Probability

* **Bernaulli Trial** → An experiment is called a bernaulli trial if it has exactly 2 outcomes, one of which is desired.

Example

1. fair coin throw → head / tail
2. fair dice roll → get a 5
3. Success / failure

2

$x = \frac{1}{6}(1) + \frac{5}{6}(x+1)$

$\boxed{x = 6}$

Q: Given that success probability of a Bernoulli trial is 'p', then what is the expected no of **trials** to get a **success**.

$$= \frac{1}{p}$$

✓

Expectation / Expected value

→ It is the average value of random variable where each value is weighted acc to probability

→ probability of event multiplied by the amount of times event happen

→ $p(x) \times x$

Q. If you toss a coin ten times, the coin is fair, How many heads you expect in 10 tosses.

$\rightarrow P(x) \times x \rightarrow \frac{1}{2} \times 10 \Rightarrow 5$

for multiple events

discrete

$$\sum p(x_i) * x_i$$

continuous

$$\int x\, p(x)\, dx$$

Qn If probability of success in Bernoulli trial is 'p', find the expected no of times we will get success in n **trials**.

↳ $\underline{\underline{np}}$

Q n friends have to choose a number in the range $[1, 100]$. find the expected no. of friends that would choose a single digit no.

$$p_{success} = \frac{9}{100}$$

ans $\rightarrow$ $\frac{9n}{100}$

Q→ You are choosing which choco to buy. Prob of buying a choc is 0.16. Find the expected no. of choc you will consider before selecting a choco.

$$p = \frac{16}{100} \longrightarrow \frac{1}{p} \Rightarrow \frac{100}{16}$$

$$x = 0.16(1) + (1-0.16)(x+1)$$

$$x = 100/16$$

Qn You've a chips company that stores coupon inside packets. The coupon chosen for each pack is chosen randomly from a set of 'n' distinct coupons. Find expected no. of packets customer should buy to get all n unique coupons.

(Coupon collector problem)

$E[X_i] \rightarrow$ Expectation of # of packets to get i^{th} new coupon

$\rightarrow \dfrac{1}{p}$

$p = \dfrac{n-(i-1)}{n}$

$\underline{\underline{(i-1)}}$

$\underline{\underline{4}}$

$\underline{\underline{10}}$

$\rightarrow \underline{\underline{\dfrac{6}{10}}} \quad \underline{\dfrac{4}{10}}$

$$E[X_i] = \underline{\underline{\dfrac{n}{n-i+1}}}$$

$$E[x] = E[x_1 + x_2 + x_3 - - - - - x_n]$$

$$= E[x_1] + E[x_2] - - ——— \underline{\underline{E[x_n]}}$$

$$= 1 + \frac{n}{n-1} + \frac{n}{n-2} + \frac{n}{n-3} \cdots\cdots \frac{n}{1}$$

$$\underline{\underline{E[x]}} = n\left(1 + \frac{1}{2} + \frac{1}{3} \cdots\cdots \frac{1}{n}\right)$$

$$p_1 = \underline{\underline{1}} \qquad p_2 \Rightarrow \frac{n-1}{n}$$

Q. Find the expected no. of coin flips to get N consecutive heads.

$$x = \frac{1}{2}(1) + \frac{1}{2}(x+1)$$

for any head.

$$x = \frac{1}{2}(x+2)$$

$$2x = x + 2$$

$$\boxed{x = 2}$$

$$T^2 \qquad TT \cdots T \qquad TT^2 \qquad TT \cdots T^2$$

$$X = \frac{1}{2^n} x\, n + \frac{1}{2}(x+1) + \frac{1}{2^2}(x+2) \cdots\cdots \frac{1}{2^n}(x+n)$$

$$X = x\left(\overbrace{\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} \cdots\cdots \frac{1}{2^n}}^{GP}\right) +$$

$$\underbrace{\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} \cdots\cdots \frac{n}{2^n}}_{AGP}$$

Rule of Linearity Of Expectation

$$E[X+Y] = \underbrace{E[X] + E[Y]}$$

expected value of sum of random variable

sum of individual expected value irrespective they're dependent or not.

Q→ Let's say we have 2 die, find the expected sum of no. from 2 die.

↳ independent

$E[D_1 + D_2] =$

$E[D_1] + E[D_2]$

$3.5 + 3.5$

→ 7

	1	2	-	-	-	-
1	2	3				

→ If the sum of no. rolled on the 2 dice is A & product is B.

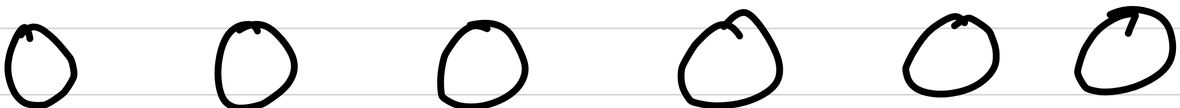
Calc $E[A+B] = E[A] + E[B]$

$7 + 3.5 \times 3.5$

$\Rightarrow$ //

$E[B] = E[D_1 \times D_2] \Rightarrow \underline{\underline{E[D_1] \times E[D_2]}}$

Q=> We throw a fair coin, N times. find expected no. of heads.



$x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_n$

$$E[x_1 + x_2 + x_3 \cdots x_n] = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \cdots \frac{1}{2}$$

$$\Rightarrow \frac{n}{2}$$