2}} \quad \left\{\because z\{a^k\} = \frac{z}{z-a}\right\}$$

$$= \frac{e^2 z}{e^2 z - 1}$$

$$z\{k^2 e^{-2k}\} = \cancel{(-1)^2 \frac{d}{}}\left(-z\frac{d}{dz}\right)^2 f(z)$$

$$= (-1)^2\left\{z\frac{d}{dz}\right\}^2\left\{\frac{e^2 z}{e^2 z-1}\right\}$$

$$= \left\{z\frac{d}{dz}\right\}\left\{z\frac{d}{dz}\right\}\left\{\frac{e^2 z}{e^2 z-1}\right\}$$

$$\cancel{= \left\{z.\frac{d}{dz}\right\}\left\{\frac{(e^2 z-1)(e^2(1)) - e^2 z . e^2}{(e^2 z-1)^2}\right\}}$$

$$\cancel{= \left\{z.\frac{d}{dz}\right\}\left\{(e^2 z-1)e^2 -\right.}$$

$$= z.e^2 \frac{d}{dz}\left\{\frac{z}{e^2 z-1}\right\}$$

$$\left\{z\frac{d}{dz}\right\} ze^2 \left(\frac{(e^2 z-1)(1) - z.e^2}{(e^2 z-1)^2}\right)$$

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$$\left\{ z \frac{d}{dz} \left\{ \cancel{z e^2 \left(\frac{(-1)}{(e^2 z - 1)^2} \right)} \right. \right.$$

$$= \cancel{-z e^2 \cdot z \frac{d}{dz} \left(\frac{1}{(e^2 z - 1)^2} \right)}$$

$$= \cancel{-z e^2 \left[\frac{(e^2 z - 1)^2 \cdot 0 - 1 \cdot 2(e^2 z - 1) \cdot e^2}{(e^2 z - 1)^4} \right]}$$

$$= \cancel{-z e^2 \left[\frac{-2(e^2 z - 1)}{(e^2 z - 1)^4} \right]}$$

$$=$$

$$\left\{ z \cdot \frac{d}{dz} \right\} z \cdot e^2 \left(\frac{-1}{(e^2 z - 1)^2} \right)$$

$$= -z e^2 \frac{d}{dz} \left[\frac{z}{(e^2 z - 1)^2} \right]$$

$$= -z e^2 \left[\frac{(e^2 z - 1)^2 \cdot (1) - z \cdot 2(e^2 z - 1) \cdot e^2}{(e^2 z - 1)^4} \right]$$

$$= -z e^2 \left[\frac{(e^2 z - 1) - z 2 e^2}{(e^2 z - 1)^3} \right]$$

$$= -z e^2 \left[\frac{-(e^2 z + 1)}{(e^2 z - 1)^3} \right]$$

$$= + z e^2 \left[\frac{e^2 z + 1}{(e^2 z - 1)^3} \right] \quad //$$

Q.6) Find $z\{2^k \cos(3k+2)\}$ $k \geq 0$

$\rightarrow$ $\because \cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$

$\cos(3k+2) = \cos 3k \cdot \cos 2 - \sin 3k \cdot \sin 2$

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$$Z[\cos(3k+2)] = Z[\cos 3k.\cos 2] - Z[\sin 3k.\sin 2]$$

$$= \cos 2\ Z[\cos 3k] - \sin 2\ Z[\sin 3k]$$

$$= \cos 2\left(\frac{z^2 - z\cos 3}{z^2 - 2z\cos 3 + 1}\right) - \frac{\sin 2.z\sin 3}{z^2 - 2z\cos 3 + 1}$$

$$= \frac{z^2\cos 2 - z\cos 3\cos 2 - \sin 2.z.\sin 3}{z^2 - 2z\cos 3 + 1}$$

$$= z\left[\frac{z\cos 2 - \cos 2\cos 3 - \sin 2\sin 3}{z^2 - 2z\cos 3 + 1}\right]$$

$$Z[\cos(3k+2)] = z\left[\frac{z\cos 2 - \cos(2-3)}{z^2 - 2z\cos 3 + 1}\right]$$

$$= z\left[\frac{z\cos 2 - \cos 1}{z^2 - 2z\cos 3 + 1}\right]$$

$$Z[2^k\cos(3k+2)] = \frac{z}{2}\left[\frac{\frac{z}{2}\cos 2 - \cos 1}{\left(\frac{z}{2}\right)^2 - \frac{2z\cos 3}{2} + 1}\right]$$

$$= \frac{z}{2}\left[\frac{\frac{z\cos 2}{2} - \cos 1}{\frac{z^2}{4} - z\cos 3 + 1}\right]$$

$$= z\left[\frac{\frac{z\cos 2 - 2\cos 1}{\cancel{4}}}{\frac{z^2 - 4z\cos 3 + 4}{\cancel{4}}}\right]$$

Q.7) F

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$$= z\left[\frac{z\cos 2 - 2\cos 1}{z^2 - 4z\cos 3 + 4}\right]$$

sin2]

Q.7) Find $z\{f(k) * g(k)\}$ if $f(k) = \frac{1}{5^k}$ & $g(k) = \frac{1}{7^k}$

→ $$z\left\{\frac{1}{5^k}\right\} = \sum_{k=0}^{\infty} \frac{1}{5^k} z^{-k}$$

$$= \sum_{k=0}^{\infty} \frac{1}{(5z)^k}$$

$$= 1 + \frac{1}{(5z)} + \frac{1}{(5z)^2} + \frac{1}{(5z)^3} + \cdots$$

$$= \frac{1}{1 - 1/5z}$$

$$f(k) = z\left\{\frac{1}{5^k}\right\} = \frac{5z}{5z-1}$$

$$z\left\{\frac{1}{7^k}\right\} = \sum_{k=0}^{\infty} \frac{1}{7^k} \cdot z^{-k}$$

$$= \sum_{k=0}^{\infty} \frac{1}{(7z)^k}$$

$$= 1 + \frac{1}{7z} + \frac{1}{(7z)^2} + \frac{1}{(7z)^3} + \cdots$$

$$= \frac{1}{1 - 1/7z}$$

$$g(k) = z\left\{\frac{1}{7^k}\right\} = \frac{7z}{7z-1}$$

$$z\{f(k) * g(k)\} = \left(\frac{5z}{5z-1}\right)\left(\frac{7z}{7z-1}\right)$$

Q.8) Find inverse z-transform of f(z)

$$f(z) = \frac{z}{(z-1)(z-2)} \; ; \; |z|>2 \quad \text{------}(I)$$

→ $$\frac{f(z)}{z} = \frac{1}{(z-1)(z-2)}$$

$$\frac{1}{(z-1)(z-2)} = \frac{A}{(z-1)} + \frac{B}{(z-2)}$$

$$1 = A(z-2) + B(z-1)$$

Put $z=2$, $\boxed{1=B}$ Put $z=1$ $\boxed{A=-1}$

$$\frac{f(z)}{z} = \frac{-1}{z-1} + \frac{1}{z-2}$$

$$f(z) = \frac{-z}{z-1} + \frac{z}{z-2}$$

$$z^{-1}[f(z)] = -1\, z^{-1}\left[\frac{z}{z-1}\right] + z^{-1}\left[\frac{z}{z-2}\right]$$

$$\because z^{-1}\left[\frac{z}{z-a}\right] = a^k$$

$\because z>2 \; \therefore z>1$

$$= -1 \cdot 1^k + 2^k$$

$$= 2^k - 1^k$$

$$= 2^k - 1 \quad k \geq 1$$

Q.9) Find the Inverse z-transform of $\frac{z^2}{(z-1)(z-2)}$ using convolution theorem

→

$$\frac{z^2}{(z-1)(z-2)} = \frac{z}{(z-1)} \cdot \frac{z}{(z-2)}$$

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$$Z^{-1}\left\{\frac{z}{z-1}\right\} = 1^k \quad k \geq 0$$

$$Z^{-1}\left\{\frac{z}{z-2}\right\} = 2^k \quad k \geq 0$$

$$Z^{-1}\left\{\frac{z^2}{(z-1)(z-2)}\right\} = Z^{-1}\left\{\frac{z}{z-1} \cdot \frac{z}{z-2}\right\} = 1^k * 2^k$$

z^{-1}

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$$= \sum_{m=0}^{k} 1^m 2^{k-m} = \sum_{m=0}^{k} 1^m \cdot \frac{2^k}{2^m} = \sum_{m=0}^{k} 2^k \left(\frac{1}{2}\right)^m$$

$$= 2^k \sum_{m=0}^{k} \left(\frac{1}{2}\right)^m$$

$$= 2^k \left[1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots \left(\frac{1}{2}\right)^k \right]$$

$$= 2^k \left[\frac{\left(\frac{1}{2}\right)^{k+1} - 1}{\frac{1}{2} - 1} \right]$$

$$= 2^k \left[\frac{\frac{1^{k+1} - 2^{k+1}}{2^{k+1}}}{\frac{1-2}{2}} \right]$$

$$= \frac{1^{k+1} - 2^{k+1}}{-1}$$

$$= 2^{k+1} - 1$$

//

Q.1)

Ans:-