$$f(i) = f(i-1) + f(i-2)$$

fib

→ $O(1)$ per state

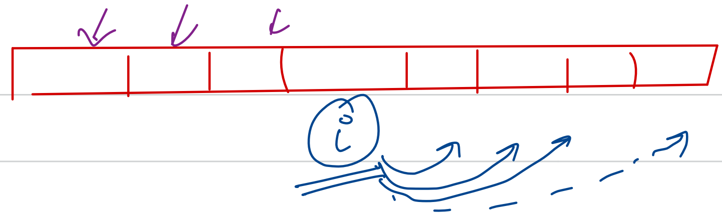
$O(1) \times (n+1)$

$O(n+1)$

$\downarrow$

$O(n)$

P :)



1. From any i^{th} index I can jump to

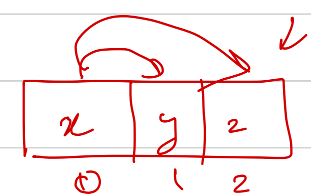
$(i+1)$ $(i+2)$ $(i+3)$ ...... $(i+k)$

2. Min cost to reach from $(1 \to N)$

$k = 2$

3. Similarly to reach at any i^{th} index we can come from

$(i-1)$ $(i-2)$ $(i-3)$ .... $(i-k)$



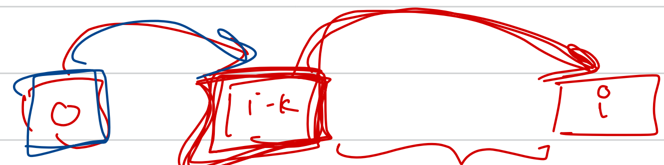
$$f(i) = \min\Big(\underbrace{|h_i - h_{i-1}| + f(i-1)}, \underbrace{|h_i - h_{i-2}| + f(i-2)}, \overbrace{|h_i - h_{i-3}| + f(i-3) \;\dots\; \underbrace{|h_i - h_{i-k}| + f(i-k)}}^{k \text{ terms}}\Big)$$

calc. min cost to reach i^{th} index

min cost to go from $0-i$ via $i-1$

min cost to reach $0-i$ via $i-2$

min cost to reach $0-i$ via $i-k$



skipped

1

1D

n subproblems

final ans → $f(n-1)$

$f(i-k) + |h_i - h_{i-k}|$

$$\underbrace{f(i) = \min\big(|h_i - h_{i-j}| + f(i-j)\big)} \quad \forall j \in (1 \dots k)$$

$f(n)$

$(|h_n - h_{x-1}|)$ $\quad \boxed{f(n-1)} \quad f(n-2) \quad f(n-3) \quad \ldots \quad f(n-k)$

for i^{th} state $\longrightarrow O(k)$

$\longrightarrow O(nk)$

TC $\longrightarrow$ Time reqd to solve one state $\times$ no of unique states

$0 + |h_0 - h_1|$

4, 4, 2, 5, 6 $\quad n = 5$ $\quad k = 3$

dp → | 0 | 0 | 2 | 1 | 2 |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |

final ans

dp(1)

dp[i] → f(i) → min cost to reach i from 0

f(4)

f(3) $\quad$ f(2) $\quad$ f(1)

1 + 1 $\quad$ 4 + 2 $\quad$ 2 + 0

Q. You are given integer n. On each step, you may subtract from it any one digit that appears on it.

Calc. min steps to make number n equal to 0.

$$\underbrace{27 \xrightarrow{-7} 20 \xrightarrow{-2} 18 \xrightarrow{-8} 10 \xrightarrow{-1} 9 \xrightarrow{-9} 0}_{5}$$

5 ans

$n \le 10^6$

greedy

proof

$n = 27 \rightarrow$ min steps to reach 0

We don't know how calc min steps of $27 \rightarrow 0$

$27 \xrightarrow{1} 20 \xrightarrow{x} 0$

$x+1$

$$27 \xrightarrow{-2} 25 \qquad 27 \xrightarrow{-7} 20$$

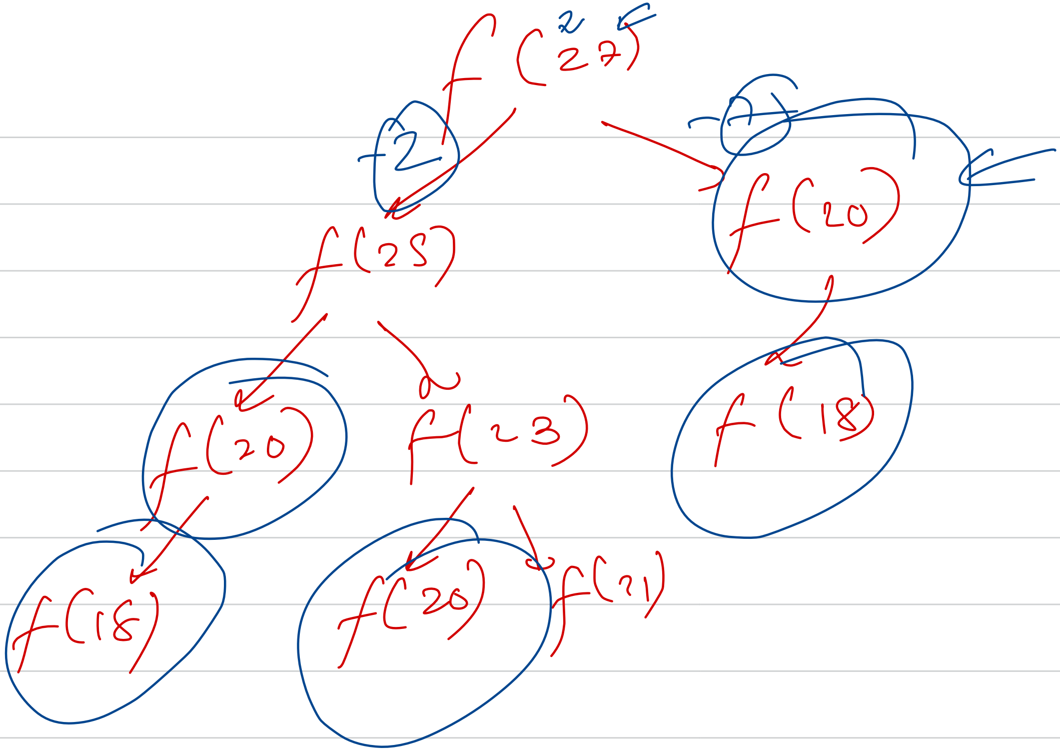
we know this

$$f(27) = 1 + \min\left(f(25),\ f(20)\right)$$

$f(27)$: min steps to reach 0 from 27

$f(25)$: min steps from $25 \rightarrow 0$

$f(20)$: min steps from $20 \rightarrow 0$ = x



f(27)
f(25)
f(20)
f(20)
f(23)
f(18)
f(18)
f(20)
f(21)

$$f(n) = 1 + \min\left(dp[n - j]\right)$$

$\forall j \in$ digits of n

and

$j \neq 0$

$27 \to [2, 7] \to$ digits

$27 \to 25$

$27 \to 20$

$n \in [1, 9]$

$f(n) \to 1$

1 → 0

2

3

... 9

Q. You have a system of coins, of n coins, Each coin has a +ve value. Calc the no. of uniq ways you can produce a money sum x using the coins.

[2, 3, 5] n — infinite supply

permutations

x = 9 → 8

2, 2, 5
2, 5, 2
5, 2, 2
3, 3, 3
2, 2, 2, 3
2, 2, 3, 2
2, 3, 2, 2
3, 2, 2, 2

→ 8

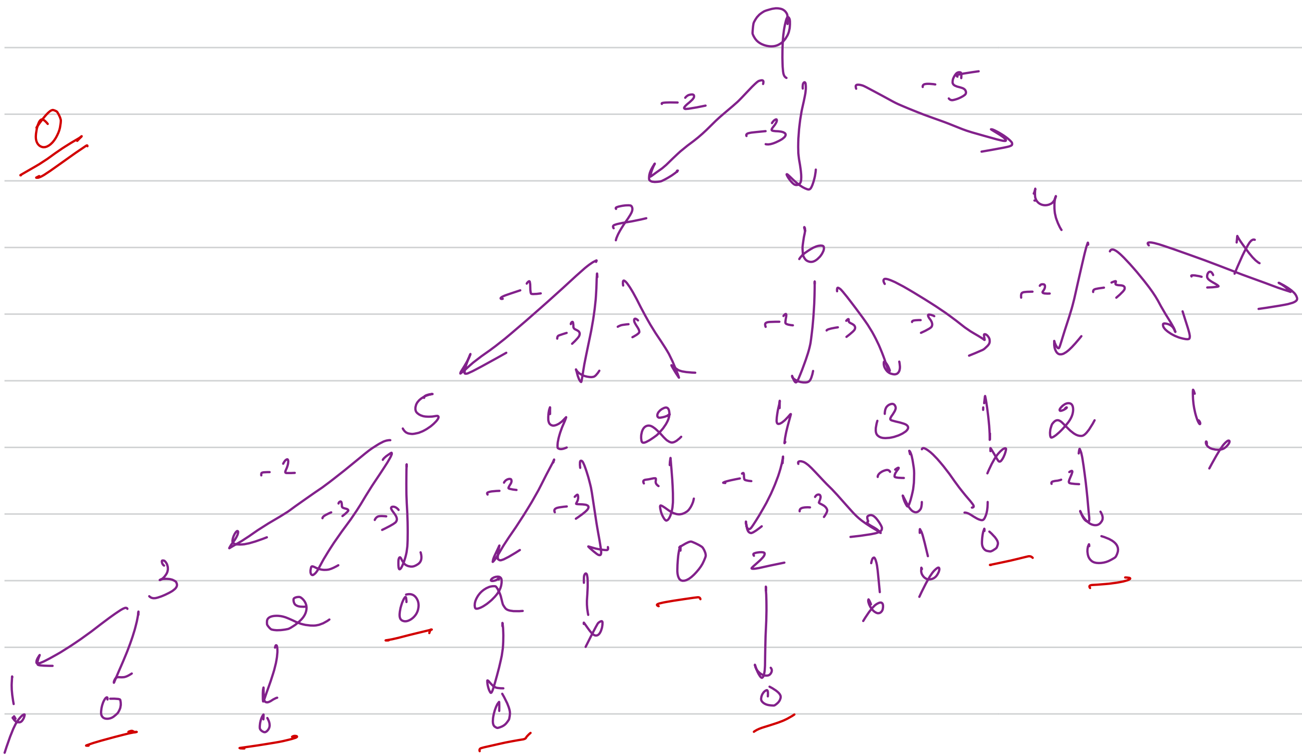
$x \le 10^6$ $n \le 10^2$ $c_i \le 10^6$

→ $x \to 9$

$[2, 3, 5]$

→ 8 ways

0



8 zeroes

We don't know the total ways to reduce 9 → 0 using [2, 3, 5]

Count permutation

$f(9) = f(7) + f(6) + f(4)$

no. of ways to reduce 9 → 0

permutation

no of ways with first element as 2 and reduce 9 → 0

2, 5, 2
2, 3, 2, 2
2, 2, 3, 2
2, 2, 2, 3
2, 2, 5

→ ways to go from 7 → 0

$[c_1, c_2 \; c_3 \; \ldots \ldots \; c_n]$

$$f(x) = \sum_{i=1}^{N} f(x - c_i)$$

$$dp(x) = \sum_{i=1}^{N} dp(x - c_i)$$

$x = 0 \longrightarrow$ 1 ways $\rightarrow$ always for country

$\{2, 3\} \rightarrow \{2\}$, $\{3\}$, $\{2, 3\}$, $\{0\}$

$\rightarrow 3$

$5 \rightarrow 5$

$\rightarrow 2, 3$

2, 3

$f(2)$

$f(3)$

$f(5)$

$\underline{f(2)} = f(0)$

1

$f(3) = f(0) + f(1)$

1 + 0

$f(5) = f(0) + f(2) + f(3)$

1

3

Q→ Consider the same prev question, this time calculate the no. of distinct ordered ways to produce money

[2, 3, 5]

$x = 9 \longrightarrow 3$

[2, 2, 2, 3]
[2, 2, 5]
[3, 3, 3]

→ all unique combination

0	1	2	3	4	5	6	7	8	9
1	0	1	1	1	2	2	2	3	3

ans

3 2 2 2

2 3 3

2 2 2 2

3, 2, 3

[2, 3, 5]

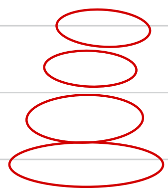
① with only 2 what all combination you can make

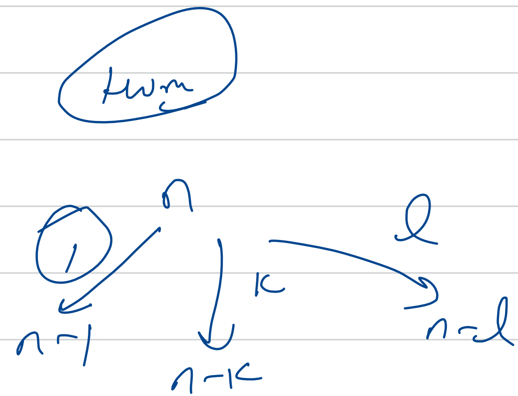
② with only 2, 3 what all combination you can make

money(9) = 5, money(4)

t = 1

Q. N coins





A, B, A, B, A - - - -

winner → who takes last

1 coin or K coins

or l coins

n → no. of coins

k, l → On each turn you can have 1, k, l coins picked

$$\boxed{f(n,k,l)} = f(n-1,k,l) \quad f(n-k,k,l)$$

$$f(n-l,k,l)$$

if we have n coins whether A will win or B will win

Sub problem

				Bygapr
0	0	0	→	1
0	1	0	→	1
0	1	1	→	1
1	1	1	→	0
1	0	1	→	1

0 → loosing

1 → wing

K =

$K = 3$ $\qquad$ $l = 3$

$n = 7$

$n = 2 \rightarrow A$

$n = 7 \rightarrow A$

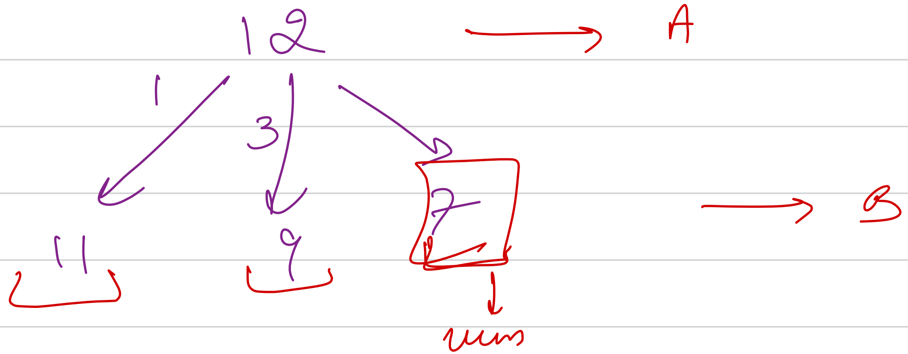
$n = 2 \rightarrow B$

$n = 1 \rightarrow A$

$n = 0$

It doesn't only depend on what A is picking

it also depends on what B gets



1.

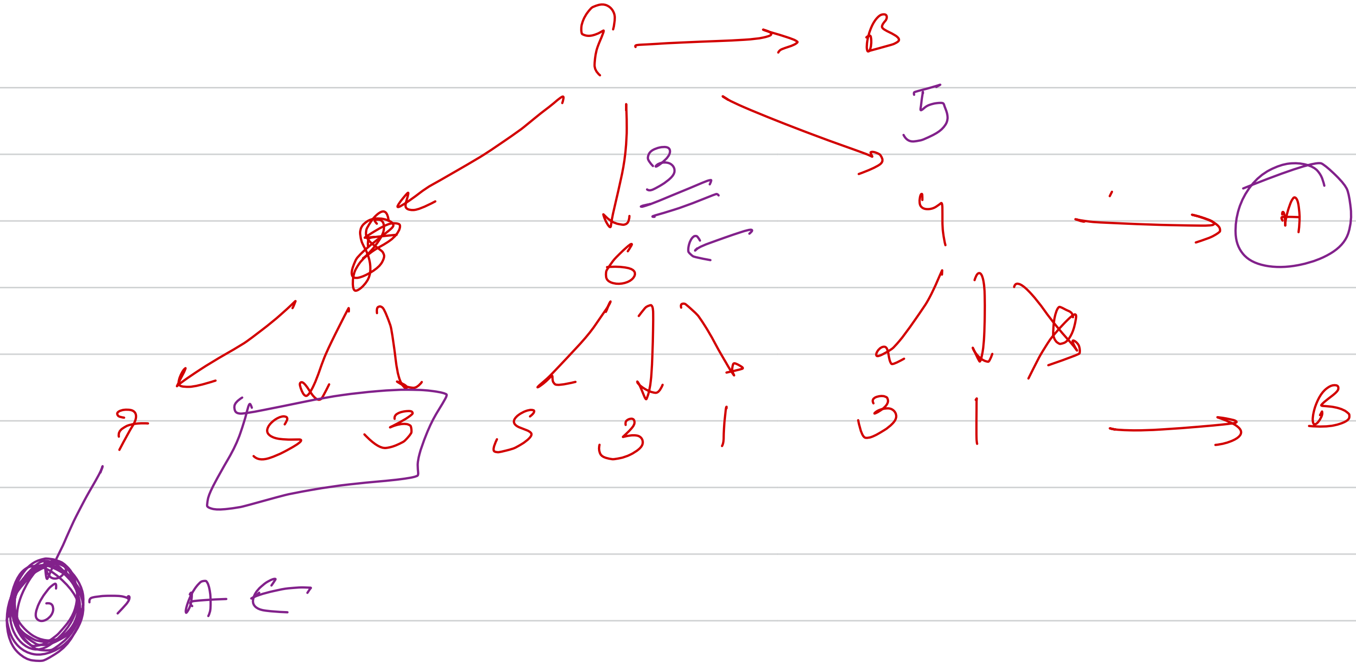
$7 \rightarrow B$

$\downarrow 5$

$2 \rightarrow A$

$\downarrow 1$

$1 \rightarrow B$ ✓



9
B
5
3
8
6
4
A
7
4
5
3
5
3
1
3
1
B
6
A

$f(n) \rightarrow$ whether n is a winning state

or losing state

(x, d)

$m \rightarrow$ $dp(10^8)$

$O(m_i + n)$

$$dp(n) = dp(n-k) \,\&\, dp(n-l) \,\&\, dp(n-1)$$

$$dp(n) = \begin{cases} dp[n-k] == 0 \longrightarrow 1 \\ \text{or} \\ dp[n-l] == 0 \longrightarrow 1 \\ \text{or} \\ dp[n-1] == 0 \longrightarrow 1 \\ \text{otherwise} \longrightarrow 0 \end{cases}$$

$dp[1] \rightarrow 1$ $\quad dp[k] \rightarrow 1$ $\quad dp[l] \rightarrow 1$

https://www.spoj.com/problems/MCOINS/