Q :-1 $RS(a)R^T = S(Ra)$ shows that ??

LHS $= RS(a)R^T$

$$R = \begin{bmatrix} C\theta & S\theta & 0 \\ -S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R^T = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{a} = [a_1\ a_2\ a_3]^T$$

$$S(a) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

$$RS(a)R^T = \begin{bmatrix} C\theta & S\theta & 0 \\ -S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a_3S\theta & -a_3C\theta & a_2C\theta - a_1S\theta \\ a_3C\theta & a_3S\theta & -a_2S\theta - a_1C\theta \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -a_3 & a_2C\theta - a_1S\theta \\ a_3 & 0 & -a_2S\theta - a_1C\theta \\ -a_2C\theta + a_1S\theta & a_2S\theta + a_1C\theta & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -a_3 & (a_2C\theta - a_1S\theta) \\ a_3 & 0 & -(a_2S\theta + a_1C\theta) \\ -(a_2C\theta - a_1S\theta) & (a_2S\theta + a_1C\theta) & 0 \end{bmatrix}$$

RHS $\Rightarrow S(Ra)$

$$Ra = \begin{bmatrix} C\theta & S\theta & 0 \\ -S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 C\theta + a_2 S\theta \\ -a_1 S\theta + a_2 C\theta \\ a_3 \end{bmatrix}$$

$$S(Ra) = \begin{bmatrix} 0 & -a_3 & a_2 C\theta - a_1 S\theta \\ +a_3 & 0 & -(a_1 C\theta + a_2 S\theta) \\ -(a_2 C\theta - a_1 S\theta) & (a_1 C\theta + a_2 S\theta) & 0 \end{bmatrix}$$

$$LHS = RHS$$

$$\boxed{R S(a) R^T = S(Ra)}$$

Q:-6 Ans:- Different types of gear boxes used in Robotics- for Speed Control, torque and dirn of motion ~~of~~ in robotic system.

① Planetary Gearbox -

It Consist a Centeral sun gear, planet gears & ring gears. These gears provide high torque transmission with low backlash. Planetary gear boxes are often used in robotics arms and manipulator where space is limited and precision is crucial.

② Spur gearbox:- Spur gears are Simple and widely used. They have straight teeth and are designed for parallel shafts. These gear boxes are Commonly found in robots that require low precision, like wheeled robots.

③ Helical Gearbox :- These have inclined teeth, which allows for Smoother and quieter operation Compared to

Spur gears. Helical gearboxes are used when higher torque and precision are required, such as robotic arm and CNC machine.

(4) Bevel gearbox :- These gear boxes are used where dirn of rotation needs to be change. It has intersecting shaft.

(5) Worm gearbox :- It has higher reduction ratio. Its applications where the motor needs to drive heavy loads.

(6) Cycloidal gearbox :- These have a unique mechanism involving pins and roller to achieve high torque transmission with low backlash. They are used in applications requiring high shock tolerance and precise control.

(7) Harmonic Drive :- It use a flexspline that is deformed by an elliptical wave generator. This enables high reduction ratios with zero backlash. These are used in applications requiring precision, such as robotic Arm in medical procedures and space exploration.

(8) Parallel shaft gearbox :- These gear box have parallel shaft like (spur and helical gear. This gear box is used where compact size is not concern, such as Industrial robots.

(9) Hybrid gearbox :- These gearbox made with combination of different gears to achieve specific performance.

① Planetary Gearbox

Pros :- Compact design, high torque, Low backlash
Cons :- Complex assembly and maintenance

Application :- Robotics arm, precision positioning system, Satellite mechanism.

② Spur Gearbox

pros :- Simple design, less cost
Cons :- High impact stress, High noise, Low torque

Application :- Wheeled robots, Conveyor systems

③ Helical Gearbox.

pros :- Smoother Operation, less noise, High torque
Cons :- Complex design

Application :- Robotic arm, CNC Machines

④ Bevel gearbox.

pros :- Change the dir^n of shaft, Compact design
Cons :- Limited torque

Application :- Robotic joints

⑤ Worm gearbox

pros :- Higher speed reduction, Self locking ability
Cons :- friction worm and Wormwheel

Application :- Robotic grippers, heavy load handling

⑥ Cycloidal gearbox

pros :- High shock tolerance, low backlash
Cons :- Complex mechanism
App^n :- Industrial Robots.

⑦ Harmonic drive.

pros :- High precision, ~~Hig~~ light weight
Cons :- expensive
Appⁿ :- Robotic arm in medical procedures, space exploration

⑧ parallel shaft gearbox.

pros :- Versatile, low cost
Cons :- limited torque
Appⁿ :- packaging Machinary.

⑨ Hybrid gearbox

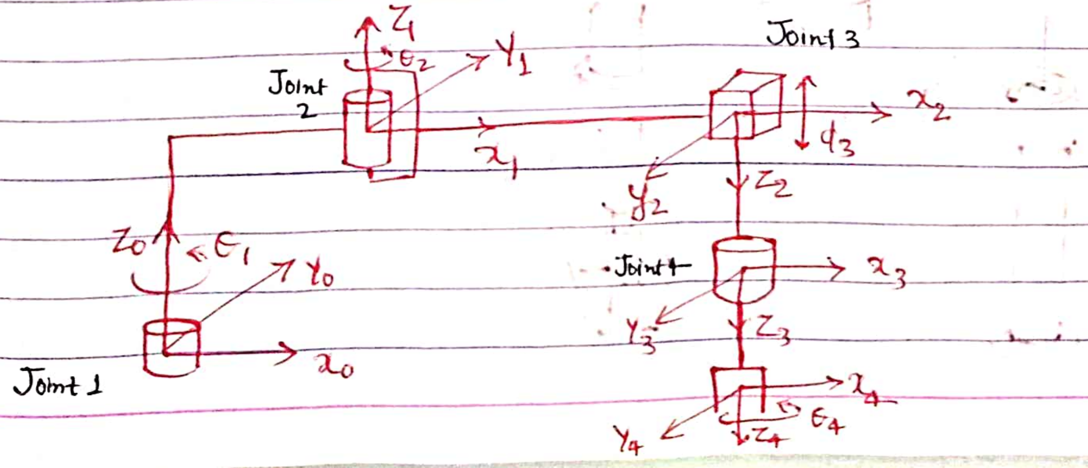
pros :- Advantages of multiple gears
Cons :- Design complexity can increase.
Appⁿ :- Used for specific applications.

Gearbox used in Drone application -

Gearbox with brushless Dc Motor.

Reasons :- ⓐ Torque and thrust
ⓑ Rotor speed reduction
ⓒ propeller matching
ⓓ Control precision
ⓔ Motor size

Q:-7 Ans :- Manipulator Jacobian for RRP SCARA Configration



Joint 1, 2 & 4 are revolute and joint 3 is prismatic.

$\therefore$ $O_4 - O_3$ is parallel to Z_3

then $Z_3 \times (O_4 - O_3) = 0$

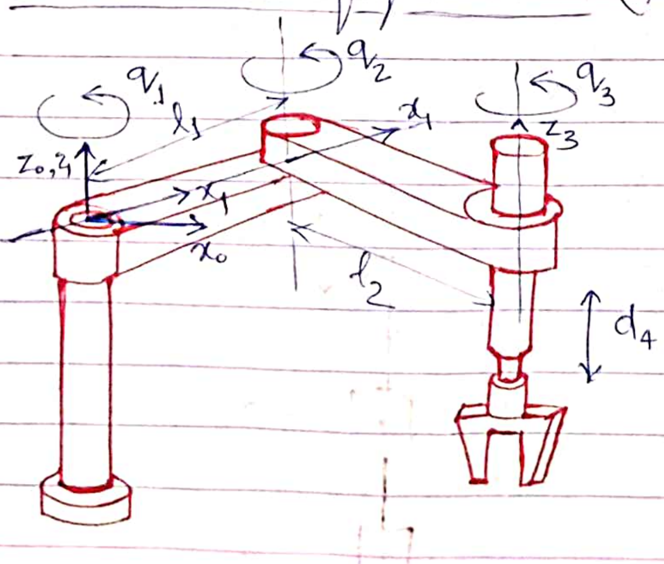
$$J = \begin{bmatrix} Z_0 \times (O_4 - O_0) & Z_1 \times (O_4 - O_1) & Z_2 & 0 \\ Z_0 & Z_1 & 0 & Z_3 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad O_1 = \begin{bmatrix} a_1 C_1 \\ a_1 S_1 \\ 0 \end{bmatrix} \quad O_2 = \begin{bmatrix} a_1 C_1 + a_2 C_{12} \\ a_1 S_1 + a_2 S_{12} \\ 0 \end{bmatrix}$$

$$O_4 = \begin{bmatrix} a_1 C_1 + a_2 C_{12} \\ a_1 S_1 + a_2 S_{12} \\ d_3 - d_4 \end{bmatrix} \qquad \begin{aligned} Z_0 = Z_1 = K &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ Z_2 = Z_3 = -K &= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \end{aligned}$$

$$J = \begin{bmatrix} -a_1 S_1 - a_2 S_{12} & -a_2 S_{12} & 0 & 0 \\ a_1 C_1 + a_2 C_{12} & a_2 C_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

Q:-2 SCARA Configration (RRP)



$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 H_3^4 \begin{bmatrix} P_4 \\ 1 \end{bmatrix}$$

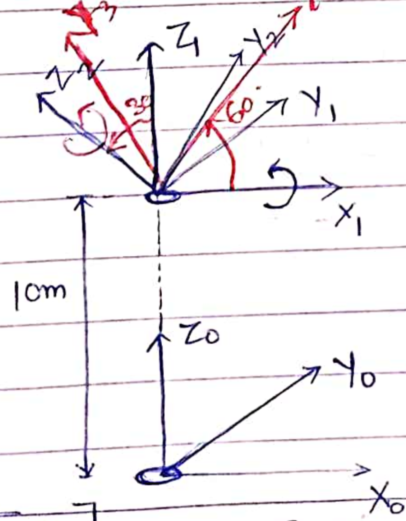
$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_1 & sq_1 & 0 & 0 \\ -sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_2 & sq_2 & 0 & l_1 \\ -sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_3 & sq_3 & 0 & l_2 \\ -sq_3 & cq_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$H_3^4 = \begin{bmatrix} I & d_3^4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad d_3^4 = \begin{bmatrix} 0 \\ 0 \\ -d_4 \end{bmatrix}$$

Q:- 5



$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} I & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\theta & s\theta & 0 \\ 0 & -s\theta & c\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C30 & S30 & 0 \\ 0 & -S30 & C30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta & S\theta & 0 & 0 \\ -S\theta & C\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C60 & S60 & 0 & 0 \\ -S60 & C60 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

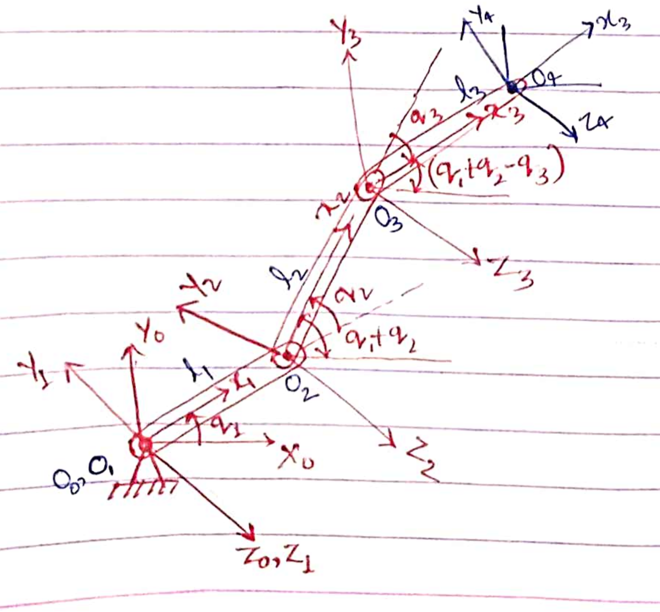
$$= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.5 \\ 12.598 \\ 1 \end{bmatrix}$$

Q:9 Manipulator Jacobian for RRR Configuration With all Rotational axis parallel.



Since all are revolute Joint -

$$J = \begin{bmatrix} z_1 \times (O_4 - O_1) & z_2 \times (O_4 - O_2) & z_3 \times (O_4 - O_3) \\ z_1 & z_2 & z_3 \end{bmatrix}$$

~~$z_1 = z_2 = z_3 = k$~~

$$z_{i-1} = R_0^{i-1} \hat{k}$$

$$z_1 = R_0^1 \hat{k}$$

$$= \begin{bmatrix} cq_1 & sq_1 & 0 \\ -sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_2 = R_0^2 \hat{k} = R_0^1 R_1^2 \hat{k}$$

$$= \begin{bmatrix} cq_1 & -sq_1 & 0 \\ -sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & sq_2 & 0 \\ -sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Similarly $Z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$Z_1 = Z_2 = Z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$O_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad O_2 = \begin{bmatrix} l_1 \cos q_1 \\ l_1 \sin q_1 \\ 0 \end{bmatrix} \qquad O_3 = \begin{bmatrix} l_1 cq_1 + l_2 c(q_1+q_2) \\ l_1 sq_1 + l_2 s(q_1+q_2) \\ 0 \end{bmatrix}$$

$$O_4 = \begin{bmatrix} l_1 cq_1 + l_2 c(q_1+q_2) + l_3 c(q_1+q_2-q_3) \\ l_1 sq_1 + l_2 s(q_1+q_2) + l_3 s(q_1+q_2-q_3) \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} l_1 cq_1 + l_2 c(q_1+q_2) + l_3 c(q_1+q_2-q_3) \\ l_1 sq_1 + l_2 s(q_1+q_2) + l_3 s(q_1+q_2-q_3) \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} l_2 c(q_1+q_2) + l_3 c(q_1+q_2+q_3) \\ l_2 s(q_1+q_2) + l_3 s(q_1+q_2+q_3) \\ 0 \end{bmatrix} \right) & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_3 c(q_1+q_2-q_3) \\ l_3 s(q_1+q_2-q_3) \\ 0 \end{bmatrix} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$