Q:-

N integers, q queries (L, R)

$\gcd(F(a_L), F(a_{L+1}), \dots\dots\dots F(a_R)) \% 10^9+7$

$F(x) \rightarrow x^{th}$ fibonacci

$\rightarrow$ m, n & $m \mid n$ $\rightarrow$ $F_m \mid F_n$

Proove that any two consecutive fib are co-prime. $(\gcd(a,b) == 1)$

$\hookrightarrow \gcd(f_n, f_{n+1}) = 1$

$\gcd(f_1, f_2) \rightarrow 1$

assume $\gcd(f_n, f_{n+1}) = 1$

proove that $\gcd(f_{n+1}, f_{n+2}) = 1$

$$\gcd(f_{n+1}, f_{n+2}) = \gcd(f_{n+1}, f_{n+1}+f_n)$$

$$\gcd(a, a+b) \rightarrow \underline{\underline{\gcd(a, b)}}$$

$$\gcd(f_{n+1}, f_{n+1}+f_n) = \gcd(f_{n+1}, f_n) = 1$$

<u>Hence proved</u>

1. $\gcd(f_n, f_{n+1}) = 1$

2. $f_{m+n} = f_{m+1} f_n + f_m f_{n-1}$

3. $m \mid n \longrightarrow f_m \mid f_n$

Let's proove, $\gcd(f_m, f_n) = f_{\gcd(m,n)}$

$n = qm + r$ $\quad r \to$ remainder

$$\gcd(f_m, f_n) = \gcd(f_m, f_{qm+r})$$

$m \mid qm$

$f_m \mid f_{qm}$

$$= \gcd(f_m,\ f_{qm+1} f_r + f_{qm} f_{r-1})$$

$$= \gcd(f_m,\ f_{qm+1} f_r)$$

$m \mid qm \rightarrow f_m \mid f_{qm}$ and $\gcd(f_{qm}, f_{qm+1}) = 1$

$\Rightarrow \gcd(f_m, f_r)$

r = remainder

$$\gcd(f_n, f_m) = \gcd(f_m, f_{n \% m})$$

$\rightarrow \gcd(a,b) = \gcd(b, a \% b)$

$$\gcd(100, 80) \to \gcd(80, 20) \to \gcd(20, 0) \to 20$$

$$\gcd(f_{100}, f_{80}) \to \gcd(f_{80}, f_{20}) \to \gcd(f_{20}, f_0)$$

$$f_0 = 0$$

$$f_{20}$$

$$\gcd(f_m, f_n) = f_{\gcd(m,n)}$$

$\rightarrow (a_1, a_2 \text{ --- --- } a_n)$

$\gcd(f_{a_l}, f_{a_{l+1}} \text{ - - - - } f_{a_r}) \% 10^9+7$

$\downarrow$

$\gcd(a_l, a_{l+1} \text{ - - - - - } a_r) \% 10^9+7$

y

Segment tree

$y \approx 10^9$

ans $\rightarrow f_y$

f_y

DP

10^9

2018 $\rightarrow$ online Intern test

Microsoft

→ **Matrix Exponentiation** ⇒ logarithmic factor

Linear recurrence is a func^n in which each term of the seq, is a linear combination of prev term.

Ex $f_n = f_{n-1} + f_{n-2}$

general eq^n → $f_K = a f_{K-1} + b f_{K-2} + c f_{K-3} - - - - - -$

Step 1 → Define the no. of prev terms you are dependent on. Let's call this $\underline{K}$.

$$f_n = f_{n-1} + f_{n-2} \qquad \boxed{K = 2}$$

$$f_n = 2f_{n-1} + \underline{f_{n-2} + 0 f_{n-3} + 3f_{n-4}}$$

$K = 4$

Step 2

Find the first k terms - & store them in a column matrix

$F_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{(k,1)}$ ← fib

Step 3 Define transformation matrix

$\begin{bmatrix} & \\ & \end{bmatrix}_{k \times k} \begin{bmatrix} - \end{bmatrix}_{k \times 1}$

transformation matrix → fib

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} f_{i-1} \\ f_i \end{bmatrix} = \begin{bmatrix} f_i \\ f_{i+1} \end{bmatrix}$$

f_i $\quad f_{i+1}$

$$f_n = T \times f_{n-1} = T(T f_{n-2}) = T^2 f_{n-2}$$

$$= T^3 f_{n-3}$$

$$\vdots$$

$$\boxed{f_n = T^{n-1} f_1} \longrightarrow O(k^3 \log n)$$

$k \times k$ → $O(k^3)$

$$\underbrace{f(i) = f(i-1) + 2f(i-2) + 0f(i-3) + 4f(i-4)}$$

identity

$$\begin{array}{cccc} T & F_i & & F_{i+1} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 4 & 0 & 2 & 1 \end{bmatrix}_{4\times4} & \begin{bmatrix} f(i-4) \\ f(i-3) \\ f(i-2) \\ f(i-1) \end{bmatrix}_{4\times1} & = & \begin{bmatrix} f(i-3) \\ f_{i-2} \\ f_{i-1} \\ f_i \end{bmatrix}_{4\times1} \end{array}$$

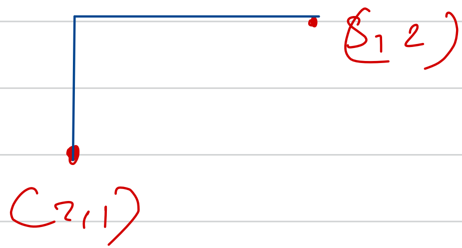
$$f(i) = f(i-1) + 2f_{i-2} + 0f_{i-3} + 4f_{i-4} + d$$

constant

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 4 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{S \times S} \underbrace{\begin{bmatrix} f_{i-4} \\ f_{i-3} \\ f_{i-2} \\ f_{i-1} \\ d \end{bmatrix}}_{S \times 1} = \underbrace{\begin{bmatrix} f_{i-3} \\ f_{i-2} \\ f_{i-1} \\ f_{i} \\ d \end{bmatrix}}_{S \times 1}$$

→ Euclidean dist → $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

→ Manhattan dist → $|x_1-x_2|+|y_1-y_2|$



Eqn of line → $y = mx + c$ $\qquad \frac{x}{a}+\frac{y}{b}=1$

→ $(y-y_1) = m(x-x_1)$

Rotation of origin

$180^\circ \rightarrow (x, y) \longrightarrow (-x, -y)$

clockwise 90° $(x, y) \longrightarrow (y, -x)$

anticlockwise 90° $(x, y) \rightarrow (-y, x)$

Given a line and a point, find the $\perp^r$ distance of point from line.

(x_0, y_0)

$$d = \frac{|a x_0 + b y_0 + d|}{\sqrt{a^2 + b^2}}$$