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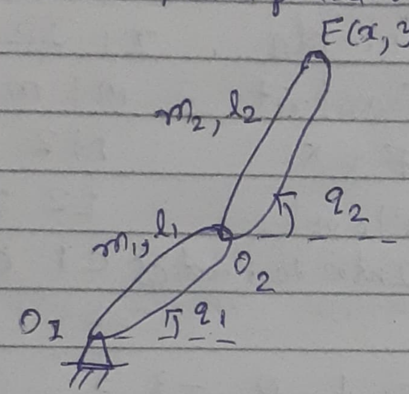
* Elbow Manipulator



E = end effector
(x, y) = end effector position
(q_1, q_2) = angles w.r.t horizontal

Assume → origin at O_1
→ Motors are connected to each link at O_1 & O_2 respectively.

$$\left.\begin{aligned} x &= l_1 \cos q_1 + l_2 \cos q_2 \\ y &= l_1 \sin q_1 + l_2 \sin q_2 \end{aligned}\right\} \quad \text{— (1)}$$

Let's differentiate eq^n (1),

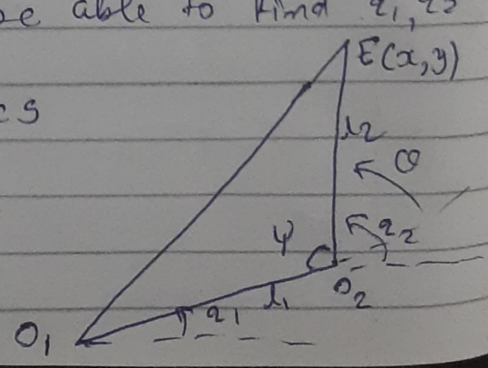
$$\dot{x} = -l_1 \sin q_1 \, \dot{q}_1 - l_2 \sin q_2 \, \dot{q}_2$$
$$\dot{y} = l_1 \cos q_1 \, \dot{q}_1 + l_2 \cos q_2 \, \dot{q}_2$$

So, End effector velocity

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & -l_2 \sin q_2 \\ l_1 \cos q_1 & l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad \text{— (2)}$$

We will need the reverse relationships.
Given x, y, & we need to be able to find q_1, q_2

Taking ~~reve~~ inverse kinematics

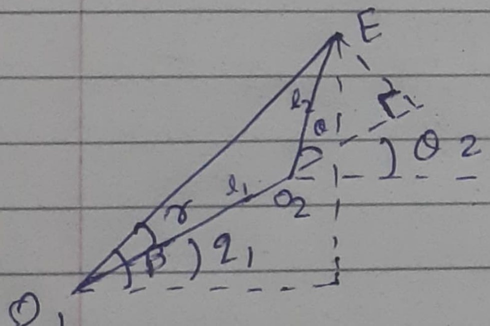


In $\Delta O_1 O_2 E$,
using cosine rule

$$x^2+y^2 = l_1^2 + l_2^2 - 2l_1l_2\cos\psi$$
$$x^2+y^2 = l_1^2 + l_2^2 + 2l_1l_2\cos\theta$$

$$\therefore \cos\theta = \frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}$$

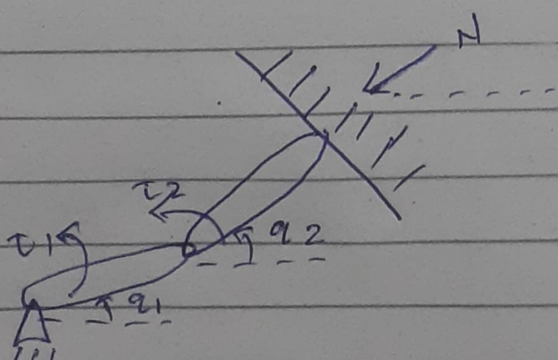
$$\therefore \theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right) \quad \text{— (3a)}$$



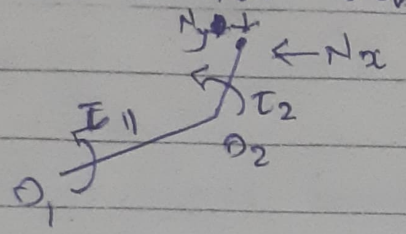
$$q_1 = \beta - \gamma$$
$$q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\theta}{l_1+l_2\cos\theta}\right) \quad \text{(3b)}$$

$$q_2 = q_1 + \theta \longrightarrow \text{(3c)}$$

*



FBD of entire robot

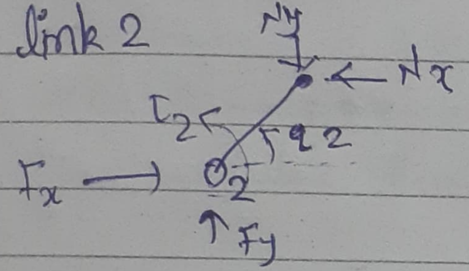


Forces applied by the manipulator
$F_x = -N_x, \; F_y = -N_y$
Neglect gravity

(static equilibrium)

* FBD of each link separately.

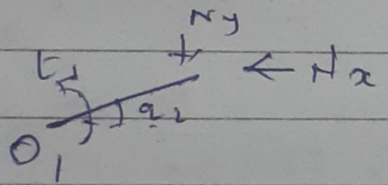
link 2



$\Sigma M_{\theta_2} = 0$ C.C.W (+ve)

$\Rightarrow N_y l_2 cq_2 - N_x l_2 sq_2 = \tau_2$

* FBD of link 1



$\Sigma M_{\theta_1} = 0$

$\Rightarrow N_y l_1 cq_1 - N_x l_1 sq_1 = \tau_1$

& $N_y l_2 cq_2 - N_x l_2 sq = \tau_2$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 sq_1 & l_1 cq_1 \\ -l_2 sq_2 & l_2 cq_2 \end{bmatrix} \begin{pmatrix} N_x \\ N_y \end{pmatrix} \quad \text{--- (4)}$$

* For τ_3 and next level answer to τ_1 Need to understand dynamics.

* Lagrange's Equations
Lagrangian $\mathcal{L} = K - V$

K - kinetic energy , V = potential energy

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i' \quad \text{—} \ (\text{J})$$

$\Rightarrow$ Q_i are generalized forces derived using principle of virtual work.

K.E of link 2

$$K = \frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2 + \frac{1}{2} m_2 v_{c_2}^2$$

pure rotation of l_1

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2}\dot{q}_2 \cos(q_2 - q_1)$$

$$V = m_1 g \frac{l_1}{2} s q_1 + m_2 g \left(l_1 s q_1 + \frac{l_2}{2} s q_2\right)$$

putting value in eq (5)

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 + q_1)$$

$$- m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$

$$+ m_1 g \frac{l_1}{2} c q_1 + m_2 g l_1 c q_1 = \tau_1$$

(6a)

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1)$$

$$- m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} s q_2 = \tau_2$$