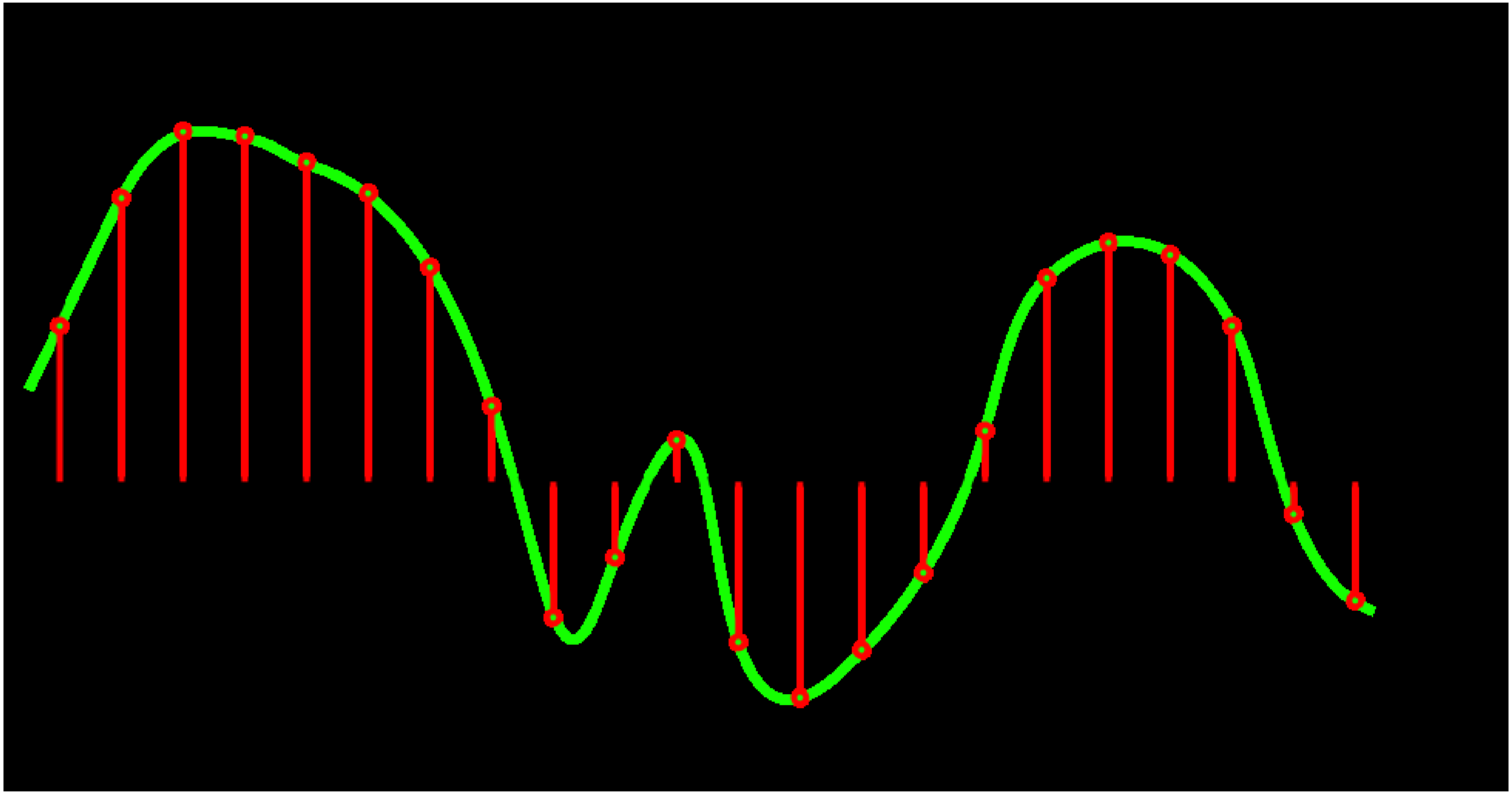
Sampling and Aliasing

Applied DSP practice lab

Sampling

- Process of converting the continuous time signal into a discrete time signal.
- These discrete points are called sample points and we have to choose the points appropriately so that the discrete points when joined together form the original signal reliably.
- The frequency with which these samples are taken from time domain is called sampling frequency denoted as *fs (samples/sec).*

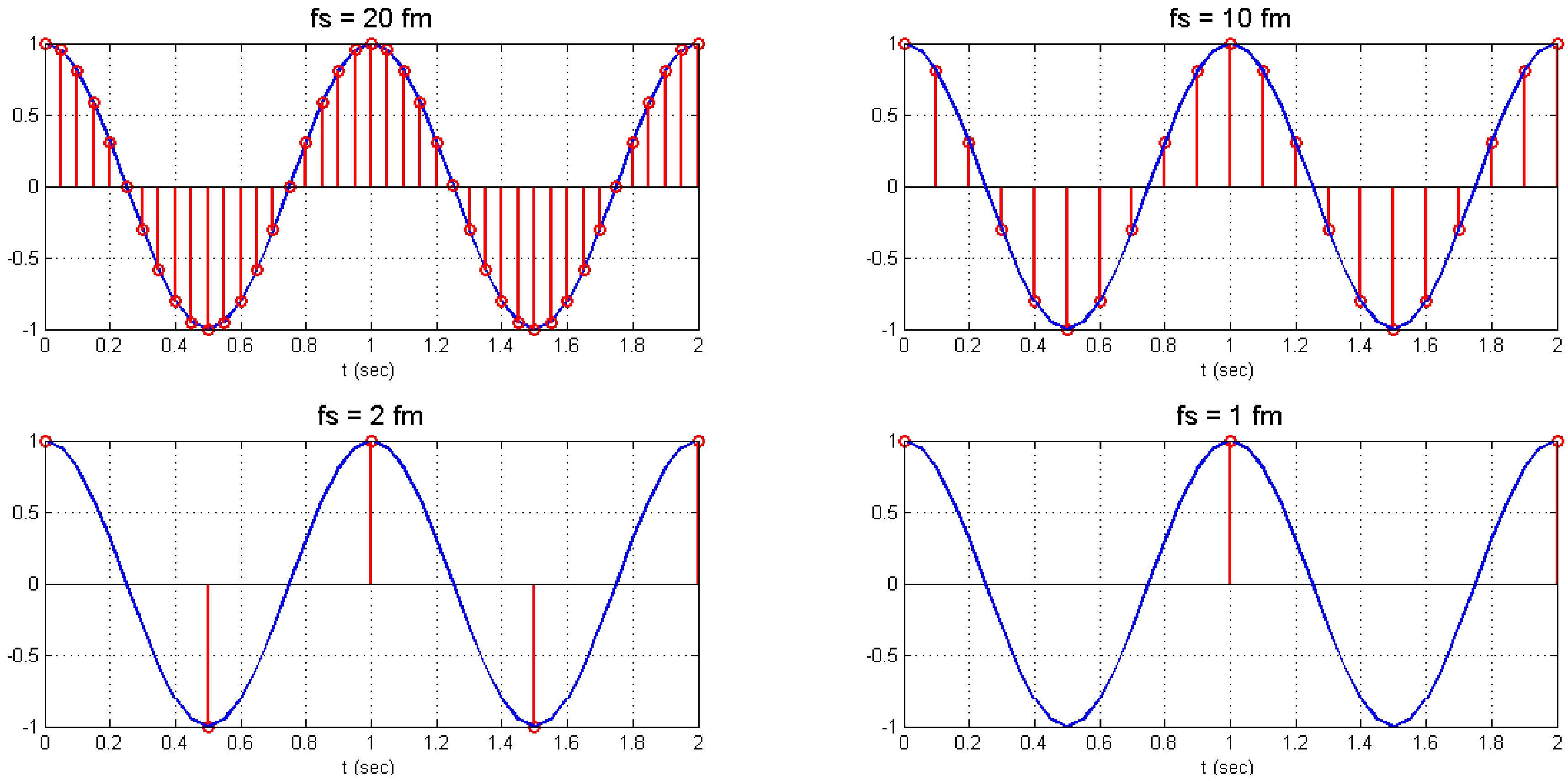
Sampling a continuous time signal



Problem with Sampling

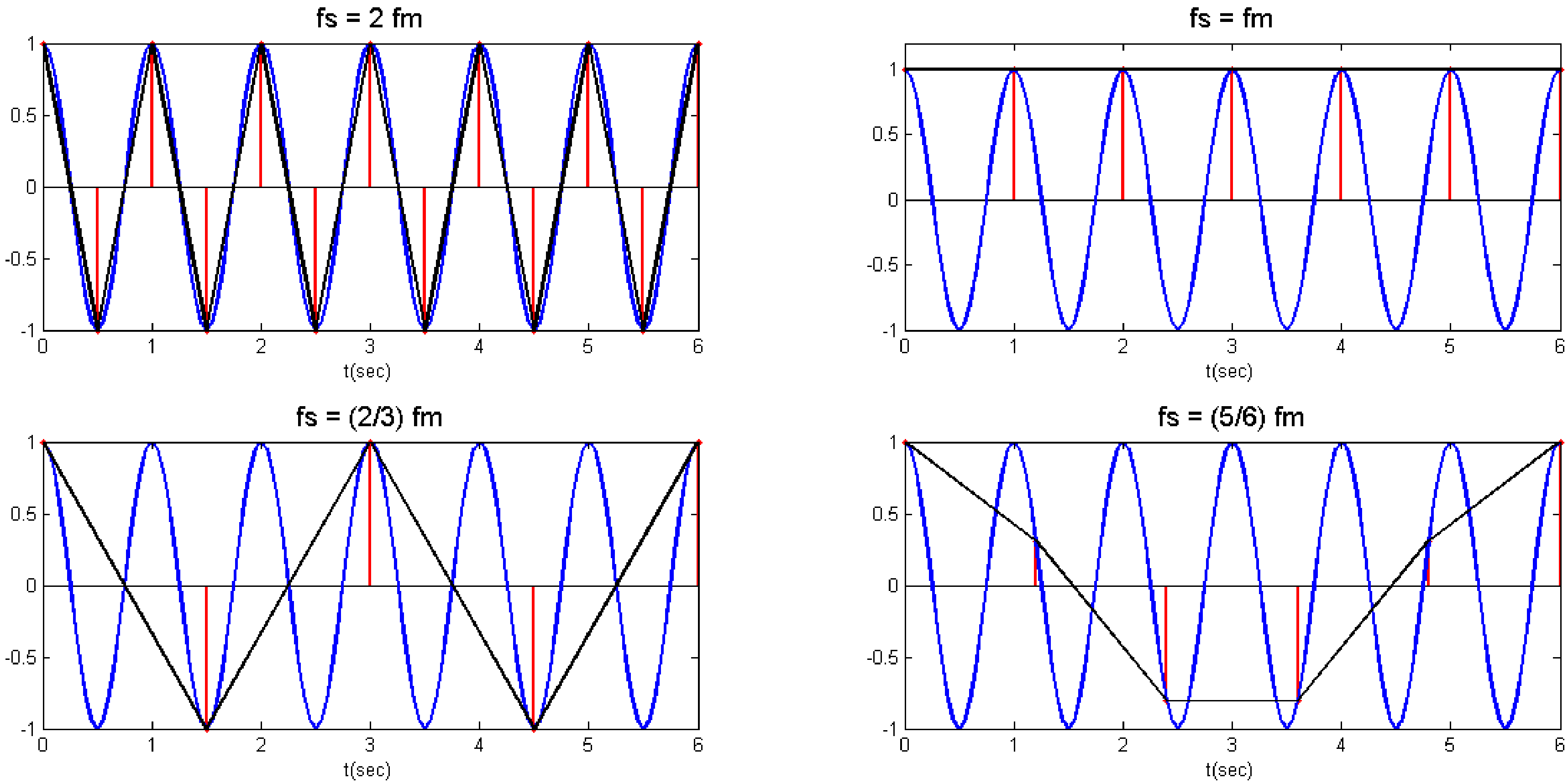
- One main problem that stems when sampling the signal without appropriate *fs is* ***"aliasing".***
- When a sampled signal is interpolated (joining the sample points together), the reconstructed signal may resemble a lower frequency signal if sampling frequency is not properly chosen

Problem with Sampling(contd..)



Sampling a cosine signal of frequency *fm=1 Hz*
with various sample frequency fs.

Problem with Sampling(contd..)



Reconstruction of signal from sampled signal.

To avoid aliasing in sampling composite signals

- *Select* $f_s \geq 2 f_m$ *(Sampling Theorem,* f_m *should be maximum of all available frequencies)*

where

f_s*=sampling frequency*

f_m*=maximum frequency*

$f_{alias} = f_m \pm n f_s$

Process of sampling

1. Decide sampling frequency fs.
2. Find the interval of time domain.
3. Assign values to time variable.
4. Find *f(t)*

Process of sampling

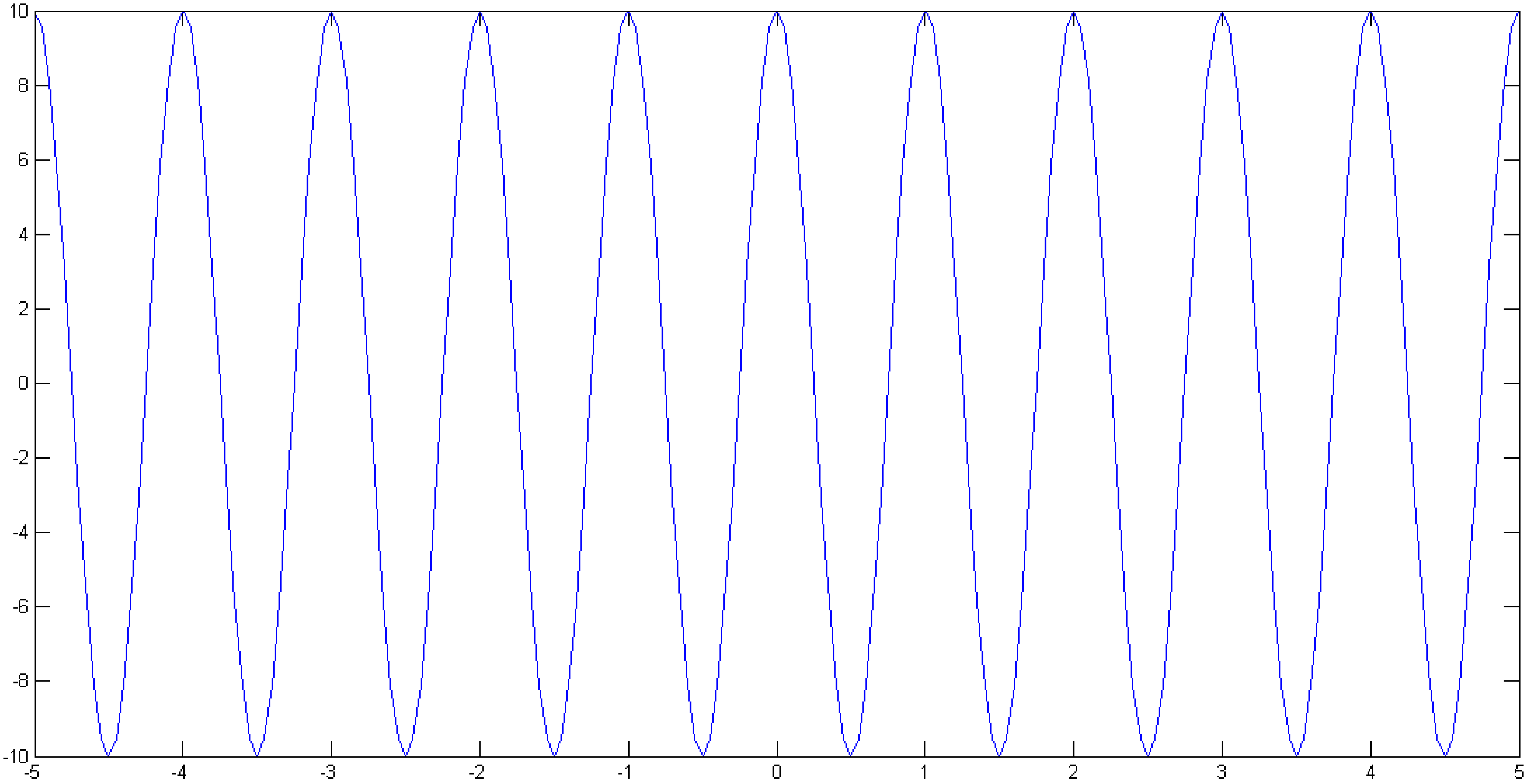
- Example

Plot $x(t) = A_m \cos(2\pi f_m t)$ for $\frac{-c}{f_m} \leq t \leq \frac{c}{f_m}$. Where $A_m = 10$, $f_m = 1$ Hertz, number of cycles needed is $2c = 10$.

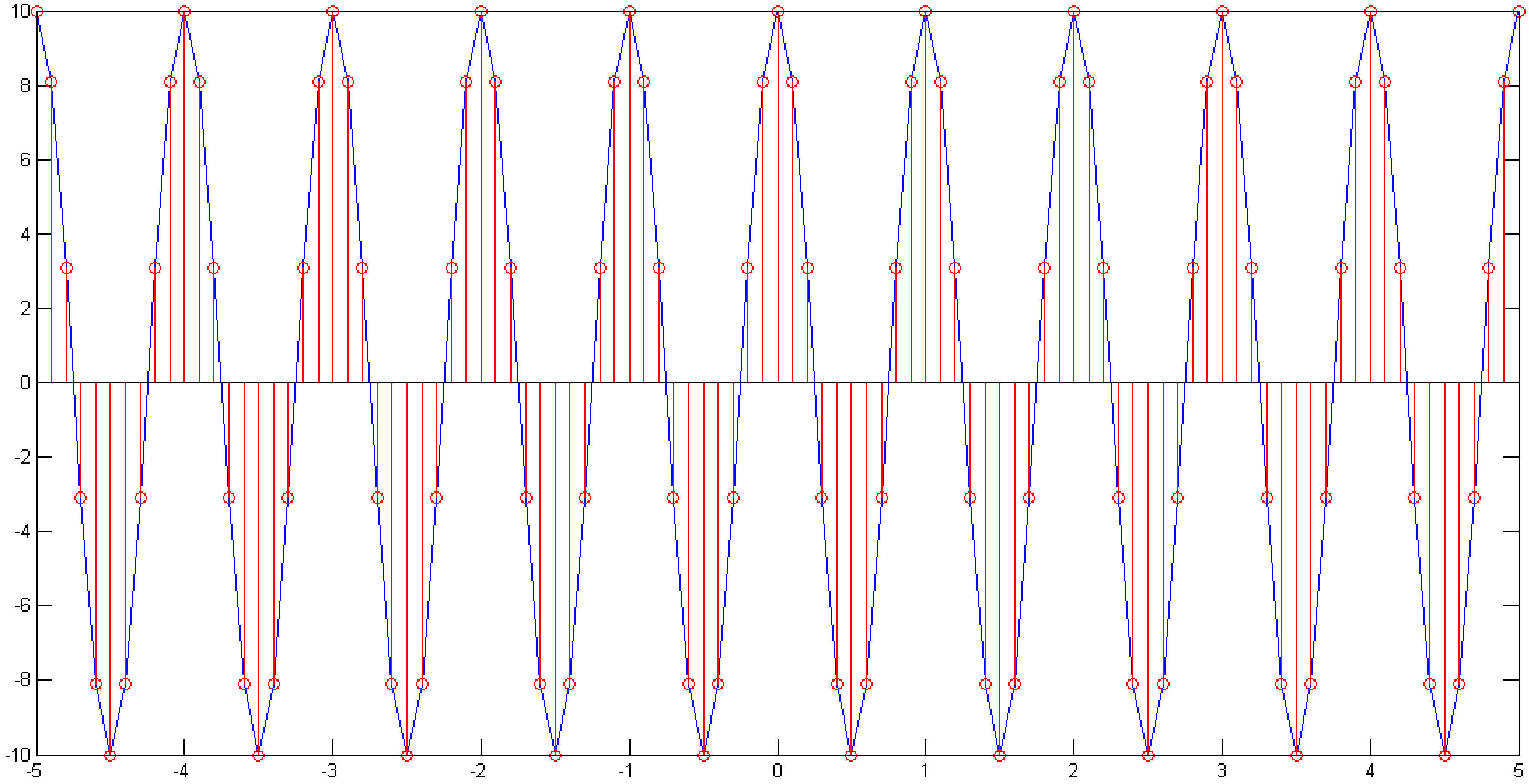
1. $f_s \geq 2f_m = 2$ Samples/sec. Decide $f_s = 10 > 2$.
2. $c = 5$, $T_m = 1/f_m = 1$ sec/cycle. Time starts from $\frac{-c}{f_m}$ till $\frac{c}{f_m}$. The duration is $\frac{2c}{f_m} = 10$ sec.
3. Time variable changes by steps of $T_s = 1/f_s$ sec/sample. So, $t = \frac{-c}{f_m} : T_s : \frac{c}{f_m}$ in MATLAB.
4. Find $x = A_m \cos(2\pi f_m t)$ in MATLAB.
5. Plot with *stem(t,x)* in MATLAB.

Remember the units of variables!

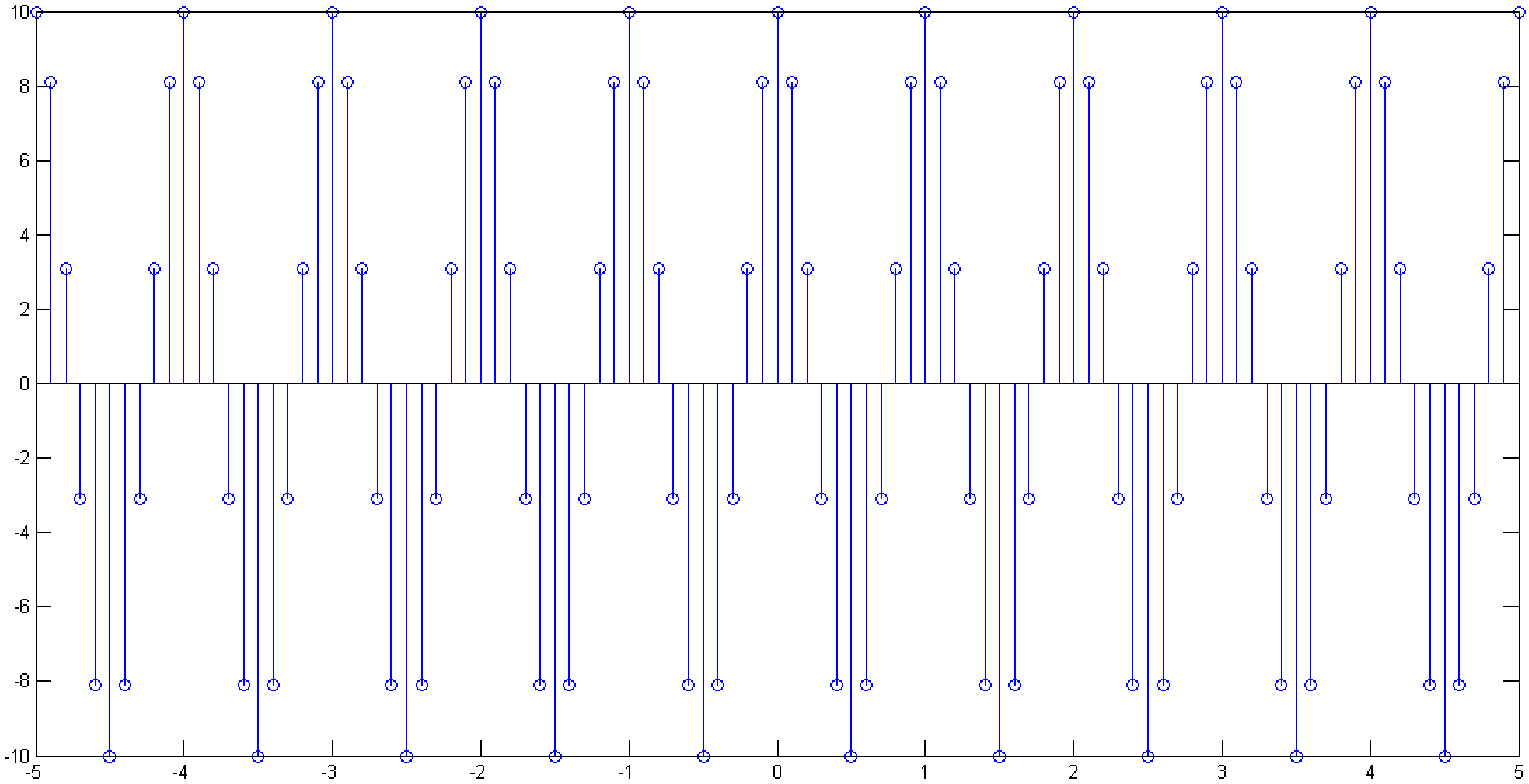
Process of sampling



Process of sampling



Process of sampling



Some formulae

1. Sampling frequency fs (sample/sec)

2. Total duration of signal T (sec)

3. Total number of samples in T (sec) = fs x T

4. fs ≥ 2 fm (fm being max. of available freq.)

5. $f_{alias} = f_m \pm nf_s$

Further Reading

1. Introduction To Signal Processing, Sophocles J. Orfanidis
   a. Title 1.3 and Examples 1.3.1, 1.3.2
   b. Title 1.4, Examples 1.4.1 – 1.4.5