

Assignment 3 Part-1

$$1) T(n) = \begin{cases} 1, & n=1 \\ 2T(n/2) + n, & n>1 \end{cases}$$

$$T(n) = 2T(n/2) + n$$

$$\begin{aligned} T(n/2) &= 2T\left(\frac{n}{2 \times 2}\right) + n/2 \\ &= 2T(n/4) + n/2 \end{aligned}$$

$$T(n/2) = 2T(n/2^2) + n/2 \quad \text{--- (1)}$$

Sub (1) in (1)

$$\begin{aligned} T(n) &= 2 \times (2T(n/2^2) + n/2) + n \\ &= 4T(n/2^2) + n + n \end{aligned}$$

$$T(n) = 2^2 T(n/2^3) + 2n \quad \text{--- (2)}$$

$$T(n/2^2) = 2T(n/2 \times 4) + n/4$$

Sub (a) in (b)

$$T(n) = 2^3 \left(2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2} \right) + 2n - (c)$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + n + 2n$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + 3n$$

⋮

k times

$$= 2^k T\left(\frac{n}{2^k}\right) + kn$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log_2 n = k$$

$$k = \log_2 n$$

$$= 2^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + \log_2 n \cdot n$$

$$= T(1) + \log_2 n \cdot n$$

$$= 1 + n \log_2 n = O(n \log_2 n)$$

$$2) T(n) = \begin{cases} 1, & n=1 \\ 8T(n/2) + n^2, & n>1 \end{cases}$$

$$T(n) = 8T(n/2) + n^2 \quad \text{--- (A)}$$

$$T(n/2) = 8T(n/4) + (n/2)^2 \quad \text{--- (B)}$$

Sub (B) in (A)

$$\begin{aligned} T(n) &= 8 \left(8T(n/4) + (n/2)^2 \right) + n^2 \\ &= 8^2 T(n/4) + 3n^2 \\ &= 8^2 T(n/4) + 3n^2 \quad \text{--- (C)} \end{aligned}$$

$$T(n/4) = 8T(n/8) + (n/4)^2 \quad \text{--- (D)}$$

Sub (D) in (C)

$$\begin{aligned} T(n) &= 8^2 \left(8T(n/8) + (n/4)^2 \right) + 3n^2 \\ &= 8^3 T(n/8) + 8^2 \frac{n^2}{4} + 3n^2 \\ &= 8^3 T(n/8) + 64 \frac{n^2}{16} + 3n^2 \\ &= 8^3 T(n/8) + 4n^2 + 3n^2 \end{aligned}$$

$$= 8^3 T(n/2^3) + 7n^2 - 2$$

⋮
k times

$$= 8^k T(n/2^k) + 7n^k$$

$$\frac{d}{dx} = 1$$

$$= 8^k T(n/2^k) + (2^k - 1) n^2$$

$$n = 2^k$$

$$\log_2 n = k \Rightarrow 2^k = n$$

$$= 8^{\log_2 n} T(n/2^{\log_2 n}) + (2^{\log_2 n} - 1) n$$

$$= n^{\log_2 8} T(n/n) + (2^{\log_2 n} - 1) n$$

$$= n^3 T(1) + (2^{\log_2 n} - 1) n$$

$$= n^3 T(1) + 2n^{\log_2 2} - n^{\log_2 2}$$

$$= n^3 + n^3 - n^2 = O(n^3)$$