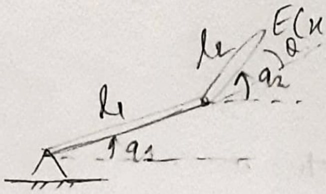
7/8

Mini Project → next Monday.   wj2p6e4 → classroom code.



2R manipulator

T_1 → trajectory following.

T_2 → apply a force on a wall

T_3 → act like a spring.

$$\left.\begin{aligned} x &= l_1 \cos q_1 + l_2 \cos q_2 \\ y &= l_1 \sin q_1 + l_2 \sin q_2 . \end{aligned}\right\} \rightarrow (1)$$

Differentiating (1)

$$\dot{x} = -l_1 \sin q_1 \, \dot{q}_1 - l_2 \sin q_2 \, \dot{q}_2$$

$$\dot{y} = l_1 \cos q_1 \, \dot{q}_1 + l_2 \cos q_2 \, \dot{q}_2$$

End effector velocity $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & -l_2 s q_2 \\ l_1 c q_1 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \rightarrow (2)$

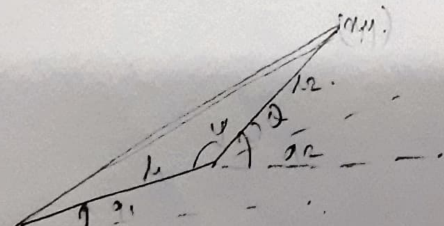
Given x, y; we need to be able to find q_1, q_2.

Option 1 → Solve numerically.

Option 2 → Derive a closed-form expression.

→ Hard in general.

→ multiple solutions.

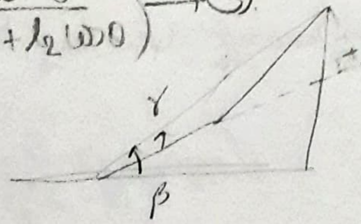


cosine rule + switching to acute angle

$$x^2 + y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right) \rightarrow (3)$$

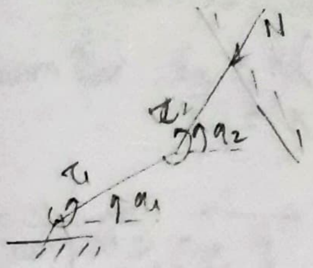
$$q_1 = \beta - \gamma = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right) \rightarrow (3)$$



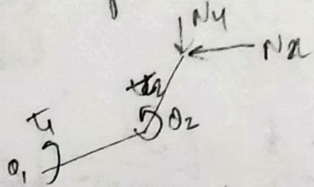
$$q_2 = q_1 + \theta$$

T2

(x_d, y_d) ; $(q_{1d}, q_{2d}) \rightarrow$ desired value.



FBD of entire robot

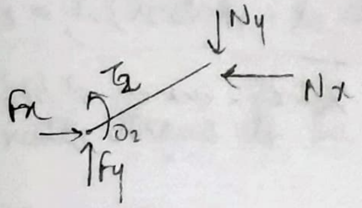


Forces applied by the manipulator will be equal & opposite.

Neglect gravity

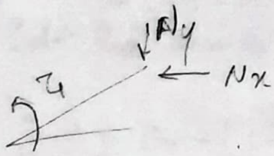
Static equilibrium,

FBD of each ~~link~~ link separately,



$$-N_y l_2 c q_2 + N_x l_2 s q_2 = -\tau_2$$

FBD of link 1,



$$\Sigma M_{O_1} = 0$$

$$N_y l_1 c q_1 - N_x l_1 s q_1 = \tau_1$$

(4)

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & l_1 c q_1 \\ -l_2 s q_2 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} N_x \\ N_y \end{bmatrix}$$

T_3 & next level ans to T_1

Lagrange's Equations

Lagrangian $\quad \mathcal{L} = K - V$

K - Kinetic Energy ; V - potential energy.

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i' \longrightarrow \text{generalised forces derived using principle of virtual work}$$

$\longrightarrow$ (5)

$$K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right) \dot{q}_1^2}_{\text{pure rotation of } l_1} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right) \dot{q}_2^2 + \frac{1}{2} m_2 v_{c_2}^2}_{\text{kinetic energy of link 2}}$$

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2} \dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

$$V = m_1 g \frac{l_1}{2} s q_1 + m_2 g \left(l_1 s q_1 + \frac{l_2}{2} s q_2\right)$$

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$
$$+ m_1 g \frac{l_1}{2} c q_1 + m_2 g l_1 c q_1 = \tau_1$$

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$
$$+ m_2 g \frac{l_2}{2} c q_2 = \tau_2$$

$\longrightarrow$ (6)

Equation 4 is valid for any force f_x, f_y for not just wall forces but for other external forces

$$\left.\begin{aligned} f_x &= Kx \\ f_y &= Ky \end{aligned}\right\} \text{ more general } \quad \begin{aligned} f_x &= K_x(x - x_0) \\ f_y &= K_y(y - y_0) \end{aligned}$$

From ① $f_x = K(l_1 \cos q_1 + l_2 \cos q_2)$
$f_y = K(l_1 \sin q_1 + l_2 \sin q_2)$

From ④
$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & l_1 \cos q_1 \\ -l_2 \sin q_2 & l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} K(l_1 \cos q_1 + l_2 \cos q_2) \\ K(l_1 \sin q_1 + l_2 \sin q_2) \end{bmatrix}$$

$$\left.\begin{aligned} \tau_{2s} &= K(l_1 \sin q_1 + l_2 \sin q_2) l_2 \cos q_2 - K(l_1 \cos q_1 + l_2 \cos q_2) \cdot l_2 \sin q_2 \\ \tau_{1s} &= K(l_1 \sin q_1 + l_2 \sin q_2) l_1 \cos q_1 - K(l_2 \cos q_1 + l_2 \cos q_2) \cdot l_1 \sin q_1 \end{aligned}\right\} ⑦$$

set motor torques to be $\tau_1 + \tau_{1s}$ & $\tau_2 + \tau_{2s}$ respectively.

τ_1, τ_2 - torques needed for basic motion of robot.

Another way to tackle Task 1
solve for q_{1d} & q_{2d} (desired angles) from eq^n 3

$$\dot{q}_{1d}, \dot{q}_{2d}, \ddot{q}_{1d}, \ddot{q}_{2d} \longrightarrow \tau_1, \tau_2 \text{ from } ⑥$$

works better when dynamic effects are significant.

* Investigate with simulation what goes wrong without feedback control?
* what goes wrong with no dynamics and only statics.
* what goes wrong with trying to achieve force and position control simultaneously?