15 PROBABILITY

Exercise-1 : Single Choice Problems

1. The boy comes from a family of two children; What is the probability that the other child is his sister ? :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

2. If A be any event in sample space then the maximum value of $3\sqrt{P(A)}+4\sqrt{P(\bar{A})}$ is :

(a) 4 (b) 2
(c) 5 (d) Can not be determined

3. Let A and B be two events, such that $P(\overline{A \cup B})=\frac{1}{6}, P(A \cap B)=\frac{1}{4}$ and $P(\bar{A})=\frac{1}{4}$, where $\bar{A}$ stands for complement of event A. Then events A and B are :

(a) equally likely and mutually exclusive (b) equally likely but not independent
(c) independent but not equally likely (d) mutually exclusive and independent

4. Let n ordinary fair dice are rolled once. The probability that at least one of the dice shows an odd number is $\left(\frac{31}{32}\right)$ than 'n' is equal to :

(a) 3 (b) 4 (c) 5 (d) 6

5. Three a's, three b's and three c's are placed randomly in a 3×3 matrix. The probability that no row or column contain two identical letters can be expressed as $\frac{p}{q}$, where p and q are coprime then $(p+q)$ equals to :

(a) 151 (b) 161 (c) 141 (d) 131

6. A set contains $3n$ members. Let P_n be the probability that S is partitioned into 3 disjoint subsets with n members in each subset such that the three largest members of S are in different subsets. Then $\lim_{n \to \infty} P_n =$

(a) 2/7 (b) 1/7 (c) 1/9 (d) 2/9

7. Three different numbers are selected at random from the set $A = \{1, 2, 3, \ldots\ldots, 10\}$. Then the probability that the product of two numbers equal to the third number is $\frac{p}{q}$, where p and q are relatively prime positive integers then the value of $(p + q)$ is :

(a) 39 (b) 40 (c) 41 (d) 42

8. Mr. A's T.V. has only 4 channels ; all of them quite boring so he naturally desires to switch (change) channel after every one minute. The probability that he is back to his original channel for the first time after 4 minutes can be expressed as $\frac{m}{n}$; where m and n are relatively prime numbers. Then $(m + n)$ equals :

(a) 27 (b) 31 (c) 23 (d) 33

9. Letters of the word TITANIC are arranged to form all the possible words. What is the probability that a word formed starts either with a T or a vowel ?

(a) $\frac{2}{7}$ (b) $\frac{4}{7}$ (c) $\frac{3}{7}$ (d) $\frac{5}{7}$

10. A mapping is selected at random from all mappings $f : A \to A$ where set $A = \{1, 2, 3, \ldots\ldots, n\}$

If the probability that mapping is injective is $\frac{3}{32}$, then the value of n is :

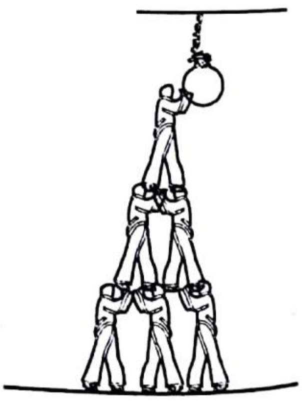
(a) 3 (b) 4 (c) 8 (d) 16

11. A 4 digit number is randomly picked from all the 4 digit numbers, then the probability that the product of its digit is divisible by 3 is :

(a) $\frac{107}{125}$ (b) $\frac{109}{125}$

(c) $\frac{111}{125}$ (d) None of these

12. To obtain a gold coin; 6 men, all of different weight, are trying to build a human pyramid as shown in the figure. Human pyramid is called "stable" if some one not in the bottom row is "supported by" each of the two closest people beneath him and no body can be supported by anybody of lower weight. Formation of 'stable' pyramid is the only condition to get a gold coin. What is the probability that they will get gold coin ?



(a) $\frac{1}{45}$ (b) $\frac{2}{45}$

(c) $\frac{4}{45}$ (d) $\frac{1}{\underline{|5}}$

13. From a pack of 52 playing cards; half of the cards are randomly removed without looking at them. From the remaining cards, 3 cards are drawn randomly. The probability that all are king.

(a) $\frac{1}{(25)(17)(13)}$ (b) $\frac{1}{(25)(15)(13)}$

(c) $\frac{1}{(52)(17)(13)}$ (d) $\frac{1}{(13)(51)(17)}$

14. A bag contains 10 white and 3 black balls. Balls are drawn one by one without replacement till all the black balls are drawn. The probability that the procedure of drawing balls will come to an end at the seventh draw is :

(a) $\frac{15}{286}$ (b) $\frac{105}{286}$ (c) $\frac{35}{286}$ (d) $\frac{7}{286}$

15. Let S be the set of all function from the set $\{1, 2, \ldots, 10\}$ to itself. One function is selected from S, the probability that the selected function is one-one onto is :

(a) $\frac{9!}{10^9}$ (b) $\frac{1}{10}$ (c) $\frac{100}{10!}$ (d) $\frac{9!}{10^{10}}$

16. Two friends visit a restaurant randomly during 5 pm to 6 pm. Among the two, whoever comes first waits for 15 min and then leaves. The probability that they meet is :

(a) $\frac{1}{4}$ (b) $\frac{1}{16}$ (c) $\frac{7}{16}$ (d) $\frac{9}{16}$

17. Three numbers are randomly selected from the set $\{10, 11, 12, \ldots\ldots, 100\}$. Probability that they form a Geometric progression with integral common ratio greater than 1 is :

(a) $\frac{15}{^{91}C_3}$ (b) $\frac{16}{^{91}C_3}$ (c) $\frac{17}{^{91}C_3}$ (d) $\frac{18}{^{91}C_3}$

Answers

1.	(a)	**2.**	(c)	**3.**	(c)	**4.**	(c)	**5.**	(c)	**6.**	(d)	**7.**	(c)	**8.**	(b)	**9.**	(d)	**10.**	(b)
11.	(a)	**12.**	(a)	**13.**	(a)	**14.**	(a)	**15.**	(a)	**16.**	(c)	**17.**	(d)						

Exercise-2 : One or More than One Answer is/are Correct

1. A consignment of 15 record players contain 4 defectives. The record players are selected at random, one by one and examined. The one examined is not put back. Then :

(a) Probability of getting exactly 3 defectives in the examination of 8 record players is $\frac{{}^4C_3\ {}^{11}C_5}{{}^{15}C_8}$.

(b) Probability that 9^{th} one examined is the last defective is $\frac{8}{197}$.

(c) Probability that 9^{th} examined record player is defective, given that there are 3 defectives in first 8 players examined is $\frac{1}{7}$.

(d) Probability that 9^{th} one examined is the last defective is $\frac{8}{195}$.

2. If $A_1, A_2, A_3, \ldots\ldots, A_{1006}$ be independent events such that $P(A_i) = \frac{1}{2i}$ $(i = 1, 2, 3, \ldots\ldots, 1006)$ and probability that none of the events occurs be $\frac{\alpha!}{2^{\alpha}(\beta!)^2}$, then :

(a) β is of form $4k + 2$, $k \in I$

(b) $\alpha = 2\beta$

(c) β is a composite number

(d) α is of form $4k$, $k \in I$

3. A bag contains four tickets marked with 112, 121, 211, 222 one ticket is drawn at random from the bag. let E_i $(i = 1,2,3)$ denote the event that i^{th} digit on the ticket is 2. Then :

(a) E_1 and E_2 are independent

(b) E_2 and E_3 are independent

(c) E_3 and E_1 are independent

(d) E_1, E_2, E_3 are independent

4. For two events A and B let, $P(A) = \frac{3}{5}$, $P(B) = \frac{2}{3}$, then which of the following is/are correct ?

(a) $P(A \cap \bar{B}) \le \frac{1}{3}$

(b) $P(A \cup B) \ge \frac{2}{3}$

(c) $\frac{4}{15} \le P(A \cap B) \le \frac{3}{5}$

(d) $\frac{1}{10} \le P(\bar{A}/B) \le \frac{3}{5}$

Answers

1.	(a, c, d)	**2.**	(a, b, c, d)	**3.**	(a, b, c)	**4.**	(a, b, c, d)				

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

There are four boxes B_1, B_2, B_3 and B_4. Box B_i has i cards and on each card a number is printed, the numbers are from 1 to i. A box is selected randomly, the probability of selecting box B_i is $\frac{i}{10}$ and then a card is drawn.

Let E_i represent the event that a card with number 'i' is drawn. Then :

1. $P(E_1)$ is equal to :

(a) $\frac{1}{5}$ (b) $\frac{1}{10}$ (c) $\frac{2}{5}$ (d) $\frac{1}{4}$

2. $P(B_3|E_2)$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

Paragraph for Question Nos. 3 to 5

Mr. A randomly picks 3 distinct numbers from the set {1, 2, 3, 4, 5, 6, 7, 8, 9} and arranges them in descending order to form a three digit number. Mr. B randomly picks 3 distinct numbers from the set {1, 2, 3, 4, 5, 6, 7, 8} and also arranges them in descending order to form a 3 digit number.

3. The probability that Mr. A's 3 digit number is always greater than Mr. B's 3 digit number is :

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

4. The probability that A and B has the same 3 digit number is :

(a) $\frac{7}{9}$ (b) $\frac{4}{9}$ (c) $\frac{1}{84}$ (d) $\frac{1}{72}$

5. The probability that Mr. A's number is larger than Mr. B's number, is :

(a) $\frac{37}{56}$ (b) $\frac{39}{56}$ (c) $\frac{31}{56}$ (d) none of these

Paragraph for Question Nos. 6 to 7

In an experiment a coin is tossed 10 times.

6. Probability that no two heads are consecutive is :

(a) $\frac{143}{2^{10}}$ (b) $\frac{9}{2^6}$ (c) $\frac{2^7-1}{2^{10}}$ (d) $\frac{2^6-1}{2^6}$

7. The probability of the event that "exactly four heads occur and occur alternately" is :

(a) $1-\frac{4}{2^{10}}$ (b) $1-\frac{7}{2^{10}}$ (c) $\frac{4}{2^{10}}$ (d) $\frac{5}{2^{10}}$

Paragraph for Question Nos. 8 to 10

The rule of an "obstacle course" specifies that at the n^{th} obstacle a person has to toss a fair 6 sided die n times. If the sum of points in these n tosses is bigger than 2^n, the person is said to have crossed the obstacle.

8. The maximum obstacles a person can cross :

(a) 4 (b) 5 (c) 6 (d) 7

9. The probability that a person crosses the first three obstacles :

(a) $\frac{143}{216}$ (b) $\frac{100}{243}$ (c) $\frac{216}{243}$ (d) $\frac{100}{216}$

10. The probability that a person crosses the first two obstacles but fails to cross the third obstacle.

(a) $\frac{36}{243}$ (b) $\frac{116}{216}$ (c) $\frac{35}{243}$ (d) $\frac{143}{243}$

Paragraph for Question Nos. 11 to 12

In an objective paper, there are two sections of 10 questions each. For 'section 1', each question has 5 options and only one option is correct and 'section 2' has 4 options with multiple answers and marks for a question in this section is awarded only if he ticks all correct answers. Marks for each question in 'section 1' is 1 and in 'section 2' is 3. (There is no negative marking).

11. If a candidate attempts only two questions by gussing, one from 'section 1' and one from 'section 2', the probability that he scores in both questions is :

(a) $\frac{74}{75}$ (b) $\frac{1}{25}$ (c) $\frac{1}{15}$ (d) $\frac{1}{75}$

12. If a candidate in total attempts 4 questions all by gussing, then the probability of scoring 10 marks is :

(a) $\frac{1}{15}\left(\frac{1}{15}\right)^2$ (b) $\frac{4}{5}\left(\frac{1}{15}\right)^3$ (c) $\frac{1}{5}\left(\frac{14}{15}\right)^3$ (d) None of these

Answers

1.	(c)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(a)	**6.**	(b)	**7.**	(c)	**8.**	(a)	**9.**	(b)	**10.**	(c)
11.	(d)	**12.**	(d)																

Exercise-4 : Matching Type Problems

1. A is a set containing n elements, A subset P (may be void also) is selected at random from set A and the set A is then reconstructed by replacing the elements of P. A subset Q (may be void also) of A is again chosen at random. The probability that

	Column-I		Column-II
(A)	Number of elements in P is equal to the number of elements in Q is	(P)	$\frac{^{2n}C_n}{4^n}$
(B)	The number of elements in P is more than that in Q is	(Q)	$\frac{(2^{2n} - {^{2n}C_n})}{2^{2n+1}}$
(C)	$P \cap Q = \phi$ is	(R)	$\frac{^{2n}C_{n+1}}{4^n}$
(D)	Q is a subset of P is	(S)	$\left(\frac{3}{4}\right)^n$
		(T)	$\frac{^{2n}C_n}{4^{n-1}}$

Answers

1.	A $\rightarrow$ P ; B $\rightarrow$ Q ; C $\rightarrow$ S ; D $\rightarrow$ S

Exercise-5 : Subjective Type Problems

1. Mr. A writes an article. The article originally is error free. Each day Mr. B introduces one new error into the article. At the end of the day, Mr. A checks the article and has $\frac{2}{3}$ chance of catching each individual error still in the article. After 3 days, the probability that the article is error free can be expressed as $\frac{p}{q}$ where p and q are relatively prime positive integers. Let $\lambda = q - p$, then find the sum of the digits of λ.
2. India and Australia play a series of 7 one-day matches. Each team has equal probability of winning a match. No match ends in a draw. If the probability that India wins atleast three consecutive matches can be expressed as $\frac{p}{q}$ where p and q are relatively prime positive integers. Find the unit digit of p.
3. Two hunters A and B set out to hunt ducks. Each of them hits as often as he misses when shooting at ducks. Hunter A shoots at 50 ducks and hunter B shoots at 51 ducks. The probability that B bags more ducks than A can be expressed as $\frac{p}{q}$ in its lowest form. Find the value of $(p+q)$.
4. If $a, b, c \in N$, the probability that $a^2+b^2+c^2$ is divisible by 7 is $\frac{m}{n}$ where m, n are relatively prime natural numbers, then $m+n$ is equal to :
5. A fair coin is tossed 10 times. If the probability that heads never occur on consecutive tosses be $\frac{m}{n}$ (where m, n are coprime and $m, n \in N$), then the value of $(n-7m)$ equals to :
6. A bag contains 2 red, 3 green and 4 black balls. 3 balls are drawn randomly and exactly 2 of them are found to be red. If p denotes the chance that one of the three balls drawn is green ; find the value of $7p$.
7. There are 3 different pairs (*i.e.*, 6 units say a, a, b, b, c, c) of shoes in a lot. Now three person come and pick the shoes randomly (each gets 2 units). Let p be the probability that no one is able to wear shoes (*i.e.*, no one gets a correct pair), then the value of $\frac{13p}{4-p}$, is :
8. A fair coin is tossed 12 times. If the probability that two heads do not occur consecutively is p, then the value of $\frac{[\sqrt{4096p} - 1]}{2}$ is, where [ ] denotes greatest integer function :
9. X and Y are two weak students in mathematics and their chances of solving a problem correctly are 1/8 and 1/12 respectively. They are given a question and they obtain the same answer. If the probability of common mistake is $\frac{1}{1001}$, then probability that the answer was correct is a/b (a and b are coprimes). Then $|a-b| =$

10. Seven digit numbers are formed using digits 1, 2, 3, 4, 5, 6, 7, 8, 9 without repetition. The probability of selecting a number such that product of any 5 consecutive digits is divisible by either 5 or 7 is P. Then $12P$ is equal to

11. Assume that for every person the probability that he has exactly one child, excactly 2 children and exactly 3 children are $\frac{1}{4}, \frac{1}{2}$ and $\frac{1}{4}$ respectively. The probability that a person will have 4 grand children can be expressed as $\frac{p}{q}$ where p and q are relatively prime positive integers. Find the value of $5p - q$.

12. Mr. B has two fair 6-sided dice, one whose faces are numbered 1 to 6 and the second whose faces are numbered 3 to 8. Twice, he randomly picks one of dice (each dice equally likely) and rolls it. Given the sum of the resulting two rolls is 9. The probability he rolled same dice twice is $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $(m + n)$.

Answers

1.	7	**2.**	7	**3.**	3	**4.**	8	**5.**	1	**6.**	3	**7.**	2
8.	9	**9.**	1	**10.**	7	**11.**	7	**12.**	7				

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Chapter 16 – Logarithms