Sunday, 19 October 2025 10:38 AM

Graph Coloring

Goal: consistently color vertices
minimize number of colors

Bug
states
Actual
Not actual
state change

Bug: 3 states, edges between
states mean overlapping schedules.

Study

5 → M	Bug: S_{ch}
1 → S	Physics: Lecture
4 → S	Discrete
11 → M	
1 → S	

Job: to be done without two in the same
time slot. (same day)

Graph Coloring Algorithm:

Given a graph G and k colors, assign
a color to each node so that adjacent
nodes get different colors.

color
color

Def: The minimum # colors G can be for
coloring assigns to the
chromatic number $\chi(G)$

What is the minimum # of colors here??

The state

Bug
states
Actual
Lecture
Discrete

5 → 4 → C_1
4 → 3 → C_2
11 → 1 → C_3
1 → 1 → C_4

Lecture
Discrete
→ 2 colors

2 colors?? no

Bug
S_{ch}

$\chi(G) = 3$

No one found a fast algorithm for
determining chromatic #.

For finding χ, you have to try
exponential # of possibilities.

NP complete problems

→ NP complete problems are problems whose
solution is unknown

→ No polynomial time algorithm has been
discovered for any NP complete problem.

→ If it also not satisfied, could no
polynomial time algorithm will ever
exist

Approximation Algorithm (greedy)

→ check under the different degree of
(chosen finding)

Algorithm:

1. Order the nodes $v_1, v_2, \ldots, v_n$
2. Pick the color $C_1, C_2, \ldots$
3. Previous Person use all a future
for $i = 1, \ldots, n$
Assign the lowest legal color to v_i

v_1
C_1
v_2
C_2
v_3
C_3
v_4
C_4
v_5
C_5

C_1 $\sqrt{v_1}$ C_2
C_3 C_4 C_5

Theorem: If every node in a n-node graph
G has a degree $< d$, then Greedy
algorithm uses at most $d+1$ colors for G.

(using greedy algorithm)

For two node nodes, 3 degree (max),
max color needed → 4.

degree $= n-1$
max color $= n$

C_1
C_2
C_3
C_4
C_5

$\chi(G) < d$

v_1
C_1
v_2
C_2
v_3
C_3
v_4
C_4
v_5
C_5

v_1
v_2
v_3
v_4
v_5
C_1
C_2
C_3
C_4
C_5

Bipartite graph

A graph $G = (V, E)$ is called bipartite
if V can be partitioned into V_1 & V_2 so
that all the edges connect a node
in V_1 to a node in V_2.

V_1
V_2

Applications

1. Scheduling time slots
2. Bipartite graph
3. Map coloring

Q: Check whether a graph is bipartite
or not.

V_1
V_2
Blue
Red
Blue
Red

Blue
Red
Blue
Red

Bipartite

1. Assign a color (red) to source
vertex
2. Color all the neighbors (green)
3. Color all the neighbors of
neighbors with red color
4. BFS → assign color to all nodes
5. Whenever coloring neighbors, if we
encounter which is already
colored, check that so consistent
nodes, then the
graph be colored with
2 colors.

Chromatic # for path graphs

If G has no G vertices is even, then
G is a even cycle, and to color
such a graph we only require two
colors.

If G has no G vertices is odd, then
G is a odd cycle, and to color
such a graph we require 3
colors.

Q: Given a tree with n nodes, how
many edges edges you can add
such that the tree still remain
bipartite.

A: The tree always a bipartite
graph.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

Bipartite

DFS / BFS

cut | visit

max edges
for n vertices
$\Rightarrow \binom{n-1}{2}$

Tree
$n-1$ edges

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

Minimum Spanning Tree

Given an undirected graph $G = (V, E)$,
a spanning tree is the graph G or
tree that spans G, and is a subgraph
of G.

MST

$W_T = \sum_{e \in T} w_e$ is a spanning tree where the cost
is minimum. It along all the spanning
trees

→ Routing, telephone problem
→ Partitioning, clustering
→ Studying of surfaces
→ Best heuristic, NP hard problem

a
b
c
d
e
f
g
h
i
j
k
l
m
n
o
p
q
r
s
t
u
v
w
x
y
z
a
b
c
d
e
f
g
h
i
j
k
l
m
n
o
p
q
r
s
t
u
v
w
x
y
z
a
b
c
d
e
f
g
h
i
j
k
l
m
n
o
p
q
r
s
t
u
v
w
x
y
z

Two Algorithms

1. Prim's
2. Kruskal's

Prim's Algo

→ maintains tree add V vertices

visited
Prim's
v_1
v_2
v_3
v_4
v_5
v_6
v_7
v_8
v_9
v_{10}
v_{11}
v_{12}
v_{13}
v_{14}
v_{15}
v_{16}
v_{17}
v_{18}
v_{19}
v_{20}
v_{21}
v_{22}
v_{23}
v_{24}
v_{25}
v_{26}
v_{27}
v_{28}
v_{29}
v_{30}
v_{31}
v_{32}
v_{33}
v_{34}
v_{35}
v_{36}
v_{37}
v_{38}
v_{39}
v_{40}
v_{41}
v_{42}
v_{43}
v_{44}
v_{45}
v_{46}
v_{47}
v_{48}
v_{49}
v_{50}
v_{51}
v_{52}
v_{53}
v_{54}
v_{55}
v_{56}
v_{57}
v_{58}
v_{59}
v_{60}
v_{61}
v_{62}
v_{63}
v_{64}
v_{65}
v_{66}
v_{67}
v_{68}
v_{69}
v_{70}
v_{71}
v_{72}
v_{73}
v_{74}
v_{75}
v_{76}
v_{77}
v_{78}
v_{79}
v_{80}
v_{81}
v_{82}
v_{83}
v_{84}
v_{85}
v_{86}
v_{87}
v_{88}
v_{89}
v_{90}
v_{91}
v_{92}
v_{93}
v_{94}
v_{95}
v_{96}
v_{97}
v_{98}
v_{99}
v_{100}
v_{101}
v_{102}
v_{103}
v_{104}
v_{105}
v_{106}
v_{107}
v_{108}
v_{109}
v_{110}
v_{111}
v_{112}
v_{113}
v_{114}
v_{115}
v_{116}
v_{117}
v_{118}
v_{119}
v_{120}
v_{121}
v_{122}
v_{123}
v_{124}
v_{125}
v_{126}
v_{127}
v_{128}
v_{129}
v_{130}
v_{131}
v_{132}
v_{133}
v_{134}
v_{135}
v_{136}
v_{137}
v_{138}
v_{139}
v_{140}
v_{141}
v_{142}
v_{143}
v_{144}
v_{145}
v_{146}
v_{147}
v_{148}
v_{149}
v_{150}
v_{151}
v_{152}
v_{153}
v_{154}
v_{155}
v_{156}
v_{157}
v_{158}
v_{159}
v_{160}
v_{161}
v_{162}
v_{163}
v_{164}
v_{165}
v_{166}
v_{167}
v_{168}
v_{169}
v_{170}
v_{171}
v_{172}
v_{173}
v_{174}
v_{175}
v_{176}
v_{177}
v_{178}
v_{179}
v_{180}
v_{181}
v_{182}
v_{183}
v_{184}
v_{185}
v_{186}
v_{187}
v_{188}
v_{189}
v_{190}
v_{191}
v_{192}
v_{193}
v_{194}
v_{195}
v_{196}
v_{197}
v_{198}
v_{199}
v_{200}
v_{201}
v_{202}
v_{203}
v_{204}
v_{205}
v_{206}
v_{207}
v_{208}
v_{209}
v_{210}
v_{211}
v_{212}
v_{213}
v_{214}
v_{215}
v_{216}
v_{217}
v_{218}
v_{219}
v_{220}
v_{221}
v_{222}
v_{223}
v_{224}
v_{225}
v_{226}
v_{227}
v_{228}
v_{229}
v_{230}
v_{231}
v_{232}
v_{233}
v_{234}
v_{235}
v_{236}
v_{237}
v_{238}
v_{239}
v_{240}
v_{241}
v_{242}
v_{243}
v_{244}
v_{245}
v_{246}
v_{247}
v_{248}
v_{249}
v_{250}
v_{251}
v_{252}
v_{253}
v_{254}
v_{255}
v_{256}
v_{257}
v_{258}
v_{259}
v_{260}
v_{261}
v_{262}
v_{263}
v_{264}
v_{265}
v_{266}
v_{267}
v_{268}
v_{269}
v_{270}
v_{271}
v_{272}
v_{273}
v_{274}
v_{275}
v_{276}
v_{277}
v_{278}
v_{279}
v_{280}
v_{281}
v_{282}
v_{283}
v_{284}
v_{285}
v_{286}
v_{287}
v_{288}
v_{289}
v_{290}
v_{291}
v_{292}
v_{293}
v_{294}
v_{295}
v_{296}
v_{297}
v_{298}
v_{299}
v_{300}
v_{301}
v_{302}
v_{303}
v_{304}
v_{305}
v_{306}
v_{307}
v_{308}
v_{309}
v_{310}
v_{311}
v_{312}
v_{313}
v_{314}
v_{315}
v_{316}
v_{317}
v_{318}
v_{319}
v_{320}
v_{321}
v_{322}
v_{323}
v_{324}
v_{325}
v_{326}
v_{327}
v_{328}
v_{329}
v_{330}
v_{331}
v_{332}
v_{333}
v_{334}
v_{335}
v_{336}
v_{337}
v_{338}
v_{339}
v_{340}
v_{341}
v_{342}
v_{343}
v_{344}
v_{345}
v_{346}
v_{347}
v_{348}
v_{349}
v_{350}
v_{351}
v_{352}
v_{353}
v_{354}
v_{355}
v_{356}
v_{357}
v_{358}
v_{359}
v_{360}
v_{361}
v_{362}
v_{363}
v_{364}
v_{365}
v_{366}
v_{367}
v_{368}
v_{369}
v_{370}
v_{371}
v_{372}
v_{373}
v_{374}
v_{375}
v_{376}
v_{377}
v_{378}
v_{379}
v_{380}
v_{381}
v_{382}
v_{383}
v_{384}
v_{385}
v_{386}
v_{387}
v_{388}
v_{389}
v_{390}
v_{391}
v_{392}
v_{393}
v_{394}
v_{395}
v_{396}
v_{397}
v_{398}
v_{399}
v_{400}
v_{401}
v_{402}
v_{403}
v_{404}
v_{405}
v_{406}
v_{407}
v_{408}
v_{409}
v_{410}
v_{411}
v_{412}
v_{413}
v_{414}
v_{415}
v_{416}
v_{417}
v_{418}
v_{419}
v_{420}
v_{421}
v_{422}
v_{423}
v_{424}
v_{425}
v_{426}
v_{427}
v_{428}
v_{429}
v_{430}
v_{431}
v_{432}
v_{433}
v_{434}
v_{435}
v_{436}
v_{437}
v_{438}
v_{439}
v_{440}
v_{441}
v_{442}
v_{443}
v_{444}
v_{445}
v_{446}
v_{447}
v_{448}
v_{449}
v_{450}
v_{451}
v_{452}
v_{453}
v_{454}
v_{455}
v_{456}
v_{457}
v_{458}
v_{459}
v_{460}
v_{461}
v_{462}
v_{463}
v_{464}
v_{465}
v_{466}
v_{467}
v_{468}
v_{469}
v_{470}
v_{471}
v_{472}
v_{473}
v_{474}
v_{475}
v_{476}
v_{477}
v_{478}
v_{479}
v_{480}
v_{481}
v_{482}
v_{483}
v_{484}
v_{485}
v_{486}
v_{487}
v_{488}
v_{489}
v_{490}
v_{491}
v_{492}
v_{493}
v_{494}
v_{495}
v_{496}
v_{497}
v_{498}
v_{499}
v_{500}
v_{501}
v_{502}
v_{503}
v_{504}
v_{505}
v_{506}
v_{507}
v_{508}
v_{509}
v_{510}
v_{511}
v_{512}
v_{513}
v_{514}
v_{515}
v_{516}
v_{517}
v_{518}
v_{519}
v_{520}
v_{521}
v_{522}
v_{523}
v_{524}
v_{525}
v_{526}
v_{527}
v_{528}
v_{529}
v_{530}
v_{531}
v_{532}
v_{533}
v_{534}
v_{535}
v_{536}
v_{537}
v_{538}
v_{539}
v_{540}
v_{541}
v_{542}
v_{543}
v_{544}
v_{545}
v_{546}
v_{547}
v_{548}
v_{549}
v_{550}
v_{551}
v_{552}
v_{553}
v_{554}
v_{555}
v_{556}
v_{557}
v_{558}
v_{559}
v_{560}
v_{561}
v_{562}
v_{563}
v_{564}
v_{565}
v_{566}
v_{567}
v_{568}
v_{569}
v_{570}
v_{571}
v_{572}
v_{573}
v_{574}
v_{575}
v_{576}
v_{577}
v_{578}
v_{579}
v_{580}
v_{581}
v_{582}
v_{583}
v_{584}
v_{585}
v_{586}
v_{587}
v_{588}
v_{589}
v_{590}
v_{591}
v_{592}
v_{593}
v_{594}
v_{595}
v_{596}
v_{597}
v_{598}
v_{599}
v_{600}
v_{601}
v_{602}
v_{603}
v_{604}
v_{605}
v_{606}
v_{607}
v_{608}
v_{609}
v_{610}
v_{611}
v_{612}
v_{613}
v_{614}
v_{615}
v_{616}
v_{617}
v_{618}
v_{619}
v_{620}
v_{621}
v_{622}
v_{623}
v_{624}
v_{625}
v_{626}
v_{627}
v_{628}
v_{629}
v_{630}
v_{631}
v_{632}
v_{633}
v_{634}
v_{635}
v_{636}
v_{637}
v_{638}
v_{639}
v_{640}
v_{641}
v_{642}
v_{643}
v_{644}
v_{645}
v_{646}
v_{647}
v_{648}
v_{649}
v_{650}
v_{651}
v_{652}
v_{653}
v_{654}
v_{655}
v_{656}
v_{657}
v_{658}
v_{659}
v_{660}
v_{661}
v_{662}
v_{663}
v_{664}
v_{665}
v_{666}
v_{667}
v_{668}
v_{669}
v_{670}
v_{671}
v_{672}
v_{673}
v_{674}
v_{675}
v_{676}
v_{677}
v_{678}
v_{679}
v_{680}
v_{681}
v_{682}
v_{683}
v_{684}
v_{685}
v_{686}
v_{687}
v_{688}
v_{689}
v_{690}
v_{691}
v_{692}
v_{693}
v_{694}
v_{695}
v_{696}
v_{697}
v_{698}
v_{699}
v_{700}
v_{701}
v_{702}
v_{703}
v_{704}
v_{705}
v_{706}
v_{707}
v_{708}
v_{709}
v_{710}
v_{711}
v_{712}
v_{713}
v_{714}
v_{715}
v_{716}
v_{717}
v_{718}
v_{719}
v_{720}
v_{721}
v_{722}
v_{723}
v_{724}
v_{725}
v_{726}
v_{727}
v_{728}
v_{729}
v_{730}
v_{731}
v_{732}
v_{733}
v_{734}
v_{735}
v_{736}
v_{737}
v_{738}
v_{739}
v_{740}
v_{741}
v_{742}
v_{743}
v_{744}
v_{745}
v_{746}
v_{747}
v_{748}
v_{749}
v_{750}
v_{751}
v_{752}
v_{753}
v_{754}
v_{755}
v_{756}
v_{757}
v_{758}
v_{759}
v_{760}
v_{761}
v_{762}
v_{763}
v_{764}
v_{765}
v_{766}
v_{767}
v_{768}
v_{769}
v_{770}
v_{771}
v_{772}
v_{773}
v_{774}
v_{775}
v_{776}
v_{777}
v_{778}
v_{779}
v_{780}
v_{781}
v_{782}
v_{783}
v_{784}
v_{785}
v_{786}
v_{787}
v_{788}
v_{789}
v_{790}
v_{791}
v_{792}
v_{793}
v_{794}
v_{795}
v_{796}
v_{797}
v_{798}
v_{799}
v_{800}
v_{801}
v_{802}
v_{803}
v_{804}
v_{805}
v_{806}
v_{807}
v_{808}
v_{809}
v_{810}
v_{811}
v_{812}
v_{813}
v_{814}
v_{815}
v_{816}
v_{817}
v_{818}
v_{819}
v_{820}
v_{821}
v_{822}
v_{823}
v_{824}
v_{825}
v_{826}
v_{827}
v_{828}
v_{829}
v_{830}
v_{831}
v_{832}
v_{833}
v_{834}
v_{835}
v_{836}
v_{837}
v_{838}
v_{839}
v_{840}
v_{841}
v_{842}
v_{843}
v_{844}
v_{845}
v_{846}
v_{847}
v_{848}
v_{849}
v_{850}
v_{851}
v_{852}
v_{853}
v_{854}
v_{855}
v_{856}
v_{857}
v_{858}
v_{859}
v_{860}
v_{861}
v_{862}
v_{863}
v_{864}
v_{865}
v_{866}
v_{867}
v_{868}
v_{869}
v_{870}
v_{871}
v_{872}
v_{873}
v_{874}
v_{875}
v_{876}
v_{877}
v_{878}
v_{879}
v_{880}
v_{881}
v_{882}
v_{883}
v_{884}
v_{885}
v_{886}
v_{887}
v_{888}
v_{889}
v_{890}
v_{891}
v_{892}
v_{893}
v_{894}
v_{895}
v_{896}
v_{897}
v_{898}
v_{899}
v_{900}
v_{901}
v_{902}
v_{903}
v_{904}
v_{905}
v_{906}
v_{907}
v_{908}
v_{909}
v_{910}
v_{911}
v_{912}
v_{913}
v_{914}
v_{915}
v_{916}
v_{917}
v_{918}
v_{919}
v_{920}
v_{921}
v_{922}
v_{923}
v_{924}
v_{925}
v_{926}
v_{927}
v_{928}
v_{929}
v_{930}
v_{931}
v_{932}
v_{933}
v_{934}
v_{935}
v_{936}
v_{937}
v_{938}
v_{939}
v_{940}
v_{941}
v_{942}
v_{943}
v_{944}
v_{945}
v_{946}
v_{947}
v_{948}
v_{949}
v_{950}
v_{951}
v_{952}
v_{953}
v_{954}
v_{955}
v_{956}
v_{957}
v_{958}
v_{959}
v_{960}
v_{961}
v_{962}
v_{963}
v_{964}
v_{965}
v_{966}
v_{967}
v_{968}
v_{969}
v_{970}
v_{971}
v_{972}
v_{973}
v_{974}
v_{975}
v_{976}
v_{977}
v_{978}
v_{979}
v_{980}
v_{981}
v_{982}
v_{983}
v_{984}
v_{985}
v_{986}
v_{987}
v_{988}
v_{989}
v_{990}
v_{991}
v_{992}
v_{993}
v_{994}
v_{995}
v_{996}
v_{997}
v_{998}
v_{999}
v_{1000}