Assignment 1

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2. The seven types of robots as discussed:

   Manipulators: These are stationary robots that have linked arms which can move and access different positions, at different angles depending on the degree of freedom. The following link shows a manipulator with 6 degrees of freedom.
   [Universal Robots - UR10 - Demonstration]

   Mobile Robots: These robots can move about while moving. These usually refer to ground-based robots that move on rails or wheels. A highly popular example is the Roomba vacuum cleaner shown below:
   [Clean Floors with the Press of a Button | Roomba® 900 series | iRobot®]

   Limbed Robots: These robots have the ability to move by using jointed limbs. Example: [Festo – BionicCobot (English)]

   UAVs: Unmanned Aerial Vehicle and they are autonomous robots that fly through the air. Example: [DJI - Introducing DJI Avata]

   AUVs: These are Autonomous Underwater Vehicles. They are autonomous mobile robots that operate underwater. The following is an example of an AUV made in IIT Bombay
   Matsya (Autonomous Underwater Vehicle)

   Humanoid Robots: These are robots that have an upright orientation and have human-like limbs attached to a torso.

   Microbots: As the name suggests, these are very small autonomous machines. They can be small enough to operate at a cellular level and are hence of great interest in the medical field.

3.

AC

Synchronous

Operate at a constant speed and maintain synchrony with the supply frequency.
Used in applications requiring precise speed control, like industrial machinery and synchronous clocks.
Higher efficiency and power factor than induction motors.

Asynchronous

Induction motor- operates by electromagnetic induction.
The rotor is not connected to an electrical power supply and relies only on the stator's magnetic field to induce a current in the rotor.
Widely used in industrial applications.

Servo

incorporates a feedback mechanism using an encoder or potentiometer, to continuously monitor the motor's actual position and compare it to the desired position.
Adjust their behavior in real-time, which makes them suitable for applications requiring accurate and controlled movement, such as robotics, automation, CNC machines, and remote-controlled vehicles.

DC

Brushed

-operates using a system of brushes and a commutator to change the direction of current flow in the motor's coils.
-the motor's rotation is driven by a rotating magnetic field that.
-Used in applications where cost is a consideration and precise control is not crucial, such as household appliances, power tools, and toys.

Brushless

BLDC

-Uses electronic commutation to switch the current direction in the windings.
-Offers high efficiency, longer lifespan, and low maintenance, making them suitable for applications like electric vehicles and appliances.

Stepper

-Rotates in fixed angular increments known as steps.
-Driven by sending a series of electrical pulses to their coils, causing the rotor to move one step at a time.
-Offer precise control over position and are commonly used in applications that require accurate positioning, such as 3D printers, CNC machines, and robotics.

6. $$R_0^1 = \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 & \hat{j}_1 \cdot \hat{i}_0 & \hat{k}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 & \hat{j}_1 \cdot \hat{j}_0 & \hat{k}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 & \hat{j}_1 \cdot \hat{k}_0 & \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix}$$

$$\downarrow \quad \downarrow \quad \downarrow$$
$$c_1 \quad c_2 \quad c_3$$

For columns to be orthogonal, dot product of the columns taken 2 at a time equals zero.

$$c_1 \cdot c_2 = 0 \qquad c_2 \cdot c_3 = 0 \qquad c_3 \cdot c_1 = 0$$

$$= R_{x,\alpha} R_{y,\beta} R_{z,\gamma}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \ldots \end{bmatrix} \begin{bmatrix} C_\beta & 0 & +S_\beta \\ & 1 & 0 \end{bmatrix} \begin{bmatrix} C_\gamma & -S_\gamma & 0 \\ & C_\gamma & 0 \end{bmatrix} = \begin{bmatrix} C_\beta & 0 & S_\beta \\ S_\alpha S_\beta & C_\alpha & -S_\alpha C_\beta \end{bmatrix} \begin{bmatrix} C_\gamma & -S_\gamma & 0 \\ S_\gamma & C_\gamma & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\alpha & -S_\alpha \\ 0 & S_\alpha & C_\alpha \end{bmatrix} \begin{bmatrix} C_\beta & 0 & +S_\beta \\ 0 & 1 & 0 \\ -S_\beta & 0 & C_\beta \end{bmatrix} \begin{bmatrix} C_\gamma & -S_\gamma & 0 \\ S_\gamma & C_\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_\beta & 0 & S_\beta \\ S_\alpha S_\beta & C_\alpha & -S_\alpha C_\beta \\ -S_\beta C_\alpha & S_\alpha & C_\alpha C_\beta \end{bmatrix} \begin{bmatrix} C_\gamma & -S_\gamma & 0 \\ S_\gamma & C_\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{array}{ccc} \left[\begin{matrix} C_\beta C_\gamma \\ (S_\alpha S_\beta C_\gamma + C_\alpha S_\gamma) \\ (-C_\alpha S_\beta C_\gamma + S_\alpha S_\gamma) \end{matrix}\right. & \begin{matrix} -C_\beta S_\gamma \\ (-S_\alpha S_\beta S_\gamma + C_\gamma C_\alpha) \\ (C_\alpha S_\beta S_\gamma + S_\alpha C_\gamma) \end{matrix} & \left.\begin{matrix} S_\beta \\ -S_\alpha C_\beta \\ C_\alpha C_\beta \end{matrix}\right] \\ \downarrow & \downarrow & \downarrow \\ C_1 & C_2 & C_3 \end{array}$$

$$-S_\alpha^2 S_\beta C_\beta C_\gamma - S_\alpha S_\gamma C_\alpha C_\beta + C_\alpha^2 C_\beta C_\gamma S_\beta + C_\alpha C_\beta S_\alpha S_\gamma$$

$$C_1 \cdot C_2 = C_\beta C_\gamma \cdot (-C_\beta S_\gamma) + (S_\alpha S_\beta C_\gamma + C_\alpha S_\gamma)(-S_\alpha S_\beta S_\gamma + C_\gamma C_\alpha) + (-C_\alpha S_\beta C_\gamma + S_\alpha S_\gamma)(C_\alpha S_\beta S_\gamma + S_\alpha C_\gamma)$$

$$= -C_\beta^2 C_\gamma S_\gamma - S_\alpha^2 S_\beta^2 C_\gamma S_\gamma + S_\alpha S_\beta C_\gamma^2 C_\alpha - S_\alpha S_\beta S_\gamma^2 C_\alpha + C_\alpha^2 C_\gamma S_\gamma - C_\alpha^2 S_\beta^2 C_\gamma S_\gamma - C_\alpha S_\alpha S_\beta C_\gamma^2$$
$$+ S_\alpha S_\beta S_\gamma^2 C_\alpha + S_\alpha^2 S_\gamma C_\gamma$$

$$= C_\gamma S_\gamma (-C_\beta^2 - S_\alpha^2 S_\beta^2 - C_\alpha^2 S_\beta^2 + S_\alpha^2) + S_\alpha S_\beta C_\alpha (\cancel{C_\gamma^2} - \cancel{S_\gamma^2} - \cancel{C_\gamma^2} + \cancel{S_\gamma^2}) + C_\alpha^2 S_\gamma C_\gamma$$

$$= C_\gamma S_\gamma (S_\alpha^2 - C_\beta^2 - S_\beta^2) + C_\alpha^2 S_\gamma C_\gamma$$

$$= C_\gamma S_\gamma (S_\alpha^2 - 1) + C_\alpha^2 S_\gamma C_\gamma$$

$$= -C_\alpha^2 S_\gamma (C_\gamma - C_\gamma)$$

$$C_1 \cdot C_2 = 0 \,//$$

$$C_2 \cdot C_3 = -S_\beta C_\beta S_\gamma + S_\alpha^2 C_\beta S_\beta S_\gamma - \cancel{S_\alpha C_\alpha C_\beta C_\gamma} + C_\alpha^2 S_\beta C_\beta S_\gamma + \cancel{S_\alpha C_\alpha C_\beta C_\gamma}$$

$$= (C_\alpha^2 + S_\alpha^2 - 1) S_\beta C_\beta S_\gamma$$

$$C_2 \cdot C_3 = 0 \,//$$

$$C_3 \cdot C_1 = S_\beta C_\beta C_\gamma - S_\alpha C_\beta (S_\alpha S_\beta C_\gamma + C_\alpha S_\gamma) + C_\alpha C_\beta (-C_\alpha S_\beta C_\gamma + S_\alpha S_\gamma)$$

$$= (1 - S_\alpha^2 - C_\alpha^2) S_\beta C_\beta C_\gamma - \cancel{S_\alpha C_\alpha C_\beta S_\gamma} + \cancel{S_\alpha C_\alpha C_\beta S_\gamma}$$

$$C_3 \cdot C_1 = 0 \,//$$

$\therefore$ The columns of the rotational matrix R_0^1 are orthogonal.

c_3 c_1 //

$\therefore$ The columns of the rotational matrix R_0^1 are orthogonal.

7. $R_0^1 = \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 & \hat{j}_1 \cdot \hat{i}_0 & \hat{k}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 & \hat{j}_1 \cdot \hat{j}_0 & \hat{k}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 & \hat{j}_1 \cdot \hat{k}_0 & \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix}$

$R_0^1 = R_{x,\alpha} R_{y,\beta} R_{z,\gamma}$ Representing as product of basic rotation matrices.

$\det(R_0^1) = \det(R_{x,\alpha} R_{y,\beta} R_{z,\gamma})$

$= \det(R_{x,\alpha}) \cdot \det(R_{y,\beta}) \cdot \det(R_{z,\gamma})$

$\det(R_{x,\alpha}) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & C_\alpha & -S_\alpha \\ 0 & S_\alpha & C_\alpha \end{vmatrix} = 1(\cos^2\alpha + \sin^2\alpha) = 1$

$\det(R_{y,\beta}) = \begin{vmatrix} C_\beta & 0 & S_\beta \\ 0 & 1 & 0 \\ -S_\beta & 0 & C_\beta \end{vmatrix} = C_\beta(C_\beta - 0) + S_\beta(0 + S_\beta)$

$= \cos^2\beta + \sin^2\beta$

$= 1$

$\det(R_{z,\gamma}) = \begin{vmatrix} C_\gamma & -S_\gamma & 0 \\ S_\gamma & C_\gamma & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$= C_\gamma(C_\gamma - 0) + S_\gamma(S_\gamma - 0) + 0$

$= \cos^2\gamma + \sin^2\gamma$

$= 1$

$\therefore \boxed{\det(R_0^1) = 1}$