Thursday, 28 October 2021 8:15 PM

Game theory

→ Cricket ✗
→ Hockey ✗

→ coin games
→ number games

- → 2 player games (can be more than that also)
- → Perfect information
- → optimal moves
- → win or lose or draw

→ Simple games
→ Nim game
→ Minimax
→ grundy numbers
→ sprague grundy theorem

* A lot of questions.

Q Two players Alice & Bob are playing a game. They have a pile of n coins in it. They can pick either 1 or 2 coins in one turn.
Alice will go first and they take alternate turns.

→ The player who picks the last coin is winner.

Find the winner

Eg: n = 10 $\boxed{\text{Alice}}$
Eg: $\boxed{n = 4}$ → $\boxed{\text{Alice.}}$

last coin → winner

n = 1	[first person]	→ Alice	W
n = 2	[first person]	→ Alice	W
n = 3	[second person]	→ Bob wins	L
n = 4	[first person]	→ Alice	W
n = 5	[first person]	→ Alice	W
n = 6	[second person]	→ Bob	L

```
If (n % 3 == 0)
    print ("Bob");
else
    print ("Alice");
```

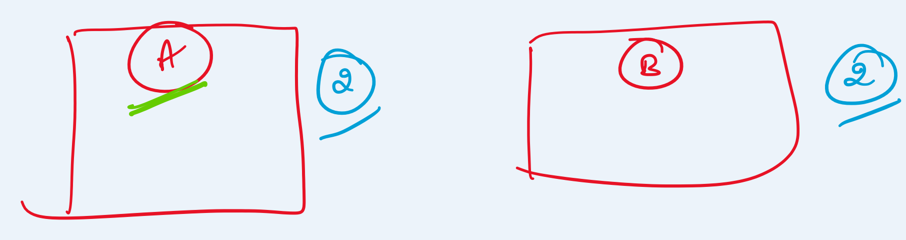
Types

① Cooperative games (ultimate cooperation)
→ possibility of cooperation / coalition
→

② Non-cooperative games (ultimate competition)
→ no cooperation b/w players
→ everyone is playing rationally with their strategies to accomplish their own desires & goals.

Prisoner's Dilemma

A B



① A ✓ (free) B ✗ (10)
② B ✓ (free) A ✗ (10)
③ A ✓ (5) B ✓ (5)
④ A ✗✗ (2) B ✗✗ (2)

Possible results

A/B	Silence	Confess
Silence	A 2 | B 2	10 | 0
Confess	A 0 | B 10	A 5 | B 5 ✓

Nash equilibrium

Nash equilibrium

→ Nash equilibrium is the state where an individual is always better off or equal to its opponent rather than facing a loss and benefitting the other.

→ optimal outcome from this situation where the players are rational and only make a decision after considering the adv & disadv of every party involved.

Example: Battle of Sexes

Couple → date
husband → football
wife → movie

Wife/Hus	Movie	Football
Movie	(2, 1)	(0,0)
Football	(0,0)	(1,2)

Nim game

n piles
n_1 0 0 0 0 0 0 6
n_2 0 0 0 0 0 0 0 0 0 9
⋮
n_n 0 0 0 3

Keep Thinking

→ chooses one pile and takes atleast one coin from it.

→ last coin whoever removes is the winner.

It is a losing position for the player whose turn it is if & only if
$n_1 \wedge n_2 \ldots n_k = 0$

1 → 6 1 1 0
2 → 9 1 0 0 1
3 → 3 1 1
4
5
①

$a \wedge b$ →
$a \wedge a = 0$

1
1 0
1 1

$7 \quad \frac{19}{4} = 3$

1 1 1
1 0 0
0 0 1
0 1 0 = ②