

Z - Algorithm

- Naive $\rightarrow O(P * T)$
- Rabin-Karp \rightarrow Worst case $\Rightarrow O(P * T)$
Avg case $\Rightarrow O(T)$

→ Now if the algorithm is the function

$z(k) = \text{longest substring starting at } k \text{ which is also a prefix of the string.}$

String $\rightarrow a^2 a b x a a y a a z b$

$\rightarrow z \rightarrow 0 \ 1 \ 0 \ 0 \ 2 \ 1 \ 0 \ 3 \ 1 \ 0$

z-array Starting at index ①, there is a string of length 1 equivalent to prefix of the string.

1. To use this information for pattern matching

2. How to compute this z - array effectively.

Q text $\rightarrow x \underline{a} b c a b z \underline{a} b c$
pattern $\rightarrow abc$

1. Concatenate pattern + text and separate them by a character which is not present in either of the strings.
 $abc \neq x a b c a b z a b c$

2. Calculate the z - array of this new concatenated string.

$\begin{array}{ccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \text{pat} & a & b & c & & & & & \\ \text{text} & & & & x & a & b & c & a \\ \hline z & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \end{array}$

3. Look for positions in the z - array where the value is equal to the length of the pattern.

To find the original index in text, where pattern is present,

$$\text{index} = (\text{length of pattern} + \text{special char})$$

$$5 = (3 + 1) \Rightarrow \boxed{1}$$

$$11 = (3 + 1) \Rightarrow \boxed{7}$$

Time Complexity : $O(|P| + |T|)$

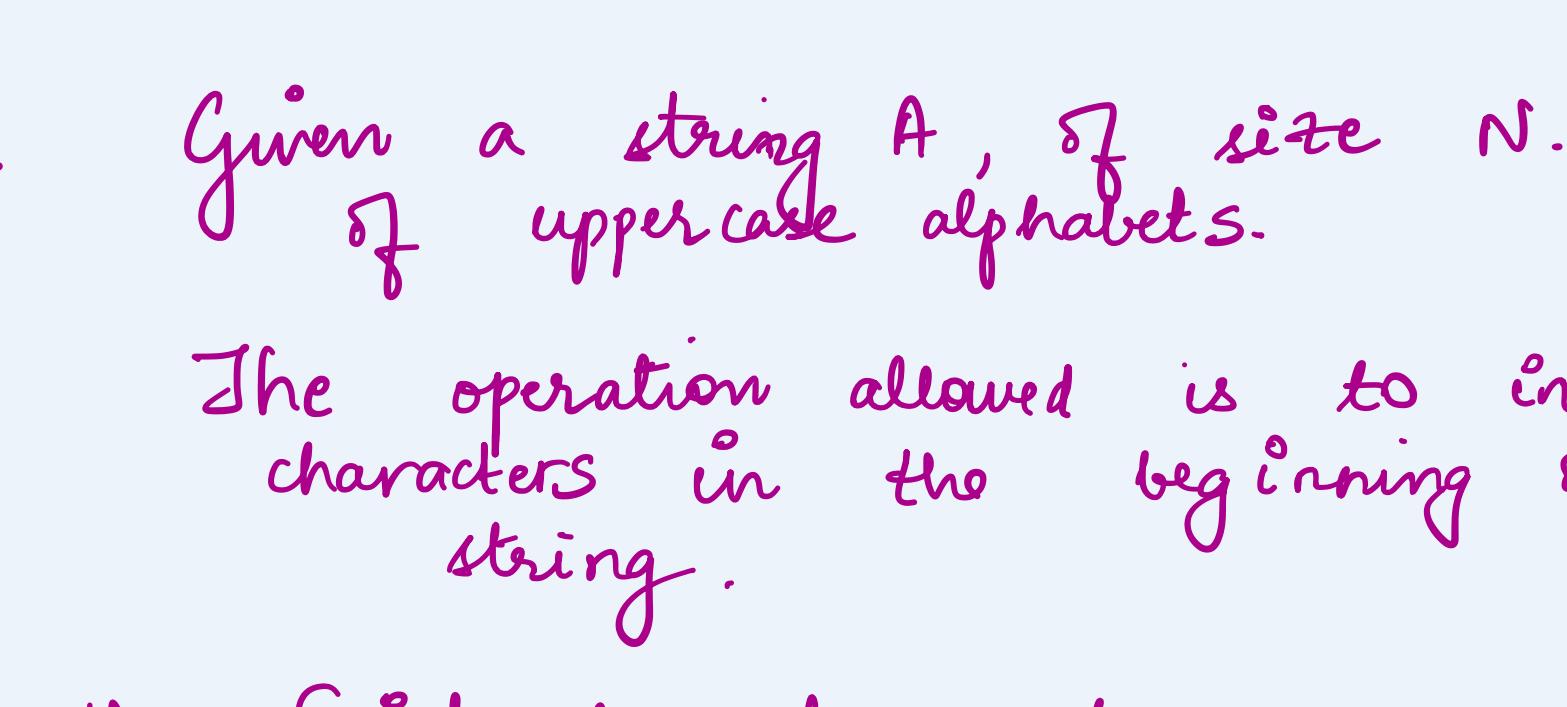
Quick ques :

$$S = P \neq T$$

$$\Rightarrow S = T \neq P \quad ? \quad \text{No}$$

→ This is not a symmetric property.

Now Z - algorithm,



$$z_i > 0$$

$\alpha \rightarrow$ length till the point there is a match

Now, for a given position i , where $z_i > 0$

→ starting at position i , there is some substring (s) that matches the prefix of the string

$[y \neq x]$

String of length $|s|$

$$i=0 \longrightarrow i=|s|$$

Compute the $|z|$ from $i=1$ to $i=|s|$

Naive approach

① Compute z_1 ,

$\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ a & a & a & a & c & b & a & b & c \\ \downarrow & & & & & & & & \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}$

for just computing $z_1 \rightarrow 4$ comparisons

worst case

$\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ a & a & a & a & a & a & a & a & a \\ \downarrow & & & & & & & & \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}$

$z \rightarrow 0 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4$

$z_1 = 4$

$z_2 = 3$

$z_3 = 2$

$z_4 = 1$

$z_5 = 0$

$z_6 = 0$

$z_7 = 0$

$z_8 = 0$

$z_9 = 0$

$z_{10} = 0$

$z_{11} = 0$

$z_{12} = 0$

$z_{13} = 0$

$z_{14} = 0$

$z_{15} = 0$

$z_{16} = 0$

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