ASSIGNMENT-2 REPORT

Optimization Methods

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1 Matyas Function

$$f(\mathbf{x}) = 0.26(x_1^2 + x_2^2) - 0.48x_1x_2$$

1.1 Jacobian

$$\nabla f(\mathbf{x}) = \begin{bmatrix} 0.52x_1 - 0.48x_2 \\ 0.52x_2 - 0.48x_1 \end{bmatrix}$$

1.2 Hessian

$$\nabla^2 f(\mathbf{x}) = \begin{bmatrix} 0.52 & -0.48 \\ -0.48 & 0.52 \end{bmatrix}$$

1.3 Minima

$$f(\mathbf{x}) = 0.26(x_1^2 + x_2^2) - 0.48x_1x_2$$

For minimum $\nabla f(x) = 0$ By equating the derivative we get the point as

$$x = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$$

$\therefore$ The minimum value of f is 0 at x=$\begin{bmatrix} 0 & 0 \end{bmatrix}^T$

1.4 Algorithms that failed to converge

All the given algorithms from all the given initial points were able to converge for this function.

1.5 Graphs

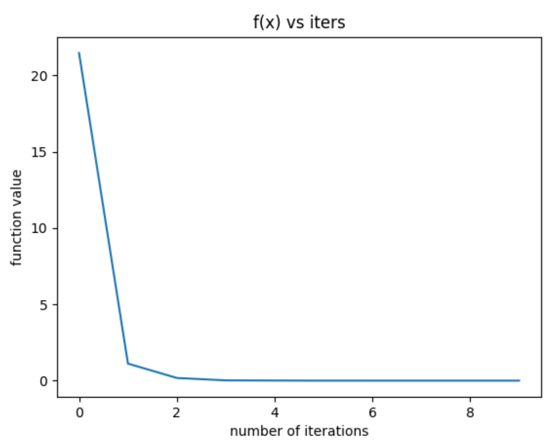

Figure 1: matyas function [1. 10.] BFGS vals

2 Rotated Hyper-Ellipsoid Function

$$f(\mathbf{x}) = \sum_{i=1}^{d} \sum_{j=1}^{i} x_j^2$$

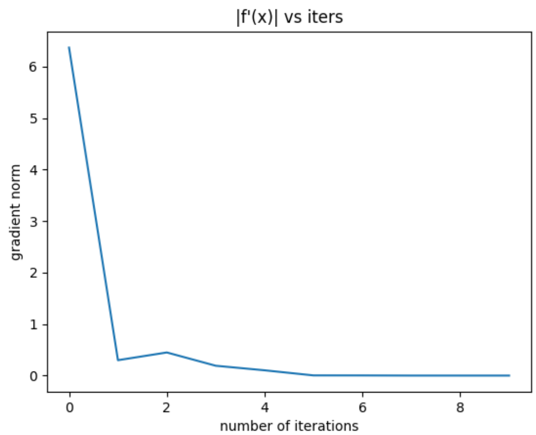

Figure 2: matyas function [1. 10.] BFGS grad

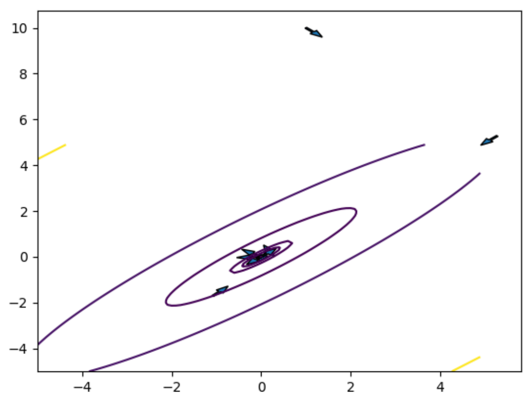

Figure 3: matyas function[1. 10.] BFGS contour

2.1 Jacobian

$$\nabla f(\mathbf{x}) = \begin{bmatrix} 2dx_1 \\ 2(d-1)x_2 \\ 2(d-2)x_3 \\ 2(d-3)x_4 \\ .. \\ 4x_{d-1} \\ 2x_d \end{bmatrix}$$

2.2 Hessian

$$\nabla^2 f(\mathbf{x}) = \begin{bmatrix} 2d & 0 & 0 & 0 & & . & . & . & . & . \\ 0 & 2(d-1) & 0 & 0 & & . & . & . & . & . \\ 0 & 0 & 2(d-2) & 0 & & . & . & . & . & . \\ 0 & 0 & 0 & 2(d-3) & & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & \\ . & . & . & . & . & . & . & . & . & \\ . & . & . & . & . & . & . & 4 & . & \\ 0 & 0 & 0 & 0 & & . & . & . & . & 2 \end{bmatrix}$$

2.3 Minima

$$f(\mathbf{x}) = \sum_{i=1}^{d} \sum_{j=1}^{i} x_j^2$$

for minimum $\nabla f(x) = 0$ By equating the derivative we get the points as

$$x = \begin{bmatrix} 0 & 0 & . & . & . & . & 0 \end{bmatrix}^T$$

$\therefore$ the minimum value of f is 0 at x=$\begin{bmatrix} 0 & 0 & 0 & 0 & . & . & . & . & . & 0 \end{bmatrix}^T$

2.4 Algorithms that failed to converge:

All the given algorithms from all the given initial points were able to converge for this function.

2.5 Graphs

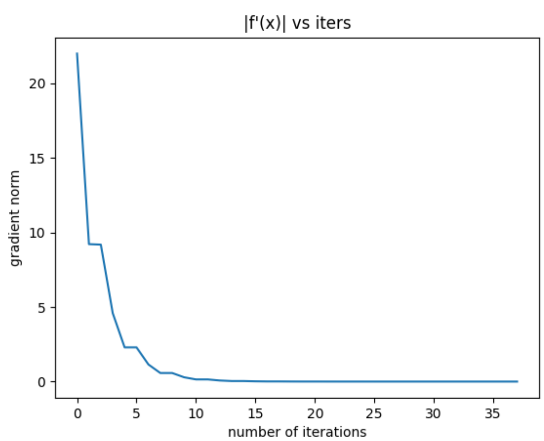

Figure 4: hyperEllipsoid function [-3. 3. 2.] Polak Ribiere vals

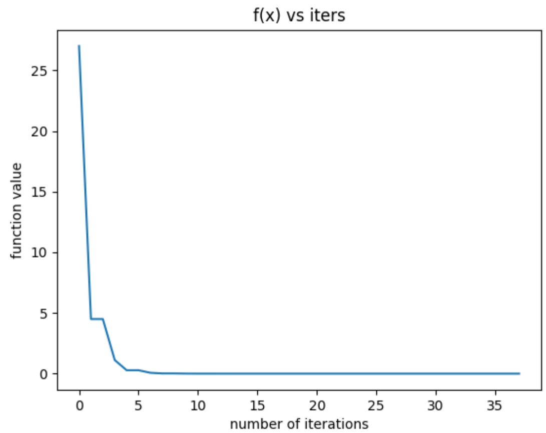

Figure 5: hyperEllipsoid function [-3. 3. 2.] Polak Ribiere gradient

3 Trid Function

3.1 Algorithms that failed to converge:

All the given algorithms from all the given initial points were able to converge for this function.

3.2 Graphs

4 Three Hump Camel Function

$$f(x, y) = 2x^2 - 1.05x^4 + \frac{x^6}{6} + xy + y^2$$

4.1 Algorithms that failed to converge:

- **Conjugate-HR, Conjugate-HS, DFP, BFGS** were able to converge to some local minima.But were not able to converge to global minima from all the given initial points.

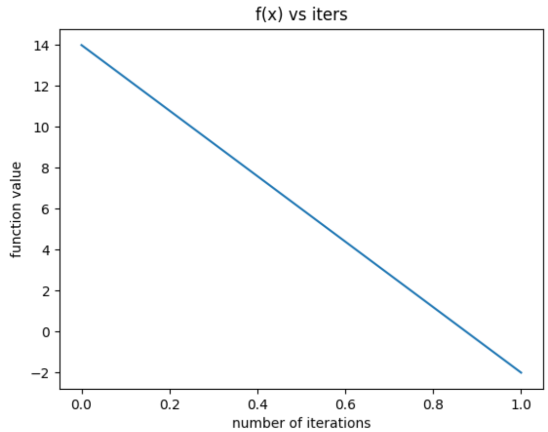

Figure 6: trid function [-2. -2.] SR1 vals

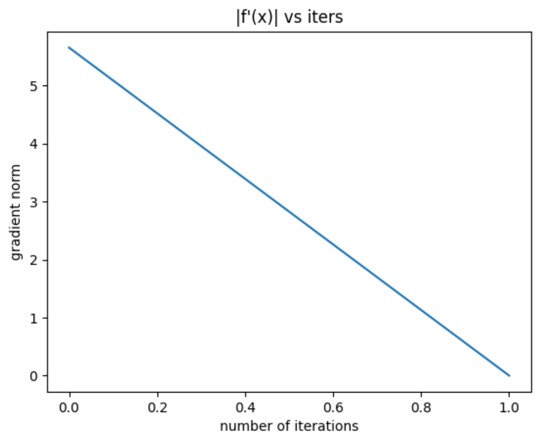

Figure 7: trid function [-2. -2.] SR1 gradient

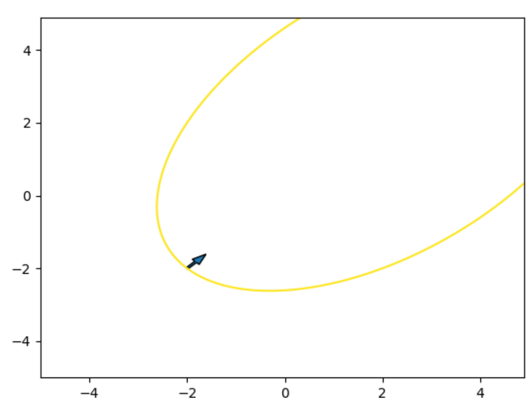

Figure 8: trid function [-2. -2.] SR1 contour

- While **Conjuage PS** was able to converge to global minima with initial point as [-2,1], [2,-1], [-2,-1], [2,1].

- **SR1** was able to converge to global minima with initial point as [2,1], [-2,-1] but not able to converge to global minima with initial points as [2,-1], [-2,1].But were able to converge to some local minima.

4.2 Graphs

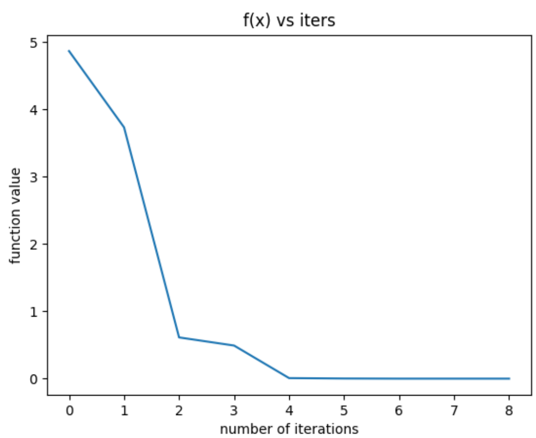

Figure 9: three hump camel function [2. 1.] SR1 vals

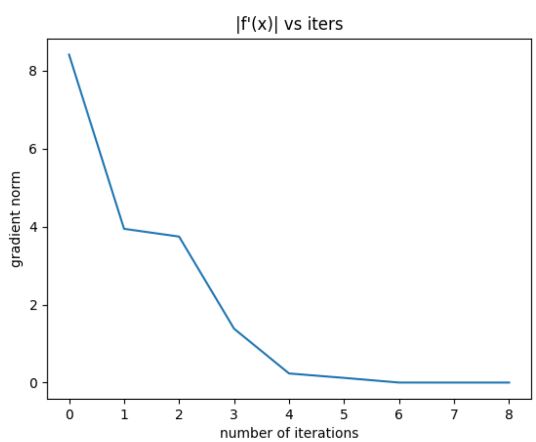

Figure 10: three hump camel function [2. 1.] SR1 grad

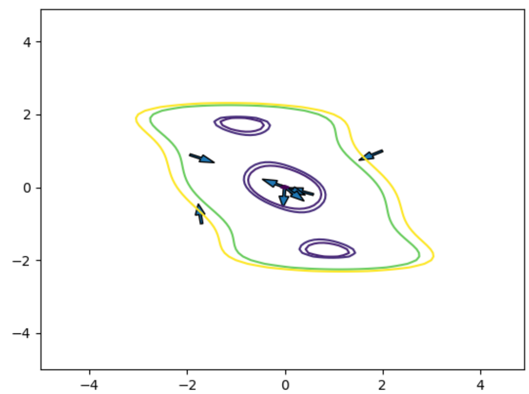

Figure 11: three hump camel function[2. 1.] SR1 cont

5 Styblinski-Tang Function

5.1 Mathematical Formulation

$$f(\mathbf{x}) = \frac{1}{2}\sum_{i=1}^{d}\left(x_i^4 - 16x_i^2 + 5x_i\right)$$

5.2 Algorithms that failed to converge:

- **Conjugate HS, Conjugate Pr, Conjugate FS, SR1, DFP, BFGS** were not able to converge to global minima with an initial point as [3, -3, 3, -3]. But were able to converge to some local minima.

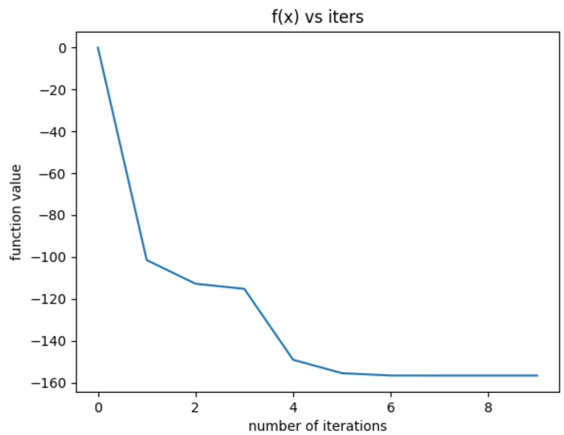

Figure 12: styblinski tang function [0. 0. 0. 0.] DFP vals

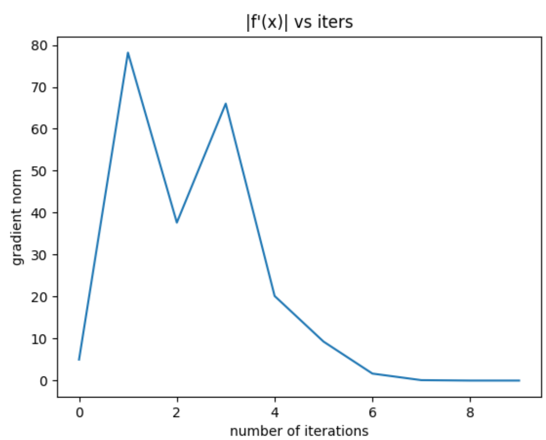

Figure 13: styblinski tang function [0. 0. 0. 0.] DFP grad

5.3 Graphs

6 Rosen Brock Function

6.1 Mathematical Formulation

$$f(\mathbf{x}) = \sum_{i=1}^{d-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right]$$

6.2 Algorithms that failed to converge

- **SR1** failed to converge to global minima with every initial point.
- **Conjugate FR** failed to converge to global minima from initial point [2,2,2,-2].
- **DFP** failed to converge to global minima from initial points [2,-2,-2,2],[-2,2,2,2].

6.3 Graphs

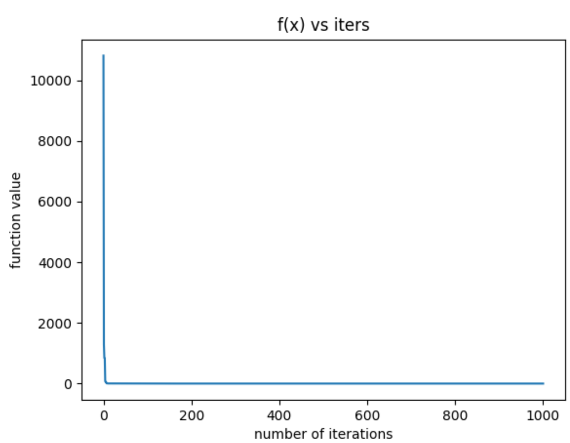

Figure 14: rosenbrock function [3. 3. 3. 3.] Polak-Ribiere vals

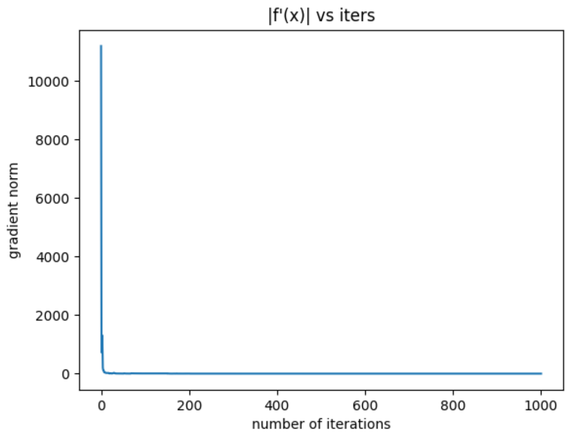

Figure 15: rosenbrock function [3. 3. 3. 3.] Polak-Ribiere gradient

7 Root of a square Function

7.1 Mathematical Formulation

$$f(\mathbf{x}) = \sqrt{1 + x_1^2} + \sqrt{1 + x_2^2}$$

7.2 Algorithms that failed to converge

- All the given algorithms were able to converge to global minima from all given initial points since it is a convex function.

7.3 Graphs

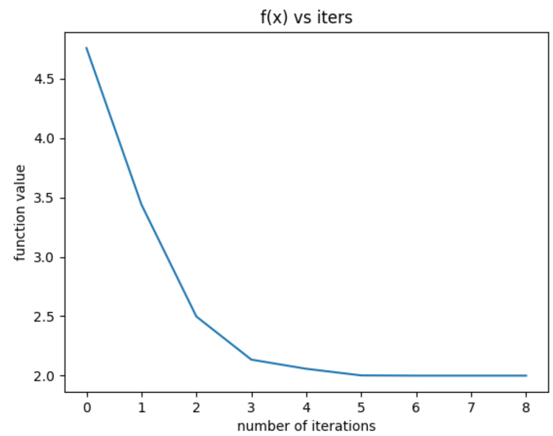

Figure 16: func 1[-3.5 0.5] SR1 vals

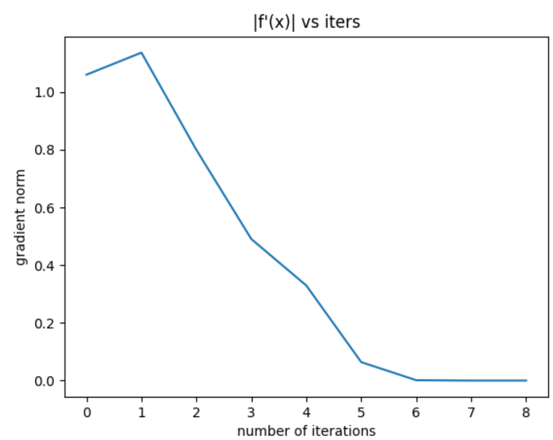

Figure 17: func 1[-3.5 0.5] SR1 grad

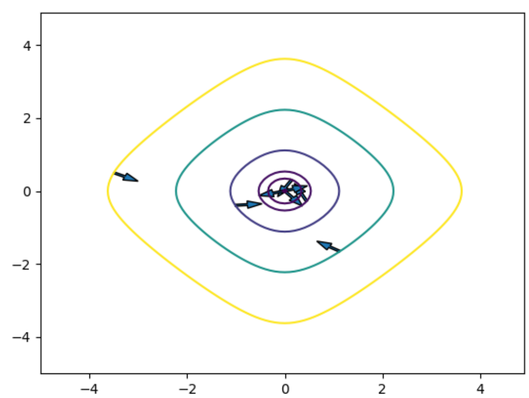

Figure 18: func 1[-3.5 0.5] SR1 grad