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# Assignment - 2

Q-1 Show that $RS(a)R^T = S(Ra)$, where R is a rotation matrix.

A-1 Given, $RS(a)R^T = S(Ra)$, here R is rotation matrix

↳ R has the property that its transpose is its inverse, i.e, $R^T = R^{-1}$, Also it preserves the length of vectors, i.e, $\|Ra\| = \|$ for any vector a.

A skew symmetric matrix $S(a)$ for a vector a in 3D is defined as

$$S(a) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

Now let's compute $R(S(a))R^T$ and $S(Ra)R$

$RS(a)R^T = R \times S(a) \times R^{-1}$

$S(Ra)R = S(R \times a) \times R$

Since R is a rotation Matrix, its rotates the vector 'a' to a new position 'Ra' but does not change its length. Therefore, the skew-symmetric matric $S(Ra)$ is just a rotated version of $S(a)$, and multipling it by R on the right corresponds to rotating the vectors it would act upon in the appositeion dir$^n$

∴ We have $R(S(a))R^T = S(Ra)R$. This shows that the action of skew symmetric matrix on vector (which corresponds to a cross product with 'a') commutes with rotation - you can either first rotate and then take cross product, or first take the cross product and then rotates, and the result will be same.

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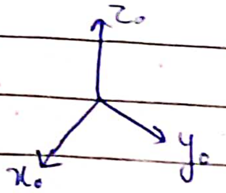
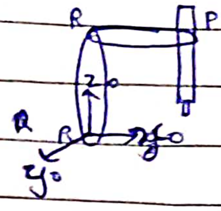
A-7 Example :- RRP - SCARA

$P_3 = \begin{bmatrix} 0 \\ 0 \\ -l_3 \end{bmatrix}$




$R_0^1 = \begin{bmatrix} R_{z,q_1} \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ , $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_1^2 = \begin{bmatrix} R_{z,q_2} \end{bmatrix} = \begin{bmatrix} cq_2 & -sq_2 & 0 & 0 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ , $d_1^2 = \begin{bmatrix} 0 \\ l_1 \\ 0 \end{bmatrix}$

$R_2^3 = \begin{bmatrix} R_{z,0} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ , $d_2^3 = \begin{bmatrix} 0 \\ l_2 \\ -q_3 \end{bmatrix}$

zero rotation means identity matrix.

$\therefore$ End effector position $\Rightarrow E = R_0^1 \cdot R_1^2 \cdot R_2^3$

↳ first row, 3 elements of E are coordinates of end effector.

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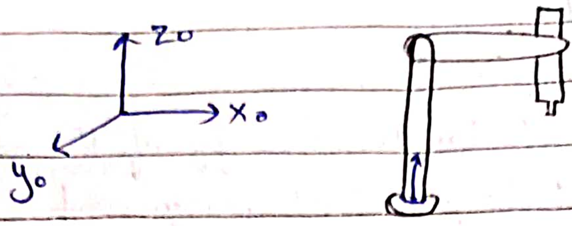
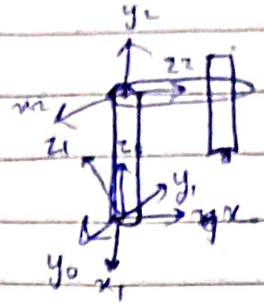
Q-2 Various Coordinate point frames of $p_0$ using a composition of homogeneous transformations for the RRP SCARA configuration.



## * Stanford RRP configuration,




0 - frame - inertial frame at base
1 - frame - frame attached to link 1
2 - frame - frame attached to link 2

$$R_0^1 = [R_{z,q_1}] = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = R_{x,\pi/2} R_{z,q_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\pi/2 & -s\pi/2 \\ 0 & s\pi/2 & c\pi/2 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_1^2$$

$$\therefore H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}, \quad H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_1^2 = \begin{bmatrix} cq_2 & -sq_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ sq_2 & cq_2 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q-7 Derive the Manipulator Jacobian for RRP SCARA configuration.

We know,

$J = [J_1 \; J_2 \ldots J_n]$,

where, $i^{th}$ column is $J_i = \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix}$ (for revolute joint)

$J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix}$ (for prismatic joint.

here $i$ is number of joint, $n$ is total joints

For RRP SCARA configuration

$1^{st}$ R $\Rightarrow$ $J_1 = \begin{bmatrix} z_{1-1} \times (o_3 - o_0) \\ z_0 \end{bmatrix}$

$2^{nd}$ R $\Rightarrow$ $J_2 = \begin{bmatrix} z_{2-1} \times (o_3 - o_1) \\ z_1 \end{bmatrix}$

1 P $\Rightarrow$ $J_3 = \begin{bmatrix} z_2 \\ 0 \end{bmatrix}$

$\therefore$ $J = [J_1 \; J_2 \; J_3]$

$\therefore$ $J = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$

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Q-7 Derive the Manipulator Jacobian for PRR config with all rotation axis parallel to each other (entire robot is planar like the elbow manipulator)

We know,

$J = [J_1 \; J_2 \; J_3]$

where, $i^{th}$ column is $J_i = \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix}$ (for revolute joint)

1R $\Rightarrow$ $J_1 = \begin{bmatrix} z_0 \times (o_3 - o_0) \\ z_0 \end{bmatrix}$

2R $\Rightarrow$ $J_2 = \begin{bmatrix} z_1 \times (o_3 - o_1) \\ z_1 \end{bmatrix}$

3R $\Rightarrow$ $J_3 = \begin{bmatrix} z_2 \times (o_3 - o_2) \\ z_2 \end{bmatrix}$

$\therefore J = [J_1 \; J_2 \; J_3]$

$\therefore J = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 \times (o_3 - o_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$

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Q-5

A-5 Let's consider an inertial frame attached at a base station with z-axis pointing straight up and x and y axes along the ground forming a right hand system. The drone takes off from the base station and travels 10m straight up, which would be 10m in the z direction. This can be represented by a translation matrix TI:

$$TI = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

At hover point, the drone orientation is as if it completed a 30° rotation about the x-axis followed by a 60° rotation about the resulting new (current) z-axis. These rotation can be represented by rotation matrixes $R_1$ & $R_2$:

$$R_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(30) & -\sin(30) & 0 \\ 0 & \sin(30) & \cos(30) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R_2 = \begin{bmatrix} \cos 60 & -\sin 60 & 0 & 0 \\ \sin 60 & \cos 60 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It is then observed using a lidar installed on the drone that an obstacle is 3 m exactly above the drone (in the drone frame). This can be represented by a translation matrix T2 /

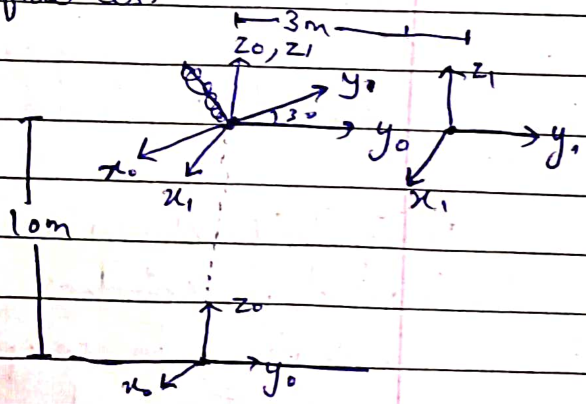
$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The position vector of obstacle w.r.t base coordinate frame can be calculated by multiplying these matrices together:

$T = T_1 . R_1 . R_2 . T_2$

∴ coordinates of are first 3 element of $I^{st}$ row.

This gives us the position vector of the obstacle with respect to the base coordinates frames.



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Q-6

A-6 There are several types of gearboxes that are typically used with motors in robotics applications. Some of the most common types include planetary gearboxes, harmonic drives, and cycloid drives.

① Planetry gearbox are ~~connect~~ compact and versatile devices that can provide high torque and speed ~~reed~~ reduction in a small packages. They are often used in industrial robots and other heavy-duty applications.

② Harmonics drives, also known as strain wave gearboxes, are lightweight and provides zero backlash, making them ideal for precise positioning applications. They are commonly used in industrial robots and other high-precision machinery.

③ In drone applications, gearboxes are not typically used along with motors. Drones ~~a~~ usually use brushless motors that provides high power to weight ratio and efficiency without the need for a gearbox.

④ Cycloid drives are another types of gearbox commonly used in robotics. They provide high torque and speed reduction in compact package and are often used in application where high precision is not required.