# Combinatorics

Motivation Problem: (In Criven infrinte supply of Ntypes of balls. find the no. of ways to choose K balls from the given set with repetition allowed.

N=3 Type A, B, C (Don't consider permutations)

K=3 (A,B,C) (CCC) (AAC) (BBA)

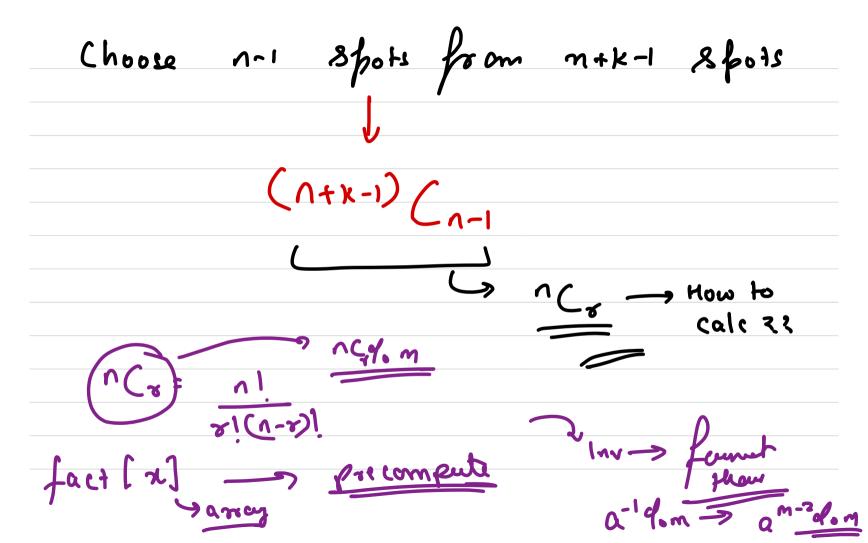
(AAA) (AAB) (BCC) Example (AAA) (MITIS) (CCA)

Berause fermutations doesn't matter, un lan take type wise decision i.e. How many balls to pick from any x type

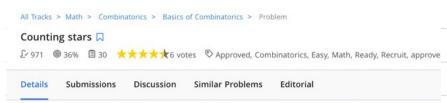
We can partition the selected choices. for n types, we have to draw 1-1

partition lines i.e., we need to count # of ways
to put (n-1) lines there.

But	how many	zositions aux	there 3.?
	U		(K+n-1)
			> (K+n-1)
			7
			Je=6
			10.50
	11		( 1:3
		11	
		-//	V
		•	
	i		



Enample Problem - You walk into a candy Diore, and got money to by 6 candies. The store has 3 diff candres C1, C2, C2. How many ways are there to select candies? n=3 K=6 6+3-1 => 8 => 28 and



# Problem

There are N stars in the sky. A manual attempt at counting yielded K stars. It is possible that the same star may have been counted more than once. You need to determine the probability that any star may have been counted more than once. Probability can be represented as a rational number  $\frac{P}{Q}$  . If Q is not divisible by  $10^9+7$  there is a unique integer  $x\mid 0\leq x<10^9+7$  where  $P\equiv Qx$ % (10<sup>9</sup> + 7). Calculate value of this integer x.

# Input Format:

First line of input consists of a single integer T denoting number of test cases. Following T lines contain two space separated integers denoting N and K.

# Output Format:

Print the answer to each test case in a new line.

## Input Constraints:

1 < T < 10

 $1 \le N, K \le 100000$ 

Sample Input	%	Sample Output	%
1 3 3		30000003	

Time Limit: 1

Memory Limit: 256 Source Limit:

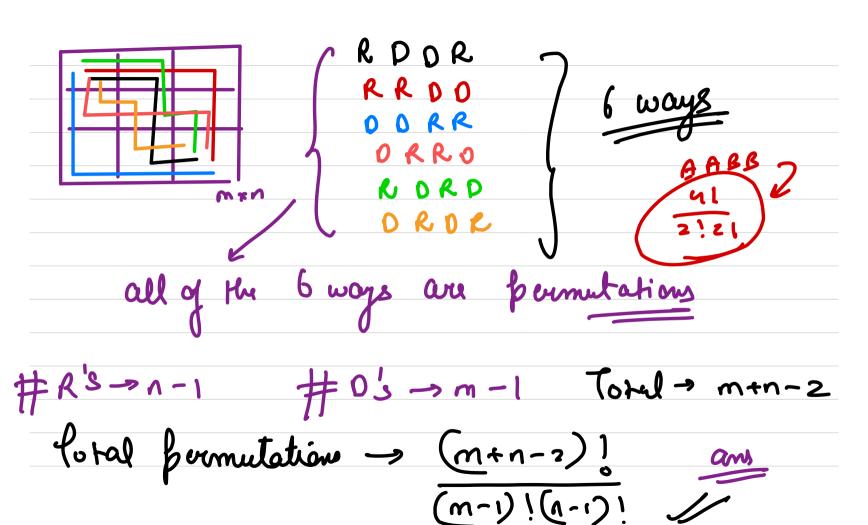
Choose k Stars

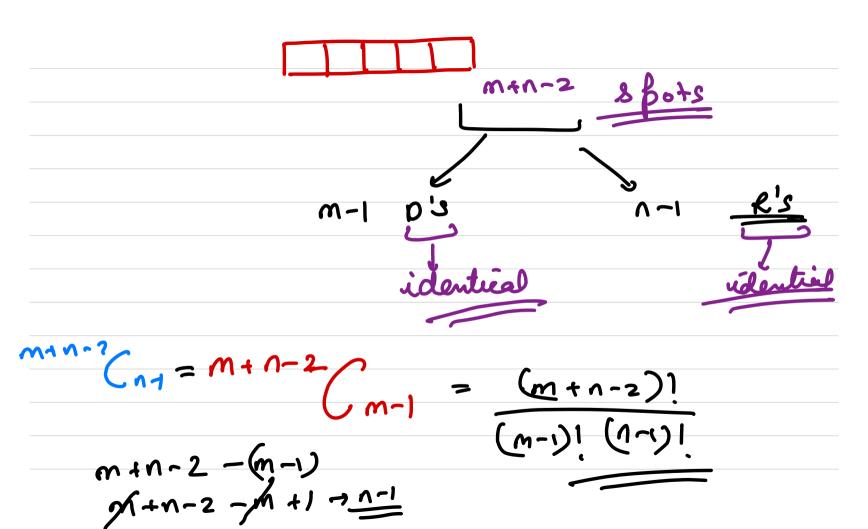
Total no. of way

probability =

Ou standing at the toplyt of the grid.

find the , 10. of ways to reach bottom right if from any cell you can only go rightwards or down words  $1 \leq m, n \leq 10^5$ m=3 , n=3  $amb \rightarrow 6$ 





# K-Special Cells

Details Submissions Discussion Similar Problems Editorial

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- You are given a  $N \times M$  matrix which has K special cells in it. You have to reach (N,M) from (1,1). From any cell, you can only move **rightwards** or **downwards**.
- The K-special cells are those cells in this grid which have special strength at them.  $i^{th}$  special cell has P[i] units of strength and if you travel through this cell, you store the strength.
- ullet Find the total strength you can store after travelling through all the possible paths in the grid to reach cell (N,M).

#### · Note:

- 1. The strength of a path is the sum of strength P[i] of all the special cells that are visited in this path.
- 2. The cells that are not special have power quotient equals to zero.

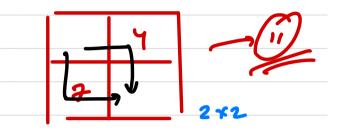
# Input format

- The first line contains the total number of test cases denoted by T.
- The first line of each test case contains three space separated integers N, M and K where N x
   M is the size of grid and K is the total number of special cells in the grid.
- Each of next K lines contains X[i], Y[i], and P[i] where (X[i],Y[i]) is the location of special cell and P[i] is the cell strength.

# W'N =10,

## Output format

• For each test case, print in a new line a single integer representing the total strength that you can store, as the total strength can be too large, print it modulo  $10^9 + 7$ .



euen make a

cell: p(i)(j)×(++ of parter it is a

if un can count the contribution of each special cell into open and, then we can solve it.

How to cake the no of forth when ling is also a part 9.2. Calle the no. of path from 0,0 to i,j, say this is Calc no. of path from i, j to n-1, m-1, say this
is y.

Total paths => xxy

$$(i-y)!(1-y)!$$

$$y = (n-i+m-j)!$$
 $(n-i)!(m-j)!$ 

Dr Commen m, n that represents a morn grid which as k blocked cells. find botal no. of ways to reach from top left to bottom sight. Total ways vlum san 61. chel cell is a fast. almost sumber to

Des Criven a value n, which represents a n-sided polygon. find the total no. of diagonals in the polygon.

# of ways to choose and rester of the diagonal verter of the diagonal first verten of the why n-3? diagonal if un han choose any ith verten fer first verten of diagonal, the a adjacent vertices, on't to the a "electron

You've n vertices, for a diagonal you need any 2 # 9 ways to chose 2 vertices from a cet of n is n Cz in 102 un have also chosen the 2 vertices who well from sides. So fend ans -> n (2 -n) # of sides

$$\frac{n!}{2!(n-2)!}$$

$$\frac{n(n-1)}{2}$$

$$\frac{n\left(n-1\right)}{2}$$



# Binomial Coefficients.

$$(x+y)^{2} = {}^{2}C_{0}x^{2}y^{2} + {}^{2}C_{1}xy^{2} + {}^{2}C_{1}x^{2}y^{3}$$

$$(x+y)^{2} = {}^{2}C_{0}x^{2}y^{2} + {}^{2}C_{1}x^{2}y^{2}$$

$$(x+y)^{2} = {}^{3}C_{0}x^{2}y^{3} + {}^{3}C_{1}x^{2}y^{4} + {}^{3}C_{2}x^{2}y^{4}$$

$$= y^{3} + 3xy^{2} + 3x^{2}y + x^{3}$$

What is 
$${}^{\circ}C_{0} + {}^{\circ}C_{1} + {}^{\circ}C_{2} - \cdots - {}^{\circ}C_{n} + {}$$

Des There are some cares of 4 diff companies There is a linear banking space of 2n-2 capacity Total cars of each type is more than 2 n-2. Calc the no. of ways in which enactly n consecution care of same type are parked there. 3 = 1 = 30 1=3 ms 24

AAAB AAAC AAAD DBBB ----

20-7 510 45 Degt choise in 3 w initial n spots we have

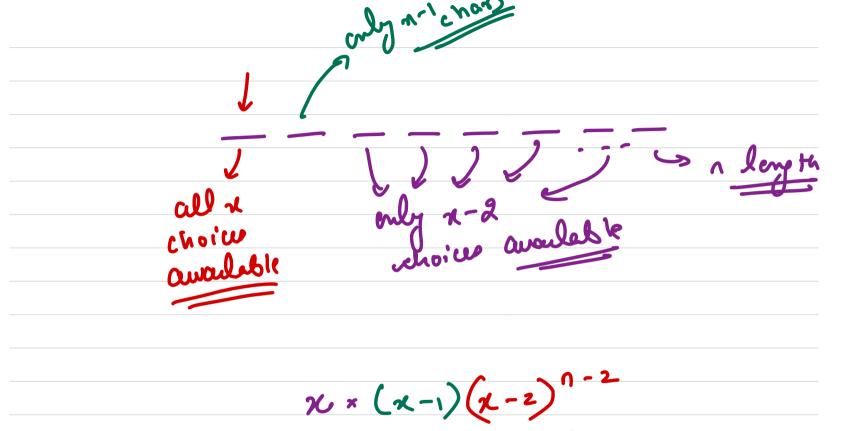
Totalways מש שייחש שי 2 × 4 × 3 × 4 +

# I. Parking Lot

time limit per test: 0.5 seconds	
memory limit per test: 64 megabytes	
input: standard input	
output: standard output	
To quickly hire highly skilled specialists one of the new IT City companies made an	
unprecedented move. Every employee was granted a car, and an employee can choose one of four different car makes.	
The parking lot before the office consists of one line of $(2n-2)$ parking spaces.	
<ul> <li>Unfortunately the total number of cars is greater than the parking lot capacity. Furthermore even amount of cars of each make is greater than the amount of parking spaces! That's why</li> </ul>	
there are no free spaces on the parking lot ever.	
Looking on the straight line of cars the company CEO thought that parking lot would be more	
beautiful if it contained exactly $n$ successive cars of the same make. Help the CEO determine	
the number of ways to fill the parking lot this way.	
Input	
The only line of the input contains one integer $n$ ( $3 \le n \le 30$ ) — the amount of successive	
cars of the same make.	
Output	
Output one integer — the number of ways to fill the parking lot by cars of four makes using	
the described way.	
Examples	
input	
3	
output	
24	
Note	
_Let's denote car makes in the following way: A — Aston Martin, B — Bentley, M —	
Mercedes-Maybach, $Z-Z$ aporozhets. For $n=3$ there are the following appropriate ways to	
fill the parking lot: AAAB AAAM AAAZ ABBB AMMM AZZZ BBBA BBBM BBBZ BAAA BMMM	
BZZZ MMMA MMMB MMMZ MAAA MBBB MZZZ ZZZA ZZZB ZZZM ZAAA ZBBB ZMMM	

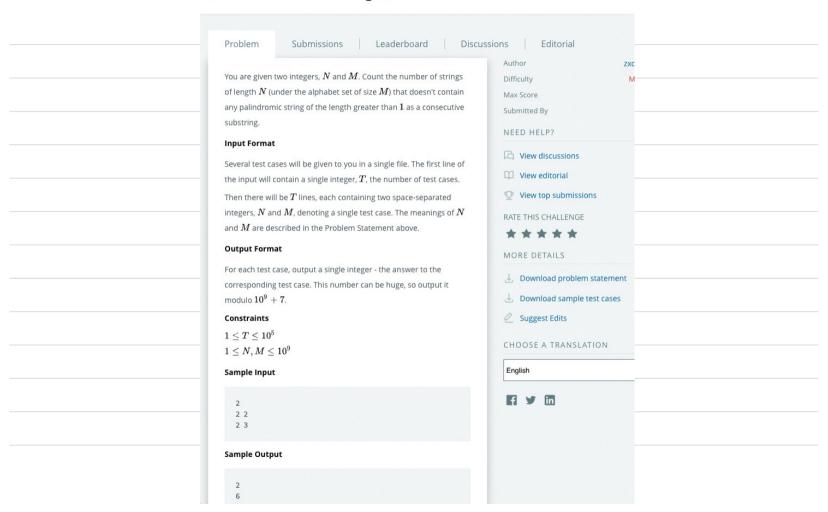
Originally it was planned to grant sport cars of Ferrari, Lamborghini, Maserati and Bugatti makes but this idea was renounced because it is impossible to drive these cars having small road clearance on the worn-down roads of IT City.

(ount the no. of strings of length n, that doesn't contain any falmdromic Substring of length greater than 1. Answer T such test coses. T = 105 x1n = 109 21 → T=1 X=2 n=2 anu - 2

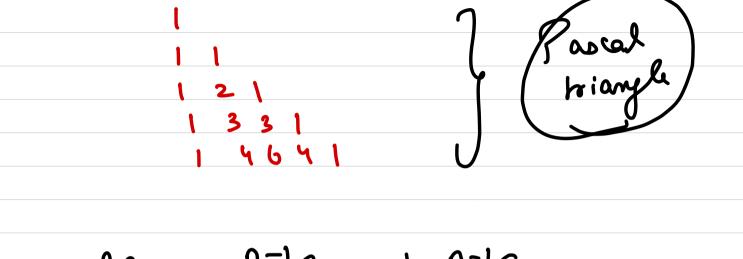


buy enpo

FUIIILD, V NGIIK, ZZ4



any palendron Samo che



 $_{0}C^{x} = _{0-1}C^{x-1} + _{0-1}C^{x}$ 

dp[n][r] = dp[n-1][r-1] + dp[n-1][r]

LO(n) - spau

Can me oplemen space & make it less then O(n) (?

$$\frac{U(x-1)}{U(x)} = \frac{A}{1} \times (U-A+1)$$

$$\frac{U(x-1)}{U(x)} = \frac{A}{1} \cdot \frac{(U-A+1)}{1}$$

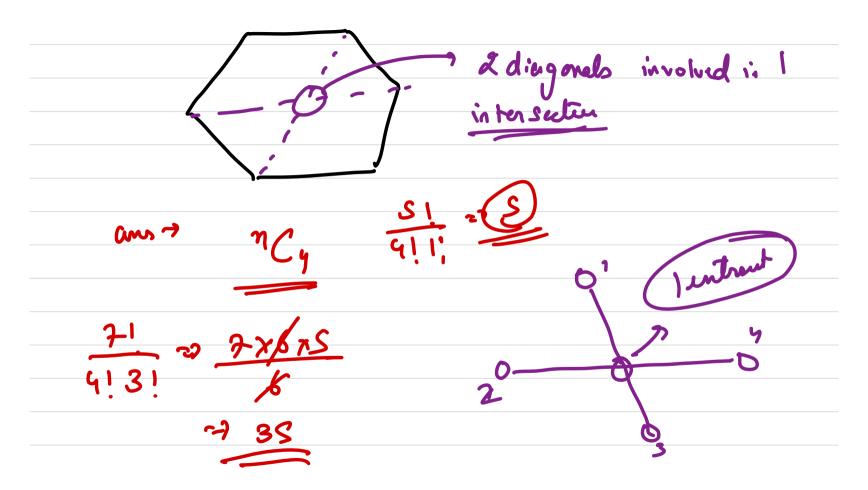
$$\frac{U(x-1)}{U(x)} = \frac{(X-1)}{1} \cdot \frac{(U-A+1)}{1}$$

$$\frac{A}{1} \cdot \frac{(U-A)}{1}$$

$$\frac{A}{1} \cdot \frac{(U-A)}{1}$$

then amoy n-1 clemets pick only of or clements Y-1 remay r element

Dr Crum a number n, find the no. of intersection points of the diagonals in a polygon of 1 voctices gener that no 3 diagonals unlesseet in the poly gon



find the value of sum of odd terms of binomial? To fund > 1(, +1(3+1(5 ----(2+y) = 1(0xy + 1(1xy + 1) (2x2y -----Put x = -1 y = 1  $O = (d-1)(+1)^{2} + (-1)(+1)^{2-1} + (2+1)^{2-1} + (2+1)(+1)^{2-1} + (2+1)^{2-1$ 

~ ( 0 + ~ ( 2 + ~ ) ( 4 - - - - · · = ~ ( 1 + ~ ( 3 + ~ ( 3 - - - · · ·

and we know

$$1(0 + 1)(1 + 1)(2 + 1)(3 - - - - - 1)(n = 2)$$
  
 $2(1(1 + 1)(3 + 1)(3 - - - - - 1) = 2^{1}$ 

10, + 103 + 10 --

2 ans