

Solving some problems, fibonacci, gcd

Binomial coefficients

Inclusion exclusion principle, problem solving

Q. fibosum

→  $[l, r]$  → Sum of fibonacci

from  $[f_l^{\text{th}} - f_r^{\text{th}}]$  ←  
←

range sum  $(l, r)$  → range sum  $(0, r)$  - range sum  $(0, l-1)$

$$S_n = f_0 + f_1 + f_2 + f_3 + \dots + f_n$$

add  $f_1$  on both sides

$$S_n + f_1 = f_0 + f_1 + f_1 + f_2 + f_3 + \dots + f_n$$

$$S_n + f_1 = \underbrace{f_0 + f_1}_{f_1} + f_1 + f_2 + f_3 + \dots + f_n$$

$$= \underbrace{f_1 + f_2}_{f_2} + f_2 + f_3 + \dots + f_n$$

$$\Rightarrow \underbrace{f_2 + f_3}_{f_3} + f_3 + \dots + f_n$$

$$\Rightarrow \underbrace{f_3 + f_4 + f_4}_{f_4} + \dots + f_n$$

$$\Rightarrow f_n + f_{n+1}$$

$$S_n + f_1 \Rightarrow f_{n+2}$$

$$S_n = f_{n+2} - f_1$$

$$(f_1 = 1)$$

$$S_n = f_{n+2} - 1$$

calc in log using  
matrix exponentials

## → Binomial Coefficients

It is a coefficient that denotes no. of ways to choose  $k$  objects from given  $n$  objects -

$$\hookrightarrow {}^nC_k \leftrightarrow \underline{\underline{\binom{n}{k}}}$$

$${}^nC_k = \boxed{\frac{n!}{k! (n-k)!}}$$

$${}^nC_k = {}^nC_{n-k} \rightarrow \frac{n!}{k! (n-k)!} = \frac{n!}{(n-k)! (k)!} \leftarrow$$

$$(a+b)^n \Rightarrow {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 \dots$$

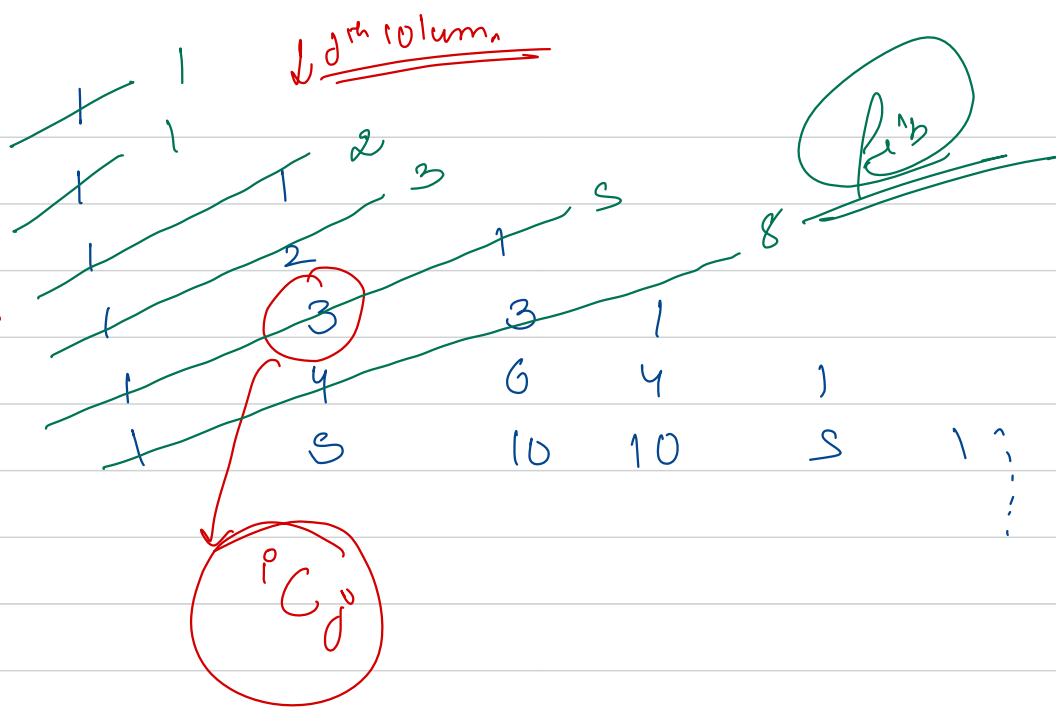
$$\dots \dots {}^nC_k a^{n-k} b^k \dots \dots {}^nC_n b^n$$

Binomial  
expansion

Pascal's Triangle

Pascal's triangle

row  
1



Given  $n$ , print first  $n$  rows of the triangle.

$${}^nC_k = \frac{{}^nC_{k-1} \times (n-k+1)}{1}$$

$${}^nC_2 = \frac{{}^nC_1 \times (4-2+1)}{2}$$

$$= \frac{4 \times 3}{2} = 6$$

↓  $2^k$

1						
1		1				
1		2		1		
1		3		3		1
1		4		6		4
1		5		10		10
						5
						1

$$f(i, j) = f(i-1, j) + f(i-1, j-1)$$

$${}^nC_k = {}^{n-1}C_k + {}^{n-1}C_{k-1}$$

→ can use formula

→ DP



$${}^nC_k = \frac{n!}{k!(n-k)!} \quad \text{--- (1)}$$

$${}^nC_{k-1} = \frac{n!}{(k-1)!(n-k+1)!} \quad \text{--- (2)}$$

$$\text{(1)} / \text{(2)}$$

$$\frac{{}^nC_k}{{}^nC_{k-1}} = \frac{(k-1)!(n-k+1)!}{k!(n-k)!} \leftarrow$$

$$\frac{{}^nC_k}{{}^nC_{k-1}} = \frac{(n-k+1)}{k} = \frac{(k-1)! (n-k+1)!}{k \times (k-1)!} \times \frac{(n-k+1) \times (n-k)!}{(n-k)!}$$

$$\hookrightarrow \boxed{{}^nC_k = \frac{{}^nC_{k-1} \times (n-k+1)}{k}} \quad \rightarrow \boxed{{}^nC_0 = 1} \quad \text{--- number}$$

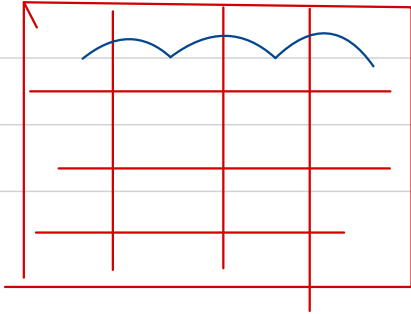
→ Calc  ${}^nC_k$  → TC →  $O(n)$

Q<sub>2</sub> Given  $n$  numbers, find the no. of ways in which we can create some subset for

these no.s.

$$\rightarrow \underline{2^n} = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n$$

→ Q<sub>n</sub> Given a  $n \times m$  grid and a tourist, starting from  $(0,0)$ , who wants to reach  $(n-1, m-1)$  with a constraint that from each cell he can go right/down only. Count the no. of ways for tourist to complete journey from  $(0,0) \rightarrow \underline{(n-1, m-1)}$



$(m-1)$                        $(n-1)$   
 RRR... DDDD...  
 RRRD...

$$n-1 + m-1 \Rightarrow \boxed{n+m-2} \rightarrow \text{total steps}$$

$$\frac{(n+m-2)!}{(m-1)!(n-1)!}$$

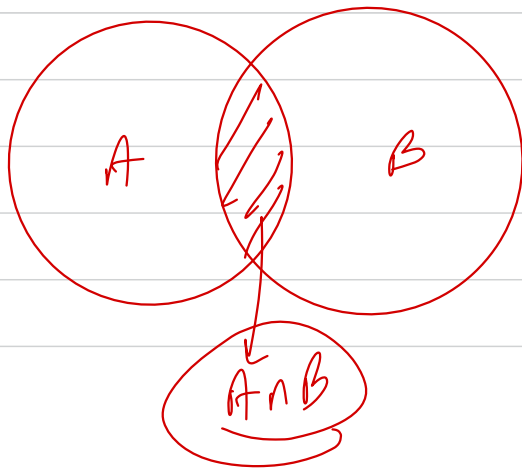
$\rightarrow (n+m-2) (m-1)$

Q<sup>n</sup> Given a no  $n$ , find out the count of no's from  $1 \rightarrow n$  which are divisible by any of the first 20 prime no.s

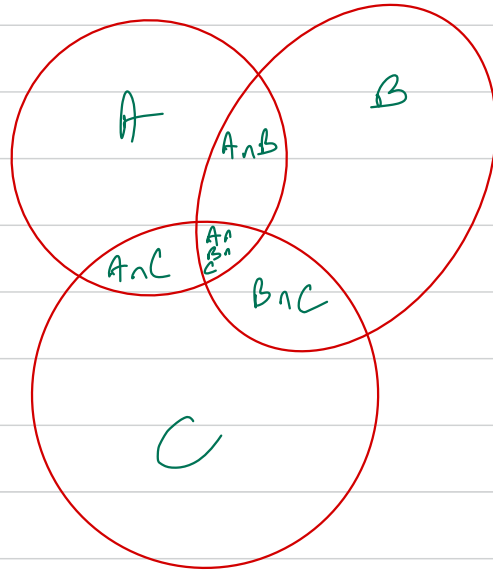
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## Principal of inclusion and exclusion

It is a counting technique which generalises for elements in union of finite sets.



$$\rightarrow |A \cup B| = |A| + |B| - |A \cap B|$$



$$\frac{2^n - 1}{2}$$

$$\begin{aligned} |A \cup B \cup C| &= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| \\ &\quad + |A \cap B \cap C| \end{aligned}$$

Q →

$$\frac{n!}{K^x}$$

$$\underline{\underline{x}}$$

$$n = 5$$

$$K = 2$$

$$\rightarrow \frac{5!}{2^x} \rightarrow \frac{5 \times 4 \times 3 \times 2 \times 1}{2^x}$$

$$\rightarrow \frac{5 \times 3 \times 2^3 \times 1}{2^x}$$

$$\underline{\underline{x = 3}}$$

every no. can be represented  
as product of power of primes



$$K = p_1^{k_1} p_2^{k_2} p_3^{k_3} \dots p_l^{k_l}$$

$$n! = p_1^{c_1} p_2^{c_2} p_3^{c_3} \dots p_l^{c_l}$$

$$\frac{n!}{K^\alpha} = \frac{p_1^{c_1} p_2^{c_2} p_3^{c_3} \dots p_l^{c_l}}{p_1^{k_1 \alpha} p_2^{k_2 \alpha} p_3^{k_3 \alpha} \dots p_l^{k_l \alpha}}$$

$$c_i \geq k_i \alpha$$

$$\alpha \leq \frac{c_i}{k_i}$$

$$\alpha \leq \frac{c_2}{k_2}$$

$$\alpha \leq \frac{c_3}{k_3} \dots \alpha \leq \frac{c_l}{k_l}$$

$$\alpha \rightarrow \min \left( \frac{c_i}{k_i} \right)$$

$$\forall i \in [1, l]$$

$$K = \underline{\underline{16}}$$

$$K \Rightarrow p_1^{k_1} p_2^{k_2} \dots$$

$$\underline{\underline{\sqrt{K}}}$$

$$2 - \sqrt{K}$$

↓ factor

$$K \Rightarrow K f_C^2$$

$$C(1+x)$$

$$\frac{C(1+x)}{L}$$

$$n!$$

$$\rightarrow p_1^{c_1} p_2^{c_2} \dots p_n^{c_n}$$

$$\left[ \frac{n}{p_1} \right] + \left[ \frac{n}{p_1^2} \right] + \left[ \frac{n}{p_1^3} \right] \dots$$

$n = 100 \rightarrow 1 \times 2 \times 3 \times 4 \times 5 \dots 10 \dots 15 \dots 20 \dots 25 \dots 30 \dots 75 \dots 100$

$$\underbrace{f_1 \rightarrow s}_{\text{---}} \rightarrow \underbrace{\left[ \frac{100}{s} \right]}_{\text{---}} + \left[ \frac{100}{s^2} \right] \text{---}$$