Q: Distinct Subsequences

"abab"

a
b
ab
aba
aab
abab
bab
ba
bb
abb

abc → 7

a b a b

1 2 3 4

$f(i) \Rightarrow 1$ $i = 0$

$f(i) = 2$ $i = 1$

$f(i) = 4$ $i = 2$

$f(i) = 4 * 2 - 1$, $i = 3$

$\rightarrow 7$

$f(i) = 7 * 2 - 2$ $i = 4$

$\hookrightarrow 12$

$f(i)$
$\downarrow$
count of distinct subsequences of string $s[0, i]$

a' b a''

[a' a'b a'ba''
b a'a'' ....
ba''] b'

x b'

[~~a' b'~~ a'bb' a'ba''b'
b b' a'a''b' ~~b'~~
ba''b']

1D DP

$f(i) = 2 * f(i-1) - f(\text{last}[s[i]] - 1)$

func^n that return no. of distinct subsequent of $s[0, i]$

why i did this ??

$\text{last}[s[i]] \rightarrow$ the just last index when we same $s[i]$

10

b b a b

i=0 1 2 3 4

$f(0) = 1$

$f(1) = 2$

$f(2) = 2*2-1 \Rightarrow 3$

$f(3) = 6$

$f(4) = 6*2 - 2$

$\Rightarrow 10$

"" b bb

a ✓ → a, ba, bba

ϕ → "", b, bb

→ ab, bab, bbab, b, bb, bbb

a, ba, bba, "", b, bb

"" b bb → "" b bb

$10 - 1 \rightarrow 9$

Q =)

(4x2) (2x3) (3x1) (1x3)

24 (4x3) 9 (3x3)

36 (4x3)

24 + 36 + 9 → 69

4x2 , 2x3 , 3x1 , 1x3

6 (2x1)

8 (4x1)

12 (4x3)

6 + 8 + 12

→ 26

$(4\times 2)\quad(2\times 3)\quad(3\times 1)\qquad(1\times 3)$

$[\underbrace{4,\ \underbrace{2,\ \underbrace{3,\ \underbrace{1,\ 3}_{1\times 3}}_{3\times 1}}_{2\times 3}}_{4\times 2}]$

min operatn reqd to multiply these matrices

4 2 3 1 3

( ) (4x2, 2x3, 3x1, 1x3) (4x2, 2x3, 3x1, 1x3) ( )

(4x2) (2x3, 3x1, 1x3) (4x2, 2x3, 3x1) (1x3)

$\boxed{(4x2, 2x3)}$ (3x1, 1x3) ( ) (4x2, 2x3, 3x1) (4x2) (2x3, 3x1)

(4x2, 2x3) (3x1)

minimum of all cases → ans (4x2, 2x3, 3x1) ( )

To calc ans for n matrices

n-1, n-2, n-3, - - - -

4, 2, 3, 1, 3

n = 4

(4x2, 2x3, 3x1)(1x3) (4x2)(2x3, 3x1, 1x3) n - 1 → 3

(4x2)(2x3, 3x1)(1x3) (2x3)(3x1, 1x3) n - 2

$i=0$ $\quad$ $j=4$

4 $\quad$ 2 $\quad$ 3 $\quad$ 1 $\quad$ 3

$(4\times2,\ 2\times3,\ 3\times1)\ (1\times3)$

$(4\times2)\ (2\times3,\ 3\times1,\ 1\times3)$

$4\times3 \qquad 3\times3$

$(4\times2,\ 2\times3)\ (3\times1,\ 1\times3)$

$\boxed{\begin{gathered}\text{min cost}\,(4\times2,\ 2\times3)\ + \\ \text{min cost}\,(3\times1,\ 1\times3)\ + \\ 4\times3\times3\end{gathered}}$

$i = 1$ $\qquad\qquad$ $j = 4$

$a \longrightarrow 4 \quad 2 \quad 3 \quad 1 \quad 3$

$(4 \times 2) \quad ((2 \times 3,\ 3 \times 1,\ 1 \times 3)) \longrightarrow 4 \times 2 \times 3$

$$f(i, j) = f(i, k) + f(k+1, j) + a[i-1] * a[k] * a[j]$$

$$\forall k \in [i, j-1]$$

min cost to multiply all the matrices from $[i, j]$

$$f(1, 1) + f(1, 4) +$$

$f(1,1) \to j$ $\qquad$ $f(1,4) \to 2$

$k \to 1$

		0	1	2	3	4
		4	2	3	1	3
0	4	0	0	24	14	26
1	2		0	0	6	12
2	3			0	0	9
3	1				0	0
4	3					0

ans

$l = 5$

$0 + 5 - 1$

4

$j = i + l - 1$

$\Rightarrow 2 + \ \ -1$

$\Rightarrow$

$f(i, j)$

$\downarrow$

min cost

to multiply

(i, j)

$(4, 2, 3, 1, 3)$

$\boxed{(4 \times 2)}(2 \times 3, 3 \times 1, 1 \times 3)$

$0 + 12 + 24$

$((4 \times 2)(2 \times 3))((3 \times 1)(1 \times 3))$

$24 + 9 +$

30

$(4 \times 2, 2 \times 3, 3 \times 1)$

(1×3)

$12 + 14 + 0$

0 1 2 3 4

$i=1$ $j=n-1$

4 2 3 1 3

$n=5$

$K \in [i, j-1]$

$n-1$

$f(i,j)$

↓

min cost of matrix chain multiply

$(1,1)\,(2,4)$

0

$(2,2)\,(3,4)$ $(2,3)\,(4,4)$

$f(2,4)$

$(3,3)\,(4,4)$

$K=2$, $K=3$

mcm(

$$f(i,j) = f(i,k) + f(k+1,j)$$
$$+ \underbrace{a[i-1] * a[k] * a[j]}_{\text{Cost of multiplicat}}$$

$f(1, n-1)$

final ans

[4 2 3 1 3]

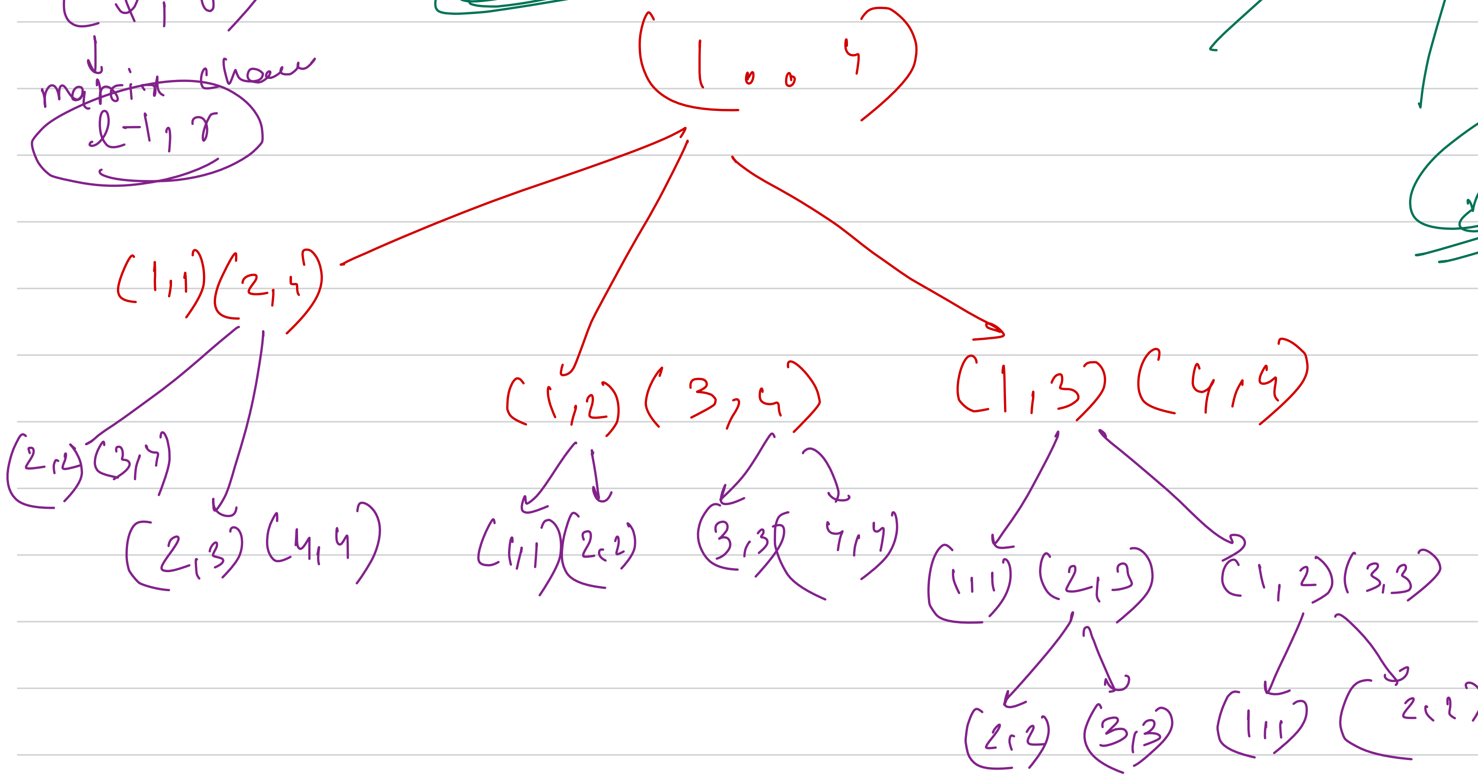
(4×2) (2×3) (3×1) (1×3)

n^3

(l, r)

↓ matrix chain

$l-1, r$



$f(0, n-1)$

A | B | C | D | E

(A) (B, C, D, E)

(A, B) (C D E)

(ABC) (D E)

(ABCD) (E)

$$f(i, j) = f(i, k) + f(k+1, j) + \text{colour}(i, k) * \text{colour}(k+1, j)$$

min smoke

Smoke

40 60 20

(40) (60, 20)

0 + 1200 + 3200

4400

(40, 60) (20)

0 + 2400 + 0

2400

$i = 0$

$j = n-1$

→ when we want ans from contiguous cross section

→ when we see cross-section can be <u>partition</u>