Combinatorics

Motivation Problem :

→ only combinations

Q: Given infinite supply of N types of balls. find the no. of ways to choose K balls from the given set with repetition allowed.

(Don't consider permutations)

$N = 3$
$K = 3$

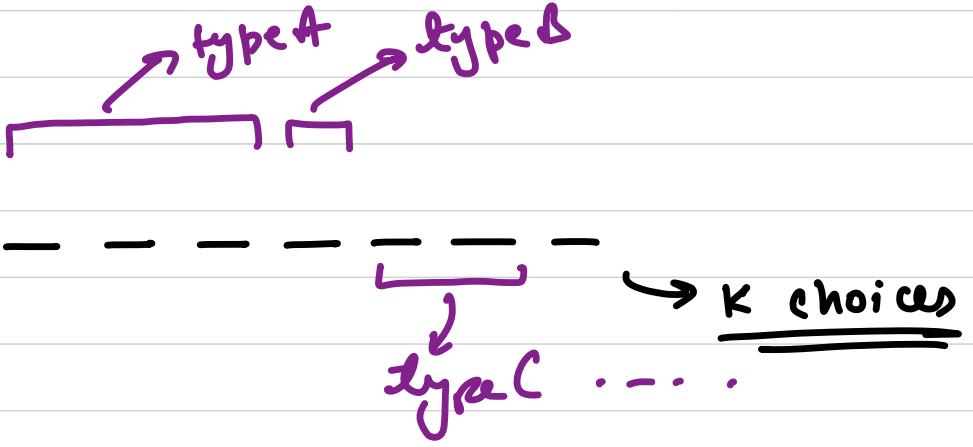
Example

Type A, B, C

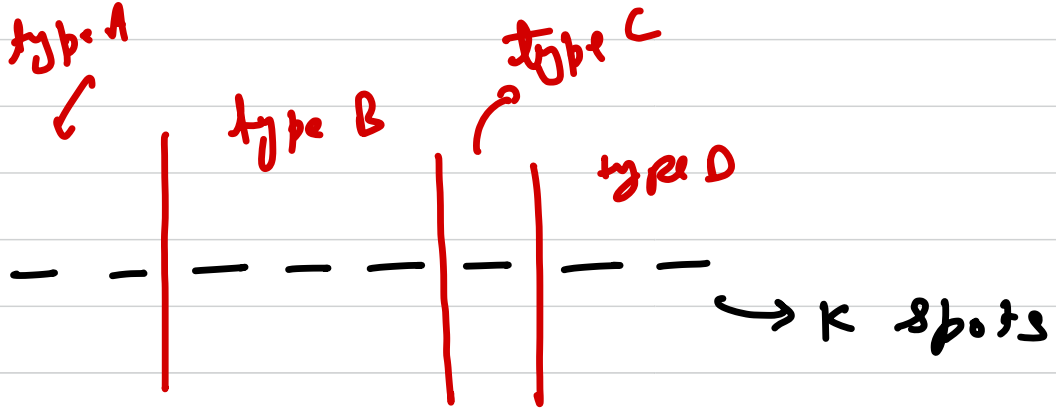
(A, B, C) (CCC) (AAC) (BBA)
(AAA) (AAB) (BCC)
(BBB) (BBC) (CCA)

Because permutations doesn't matter, we can take type wise decision i.e.

How many balls to pick from any x type



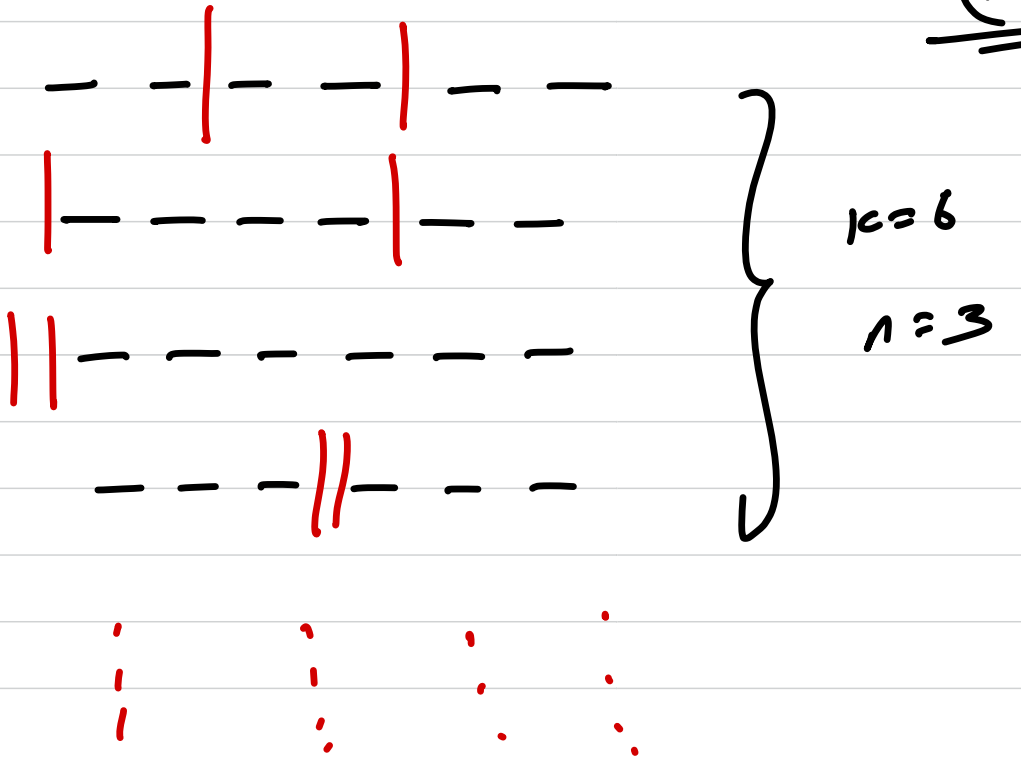
We can partition the selected choices.



for n types, we have to draw n-1 partition lines i.e, we need to count # of ways to put (n-1) lines here.

But how many positions are there ?

↳ $(K+n-1)$



Choose $n-1$ spots from $n+k-1$ spots

$$(n+k-1)C_{n-1}$$

nC_r → How to calc ??

$nC_r = \frac{n!}{r!(n-r)!}$ → $nC_r \% m$

fact[x] → precompute → array

Inv → fermat theorem

$a^{-1} \% m \rightarrow a^{m-2} \% m$

Example Problem → You walk into a candy store, and got money to by 6 candies. The store has 3 diff candies C_1, C_2, C_3.

How many ways are there to select candies?

$n = 3 \qquad k = 6$

$$6+3-1C_{3-1} \Rightarrow {}^{8}C_2 \Rightarrow 28 \text{ ans}$$

All Tracks > Math > Combinatorics > Basics of Combinatorics > Problem

Counting stars

971 36% 30 ★★★★½ 6 votes Approved, Combinatorics, Easy, Math, Ready, Recruit, approve

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Problem

There are ***N*** stars in the sky. A manual attempt at counting yielded ***K*** stars. It is possible that the same star may have been counted more than once. You need to determine the probability that any star may have been counted more than once. Probability can be represented as a rational number $\frac{P}{Q}$. If Q is not divisible by $10^9 + 7$ there is a unique integer $x \mid 0 \leq x < 10^9 + 7$ where $P \equiv Qx \ \% \ (10^9 + 7)$. Calculate value of this integer x.

Input Format:
First line of input consists of a single integer T denoting number of test cases.
Following T lines contain two space separated integers denoting N and K.

Output Format:
Print the answer to each test case in a new line.

Input Constraints:
$1 \leq T \leq 10$
$1 \leq N, K \leq 100000$

Sample Input	Sample Output
1 3 3	300000003

Time Limit: 1
Memory Limit: 256
Source Limit:

N type of stars

Choose k stars

Total no. of ways → $n+k-1C_{n-1}$

# of way for only unique

count → nC_k $(k \leq n)$

# of ways for repeat only

$n+k-1C_{n-1} - nC_k$

$$\text{probability} = \frac{{}^{n+k-1}C_{n-1} - {}^{n}C_{k}}{{}^{n+k-1}C_{n-1}}$$

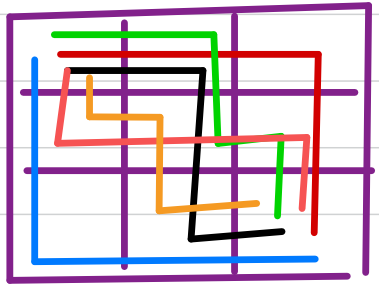
→ formula this

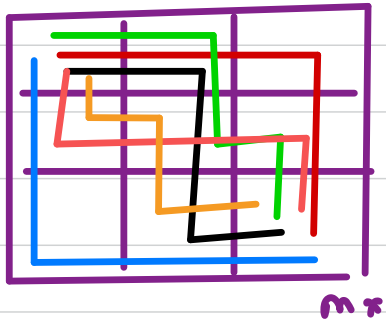
Q: Given a grid of m x n dimensions. You are standing at the topleft of the grid. Find the total no. of ways to reach bottom right if from any cell you can only go rightwards or downwards

$1 \le m, n \le 10^5$

$m = 3$, $n = 3$

ans → 6





RDDR
RRDD
DDRR
DRRD
RDRD
DRDR

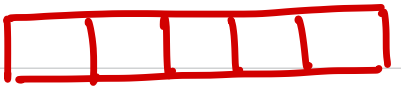
6 ways

AABB
4!
2!2!

all of the 6 ways are permutations

R's → n-1 # D's → m-1 Total → m+n-2

Total permutations → $\frac{(m+n-2)!}{(m-1)!(n-1)!}$ ans



$m+n-2$ spots

$m-1$ D's → identical

$n-1$ R's → identical

$$^{m+n-2}C_{n-1} = {}^{m+n-2}C_{m-1} = \frac{(m+n-2)!}{(m-1)!\,(n-1)!}$$

$m+n-2-(m-1)$

$m+n-2-m+1 \rightarrow n-1$

K-Special Cells

- You are given a $N \times M$ matrix which has K special cells in it. You have to reach (N, M) from $(1, 1)$. From any cell, you can only move **rightwards** or **downwards**.
- The $K - special$ cells are those cells in this grid which have special strength at them. i^{th} special cell has $P[i]$ units of strength and if you travel through this cell, you store the strength.
- Find the total strength you can store after travelling through all the possible paths in the grid to reach cell (N, M).

- **Note:**
  1. The strength of a path is the sum of strength $P[i]$ of all the special cells that are visited in this path.
  2. The cells that are not special have power quotient equals to zero.

Input format

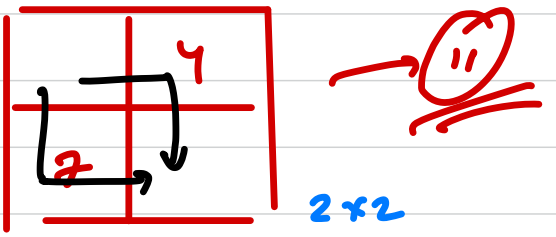
- The first line contains the total number of test cases denoted by **T**.
- The first line of each test case contains three space separated integers **N**, **M** and **K** where **N** x **M** is the size of grid and **K** is the total number of special cells in the grid.
- Each of next **K** lines contains **X[i]**, **Y[i]**, and **P[i]** where (X[i],Y[i]) is the location of special cell and P[i] is the cell strength.

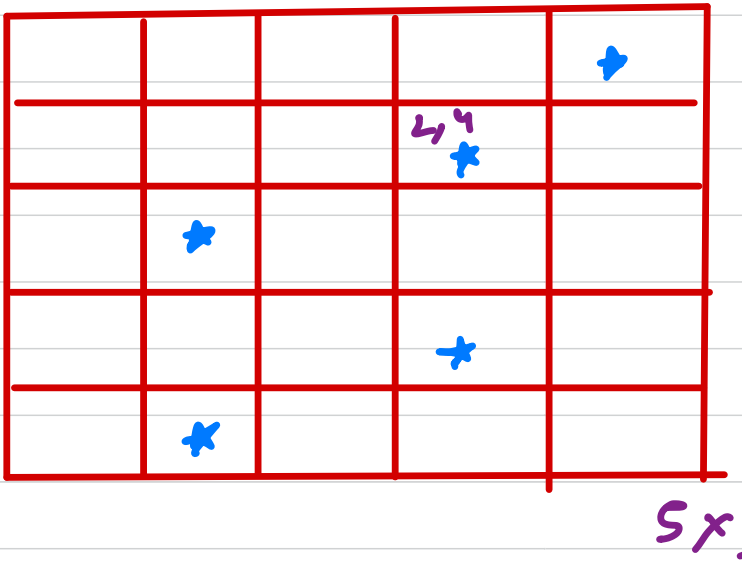
$K \leq 10^5$

$N, M \leq 10^9$ → We can't even make a grid.

Output format

- For each test case, print in a new line a single integer representing the total strength that you can store, as the total strength can be too large, print it modulo $10^9 + 7$.





contri of 1 special cell =

p[i][j] × (# of path it is a part of)

if we can count the contribution of each special cell into our ans, then we can solve it.

How to calc the no of path when i,j is also a part ?.?

Calc the no. of path from 0,0 to i,j, say this is x.

Calc no. of path from i,j to n-1, m-1, say this is y.

Total paths ⇒ x * y

$$x \rightarrow \frac{(i+j-2)!}{(i-1)!(j-1)!}$$

$$y = \frac{(n-i+m-j)!}{(n-i)!(m-j)!}$$

Q→ Given m, n that represents a m×n grid which has k blocked cells. Find total no. of ways to reach from top left to bottom right.

Total ways − Total ways when some blocked cell is a part.

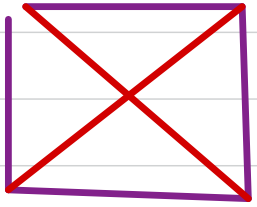
↓

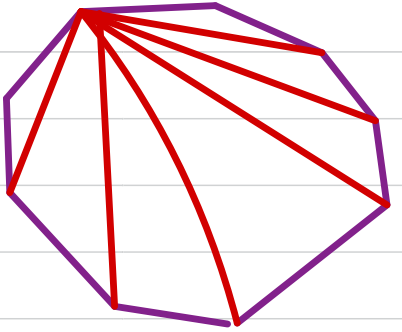
almost similar to prev

Q: Given a value n, which represents a n-sided polygon. find the total no. of diagonals in the polygon.

n = 4

ans → 2





$\frac{n \times (n-3)}{2}$

ans

$$\frac{n \times n-3}{2}$$ to avoid double count

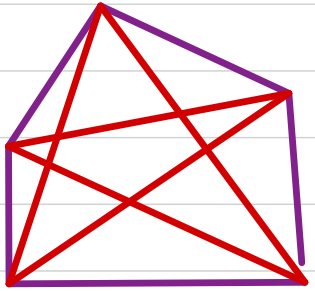
# of ways to choose first vertex of the diagonal

# of ways to choose 2nd vertex of the diagonal

why $n-3$??

if we have chosen any i^{th} vertex for first vertex of diagonal, the 2 adjacent vertices, can't be the 2nd vertex

2nd way



You've n vertices, for a diagonal you need any 2

# of ways to choose 2 vertices from a set of n is ${}^nC_2$

in ${}^nC_2$ we have also chosen the 2 vertices who will form sides. So

final ans $\rightarrow$ ${}^nC_2 - n$ $\hookrightarrow$ # of sides

$$\frac{n!}{2!(n-2)!} \quad - n$$

$$\frac{n(n-1)}{2} \quad - n$$

$$n\left(\frac{n-1}{2} - 1\right)$$

$$n\left(\frac{n-1-2}{2}\right)$$

$$\frac{n(n-3)}{2}$$

Binomial Coefficients.

$$(x+y)^n = {}^nC_0 x^0 y^n + {}^nC_1 x y^{n-1} \cdots\cdots {}^nC_n x^n y^0$$

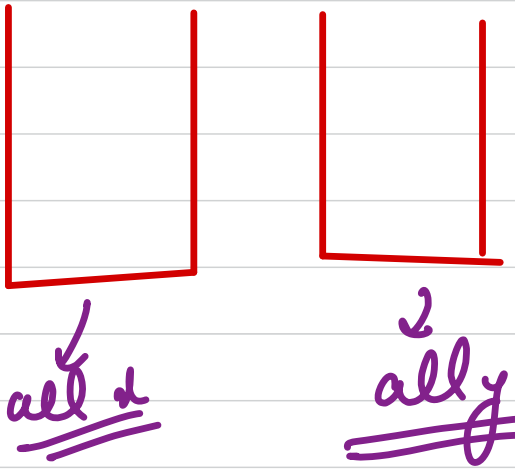
$$(x+y)^n = \sum_{r=0}^{n} {}^nC_r x^r y^{n-r}$$

Example

$$(x+y)^3 = {}^3C_0 x^0 y^3 + {}^3C_1 x y^2 + {}^3C_2 x^2 y + {}^3C_3 x^3 y^0$$

$$= y^3 + 3xy^2 + 3x^2 y + x^3$$

$$(x+y)^n = {}^nC_0 x^0 y^n + {}^nC_1 x y^{n-1} \cdots {}^nC_n x^n y^0$$



Total n objects chosen

Let's say pick r x's

$$\sum {}^nC_r x^r y^{n-r}$$

What is $^nC_0 + {}^nC_1 + {}^nC_2 \cdots\cdots {}^nC_n$??

$\downarrow$

2^n

$(x+y)^n = {}^nC_0 x^0 y^n + {}^nC_1 x y^{n-1} \cdots\cdots {}^nC_n x^n y^0$

Put $x=1, y=1$

$(1+1)^n = {}^nC_0 1^0 1^n + {}^nC_1 1^1 1^{n-1} \cdots\cdots {}^nC_n 1^n 1^0$

$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 \cdots\cdots {}^nC_n$

HP