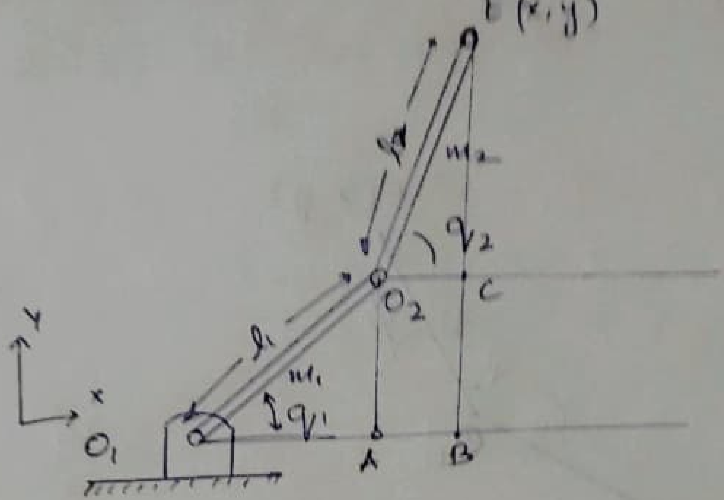
2R Manipulator



we have,

q_1, q_2 = joint angles

$E(x, y)$ = Quardinates of End-Effector positions.

Assume origin at O_1 and motor's are connected at O_1 & O_2.

Let τ_1 & τ_2 be the torques applied by motor's at the joint O_1 & O_2 to control q_1 & q_2.

End effector position can be given by:

$x = OA + AB$; $Y = CB + EC$

$$\Rightarrow \left.\begin{array}{l} x = l_1 \cos q_1 + l_2 \cos q_2 \\ y = l_2 \sin q_1 + l_2 \sin q_2 \end{array}\right\} \quad \text{—— (1)}$$

Now if we differentiale (1) w.r.t. time we get the relation b/w End effector velocity and angular velocity of arms.

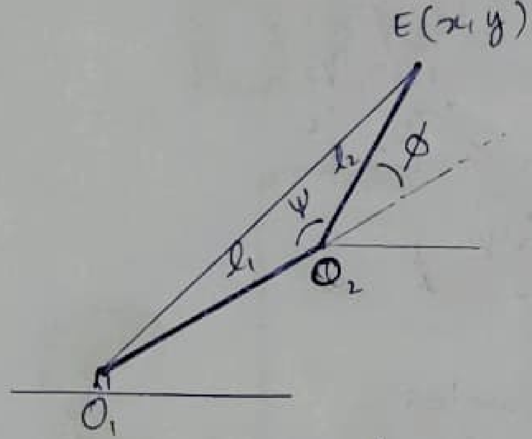
$$\rightarrow \dot{x} = -l \sin q_1 \dot{q}_1 - l_2 \sin q_2 \cdot \dot{q}_2$$

$$\dot{y} = l_2 \cos q_2 \cdot \dot{q}_1 + l_2 \cos q_3 \cdot \dot{q}_2$$

$$\rightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l \sin q_1 & -l_1 \sin q_2 \\ l_1 \cos q_1 & l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad \text{—— (2)}$$

Inverse Kinematics

Now we will drive the inverse relation b/w (x, y) and (q_1, q_2) i.e. given (x, y) we will find (q_1, q_2)



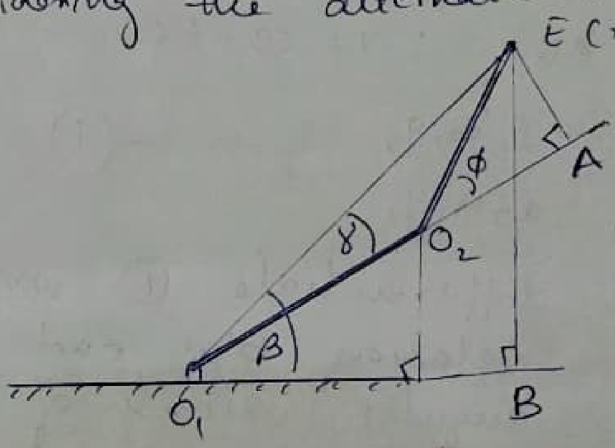
using Cosine rule:

$$(x^2+y^2) = l_1^2 + l_2^2 - 2 l_1 l_2 \cos\psi$$

$$x^2 + y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \cos\phi \quad (\because \psi + \phi = \pi)$$

$$\Rightarrow \quad \phi = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2 l_1 l_2}\right)$$

Now considering the alternate triangles.



Now & clearly $q_1 = \beta - \gamma$

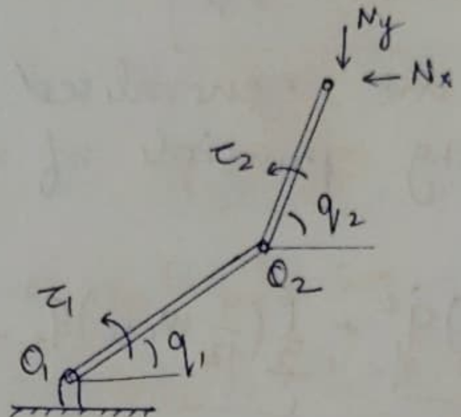
$\therefore$ from ΔOEB & ΔOEA

$$\beta = \tan^{-1}\left(\frac{y}{x}\right) \qquad \gamma = \tan^{-1}\left(\frac{l_2 \sin\phi}{l_1 + l_2 \cos\phi}\right)$$

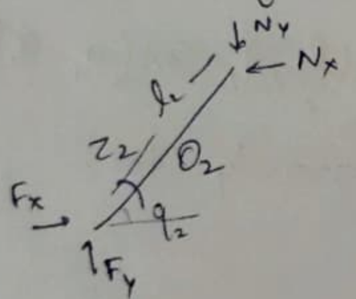
Thus,

$$q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\phi}{l_1 + l_2 \cos\phi}\right) \quad \text{--- (3)}$$

Now developing relationship b/w motor torques (τ_1 & τ_2) & End-effector force (F_x, F_y)



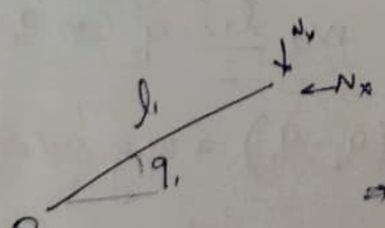
FBD of each link



Taking Moment about O_2

$$\Rightarrow \Sigma M_{O_2} = 0$$

$$\Rightarrow N_y l_2 \cos q_2 - N_x l_2 \sin q_2 = \tau_2$$



Taking moment about O_1

$$\Rightarrow \Sigma M_{O_1} = 0$$

$$\Rightarrow N_y l_1 \cos q_1 - N_x l_1 \sin q_1 = \tau_1$$

Thus,

$$\begin{bmatrix} -\tau_1 \\ -\tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & -l_1 \cos q_1 \\ l_2 \sin q_2 & l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} N_x \\ N_y \end{bmatrix} \quad \text{--- (4)}$$

To formulate dynamic equations for this robotic arms we use Eular-Langrange method.

$L = K - V$, $K = K.E$, $V = P.E$

$$\boxed{Q_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}} \quad -5$$

where, Q_i are generalised forces derived using principle of virtual work.

$$K = \underbrace{\tfrac{1}{2}\left(\tfrac{1}{3} m_1 l_1^2\right)\dot{q}_1^2}_{\text{Pure rotation of } L_1} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2 + \frac{1}{2} m_2 v_{c_2}^2}_{\text{Pure rotation of } L_2}$$

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \left(\frac{l_2}{2}\right)\dot{q}_2 \cdot \cos(q_2 - q_1)$$

$V = 0$ ($\because$ since we neglect gravity for simplicity).

$$\boxed{\begin{aligned}\tau_1 &= \frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1) \\ &\quad - m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_1 g \frac{l_1}{2} \cos q_2 \\ &\quad + m_2 g l_1 \cos q_1 \\ \tau_2 &= \frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1) \\ &\quad - m_2 \frac{l_1 l_2}{4} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} \sin q_2\end{aligned}}$$

equ -6

To make arm behave as spring.

using eqn ④ & ①

$$F_x = kx \quad , \quad F_y = ky$$

$$\rightarrow F_x = k(l_1 \cos q_1 + l_2 \cos q_2)$$

$$F_y = k(l_1 \sin q_1 + l_2 \sin q_2)$$

Substituting in eqn ④

$$\tau_{s_1} = k(l_1 \sin q_1 + l_2 \sin q_2) l_1 \cos q_1 - k(l_1 \cos q_1 + l_2 \cos q_2) l_1 \sin q_1$$

$$\tau_{s_2} = k(l_1 \sin q_1 + l_2 \sin q_2) l_2 \cos q_2 - k(l_1 \cos q_1 + l_2 \cos q_2) l_2 \sin q_2$$

⑦

Now For motor to behave as spring.

Motor 1 torque $= \tau_1 + \tau_{s_1}$

Motor 2 torque $= \tau_2 + \tau_{s_2}$

Github links

Task [1] https://github.com/imofaz/ITR/blob/123770ed32be8a599ab8ffb413114eaae8beb9a0/task%201.py

Task [2] https://github.com/imofaz/ITR/blob/123770ed32be8a599ab8ffb413114eaae8beb9a0/task%202.py

Task [3] https://github.com/imofaz/ITR/blob/123770ed32be8a599ab8ffb413114eaae8beb9a0/task%203.py

Task [4] https://github.com/imofaz/ITR/blob/123770ed32be8a599ab8ffb413114eaae8beb9a0/task%204.py