Q1

$$a = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad R = R_{z,\pi/2}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\pi/2 & -\sin\pi/2 \\ 0 & \sin\pi/2 & \cos\pi/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$S(a) = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}, \quad R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$Ra = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ -z \\ y \end{bmatrix}$$

$$\Rightarrow S(Ra) = \begin{bmatrix} 0 & -y & -z \\ y & 0 & -x \\ +z & x & 0 \end{bmatrix}$$

Now

$$R\,S(a)\,R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -z & +y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

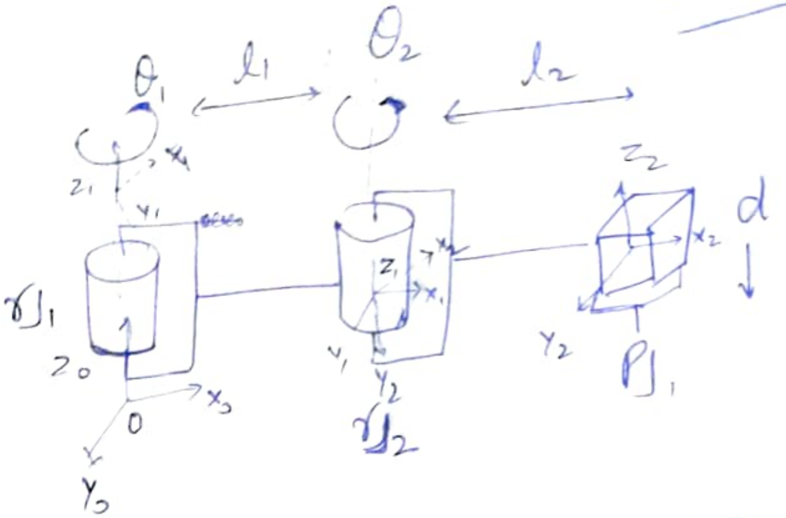
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -y & -z \\ z & x & 0 \\ -y & 0 & x \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -y & -z \\ +y & 0 & -x \\ z & x & 0 \end{bmatrix}$$

Thus $\quad R\,S(a)\,R^T = S(Ra)$

SCARA RRP

Q2



For rotation of rJ_1 and translation by l_1

$$P_1^0 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & l_1 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & l_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

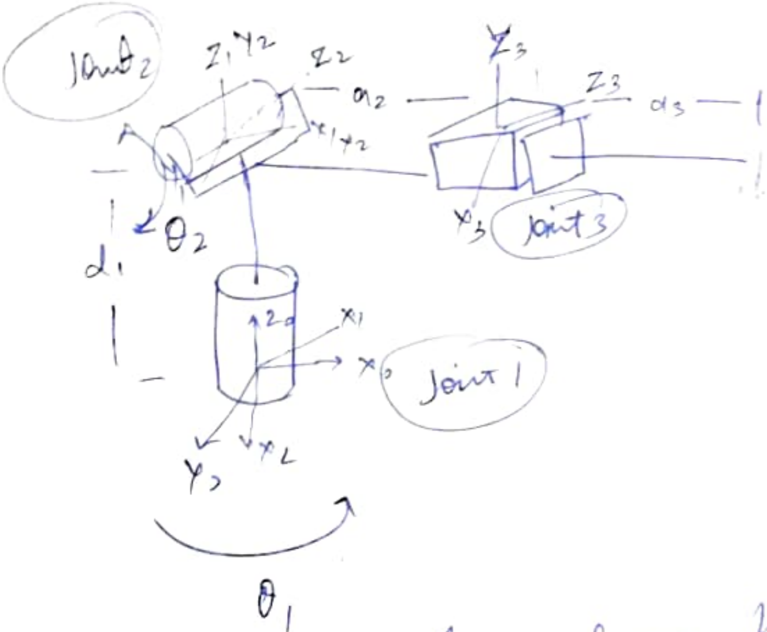
For rotation of rJ_2 and translation by l_2

$$P_2^1 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & l_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For translation along PJ by -d.

$$P_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus $P_3^0 = P_1^0 * P_2^1 * P_3^2$



Q4

Stanford type
RRP

1 - Transformation from base to joint 1:

T_1^0 = Rotation(z θ_1) * translation(z, d_1)

$$= \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & \theta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

T_2^1 = Rotation(x, $\pi/2$) * Rotation (z, θ_2) * tran(x, d_2)

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

T_3^2 = translation (x, d_3)

$$= \begin{bmatrix} 1 & 0 & 0 & d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus $T_3^0 = T_1^0 * T_2^1 * T_3^2$

Q5

① Since drone travels straight along z-axis without rotation

$$T_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

② 30° rotation about x-axis

$$T_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ & 0 \\ 0 & \sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

③ 60° rotation about new z-axis

$$T_3^2 = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

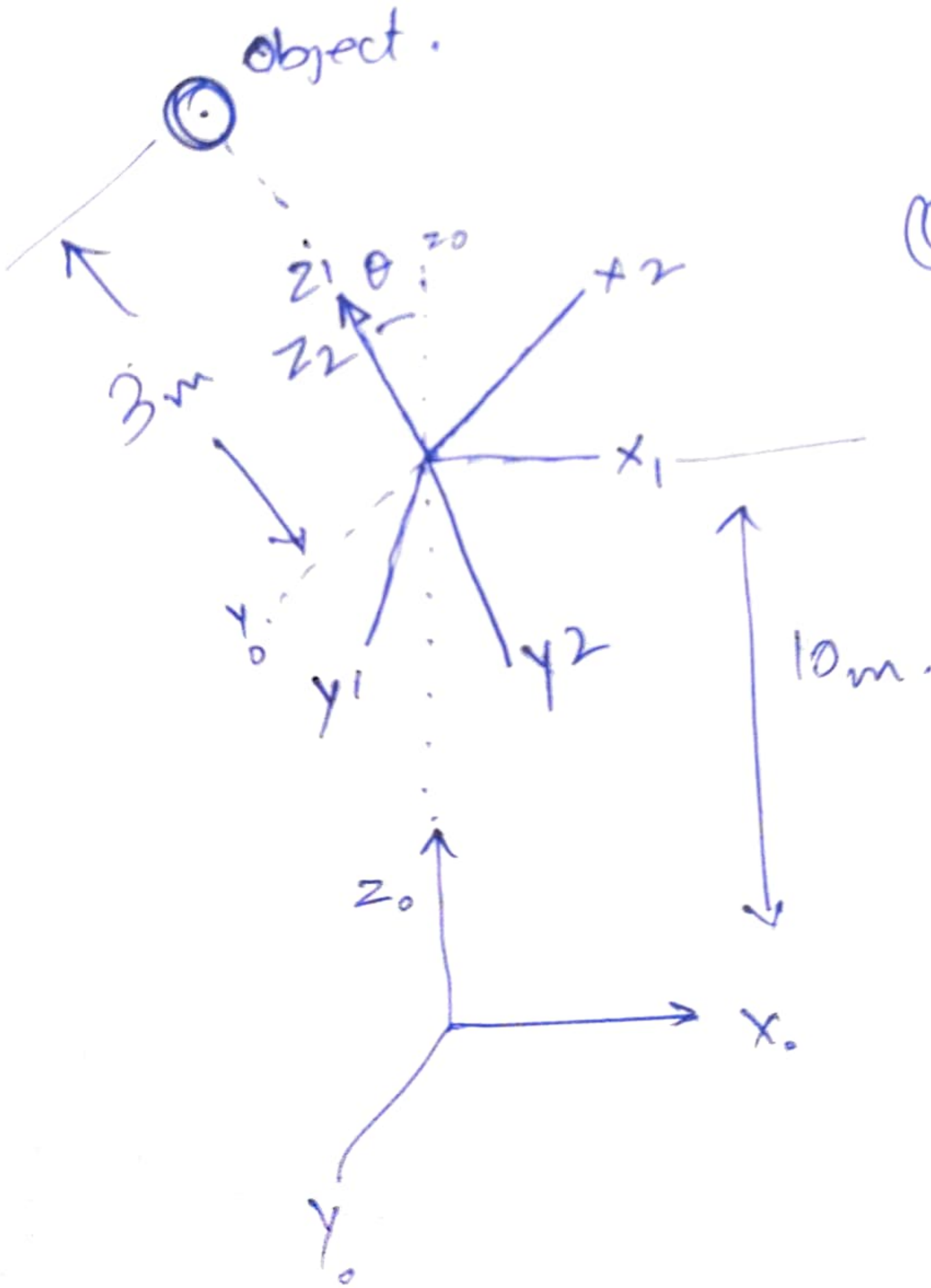
$$\therefore T_3^0 = T_1^0 * T_2^1 * T_3^2$$

$$T_3^0 = \begin{bmatrix} 0.86 & -0.5 & 0 & 0 \\ 0.43 & 0.75 & -0.5 & 0 \\ 0.25 & 0.43 & 0.86 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

④ Obstacle's position $(0,0,3,1)$ in base frame

$$P_0 = T_3^0 \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1.5 \\ 12.598 \\ 1 \end{bmatrix}$$

Q5.



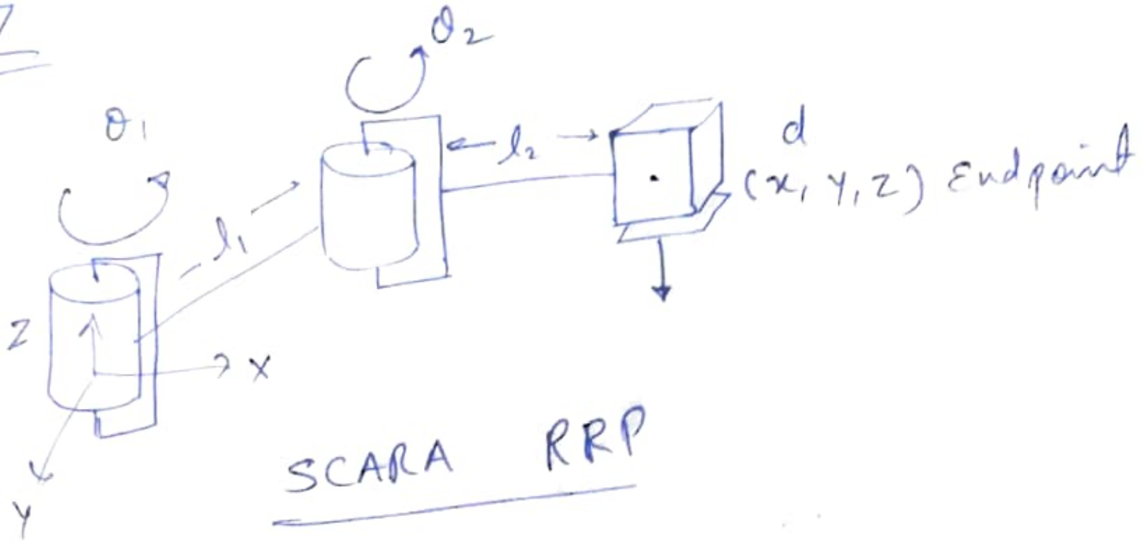
Q6 In robotic applications, various types of gearbox are utilized :

① Planetary gearbox offers high torque trasmission and compact size, making it suitable where space is limited, Such as a robotic arm. However it has some backlash error issues.

② harmonic drive gearbox offers high precision and minimal backlash. it is suitable in applications requiring accurate positioning. these gearbox are very expensive.

In drone application, a gearbox is typically not used with a motor due to emphasis on weight reduction. Geer add weight and Complexity which has negative impact on drone performance.

Q7



SCARA RRP

From the given diagram, the cordinates of the end effector from base frame are:

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$$

$$z = -d.$$

Now,

$$\frac{\partial x}{\partial \theta_1} = -l_1 \sin\theta_1 - l_2 \sin(\theta_1 + \theta_2)$$

$$\frac{\partial x}{\partial \theta_2} = -l_2 \cdot \sin(\theta_1 + \theta_2)$$

$$\frac{\partial x}{\partial z'} = 0$$

$$\frac{\partial y}{\partial \theta_1} = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$$

$$\frac{\partial y}{\partial \theta_2} = l_2 \cos(\theta_1 + \theta_2)$$

$$\frac{\partial y}{\partial z'} = 0$$

$$\frac{\partial z}{\partial \theta_1} = 0$$

$$\frac{\partial z}{\partial \theta_2} = 0$$

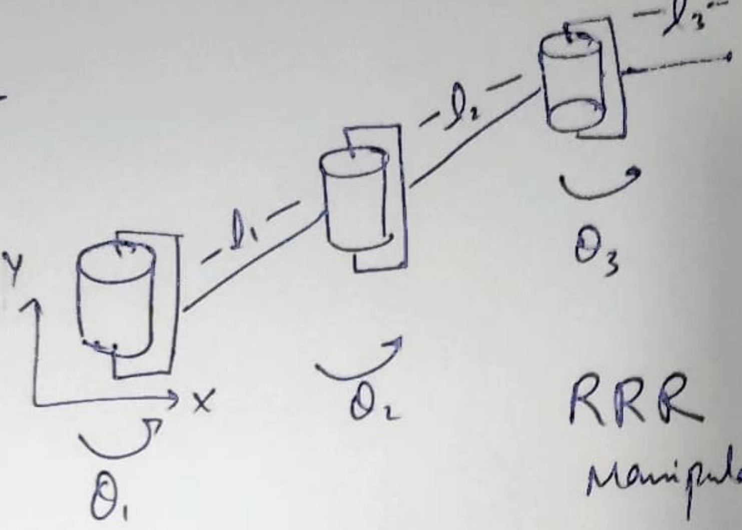
$$\frac{\partial z}{\partial z'} = 1$$

$$\therefore \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = [J] \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial z'} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial z'} \\ \frac{\partial z}{\partial \theta_1} & \frac{\partial z}{\partial \theta_2} & \frac{\partial z}{\partial z'} \end{bmatrix}$$

$$= \begin{bmatrix} (-l_1 s\theta_1 - l_2 s(\theta_1+\theta_2)) & -l_2 s(\theta_1+\theta_2) & 0 \\ (l_1 c\theta_1 + l_2 c(\theta_1+\theta_2)) & l_2 c(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q9



RRR planar elbow Manipulator.

$$x = l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) + l_3 c(\theta_1+\theta_2+\theta_3)$$

$$y = l_1 s\theta_1 + l_2 s(\theta_1+\theta_2) + l_3 s(\theta_1+\theta_2+\theta_3)$$

$$\frac{\partial x}{\partial \theta_1} = -l s\theta_1 - l_2 s(\theta_1+\theta_2) - l_3 s(\theta_1+\theta_2+\theta_3)$$

$$\frac{dx}{\partial \theta_2} = -l_2 s(\theta_1+\theta_2) - l_3 s(\theta_1+\theta_2+\theta_3)$$

$$\frac{\partial x}{\partial \theta_3} = -l_3 s(\theta_1+\theta_2+\theta_3)$$

$$\frac{\partial y}{\partial \theta_1} = l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) + l_3 c(\theta_1+\theta_2+\theta_3)$$

$$\frac{\partial y}{\partial \theta_2} = l_2 c(\theta_1+\theta_2) + l_3 c(\theta_1+\theta_2+\theta_3)$$

$$\frac{\partial y}{\partial \theta_3} = l_3 c(\theta_1+\theta_2+\theta_3)$$

$$\Rightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \end{bmatrix}}_{\text{Jacobian matrix.}} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$