$R_2^3 = I$, $d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$, $P_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix}$

So, $\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & \sqrt{3}/2 & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Ques 6

Ans Spur Gearbox → used in wide range of robotics applicati
- pros → provide high torque, cost-effective, efficient
- Cons → have some backlash, can be noisy.

Planetary Gearbox → commonly used in robotic arm.
- pros → compact, heigh torque density.
- Cons → expensive, complex to manufacture

Harmonic Drive gearbox. → used in robotic surgery
- Pros - high precision, offer zero-backlash.
- Cons - more expensive, not handle much torque

$$J = \left[R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right.$$

$$R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_0^0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) + l_1 c q_1 \\ l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2) + l_1 s q_1 \\ 0 \end{bmatrix}$$

$$R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_1^0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) \\ l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2) \\ 0 \end{bmatrix}$$

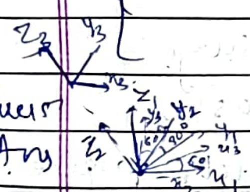
$$R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (d_3^0 - d_2^0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_3 c(q_1+q_2+q_3) \\ l_3 s(q_1+q_2+q_3) \\ 0 \end{bmatrix}$$

$$R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} ; \quad R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} ; \quad R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} -(l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2) + l_1 s q_1) & -[l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2)] & -l_3 s(q_1+q_2+q_3) \\ l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) + l_1 s q_1 & l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) & l_3 c(q_1+q_2+q_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Ques

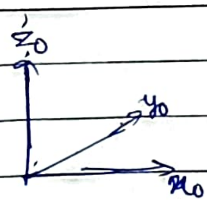
Ans



so, $H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$ — $R_0^1 \rightarrow I$

$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$

$R_1^2 = R_{x,\pi/6} \, R_{z,\pi/6}$, $d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$



so $R_1^2 = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 \end{bmatrix}$

$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$

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$n=3$

so, $$R_0^1 \begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_3^0 - d_0^0) = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix} cq_1 l_1 + c(q_1+q_2)l_2 \\ sq_1 l_2 + s(q_1+q_2)l_2 \\ 0\end{bmatrix}$$

$$= \begin{bmatrix} -sq_1 l_1 - s(q_1+q_2)l_2 \\ cq_1 l_1 + c(q_1+q_2)l_2 \\ 0 \end{bmatrix}$$

$$R_1^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_3^0 - d_1^0) = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix} cq_1 l_1 + c(q_1+q_2)l_2 \\ sq_2 l_2 + s(q_1+q_2)l_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -sq_1 l_1 - s(q_1+q_2)l_2 \\ cq_1 l_1 + c(q_1+q_2)l_2 \\ 0 \end{bmatrix}$$

$$R_2^0 \begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$J = \begin{bmatrix} -sq_1 l_1 - s(q_1+q_2)l_2 & -s(q_1+q_2)l_2 & 0 \\ cq_1 l_1 + c(q_1+q_2)l_2 & c(q_1+q_2)l_2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Ques 9

Ans we know R_0^0, R_1^0, $R_2^0 = R_0^0 R_1^0$

$$d_3^0 = \begin{bmatrix} l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) + l_1 cq_1 \\ l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2) + l_1 sq_1 \\ 0 \end{bmatrix}$$

so

Quest 7

Ans we know for RRP SCARA

Here we need to find R_0^3

R_0^1

so, $R_0^3 = R_0^1 R_1^2 R_2^3$, we know R_0^1, R_1^2, R_2^3 for SCARA

so, $$R_0^3 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By solving,
$$R_0^3 = \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 \\ s(q_1+q_2) & c(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

also, $d_0^2 = \begin{bmatrix} cq_1 l_1 \\ sq_1 l_1 \\ 0 \end{bmatrix}$ and $d_0^3 = \begin{bmatrix} cq_1 l_1 + c(q_1+q_2) l_2 \\ sq_1 l_1 + s(q_1+q_2) l_2 \\ 0 \end{bmatrix}$

~~also, $H_0^3 = H_0^1 H_1^2 H_2^3$~~

so, $H_0^3 = \begin{bmatrix} R_0^3 & d_0^3 \\ 0 & 1 \end{bmatrix}$

$$H_0^3 = \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 & cq_1 l_1 + c(q_1+q_2) l_2 \\ s(q_1+q_2) & c(q_1+q_2) & 0 & sq_1 l_1 + s(q_1+q_2) l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

now, $J = \begin{bmatrix} J_v \\ J_w \end{bmatrix}$

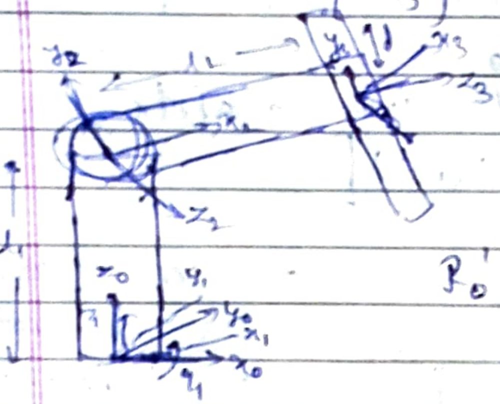
$J_v \rightarrow$ prismatic $R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, Revolute $R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_n^0 - d_{i-1}^0)$

$J_w \rightarrow$ prismatic $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, Revolute $R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$R_2^3 = R_{x,0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ d \end{bmatrix}$

so, $\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{x,0} & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 0 \end{bmatrix}$

also, $P_3 = \begin{bmatrix} 0 \\ 0 \\ -l_3 \end{bmatrix}$



so, $\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 0 \end{bmatrix}$

$R_0^1 = R_{z,q_1} = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_1^2 = R_{x,\frac{\pi}{2}} R_{z,q_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\pi/2 & -s\pi/2 \\ 0 & s\pi/2 & c\pi/2 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$

$\therefore R_2^3 = R_{x,0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $d_2^3 = \begin{bmatrix} l_2 + d \\ 0 \\ 0 \end{bmatrix}$

$P_3 = \begin{bmatrix} 0 \\ -l_3 \\ 0 \end{bmatrix}$; $\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 0 \end{bmatrix}$

Assignment - 02
Vineet K. Tatu.
21110239

Q1 we have to prove $RS(a)R^T = S(Ra)$

let start with $RS(a)(R^T b)$

so, $RS(a)R^Tb = R(a \times R^Tb)$ {$S(a)p = a \times p$}

$= \cancel{(R \times a)}$

$= (Ra) \times (RR^Tb)$ {$R \to$ is orthogonal}

$= (Ra) \times (b)$

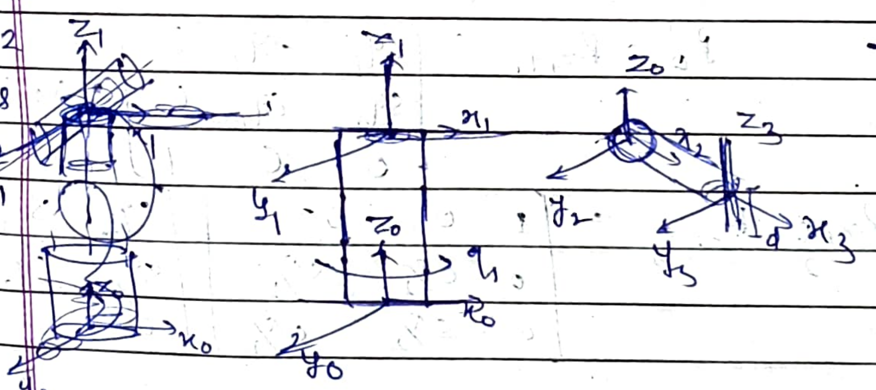
$= S(Ra)b$ {$a \times p = S(a)p$}

so, $RS(a)R^Tb = S(Ra)b$

now assume $b \to$ identity matrix

so, $\boxed{RS(a)R^T = S(Ra)}$

Que 2
Ans



$z_1, z_2 \to$ axis of rotation

$z_3 \to$ axis of translation of prismatic

also we know $\begin{bmatrix} p_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} p_3 \\ 1 \end{bmatrix}$

$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$, $R_0^1 = R_{z,q_1} = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

so, $H_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$R_1^2 = R_{z,q_2} = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $d_1^2 = \begin{bmatrix} d_1 \\ 0 \\ 0 \end{bmatrix}$

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