Assignment 2

Thursday, August 31, 2023 23:17

1. Show that $R\,S(a)\,R^T = S(Ra)$, R is a rotation matrix.

$$S(Ra)\cdot Ra = (R\cdot a)\times(Ra) \qquad [\because S(a)p = a\times p]$$

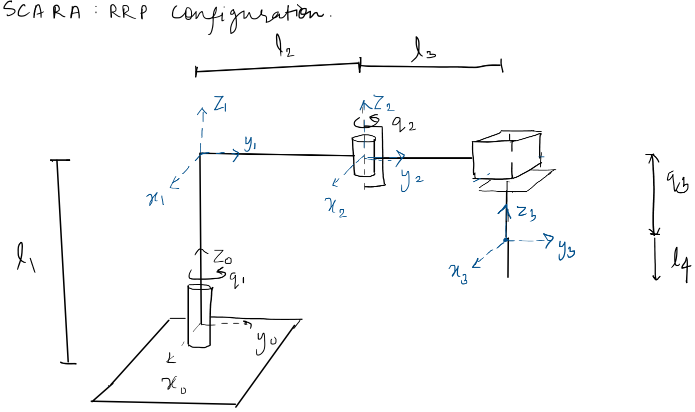
$$= R\cdot(a\times a)$$
$$= R\cdot S(a)\,a$$
$$= R\cdot S(a)\,I_{3\times3}\,a$$
$$= R\cdot S(a)\,R^TR\,a \qquad [\because R^T = R^{-1}]$$

$$S(Ra)\cdot Ra = R\,S(a)\,R^T\,Ra$$

Comparing RHS and LHS, we get:

$$S(Ra) = R\,S(a)\,R^T$$

2. SCARA : RRP configuration.



3 ⇒ Prismatic joint, 2 ⇒ revolute

$$P_3 = \begin{bmatrix}0\\0\\-l_4\end{bmatrix} \qquad R_2^3 = I_{3\times3} \qquad d_2^3 = \begin{bmatrix}0\\l_3\\-q_3\end{bmatrix}$$

$$R_1^2 = R_{z,q_2} \qquad d_1^2 = \begin{bmatrix}0\\l_2\\0\end{bmatrix}$$

$$R_0^1 = R_{z,q_1} \qquad d_0^1 = \begin{bmatrix}0\\0\\l_1\end{bmatrix}$$

$$\begin{bmatrix}P_0\\1\end{bmatrix} = H_0^1H_1^2H_2^3\begin{bmatrix}P_3\\1\end{bmatrix} = \begin{bmatrix}cq_1 & -sq_1 & 0 & 0\\ sq_1 & cq_1 & 0 & 0\\ 0 & 0 & 1 & l_1\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}cq_2 & -sq_2 & 0 & 0\\ sq_2 & cq_2 & 0 & l_2\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} H_2^3\begin{bmatrix}P_3\\1\end{bmatrix}$$

$$= \begin{bmatrix}cq_1cq_2 - sq_1sq_2 & -(cq_1sq_2 + sq_1cq_2) & 0 & -l_2sq_1\\ sq_1cq_2 + cq_1sq_2 & -sq_1sq_2 + cq_1cq_2 & 0 & l_2cq_1\\ 0 & 0 & 1 & l_1\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & l_3\\ 0 & 0 & 1 & -q_3\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}P_3\\1\end{bmatrix}$$

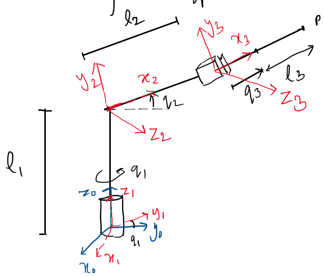
$$= \begin{bmatrix}c(q_1+q_2) & -s(q_1+q_2) & 0 & -l_2sq_1\\ s(q_1+q_2) & c(q_1+q_2) & 0 & l_2cq_1\\ 0 & 0 & 1 & l_1\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & l_3\\ 0 & 0 & 1 & -q_3\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}P_3\\1\end{bmatrix}$$

$$= \begin{bmatrix}c(q_1+q_2) & -s(q_1+q_2) & 0 & -l_3s(q_1+q_2) - l_2sq_1\\ s(q_1+q_2) & c(q_1+q_2) & 0 & l_3c(q_1+q_2) + l_2cq_1\\ 0 & 0 & 1 & l_1 - q_3\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\-l_4\\1\end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} -(l_3 s(q_1+q_2) + l_2 s q_1) \\ l_3 c(q_1+q_2) + l_2 c q_1 \\ l_1 - l_4 - q_3 \\ 1 \end{bmatrix}$$

$$\Rightarrow \boxed{P_0 = \begin{bmatrix} -(l_3 s(q_1+q_2) + l_2 s q_1) \\ l_3 c(q_1+q_2) + l_2 c q_1 \\ l_1 - l_4 - q_3 \end{bmatrix}}$$

4. Stanford-type : RRP configuration.



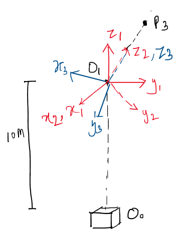
$$P_3 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix} \qquad R_2^3 = I_{3\times3} \qquad d_2^3 = \begin{bmatrix} l_2+q_3 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = R_{y,\pi/2}\, R_{z,q_2} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$R_0^1 = R_{z,q_1} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_1^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

5.



$$0\text{-frame} \xrightarrow{\text{translation } z = 10m} 1\text{-frame} \qquad 1\text{-frame} \xrightarrow{\text{rot by } 30^\circ \text{ abt x axis}} 2\text{-frame}$$

$$2\text{-frame} \xrightarrow{\text{rot about z axis by } 60^\circ} 3\text{-frame}$$

Position of Obstacle in 3-frame :

$(O_1P_3 = 3m)$

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \qquad R_2^3 = R_{z,60^\circ} \qquad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = R_{x,30^\circ} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_0^1 = I_{3\times3} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\pi/6 & -s\pi/6 & 0 \\ 0 & s\pi/6 & c\pi/6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\pi/3 & -s\pi/3 & 0 & 0 \\ s\pi/3 & c\pi/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\pi/6 & -s\pi/6 & 0 \\ 0 & s\pi/6 & c\pi/6 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\pi/3 & -s\pi/3 & 0 & 0 \\ s\pi/3 & c\pi/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\pi/3 & -s\pi/3 & 0 & 0 \\ c\pi/6 \cdot s\pi/3 & c\pi/6\, c\pi/3 & -s\pi/6 & 0 \\ s\pi/6\, s\pi/3 & c\pi/3\, s\pi/6 & c\pi/6 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -3\sin\pi/6 \\ 3\cos\pi/6 + 10 \\ 1 \end{bmatrix} \qquad \therefore\ p_0 = \frac{-3}{2}\hat{j} + \left(\frac{3\sqrt{3}}{2} + 10\right)\hat{k}$$

position of obstacle wrt base frame.

6. A gearbox is a mechanical component used to change the speed (RPM) and increase the motor's torque. Some types of gearboxes commonly used in robotics:

- Spur Gearbox: used in applications that require low to medium torque transmission like in small robotic arms.
  Pros- Simple design, high efficiency and compact size.
  Cons- Lower torque transmission capability, noise generation.

- Planetary Gearbox: used in applications that require high torque such as heavy robotic arms.
  Pros- High torque transmission capacity, compact size, and efficient power distribution
  Cons- Higher cost and complexity compared to spur gearboxes.

- Worm Gearbox: used in applications with the requirement of high-torque transmission and self-locking, such as robotic lifting mechanisms.
  Pros: High torque transmission, self-locking feature, and ability to handle heavy loads. Cons: Lower efficiency, higher wear and tear, and limited speed reduction capability.

Typically, gearboxes are not used in drones. This is mainly because drones require lightweight and efficient power transmission systems, and direct drive motors or specialized propeller attachments are more suitable for their needs. Adding an additional gearbox would introduce unnecessary weight, complexity, and potential points of failure.

7. Manipulator Jacobian for SCARA: RRP

$$J = [\,J_1 \quad J_2 \quad J_3\,]_{6\times 3}$$

$$J_i = \begin{bmatrix} Z_{i-1} \times (o_n - o_{i-1}) \\ Z_{i-1} \end{bmatrix}, \text{ for revolute joint}$$

$$= \begin{bmatrix} Z_{i-1} \\ 0 \end{bmatrix}, \text{ for prismatic joint}$$

$$o_n = p_0 = \begin{bmatrix} -(l_3 s(q_1+q_2) + l_2 s q_1) \\ l_3 c(q_1+q_2) + l_2 c q_1 \\ l_1 - l_4 - q_3 \end{bmatrix} \quad [\text{from Ans 2}]$$

$Z_{i-1} = R_0^{i-1}\hat{k}$ (Assuming all rotations about z-axis by convention)

$o_n - o_{i-1} = R_0^{i-1}\, d_{i-1}^n$

Joint (1): (R) $Z_0 = R_0^0\hat{k} = \hat{k}$

$$o_n - o_0 = R_0^0\, d_0^n = p_0$$

$$J_1 = \begin{bmatrix} \hat{k} \times p_0 \\ \hat{k} \end{bmatrix} = \begin{bmatrix} -(l_3 s(q_1+q_2) + l_2 s q_1)\hat{j} - (l_3 c(q_1+q_2) + l_2 c q_1)\hat{i} \\ \hat{k} \end{bmatrix} = \begin{bmatrix} -(l_3 c(q_1+q_2) + l_2 s q_1) \\ -(l_3 s(q_1+q_2) + l_2 s q_1) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Joint (2) (R): $Z_1 = R_0^1\hat{k} = \hat{k}$

$$o_1 = R_{zq_1}\begin{bmatrix} 0 \\ l_2 \\ l_1 \end{bmatrix} = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ l_2 \\ l_1 \end{bmatrix} = \begin{bmatrix} -l_2 s q_1 \\ l_2 c q_1 \\ l_1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} \hat{k} \times (o_n - o_1) \\ \hat{k} \end{bmatrix}$$

$$o_n - o_1 = \begin{bmatrix} -l_3 s(q_1+q_2) \\ l_3 c(q_1+q_2) \\ -l_4 - q_3 \end{bmatrix} \qquad J_2 = \begin{bmatrix} -l_3 s(q_1+q_2)\hat{j} - l_3 c(q_1+q_2)\hat{i} \\ \hat{k} \end{bmatrix}$$

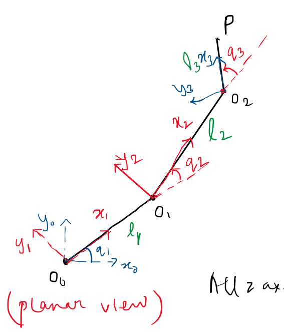
$$o_n - o_1 = \begin{bmatrix} -l_3 s(q_1+q_2) \\ l_3 c(q_1+q_2) \\ -l_4 - q_3 \end{bmatrix} \qquad J_2 = \begin{bmatrix} -l_3 s(q_1+q_2)\hat{j} - l_3 c(q_1+q_2)\hat{i} \\ \hat{k} \end{bmatrix}$$

Joint ③ (P): $Z_2 = \hat{k}$ $\qquad J_3 = \begin{bmatrix} Z_2 \\ 0 \end{bmatrix}$

$$\boxed{J = \begin{bmatrix} -(l_3 c(q_1+q_2) + l_2 s q_1) & -l_3 c(q_1+q_2) & 0 \\ -(l_3 s(q_1+q_2) + l_2 s q_1) & -l_3 s(q_1+q_2) & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}_{6\times 3}}$$

9. RRR configuration w/ all axes parallel. $(z_0 = z_1 = z_2 = z_3 = \hat{k})$ $\qquad l_1, l_2, l_3 \rightarrow$ lengths of links

$$J = \begin{bmatrix} Z_0 \times (o_3 - o_0) & Z_1 \times (o_3 - o_1) & Z_2 \times (o_3 - o_2) \\ Z_0 & Z_1 & Z_2 \end{bmatrix}$$



All z axes are out of plane. (⊥)

$$\begin{bmatrix} O_3 \\ 1 \end{bmatrix} = \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} \qquad P_3 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2^3 = R_{z,q_3} \qquad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = R_{z,q_2} \qquad d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$R_0^1 = R_{z,q_1} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(Take $C(q_1+q_2) = cq_1q_2$)

$$\begin{bmatrix} O_3 \\ 1 \end{bmatrix} = \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & l_1 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} cq_1q_2 & -sq_1q_2 & 0 & l_1cq_1 \\ sq_1q_2 & cq_1q_2 & 0 & l_1sq_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_3 & -sq_3 & 0 & l_2 \\ sq_3 & cq_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} cq_1q_2q_3 & -sq_1q_2q_3 & 0 & l_2cq_1q_2 + l_1cq_1 \\ sq_1q_2q_3 & cq_1q_2q_3 & 0 & l_2sq_1q_2 + l_1sq_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} l_3cq_1q_2q_3 + l_2cq_1q_2 + l_1cq_1 \\ l_3sq_1q_2q_3 + l_2sq_1q_2 + l_1sq_1 \\ 0 \\ 1 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} l_3cq_1q_2q_3 + l_2cq_1q_2 + l_1cq_1 \\ l_3sq_1q_2q_3 + l_2sq_1q_2 + l_1sq_1 \\ 0 \end{bmatrix} \qquad O_2 = \begin{bmatrix} l_1cq_1 + l_2cq_1q_2 \\ l_1sq_1 + l_2sq_1q_2 \\ 0 \end{bmatrix}$$ joint 2 position wrt o-frame

$$O_1 = \begin{bmatrix} l_1cq_1 \\ l_1sq_1 \\ 0 \end{bmatrix} \qquad O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad Z_1 = Z_2 = Z_3 = \hat{k}$$

$$O_3 - O_0 = O_3 \quad , \quad O_3 - O_1 = \begin{bmatrix} l_2 c q_1 q_2 + l_3 c q_1 q_2 q_3 \\ l_2 s q_1 q_2 + l_3 s q_1 q_2 q_3 \\ 0 \end{bmatrix} \quad , \quad O_3 - O_2 = \begin{bmatrix} l_3 c q_1 q_2 q_3 \\ l_3 s q_1 q_2 q_3 \\ 0 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} -(l_3 s q_1 q_2 q_3 + l_2 s q_1 q_2 + l_1 s q_1)\,\hat{i} + (l_3 c q_1 q_2 q_3 + l_2 c q_1 q_2 + l_1 c q_1)\,\hat{j} \\ \hat{k} \end{bmatrix}$$

$$J_2 = \begin{bmatrix} -(l_2 s q_1 q_2 + l_3 s q_1 q_2 q_3)\,\hat{i} + (l_2 c q_1 q_2 + l_3 c q_1 q_2 q_3)\,\hat{j} \\ \hat{k} \end{bmatrix}$$

$$J_3 = \begin{bmatrix} -l_3 s q_1 q_2 q_3\,\hat{i} + l_3 c q_1 q_2 q_3\,\hat{j} \\ \hat{k} \end{bmatrix}$$

$$J = \begin{bmatrix} -(l_3 s q_1 q_2 q_3 + l_2 s q_1 q_2 + l_1 s q_1) & -(l_2 s q_1 q_2 + l_3 s q_1 q_2 q_3) & -l_3 s q_1 q_2 q_3 \\ (l_3 c q_1 q_2 q_3 + l_2 c q_1 q_2 + l_1 c q_1) & (l_2 c q_1 q_2 + l_3 c q_1 q_2 q_3) & l_3 c q_1 q_2 q_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$