Q:- Given two numbers n and m such that m divides n, then prove the f_m divides f_n where f_x is the x^{th} fibonacci.

$m \geq 0 \quad n > 0$

$m \mid n \longrightarrow m$ divides n, $\Rightarrow n$ is divisible by m

To prove $\rightarrow f(m)$ divides $f(n) \rightarrow f(n)$ is divisible by $f(m)$

for any no, a, b $\boxed{b = ak}$ $\boxed{PMI}$

$f_n = f_{n-1} + f_{n-2}$ ✓ → recurrence of fibonacci

$\boxed{\text{for } K=1}$ → $f(b)$ is divisible by $f(a)$

assume for some value of k, the relation holds true.

$f(ka)$ is divisible by $f(a)$

Proove for $f_{(k+1)a}$ is also divisible by f_a

$f_{(k+1)a}$ is divisble by f_a $\leftrightarrows$ $f_{a(k+1)}$

$$f_{x+1} = f_{x+1} \times f_1 + f_x \times f_0 \qquad \boxed{f_0 = 0,\ f_1 = 1}$$

$$f_{x+k+1} = f_{x+k} + \underline{f_{x+k-1}} \qquad \underline{(\text{general fib eq}^n)}$$

$$= (f_{x+1} \times f_k + f_x \times f_{k-1}) + (f_{x+1} f_{k-1} + f_x f_{k-2})$$

$$= (f_{x+1}(f_k + f_{k-1}) + f_x(f_{k-1} + f_{k-2}))$$

$$= f_{x+1} f_{k+1} + f_x f_k$$

$f_{(k+1)a}$ is divisible by f_a

$$f_{ak+a} = f_{a+ak} = f_{a+1} f_{ak} + f_a f_{ak-1}$$

$\rightarrow$ f_{ak} is divisible by f_a

$$f_{(k+1)a} = \underbrace{f_{a+1} f_{ak}}_{x} + \underbrace{f_a f_{ak-1}}_{y}$$

because x & y are divisible by f_a $\rightarrow$ $f_{(k+1)a}$ is also divisible by f_a.

M.P.

Q3 Given t pairs of integers a, b. Find gcd of a, b.

b → large $t \le 10^2$

no. of digits in b . $a \to 10^5$

can be at max 250 digits

$a > b$ $\quad \gcd(a, b) = \gcd(b, a \% b)$

$b > a$ $\quad \gcd(b, a) = \gcd(a, b \% a)$

$b = x_1 x_2 x_3 x_4 \ldots x_n$ (string)

$b = 2345$
$= 2 \times 10^3 + 3 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

$b \% a \rightarrow \left((2 \times 10^3) \% a + (3 \times 10^2) \% a + (4 \times 10) \% a + 5 \% a \right) \% a$

$$b = x_1 x_2 x_3 \cdots\cdots x_n$$

$(b[i] - '0') + \text{num}$

$$b \% a \rightarrow \left((x_1 \times 10^{n-1} \% a) + x_2 \times 10^{n-2} \% a \cdots\cdots \right) \% a$$

2345

$$\hookrightarrow \quad 2$$

$$(2 \times 10 + 3) \Rightarrow 4$$

$$((4 \times 10) + 4) \rightarrow 2$$

$$(2 \times 10) + 5) \rightarrow$$

n = 0

for (i = 0 ; i < b.size() ; i++) {

n = ((n%a × 10%a)%a + (b[i] - '0')%a)%a

Q →

$ax + by = d$

$\boxed{x, y \geq 0}$

if we try all possible values of x, y

$O\left(\frac{d^2}{ab}\right) \to$ TLE

Brute force

$0 \leq x \leq \frac{d}{a}$

$0 \leq y \leq \frac{d}{b} \to$ TLE

$ax + by = d$

$O\left(\frac{d}{b}\right)$

?? TLE

$ax = d - by$

$\boxed{0 \leq y \leq \frac{d}{b}}$

→ for all those values of y for which RHS is divisible by a.

$$ax + by = d$$

$$ax = d - by$$

Instead of going to all possible values of y, can we go to fewer values of y that can make RHS divisible by a ??

$$ax + by = d$$

$$ax = d - by$$ → any how any cost, you got the first value of y for which RHS is divisible by a.

$$ax = d - by_1 \quad \rightarrow \underline{\underline{1}}$$

Let's say the first value is $\underline{\underline{y_1}}$

$$ax = d - b(y_1 + a)$$

$$ax = d - b(y_1 + 2a)$$

$$ax = d - b(y_1 + 3a)$$

$$\vdots$$

$$\cancel{ax + by = d \, (\gcd(a, b))}$$

if y_1 is the first value of y for which $d - by$ is divisible by a.

$\boxed{y_1 \to 1}$

$y_1 + a$

$y_1 + 2a$

$\vdots$

$y_1 + na$

n terms

ans $\to$ $n+1$

max value of $y \to \frac{d}{b}$

$$y_1 + na \le \frac{d}{b}$$

$$na = \frac{d}{b} - y_1$$

$$n = \frac{\frac{d}{b} - y_1}{a}$$

How to get y_1

for y_1

$$(d - by_1)\, d_a a == 0$$

$$d\, d_a a - (b\, d_a a)(y_1 d_a a) = 0$$

we know $\quad ax + by = d$

$$y = \frac{d - ax}{b}$$

$$y\, d_a a \Rightarrow \left(\frac{d}{b}\right) d_o a - \left(\frac{ax}{b}\right) d_o a$$

zero

$$d \% a - (b \% a)\left(\left(\frac{d}{b}\right) \% a\right) == 0$$

This is the first value of y

$$y_1 \rightarrow \left(\frac{d}{b}\right) \% a$$

$$y_1 \Rightarrow d \% a \times \text{ModInv}(b, a) \% a$$

extended euclid

log