Q» Calculate sum of xor of all possible pairs of an array.

Ex [7, 3, 5] → 12

$0 \wedge 0 \to 0$
$0 \wedge 1 \to 1$
$1 \wedge 0 \to 1$
$1 \wedge 1 \to 0$

$\underbrace{7 \wedge 3}_{4}$ $\underbrace{3 \wedge 5}_{6}$ $\underbrace{7 \wedge 5}_{2}$ → 12

7 → 1 1 1
3 → 0 1 1
5 → 1 0 1

$\underbrace{b^{th} \text{ pair}}$ → 1/0

A	B
1	0
0	1

→ p → 1

i^{th} bit

$7 \rightarrow 1\ 1\ 1$

$3 \rightarrow 0\ 1\ 1$

$5 \rightarrow 1\ 0\ 1$

k^{th} bit

p_1 1

p_2 1

p_3 0

c_0, c_1 ; i

cnt + $(c_0 \times c_1) \times 2$; i

1 → i^{th} bit

count of 1 at i^{th} bit × count of 0 at i^{th} bit

$p_1 + p_2 + p_3$ ⇒ x

k^{th}

How many pairs will have i^{th} bit 1.

$4+6+2 = 12$

$$(1\times 2^2 + 0\times 2^1 + 0\times 2^0) + (1\times 2^2 + 1\times 2^1 + 0\times 2^0) +$$
$$(0\times 2^2 + 1\times 2^1 + 0\times 2^0) = 12$$

$$(0+1+1)2^2 + (0+1+1)2^1 + (0+0+0)2^0$$

$$2\times 2^2 + 2\times 2^1 + 0$$

$$\to 8+4 \to 12$$

count of pairs with i^{th} bit after xor is 1 is m.

ans $+= m\times 2^i$

Recurrence Relations

Method of guessing

$$T(n) = \sqrt{n}\, T(\sqrt{n}) + n$$

assume $T(n) = \Theta(n \log n)$

$$T(n) \le c n \log n$$

$$T(n) = \sqrt{n}\, T(\sqrt{n}) + n$$

$$T(n) \le (\sqrt{n} \times c\sqrt{n} \log \sqrt{n} + n)$$

$$T(n) \le \frac{1}{2} c n \log n + n$$

$$T(n) \le c n \log n$$

$$T(n) = \sqrt{n}\, T(\sqrt{n}) + n$$

$$T(n) \ge \sqrt{n}\, k \sqrt{n} \log \sqrt{n} + n$$

$$T(n) \ge \frac{nk \log n}{2} + n$$

No

Assume → $T(n) = \Theta(n)$

$T(n) \le cn$

$T(n) = \sqrt{n}\, T(\sqrt{n}) + n$ $\times \times \times$

$T(n) \le \sqrt{n} \times c\sqrt{n} + n$

$T(n) \le cn + n$

$T(n) \le (c+1)n$

$n\sqrt{\log n}$

$T(n) \leq c n \sqrt{\log n}$

$T(n) = \sqrt{n}\, T(\sqrt{n}) + n$

$T(n) \leq \sqrt{n} \times c \sqrt{n} \sqrt{\log \sqrt{n}} + n$

$T(n) \leq n c \frac{1}{\sqrt{2}} \sqrt{\log n} + n$

HP

$T(n) \geq \sqrt{n}\, k \sqrt{n} \sqrt{\log \sqrt{n}} + n$

$\geq k n \frac{1}{\sqrt{2}} \sqrt{\log n} + n$

$T(n) \geq k n \sqrt{\log n}$

case → $n \log \log n$

$$T(n) = \sqrt{n}\, T(\sqrt{n}) + n$$

$$T(n) \le c\, n \log\log n$$

$$T(n) \ge k\, n \log\log n$$

$\Theta(n \log \log n)$

$$T(n) \le \sqrt{n}\, c\sqrt{n} \log\log\sqrt{n} + n$$

$$\le c n \log\log\sqrt{n} + n$$

$$\le c n \log\left(\frac{\log n}{2}\right) + n$$

$$\le c n \log\log n - c n \log 2 + n$$

$c \ge 1$

$$T(n) \le cn\log\log n + n(1 - \underline{c\log 2})$$

$$T(n) \le \underline{cn\log\log n}$$

[illegible] $\epsilon > 1$

[illegible]

$$T(n) \geq \sqrt{n}\, k\sqrt{n}\log\log\sqrt{n} + n$$

$$\geq kn\log\left(\frac{\log n}{2}\right) + n$$

$$\geq kn\log\log n - kn\log 2 + n$$

$$T(n) \geq kn\log\log n + n(1 - k\log 2)$$

$$T(n) \geq kn\log\log n$$

$$k \geq 1$$

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

$n \log n$

Q-1 $T(n) = \sum_{i=1}^{n} \log i$

$T(n) = \log 1 + \log 2 + \log 3 \ldots \log n$

$T(n) = \log n!$

$\log n! \leq \log n^n$

$T(n) \leq \log n^n$

$T(n) \leq n \log n$

$O(n \log n)$

Q: $T(n) = \begin{cases} 1 & n=1 \\ T(n-1) + n(n-1) & n \geq 1 \end{cases}$

$$T(n) = T(n-1) + n(n-1)$$

$$= T(n-2) + (n-2)(n-1) + n(n-1)$$

$$= T(n-3) + (n-3)(n-2) + (n-2)(n-1) + n(n-1)$$

$$T(n) = T(1) + \sum_{i=1}^{n} i(i-1)$$

$$T(n) = 1 + \sum_{i=1}^{n} i^2 - i$$

$$T(n) = 1 + \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$T(n) \rightarrow \theta(n^3)$

$$= \frac{6 + (n^2+n)(2n+1) - 3n^2 - 3n}{6}$$

$$\Rightarrow \frac{6 + 2n^3 + \cancel{2n^2} + \cancel{n^2} + n - \cancel{3}n^2 - 3n}{6}$$

$$T(n) \Rightarrow \frac{2n^3 - 2n + 6}{6} \leq Kn^3$$

$$T(n) = T(0.8n) + O(n)$$

$$T(n) = T\left(\frac{4}{5}n\right) + O(n)$$

$a = 1 \qquad b = \frac{5}{4} \qquad k = 1$

$$a < b^k \longrightarrow \underline{\underline{O(n)}}$$

$n!$

2^n

n^4

n^3

n^2

$n\sqrt{n}$

$n \log n$

$n\sqrt{\log n}$

$n \log \log n$

$\uparrow$

$\sqrt{n}$

$\log n$

1

Master theorem of subtraction

$$T(n) = \begin{cases} c & n \le 1 \\ aT(n-b) + f(n) & n > 1 \end{cases}$$

$f(n) = O(n^k)$

$c \rightarrow$ const

$a > 0$ $b > 0$ $k \ge 0$

Solution $\rightarrow$ $T(n) = \begin{cases} O(n^k) & a < 1 \\ O(n^{k+1}) & a = 1 \\ O(n^k a^{n/b}) & a > 1 \end{cases}$