

Q.2 Given an array of n integers (positive)
 Calculate the sum of product of elements
 of all possible subsets. (ans $10^{10} + 2$) $n \leq 10^5$
 $a_i \leq 10^9$

2, 3, 5

(2) (3) (5)

(2, 3) (3, 5) (2, 5)

(2, 3, 5)

$$= 2 + 3 + 5 + 6 + 10 + 15 + 30$$

71 ans

$$\underline{\underline{a}} \longrightarrow a$$

$$ab \longrightarrow a + b + ab \rightarrow a(1+b) + b$$

$$\rightarrow a(1+b) + b + 1 - 1 \Rightarrow (1+b)(1+a) - 1$$

$$abc \rightarrow a + b + c + ab + bc + ac + abc$$

$$a(1+c) + b(1+c) + ab(1+c) + c + 1 - 1$$

$$(a + b + ab + 1)(1+c) - 1$$

$$(a(1+b) + b + 1)(1+c) - 1$$

$$(1+a)(1+b)(1+c) - 1$$

$$\text{ans} \rightarrow \left(\prod_{i=0}^{n-1} (1 + a_i) \right) - 1$$

Q How many distinct values can be represented with 5 decimal digits?

0, 1, 2, ..., 9

$\overline{10}$ $\overline{10}$ $\overline{10}$ $\overline{10}$ $\overline{10}$

10^5 digits

Combinatorics

Q₂
Came walk

ans 16

1	1
1	1

grid of integers

$m \times n$

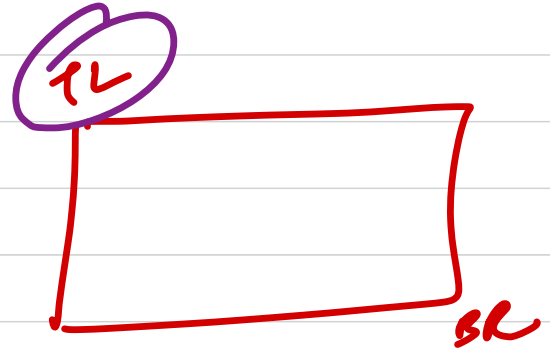
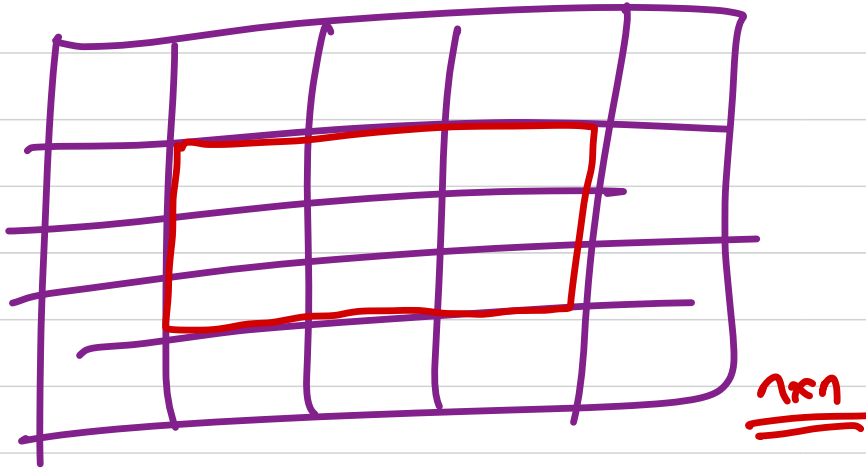
$$m \leq 10^3$$

$$n \leq 10^3$$

Calculate sum of all submatrices.

$$1+1+1+1 + 2+2 + 2+2 + 4$$

Brute force \rightarrow calc all possible submatr-
and add them.



$$\underline{\underline{n^2}} \times n^2 \times n^2 \rightarrow \underline{\underline{O(n^6)}}$$

for (i=0; i < n; i++)

for (j=0; j < n; j++)

for (k=i; k < n; k++)

for (l=j; l < n; l++)

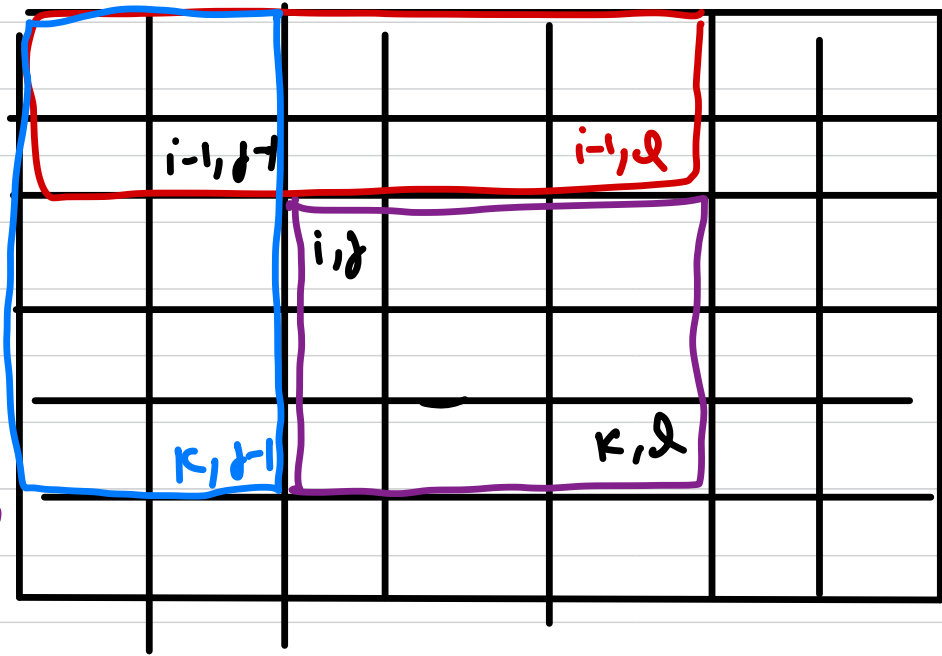
for (m=i; m <= k; m++)

for (n=j; n <= l; n++)

sum += a[m][n]

26

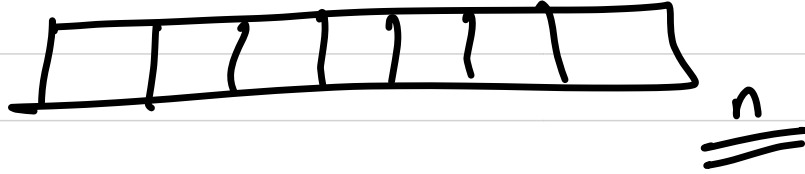
$$\text{Sum}(i, j, k, l) \rightarrow p(k, l) - p(i-1, l) - p(k, j-1) + p(i-1, j-1)$$



O(1)

p
↓

$p(i, j) \rightarrow$ it store sum of elements of submatrix
with top left @ $(0, 0)$ & BR @ (i, j)

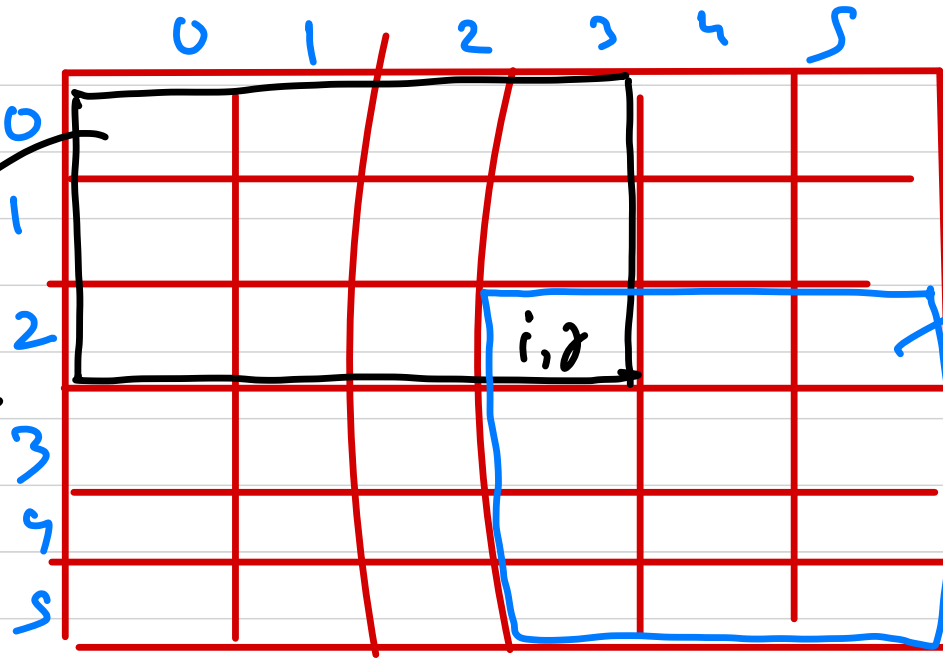


prefix sum \rightarrow

$$s[l, r] = \underline{\underline{p[r] - p[l-1]}}$$

TL, BR

$x = \#$ of
Top left
coordinates



$y = \#$ of
Bottom
right
coordinates

6x6

of submatrices $\rightarrow x \times y$ $x \rightarrow (i+1)(j+1)$ $y \rightarrow (n-i)(m-j)$
 value of cell $\rightarrow (\text{mat}[i][j] \times (i+1)(j+1)(n-i)(m-j))$
 Contri \rightarrow value \times no. of submatrices it is a part of

Qⁿ You have N balls (and we have N -types) (and we have infinite supply of each ball type). Count the no. of ways to choose k balls with repetition allowed.

$$N = 3$$

$$k = 2$$

6

A	A	B	B
A	B	B	C
A	C	C	C

$$N = 3$$

$$k = 3$$

10

$$3+2-1 \quad C_{3-1}$$

$$5C_2 \rightarrow \underline{\underline{10}}$$

n-1 partition

N=4

A A | B | C C C | D D D D

A A . . . A

B R

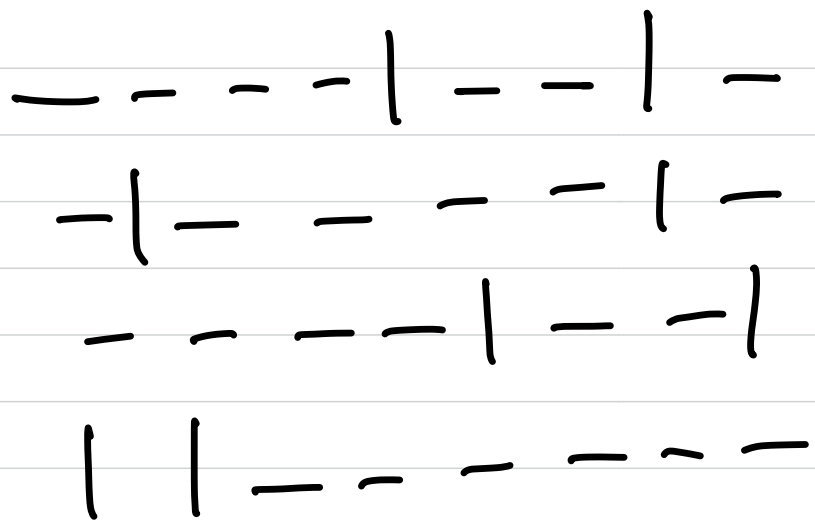
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| B B | C C | D D

to put n-1 partition

$$N=3$$

$$\underline{\underline{C=6}}$$



$$(K+n-1)C_{n-1}$$

— — — — — — — — — —

$$K=10$$

$$\underline{\underline{N=5}}$$