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Assignment :- 2

(1) Here, we need to prove $RS(a)R^T = S(Ra)$

And, we are give R as the rotational matrix

Now, let us take any $\vec{b} \in \mathbb{R}^3$

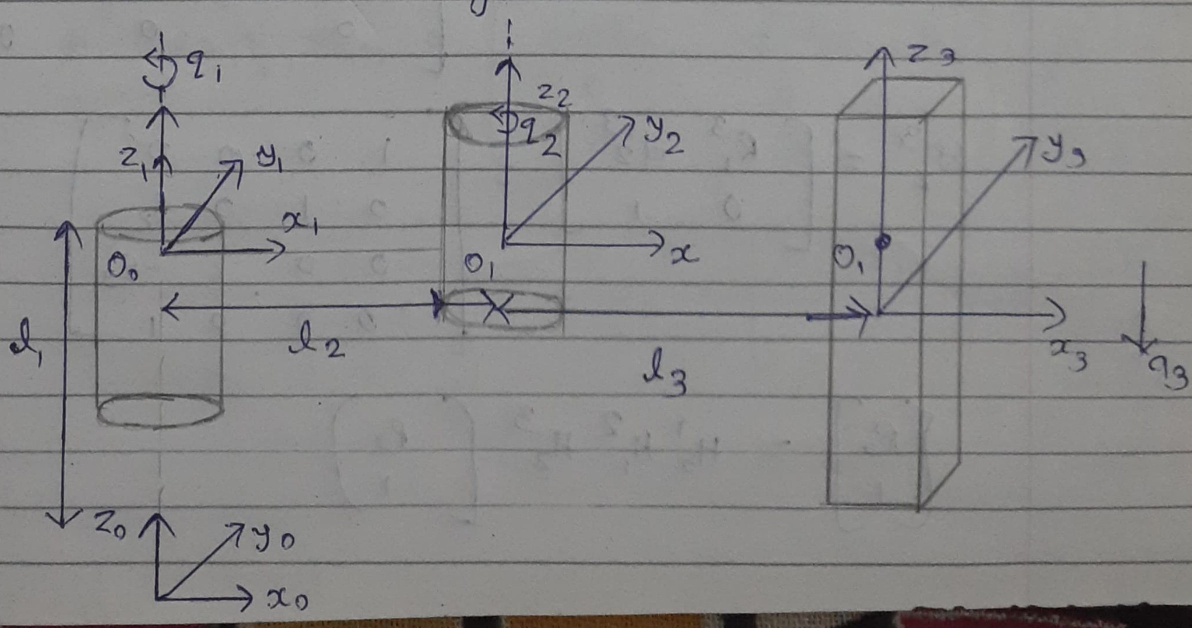
$$RS(\vec{a})R^T\vec{b} = R(\vec{a} \times R^T\vec{b}) \quad (\because S(a)\cdot\vec{p} = \vec{a}\times\vec{p})$$
$$= (R\vec{a}) \times (RR^T\vec{b})$$

but we know that $RR^T = I$ (Identity matrix)

$$RS(\vec{a})R^T\vec{b} = (R\vec{a}) \times \vec{b}$$
$$= S(R\vec{a})\vec{b}$$

Hence, $RS(\vec{a})R^T = S(R\vec{a})$

(2) SCARA (RRP) Configuration



we can write the rotation matrixes as follows:

$$R_0^1 = R_{z,q_1} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_0^1 = \begin{bmatrix} l_1 \cos q_1 \\ l_1 \sin q_1 \\ 0 \end{bmatrix}$$

$$R_1^2 = R_{z,q_2} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} l_2 \cos q_2 \\ l_2 \sin q_2 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ -l_3 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0 \\ 0 \\ -d \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & l_1 \cos q_1 \\ \sin q_1 & \cos q_1 & 0 & l_1 \sin q_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

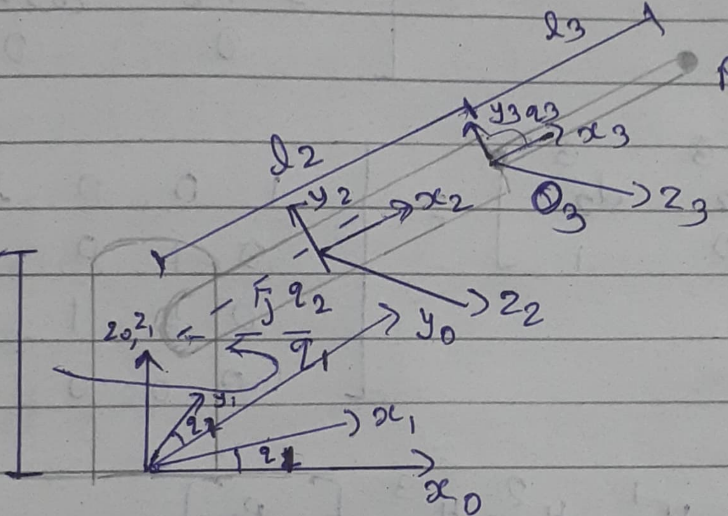
$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & l_2 \cos q_2 \\ \sin q_2 & \cos q_2 & 0 & l_2 \sin q_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} -l_1 \cos q_1 + l_2 \cos(q_1 + q_2) \\ l_1 \sin q_1 + l_2 \sin(q_1 + q_2) \\ -l_3 - d \end{bmatrix}$$

(4) Stanford-type (RRP) Configuration



We can write down the rotation matrices as following

$$R_0^1 = R_{z,q_1} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$R_1^2 = R_{x,\frac{\pi}{2}} R_{z,q_2} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ 0 & 0 & -1 \\ \sin q_2 & \cos q_2 & 0 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_2^3 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix}$$

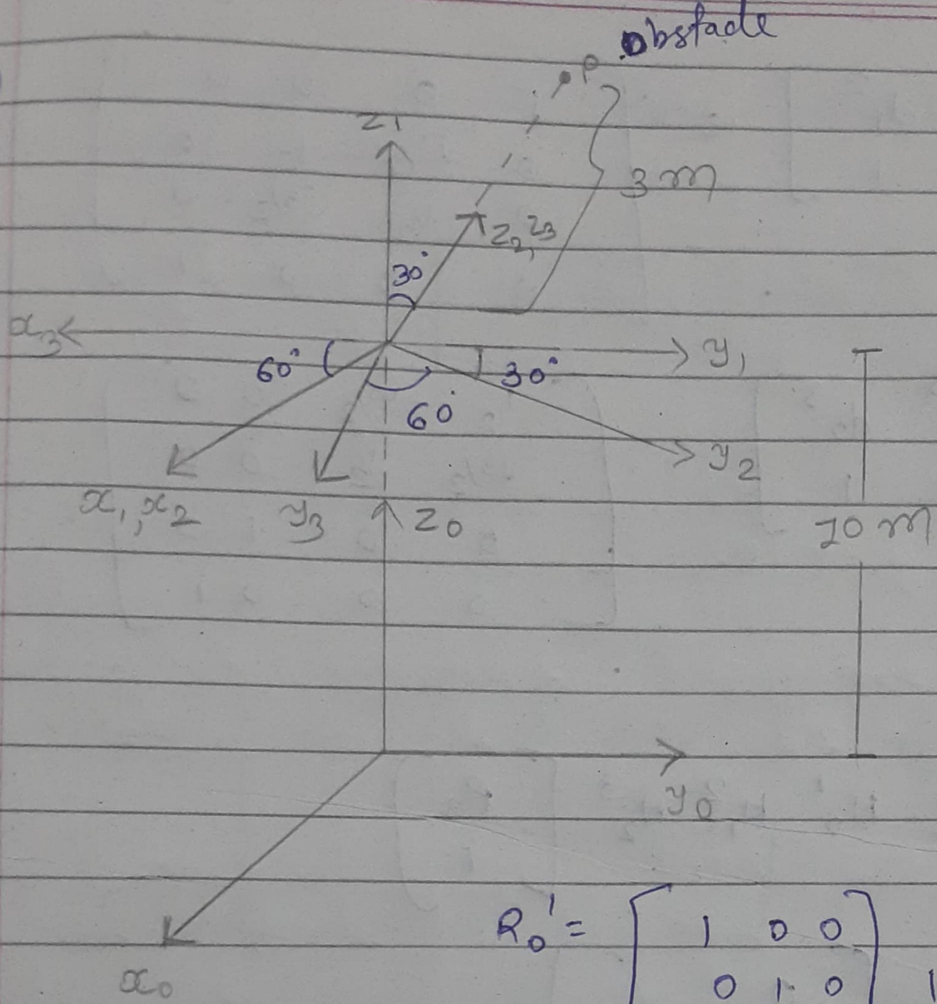
$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & l_2 \\ 0 & 0 & -1 & 0 \\ \sin q_2 & \cos q_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \cdot H_1^2 \cdot H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

(5)



$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_1^2 = R_{x,30} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}, \quad R_2^3 = R_{z,60} = \begin{bmatrix} \cos 60 & -\sin 60 & 0 \\ \sin 60 & \cos 60 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So,

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \, H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 + 10 \\ 1 \end{bmatrix}$$

So, $P_0 = \begin{bmatrix} 0 \\ -3/2 \\ 10 + \frac{3\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ -1.5 \\ 12.598 \end{bmatrix}$

Qn 6)

<u>Planetary Gearbox:</u>

- Pros: High torque output, compact size, and high efficiency due to many gear connections. Suitable for applications requiring precise motion control and limited areas, such as robotic arms and CNC machines.
- Cons: Slightly more complex and pricey than other types of gearboxes.

<u>Worm Gearbox:</u>

- Pros: The self-locking system prevents back-driving, making it ideal for applications that need retaining positions without power. It is effective in applications like conveyor belts and hoisting mechanisms.
- Cons: Lower efficiency from sliding contact can result in heat generation and a shorter lifespan.

<u>Spur Gearbox:</u>

- Pros: Simple design, low cost, and suitability for a wide range of applications. When properly developed, it provides good efficiency.
- Cons: Larger in size than planetary gears for comparable torque output. More noise and vibration may be produced.

<u>Harmonic Drive Gearbox:</u>

- Pros: zero backlashes, great precision, and an excellent torque-to-weight ratio. Ideal for high precision and compactness applications such as robotic joints and medical equipment.
- Cons: Higher cost and worse efficiency compared to other gearboxes.

Adding a gearbox would raise the cost of manufacturing a drone and complicate the mechanical system. Consumer drones are typically meant to be affordable, with manufacturers aiming to reach a balance between performance and cost-effectiveness. Drones are also designed to be lightweight in order to increase flying time and maneuverability. Adding a gearbox would increase weight, affecting the drone's overall efficiency and flight durability. Consumer drones emphasize portability when doing hovering photography or filmmaking. The use of a gearbox in a drone is determined by the application's specific requirements, which must balance considerations such as payload capacity and flight time against the extra weight and expense of the gearbox.

(7) For SCARA manipulator (RRP) Jacobian can be given as following

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 (o_3 - o_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

prismatic joint

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad O_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}, \quad O_2 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} c_1 & -s_1 & 0 & l_1 c_1 \\ s_1 & c_1 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad O_3 = \begin{bmatrix} l_1 c_1 + l_1 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ d_3 - d_4 \end{bmatrix}$$

$$H_0^2 = H_0^1 \cdot H_1^2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{12} & c_{12} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

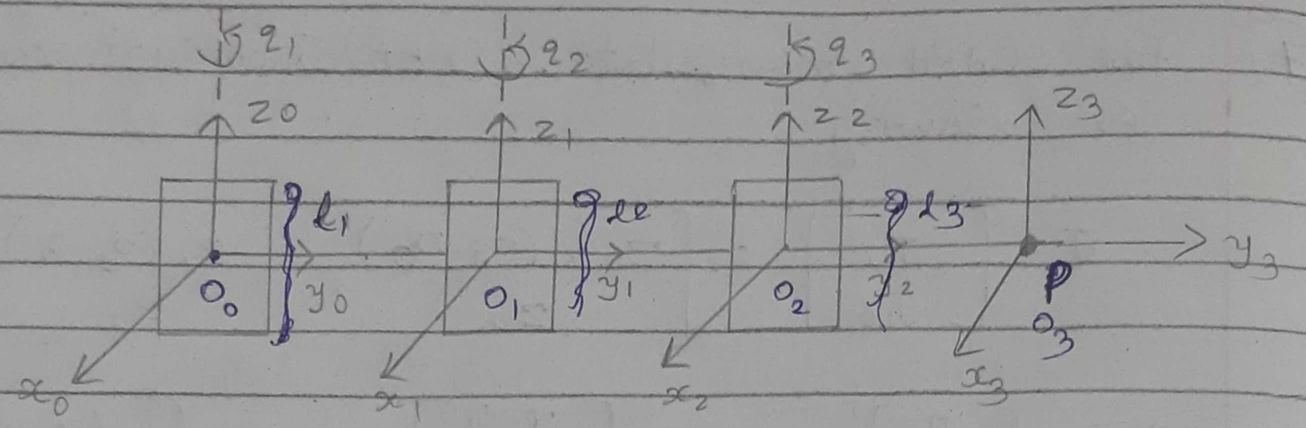
$$H_0^3 = H_0^2 \cdot H_2^3 = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{12} & c_{12} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = z_1 \quad \& \quad z_2 = z_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

So,

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

(9)



Jacobian is given as,

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \times (O_3 - O_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad O_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}, \quad O_2 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{13} \\ 0 \end{bmatrix}$$

$$z_0 = z_1 = z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$ (here all the z axis directions are parallel to each other)

So,

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$