Q => Given a list of n integers and an integer D.
find a segment of length at least D, on which arithmetic mean of elements is maximum.

↳ Ex [3, 1, 8, 5, 7, 2] $D = 2$

⟶ [8, 5, 7] ans

$d \le n \le 10^5$
$a_i \le 10^2$

$10^{0} \longleftrightarrow 10^3$

D

Integer list given ← $[a_0, a_1, \overset{l}{|} a_2, a_3 \ldots\ldots\ldots \overset{r-1}{|} \ldots a_{n-1}]$

Search space will represent range of means

$[l, r-1] \longrightarrow \text{length} \longrightarrow \underbrace{r-l}$

$$\underline{AM} \longleftarrow \frac{\sum_{i=l}^{r-1} a_i}{r-l} \geq \underline{mid} \longrightarrow \text{True}$$

True ↳ right to find bigger mean

↓ false go to left

problem boils down to find r & l

$$\frac{\sum_{i=l}^{r-1} a_i}{r-l} \geq mid$$

$$\sum_{i=l}^{r-1} a_i \geq mid * (r-l)$$

$$\sum_{i=l}^{r-1} a_i - mid * (r-l) \geq 0$$

r-l terms

$$\overbrace{\sum_{i=l}^{r-1} a_i} - \underbrace{mid} \cdot (r-l) \geq 0$$

$$\boxed{\sum_{i=l}^{r-1} (a_i - mid) \geq 0}$$

$$\boxed{r-l \geq 0}$$

Sum of a range

prefix sum

1	2	3	1	4	5	6

$l=1$ $r=5$

$\hookrightarrow$ 1, 3, 6, 7, 11, 16, 22

$$\boxed{l \qquad \underline{\underline{r-1}}}$$

We will prepare a prefix Sum list from original list

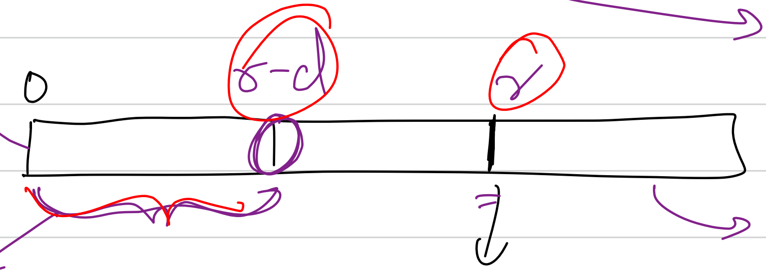
$$pre[i] = \sum_{j=0}^{i-1} (a_j - mid)$$

$$\underbrace{pre[r] - pre[l]}_{\sum_{i=l}^{r-1} \underline{(a_i - mid)}}$$

$$\begin{tikzcd} \underline{pre[l]} \arrow[d] & pre[r] \arrow[d] \\ \sum_{i=0}^{l-1} (a_i - mid) & \sum_{i=0}^{r-1} (a_i - mid) \end{tikzcd}$$

$$\left.\begin{array}{l} pre[r] - pre[l] \geq 0 \\ r - l \geq D \end{array}\right\} \longrightarrow \begin{array}{l} pre[r] \geq pre[l] \\ r - l \geq D \end{array}$$

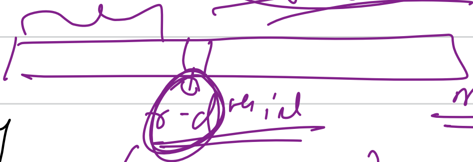
$r - d \geq l$



range of l will be from $[0, r-d]$

consider any index r

prefix



$(0 - r\text{-}d)$

min

min value idx

$$\boxed{pre[l] \leq pre[r]} \longrightarrow$$

$$\underbrace{\min[\underline{p_l} \ldots\ldots p_{r-d}]}_{} > \underline{p_r}$$

$$\left.\begin{array}{l} p[l] \le p[r] \\ l \le r - D \end{array}\right\} \longrightarrow \underline{\boxed{M_{r-D} \le p_r}}$$

m → the list that will store minimum value.

↳ the list will store index

① BS

lo mid hi

② original list → original − mid

$a_i = a_i - mid$ O(n)

For any mid

③ prefix sum list → O(n)

p

④ min prefix index list → O(n)

⑤ diff value of → ∀ r → [d−1, n−1]

at any ith index

min[i] = index of min(p0, p1, p2 ..... pi)

$$avg = \frac{p[r] - p[\min[r-d]]}{r - \min[r-d]}$$

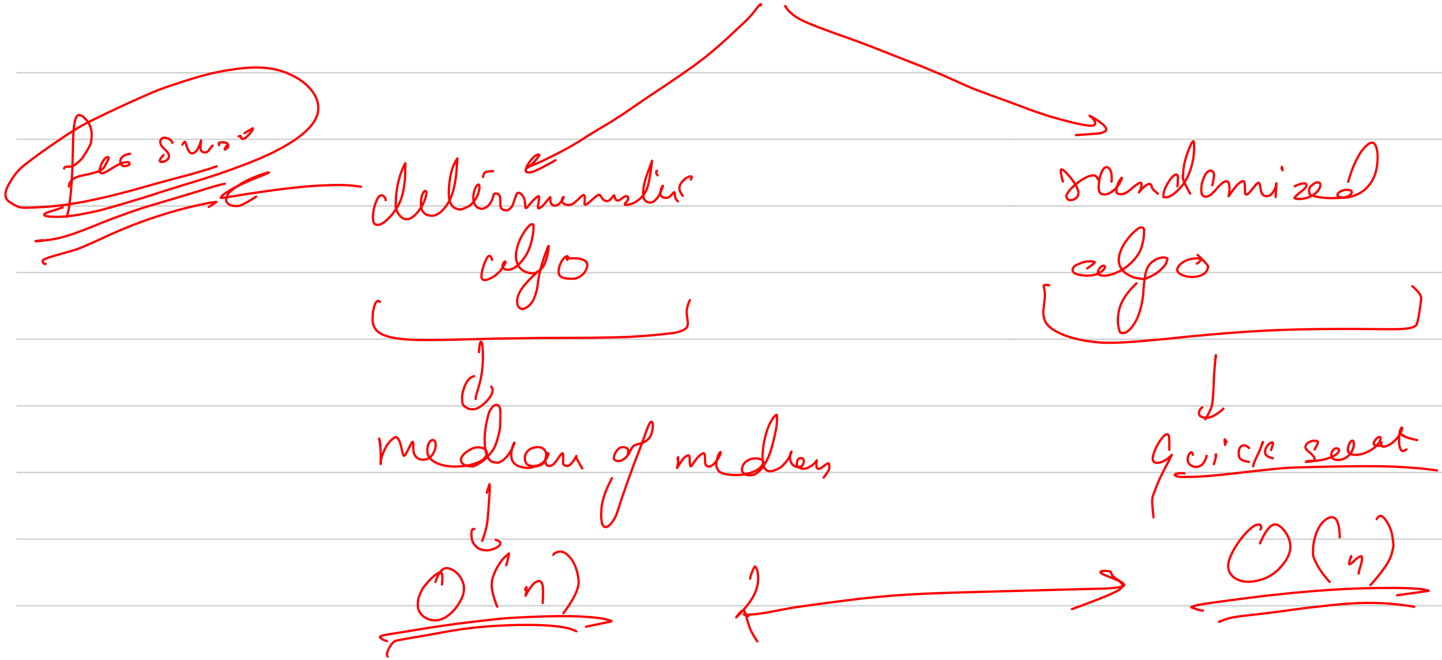
Kth order statistics / Selection algorithms

→ you have some objects find the kth object when objects will be arranged in sorted form

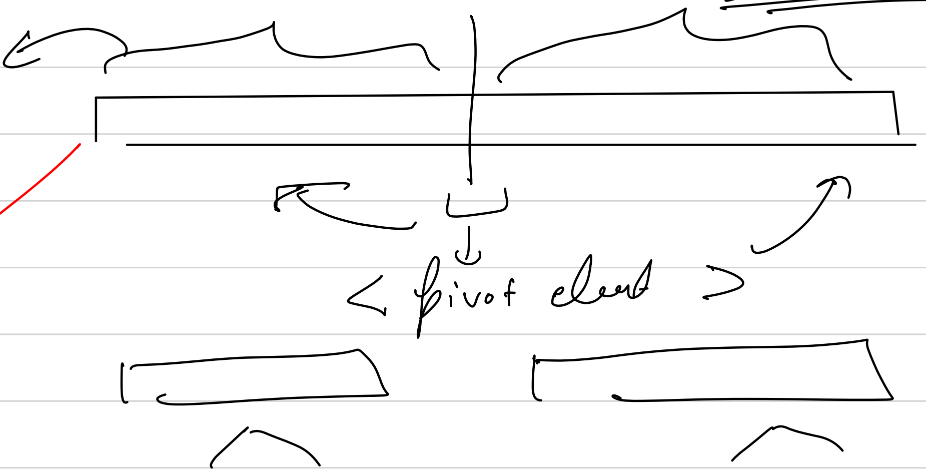
→ Basic approach

→ sort the list → $O(n\log n)$

→ return kth index


Pes sum?
deterministic algo
randomized algo
median of median
quick sort
O(n)
O(n)

Quick select



if the no of elements on the left part is > k

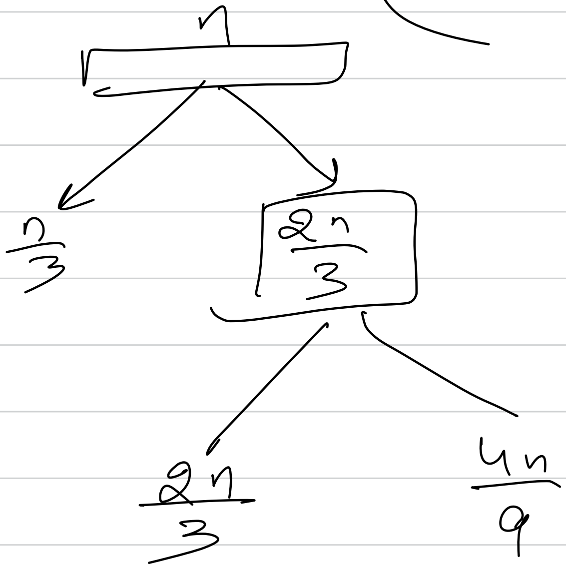
↓

kth element will be always in the left half → So no need to recursively call the right half

vice-versa

TC →

$$\left(\underbrace{n} + \frac{2n}{3} + \frac{4n}{9} + \ldots\ldots\ldots\right)$$



$$n\left(1 + \frac{2}{3} + \frac{2^2}{3} \ldots\ldots\ldots 0\right)$$

approx $\approx 3n$

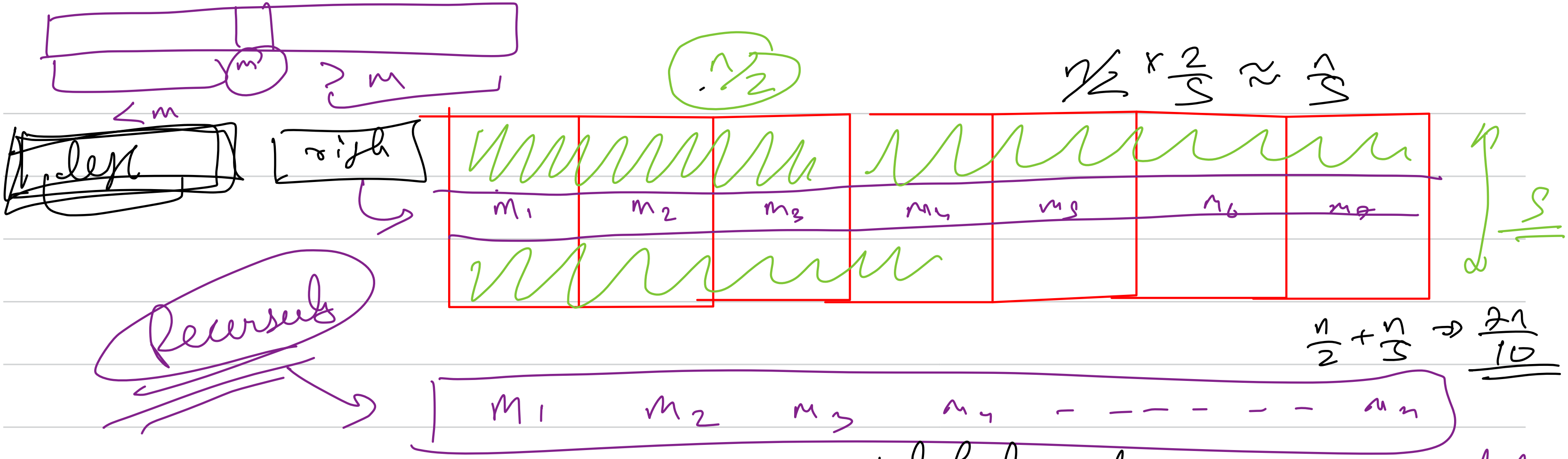
$\Theta(n)$

$O(n)$

deterministic →

divide this list into small blocks of size 5, so total $\frac{n}{5}$ blocks

we will individually sort the small blocks



median

$$T(n) = n + \overbrace{T\left(\frac{n}{5}\right)}^{\text{recursively finding median}} + T\left(\frac{7n}{10}\right) \approx O(n)$$

$$T(n) \le c \cdot n$$

$$T(n) \le \cancel{n} + \frac{c\cancel{n}}{5} + \frac{7c\cancel{n}}{10} \le c\cancel{n}$$

$$1 + \frac{c}{5} + \frac{7c}{10} \le c$$

$$1 + \frac{c}{5} + \frac{7c}{10} \leq c$$

$$1 + \frac{2c + 7c}{10} \leq c$$

$$1 + \frac{9c}{10} \leq c$$

$$1 \leq c - \frac{9c}{10}$$

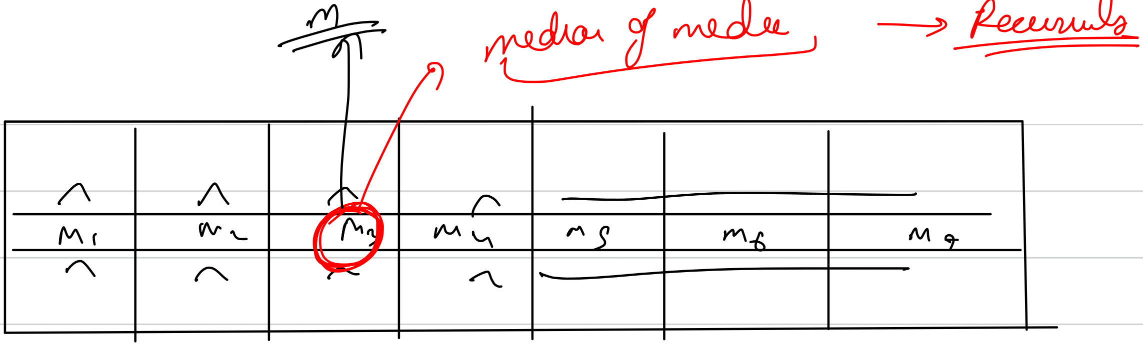
$$1 \leq \frac{c}{10}$$

$$\underline{\boxed{10 \leq c}}$$

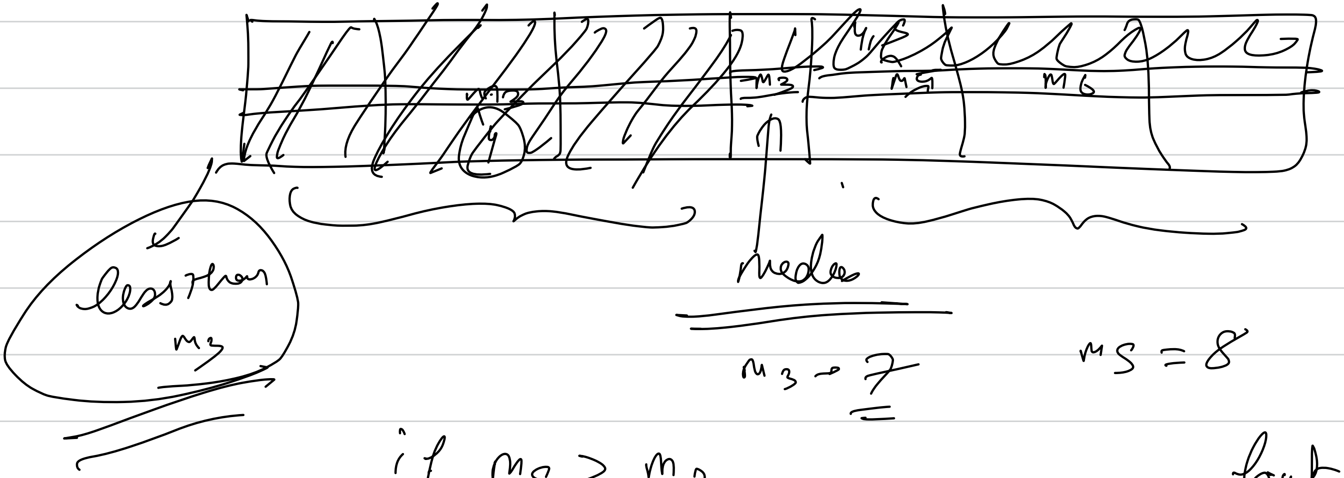
$$\underline{\boxed{10^{c}}}$$

$$10 \cdot n \rightarrow 10^{5}$$

$$\underline{\approx O(n)}$$



$M_2 = 3$



$m_3 = 7$

$m_5 = 8$

if $m_5 > m_3$

then $x > m_5$ will be also $x > m_3$

but $y < m_5$ can also be $y < m_3$

left medue right

~~dy equal~~