

# OPTWIN: DRIFT IDENTIFICATION WITH OPTIMAL SUB-WINDOWS

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## table of contents

**01** Context & Motivation

**02** OPTWIN - Concept Drift Detector

**03** Experiments

**04** Conclusion

# CONTEXT & MOTIVATION

# 1. CONTEXT & MOTIVATION

- What is Concept Drift?
  - Unforeseeable changes in the statistical properties of the incoming data stream over time.
- Concept Drift Types:
  - Sudden
  - Incremental
  - Gradual
  - Reoccurring
- Interesting use cases:

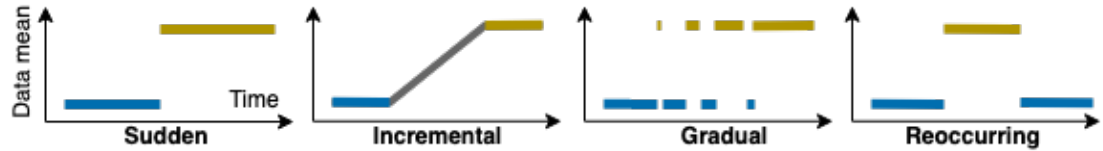


Figure 1: Concept drift types.



Stock market  
prediction



Fraud  
detection



Noise  
detection

# 1. CONTEXT & MOTIVATION: REAL-WORLD EXAMPLE

How can a concept change over time?



Figure 2: Tweet containing “Corona” term published before covid-19 pandemic.

Sentiment Analysis: 👍



Figure 3: Tweet containing “Corona” term published after covid-19 pandemic.

Sentiment Analysis: 📉

# 1. CONTEXT & MOTIVATION: CURRENT CONCEPT DRIFT DETECTORS

**Error rate-based drift detector** track the error rates produced by an Online Learning learner (e.g. loss of an ANN) in a sliding window  $W$ .

## Current drift detectors:

- Select sliding window  $W$  of the most recent errors from a learner.
- Divide  $W$  into historical and new errors.  $W_{historical}$  and  $W_{new}$ .
- Compare the mean of  $W_{historical}$  and  $W_{new}$  using a statistical test.
- If the null hypothesis is rejected a concept drift is flagged.

## Problems:

- How to identify the optimal splitting point of  $W$ ?
  - currently:  $\mathcal{O}(\log n)$
- What if the concept drift does not impact the mean of the errors?

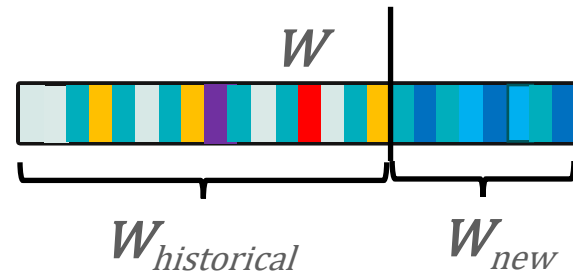


Figure 4: Sliding window illustration.

# OPTWIN - CONCEPT DRIFT DETECTOR

## 2. OPTWIN - CONCEPT DRIFT DETECTOR: INSIGHT

### Standard deviation induced concept drift:

$$W_{hist} = \{0.3; 0.7; 0.3; 0.7; 0.3; 0.7; 0.3; 0.7\}$$

$$W_{new} = \{1.0; 0.0; 1.0; 0.0; 1.0; 0.0; 1.0; 0.0\}$$

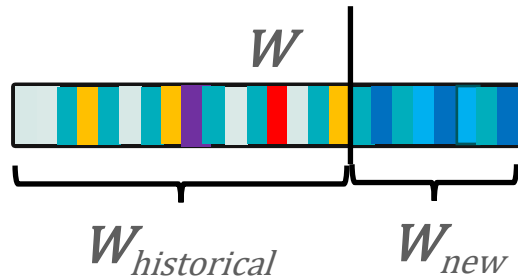
$$t_{value} = \frac{\mu_{hist} - \mu_{new}}{\sqrt{\frac{\sigma_{hist}^2}{|W_{hist}|} - \frac{\sigma_{new}^2}{|W_{new}|}}}$$

$$t_{value} = \frac{\rho}{\sqrt{\frac{1}{|W_{hist}|} - \frac{f_{value}}{|W_{new}|}}}$$

$$t_{value} = \frac{\rho \sigma_{hist}^2}{\sqrt{\frac{\sigma_{hist}^2}{|W_{hist}|} - \frac{\sigma_{new}^2}{|W_{new}|}}}$$

$$\rho = t_{value} \sqrt{\frac{1}{v|W|} - \frac{f_{value}}{(1-v)|W|}}$$

$$t_{value} = \frac{\rho \sigma_{hist}^2}{\sqrt{\frac{\sigma_{hist}^2}{|W_{hist}|} - \frac{f_{value} \sigma_{hist}^2}{|W_{new}|}}}$$



## Considering:

$$\rho\sigma_{hist}^2 = \mu_{hist} - \mu_{new}$$

$$f_{value} = \frac{\sigma_{new}^2}{\sigma_{hist}^2}$$



## 2. OPTWIN CONCEPT DRIFT DETECTOR: ALGORITHM & THEOREM

OPTWIN calculates the minimum size of  $W_{new}$  that guarantees it can identify drifts larger than  $\rho\sigma_{hist}^2$

### Algorithm 1 OPTWIN

<b>Input parameters:</b> <ul style="list-style-type: none"> <li>• <math>\delta</math> – confidence level</li> <li>• <math>\rho</math> – robustness</li> <li>• <math>w_{max}</math> – max window size</li> </ul>	<b>Global variables:</b> <ul style="list-style-type: none"> <li><math>W = \langle \rangle</math> – sliding window</li> <li><math>w_{min} = 30</math> – min window size</li> <li><math>\eta = 1e^{-5}</math> – avoids division by 0</li> </ul>
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```

1: procedure ADDELEMENT( $x_i$ )
2:    $W \leftarrow W \cup x_i$ 
3:   if  $|W| < w_{min}$  then
4:     return False
5:   else if  $|W| \geq max\_length$  then
6:      $W \leftarrow W - W_0$ 
7:    $v \leftarrow OPTIMALCUT(|W|, \rho, \delta^{\frac{1}{4}})$  cf. Equation (1)
8:    $v_{split} \leftarrow \lfloor v |W| \rfloor$ 
9:    $W_{hist} \leftarrow W_0:v_{split}$ 
10:   $W_{new} \leftarrow W_{v_{split}:|W|-1}$ 
// f-test
11:  if  $\frac{(\sigma_{W_{new}} + \eta)^2}{(\sigma_{W_{hist}} + \eta)^2} > f\_ppf(\delta^{\frac{1}{4}}, v|W| - 1, (1 - v)|W| - 1)$  then
12:    reset()
13:    return True
// t-test – cf. Equations (1) and (2)
14:  else if  $t\_value(W_{hist}, W_{new}) > t\_ppf(\delta^{\frac{1}{4}}, df)$  then
15:    reset()
16:    return True
  
```

$$\rho = t_{value} \sqrt{\frac{1}{v|W|} - \frac{f_{value}}{(1-v)|W|}}$$

### Theorem 4.5.1

- **False Positive Bound.** At every step, if  $\mu_W$  and  $\sigma_W^2$  remain constant within  $W$ , OPTWIN will flag a concept drift at this step with a confidence of at most  $1-\delta$ .
- **False Negative Bound** (for mean drift with large enough  $W$ ). For any partitioning of  $W$  into two sub-windows  $W_{hist}$   $W_{new}$ , with  $|W| \geq w_{proof}$  and  $W_{new}$  containing the most recent elements, if  $\mu_{hist} - \mu_{new} > \rho \sigma_{hist}$ , then, with confidence  $\delta$ , OPTWIN flags a concept drift in at most  $|W| - v_{split}$  steps.
- **False Negative Bound** (for mean drift with small  $W$ ). For any partitioning of  $W$  into two sub-windows  $W_{hist}$   $W_{new}$ , with  $w_{min} \leq |W| < w_{proof}$  and  $W_{new}$  containing the most recent elements, if  $\mu_{hist} - \mu_{new} > \rho_{temp} \sigma_{hist}$ , then, with confidence  $\delta$ , OPTWIN flags a concept drift in at most  $\frac{|W|}{2}$  steps.
- **False Negative Bound** (for standard-deviation drift with any  $W$ ). For any partitioning of  $W$  into two sub-windows  $W_{hist}$   $W_{new}$ , with  $|W| \geq w_{min}$  and  $W_{new}$  containing the most recent elements, if  $\frac{\sigma_{new}^2}{\sigma_{hist}^2} > f\_ppf(\delta', v|W|-1, (1-v)|W|-1)$ , then, with confidence  $\delta$ , OPTWIN flags a concept drift in at most  $v_{split}$  steps.

# EXPERIMENTS

### 3. EXPERIMENTS: DRIFT IDENTIFICATION ON MOA

- Massive Online Analysis (MOA):
  - Binary & Non-binary data
  - Gradual & Sudden Drifts
  - AGRAWL, Random RBF, and STAGGER synthetic datasets
  - Electricity and Covertypes real-world datasets
- Regression on ANN models:
  - CNN model
  - CIFAR-10 dataset (simulated concept drifts)

Detector	Avg. Delay	Avg. FP	Avg. F1
ADWIN	132.19	4.39	67%
DDM	569.37	0.73	86%
EDDM	1127.56	8.22	49%
STEPD	196.97	34.93	30%
ECDD	131.84	67.22	37%
OPTWIN $_{\rho=0.1}$	156.13	<b>0.17</b>	<b>95%</b>
OPTWIN $_{\rho=0.5}$	<b>120.98</b>	0.19	<b>95%</b>
OPTWIN $_{\rho=1.0}$	414.29	0.32	89%

TABLE I

AVERAGE STATISTICS OF DRIFT IDENTIFICATION ON THE FOLLOWING SYNTHETIC SETTINGS: GRADUAL BINARY DRIFT, GRADUAL NON-BINARY DRIFT, SUDDEN BINARY DRIFT, SUDDEN NON-BINARY DRIFT, SUDDEN STAGGER, SUDDEN RANDOM RBF, AND SUDDEN AGRAWL.

### 3. EXPERIMENTS: DRIFT IDENTIFICATION ON MOA

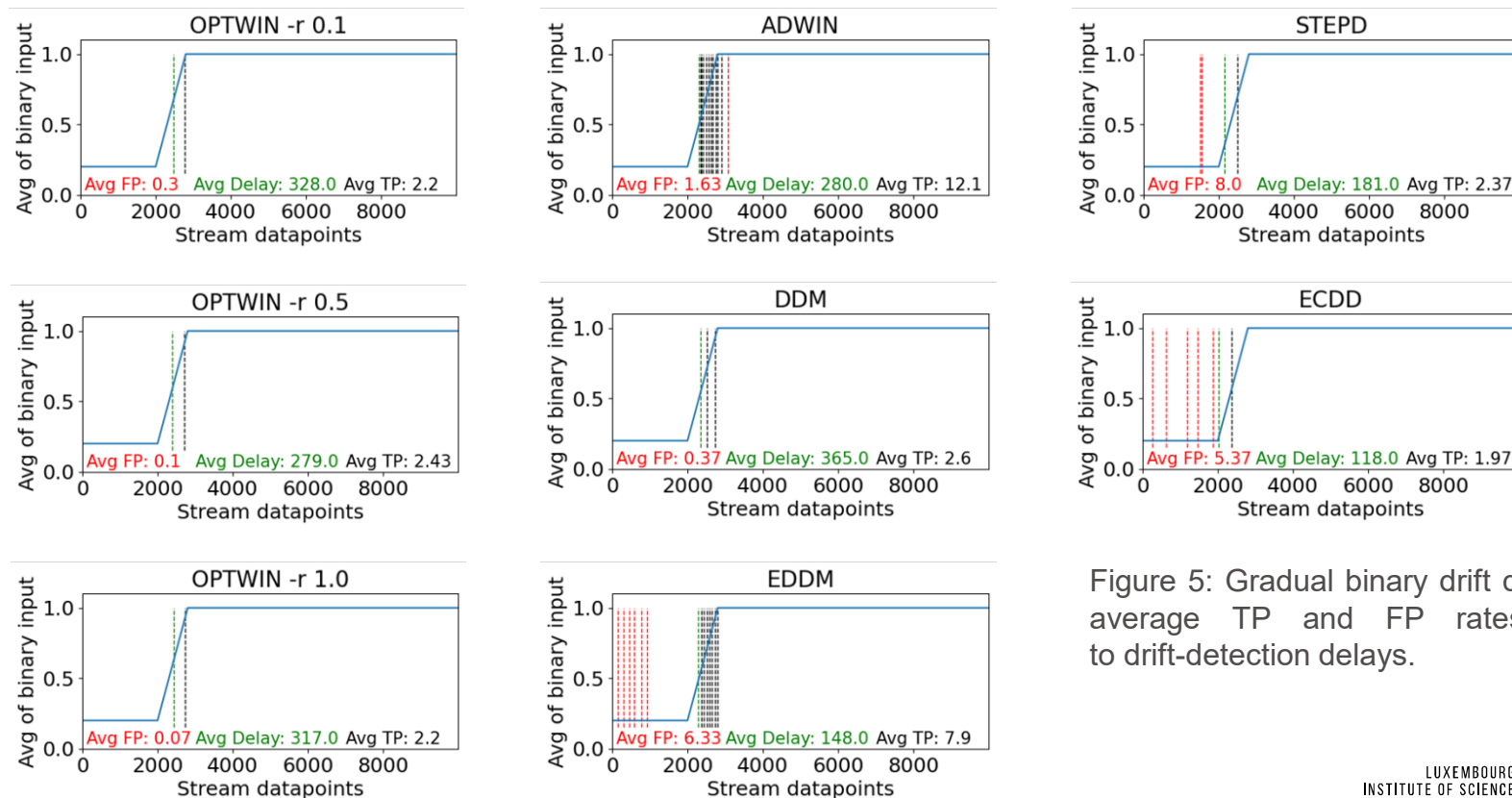


Figure 5: Gradual binary drift detection with average TP and FP rates compared to drift-detection delays.

# EXPERIMENTS: DRIFT DETECTION ON ANN MODELS

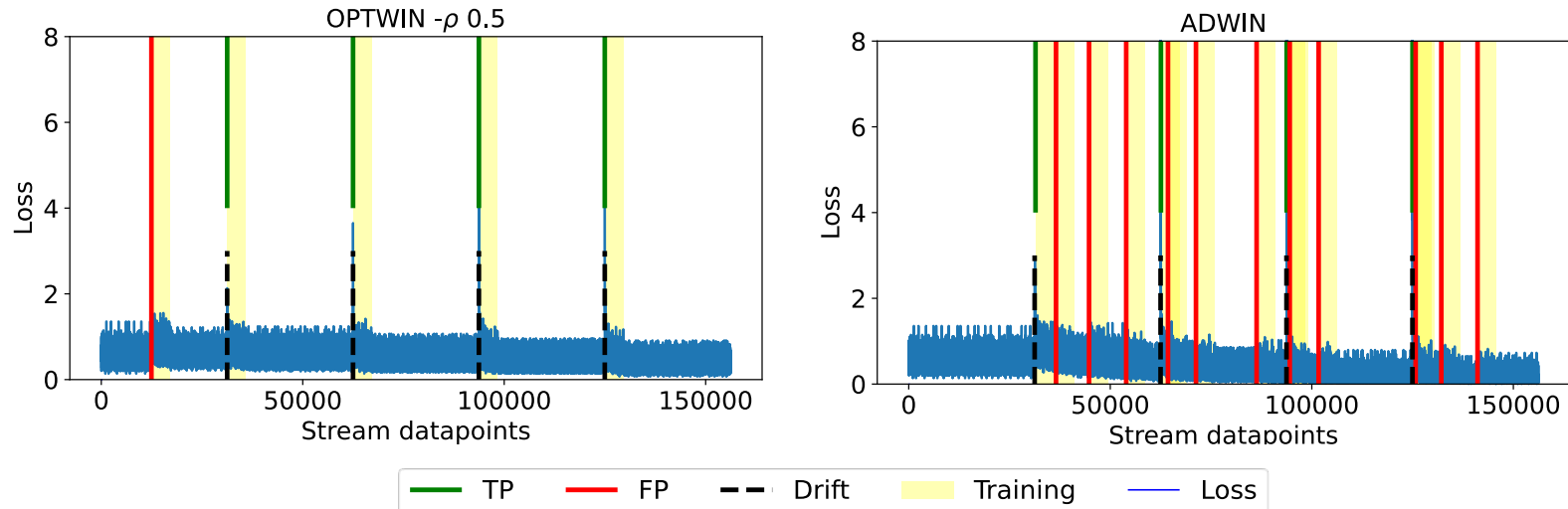


Figure 6: Sudden drift detection over the loss of a CNN.

- **OPTWIN was 21% faster than ADWIN due to a lower number of false positives.**

# CONCLUSION

## 4. CONCLUSION

- Improve optimal splitting point identification from  $\mathcal{O}(\log n)$  to  $\mathcal{O}(1)$ .
- Reduce False Positives (improve F1-score).
- Pre-computable optimal sliding window splitting point.
- Suited for regression and classification problems.
- Available in GitHub in Python, C++, and Java.



<http://github.com/maurodl/optwin>

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# thank you



[github.com/maurodl/optwin](https://github.com/maurodl/optwin)

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