Solving some problemes, fibo, gcd

Binomial coeffecients

Inclusion exclusion principle, problem solving

Q: fibosum $\rightarrow$ $\boxed{l,\ r}$ $\rightarrow$ Sum of fibonacci

from $\boxed{f_l^{th} \rightarrow f_r^{th}}$

$$\underline{\text{rangsum}(l, r)} \rightarrow \text{rangesum}(0, r) - \text{rangesum}(0, l-1)$$

$$S_n = f_0 + f_1 + f_2 + f_3 - - - - - - - - f_n$$

add f_1 on both sides

$$S_n + f_1 = f_0 + f_1 + f_1 + f_2 + f_3 - - - - - f_n$$

$$S_n + f_1 = f_0 + f_1 + f_1 + f_2 + f_3 - - - - - f_n$$

$$= f_1 + f_2 + f_2 + f_3 - - - - - f_n$$

$$\Rightarrow f_2 + f_3 + f_3 - - - - - - - f_n$$

$$\Rightarrow f_3 + f_4 + f_4 - - - - - - - f_n$$

$$\vdots$$

$$\Rightarrow f_n + f_{n+1}$$

$$S_n + f_1 \Rightarrow f_{n+2}$$

$S_n = f_{n+2} - f_1$ $(f_1 = 1)$

$$\boxed{S_n = f_{n+2} - 1}$$

$\rightarrow$ calc in log using matrix exponentiation

Binomial Coefficients

It is a coefficient that denotes no. of ways to choose k objects from given n objects.

$\rightarrow {}^nC_k \longleftrightarrow \binom{n}{k}$

$${}^nC_k = \boxed{\frac{n!}{k!\,(n-k)!}}$$

$${}^nC_k = {}^nC_{n-k}$$

$$\rightarrow \frac{n!}{k!\,(n-k)!} = \frac{n!}{(n-k)!\,(k)!}$$

$$(a+b)^n \Rightarrow {}^nC_0\, a^n + {}^nC_1\, a^{n-1} b + {}^nC_2\, a^{n-2} b^2 \text{ --- } \ldots\ {}^nC_k\, a^{n-k} b^k \text{ ---- } {}^nC_n\, b^n$$

Binomial expansion

Pascals Triangle

Pascal's triangle

j^{th} column

i^{th} row →

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

⋮

1, 1, 2, 3, 5, 8 — fib

${}^{i}C_{j}$

Given n, print first n rows of the triangle.

$${}^nC_k = \frac{{}^nC_{k-1} \times (n-k+1)}{k}$$

$${}^4C_2 = \frac{{}^4C_1 \times (4-2+1)}{2}$$

$$= \frac{\cancel{4}^{2} \times 3}{\cancel{2}} \Rightarrow 6$$

$\downarrow 2^{nd}$

$i \longrightarrow$

1					
1	1				
1	2	1			
1	3	3	1		
1	4	6	4	1	
1	5	10	10	5	1

$$f(i,j) = f(i-1,j) + f(i-1,j-1)$$

$$\boxed{{}^nC_k = {}^{n-1}C_k + {}^{n-1}C_{k-1}}$$

→ Common problem

DP

$${}^{n}C_{k} = \frac{n!}{k!\,(n-k)!} \quad \text{--- (1)}$$

$${}^{n}C_{k-1} = \frac{n!}{(k-1)!\,(n-k+1)!} \quad \text{--- (2)}$$

(1)/(11)

$$\frac{{}^{n}C_{k}}{{}^{n}C_{k-1}} = \frac{(k-1)!\,(n-k+1)!}{k!\,(n-k)!}$$

$$= \frac{\cancel{(k-1)!}}{k \times \cancel{(k-1)!}} \times \frac{(n-k+1) \times \cancel{(n-k)!}}{\cancel{(n-k)!}}$$

$$\frac{{}^{n}C_{k}}{{}^{n}C_{k-1}} = \frac{(n-k+1)}{k}$$

$$\boxed{{}^{n}C_{k} = \frac{({}^{n}C_{k-1}) \times (n-k+1)}{k}}$$

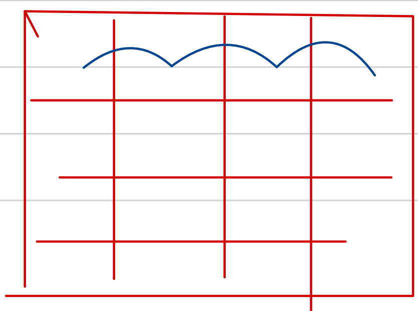
$\longrightarrow$ ${}^{n}C_{0} \to 1$ ~~zemba~~ remember

→ Calc ${}^nC_r$ → TC → $O(r)$

Qn Given n numbers, find the no. of ways in which we can create some subset from these nos.

$$\to 2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 \cdots {}^nC_n$$

→ **Q.** Given a $n \times m$ grid and a tourist, starting from $(0,0)$, who wants to reach $(n-1, m-1)$ with a constraint that from each cell he can go right / down only. Count the no. of ways for tourist to complete journey from $(0,0) \to (n-1, m-1)$



$$\underbrace{\overbrace{RRR\ldots\ldots}^{(M-1)}\ \overbrace{DDDD\ldots\ldots}^{(n-1)}}_{RDRD\text{ - - - - -}}$$

$n-1+m-1 \Rightarrow \boxed{n+m-2}$ → total steps

$$\frac{(n+m-2)!}{(m-1)!\,(n-1)!}$$

$^{(n+m-2)}C_{(m-1)}$

Q:) Given a no. n, find out the count of no's from $1 \to n$ which are divisible by any of the first 20 prime no.s

2 3 5
[a, b, c]
1 1 0

$\boxed{n \le 10^{18}}$

$n \to$ 3 and 5

no $\to$ 3
no $\to$ 5

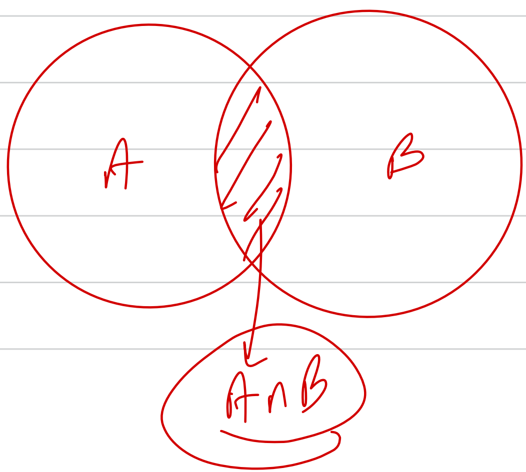
no $\to$ 15

no. divisible by 3 and 5

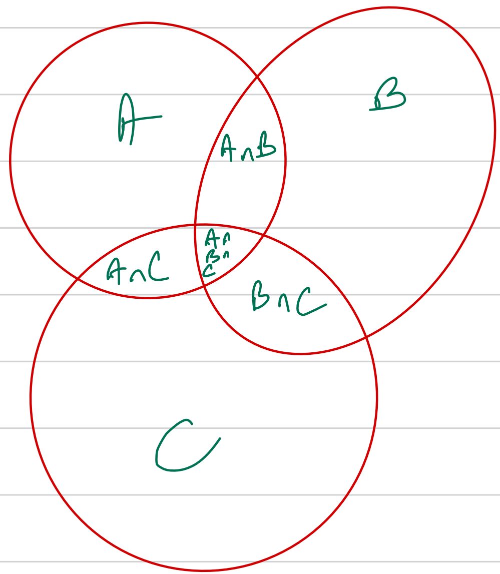
$$|A \cup B| = |A| + |B| - |A \cap B|$$

Principal of inclusion and exclusion

It is a counting technique which generalises for elements in union of finite sets.



$$|A \cup B| = |A| + |B| - |A \cap B|$$



$2^n - 1$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Q ⇒ $\dfrac{n!}{K^x}$ x

$n = 5$
$K = 2$

$\rightarrow \dfrac{5!}{2^x} \rightarrow \dfrac{5 \times 4 \times 3 \times 2 \times 1}{2^x}$

$\rightarrow \dfrac{5 \times 3 \times 2^3 \times 1}{2^x}$

$x = 3$

every no. can be represented as product of power of primes.

$$K = p_1^{k_1} p_2^{k_2} p_3^{k_3} \cdots\cdots p_l^{k_l}$$

$$n! = p_1^{c_1} p_2^{c_2} p_3^{c_3} \cdots\cdots p_l^{c_l}$$

$$\frac{n!}{K^x} = \frac{p_1^{c_1} p_2^{c_2} p_3^{c_3} \cdots\cdots p_l^{c_l}}{p_1^{k_1 x} p_2^{k_2 x} p_3^{k_3 x} \cdots\cdots p_l^{k_l x}}$$

$$c_1 \geq k_1 x$$

$$x \leq \frac{c_1}{k_1} \qquad x \leq \frac{c_2}{k_2} \qquad x \leq \frac{c_3}{k_3} \quad \cdots\cdots \quad x \leq \frac{c_l}{k_l}$$

$$x \rightarrow \min\left(\frac{c_i}{k_i}\right) \quad \forall\, i \in [1, l]$$

$K = 16$ $\qquad\qquad \sqrt{K}$

$K \Rightarrow p_1^{k_1}\, p_2^{k_2} \cdots\cdots$ $\qquad 2 - \sqrt{K}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ~~d~~ factor $\to i^{G}$

$K \Rightarrow K / c^2 \rightarrow c_1 + x$

$i \quad c_1$

$n! \longrightarrow p_1^{c_1}\, p_2^{c_2} \cdots\cdots p_d^{c_d}$

$$\left\lfloor \frac{n}{p_1} \right\rfloor + \left\lfloor \frac{n}{p_1^2} \right\rfloor + \left\lfloor \frac{n}{p_1^3} \right\rfloor \cdots$$

$n = 100 \rightarrow 1 \times 2 \times 3 \times 4 \times 5 ---- 10 ---- 15 ---- 20 ---- 25 ---- 50 -- 75 - 100$

$f_1 \rightarrow 5 \rightarrow \left\lfloor \frac{100}{5} \right\rfloor + \left\lfloor \frac{100}{5^2} \right\rfloor ------$