Solutions

CHAPTER 5 PROBABILITY

SECTION-1

SINGLE CHOICE QUESTIONS

1. $P = \dfrac{\overset{\text{both 'a'}}{1 \times 2} + \overset{\text{both 's'}}{2 \times 4} + \overset{\text{both 'i'}}{2 \times 1} + \overset{\text{both 'n'}}{2 \times 1}}{8 \times 8} = \dfrac{14}{64} = \dfrac{7}{32}$

2. $P = \dfrac{\overset{\text{selecting 3 out of 7 already drawn}}{{}^{7}C_{3}} \times {}^{3}C_{2}}{{}^{10}C_{5}} = \dfrac{5}{12}$

3. Case I F = B

0	0	4	5
1	1	3	4
2	2	2	3
3	3	1	2
4	4	0	1

Case II L = R

$$P = \frac{4\left(\frac{9!}{4!5!} + \frac{9!}{3!4!1!1!} + \frac{9!}{2!2!2!3!} + \frac{9!}{3!3!1!2!} + \frac{9!}{4!4!1!}\right)}{4^9}$$

$$= \frac{3969}{4^7}$$

4. Let he takes 'n' chances

$$P = 1 - P(\text{none correct})$$

[$\because$ Total ways to answer $= 2^5 - 1 = 31$]

$$= 1 - \frac{^{30}C_n}{^{31}C_n} = 1 - \frac{31-n}{31} = \frac{n}{31}$$

$\Rightarrow \quad \frac{n}{31} > \frac{1}{8} \Rightarrow n \geq 4$ [minimum value of n = 4]

Alternate : $\rightarrow$ P (getting correct in r^{th} trial) $= \frac{^{30}C_{r-1}}{^{31}C_{r-1}} \times \frac{1}{32-r} = \frac{1}{31}$

$\therefore$ P (correct in 1^{st} trial or 2^{nd} trial or......or n^{th} trial)

$$= \frac{1}{31} + \frac{1}{31} + \frac{1}{31} + \ldots\ldots + \frac{1}{31} = \frac{n}{31}$$

5. $\underbrace{(1-p)^n}_{\text{none defective}} + \underbrace{{}^nC_1(1-p)^{n-1}p}_{\text{1 defective}}\Big(\underbrace{(1-p)^m}_{\text{none defective in sample of next 'm' items}} + \underbrace{{}^mC_1(1-p)^{m-1}p}_{\text{1 defective in next 'm' items}}\Big) + {}^nC_2(1-p)^{n-2}p^2\,(1-p)^m$

6. $P = \frac{^3C_1 2^m - {}^3C_2(1)^m}{3^m} = \frac{2^m - 1}{3^{m-1}}$

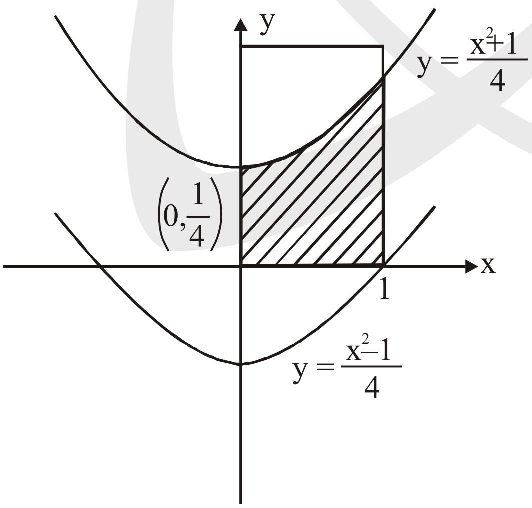
A $\rightarrow$ number of ways s.t. there is no A

B $\rightarrow$ number of ways s.t. there is no B

C $\rightarrow$ number of ways s.t. there is no C

n (A $\cup$ B $\cup$ C) = $S_1 - S_2 + S_3 = {}^3C_1\, 2^m - {}^3C_2 1^m + 0$

7.



$$|x_1 - x_2| = \sqrt{(x_1 + x_2)^2 - 4x_1x_2} = \sqrt{m^2 - 4n} \le 1$$

$$|m^2 - 4n| \le 1$$

$$\Rightarrow -1 \le x^2 - 4y \le 1$$

$$A = \int_0^1 \frac{x^2 + 1}{4} dx = \frac{1}{3}$$

$$P = \frac{1/3}{1} = \frac{1}{3}$$

8. $$P = \frac{{}^nC_0 x^n y^\circ + {}^nC_2 x^{n-2} y^2 + {}^nC_4 x^{n-4} y^4 + \ldots..}{(x+y)^n}$$

$$= \frac{(x+y)^n + (x-y)^n}{2(x+y)^n}$$

9. 4 steps or 6 steps

$$P = \frac{\frac{4!}{2!2!}}{4^4} + \frac{+2\left(\frac{6!}{3!2!1!} - \frac{4!}{2!2!} \times 2!\right)}{4^6} = \frac{3}{64}$$

10. $$\frac{x}{2} - \left\{\frac{x}{2}\right\} + \frac{x}{3} - \left\{\frac{x}{3}\right\} + \frac{x}{5} - \left\{\frac{x}{5}\right\} = \frac{31x}{30}$$

$$\Rightarrow \left\{\frac{x}{2}\right\} + \left\{\frac{x}{3}\right\} + \left\{\frac{x}{5}\right\} = 0$$

$$\Rightarrow \frac{x}{2} \in I, \frac{x}{3} \in I \ \& \ \frac{x}{5} \in I$$

$$\Rightarrow x = 30\,k, k \in I$$

$$\therefore P = \frac{33}{1000}$$

11. $P(A \cap B) = P(A) + P(B) - P(A \cup B) \ge x + y - 1$

$\& \ P(A \cap B) \le x$

$$\Rightarrow P(A/B) = \frac{P(A \cap B)}{P(B)} \in \left[\frac{x+y-1}{y}, \frac{x}{y}\right]$$

12. $$P = \frac{{}^7C_2(2^4 - 2)}{7^4} = \frac{6}{49}$$

13. Number of favourable ways = n!

(distribute n red balls over n white balls)

no. of totals ways = $\frac{(2n)!}{(n!)(2!)^n}$

(divide 2n balls into 'n' equal groups)

$\therefore \quad p = \frac{n!}{\left(\frac{(2n)!}{(n!)(2!)^n}\right)} = \frac{2^n}{{}^{2n}C_n}$

14. Let number s are x_1, $x_1 + 2d$, $x_1 + 4d$

$x_1 + 4d \le 4m + 1$

$\Rightarrow x_1 \le 4(m - d) + 1$, $d = 1, 2, 3, \ldots, m$

$\therefore$ Number of favourable ways = $(4m - 3) + (4m - 7) + \ldots + 1$

$$= \frac{m}{2}(4m - 2) = m(2m - 1)$$

Number of total ways = ${}^{4m+1}C_3 = \frac{(4m+1)4m(4m-1)}{6}$

$$P = \frac{m(2m-1)6}{(4m+1)4m(4m-1)} = \frac{3(2m-1)}{2(16m^2-1)}$$

15. $E_i \rightarrow$ the event that 'i' white are drawn from the first bag

$A \rightarrow$ one ball drawn from second bag is white.

$$P(E_2/A) = \frac{P(A/E_2)P(E_2)}{\sum_{i=1}^{5} P(E_i)P(A/E_i)}$$

$$= \frac{\frac{{}^4C_3\,{}^6C_2}{{}^{10}C_5} \times \frac{2}{5}}{\frac{4}{5} \times \frac{{}^4C_1\,{}^6C_4}{{}^{10}C_5} + \frac{3}{5}\frac{{}^4C_2\,{}^6C_3}{{}^{10}C_5} + \frac{2}{5}\frac{{}^4C_3\,{}^6C_2}{{}^{10}C_5} + \frac{{}^4C_4\,{}^6C_1}{{}^{10}C_5} \times \frac{1}{5}}$$

$$= \frac{4 \times 15 \times 2}{4 \times 4 \times 15 + 3 \times 6 \times 20 + 2 \times 4 \times 15 + 6} = \frac{20}{121}$$

16. Number of favourable ways

$$= \sum_{r=1}^{10} {}^{10}C_r^2 \quad + \quad 2\sum_{r=1}^{9} {}^{10}C_r\,{}^{10}C_{r+1} \quad + \quad 2\sum_{r=1}^{8} {}^{10}C_r\,{}^{10}C_{r+2}$$

(difference 0) (difference 1) (difference 2)

$$= ({}^{20}C_{10} - 1) + 2\,({}^{20}C_{11} - 10) + 2\,({}^{20}C_8 - {}^{10}C_2)$$

$$= {}^{20}C_{10} + 2\,{}^{20}C_{11} + 2\,{}^{20}C_{12} - 111$$

no. of total ways $= (2^{10} - 1)^2$

$$P = \frac{{}^{20}C_{10} + 2({}^{20}C_{11} + {}^{20}C_{12}) - 111}{(2^{10} - 1)^2}$$

17. $\lim\limits_{x \to 0}\left(\frac{a^x + b^x}{2}\right)^{2/x} = e^{\lim\limits_{x \to 0}\left(\frac{a^x + b^x}{2} - 1\right)\frac{2}{x}} = e^{\ln ab} = ab$

$\Rightarrow$ $ab = 6$

$(a, b) = (1, 6), (6, 1), (2, 3), (3, 2)$

$$P = \frac{4}{36} = \frac{1}{9}$$

18. $$P = \frac{\frac{1}{n} \times 1}{\frac{1}{n} \times 1 + \frac{n-1}{n} \times \frac{1}{n}} = \frac{n}{2n-1}$$

19. $$P = \frac{\frac{2}{10} \times 0.6}{\frac{2}{10} \times 0.6 + \frac{3}{10} \times 0.5 + \frac{5}{10} \times 0.4} = \frac{12}{47}$$

20. Let probability of single bacteria to die = P

$\therefore$ $P = \frac{1}{4} \times 1 + \frac{1}{2} \times P \times P + \frac{1}{4} \times P \times P \times P$

bacteria die or split

$A \rightarrow$ bacteria do not split

$B \rightarrow$ bacteria split into 2

C $\rightarrow$ bacteria split into 3

D $\rightarrow$ bacterial die

$$P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C)$$

$$P = \frac{1}{4} \times 1 + \frac{1}{2} \times P \times P + \frac{1}{4} \times P \times P \times P$$

$$\Rightarrow \quad P^3 + 2P^2 - 4P + 1 = 0$$

$$\Rightarrow \quad (P - 1)(P^2 + 3P - 1) = 0$$

$$\Rightarrow \quad P = \frac{-3+\sqrt{13}}{2}$$

$\therefore$ Probability that bacteria survives

$$= 1 - \frac{\sqrt{13}-3}{2} = \frac{5-\sqrt{13}}{2}$$

21. A $\rightarrow$ last throw get 1, 2, 3 or 4

B $\rightarrow$ last throw get 2, 3, or 4

$$P(B/A) = \frac{3}{4}$$

22. (C)

$$P = \frac{\frac{{}^5C_5}{{}^{10}C_5} \times \frac{5}{5}}{\frac{{}^5C_5}{{}^{10}C_5} \times \frac{5}{5} + \frac{{}^5C_4\,{}^5C_1}{{}^{10}C_5}\frac{4}{5} + \frac{{}^5C_3\,{}^5C_2}{{}^{10}C_5}\frac{3}{5} + \frac{{}^5C_2\,{}^5C_3}{{}^{10}C_5}\frac{2}{5} + \frac{{}^5C_1\,{}^5C_4}{{}^{10}C_5} \times \frac{1}{5}}$$

$$= \frac{5}{5+100+300+200+25}$$

$$= \frac{1}{126}$$

23. Favourable cases are (1, 4), (1, 9), (2, 8), (4, 9)

$$\therefore \quad P = \frac{4}{{}^9C_2} = \frac{1}{9}$$

24. $$P = {}^4C_1\left(\frac{3}{9}\right)\left(\frac{6}{9}\right)^3 = \frac{32}{81}$$

25. $$P = 1 - \frac{{}^{14}C_5}{{}^{15}C_5} = \frac{1}{3} = \frac{1}{3}$$

26. P_1 get paired with P_2 in 1st round = 2k

where $K(2+3+4) = 1 \quad \Rightarrow \quad k = \frac{1}{9}$

$\therefore$ P (P_2 reaches second round) = 1 – P(P_2 paired with P_1)

$$= 1 - \frac{2}{9} = \frac{7}{9}$$

27. $$x_1 + x_2 + x_3 = 8, 27$$

No. of favourable ways = $\dfrac{{}^{8-3+2}C_2 - 3\times 3}{3!} + \dfrac{{}^{27-3+2}C_2 - 3\,{}^{27-18+2}C_2 - (1+3\times 7)}{3!}$

$= 25$

$$P = \frac{25}{{}^{15}C_3} = \frac{25}{455} = \frac{5}{91}$$

28. $\underbrace{HH--H}_{m}XX--X \quad P = \dfrac{1}{2^m} + \dfrac{m+1}{2^{m+1}} - \dfrac{1}{2^{2m+1}}$

THH--HXX--X $\qquad = \dfrac{(m+3)2^m - 1}{2^{2m+1}}$

XTHH---HX---X

$\vdots$

XX ---XT$\underbrace{HH--H}_{m}$

29. $$P = \frac{4^4 - \left({}^2C_1 3^4 - 2^4\right)}{6^4} = \frac{55}{648}$$

Favourable number of ways

= Number appearing can be 2, 3, 4, 5 only – at least one of 2 or 5 excluded.

30. 7 digits are distinct and a_3 is the smallest

$\therefore$ Number of favourable ways = $\underbrace{{}^{10}C_7}_{\text{selecting 7 digit}} \times \underbrace{1}_{\text{selecting } a_3} \times \underbrace{{}^{6}C_2}_{\text{selecting } a_1, a_2}$

$$P = \frac{\left(\frac{10\times9\times8\times6\times5}{3\times2\times2}\right)}{9\times9\times8\times7\times6\times5\times4}$$

$$= \frac{5}{1512}$$

31. A $\rightarrow$ event that white balls are not among the 10 selected marbles.

B $\rightarrow$ event that blue balls are not among the 10 selected marbles.

C $\rightarrow$ event that red balls are not among the 10 selected marbles.

$$P(A\cup B\cup C) = \left(\frac{80}{100}\right)^{10} + \left(\frac{70}{100}\right)^{10} + \left(\frac{50}{100}\right)^{10} - \left(\frac{50}{100}\right)^{10} - \left(\frac{30}{100}\right)^{10} - \left(\frac{20}{100}\right)^{10}$$

$$= \frac{8^{10}+7^{10}-3^{10}-2^{10}}{10^{10}}$$

32. $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\Rightarrow \quad \frac{1}{2} = 2P(A) - (P(A))^2$

$\Rightarrow \quad P(A) = 1 - \frac{1}{\sqrt{2}}$

33. $P(\overline{A}\cap\overline{B}) = 1 - P(A\cup B) = \frac{7}{18}$

$\Rightarrow \quad P(A\cup B) = \frac{11}{18}$

$\Rightarrow \quad P(A) + P(B) - P(A\cap B) = \frac{11}{18}$

$\Rightarrow \quad P(A\cap B) = \frac{1}{18}$

$\Rightarrow \quad P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{1}{6}$

34. Odds against missing card being spade

$$= \frac{P(\text{missing card is not spade \& 2cards drawn are spades})}{P(\text{missing card is spade \& 2cards drawn are spades})}$$

$$= \frac{\frac{3}{4} \times \frac{^{13}C_2}{^{51}C_2}}{\frac{1}{4} \times \frac{^{12}C_2}{^{51}C_2}}$$

$$= \frac{3 \times 13 \times 12}{12 \times 11} = \frac{39}{11}$$

35. $|x - y| \geq 6$ holds for (1, 7),....,(1, 10), (2, 8), (2, 9), (2, 10), (3, 9), (3, 10), (4, 10)

Number of ways for which $|x - y| \geq 6 = 4 + 3 + 2 + 1 = 10$

$$P(|x - y| \leq 5) = 1 - \frac{10}{^{10}C_2} = \frac{7}{9}$$

36. Probability $= \frac{^6C_3}{6^3} = \frac{5}{54}$

37. $\underbrace{HH--H}_{7}\underbrace{XX---X}_{\text{H or T}}$ $\qquad P = \frac{1}{2^7}$

T H H -----H X - - X $\qquad P = \frac{1}{2^8}$

X T H -----H X - - X $\qquad P = \frac{1}{2^8}$

$\vdots$

X- - XT$\underbrace{HH--H}_{\text{last}7}$ $\qquad P = \frac{1}{2^8}$

$$\Rightarrow \qquad P = \frac{1}{2^7} + \underbrace{\frac{1}{2^8} + \frac{1}{2^8} + \frac{1}{2^8} + ... + \frac{1}{2^8}}_{5\text{times}} = \frac{1}{2^7} + \frac{5}{2^8}$$

$$= \frac{7}{2^8}$$

38. $$\frac{2}{7}=\frac{{}^{n}C_{1}\,{}^{n-4}C_{2}}{3\;{}^{n}C_{3}}$$

$$\frac{2}{7}=\frac{n(n-4)(n-5)}{n(n-1)(n-2)}=\frac{(n-4)(n-5)}{(n-1)(n-2)}$$

$\Rightarrow 5n^2 - 57n + 136 = 0$

$\Rightarrow \qquad n = 8.$

39. $(x - y) = n \qquad$ number of $(x, y) = n + 1$

$= n - 1 \qquad$ number of $(x, y) = n + 2$

$\vdots$

$= 1 \qquad = 2n$

$= 0 \qquad = 2n + 1$

$\therefore$ Number of favourable ways $= 2((n + 1) + (n + 2) + (n + 3)+...(2n)) + 2n + 1$

$= 3n^2 + 3n + 1$

$$P=\frac{3n^2+3n+1}{(2n+1)^2}$$

40. $$P=\frac{\frac{1}{2}\times\frac{2}{5}}{\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{2}{5}}=\frac{12}{17}$$

41. Clearly his car is at one of the crosses

|× × × × .....×|

The number of ways in which the remaining (m – 1) cars can take their places (excluding the car of the man)

$$= {}^{n-1}C_{m-1}$$

The number of ways in which the remaining (m – 1) cars can take places leaving the two places on two sides of his car $= {}^{n-3}C_{m-1}$

$$\therefore \qquad P=\frac{{}^{n-3}C_{m-1}}{{}^{n-1}C_{m-1}}=\frac{(n-m)(n-m-1)}{(n-1)(n-2)}$$

42. $$P = {}^6C_2\left(\frac{6}{12}\right)^2\left(\frac{6}{12}\right)^4\frac{6}{12} = \frac{15}{128}$$

43. $$\log_a b = \log_{2^m} 2^n = \frac{n}{m}$$

Let	m = 1,	n = 2, 3,......,25	number of (m, n) = 24
	m = 2,	n = 4, 6,......,24	number of (m, n) = 11
	m = 3,	n = 6, 9,......,24	number of (m, n) = 7
	m = 4,	n = 8, 12,......,24	number of (m, n) = 5
	m = 5,	n = 10, 15, 20, 25	number of (m, n) = 4
	m = 6,	n = 12, 18, 24	number of (m, n) = 3
	m = 7,	n = 14, 21	number of (m, n) = 2
	m = 8,	n = 16, 24	number of (m, n) = 2
	m = 9,	n = 18	number of (m, n) = 1
	m = 10,	n = 20	number of (m, n) = 1
	m = 11,	n = 22	number of (m, n) = 1
	m = 12,	n = 24	number of (m, n) = 1

$$P = \frac{62}{25\times 24} = \frac{31}{100}$$

44. A = number of persons going to hotel A = 1

B = number of persons going to hotel A = 0

C = number of persons going to hotel B = 0

D = number of persons going to hotel C = 0

$$P = 1 - \frac{n(A\cup B\cup C\cup D)}{n(s)}$$

$$n(A\cup B\cup C\cup D) = \sum n(A) - \sum n(A\cap B) + \sum n(A\cap B\cap C) - n(A\cap B\cap C\cap D)$$

$$= ({}^{20}C_1\, 2^{19} + 3.2^{20}) - ({}^{20}C_1\, 1^{19}\times 2 + 1\times 2 + 1)$$

$\therefore$ $$P = 1 - \left(\frac{13.2^{20} - 43}{3^{20}}\right)$$

45. $$P = \frac{\frac{7!}{3!2!2!2!} \times 2!}{7^7} = \frac{30}{7^6}$$

46. $$P = \frac{{}^8C_2 + {}^7C_2}{{}^{15}C_3} = \frac{7}{65}$$

47. Probability of getting prime outcome in any throw = P(2, 3, 5) = $\frac{3}{6} = \frac{1}{2}$

$$\therefore \quad P = \left(\frac{1}{2}\right)^{16} \left({}^8C_0^2 + {}^8C_1^2 + {}^8C_2^2 + \dots + {}^8C_8^2\right)$$

$$= \frac{{}^{16}C_8}{2^{16}}$$

48. $$P(A \cup B \cup C) = 1,\ P(A \cap B \cap C) = 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad P(A) + P(B) = 1$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - (P(A \cap B) + P(B \cap C) + P(C \cap A)) + P(A \cap B \cap C)$$

$$\Rightarrow \quad 1 = 1 + \frac{7}{15} - \left(0 + P(B \cap C) + \frac{1}{5}\right) + 0$$

$$\Rightarrow \quad P(B \cap C) = \frac{4}{15}$$

50. $$P = \frac{{}^6C_3 \times 1}{{}^7C_4} = \frac{4}{7}$$

51. 3H ; (2H, 2T) ; (1H, 4T) ; 6T

$$P = \left(\frac{1}{2}\right)^3 + \frac{4!}{2!2!}\left(\frac{1}{2}\right)^4 + \frac{5!}{4!}\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^6$$

$$= \frac{8 + 24 + 10 + 1}{64} = \frac{43}{64}$$

52.

x	x^4
5k	5λ
5k + 1	$5\lambda + 1$
5k + 2	$5\lambda + 1$
5k + 3	$5\lambda + 1$
5k + 4	$5\lambda + 1$

$$P = \frac{{}^{80}C_2 + {}^{20}C_2}{{}^{100}C_2}$$

$$= \frac{20\times19 + 80\times79}{100\times99} = \frac{19+316}{5\times99}$$

$$= \frac{335}{5\times99} = \frac{67}{99}$$

53.
$$P = \frac{{}^{7}C_3 \times 9}{7!}$$

$$= \frac{1}{16}$$

54.
$$P = \frac{{}^{n}C_3 \times 1}{{}^{n}C_3 \times 3} = \frac{1}{3}$$

55.
$$P = \frac{1}{2}\left(\frac{6+5}{36} + \frac{2}{11}\right) = \frac{193}{792}$$

SECTİON-2

ONE OR MORE THAN ONE CORRECT

1. **(a)** $\dfrac{7\times8 + 7\times8}{{}^{64}C_2} = \dfrac{7\times8\times2\times2}{64\times63} = \dfrac{1}{18}$

(b) $p = \dfrac{7\times7 + 7\times7}{{}^{64}C_2} = \dfrac{2\times7\times7\times2}{64\times63} = \dfrac{7}{144}$

Alternate : (a) $\dfrac{\underset{\substack{\downarrow\\ \text{Corner}\\ \text{squares}}}{4(2)} + \underset{\substack{\downarrow\\ \text{Corner row}\\ \text{\& column}\\ \text{squares}\\ \text{excluding}\\ \text{4 corner}\\ \text{squares}}}{24\times 3} + \underset{\substack{\downarrow\\ \text{remaining}\\ \text{squares}}}{36\times 4}}{64 \times 63}$

$$= \frac{1+9+1}{8\times 63} = \frac{1+9+1}{2\times 9} = \frac{1}{18}$$

(b) $\dfrac{36\times 4 + 24\times 2 + 4\times 1}{64\times 63} = \dfrac{7}{144}$

2. Let $P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) = x = P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_3)$

$P(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = a = P(E_1)P(\bar{E}_2)P(\bar{E}_3)$

$P(\bar{E}_1 \cap E_2 \cap \bar{E}_3) = b = P(\bar{E}_1)P(E_2)P(\bar{E}_3)$

$P(\bar{E}_1 \cap \bar{E}_2 \cap E_3) = x = P(\bar{E}_1)P(\bar{E}_2)P(E_3)$

$$\frac{a}{x} = \frac{P(E_1)}{P(\bar{E}_1)} = \frac{P(E_1)}{1-P(E_1)} \Rightarrow P(E_1) = \frac{a}{a+x}$$

$$P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_2) = x = \frac{x}{a+x}\frac{x}{b+x}\frac{x}{c+x}$$

$\Rightarrow$ $(a + x)(b + x)(c + x) = x^2$

3. (A) pool-A pool-B

pool-A	pool-B
P_4	×
×	×
×	×
×	×

$$P = \frac{{}^4C_3}{{}^7C_3} = \frac{4}{35}$$

(B) $P = 0$ (obvious)

(C) $P = 1$ (obvious)

(D) Pool-A	Pool-B	Pool -C	Pool-D
P_6	$\times$	$\times$	$\times$
$\times$	$\times$	$\times$	$\times$

$$P = \frac{{}^2C_1}{{}^7C_1} = \frac{2}{7}$$

4. D $\rightarrow$ event that person keeps driver

$E_1 \rightarrow$ event that person own sedan

$E_1 \rightarrow$ event that person own SUV

$$P(D) = P(E_1 \cap \bar{E}_2)P(D/E_1 \cap \bar{E}_2) + P(\bar{E}_1 \cap E_2)\, P(D/\bar{E}_1 \cap E_2) + P(E_1 \cap E_2)\, P(D/E_1 \cap E_2)$$

$$= \frac{3}{10} \times \frac{6}{10} \times \frac{6}{10} + \frac{7}{10} \times \frac{4}{10} \times \frac{7}{10} + \frac{3}{10} \times \frac{4}{10} \times \frac{9}{10}$$

$$= \frac{103}{250}$$

$$P(E_1/D) = \frac{P(E_1 \cap \bar{E}_2)P(D/E_1 \cap \bar{E}_2) + P(E_1 \cap E_2)P(D/E_1 \cap E_2)}{P(D)}$$

$$= \frac{\left(\frac{54}{250}\right)}{\left(\frac{103}{250}\right)} = \frac{54}{103}$$

6. A $\rightarrow$ event that candidate passes in exam A.

B $\rightarrow$ event that candidate passes in exam B.

C $\rightarrow$ event that candidate passes in exam C.

$$P(A \cap B \cap C) = \frac{1}{2} \cdot \frac{2}{5} = \frac{1}{10} = abc$$

$$\frac{2}{5} = \Sigma P(A \cap B) - 3P(A \cap B \cap C)$$

$$\Rightarrow \qquad \Sigma P(A \cap B) = \frac{7}{10}$$

$$\frac{3}{4} = \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$$

$$\Rightarrow \qquad \sum P(A) = \frac{27}{20} = a + b + c$$

7. $P(A_1 \cup A_2 \cup A_3 \cup \ldots\ldots \cup A_n) = 1 - P(\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3 \cap \ldots\ldots \cap \overline{A}_n)$

$$= 1 - P(\overline{A}_1)\,P(\overline{A}_2)\ldots\ldots P(\overline{A}_n)$$

$$= 1 - ((1 - P(A_1))\,(1 - P(A_2))\ldots\ldots(1 - P(A_n)))$$

8. Let $P(A) = P$

$$P(B) = \frac{1}{2}P$$

$$P(C) = \frac{1}{2} \times \frac{1}{2} \times P$$

$$P(A) + P(B) + P(C) = 1$$

$$\Rightarrow \qquad \frac{7}{4}P = 1 \quad \Rightarrow P = \frac{4}{7}$$

9. $P(n) = 1 - \dfrac{{}^6C_n\, n!}{6^n} \qquad n \le 6$

$$P(2) = 1 - \frac{{}^6C_2\, 2!}{6^2} = \frac{1}{6}$$

$$P(3) = 1 - \frac{{}^6C_3\, 3!}{6^3} = \frac{4}{9}$$

$$P(4) = 1 - \frac{{}^6C_4\, 4!}{6^4} = \frac{13}{18}$$

$$P(6) = 1 - \frac{6!}{6^6} = \frac{319}{324}$$

10. Probability of getting more even than odd = probability of getting less even than odd outcomes

$$= \frac{1}{2}\,(1 - \text{probability of getting equal odd \& even outcomes})$$

$$= \frac{1}{2}\left(1 - \frac{{}^8C_0^2 + {}^8C_1^2 + {}^8C_2^2 \ldots\ldots + {}^8C_8^2}{2^{16}}\right)$$

$$= \frac{1}{2}\left(1 - \frac{{}^{16}C_8}{2^{16}}\right)$$

11. $P(n) = (1 - P(n-1))\frac{1}{3}$

$P(2) = 0$

$P(3) = \frac{1}{3}$, $P(4) = \frac{2}{9}$, $P(5) = \frac{7}{27}$

$P(6) = \frac{20}{81}$, $P(7) = \frac{61}{243}$

12. $Q = 1 - ({}^4C_1 P(1-P)^3 + {}^4C_3 P^3(1-P)$

$$= 1 - \frac{1}{2}\left[(P + (1-P))^4 - ((-P + (1-P))^4\right]$$

$$= \frac{1}{2}(1 + (1 - 2P)^4)$$

13. $P(A) = 6$ or $\bar{6}\ \bar{7}\ 6$ or $\bar{6}\ \bar{7}\ \bar{6}\ \bar{7}\ 6$ or ....∞

$$= \frac{5}{36} + \frac{31}{36} \times \frac{30}{36} \times \frac{5}{36} + \left(\frac{31}{36} \times \frac{30}{36}\right)^2 \times \frac{5}{36} + \ldots\infty$$

$$= \frac{5/36}{1 - \frac{30 \times 31}{(36)^2}} = \frac{5 \times 6}{216 - 155} = \frac{30}{61}$$

$$P(B) = \frac{31}{36} \times \frac{6}{36} + \frac{31}{36} \times \left(\frac{30}{36} \times \frac{31}{36}\right) \times \frac{6}{36} + \frac{31}{36} \times \left(\frac{30}{36} \times \frac{31}{36}\right)^2 \times \frac{6}{36} + \ldots\infty$$

$$= \frac{\frac{31}{36} \times \frac{6}{36}}{1 - \frac{30}{36} \times \frac{31}{36}}$$

$$= \frac{31}{61}$$

14. ad – bc ≠ 0 for unique soln.

ad	bc	(a, d, b, c)
1	2 or 4	(1, 1, 1, 2), (1, 1, 2, 1), (1, 1, 2, 2)
2	1 or 4	(1, 2, 1, 1), (2, 1, 1, 1), (1, 2, 2, 2), (2, 1, 2, 2)
4	1 or 2	(2, 2, 1, 1), (2, 2, 1, 2), (2, 2, 2, 1)

$$P = \frac{10}{16} = \frac{5}{8}$$

∴ for non trivial soln.

$$P = \frac{3}{8}$$

15. $$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.25$$

$$0.75 \le P(A \cup B \cup C) = P(A) + P(B) + P(C) - (P(A \cap B) + P(B \cap C) + P(C \cap A)) + P(A \cap B \cap C) \le 1$$

$$\Rightarrow 0.1 \le P(B \cap C) \le 0.35$$

17. (b) $$P = \frac{\frac{2}{3}\times\frac{2}{3}\times\frac{2}{5}}{\frac{1}{3}\times\frac{1}{3}\times\frac{2}{5}+\frac{2}{3}\times\frac{2}{3}\times\frac{2}{5}+\frac{2}{3}\times\frac{1}{3}\times\frac{3}{5}+\frac{2}{3}\times\frac{1}{3}\times\frac{2}{5}} = \frac{8}{2+8+6+4} = \frac{2}{5}$$

(d) $$P = \frac{8+6}{2+8+6+4} = \frac{7}{10}$$

SECTION-3

COMPREHENSION BASED QUESTIONS

COMPREHENSION (Q.1 TO Q.3):

1. $$\frac{(n-1)!2!}{n!} = \frac{2}{n}$$

2. $$A\underbrace{\cdots\cdots\cdots\cdots}_{\text{'m' men}}B$$

$$\frac{{}^{n-2}C_m(n-m-1)!2!\times m!}{n!} = \frac{2(n-2)!(n-m-1)}{n!} = \frac{2(n-m-1)}{n(n-1)}$$

alt → $$\frac{(n-m-1)2!(n-2)!}{n!} = \frac{2(n-m-1)}{n(n-1)}$$

3. $\frac{2}{n(n-1)}((n-1)-0)+((n-1)-1)+((n-1)-2)+....((n-1)-m)$

$$=\frac{2}{n(n-1)}\left[\frac{(m+1)}{2}(2n-2-m)\right]$$

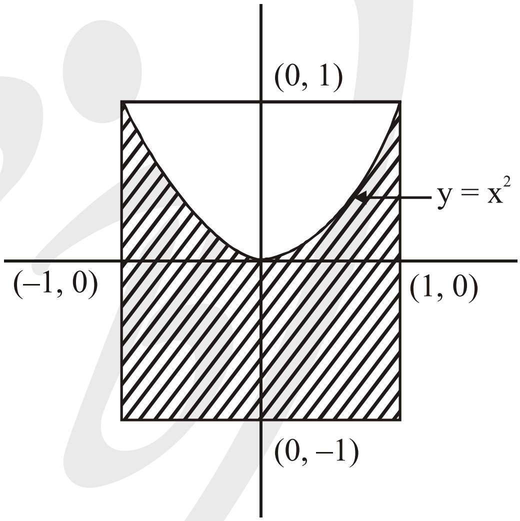
$$=\frac{(m+1)(2n-m-2)}{n(n-1)}$$

Comprehension (Q.4 to Q.5)

4.

$4(a^2-b)\geq 0$

$b\leq a^2$



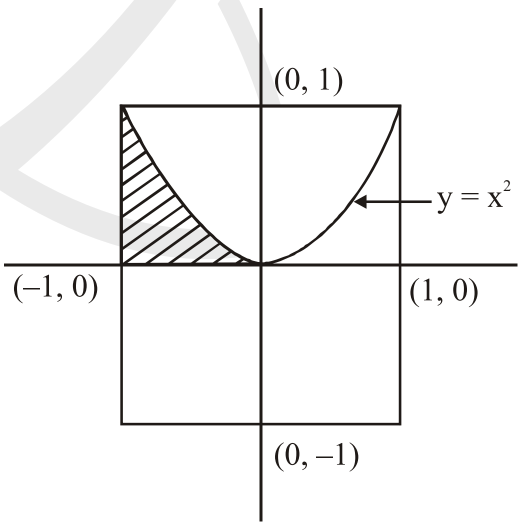
5. $D\geq 0\Rightarrow b\leq a^2$

$-a>0\Rightarrow a<0$

& $f(0)>0\Rightarrow b>0$

$$P(B)=\frac{\frac{1}{3}}{(2)^2}=\frac{1}{12}$$

$$P(A)=\frac{2\left(\frac{1}{3}\right)+2(1)}{2(2)}=\frac{\frac{8}{3}}{4}=\frac{2}{3}$$



Comprehension (Q.6 to Q.7)

6. $P\left(r \text{ or } (1, r-1) \text{ or } (1, 1, r-2), \ldots\ldots, (\underbrace{1, 1 \ldots\ldots 1}_{r-2 \text{ times}}, 2)\right)$

$$= \frac{1}{6} + \frac{1}{6^2} + \frac{1}{6^3} + \ldots + \frac{1}{6^{r-1}}$$

$$= \frac{\frac{1}{6}\left(1 - \frac{1}{6^{r-1}}\right)}{1 - \frac{1}{6}} = \frac{1}{5}\left(1 - \frac{1}{6^{r-1}}\right)$$

7. $P(\underbrace{1, 1, \ldots\ldots, 1}_{(r-2) \text{ times}}, 2)$ or $(\underbrace{1, 1, \ldots\ldots, 1}_{(r-3)}, 3)$ .... or $(\underbrace{1, 1, \ldots\ldots, 1}_{(r-6) \text{ times}}, 6)$

$$= \left(\frac{1}{6}\right)^{r-1} + \left(\frac{1}{6}\right)^{r-2} + \left(\frac{1}{6}\right)^{r-3} + \ldots\ldots + \left(\frac{1}{6}\right)^{r-5}$$

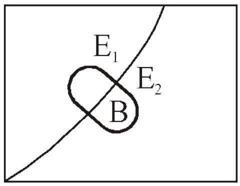
$$= \frac{\left(\frac{1}{6}\right)^{r-1}\left(6^5 - 1\right)}{5} = \frac{1}{5}\left(\left(\frac{1}{6}\right)^{r-6} - \left(\frac{1}{6}\right)^{r-1}\right)$$

Comprehension (Q.8 to Q.9)

8. $P(i) = Ki \quad i = 1, 2, 3, 4, 5, 6$

where $P(i)$ = prob. of obtaining no. equal to i

$$\sum_{i=1}^{6} P(i) = 1 \Rightarrow k\left(\frac{6 \times 7}{2}\right) = 1 \Rightarrow k = \frac{1}{21}$$



$P(B)$ = Probability that drawn ball is black

$E_1 \Rightarrow$ ball is drawn from urn A

$E_2 \Rightarrow$ ball is drawn from urn B

$$P(E_1) = \frac{1}{21}(2 + 3 + 5) = \frac{10}{21}$$

$$P(E_2) = \frac{1}{21}(1 + 4 + 6) = \frac{11}{21}$$

$$P(B) = P(E_1 \cap B) + P(E_2 \cap B) = P(E_1)P(B/E_1) + P(E_2)P(B/E_2)$$

$$= \frac{10}{21} \times \frac{3}{5} + \frac{11}{21} \times \frac{2}{5} = \frac{52}{105}$$

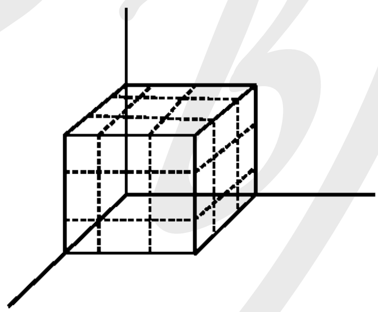
9. $P(W)$ = probability that drawn ball is white

$$P(E_2/W) = \frac{P(E_2 \cap W)}{P(W)} = \frac{P(E_2)P(W/E_2)}{P(E_1)P(W/E_1) + P(E_2)P(W/E_2)}$$

$$= \frac{\frac{11}{21} \times \frac{3}{5}}{\frac{10}{21} \times \frac{2}{5} + \frac{11}{21} \times \frac{3}{5}} = \frac{33}{53}$$

Comprehension (Q.12 to Q.14)

Observing from fig.



12. $P = \frac{1}{27}$

13. $P = \frac{12}{27} = \frac{4}{9}$

14. $P = \frac{6}{27} = \frac{2}{9}$

Comprehension (Q.15 to Q.16)

$$P(2) = \frac{3}{6} \times \frac{1}{6} = \frac{3}{36}$$

$$P(3) = \frac{3}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{1}{6} = \frac{8}{36} = P(1,2 \text{ or } 2,1)$$

$$P(4) = \frac{3}{6} \times \frac{3}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{2}{6} \times \frac{2}{6} = \frac{14}{36} = P(1, 3 \text{ or } 3, 1 \text{ or } 2, 2)$$

$$P(5) = \frac{2}{6} \times \frac{3}{6} + \frac{1}{6} \times \frac{2}{6} = \frac{8}{36} = P(2, 3 \text{ or } 3, 2)$$

$$P(6) = \frac{1}{6} \times \frac{3}{6} = \frac{3}{36}$$

Comprehension (Q.17 to Q.19) :

17. Let W $\rightarrow$ Win, D $\rightarrow$ Draw, L $\rightarrow$ Lose for X

P(X wins in n games) $= P((2W, (n-2)D) \text{ or } (2W,, 1\,L, (n-3)D))$

$$= {}^{n-1}C_1\, p^2 q^{n-2} + (n-2)(n-1)\, p^2 r\, q^{n-3}$$

$$= (n-1)\, p^2 q^{n-2} + (n-1)\,(n-2)\, p^2 q^{n-3}\, r$$

$$= (n-1)\, p^2 q^{n-3}\,(q + (n-2)r)$$

18. P(Y wins) $= P(2L, (4)D) \text{ or } (2L, 1W, 3D)$

$$= {}^5C_1\, r^2 q^4 + 5 \times 4\,(r^2 p\, q^3)$$

$$= 5q^3r^2(q + 4p)$$

19. P(Y wins) $= \dfrac{r^2}{(1-q)^2} + \dfrac{{}^2C_1\, pr^2}{(1-q)^3}$

$$= \frac{r^2((p+r)+2p)}{(1-q)^3} = \frac{r^2(3p+r)}{(1-q)^3}$$

Comprehension (Q.20 to Q.22)

20. Game ends with last two tosses resulting in 2 heads or 2 tails.

$\Rightarrow$ P (ends with 2 heads/ends with 2 heads or 2 tails)

$$= \frac{\frac{2}{3} \times \frac{2}{3}}{\frac{1}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3}} = \frac{4}{5}$$

Alternate : Let P = probability that no. of heads exceed no. of tails by 2.

$\Rightarrow$ $$P = P(H)\,P(H) + P(H)\,P(T)\,P + P(T)P\,(H)\,P$$

$$P = \frac{4}{9} + \frac{2}{9}P + \frac{2}{9}P \;\Rightarrow P = \frac{4}{5}$$

21. P(min. throws/ends with head) $= \dfrac{\frac{2}{3} \times \frac{2}{3}}{\frac{4}{5}} = \dfrac{5}{9}$

22. Obvious

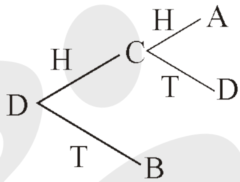
Comprehension (Q.23 to Q.24)

23. Let P = I start in field D & win

$$P = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}P\right)$$

$$\Rightarrow \quad \frac{3}{4}P = \frac{1}{4}$$

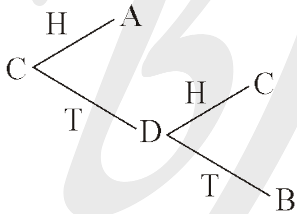
$$\Rightarrow \quad P = \frac{1}{3}$$



24. Let P = I start in field C & win

$$P = \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2} \times P\right)$$

$$\Rightarrow \quad P = \frac{2}{3}$$



Comprehension (Q.25 to Q.26) :

25.

x	x^2	Number of x
7k	7μ	3
7k + 1	7μ + 1	4
7k + 2	7μ + 4	4
7k + 3	7μ + 2	4
7k + 4	7μ + 2	4
7k + 5	7μ + 4	3
7k + 6	7μ + 1	3

$$P = \frac{{}^{3}C_2 + {}^{7}C_2 \times 2 + {}^{8}C_2}{{}^{25}C_2} = \frac{73}{300}$$

26.

x	x^2	Number of x
$5k$	5μ	5
$5k+1$	$5\mu+1$	5
$5k+2$	$5\mu+4$	5
$5k+3$	$5\mu+4$	5
$5k+4$	$5\mu+1$	5

$$P = \frac{{}^{5}C_2 + {}^{10}C_2 \times 2}{{}^{25}C_2} = \frac{1}{3}$$

Comprehension (Q.27 to Q.29)

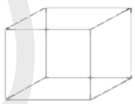
27. $P = \dfrac{\dfrac{4!}{2!2!}}{4^4} = \dfrac{3}{128}$

28. $P = \dfrac{2\left(\dfrac{6!}{3!2!} - \dfrac{4!}{2!2!} \times 2!\right)}{4^6} = \dfrac{3}{128}$

29. $P = 0$

Comprehension (Q.30 to Q.31)

30. $P = 1 \times \dfrac{2}{3} \times \dfrac{1}{3} \times \dfrac{1}{3} \times \dfrac{1}{3} \times \dfrac{1}{3}$

Alternate: For this to happen each pair of opposite faces  must be painted with the same colour. probability that first pair of opp. faces are coloured with same colour $= 1 \times \dfrac{1}{3}$

2nd $= \dfrac{2}{3} \times \dfrac{1}{3}$

3rd $= \dfrac{1}{3} \times \dfrac{1}{3}$

31. $P = 1 - \dfrac{{}^3C_2(2!)^3}{\left({}^3C_2\,2!\right)^3} = \dfrac{8}{9}$

Probability that only two colours are used given that opp. faces have different colours.

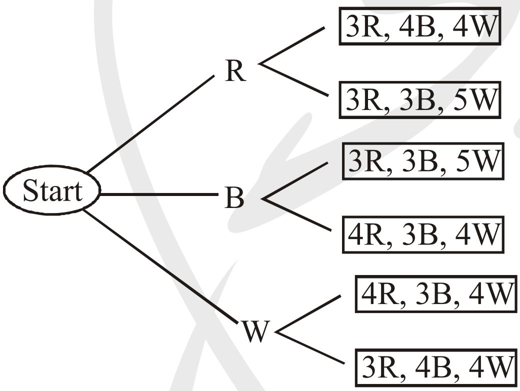
Alternate : P = 1 – P (exactly two colours are used given that each pair of opposite faces are painted with a different colour)

$$= 1 - 1 \times \frac{1}{3} \times \frac{1}{3} = \frac{8}{9}$$

($\because$ Prob. that a definite pair is selected to paint opposite faces = $\frac{1}{3}$)

Comprehension (Q.32 to Q.33)

32.



$$P = \frac{3}{10}\times 1 + \frac{3}{10}\times\frac{1}{2} + \frac{4}{10}\times\frac{1}{2} = \frac{13}{20}$$

33. $$P = \frac{\frac{3}{10}\times\left(\frac{3}{11}+\frac{3}{11}\right)}{\frac{3}{10}\times\left(\frac{3}{11}+\frac{3}{11}\right)+\frac{3}{10}\times\left(\frac{3}{11}+\frac{4}{11}\right)+\frac{4}{10}\times\left(\frac{3}{11}+\frac{4}{11}\right)}$$

$$= \frac{18}{67}$$

Comprehension (Q.34 to Q.36)

Either both a & b are divisible by p or both not divisible by p

$$\therefore \quad P_n(p) = \frac{\left[\frac{n}{p}\right]^2 + \left(n - \left[\frac{n}{p}\right]\right)^2}{n^2}$$

$$= 1 - \frac{2}{n}\left[\frac{n}{p}\right] + \frac{2}{n^2}\left[\frac{n}{p}\right]^2$$

$$\lim_{n\to\infty} P_n(p) = 1 - \frac{2}{p} + \frac{2}{p^2}$$

$$P_{25}(3) = \frac{8^2 + 17^2}{(25)^2} = \frac{353}{625}$$

Comprehension (Q.37 to Q.39)

37. $P = \frac{{}^{52}C_2 \times {}^2C_1 \times {}^{50}C_1}{({}^{52}C_2)^2} = \frac{50}{663}$

38. $P = \frac{{}^{13}C_1 \times 2 \times {}^{39}C_1 \times {}^{39}C_2}{({}^{52}C_2)^2} = \frac{247}{578}$

39. $P = \frac{{}^{52}C_3 \times 2}{{}^{104}C_3} = \frac{25}{103}$

Comprehension (Q.40 to Q.41)

40. A(n) = P(BB or BWB or WBB)

$$= \frac{n}{n+2}\frac{n-1}{n+1} + \frac{n}{n+2}\frac{2}{n+1}\frac{n-1}{n} + \frac{2}{n+2}\frac{n}{n+1}\frac{n-1}{n}$$

$$= \frac{(n+4)(n-1)}{(n+2)(n+1)}$$

$$\lim_{n\to\infty}\prod_{r=2}^{n} A(r) = \lim_{n\to\infty}\left(\prod_{r=2}^{n}\frac{r+4}{r+2}\prod_{r=2}^{n}\frac{r-1}{r+1}\right)$$

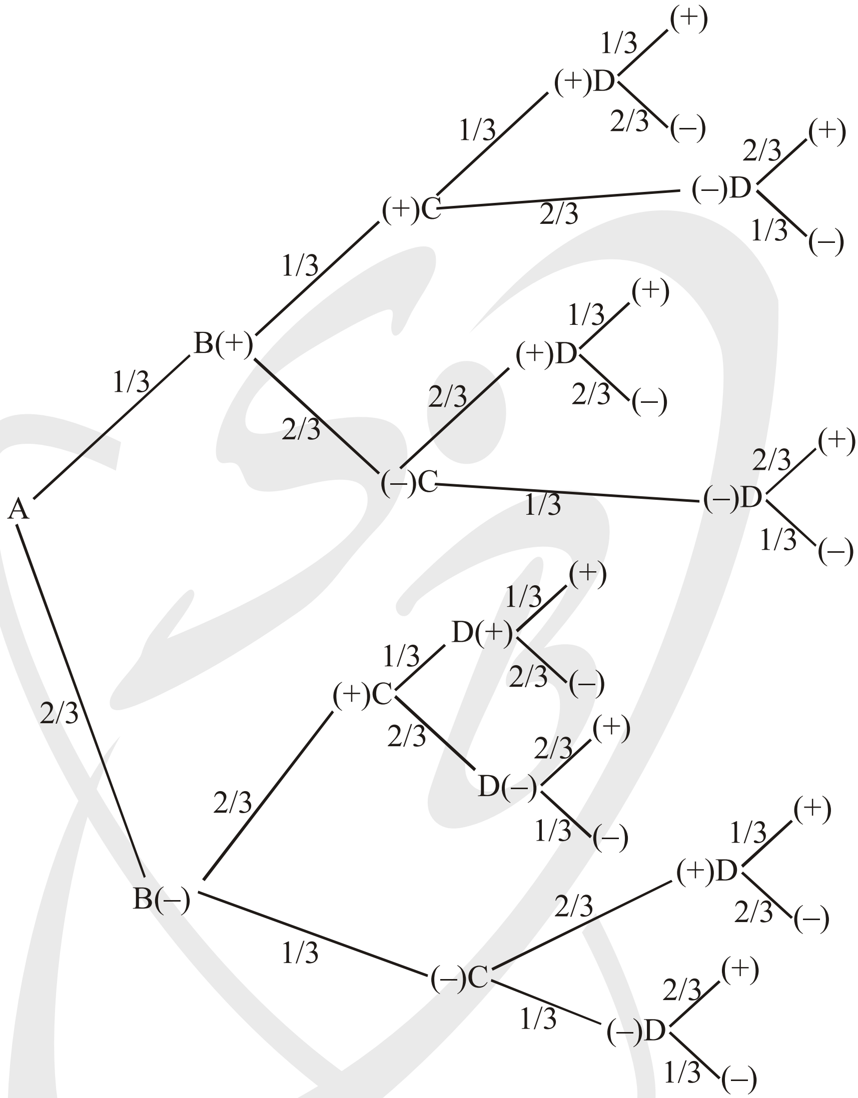
$$= \lim_{n\to\infty}\left(\frac{(n+4)(n+3)}{4\times 5}\frac{1\times 2}{(n+1)(n)}\right) = \frac{1}{10}$$

41. $B(n) = 1 - \frac{n^2+3n-4}{n^2+3n+2} = \frac{6}{(n+2)(n+1)}$

$$\lim_{n\to\infty}\sum_{r=1}^{n} B(r) = \lim_{n\to\infty} 6\left(\frac{1}{2} - \frac{1}{n+2}\right) = 3$$

COMPREHENSION (Q.42 TO Q.43)

Sol.



42. $P = \left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\right) + \frac{1\times1\times2\times2}{3^4} + \frac{1\times2\times(2\times1+1\times2)}{3^4} + \frac{2\times2\times(1\times1+2\times2)}{3^4} + \frac{2\times1\times(2\times1+1\times2)}{3^4}$

$$P = \frac{5+8+20+8}{81} = \frac{41}{81}$$

43. $P = \dfrac{\frac{1}{3}\times\frac{1}{3}\times\left(\frac{1}{3}\times\frac{1}{3}+\frac{2}{3}\times\frac{2}{3}\right)+\frac{1}{3}\times\frac{2}{3}\times\left(\frac{2}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}\right)}{\frac{41}{81}}$

$= \dfrac{5+2(4)}{41} = \dfrac{13}{41}$

COMPREHENSION (Q.44 TO Q.46)

44. P = P(W or BBW or BBBBW)

$= \dfrac{3}{10}+\dfrac{5}{10}\times\dfrac{4}{9}\times\dfrac{3}{8}+\dfrac{5}{10}\dfrac{4}{9}\dfrac{3}{8}\dfrac{2}{7}\dfrac{3}{6}$

$= \dfrac{83}{210}$

45. P = P(BW or BBBW or BBBBBW)

$= \dfrac{5}{10}\dfrac{3}{9}+\dfrac{5}{10}\dfrac{4}{9}\dfrac{3}{8}\dfrac{3}{7}+\dfrac{5}{10}\dfrac{4}{9}\dfrac{3}{8}\dfrac{2}{7}\dfrac{1}{6}\dfrac{3}{5} = \dfrac{43}{210}$

46. P = P(R or BR or BBR or BBBR or BBBBR or BBBBBR)

$= \dfrac{2}{10}+\dfrac{5}{10}\dfrac{2}{9}+\dfrac{5}{10}\dfrac{4}{9}\dfrac{2}{8}+\dfrac{5}{10}\dfrac{4}{9}\dfrac{3}{8}\dfrac{2}{7}+\dfrac{5}{10}\dfrac{4}{9}\dfrac{3}{8}\dfrac{2}{7}\dfrac{2}{6}+\dfrac{5}{10}\dfrac{4}{9}\dfrac{3}{8}\dfrac{2}{7}\dfrac{1}{6}\dfrac{2}{5}$

$= \dfrac{2}{5}$

SECTION-4

MATCH THE COLUMN

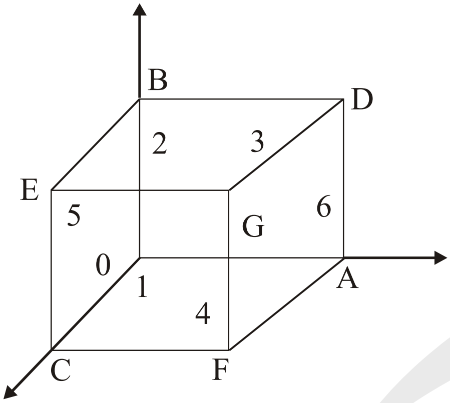
1. (A) $P = \dfrac{1\times{}^{11}C_5}{{}^{12}C_6} = \dfrac{1}{2}$

(B) $P = \dfrac{{}^{2}C_1\times{}^{10}C_5}{{}^{12}C_6} = \dfrac{6}{11}$

(C) $P = \dfrac{{}^{10}C_4}{{}^{12}C_6} = \dfrac{5}{22}$

(D) $P = \dfrac{10}{11}$

2.

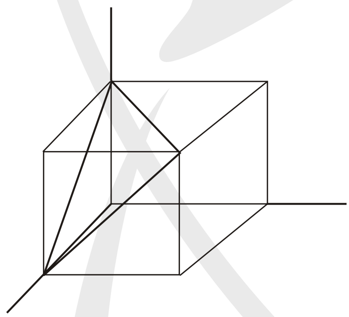


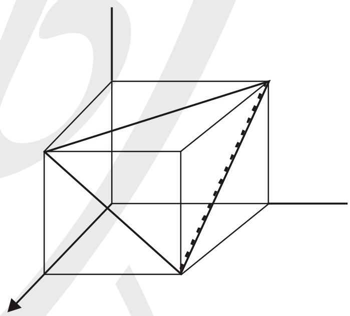
(A) P = $\dfrac{{}^4C_3 \times 6 + 4 \times 2}{{}^8C_3} = \dfrac{4}{7}$

(3 vertices in single phase) + (selecting adjacent faces 1-2 or 2-3 or 3-4 or 4-5 and one side 5 or 6 is selected automatically)

$1 \to$ CEGF, $2 \to$ EGDB, $3 \to$ OADB, $4 \to$ OCFA, $5 \to$ OCEB, $6 \to$ AFGD

e.g. $\to$





(B) $P = \dfrac{{}^4C_3 \times 6}{{}^8C_3} = \dfrac{3}{7}$

(C) $P = 1 - \dfrac{4}{7} = \dfrac{3}{7}$

(D) $P = \dfrac{{}^4C_3 \times 6 + 6 \times 4}{{}^8C_3} = \dfrac{48}{56} = \dfrac{6}{7}$

3. (A) $P = \dfrac{{}^2C_1 \times {}^{14}C_3}{{}^{16}C_4} = \dfrac{2}{5}$

(B) $P = \dfrac{{}^{14}C_2}{{}^{15}C_2} \times \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{20}$

(C) $P = \dfrac{1}{^{15}C_1} \times \dfrac{1}{2} = \dfrac{1}{30}$

(D) $P = \dfrac{^{14}C_6}{^{15}C_7} \times \dfrac{1}{2} = \dfrac{7}{30}$

4. (A) 7 digits not containing

(1, 8), (2, 7), (3, 6), (4, 5)

$\Rightarrow P = \dfrac{4 \times 7!}{^{9}C_7 \times 7!} = \dfrac{1}{9}$

(B) $P = 1 - \dfrac{^{12}C_2}{^{16}C_2} = \dfrac{9}{20}$

(C) 7^m ends with 7, 9, 3, 1

3^n ends with 3, 9, 7, 1

$(7^m, 3^n)$ ends with (7, 3), (9, 1), (3, 7), (1, 9)

$$P = \frac{25}{98} \times \frac{25}{98} + \frac{25}{98} \times \frac{24}{98} + \frac{24}{98} \times \frac{24}{98} + \frac{24}{98} \times \frac{25}{98} = \frac{(25+24)^2}{(98)^2} = \frac{1}{4}$$

(D) favourable ways are HHHHT or TTTTH $\Rightarrow P = \dfrac{2}{32} = \dfrac{1}{16}$

5. **(A)** $P = \dfrac{6![6! - (^6C_1 5! - ^6C_2 4! + ^6C_3 3! - ^6C_4 2! + ^6C_5 1! - ^6C_6)]}{6!6!}$

$= 1 - \dfrac{6! - 3 \times 5! + 20 \times 3! - 30 + 6 - 1}{6!}$

$= \dfrac{265}{720} = \dfrac{53}{144}$

(B) $P = \dfrac{6! \times 1}{6!6!} = \dfrac{1}{6!}$

(C) $P = \dfrac{6!\left[^6C_1 \times 1 \times (5! - ^5C_1 4! + ^5C_2 3! - ^5C_2 2! + ^5C_4 1! - ^5C_5)\right]}{6!6!}$

$= \dfrac{6 \times (44)}{6!} = \dfrac{44}{120} = \dfrac{11}{30}$

(D) $P = 1 - \left(\dfrac{53}{144} + \dfrac{11}{30}\right) = \dfrac{691}{720}$

6. **(A)** $P = \dfrac{{}^3C_2 + {}^3C_2}{{}^6C_3} = \dfrac{3}{10}$

(B) $b^2 = ac$

$(a, b, c) = (1, 2, 4)$

$\Rightarrow p = \dfrac{1}{{}^6C_3} = \dfrac{1}{20}$

(C) $\dfrac{1}{a} + \dfrac{1}{c} = \dfrac{2}{b}$

$b = 3, (a, c) = (2, 6)$

$b = 4, (a, c) = (3, 6)$

$\Rightarrow p = \dfrac{2}{20} = \dfrac{1}{10}$

(D) $a + b > c, \quad a < b < c$

$(a, b, c) = (2, 3, 4), (2, 4, 5), (3, 4, 5), (2, 5, 6)$

$(3, 4, 6), (3, 5, 6), (4, 5, 6)$

$\Rightarrow P = \dfrac{7}{20}$

7. (A) $P = \dfrac{1}{13}\left(\dfrac{{}^4C_4 + {}^5C_4 + \ldots.. + {}^{12}C_4}{{}^{12}C_4}\right) = \dfrac{1}{13}\dfrac{{}^{13}C_5}{{}^{12}C_4} = \dfrac{1}{5}$

(B) $P = \dfrac{{}^{10}C_4}{{}^{12}C_4} = \dfrac{14}{33}$

(C) $P = \dfrac{1}{13}\left(\dfrac{{}^2C_2\,{}^{10}C_2 + {}^3C_2\,{}^9C_2 + {}^4C_2\,{}^8C_2 + \ldots.. + {}^{10}C_2\,{}^2C_2}{{}^{12}C_4}\right)$

$= \dfrac{1}{13}\dfrac{2({}^2C_2\,{}^{10}C_2 + {}^3C_2\,{}^9C_2 + {}^4C_2\,{}^8C_2 + {}^5C_2\,{}^7C_2) + {}^6C_2\,{}^6C_2}{{}^{12}C_4} = \dfrac{1}{5}$

(D) $P = \dfrac{{}^{10}C_4}{{}^4C_4 + {}^5C_4 + \ldots. + {}^{12}C_4} = \dfrac{{}^{10}C_4}{{}^{13}C_5} = \dfrac{70}{429}$

8. (A) $P(m = 3) = \dfrac{\text{Throw resulting in}(3,4,5,6)-(4,5,6)}{\text{Total ways}} = \dfrac{4^5-3^5}{6^5}\left(\dfrac{2}{5}\right)^5-\left(\dfrac{1}{2}\right)^5$

(B) $P(n = 4) = \dfrac{\text{Throw resulting in}(1,2,3,4)-(1,2,3)}{\text{Total ways}} = \dfrac{4^5-3^5}{6^5}$

(C) P(m = 2, n = 5) = Throw resulting 2, 3, 4, 5 – atleast one of 2 or 5 not attained

$$= \frac{4^5-(3^5+3^5-2^5)}{6^5} = \left(\frac{2}{3}\right)^5-\left(\frac{1}{2}\right)^4+\left(\frac{1}{3}\right)^5$$

(D) Smallest number is 2 – number are 5 or 6

$$P = \frac{5^5-2^5}{6^5} = \left(\frac{5}{6}\right)^5-\left(\frac{1}{3}\right)^5$$

9. (A) P = 1 – P (all numbers are ≥ 10)

$$= 1-\frac{{}^{16}C_4}{{}^{25}C_4} = 1-\frac{182}{1265} = \frac{1083}{1265}$$

(B) $P = \dfrac{{}^{9}C_1\times 1\times {}^{15}C_2}{{}^{25}C_4} = \dfrac{189}{2530}$

(C) P = P(1 even, 3 odd) + P(3 even, 1 odd)

$$= \frac{{}^{13}C_3\,{}^{12}C_1+{}^{13}C_1\,{}^{12}C_3}{{}^{25}C_4} = \frac{286}{575}$$

(D) P = 1 – P(abcd is odd)

$$= 1-\frac{{}^{13}C_4}{{}^{25}C_4} = \frac{217}{230}$$

SECTİON-5

SUBJECTIVE TYPE PROBLEMS

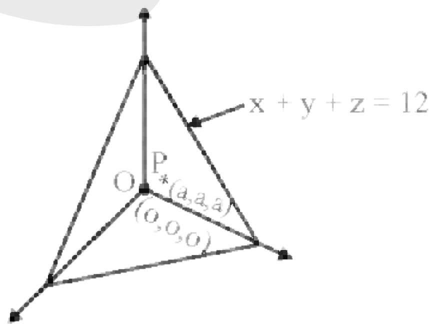
1. $((a-b)^2+(b-c)^2+(c-a)^2) \le 0$

$\Rightarrow a = b = c$

$\because$ Points O (0, 0, 0) and P(a, a, a)

lie on name side of plane x + y + z = 12

$\Rightarrow\ -12\,(a + a + a - 12) > 0$



$\Rightarrow \qquad 0 < a < 4$

$\Rightarrow \qquad a = \{1, 2, 3\}$

$$P = \frac{3}{6} = \frac{1}{2}$$

2. $f'(x) = 3x^2 + 2ax + b \geq 0 \quad \forall\, x \in R$

$\Rightarrow\ D \leq 0 \Rightarrow a^2 \leq 3b$

a	b
1	1
1, 2	2
1, 2, 3	3
1, 2, 3	4
1, 2, 3	5
1, 2, 3, 4	6

No. of favourable ways = 6 (1 + 2 + 3 (3) + 4) = 6 × 16

Total no. of ways = 6^3

$$P = \frac{16 \times 6}{6^3} = \frac{4}{9}$$

3. $$P = 7\left[\left(\frac{1}{7}\times\frac{1}{2}\right)\times\frac{1}{2}\times\frac{1}{2}+\left(\frac{6}{7}\times\frac{1}{2}\times\frac{1}{2}\right)\times\left(\frac{1}{3}\times\frac{1}{2}\right)\frac{1}{2}+\left(\frac{6}{7}\times\frac{1}{2}\times\frac{1}{2}\right)\left(\frac{2}{3}\times\frac{1}{2}\times\frac{1}{2}\right)\times\frac{1}{2}\right]$$

$$= \frac{3}{8}$$

= (paired in 1st round) + (paired in 2nd round) + (paired in 3rd round)

4. $$P = \frac{\dfrac{3}{13}\times\dfrac{{}^{11}C_1\,{}^{40}C_3}{{}^{51}C_4}}{\dfrac{3}{13}\times\dfrac{{}^{11}C_1\,{}^{40}C_3}{{}^{51}C_4}+\dfrac{10}{13}\times\dfrac{{}^{12}C_1\,{}^{39}C_3}{{}^{51}C_4}}$$

$$= \frac{11}{48}$$

5. _ _ _ 5 _ _ _ _

$$P = 1 - (\text{all } 2, 4, 6 \text{ in one side of } 5)$$

$$= 1 - \frac{3!\,3! \times 2}{7!}$$

$$= \frac{69}{70}$$

6. $E_1 \rightarrow$ boy watching doordarshan

$E_2 \rightarrow$ boy watching ten sports

$E \rightarrow$ boy fell asleep

$$P(E_1/E) = \frac{\frac{1}{5} \times \frac{3}{4}}{\frac{1}{5} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{4}} = \frac{3}{7}$$

7. $$P(E_i) = \frac{1}{2}$$

$$P(E_m) = {}^{10}C_m \left(\frac{1}{2}\right)^{10}$$

$$P(E_i \cap E_m) = {}^{9}C_{m-1} \left(\frac{1}{2}\right)^{10}$$

$$P(E_i \cap E_m) = P(E_i)\,P(E_m)$$

$$\Rightarrow \quad {}^{9}C_{m-1} \left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right) {}^{10}C_m \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow \quad m = 5$$

8. $$P = \frac{26}{39} = \frac{2}{3}$$

$$p + q = 5$$

9. Let number of men = m

number of women = w

$$\Rightarrow \qquad P = {}^{\alpha}C_1\left(\frac{m}{m+w}\right)^1\left(\frac{w}{m+w}\right)^{\alpha-1} + {}^{\alpha}C_3\left(\frac{m}{m+w}\right)^3\left(\frac{w}{m+w}\right)^{\alpha-3} + \ldots$$

$$= \frac{1}{2}\left(\left(\frac{m+w}{m+w}\right)^{\alpha} - \left(\frac{w-m}{w+m}\right)^{\alpha}\right)$$

$$\Rightarrow \qquad \frac{1}{2}-\left(\frac{1}{2}\right)^{\alpha+1} = \frac{1}{2}\left(1-\left(\frac{\mu-1}{\mu+1}\right)^{\alpha}\right)$$

$$\Rightarrow \qquad \frac{\mu-1}{\mu+1} = \frac{1}{2}$$

$$\Rightarrow \qquad \mu = 3$$

10. Put $\quad x = -w, -w^2$

$$(-w)^{n+1} - (-w)^n + 1 = 0$$

$$\Rightarrow \qquad (-w)^n w^2 + 1 = 0$$

$$\Rightarrow \qquad (-1)^n w^{n+2} + 1 = 0$$

$$\Rightarrow \qquad n = 6\lambda + 1$$

Also $n = 6\lambda + 1$ satisfy for $x = -w^2$

$$\Rightarrow \qquad P = \frac{1}{6}$$

11. $p^2 \geq 4q$

q	**p**	**Number of (p, q)**
1	2,3,....,10	9
2	3,4,....,10	8
3	4,5,....,10	7
4	4,5,....,10	7
5	5,6,....,10	6
6	5,6,....,10	6

7	6,7,....,10	5
8	6,7,....,10	5
9	6,7,....,10	5
10	7,8,9,10	4

$$P = \frac{62}{10 \times 10} = \frac{31}{50}$$

12. $$P = \frac{\left(\frac{11!}{2!2!2!5!}\right)}{\left(\frac{11!}{2!2!2!}\right)} = \frac{1}{120}$$

13. $$P = \frac{{}^{10}C_1\,{}^{10}C_1 + {}^{10}C_2}{{}^{30}C_2} = \frac{1}{3}$$

14. A $\rightarrow$ there is one undefeated team

B $\rightarrow$ there is one winless team

$P = 1 - \frac{n(A \cup B)}{2^{10}}$ ($\because$ total ways = 2^{10}, number of games = 10)

$$= 1 - \frac{{}^5C_1 2^6 + {}^5C_1 2^6 - {}^5C_2 \times 2! \times 2^3}{2^{10}}$$

$$= 1 - \frac{5 \times 2^7 - 5 \times 2^5}{2^{10}} = \frac{17}{32}$$

15. $x_1Rx_2Rx_3Rx_4Wx_5Wx_6Wx_7Wx_8$

$x_1 + x_2 + ... + x_8 = 3$

$$P = \frac{{}^{3+7}C_7}{\frac{10!}{3!4!3!}} = \frac{1}{35}$$

Alternate : observe that the position of blue balls is irrelevant for success. Thus we worry only about permutations of R R R W W W W. .

$$\Rightarrow P = \frac{4!3!}{7!}$$

16. Each person moves along 8 line segments. In order to meet, the persons must meet at diagonal points (0, 4), (1, 3), (2, 2), (3, 1) or (4, 0)

$$P = \frac{{}^4C_0\,{}^4C_4 + {}^4C_1\,{}^4C_3 + {}^4C_2\,{}^4C_2 + {}^4C_3\,{}^4C_1 + {}^4C_4\,{}^4C_0}{\sum\sum_{0\le i\le j\le 4} {}^4C_i\,{}^4C_j} = \frac{{}^8C_4}{\left(\sum_{i=0}^{4} {}^4C_i\right)\left(\sum_{j=0}^{4} C_j\right)}$$

$$= \frac{{}^8C_4}{2^8} = \frac{35}{128}$$

17. The generating function for throwing both the dice is $(3x^1 + 2x^2 + x^3)(x^1 + 2x^2 + 3x^3)$

$$= 3x^2 + 8x^2 + 14x^4 + 8x^5 + 3x^6$$

$\therefore$ 4 is the most likely sum, with probability of it occurring is $\frac{14}{36} = \frac{7}{18}$

18. $x_1 + x_2 + x_3 + x_4 = 10$, $x_i \in \{1, 2, 3, 4\}$

$$P = \frac{{}^{10-4+3}C_3 - {}^4C_1\,{}^{10-8+3}C_3}{(4)^4} = \frac{{}^9C_3 - 4\,{}^5C_3}{(4)^4} = \frac{11}{64}$$

Alternate : Favourable case = (1, 1, 4, 4) ; (1, 2, 3, 4), (1, 3, 3, 3), (2, 2, 2, 4), (2, 2, 3, 3)

$$P = \frac{\frac{4!}{2!2!} + 4! + \frac{4!}{3!} + \frac{4!}{3!} + \frac{4!}{2!2!}}{4^4} = \frac{44}{256} = \frac{11}{64}$$

19. The experiment consists in observing, among all ${}^{m+n}C_n$ configurations of heads and tails, the number of configurations of the form

$$y_1x_1y_2x_2y_3x_3y_4x_4y_5$$

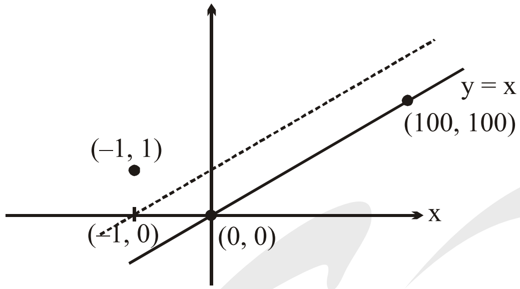
where x_k are filled with tails and the y_k are filled with tails. We need integral solutions to

$y_1 + y_2 + y_3 + y_4 + y_5 = 5$, $y_1 \ge 0, y_5 \ge 0, y_k > 0, 2 \le k \le 4$

and $x_1 + x_2 + x_3 + x_4 = 10$, $x_k > 0, 1 \le k \le 4$

$$\Rightarrow P = \frac{{}^{5-3+4}C_4 \times {}^{10-4+3}C_3}{{}^{15}C_{10}} = \frac{{}^6C_4\,{}^9C_3}{{}^{15}C_{10}} = \frac{60}{143}$$

20. Let till any point of time, there are 'x' 100 Re notes and 'y' 200 Re notes. Then for not having any problem at any time $x \geq y$.



Shift origin to (–1, 0) and reflect (0, 0) about y = x + 1.

Favourable ways = Total – going from (–1, 0) to (100, 100)

Favourable ways = ${}^{200}C_{100} - {}^{101+99}C_{101} = \frac{{}^{200}C_{100}}{101}$

$$P = \frac{\left(\frac{{}^{200}C_{100}}{101}\right)}{{}^{200}C_{100}} = \frac{1}{101}$$

21. $P = \frac{{}^{6}C_{2}\,{}^{5}C_{3}}{{}^{10}C_{5}} = \frac{5}{21}$

22. $P = \frac{{}^{30}C_{15}}{{}^{31}C_{15}} = \frac{16}{31}$

Alternate : $P = \frac{30}{31} \times \frac{14}{15} \times \frac{6}{7} \times \frac{2}{3} = \frac{16}{31}$