Exchange Arguments

The basic idea is to see your greedy sol^n w.r.t any other **solution**

Example → We have 2 positive integers 'a' & 'b' such that $a+b=n$. For a given value n, maximize $a \times b$.

In order to maximize a and b, we would like to have a and b as close as possible

↳ $n \rightarrow$ even

$a = \frac{n}{2}$ $b = \frac{n}{2}$ $\longrightarrow$ ans $\frac{n^2}{4}$

$a=x, b=x$

↳ $a > b$

$x = \frac{n}{2}$

$a = x + \Delta$

$b = x - \Delta$

$\boxed{\Delta \geq 1}$

$$a \times b = (x+\Delta)(x-\Delta)$$

$$= x^2 - \Delta^2$$

and Δ and x both are positive

$$\boxed{x^2 - \Delta^2 < x^2}$$

Coin Change → Given some denomination of coins, and a value x, find the min no. of coins we need to give a change of x.

$x = 10$ [9, 5, 1]

↳ 2 coins

5 5

9 1

But for some denomination greedy does work.

Ex

$$\underbrace{[25, 10, 5, 1]}$$

#observation

1) $\underbrace{3}_{3} \times 10 \longleftrightarrow \underbrace{25 + 5}_{2}$

We almost require 2 coin of 10

2) We at most require 1 coin of $\underline{\underline{5}}$

$2 \times 5 \longleftrightarrow 1 \times 10$

$\downarrow$ $\qquad\qquad\qquad\qquad \downarrow$

$2 \qquad\qquad\qquad\qquad \underline{\underline{1}} \leftarrow$

3) We atmost require $\underline{4}$ coins of $\underline{\underline{1}}$

$5 \times 1 \longleftrightarrow 1 \times 5$

$\downarrow$ $\qquad\qquad\qquad\qquad \downarrow$

$5 \qquad\qquad\qquad\qquad \underline{\underline{1}}$

4) We won't be have 2, 10's and 1, 5 coins because we can replace it by 25

$2 \times 10 + 1 \times 5 \longrightarrow 1 \times 25$

5) Without a 25 coin we can have at max 24 value.

$2 \times 10 + 0 \times 5 + 4 \times 1 \rightarrow 24$

$1 \times 10 + 1 \times 5 + 4 \times 1 \rightarrow 19$

Let's say we have O as an optimal solution and G is a greedy solution

→ ~~Say~~ → K = # of 25 coins in O

K' = # of 25 coins in G

$K < K'$

$K == K'$

→ $\underline{\underline{K \le K'}}$

$K = K'$

$\underline{\underline{K_n' < K_o}}$

Say d_o: # of 10 coins in O

d_n = # of 10 coins in G.

$d_o \leq d_n$ $\qquad$ $d_o = d_n$

Say n_o: # of 5 coins in O

n_n = # of 5 coins in G.

$n_o \leq n_n$ $\qquad$ $n_o = n_n$

$\underline{O} = \underline{G}$ (for <u>gen denominators</u>)

Ex

11

4

[1, 5, 7]

7 1 1 1 1

5 5 1

→ Sort the data in a given manner

→ (score, decay, time) ← Problem

↳ In what order we should solve the problem.

$$P_1 \;|\; P_2$$

$P_1 < P_2$

$\downarrow$

$P_1 \to$ sort 1

$P_2 < P_1$

$P_2 \to$ score 2

$$P_1 \to s_1 - d_1 t_1 + s_2 - d_2 (t_1 + t_2)$$

$$P_2 \to s_2 - d_2 t_2 + s_1 - d_1 (t_1 + t_2)$$

Maximize the return

$$R_1 \geq R_2$$

$$R_1 - R_2 \geqslant 0$$

$$s_1 - d_1 t_1 + s_2 - d_2 (t_1 + t_2) -$$
$$\left(s_2 - d_2 t_2 + s_1 - d_1 (t_1 + t_2) \right) \geqslant 0$$

$$\cancel{s_1} - \cancel{d_1 t_1} + \cancel{s_2} - d_2 t_1 - \cancel{d_2 t_2} -$$
$$\cancel{s_2} + \cancel{d_2 t_2} - \cancel{s_1} + \cancel{d_1 t_1} + d_1 t_2 \geqslant 0$$

$$d_1 t_2 - d_2 t_1 \geqslant 0$$

$\boxed{d_1 t_2 > d_2 t_1}$

P_1 $\quad$ P_2

↓

first

↓ R_1

$R_1 > R_2$

optimal order of the problem

$\boxed{P_i \quad P_j \quad P_k \text{ - - - - - }}$

knapsack