

## Exchange Arguments

The basic idea is to see your greedy sol<sup>n</sup>  
w.r.t any other solution

Example → we have 2 positive integers 'a' & 'b'  
Such that  $a + b = n$ . for a given value  $n$ ,  
maximize  $a \times b$ .

In order to maximize  $a$  and  $b$ , we would like to have  $a$  and  $b$  as close as possible

$\hookrightarrow n \rightarrow \underline{\text{even}}$  //

$$\underline{a=x, b=x} \quad a = \frac{n}{2} \quad b = \frac{n}{2} \quad \longrightarrow \quad \underline{\text{and}} \quad \left( \frac{n^2}{4} \right) //$$

$\hookrightarrow \underline{a > b}$

$$a = x + \Delta$$

$$b = x - \Delta$$

$$\underline{\underline{x = \frac{n}{2}}}$$

$$\boxed{\Delta \geq 1}$$

$$a \times b = (x + \Delta)(x - \Delta)$$
$$= x^2 - \Delta^2$$

and  $\Delta$  and  $x$  both are positive

$$x^2 - \Delta^2 < x^2$$

Coin Change  $\rightarrow$  Given some denominations of coins,  
and a value  $x$ , find the min no. of coins  
we need to give a change of  $\underline{x}$ .

$x=10$

$[9, 5, 1]$

$\hookrightarrow$  2 coins

(5) (5)

(9) (1)

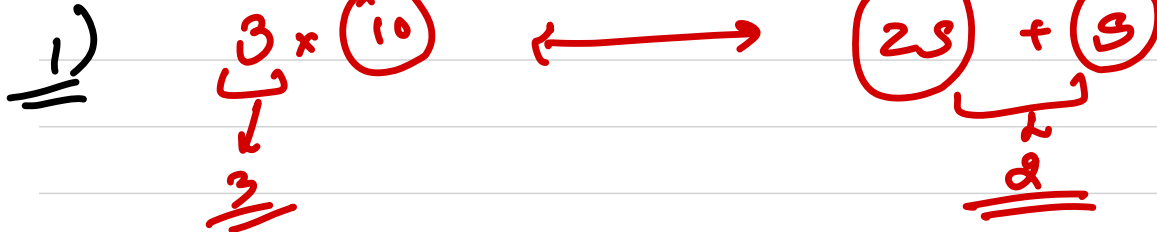
But for some denomination greedy does

work.

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$[25, 10, 5, 1]$  //

#observation

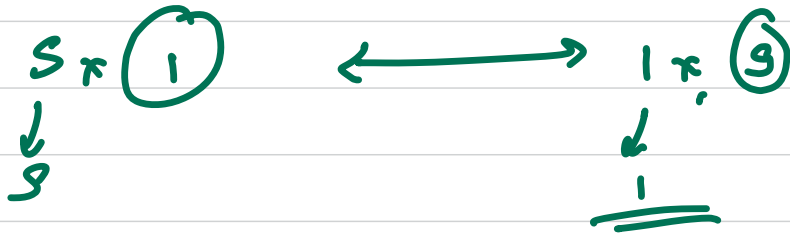


We almost require 2 coin of 10

2) We at most require 1 coin of 5



3) We at most require 4 coins of 1



4) We won't be having 2, 10's and 1, 5 coins  
because we can replace it by (25)

$$2 \times (10) + 1 \times (5) \longrightarrow 1 \times (25)$$

5) Without a (25) coin we can have at max  
24 value.

$$\begin{aligned} 2 \times (10) + 0 \times (5) + 4 \times (1) &\rightarrow (24) \\ 1 \times (10) + 1 \times (5) + 4 \times (1) &\rightarrow \underline{\underline{19}} \end{aligned} \quad \left. \vphantom{\begin{aligned} 2 \times (10) + 0 \times (5) + 4 \times (1) \\ 1 \times (10) + 1 \times (5) + 4 \times (1) \end{aligned}} \right\}$$

Let's say we have  $O$  as an optimal solution  
and  $G$  is a greedy solution

→ Say →  $K = \#$  of  $(25)$  coins in  $O$   
 $K' = \#$  of  $(25)$  coins in  $G$

$$\begin{array}{l} K < K' \\ K == K' \end{array}$$

$$\underline{\underline{K \leq K'}}$$

$\downarrow$   
 $K = K'$

$$\underline{\underline{K_n < K_o}}$$



Say  $d_0$  = # of (10) coins in  $O$

$d_n$  = # of (10) coins in  $G_n$

$$d_0 \leq d_n$$

$$d_0 = d_n$$

Say  $n_0$  = # of (5) coins in  $O$

$n_n$  = # of (5) coins in  $G_n$

$$n_0 \leq n_n$$

$$\underline{n_0 = n_n}$$

$$\underline{Q} = \underline{G_r} \quad (\text{für genau denselben})$$

En

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(4)

[1, 5, 2]

(7)

(1)

(1)

(1)

(1)



(5)

(5)

(1)



→ Sort the data in a given manner

→  $(\text{Score}, \text{decay}, \text{time}) \leftarrow \underline{\text{Problem}}$

↳ In what order we should solve the problems-

$$\begin{array}{c|c} P_1 & P_2 \\ \hline \end{array}$$

$$\begin{array}{c} \underline{P_1} < P_2 \\ \downarrow \\ P_1 \rightarrow \underline{\underline{s_{over 1}}} \end{array}$$

$$\begin{array}{c} P_2 < P_1 \\ \downarrow \\ P_2 \rightarrow \underline{\underline{s_{over 2}}} \end{array}$$

$$\underline{R_1} \rightarrow S_1 - d_1 t_1 + S_2 - d_2 (t_1 + t_2)$$

$$R_2 \rightarrow S_2 - d_2 t_2 + S_1 - d_1 (t_1 + t_2)$$

Maximize the revenue

$$\underline{\underline{R_1 \geq R_2}}$$

$$R_1 - R_2 \geq 0$$

$$s_1 - d_1 t_1 + s_2 - d_2 (t_1 + t_2) -$$

$$(s_2 - d_2 t_2 + s_1 - d_1 (t_1 + t_2)) \geq 0$$

$$\cancel{s_1} - \cancel{d_1 t_1} + \cancel{s_2} - d_2 t_1 - \cancel{d_2 t_2} -$$

$$\cancel{s_2} + \cancel{d_2 t_2} - \cancel{s_1} + \cancel{d_1 t_1} + d_1 t_2 \geq 0$$

$$d_1 t_2 - d_2 t_1 \geq 0$$

$$d_1 + t_2 \geq d_2 + t_1$$

$p_1$

$p_2$

first

$\downarrow R_1$

$$p_1 > p_2$$

optimal order of the problem

$p_1 \quad p_2 \quad p_n \quad \dots$

knapsack

