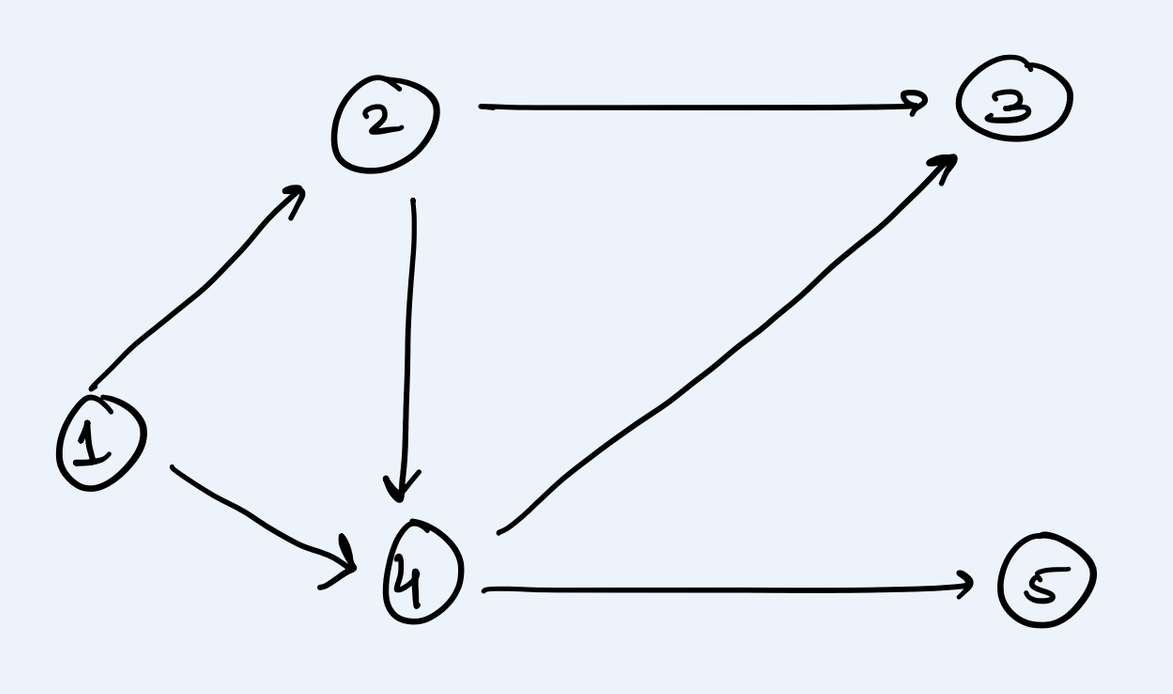
Sunday, 5 December 2021 8:26 PM

Topological Sorting

→ linear ordering of vertices such that for every directed edge (u → v) vertex u will come before vertex v in the ordering

→ TS is only valid for directed acyclic graphs. (DAG)

→ There is no unique TS ordering of a graph.



order 1 → 1 2 4 5 3

order 2 → 1 2 4 3 5

Topological sorting

① Finishing time

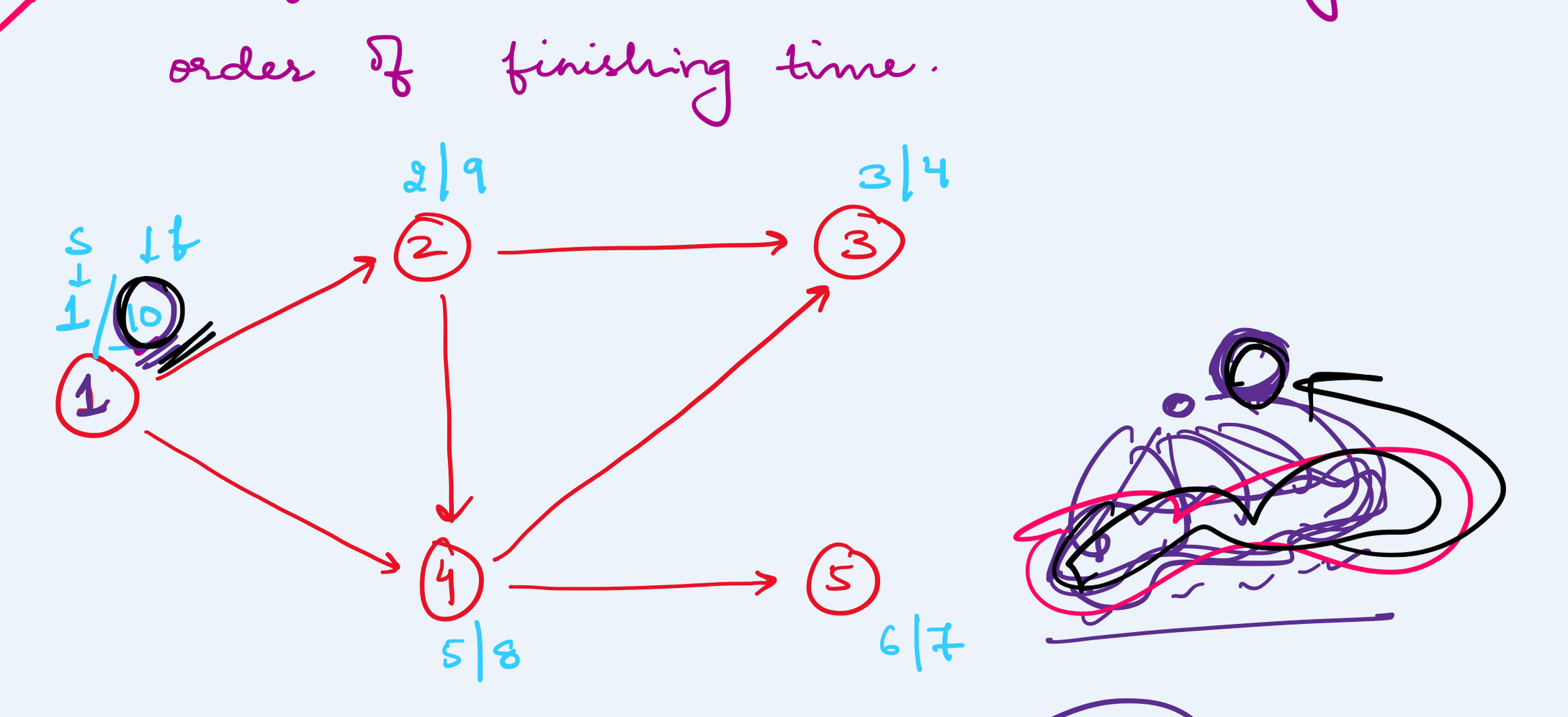
② Kahn's Algorithm

→ Finishing time

1. Postorder DFS based on finishing time ✓

① children
② myself

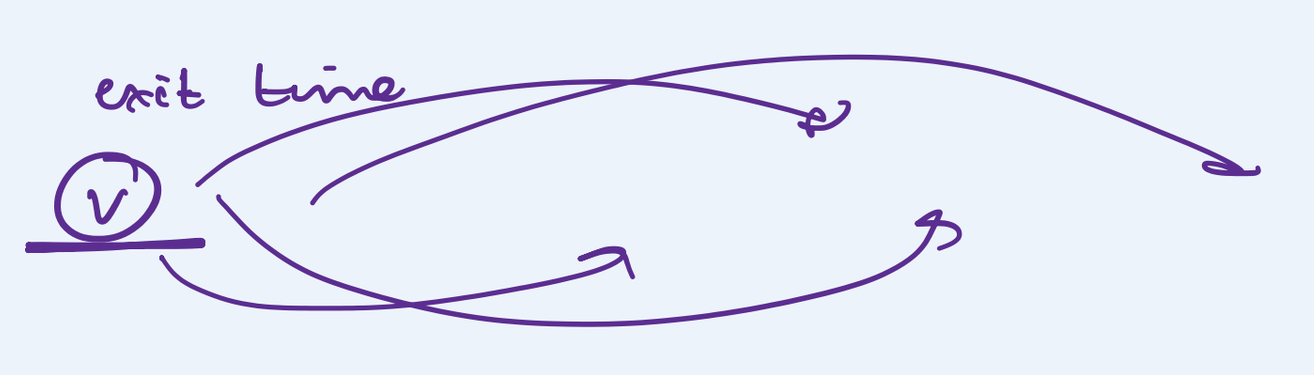
2. Arrange the nodes based on decreasing order of finishing time. ✓



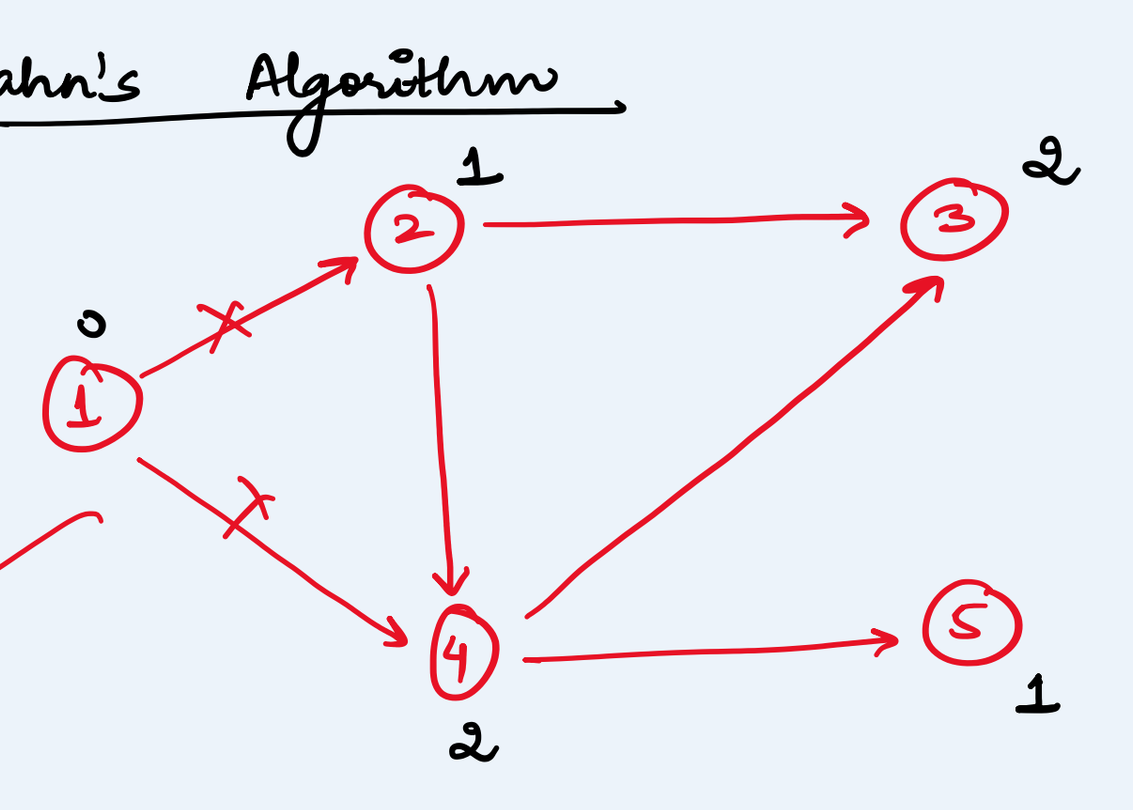
dfs(v)

1 2 4 5 3

when dfs (v) is ended, all vertices that are reachable from v either directly or indirectly would already be visited.

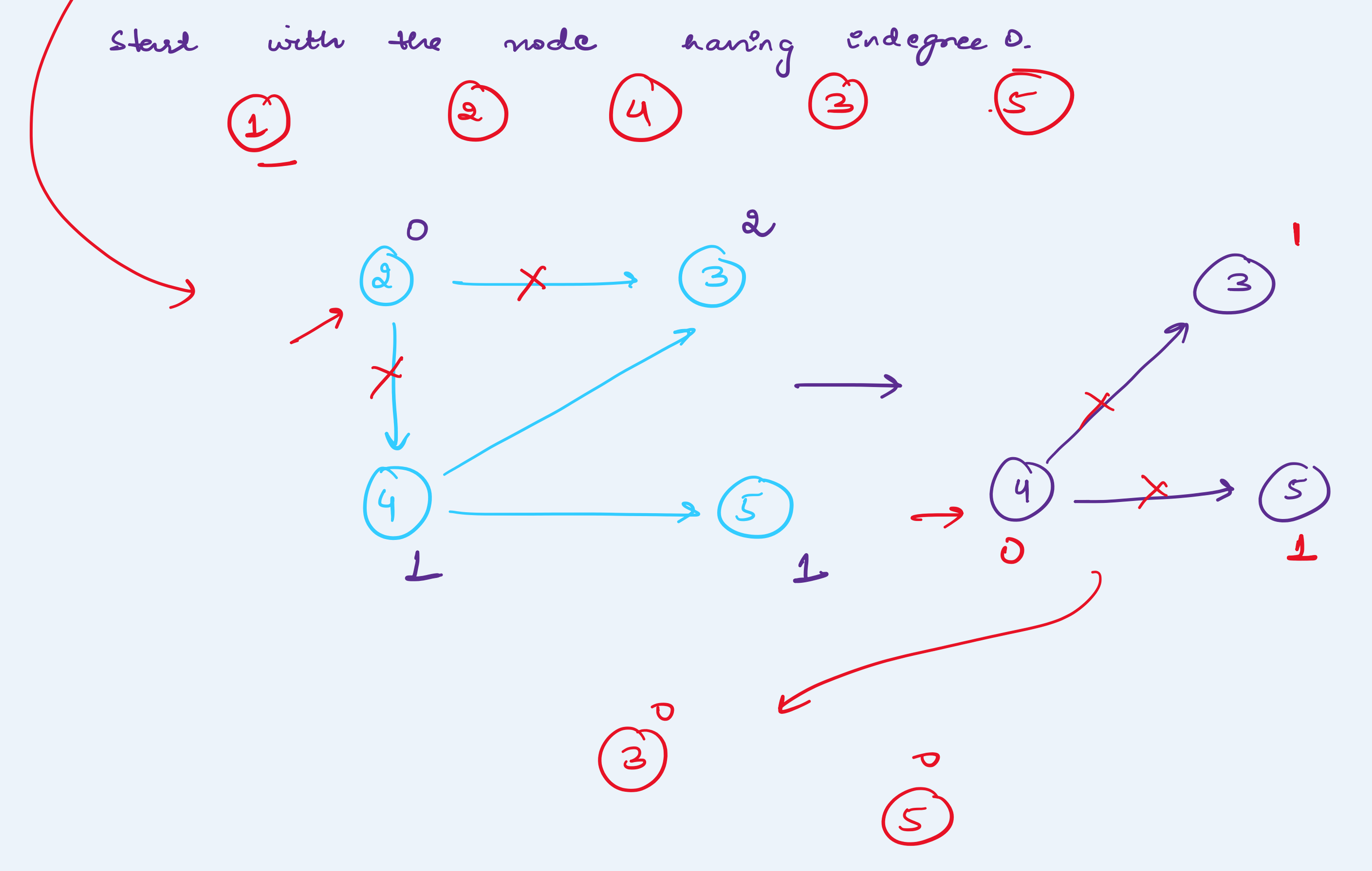


Kahn's Algorithm



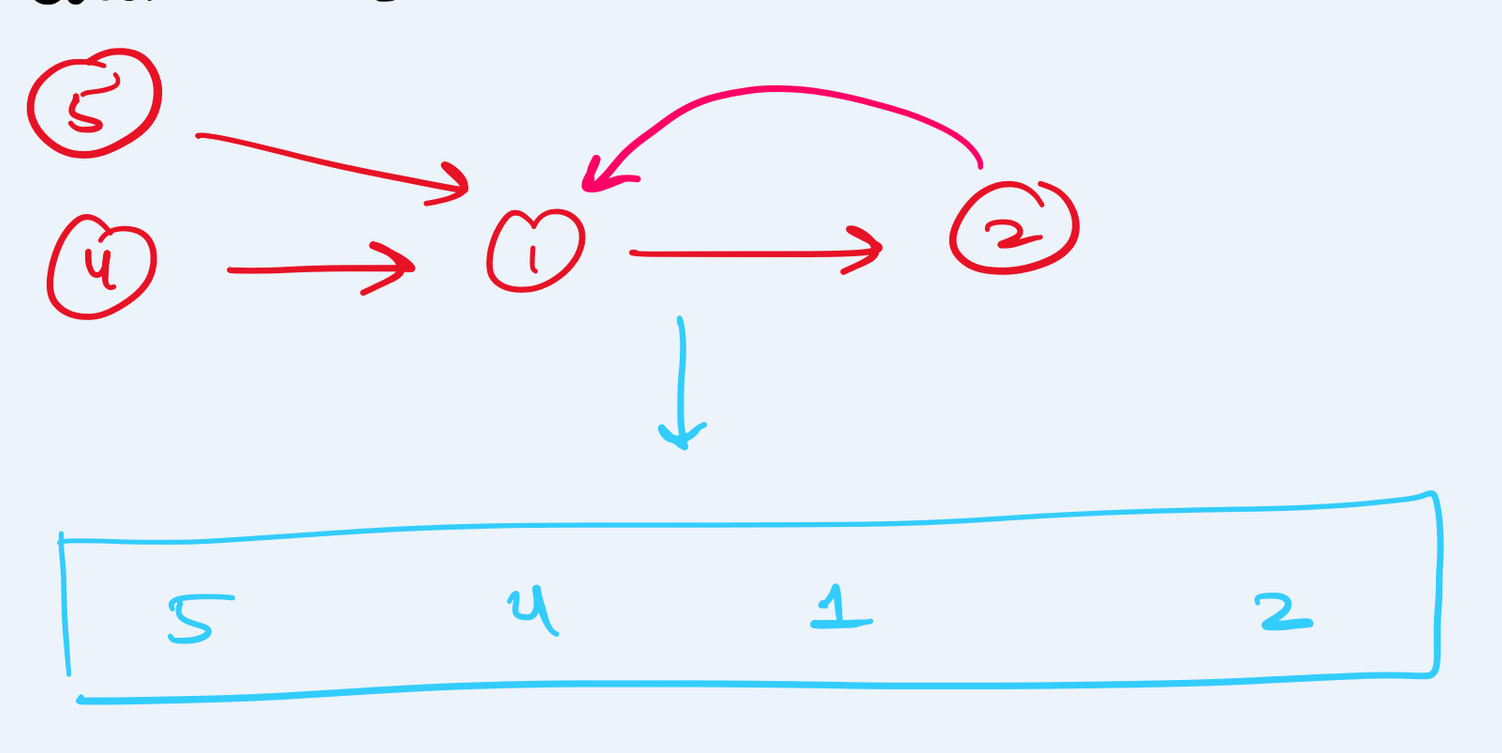
Start with the node having indegree 0.

① ② ④ ③ ⑤



```
topological sort (N, adj[N][N])
  Q
  T = [ ]
  visited = [ ]
  indegree = [ ]

  for ( i=0; i<n; i++)
          in_degree [i] = visited[i] = 0

  for  i=0 to N
      for  j=0 to N
           if adj [i][j] → t
               indegree [j] += 1

for     i=0 to N
        if   in_degree is 0
               q. enqueue (i);
               visited[i] = t;

while  Q is not empty

       vertex = q. dequeue();
       T. append (vertex);
       for  j=0 to N
            if (adj [vertex][j] → t & visited [j] → f)
                  in_degree [j] -= 1;

                  if ( in_degree [j] == 0)
                          q. enqueue (j);
                          visited [j] → t
```

Q Course schedule



5	4	1	2

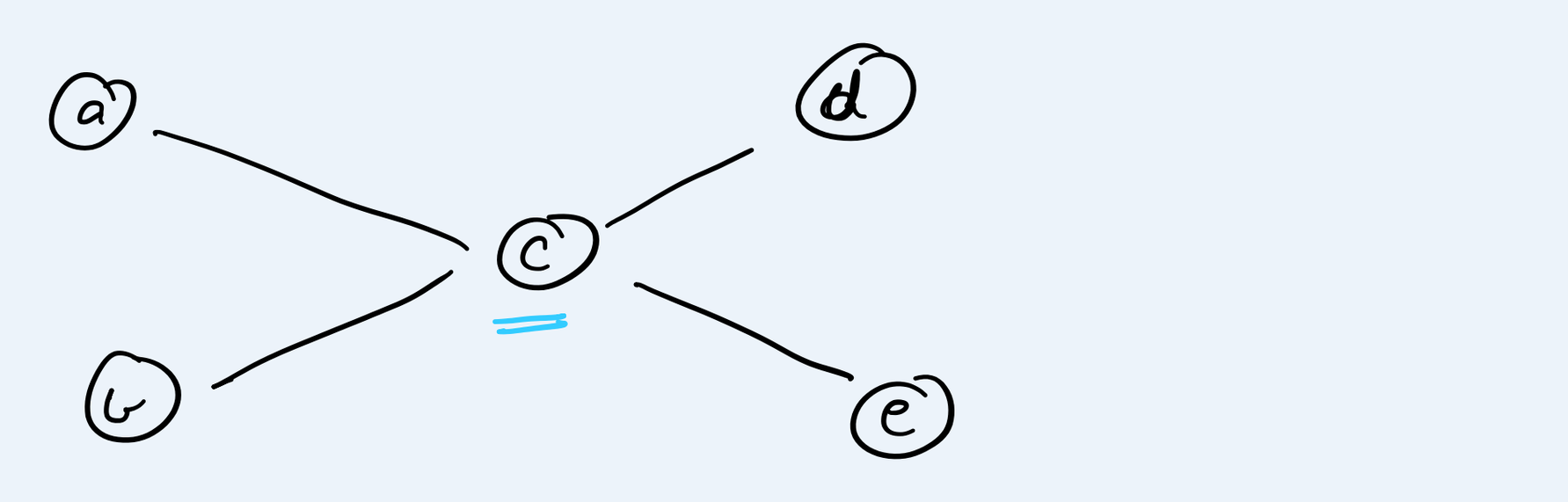
✓

Q Largest color value in a directed graph (HW)

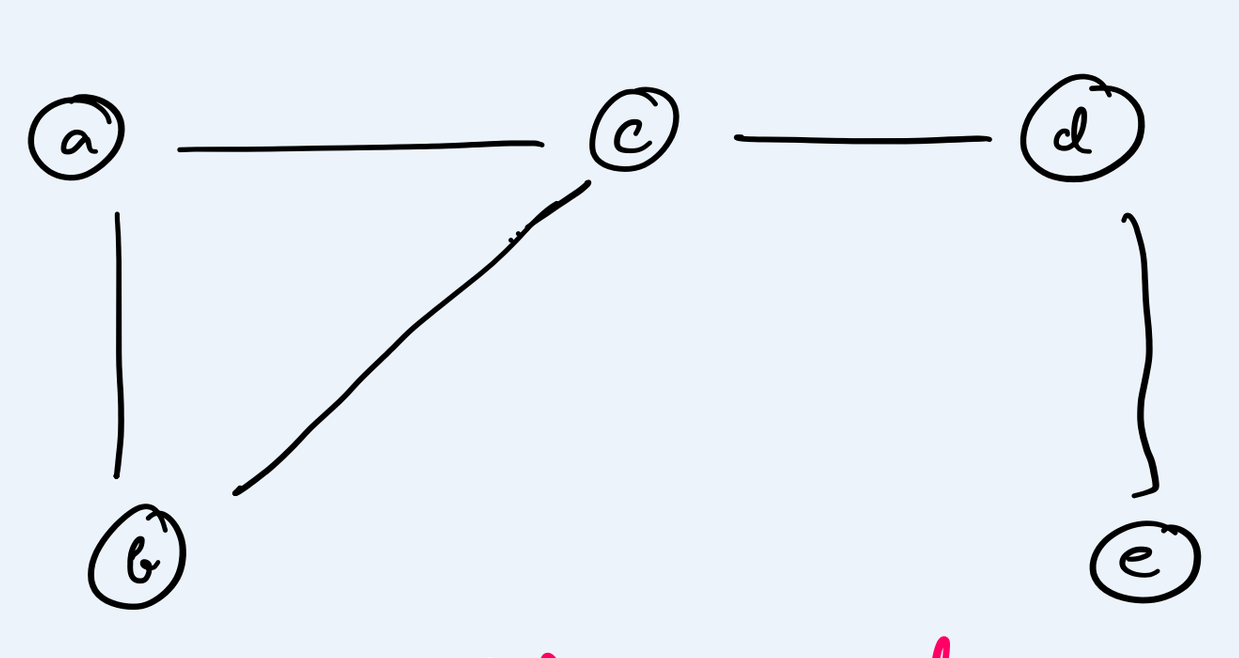
Articulation Point

A vertex is called an articulation point if removing it and all the edges associated with it, makes the graph disconnected.

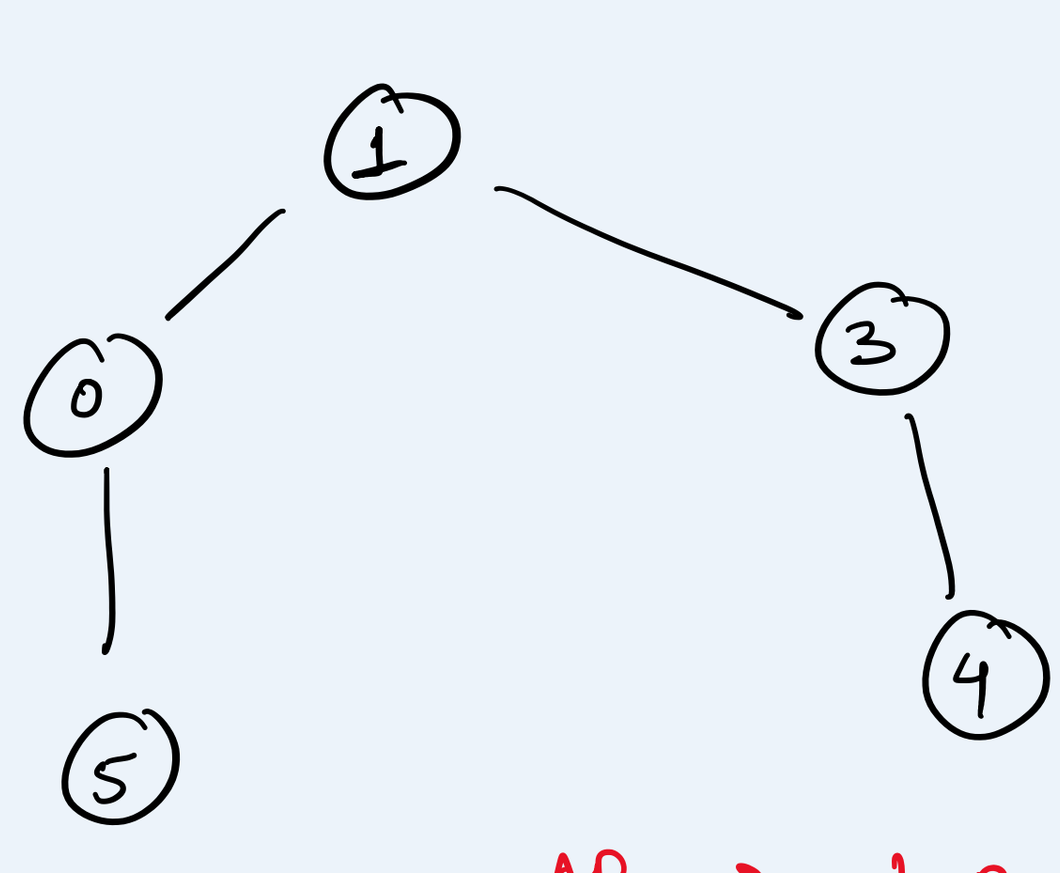
(increases the no. of connected compount in a graph)

① 

'c' is a articulation point

② 

AP → c, d

③ 

AP → 1, 0, 3

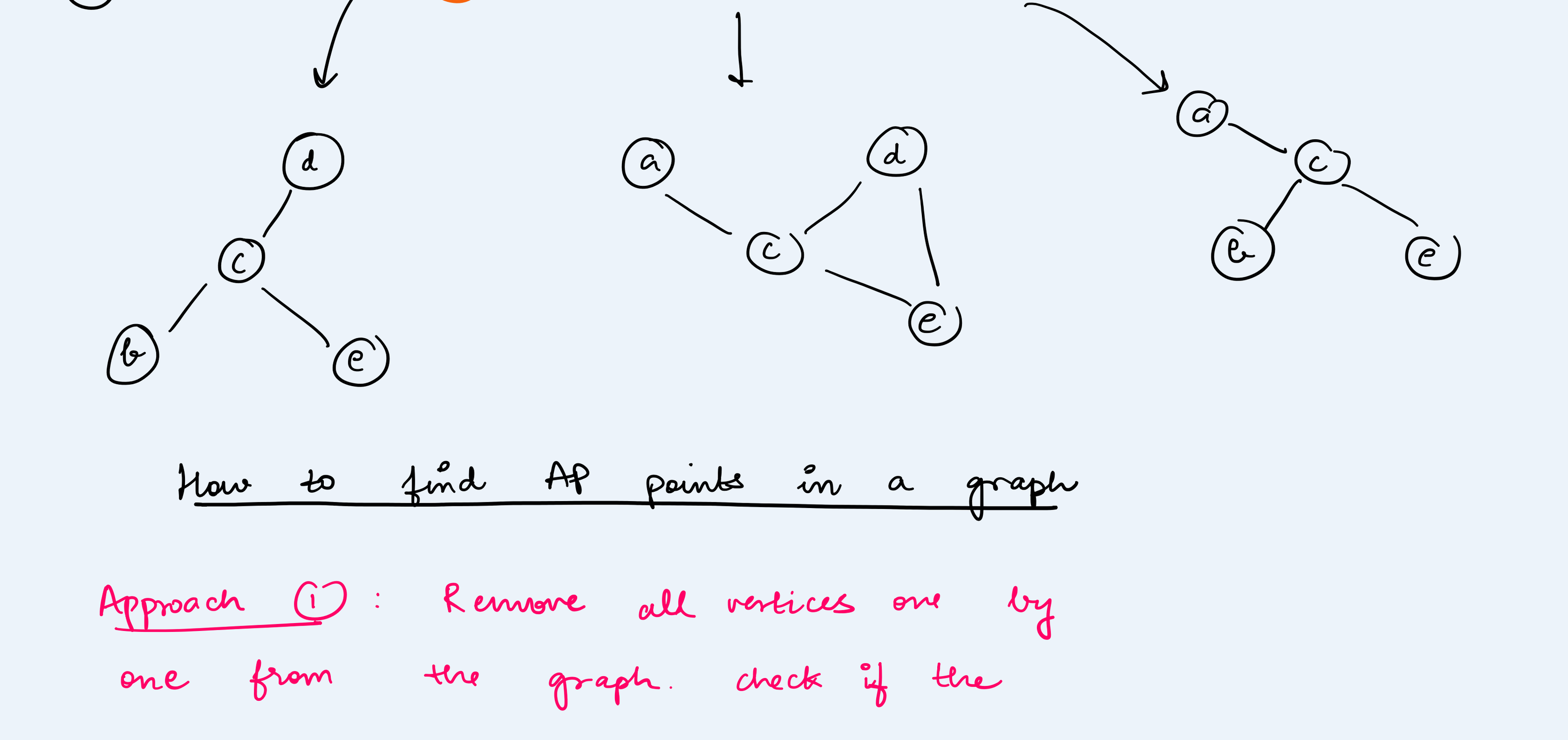
Applications

→ AP represents the vulnerabilities of a system.

→ useful for detecting critical points in any kind of network for communication, airline traffic, electronic circuit etc.

Biconnectivity (no AP)

A connected undirected graph is bioconnected if there are no vertices whose removal disconnects the rest of the graph



How to find AP points in a graph

Approach ①: Remove all vertices one by one from the graph. check if the removal of that vertex creates a disconnected graph.

pseudocode:

```
result = [ ]

for each v in V:

        remove v from graph
        if  is Connected () != true
              append v to the result
```

is Connected ()
 use BFS or DFS

BFS | DFS → $O(V + E)$

TC → $O(V \cdot (V+E))$

AP ② Tarjan's Algorithm → $O(V+E)$

Q Min no. of days to disconnect Island

Q Minimise malware spread