DL_Assignment1

Roll Number: CS23MTECH14003

Name: Hrishikesh Hemke

Q1] Perceptron learning algorithm [10 = dataset creation 2 + perceptron learning algorithm 5 + analysis 3] Implement perceptron learning algorithm for classifying a linearly separable dataset in 2D. Note that you have to create such a dataset with at least 1000 data points. Plot the dataset before and after training (with the classifier). Discuss your observations with respect to number of iterations required for perfect classification (k) by varying the level of separability (γ from the class discussions) in the dataset. (Hint: compute the average value of k for each level of γ, and do this for about 5 values of γ. Observe if you can relate to the result discussed in class)

DATASET CREATION

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
```

In [ ]:
```
def gen_linearly_separable_dataset(size, separation_level):
    # Randomly generatng class 1 points
    class1_points = np.random.normal(loc=0, scale=1, size=(size // 2, 2))
    class1_labels = np.ones((size // 2, 1))

    # Randomly generatng class 2 points with a separation
    class2_points = np.random.normal(loc=separation_level, scale=1, size=(size // 2, 2
    class2_labels = -1 * np.ones((size // 2, 1))

    # Combine the two classes
    X = np.vstack((class1_points, class2_points))
    y = np.vstack((class1_labels, class2_labels))

    # Mixing the dataset
    shuffle_indices = np.random.permutation(size)
    X = X[shuffle_indices]
    y = y[shuffle_indices]
    return X, y.flatten()
```

INITIALIZING WEIGHTS AND POLTING DATA DECISION BOUNDARY FUNC

In [ ]:
```
def initialize_weights(num_features):
    return np.random.rand(num_features)

def plot_separate_decision_boundaries(X, y, initial_weights, trained_weights, separati
    fig, axes = plt.subplots(1, 2, figsize=(12, 6))

    # Plotting initial decision boundary
    axes[0].scatter(X[y == 1, 0], X[y == 1, 1], label='Class 1', marker='o',c='c')
    axes[0].scatter(X[y == -1, 0], X[y == -1, 1], label='Class 2', marker='x',c='y')
```

```
    x_initial_boundary = np.linspace(min(X[:, 0]), max(X[:, 0]), 100)
    y_initial_boundary = (-initial_weights[0] / initial_weights[1]) * x_initial_bounda
    axes[0].plot(x_initial_boundary, y_initial_boundary, '-g', label='Initial Decisior
    axes[0].set_title(f'Initial Decision Boundary (Separation Level: {separation_level
    axes[0].set_xlabel('Feature 1')
    axes[0].set_ylabel('Feature 2')
    axes[0].legend()

    # Plotting trained decision boundary
    axes[1].scatter(X[y == 1, 0], X[y == 1, 1], label='Class 1', marker='o',c='c')
    axes[1].scatter(X[y == -1, 0], X[y == -1, 1], label='Class 2', marker='x',c='y')
    x_trained_boundary = np.linspace(min(X[:, 0]), max(X[:, 0]), 100)
    y_trained_boundary = (-trained_weights[0] / trained_weights[1]) * x_trained_bounda
    axes[1].plot(x_trained_boundary, y_trained_boundary, '-r', label='Trained Decisior
    axes[1].set_title(f'Trained Decision Boundary (Separation Level: {separation_level
    axes[1].set_xlabel('Feature 1')
    axes[1].set_ylabel('Feature 2')
    axes[1].legend()

    plt.tight_layout()
    plt.show()
```

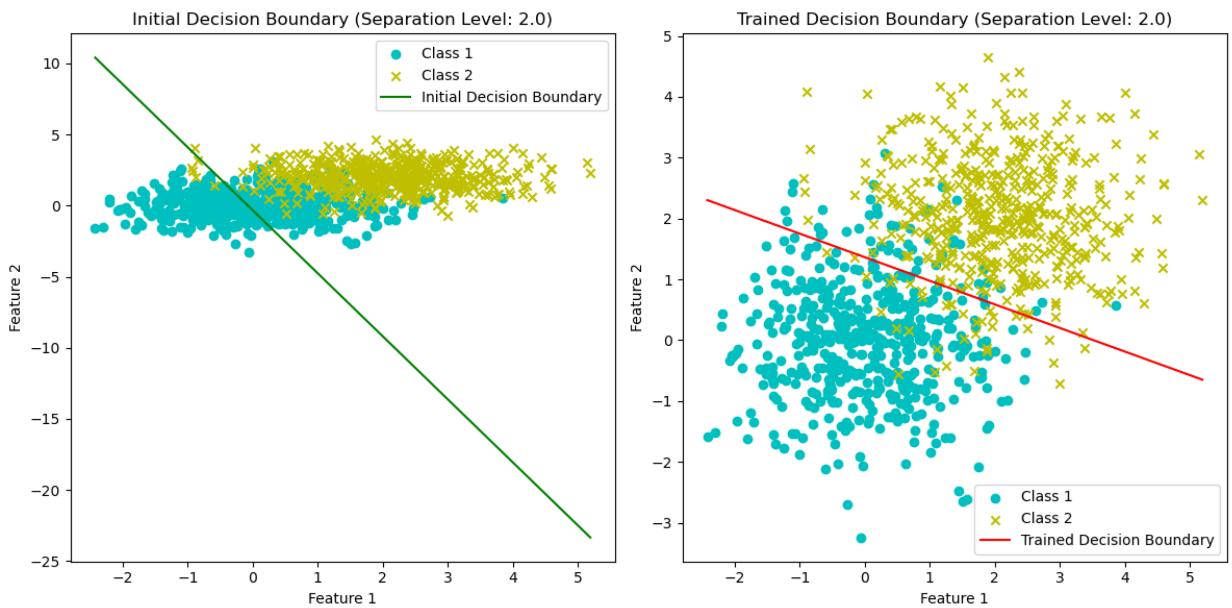
PERCEPTRON LEARNING ALGORITHM

In [ ]:
```
def train_perceptron(X, y, weights, nb_epochs_max):
    k = 0
    for epoch in range(nb_epochs_max):
        nb_changes = 0
        for i in range(X.shape[0]):
            if np.dot(X[i], weights) * y[i] <= 0:
                weights = weights + y[i] * X[i]
                nb_changes += 1
                k +=1
        if nb_changes == 0:
            break
    return weights,k
```

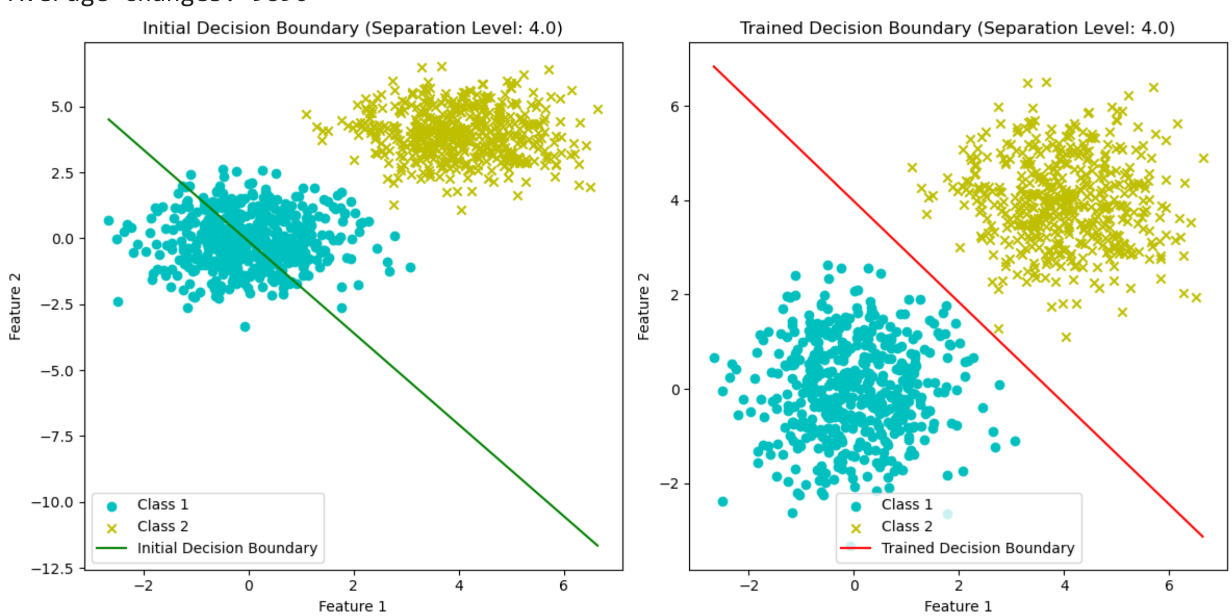
ANALYSIS

In [ ]:
```
#Initializing separation levels
separation_levels = [2.0, 4.0, 6.0, 8.0, 10.0]

# Variable to save gamma values and average k values
gamma_values = []
average_k_values = []

for separation_level in separation_levels:
    X, y = gen_linearly_separable_dataset(1000, separation_level)
    K = 0
    for z in range(5):
        # Initialize weights
        initial_weights = initialize_weights(X.shape[1] + 1)

        # Adding bias term
        X_bias = np.hstack((X, np.ones((X.shape[0], 1))))

        # calling function Train perceptron
        trained_weights, nb_changes = train_perceptron(X_bias, y, initial_weights, nb_
```

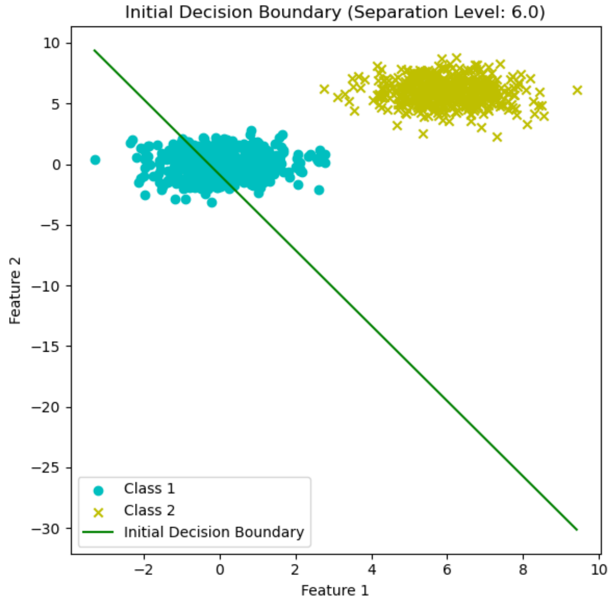
```
        K = K + nb_changes

        # Plot the dataset with separate decision boundaries
        if z == 4:
            plot_separate_decision_boundaries(X, y, initial_weights, trained_weights,
    average_k = K / 5
    gamma_values.append(separation_level)
    average_k_values.append(average_k)
    print('Average changes: %d' % (K / 5))

# Plotting graph
plt.plot(gamma_values, average_k_values, marker='o')
plt.title('Average Number of Iterations vs Separation Level')
plt.xlabel('Separation Level (Gamma)')
plt.ylabel('Average Number of Iterations (k)')
plt.show()
```

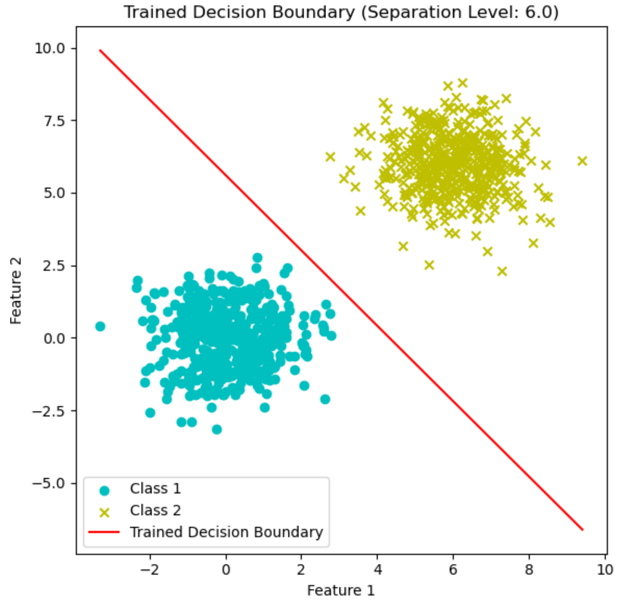



```
Average changes: 9896
```

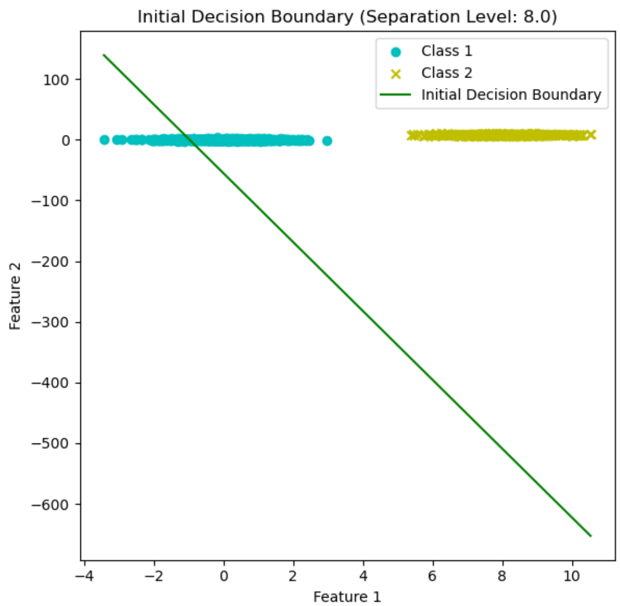


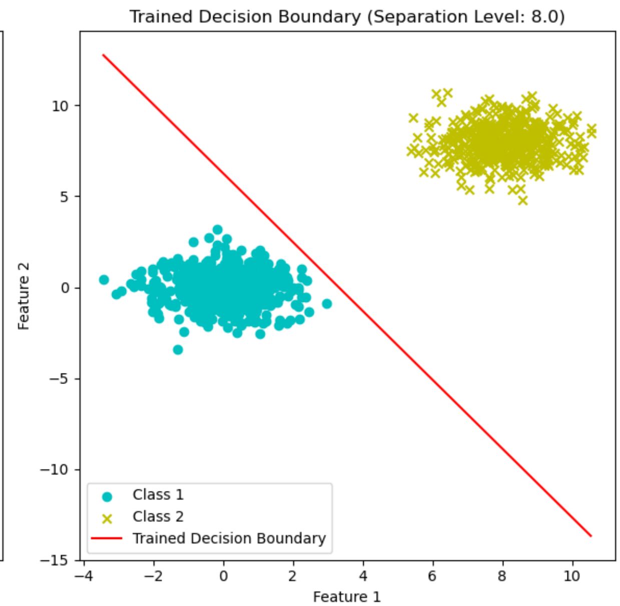

```
Average changes: 32
```



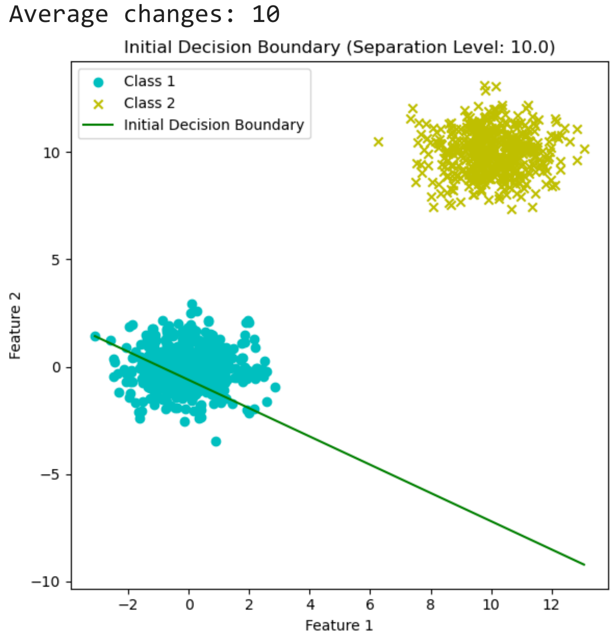


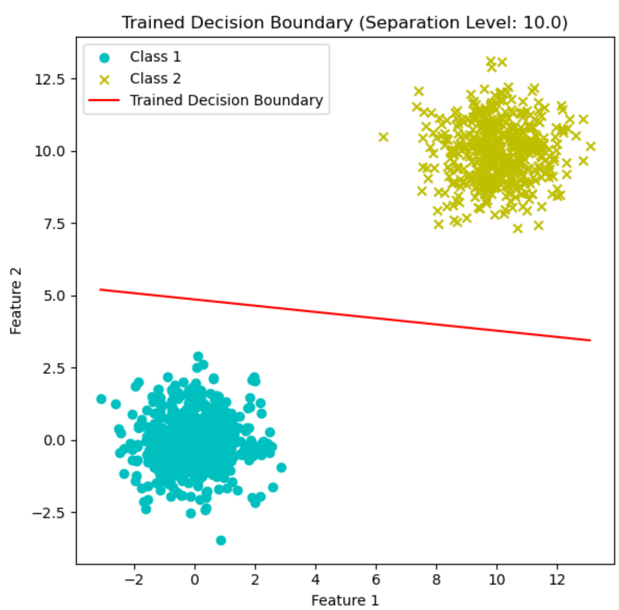

```
Average changes: 12
```



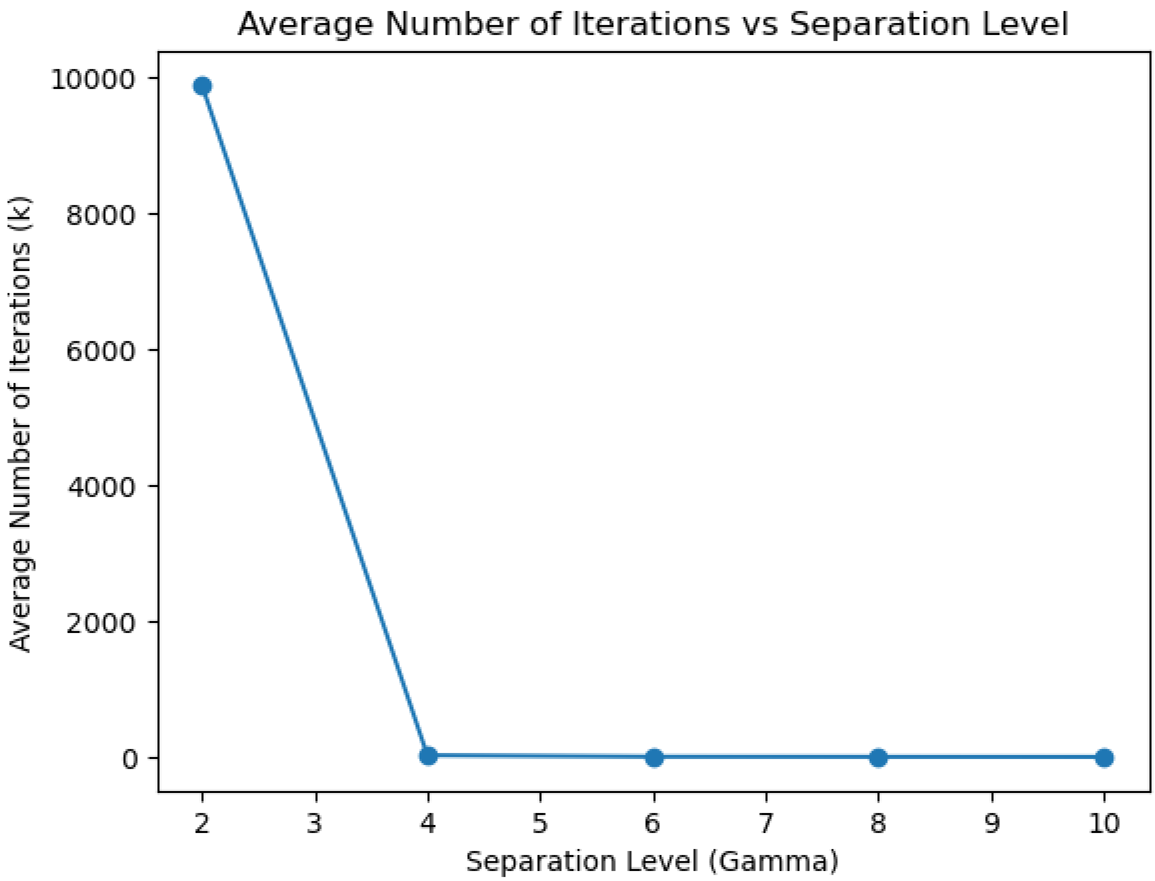



```
Average changes: 10
```





```
Average changes: 9
```



Analysis

In the above graph we can observe that as the separation level (Gamma) is increasing the average number of iteration iterations if decreasing.

Q2] Gradient descent for training a linear classifier [10 = loss formulation 4 + gradient computation 3 + update equation 3] Consider solving the above problem (training a line for classifying a linearly separable 2D dataset) using Gradient Descent algorithm. Think of a loss function (beyond simple MSE) based on our classroom discussion on the desirable properties of a loss function. You may implement the an□alytical way of finding gradient for it. You may implement the basic version of gradient descent update equation. Plot the dataset before and after training (with the classifier).

LOSS FORMATION

In [ ]:
```
# Defining sigmoid function
def sigmoid(z):
    return 1 / (1 + np.exp(-z))

# Defining cross-entropy loss function
def cross_entropy_loss(y, y_pred):
    e=1e-10
    return -np.mean(y * np.log(y_pred+e) + (1 - y) * np.log(1 - y_pred+e))
```

GRADIENT COMPUTATION

In [ ]:
```
# Function to computer gradient for loss function
def compute_gradient(X, y, y_pred):
    return np.dot(X.T, (y_pred - y)) / len(y)
```

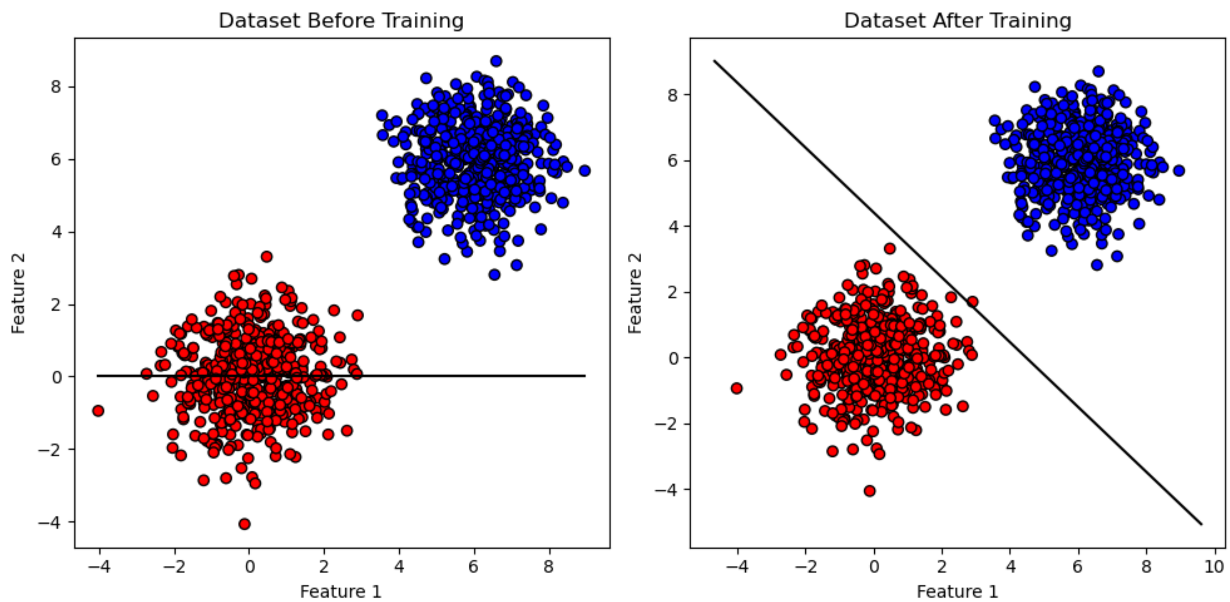
UPDATE EQUATIONS

In [ ]:
```
# Initialising parameters for data generation and training
size = 1000
separation_level = 6
learning_rate = 0.01
epochs = 3000

# Generate dataset
X, y = gen_linearly_separable_dataset(size, separation_level)
y = (y + 1) / 2  # Adjust labels to be 0 and 1

# Adding bias term to X
X_bias = np.hstack((np.ones((X.shape[0], 1)), X))

# Initializing weights
weights = np.random.randn(X_bias.shape[1])

# Training using Gradient Descent
for epoch in range(epochs):
    # Predictions using current weights
    y_pred = sigmoid(np.dot(X_bias, weights))

    # Calculating and update weights using gradient descent
    gradient = compute_gradient(X_bias, y, y_pred)
    weights -= learning_rate * gradient
```

PLOTTING OF GRAPH BEFORE TRAINING WITH DECISION BOUNDARY AND AFTER TRAINING EITH DECISION BOUNDARY

In [ ]:
```
# Plotting the dataset before training with decision boundary
plt.figure(figsize=(10, 5))
plt.subplot(1, 2, 1)
plt.scatter(X[:, 0], X[:, 1], c=y, cmap='bwr', edgecolors='k')

# Decision boundary:
x_vals = np.linspace(min(X[:, 0]), max(X[:, 0]), 100)
y_vals = np.zeros(100,)
plt.plot(x_vals, y_vals, c="k")

plt.title("Dataset Before Training")
plt.xlabel("Feature 1")
plt.ylabel("Feature 2")

# Plotting the dataset after training with decision boundary
plt.subplot(1, 2, 2)
plt.scatter(X[:, 0], X[:, 1], c=y, cmap='bwr', edgecolors='k')
ax = plt.gca()

# Decision boundary:
x_vals = np.array(ax.get_xlim())
```

```
y_vals = -(weights[0] + weights[1] * x_vals) / weights[2]
plt.plot(x_vals, y_vals, c="k")

plt.title("Dataset After Training")
plt.xlabel("Feature 1")
plt.ylabel("Feature 2")

plt.tight_layout()
plt.show()
```



Q3] MLP with a single hidden layer [20 = dataset creation 5 + MLP definition 5 + backprop 10] This question has two variations, and you are expected to attempt any one of the variations. The second variation, if implemented properly, will fetch you a 5% bonus on this assignment. Original Question: Consider a binary classification dataset that is not linearly separable in 2D (e.g. data lying on the circumference two concentric circles). Train a Multi layer perceptron (MLP) with a single hidden layer for classifying the same. You may use the loss function used in problem.You have to implement the backpropagation algorithm yourself

DATASET CREATION

In [ ]:
```
# Generating non-linearly separable dataset
def generate_non_linear_dataset(size):
    theta = np.linspace(0, 2*np.pi, size)
    inner_circle = np.column_stack((np.cos(theta), np.sin(theta)))
    outer_circle = 2 * inner_circle
    X = np.vstack((inner_circle, outer_circle))
    y = np.hstack((np.zeros(size), np.ones(size)))
    return X, y
```

In [ ]:
```
# Defining sigmoid derivative function
def sigmoid_derivative(x):
    return sigmoid(x) * (1 - sigmoid(x))
```

In [ ]:
```
# Initializing weights and biases
def initialize_parameters(input_size, hidden_size, output_size):
    np.random.seed(42)
```

```
    W1 = np.random.randn(input_size, hidden_size)
    b1 = np.zeros((1, hidden_size))
    W2 = np.random.randn(hidden_size, output_size)
    b2 = np.zeros((1, output_size))
    return W1, b1, W2, b2

# Forward pass through the network
def forward_pass(X, W1, b1, W2, b2):
    z1 = np.dot(X, W1) + b1
    a1 = sigmoid(z1)
    z2 = np.dot(a1, W2) + b2
    a2 = sigmoid(z2)
    return z1, a1, z2, a2
```

BACKPROP

In [ ]:

```
# Backward pass to update weights and biases using gradient descent
def backward_pass(X, y, z1, a1, z2, a2, W1, W2, b1, b2, learning_rate):
    m = len(X)
    # Compute gradients
    dz2 = a2 - y
    dW2 = np.dot(a1.T, dz2) / m
    db2 = np.sum(dz2, axis=0, keepdims=True) / m
    dz1 = np.dot(dz2, W2.T) * sigmoid_derivative(z1)
    dW1 = np.dot(X.T, dz1) / m
    db1 = np.sum(dz1, axis=0, keepdims=True) / m

    # Update weights and biases
    W1 -= learning_rate * dW1
    b1 -= learning_rate * db1
    W2 -= learning_rate * dW2
    b2 -= learning_rate * db2
    return W1, b1, W2, b2
```

MLP definition [train_mlp]

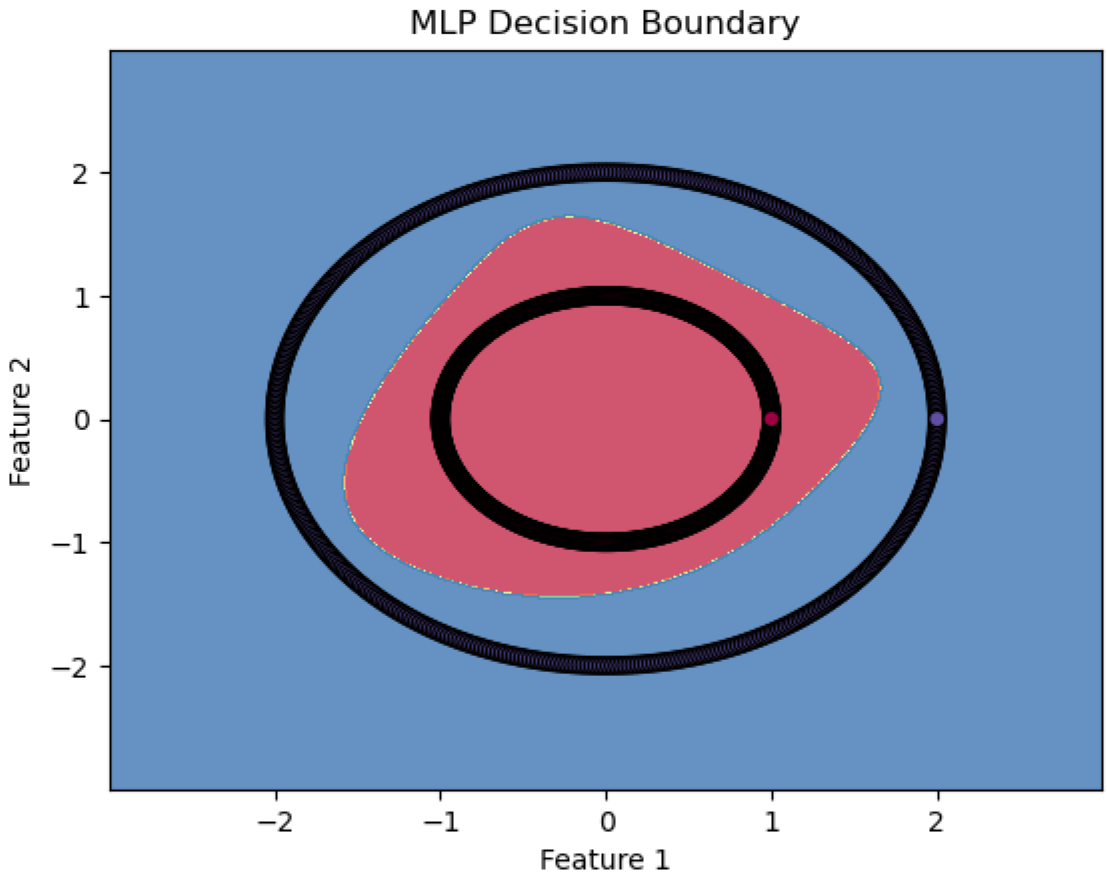
In [ ]:

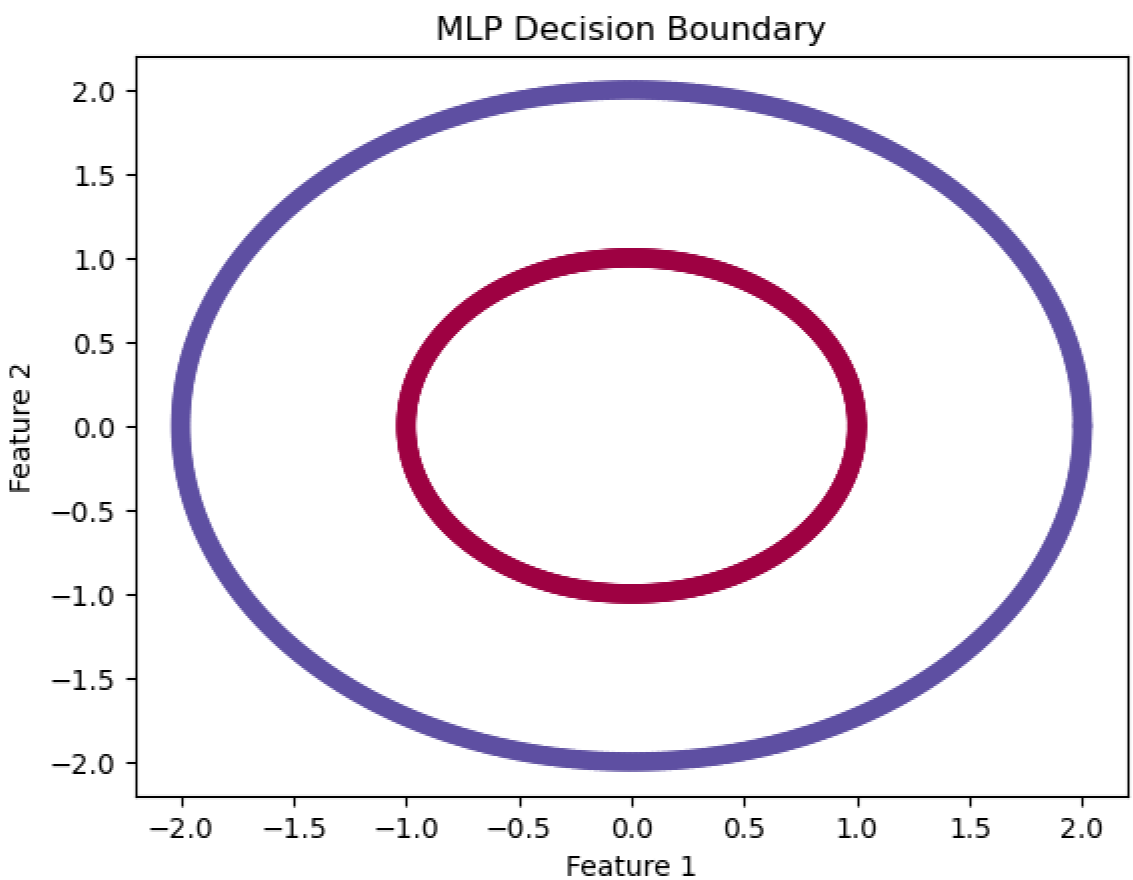
```
# Training the MLP using backpropagation
def train_mlp(X, y, hidden_size, learning_rate, epochs):
    input_size = X.shape[1]
    output_size = 1  # Binary classification

    W1, b1, W2, b2 = initialize_parameters(input_size, hidden_size, output_size)
    for epoch in range(epochs):
        z1, a1, z2, a2 = forward_pass(X, W1, b1, W2, b2)
        loss = cross_entropy_loss(y, a2)
        if epoch % 1000 == 0:
            print(f"Epoch {epoch}, Loss: {loss}")
        W1, b1, W2, b2 = backward_pass(X, y, z1, a1, z2, a2, W1, W2, b1, b2, learning_
    return W1, b1, W2, b2

# Make predictions using the trained MLP
def predict(X, W1, b1, W2, b2):
    _, _, _, a2 = forward_pass(X, W1, b1, W2, b2)
    return (a2 > 0.5).astype(int)
```

Plot decision boundary

In [ ]:
```
# Plotting decision boundary
def plot_decision_boundary(X, y, W1, b1, W2, b2):
    h = 0.01
    x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))

    points = np.c_[xx.ravel(), yy.ravel()]
    predictions = predict(points, W1, b1, W2, b2)

    plt.contourf(xx, yy, predictions.reshape(xx.shape), cmap=plt.cm.Spectral, alpha=0.
    plt.scatter(X[:, 0], X[:, 1], c=y, cmap=plt.cm.Spectral, edgecolors='k')
    plt.title("MLP Decision Boundary")
    plt.xlabel("Feature 1")
    plt.ylabel("Feature 2")
    plt.show()
    prediction_2 = predict(X, W1, b1, W2, b2)
    plt.scatter(X[:,0],X[:,1],c=prediction_2 ,cmap=plt.cm.Spectral)
    plt.title("MLP Decision Boundary")
    plt.xlabel("Feature 1")
    plt.ylabel("Feature 2")
    plt.show()

# Setting parameters
hidden_size = 7
learning_rate = 0.01
epochs = 20000

# Generating non-linear dataset
X, y = generate_non_linear_dataset(500)

# Training MLP
W1, b1, W2, b2 = train_mlp(X, y.reshape(-1, 1), hidden_size, learning_rate, epochs)

# Plotting decision boundary
plot_decision_boundary(X, y, W1, b1, W2, b2)
```

```
Epoch 0, Loss: 1.1382768532211196
Epoch 1000, Loss: 0.7152375552612651
Epoch 2000, Loss: 0.6920963031653747
Epoch 3000, Loss: 0.681956704257196
Epoch 4000, Loss: 0.6730896880555106
Epoch 5000, Loss: 0.663131519425443
Epoch 6000, Loss: 0.6514781229643029
Epoch 7000, Loss: 0.6381508951073521
Epoch 8000, Loss: 0.6234345049413771
Epoch 9000, Loss: 0.6075804118067547
Epoch 10000, Loss: 0.5906781872251226
Epoch 11000, Loss: 0.5726693409499549
Epoch 12000, Loss: 0.5533719736295221
Epoch 13000, Loss: 0.5325153889602873
Epoch 14000, Loss: 0.5098538330973493
Epoch 15000, Loss: 0.48536373728394633
Epoch 16000, Loss: 0.4594100763881744
Epoch 17000, Loss: 0.4327236399019985
Epoch 18000, Loss: 0.4061701218047626
Epoch 19000, Loss: 0.38048182879736614
```


MLP Decision Boundary
Feature 2
Feature 1
2
1
0
−1
−2
−2
−1
0
1
2


MLP Decision Boundary
Feature 2
Feature 1
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0