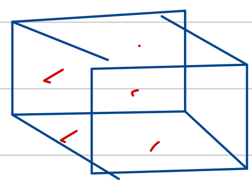
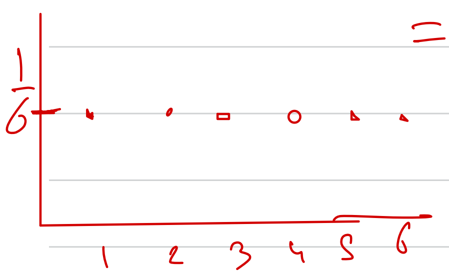
Expectation → avg value of a random variable



→ 6 faces → unbiased die

$$E(x) = \sum x_i \times p(x_i) \qquad \forall i$$

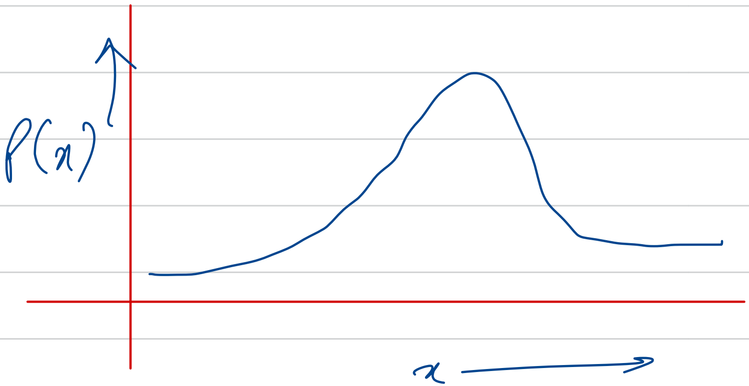
$1 \rightarrow 1/6$
$2 \rightarrow 1/6$
$3 \rightarrow 1/6$
$4 \rightarrow 1/6$
$5 \rightarrow 1/6$
$6 \rightarrow 1/6$



$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + \text{------} \; 6 \times \frac{1}{6}$$

$$= \frac{1}{6}(1+2+3+4+5+6) \rightarrow \frac{\cancel{21}^{\,7}}{\cancel{6}_{\,2}} \rightarrow \boxed{3.5}$$

$x = \{20^\circ, \; - - - - - - - \; 50^\circ\}$



$$\boxed{E(x) = \int x \cdot p(x)\, dx}$$

Rule of linearity of expectation

$$E[X_1 + X_2 + X_3 \ldots\ldots] = E[X_1] + E[X_2] \ldots\ldots$$

↳ Expected outcome for sum of 2 dice throws.

	1	2	3	4	5	6
1	2	–	–	–	–	–
2	3	–		–		
3	⋮					
4						
5						
6						

$$\rightarrow E[X_1 + X_2] = E[X_1] + E[X_2]$$
$$= 3.5 + 3.5$$
$$= 7$$

Qn If the sum of numbers rolled on the dice A and the product of the no. rolled is B, compute $\underline{\underline{E[A+B]}} = E[A] + E[B]$

$A \rightarrow x + y \rightarrow \boxed{3.5 + 3.5} \rightarrow 7$

$B \rightarrow x \times y$

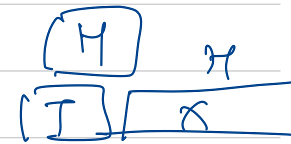
$\underline{(3.5) \times (3.5)} \rightarrow 12.25$

$7 + 12.25$

$\Rightarrow \boxed{19.25}$

Q) What is the expected no. of coin flips to get a head ??

$$\rightarrow \quad \underbrace{X}_{\text{expected no. of flips}} = \underbrace{\overset{\text{no of flip}}{1} \times \overset{p}{\frac{1}{2}}}_{\text{head in the 1st flip}} + \overbrace{(1 + X)}^{\text{wasted flip}} \times \frac{1}{2}$$



$$2X = 1 + x + 1$$

$$\boxed{x = 2}$$

Qn – Instead of getting expected value of 1 head, coin flips for return, the expected value of 2 consecutive heads.

$$X = \frac{2}{4} + \frac{x+2}{4} + \frac{1+x}{2}$$

$$4x = 2 + x + 2 + 2 + 2x$$

$$\boxed{x = 6}$$ ans

HH $\rightarrow \frac{1}{2} \times \frac{1}{2} (2)$

HT $\rightarrow (2 + x) \times \frac{1}{2} \times \frac{1}{2}$

T $\boxed{x}$

Q) What is the expected no. of coin flips reqd for getting N consecutive heads??

$$X = (X+1)\frac{1}{2} + (X+2)\frac{1}{4} + (X+3)\frac{1}{8} \cdots \frac{(X+N)}{2^n} + \frac{1}{2^n}(N)$$

$$\boxed{X = 2^{N+1} - 2}$$

$\boxed{T}$

HT

$\underbrace{HHH\cdots H}_{n}T \rightarrow HH\cdots H$

Q => The queen of honey bee nest produces off-springs one after another till she makes a male off spring. The probability of producing a male is p, what is the expected no. of off springs reqd to be produced to get a male offspring.

$$X = p \times 1 + (1-p)(X+1)$$

$$X = 1/p$$

Q.) x balls are placed in y boxes. Find the expected no. of empty box at end.

Expected no. of empty boxes

$$X_i = \begin{cases} 1 \longrightarrow i^{th} \text{ box has no balls} \\ 0 \longrightarrow \text{otherwise} \end{cases}$$

X = total no. of boxes that are empty

$$X = X_1 + X_2 + X_3 - - - - - X_y$$

$$E(X) = E(X_1 + X_2 + X_3 \text{------} X_y)$$

$$= E(X_1) + E(X_2) \text{------} E(X_y)$$

for any X_i $\longrightarrow$ 1 $\longrightarrow$ empty box

x balls $\rightarrow$ first ball $\longrightarrow$ $\left(1 - \frac{1}{y}\right)$

second ball $\longrightarrow$ $\left(1 - \frac{1}{y}\right) \times \left(1 - \frac{1}{y}\right) \rightarrow \left(1 - \frac{1}{y}\right)^2$

⋮

x^{th} ball $\longrightarrow$ $\left(1 - \frac{1}{y}\right)^x$

$$E(x_i) = \left(1 - \frac{1}{y}\right)^x \times 1$$

$$= \left(1 - \frac{1}{y}\right)^x$$

$$E(x) = E(x_1) + E(x_2) \cdots\cdots E(x_y)$$

$$= \left(1 - \frac{1}{y}\right)^x + \left(1 - \frac{1}{y}\right)^x \cdots\cdots \left(1 - \frac{1}{y}\right)^x$$

$$\boxed{E(x) = y \times \left(\frac{y-1}{y}\right)^x}$$

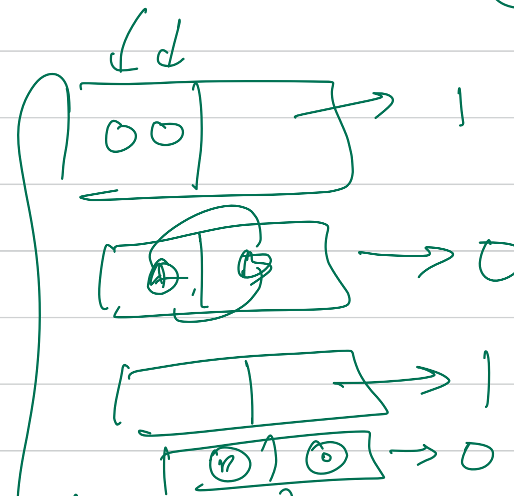
$\boxed{10 \text{ balls} \quad 5 \text{ boxes}}$

$\rightarrow 5 \times \left(\frac{5-1}{5}\right)^{10}$

$\frac{\cancel{2}}{4} \;\; \frac{1}{2}$

$\Rightarrow 5 \times \left(\frac{4}{5}\right)^{10}$

$\Rightarrow 5 \times 0.8^{10}$



2 balls $\rightarrow$ 2 box

$\rightarrow 2 \times \left(\frac{1}{2}\right)^2 \Rightarrow 2 \times \frac{1}{4} \Rightarrow \frac{1}{2}$

Q: A fair coin is flipped N times. What is the expected no of heads.

→ $E(X_1) + E(X_2) + - - - - E(X_n)$

$\Rightarrow \frac{1}{2} + \frac{1}{2} - - - - - - \frac{1}{2}$

$= \frac{n}{2}$

3 flips

HHH

HTH

THH

TTH

TTT

HHT

THT

HTT

$$\frac{3+2+2+1+0+2+1+1}{8}$$

$$\Rightarrow \frac{\cancel{12}\ \cancel{6}\ 3}{\cancel{8}\ \cancel{4}\ 2} \Rightarrow \frac{3}{2}$$

→ Bernaulli's trial

An experiment is called a bernaulli trial,

if it has exactly 2 outcomes.

Ex → coin flip

success / failure

→ Roll a dice to get 4

1. If prob of success of a bernaulli trial is p, the expected no. of trials to get success is $1/p$

(2) If prob of success in bernoulli trial is p, then expected no. of successes in n trials is

$$np$$

Q. N students, who are asked to choose a no. btw 1-100 inclusive. What is the expected no. of students that would choose single digit no.

$\rightarrow p \rightarrow \frac{9}{100} \rightarrow \frac{9N}{100}$ ans

Q. Uncle chips always distribute a coupon in a packet of chips. The coupon chosen for each pack is choosen randomly from a set of n distinct coupons. What is the expected no. of chips packet one must buy so that they get all n coupons.

$X_i \to$ No of packets to get the i^{th} new coupon.

$$E(X_i) = \frac{1}{p}$$

$$p = \frac{n-(i-1)}{n}$$

↳ that in the new packet we get i^{th} coupon

$$E(x_i) = \frac{1}{p} = \frac{n}{n-i+1}$$

$$E(x) = E(x_1) + E(x_2) + E(x_3) - - - - - - E(x_n)$$

$$= 1 + \frac{n}{n-1} + \frac{n}{n-2} - - - - - \frac{n}{2} - \frac{n}{1}$$

ans $\rightarrow$ $$= n\left(1 + \frac{1}{2} + \frac{1}{3} - - - - - - \frac{1}{n}\right)$$