

Q» Calculate sum of xor of all possible pairs of an array.

Ex

[7, 3, 5]

→ 12

0	0	→ 0
0	1	→ 1
1	0	→ 1
1	1	→ 0

$7 \wedge 3$
4

$3 \wedge 5$
6

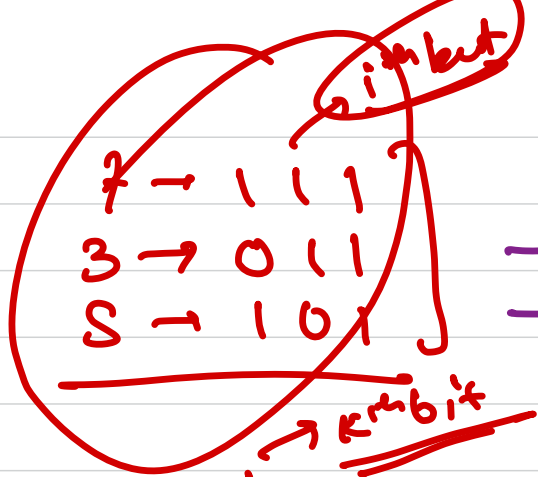
$7 \wedge 5$
2

→ 12

$7 \rightarrow 111$
 $3 \rightarrow 011$
 $5 \rightarrow 101$

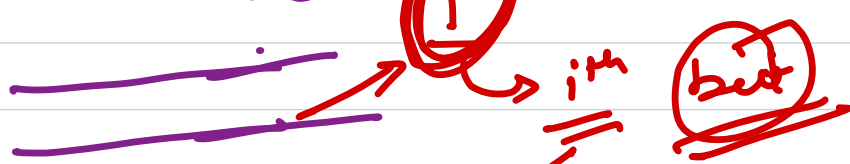
\downarrow pair → \swarrow
 \searrow 10

A B
 1 0
 0 1
 → 1 → 1



p_1
 p_2
 p_3

$\frac{1}{1}$
 $\frac{1}{0}$



Count of 1 x Count of 0 at ith bit

$p_1 + p_2 + p_3 \Rightarrow \underline{2} \quad \underline{km}$

How many pairs will have ith bit 1.

$$4 + 6 + 2 = 12$$

$$\begin{aligned} & \left(\underline{1 \times 2^2} + \underline{0 \times 2^1} + \underline{0 \times 2^0} \right) + \left(\underline{1 \times 2^2} + \underline{1 \times 2^1} + \underline{0 \times 2^0} \right) + \\ & \left(\underline{0 \times 2^2} + \underline{1 \times 2^1} + \underline{0 \times 2^0} \right) = \underline{\underline{12}} \end{aligned}$$

$$(0+1+1)2^2 + (0+1+1)2^1 + (0+0+0)2^0$$

$$2 \times 2^2 + 2 \times 2^1 + 0$$

$$= 8 + 4 = \underline{\underline{12}}$$

count of pairs with i^{th} bit
after xor is 1 is m.

$$\text{ans} += \underline{\underline{m \times 2^i}}$$

Recurrence Relations

Method of guessing

$$T(n) = \sqrt{n} T(\sqrt{n}) + n$$

Assume $T(n) = \Theta(n \log n)$ //

$$T(n) \leq C n \log n$$

$$T(n) = \sqrt{n} T(\sqrt{n}) + n$$

$$T(n) \leq (\sqrt{n} \times C \sqrt{n} \log \sqrt{n} + n)$$

$$T(n) \leq \frac{1}{2} C n \log n + n$$

$$\tau(n) \leq cn \log n$$

$$\tau(n) = \sqrt{n} \tau(\sqrt{n}) + n$$

$$\tau(n) \geq \sqrt{n} k \sqrt{n} \log \sqrt{n} + n$$

$$\tau(n) \geq \frac{nk \log n}{2} + n$$

No

Assume $\rightarrow T(n) = \underline{\underline{O(n)}}$

$$\underline{\underline{T(n) \leq cn}}$$

$$T(n) = \sqrt{n} T(\sqrt{n}) + n$$

~~X X X~~

$$T(n) \leq \sqrt{n} \cdot c\sqrt{n} + n$$

$$T(n) \leq cn + n$$

$$T(n) \leq (c+1)n$$

$$n\sqrt{\log n}$$

$$T(n) \leq cn\sqrt{\log n}$$

$$T(n) = \sqrt{n} T(\sqrt{n}) + n$$

$$T(n) \leq \sqrt{n} \times c\sqrt{n}\sqrt{\log \sqrt{n}} + n$$

$$T(n) \leq nc \frac{1}{\sqrt{2}} \sqrt{\log n} + n \quad \underline{\underline{HP}}$$

$$\begin{aligned} T(n) &\geq \sqrt{n} k \sqrt{n} \sqrt{\log \sqrt{n}} + n \\ &\geq kn \frac{1}{\sqrt{2}} \sqrt{\log n} + n \end{aligned}$$

$$\underline{\underline{T(n) \geq kn\sqrt{\log n}}}$$

$$\tau(n) = \sqrt{n} \tau(\sqrt{n}) + n$$

$$\tau(n) \leq C n \log \log n$$

$$\tau(n) \geq \kappa n \log \log n$$

case $\rightarrow \underline{n \log \log n}$

$\rightarrow \mathcal{O}(n \log \log n)$

$$\tau(n) \leq \sqrt{n} C \sqrt{n} \log \log \sqrt{n} + n$$

$$\leq C n \log \log \sqrt{n} + n$$

$$\leq C n \log \left(\frac{\log n}{2} \right) + n$$

$$\leq C n \log \log n - C n \log 2 + n$$

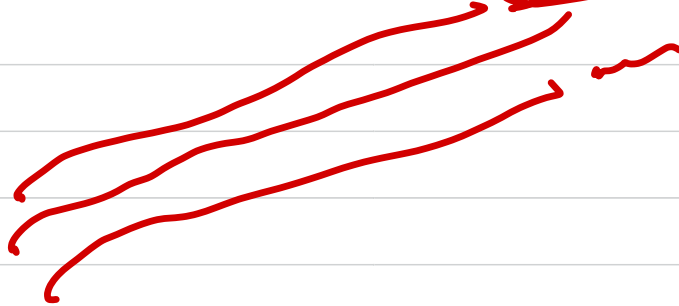
$C \geq 1$

$$T(n) \leq cn \log \log n + n(1 - \underline{\underline{c \log 2}})$$

$$T(n) \leq \underline{\underline{cn \log \log n}} \quad \checkmark \checkmark$$

yes

$\epsilon > 1$



$$\begin{aligned}\tau(n) &\geq \sqrt{n} \ll \sqrt{n} \log \log \sqrt{n} + n \\ &\geq \kappa n \log \left(\frac{\log n}{2} \right) + n\end{aligned}$$

$$\geq \kappa n \log \log n - \kappa n \log 2 + n$$

$$\tau(n) \geq \kappa n \log \log n + n(1 - \kappa \log 2)$$

$$\tau(n) \geq \kappa n \log \log n$$

$\kappa \geq 1$

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

$n \log n$

Q-1 $T(n) = \sum_{i=1}^n \log i$

$$T(n) = \log 1 + \log 2 + \log 3 + \dots + \log n$$

$$T(n) = \log n!$$

$$\log n! \leq \log n^n$$

$$T(n) \leq \log n^n$$

$$T(n) \leq n \log n$$

$$\underline{O(n \log n)}$$

Q₂ $\tau(n) = \begin{cases} 1 & n=1 \\ \tau(n-1) + n(n-1) & n \geq 1 \end{cases}$

$$\begin{aligned}
 \tau(n) &= \tau(n-1) + n(n-1) \\
 &= \tau(n-2) + (n-2)(n-1) + n(n-1) \\
 &= \tau(n-3) + (n-3)(n-2) + (n-2)(n-1) + n(n-1)
 \end{aligned}$$

$$\tau(n) = \tau(1) + \sum_{i=1}^n i(i-1)$$

$$\tau(n) = 1 + \sum_{i=1}^n i^2 - i$$

$$\tau(n) = 1 + \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$\tau(n) \rightarrow O(n^3)$

$$= \frac{6 + (n^2+n)(2n+1) - 3n^2 - 3n}{6}$$

$$\Rightarrow \frac{6 + 2n^3 + 2n^2 + n^2 + n - 3n^2 - 3n}{6}$$

$$\underline{\underline{\tau(n)}} \Rightarrow \frac{2n^3 - 2n + 6}{6} \leq \underline{\underline{Kn^3}}$$

$$T(n) = T(0.8n) + O(n)$$

$$T(n) = T\left(\frac{4}{5}n\right) + O(n)$$

$$a = 1$$

$$b = \frac{5}{4}$$

$$K = 1$$

$$\underbrace{a < b^K}_{\text{true}}$$



$$\underline{\underline{O(n)}}$$

$$n!$$

$$2^n$$

$$n^4$$

$$n^3$$

$$n^2$$

$$n\sqrt{n}$$

$$n \log n$$

$$n \sqrt[3]{n}$$

$$n \log \log n$$

$$\sqrt[3]{n}$$

$$n \log 1$$

Master theorem of
subtraction

$$T(n) = \begin{cases} C & n \leq 1 \\ aT(n-b) + f(n) & n > 1 \end{cases}$$

$C \rightarrow \text{const}$

$a > 0$

$b > 0$

$f(n) = \underline{\underline{O(n^k)}}$

$k \geq 0$

Solution

$$\Rightarrow T(n) = \begin{cases} O(n^k) & a < 1 \\ O(n^{k+1}) & a = 1 \\ O(n^k a^{n/b}) & a > 1 \end{cases}$$