Friday, 3 December 2021 9:05 PM

Graph Coloring

→ scheduling final exams

mathematics	Geometry	Sat	$c_1, c_2, c_3 \ldots$
E	C	subjects	

✓ Graph can also represent dissociation b/w nodes as well.

For 5 subjects, edge between two subjects represents that those subjects have overlapping students.

Goal is to figure out how to use least amount of exam slots available.

Slots:

8 – 9	am	Eng, Sat
9 – 10	am	Maths, Sat & Science
9 – 11	pm	Hist, etc.
11 – 1	pm	
1 – 3	pm	

OR → 3 slots ✓✓

Graph Coloring Algorithms

Given a graph G and k colors, assign a color to each node so that the adjacent nodes get different colors.

color of same

Def. The minimum # of colors required to fix the coloring rule is called chromatic number. $\chi(G)$

what is the minimum # of colors here??

3 colors

S → 8 am → C_1
E → 9 am → C_2
H → 11 am → C_3
M → 1 pm → C_1
Sci → 3 pm → C_2

2 colors ?? (No)

Bipartite

So $\chi(G) = 3$

No one knows a fast algorithm for determining chromatic no.

For finding χ, you have to try exponential no. of possibilities.

NP complete problems

→ NP complete problems are problems whose status is unknown.

→ No polynomial time algorithm has been discovered for any NP complete algorithm.

→ It is also not established that no polynomial time algorithm will ever exist.

✓ Approximation algorithm (greedy)

→ check when one higher degree of graph

[Greedy: Provide good ideas]

Algorithm

1. order the nodes $v_1, v_2, \ldots, v_n$ (based on degree)
2. while: use color $c_1, c_2, \ldots$
3. choose node v_i, assign a color, for $i = 1, 2, \ldots, n$

assign the lowest legal color to v_i

✓ Theorem: If every node in a n-node graph G has a degree ≤ d, then greedy algorithm use about d+1 colors in G.

[very useful algo]

For 1000 nodes, 3 degree (max) max color needed → 4

max degree → 4
max color needed → 5

$\chi(G) = \underline{d}$

→ Good coloring

→ Good coloring

$\chi(G) = 2$

Bipartite graph

A graph G = (V, E) is called bipartite if V can be split into $V_1 \cup V_2$ so that all the edges connected a node in V_1 to a node in V_2.

$\chi(G) = 2$ always

Applications of graph coloring.

1. Scheduling - time table
2. Bipartite graph
3. Map coloring

Q. Decide whether a graph is bipartite or not.

1.

Bipartite

2.

Bipartite

3.

Bipartite

① Assign a color (hue) to every vertex.

② Color all the neighbors (green)

③ Color all the neighbors's neighbor Red

④ BFS → assign color to all nodes

⑤ When assigning neighbor, if we a neighbor which is colored with same color, the graph cannot be bipartite.

× Bipartite

× Bipartite

Chromatic no. for cyclic graph

* If the no. of nodes is even, then it is a even cycle and to color each a graph, we require 2 colors

2 colors

* If the no. of nodes is odd, then it is an odd cycle and to color each a graph, we require 3 colors

3 colors

Minimum Spanning Tree

MST is a spanning tree where the cost is minimum among all the spanning tree.

→ Travelling salesman problem
→ Handwriting recognition
→ Clustering analysis
→ Max bottleneck min cost problem

Two Algorithms

① Prim
② Kruskal's

Both greedy algorithms