Q:) There are 2 players, and n coins ($n \to$ even). They play a game where they have alternate turns. In each turn a player can pick a coin from the start or the end of the n coins. Determine the max profit for the player playing the first turn. Consider both the players plays optimally.

P_2 P_1 P_2 P_1

8, 15, 3, 7

$P_1 \longrightarrow 22$ max

C_1	C_2	C_3	.	. -	-	- -	C_n

n coins

$P_1 \rightarrow$ plays first

$P_2 \rightarrow$ plays second

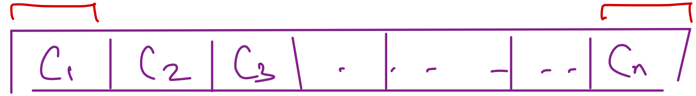
$\rightarrow$ optimal play

$P_1 \rightarrow 8+7 \rightarrow 15 \rightarrow 28$

$P_2 \rightarrow 15+3 \rightarrow 18$

8, 15, 3, 7

wrong approach of picking greater value first.



P_1 → plays first

P_2 → plays second

→ optimal play

P_1 → Sanket

P_2 → Ankit

→ Explore all possible paths

15, 22

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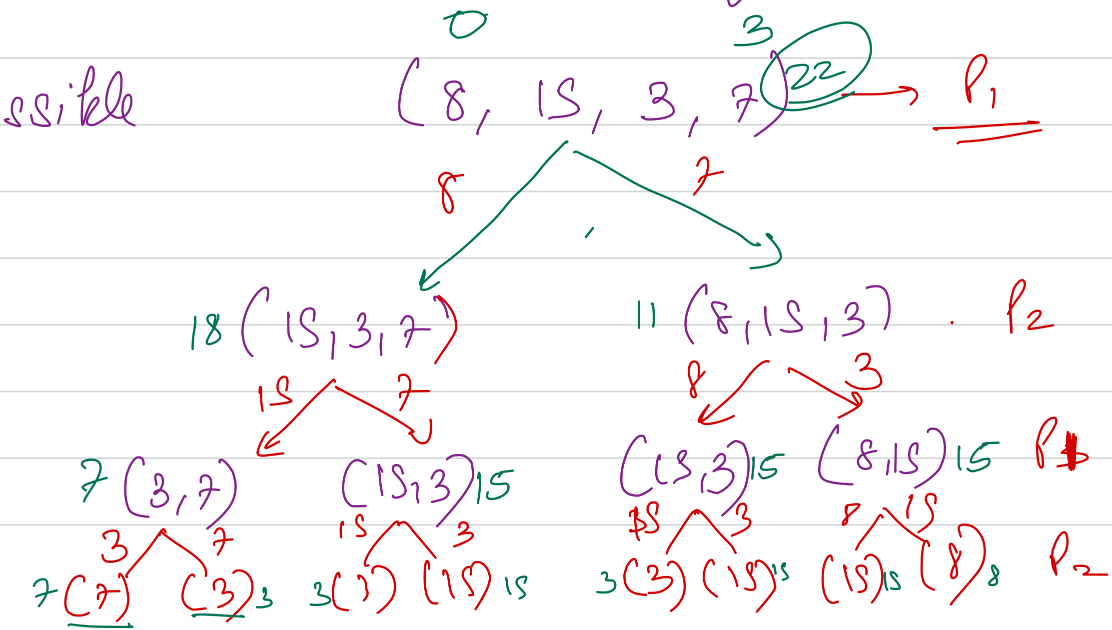

$$\begin{array}{|c|c|c|c|c|c|c|}\hline C_1 & C_2 & C_3 & \cdot & \cdots & \cdots & C_n \\ \hline \end{array}$$

0 … $n-1$; P_1

Get ans w.r.t P_1. $P_1 \rightarrow$ goes first

$f(i,j) \rightarrow$ max profit if we choose the first or last element from $[i,j]$

$$f(i,j) = \max \begin{cases} C_i + \min\big(f(i+2,j),\ f(i+1,j-1)\big) & \text{picks } i^{th} \text{ coin} \\ C_j + \min\big(f(i+1,j-1),\ f(i,j-2)\big) & \text{pick } j^{th} \text{ coin} \end{cases}$$

Base Case $\rightarrow$ $i+1 == j$ $\rightarrow$ 2 elements $\rightarrow$ $\max(i,j)$

$i = j \rightarrow C_i$

final ans $\rightarrow f(0, n-1)$

items

capacity of knapsack

6	15
6	5
5	6
6	4
6	6
3	5
7	2

$\rightarrow i1$
$\rightarrow i2$
$\rightarrow i3$
$\rightarrow i4$
$\rightarrow i5$
$\rightarrow i6$

	1	2	3	4	5	6
$w \rightarrow$	6	5	6	6	3	7
$v \rightarrow$	5	6	4	6	5	2

n

we can pick any subset of elements

$5+6+3 \rightarrow 14 \le 15$

$6+6+\cancel{8} \rightarrow \cancel{8}$

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knapsack wt

$f(i, W) = \max \begin{cases} f(i-1, W) & \text{if } wt[i] > W \text{ ↳ we can only exclude} \\ \max\left(v[i] + f(i-1, W - w_t[i]),\ f(i-1, W) \right) & \text{else include + exclude} \end{cases}$

max profit by considering choices on the first i elements with target wt as W.

$w_t \rightarrow$ weight array

$v \rightarrow$ value array

tw == 0

3 8

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	30	30	30	30	30	30
2	0	0	0	30	50	50	50	50	80
3	0	0	0	30	50	60	60	60	90

8 → target knapsack wt

3	30
4	50
5	60

we don't pick any → 0

→ 2

2nd elect

exclud

includ

4 - 4

↓ items

$50 + dp[1][0]$

$dp[i][j]$ → max profit

max profit by considering choices on the first i elements with target wt as j

$dp[n][W]$

$O(W)$ space

$O(nW)$ time

$$f(i, V) = \min \begin{cases} f(i-1, V) & \text{if } V[i] > V \\ \min\left(f(i-1, V),\ f(i-1, V - v[i]) + wt[i]\right) & \text{else} \end{cases}$$

Till the i^{th} element, to get a profit V, what is the min wt, we can pick.

max → V for which $f(n, V) \leq W$ → Target wt

ans

$n \times V \to 100 \times 10^3$

$\to 10^5$

Coin change

Q ⇒ Given a knapsack of wt W, you have n items, with value v_i & weight w_i, we need to find the max amount of profit we can make, with one condition that here we can pick multiple instances of one item.

W = 100
v → [1, 30]
w → [1, 50]
ans → 100 → pick 100 instances of wt 1

W = 8
find ans → 110

v → [10, 40, 50, 70]
w → [1, 3, 4, 5]
1 instance of 5 wt & 1 of 3 wt
70 + 40

→ permutation

→ combination

→ min coin

Qn

TL

-2	-3	3
-5	-10	1
10	30	-5

7

1

BR

right

down

Minimum initial health/energy you need to have from (0,0) to reach bottom right. if your energy drops to 0, you loose.

Backward → grid[i][j] >= 0 ? 1 : 1 - grid[i][j]

min hell

-2	-3	3
-5	11 -10	1 5
10	1 30	6 -5

$11 + 5 \rightarrow 16$

$5 - (-10)$

15

$1 - (-10)$

11

$dp[i+1][j] \rightarrow x$

min hel → $\max(1, x - grid[i][j])$

$dp[i][j]$ → min health$(dp[i+1][j])$, min hell$(dp[i][j+1])$ } min

min health from cell $(i, j) \rightarrow (n-1, m-1)$