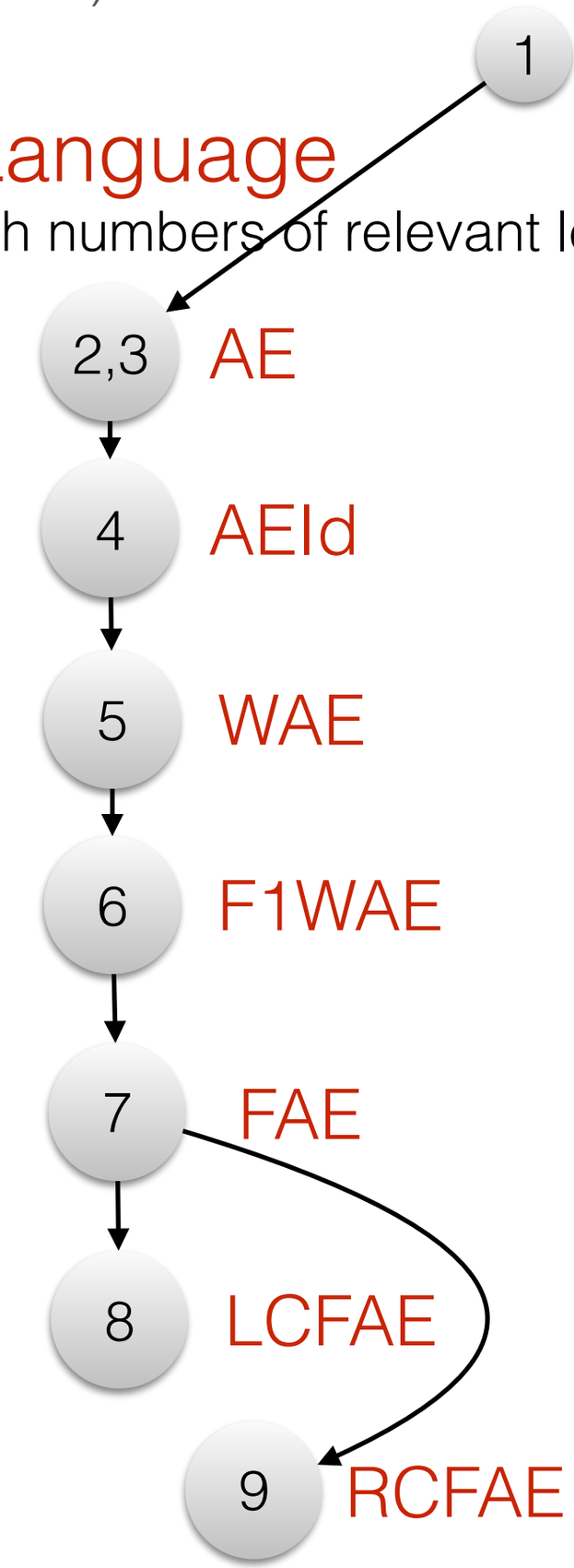


→ = possible reading order

Concepts covered

Interpretation, Desugaring

Environments, Visitors (+ Currying)

Binding (+ Scope), Substitution

First-Order Functions,
Static & Dynamic Scoping

Higher-Order Functions,
Substitution- Vs. Environment-Based Interpreter,
Closures (!)

Call-By-Name, Call-By-Need,
Thunks

Recursion with **`Letrec`**

2,3 AE

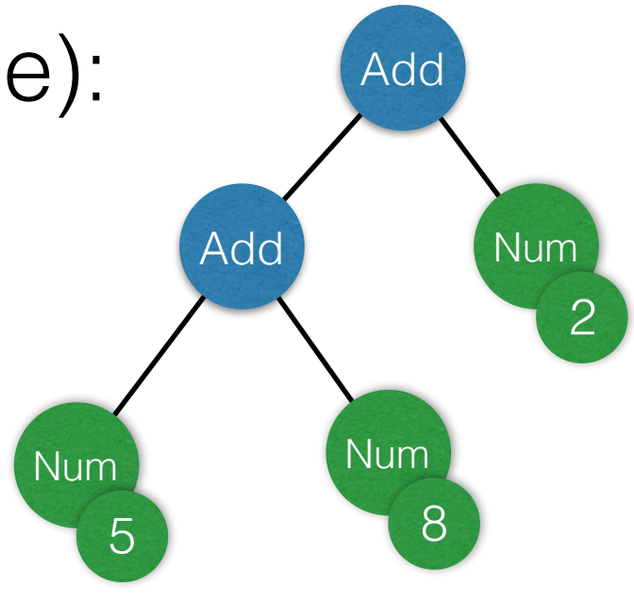
A formal language consists of:

- types of nodes, each with
- arity, and possibly
- other information (data)

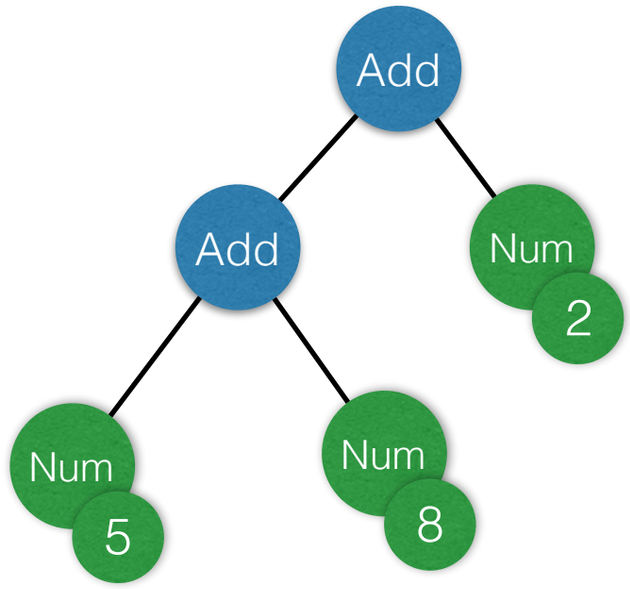
In AE, those are

- Num: arity 0, other data: a number
- Add: arity 2

An example term in tree form (Abstract Syntax Tree):



2,3 AE

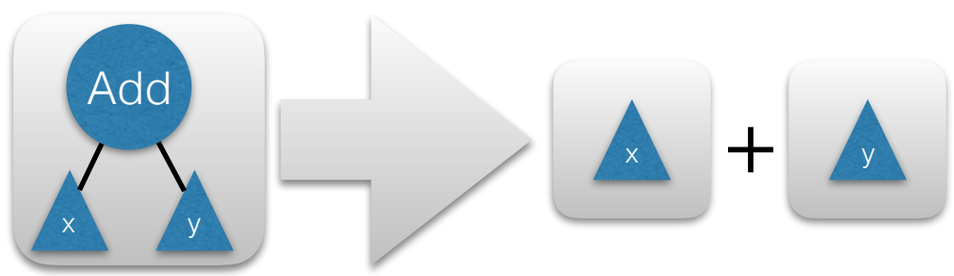


Interpretation: assigns a **value** to each term of the language

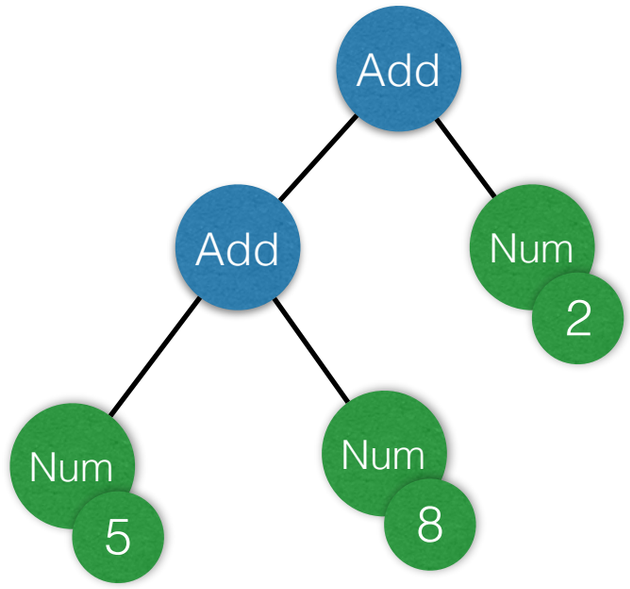
Example (Scala):

```
def eval(e: Exp): Int =
    e match {
      case Num(n) => n
      case Add(lhs, rhs) =>
        eval(lhs) + eval(rhs)
    }
```

visually:



2,3 AE



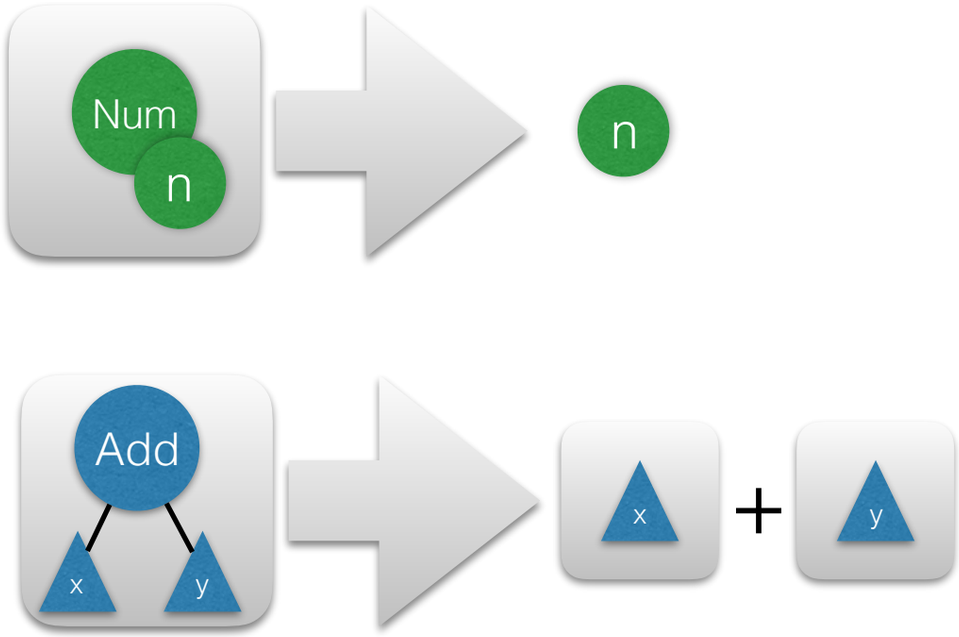
Interpretation: assigns a **value** to each term of the language

Example (Scala):

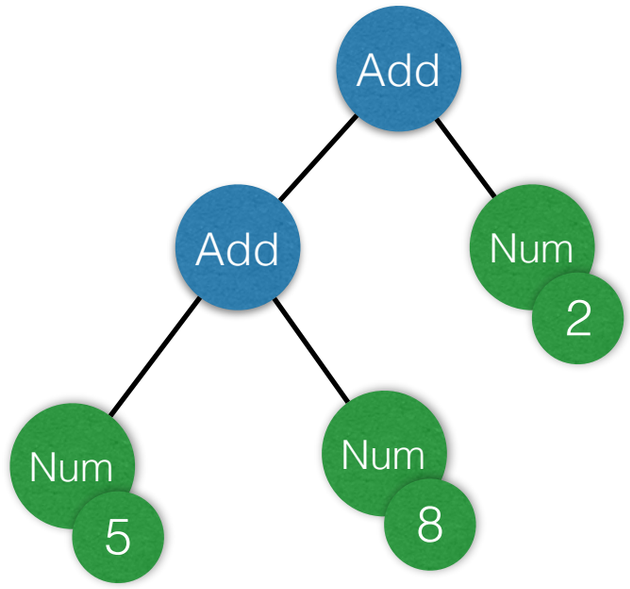
```
def eval(e: Exp): Int =
    e match {
      case Num(n) => n
      case Add(lhs, rhs) =>
        eval(lhs) + eval(rhs)
    }
```

Matching on the **node type**

visually:



2,3 AE



Interpretation: assigns a **value** to each term of the language

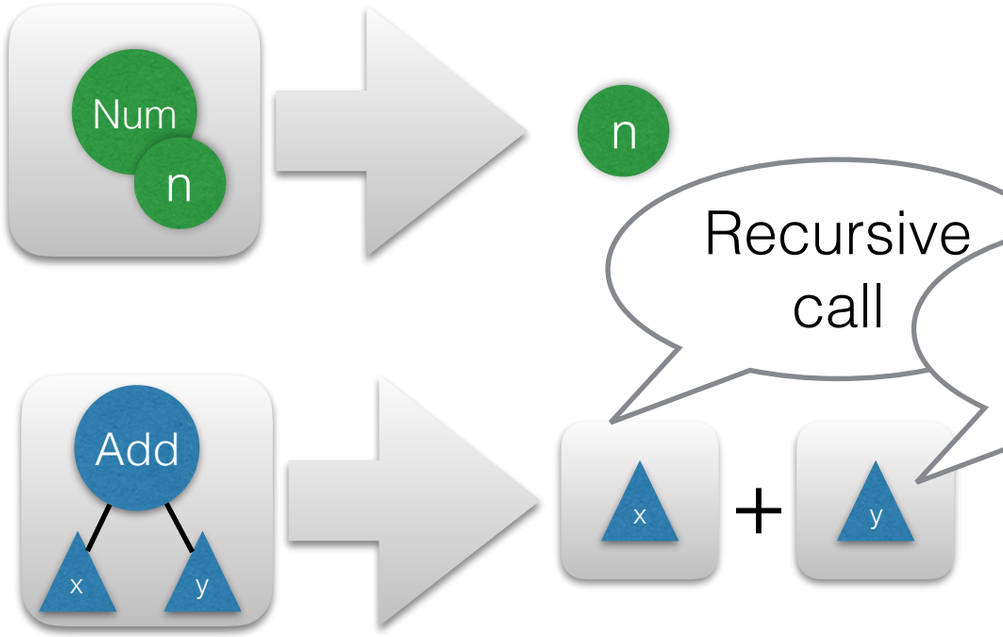
Example (Scala):

```
def eval(e:                =
    e mat
      case
      case                ad(lhs, rhs)  =>
        eval(lhs) + eval(rhs)
    }
```

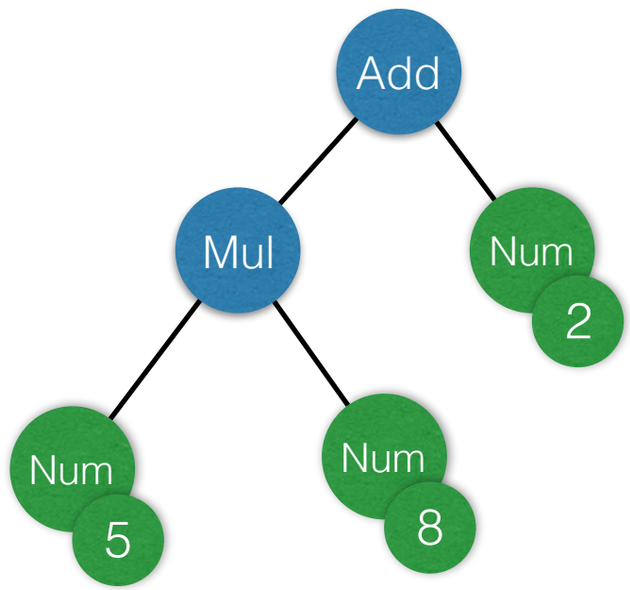
Recursive call

Recursive call

ually:



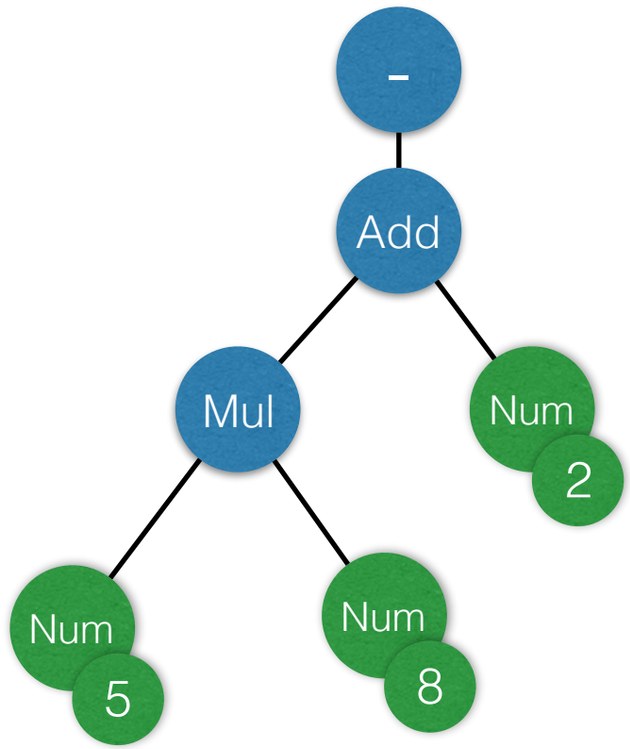
2,3 AE



Mul

We add a new node type for multiplication

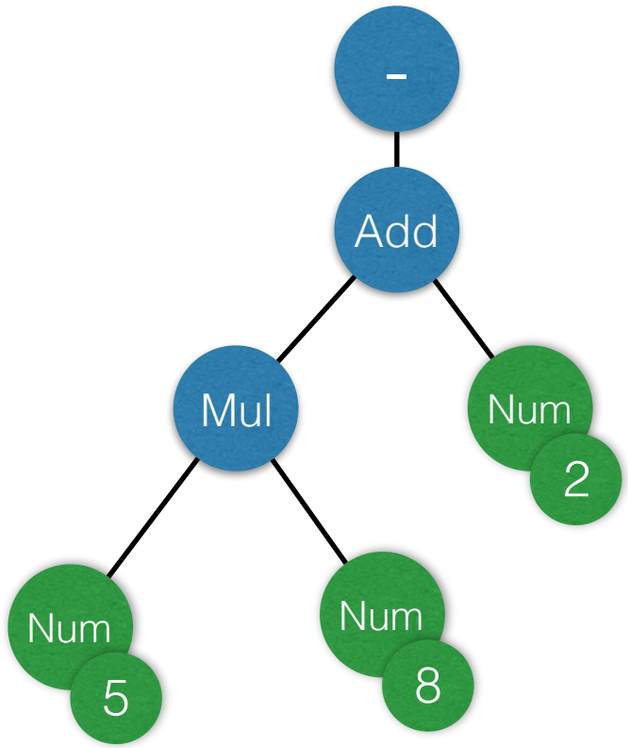
2,3 AE



-

Then we add another new node type for unary minus

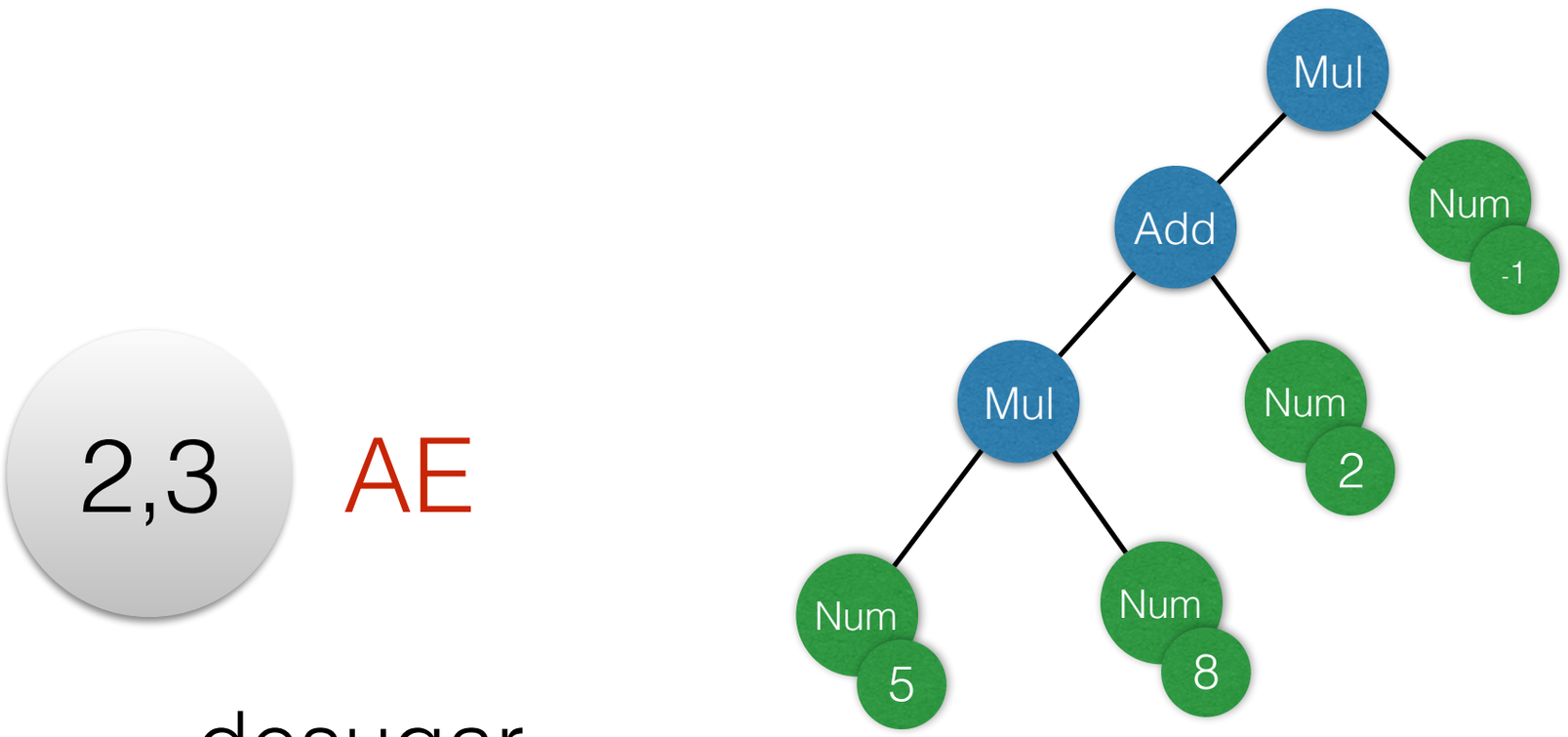
2,3 AE



But we do not really need a new node type for this.

Desugaring: transforms the AST to eliminate a node type

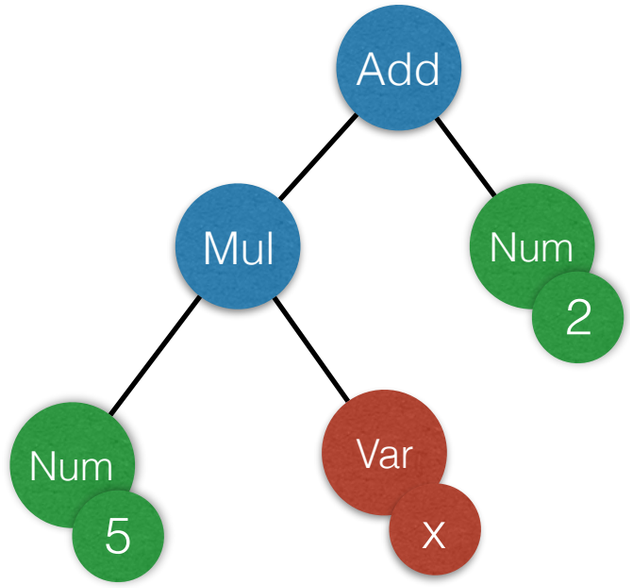
1. Semantics must stay the same.
2. Ideally we do not want to look into the child nodes for this.



Desugaring: transforms the AST to eliminate a node type

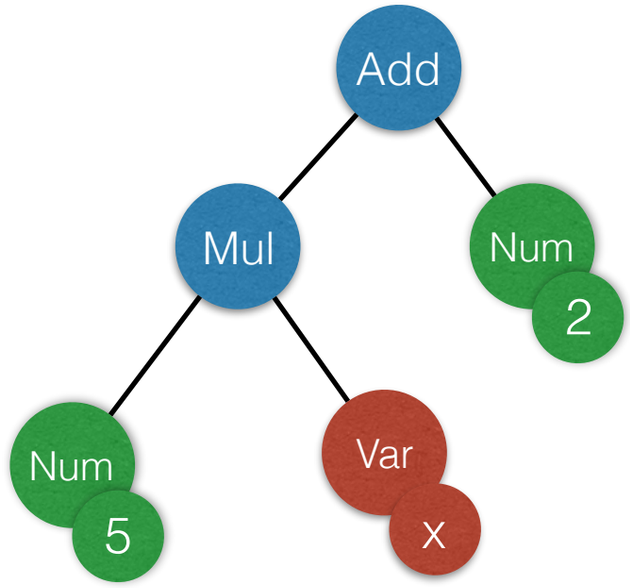
1. Semantics must stay the same.
2. Ideally we do not want to look into the child nodes for this.

4 AEId



We add a new node for a new feature: **variables**

4 AEId

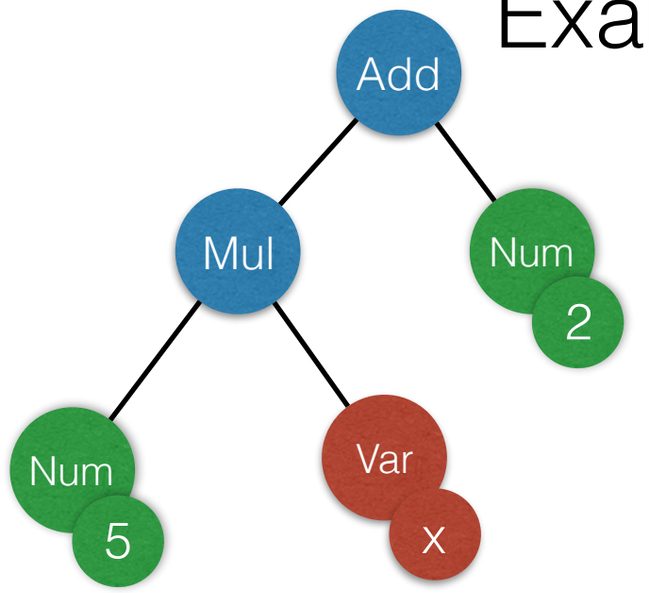


We add a new node for a new feature: **variables**

On its own, a term does not have meaning anymore.

> We need to give the interpreter a map from variable names to values.
> This map is called an environment.

4 AEId



Example environment:

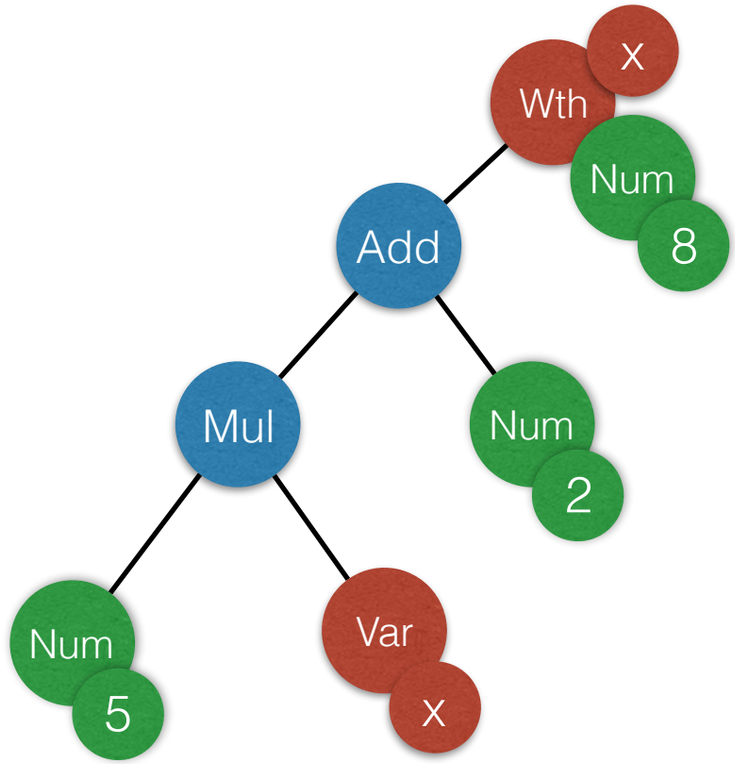
x —> 8

On its own, a term does not have meaning anymore.

We need to give the interpreter a map
from variable names to values.
This map is called an environment.

5

WAE

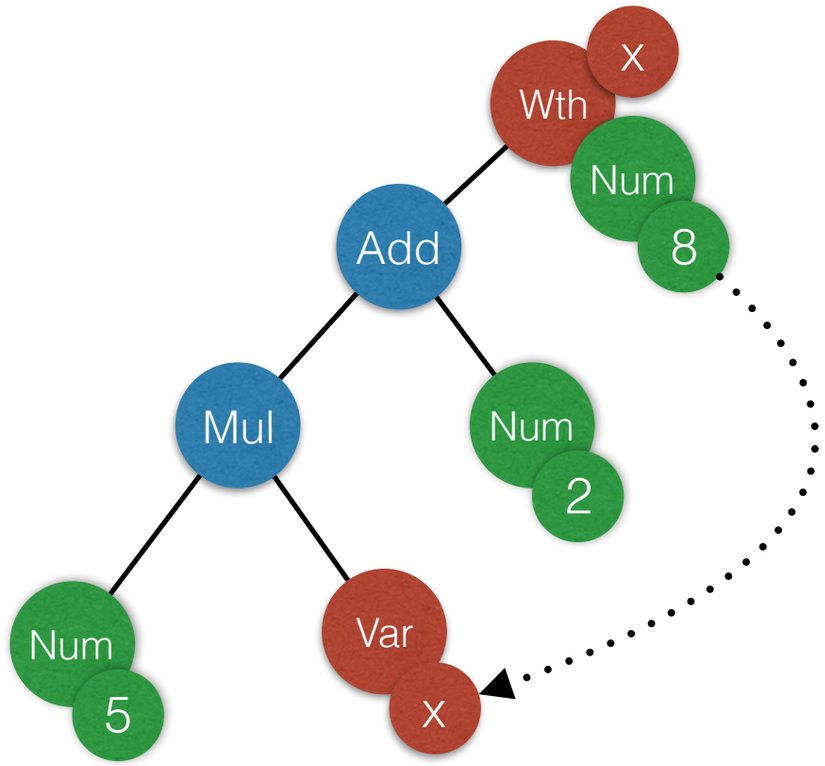


Wth

We add a new node for binding variables

5

WAE

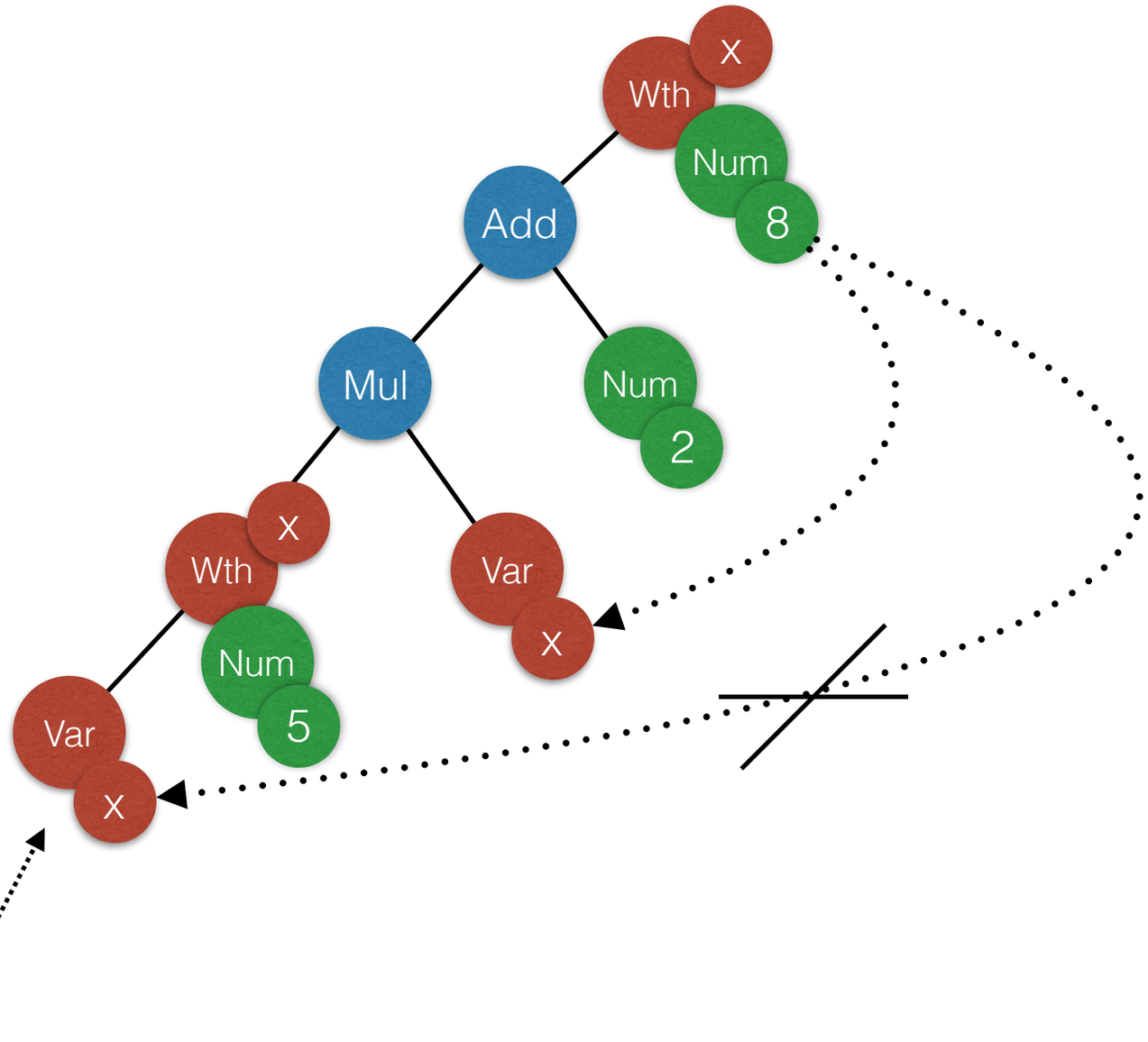


Wth

We add a new node for binding variables

An interpreter can produce a value from such a term by: substituting the value from evaluating the bound term for all **free** variables with the name

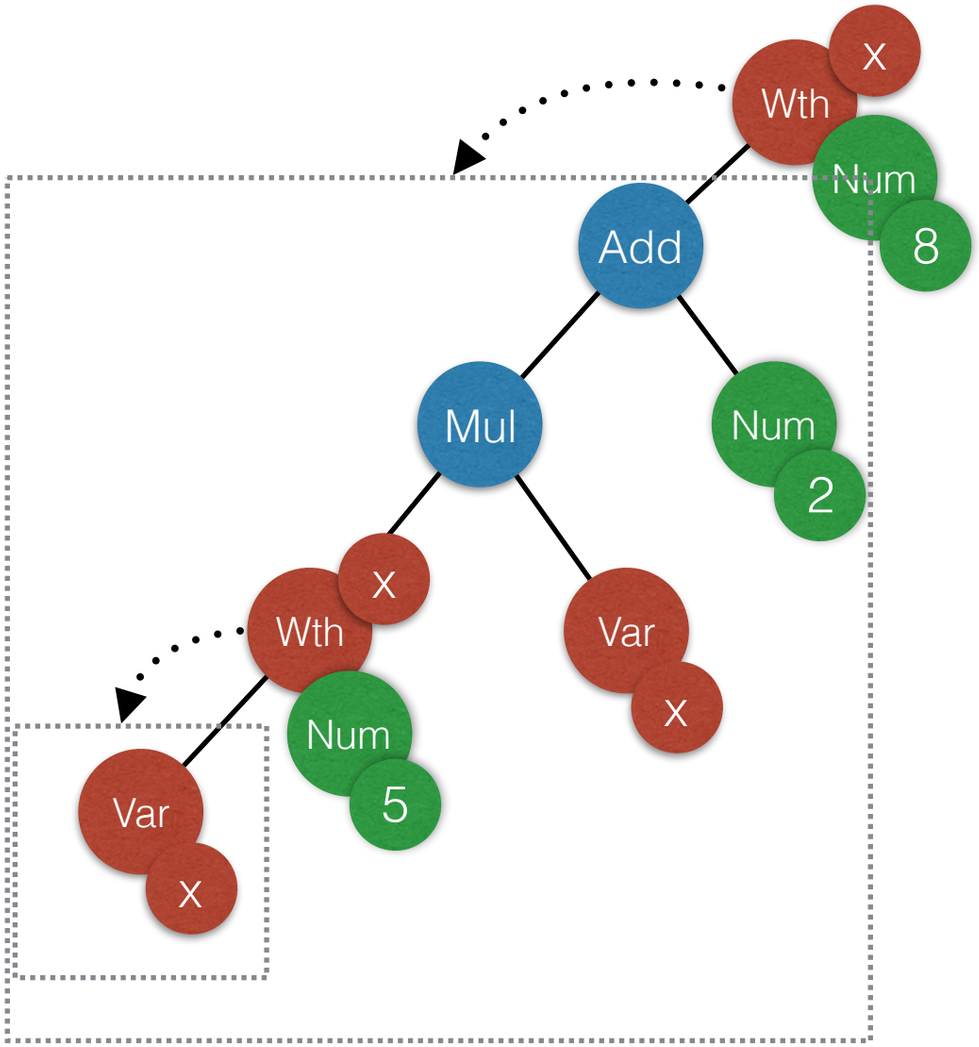
5 WAE



not free in the overall term

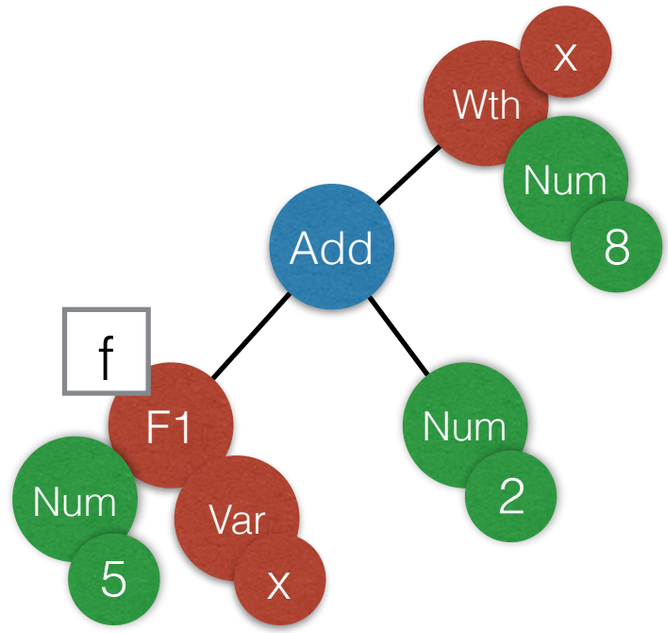
5

WAE



Static (lexical) scope

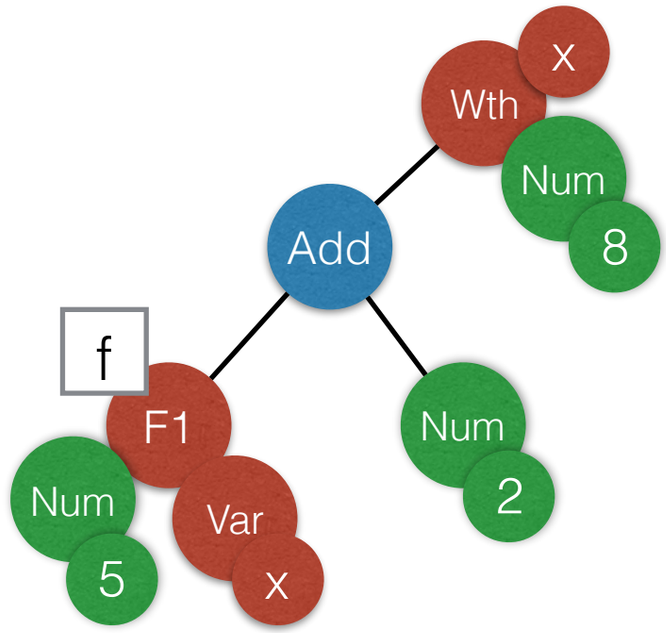
6 F1WAE



F1

We add a new node for calling first-order functions

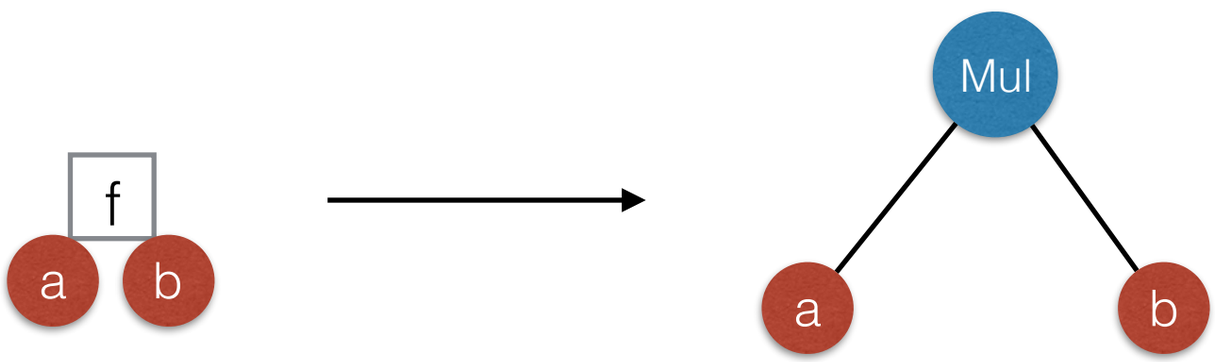
6 F1WAE



F1

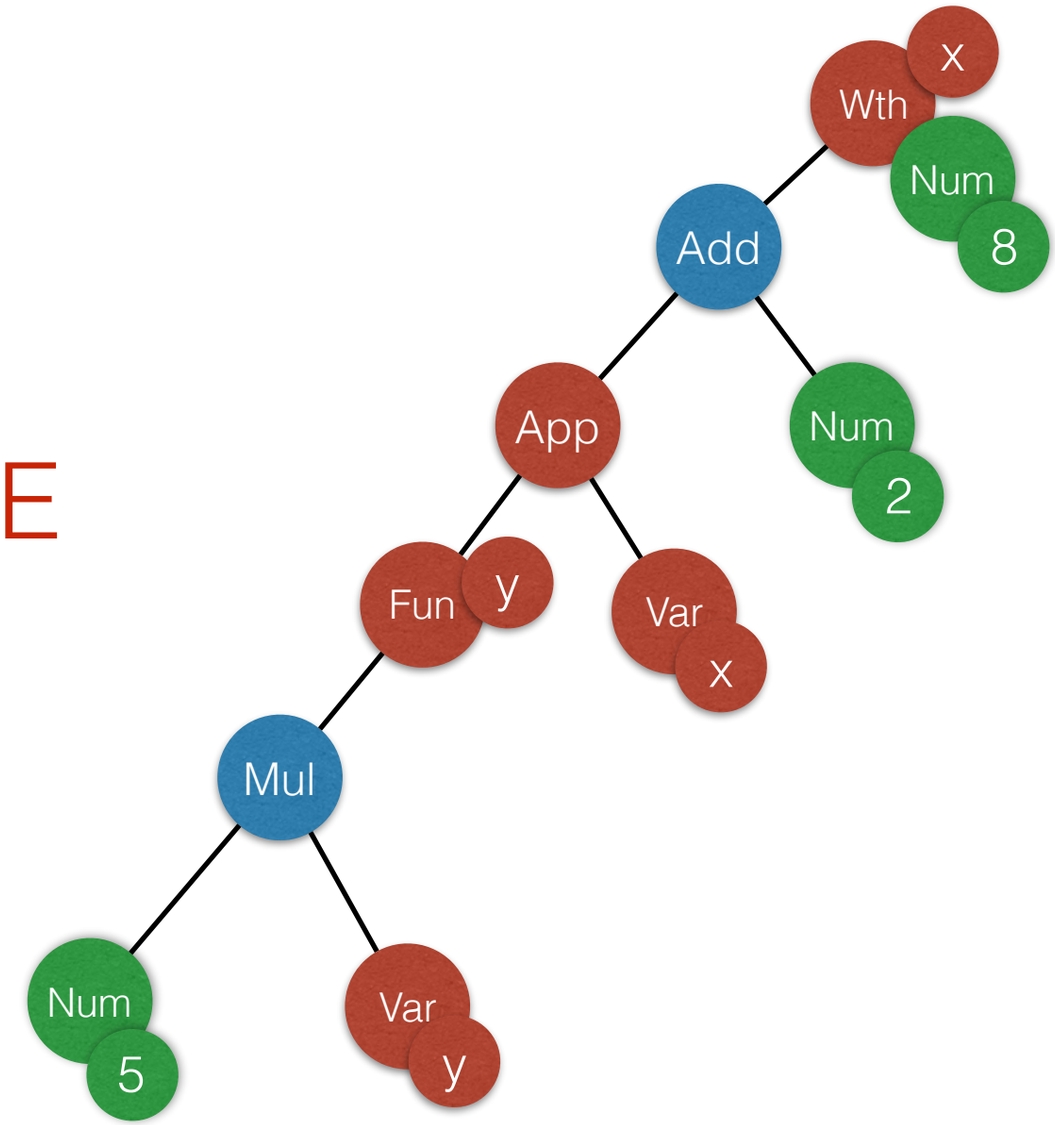
We add a new node for calling first-order functions

We have a dictionary with an entry per function:



7

FAE

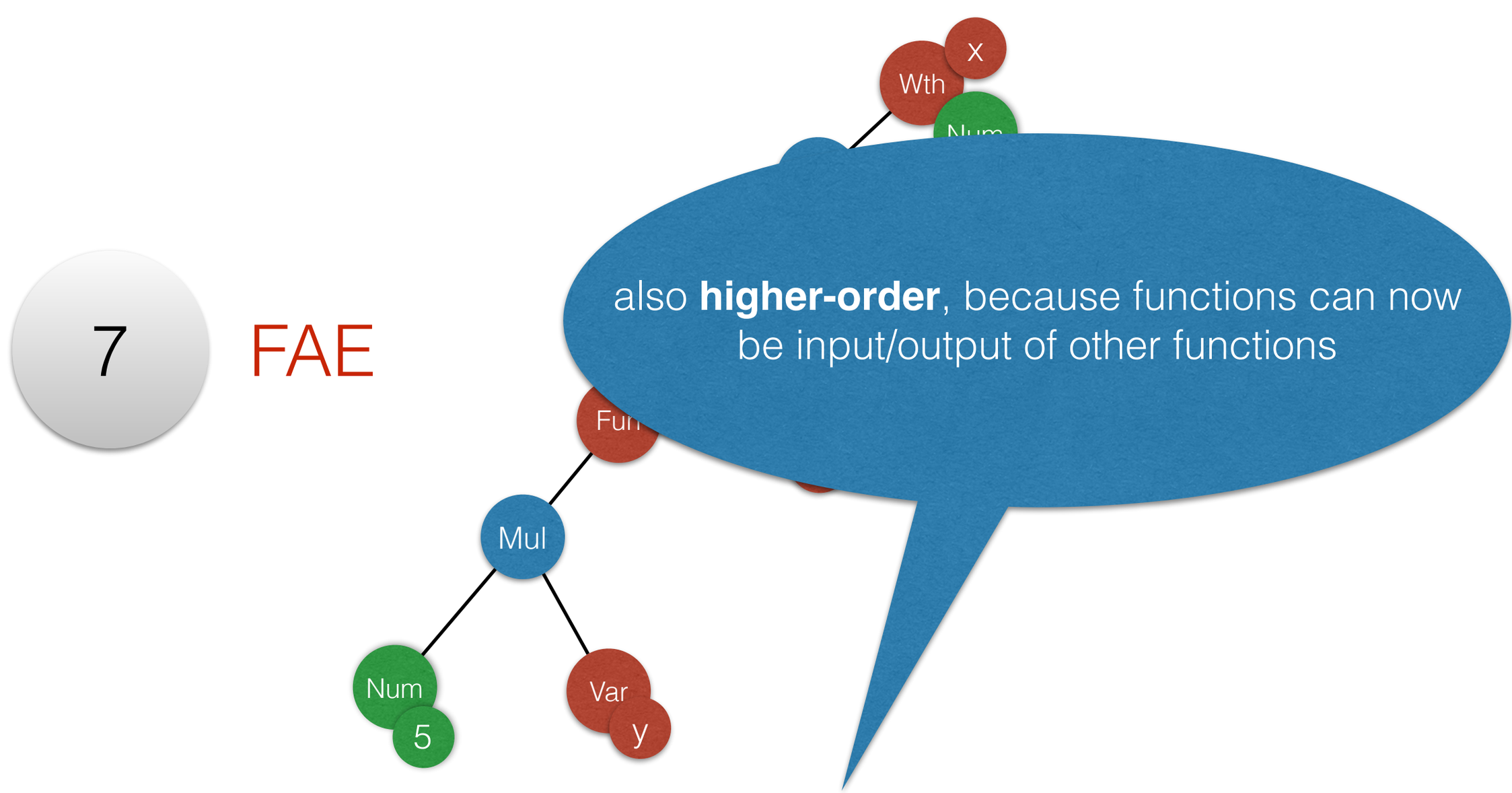


Fun Now we turn functions into **first-class** objects,
that is, their definition happens locally in the AST

App Thus we also need a new node to call, or **apply**, them

7

FAE

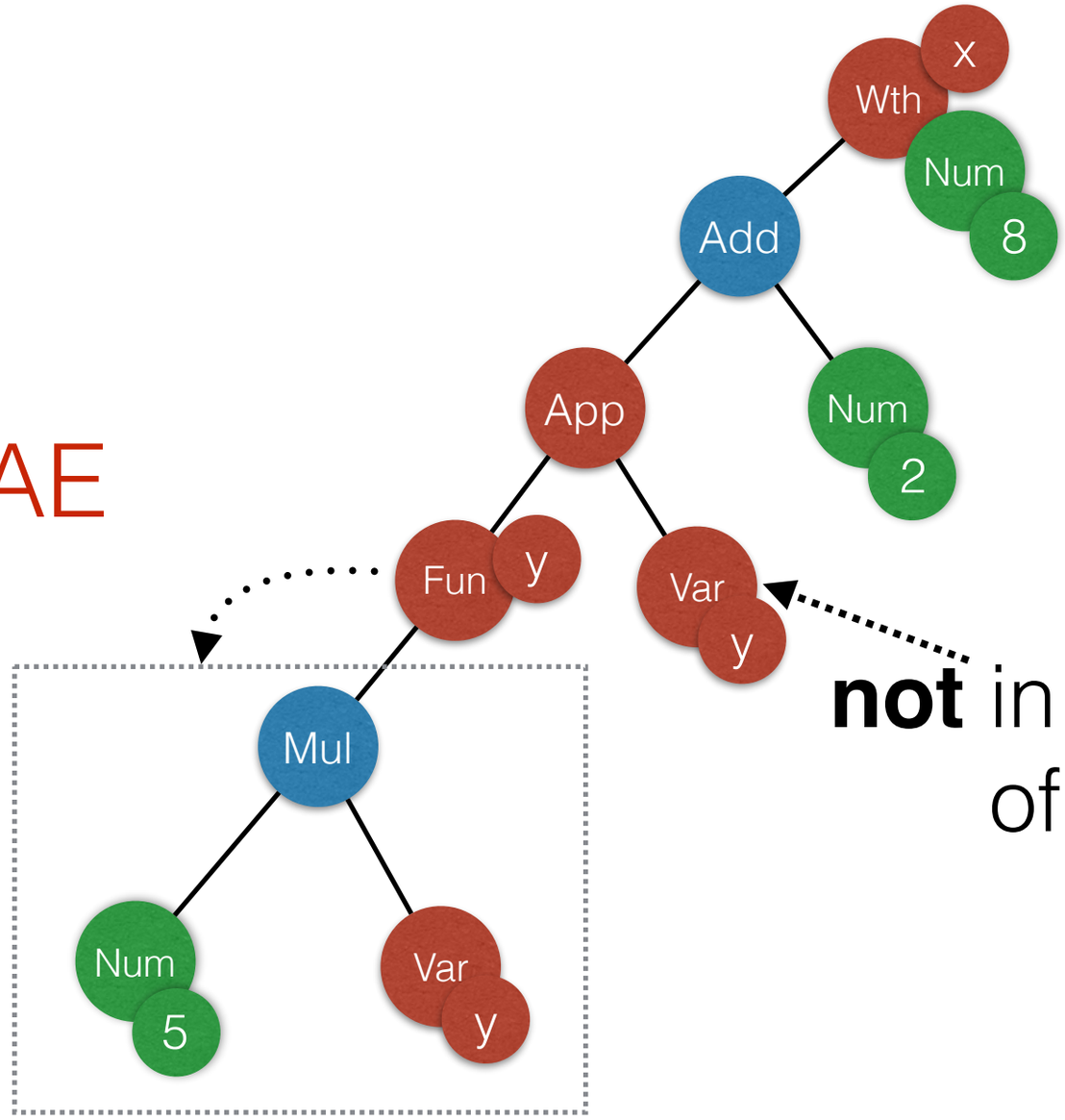


Fun Now we turn functions into **first-class** objects, that is, their definition happens locally in the AST

App Thus we also need a new node to call, or **apply**, them

7

FAE

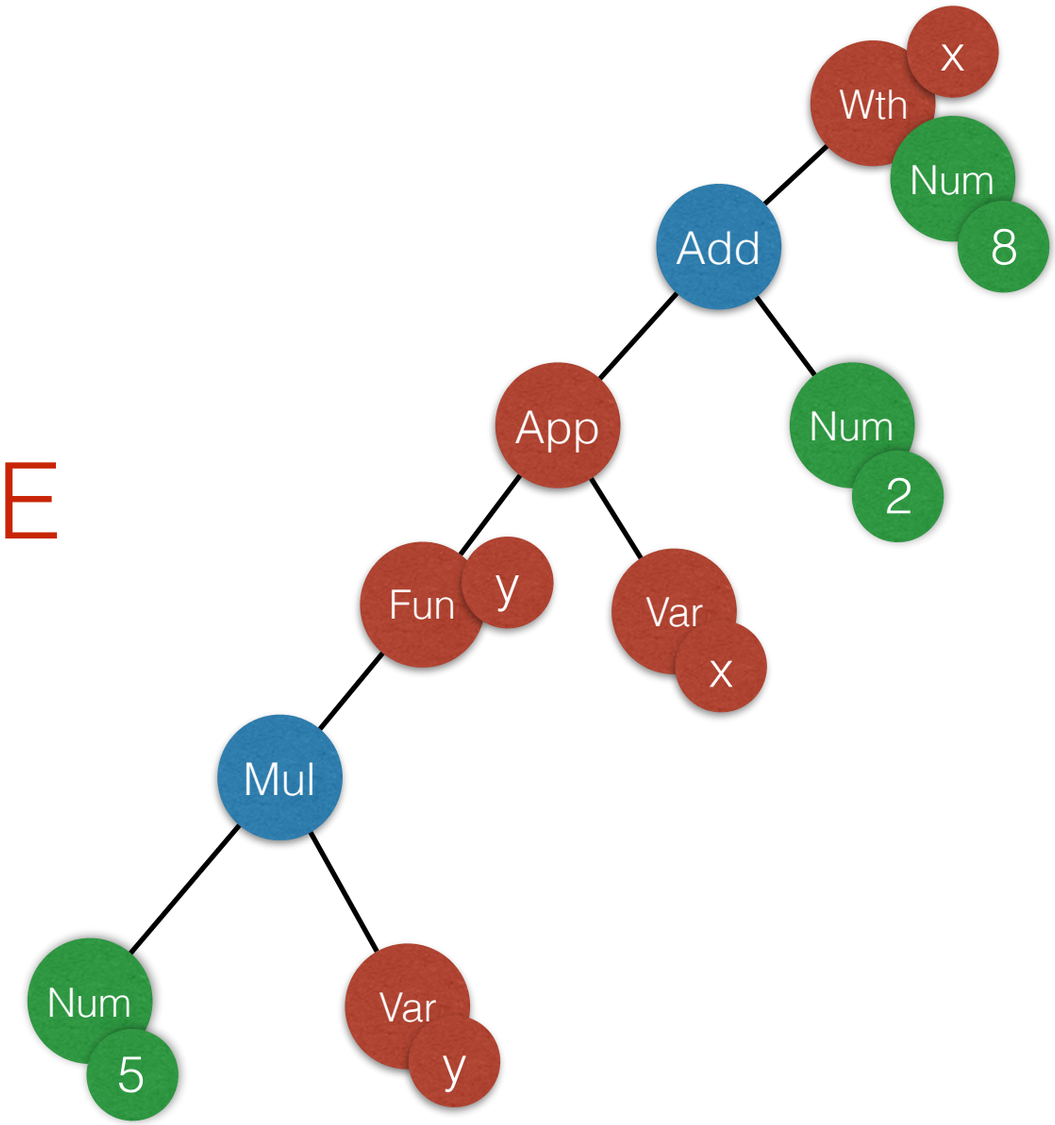


not in the static scope of the function

static scope of the function

7

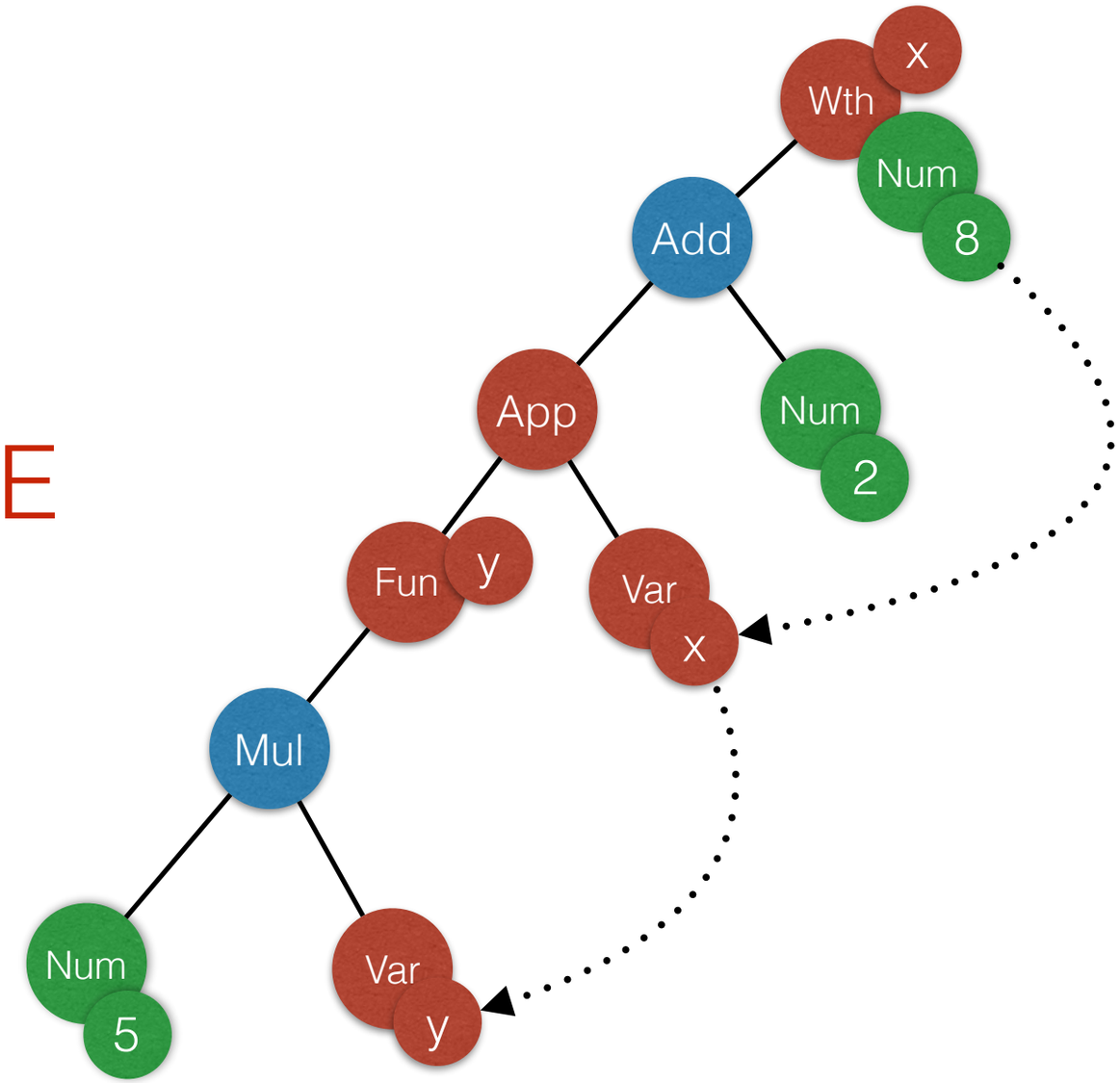
FAE



Two possible interpreters:
substitution-based and environment-based

7

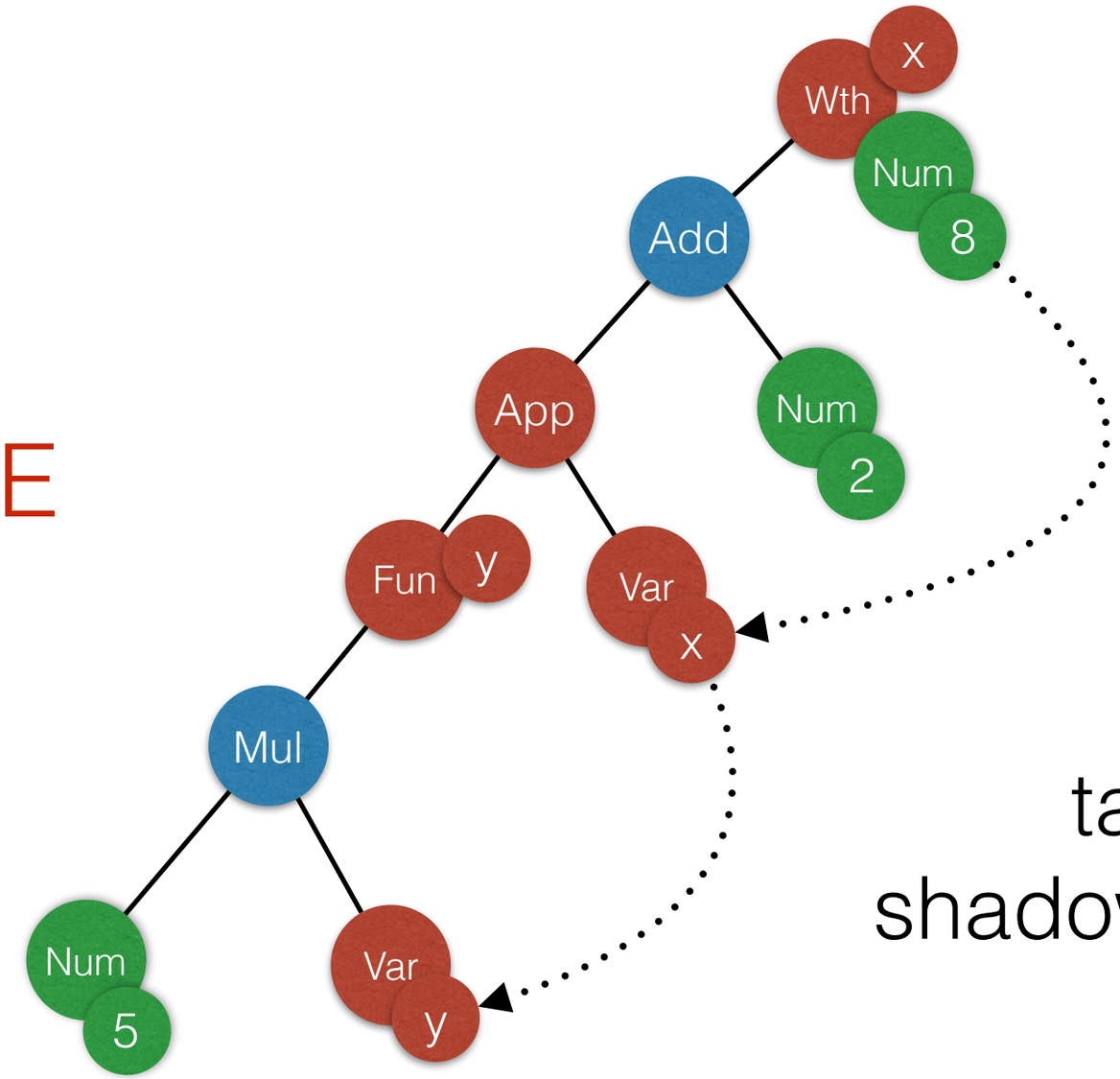
FAE



Substitution-based: just substitutes step-by-step

7

FAE

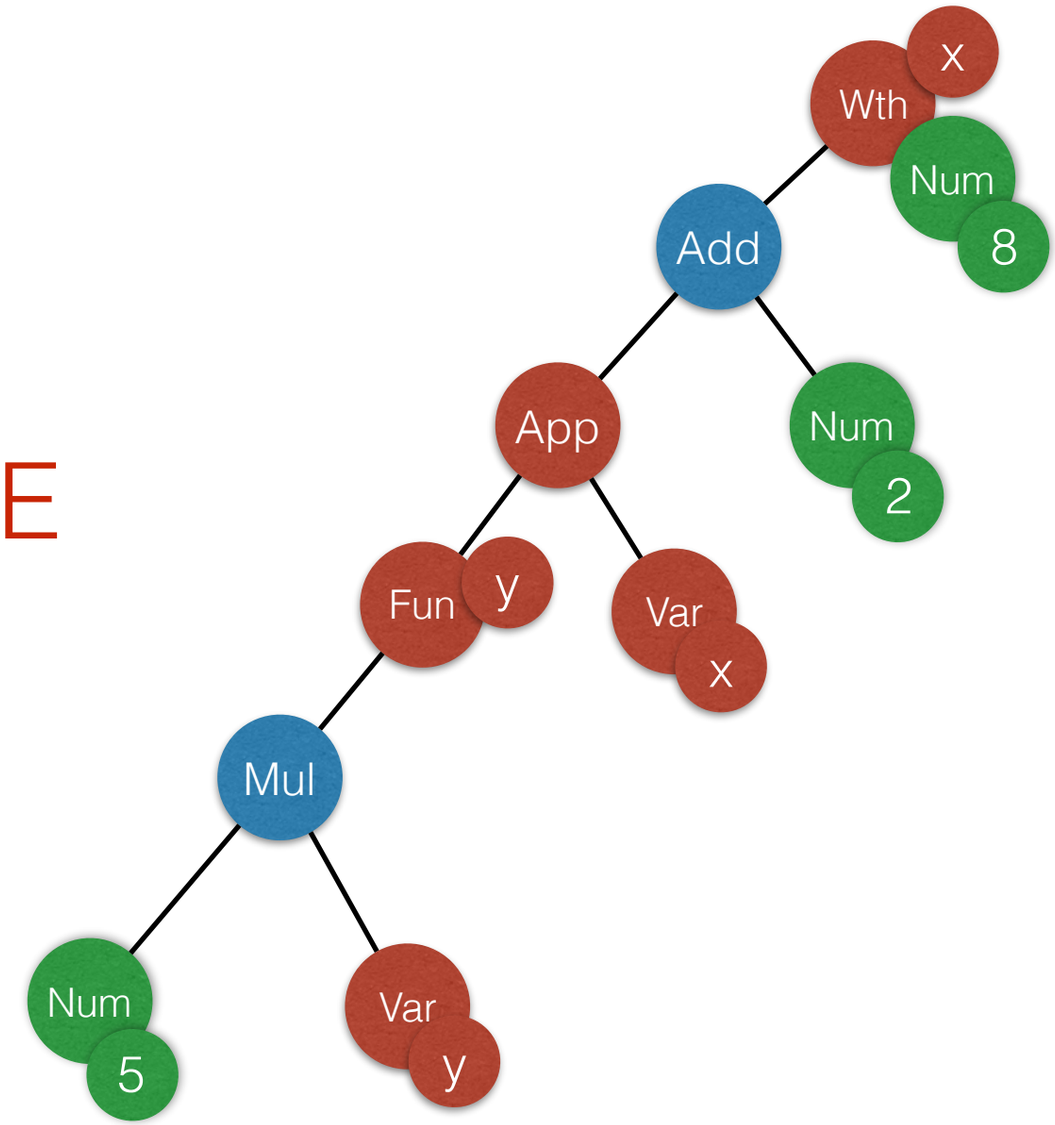


take care:
shadowed variables

Substitution-based: just substitutes step-by-step

7

FAE

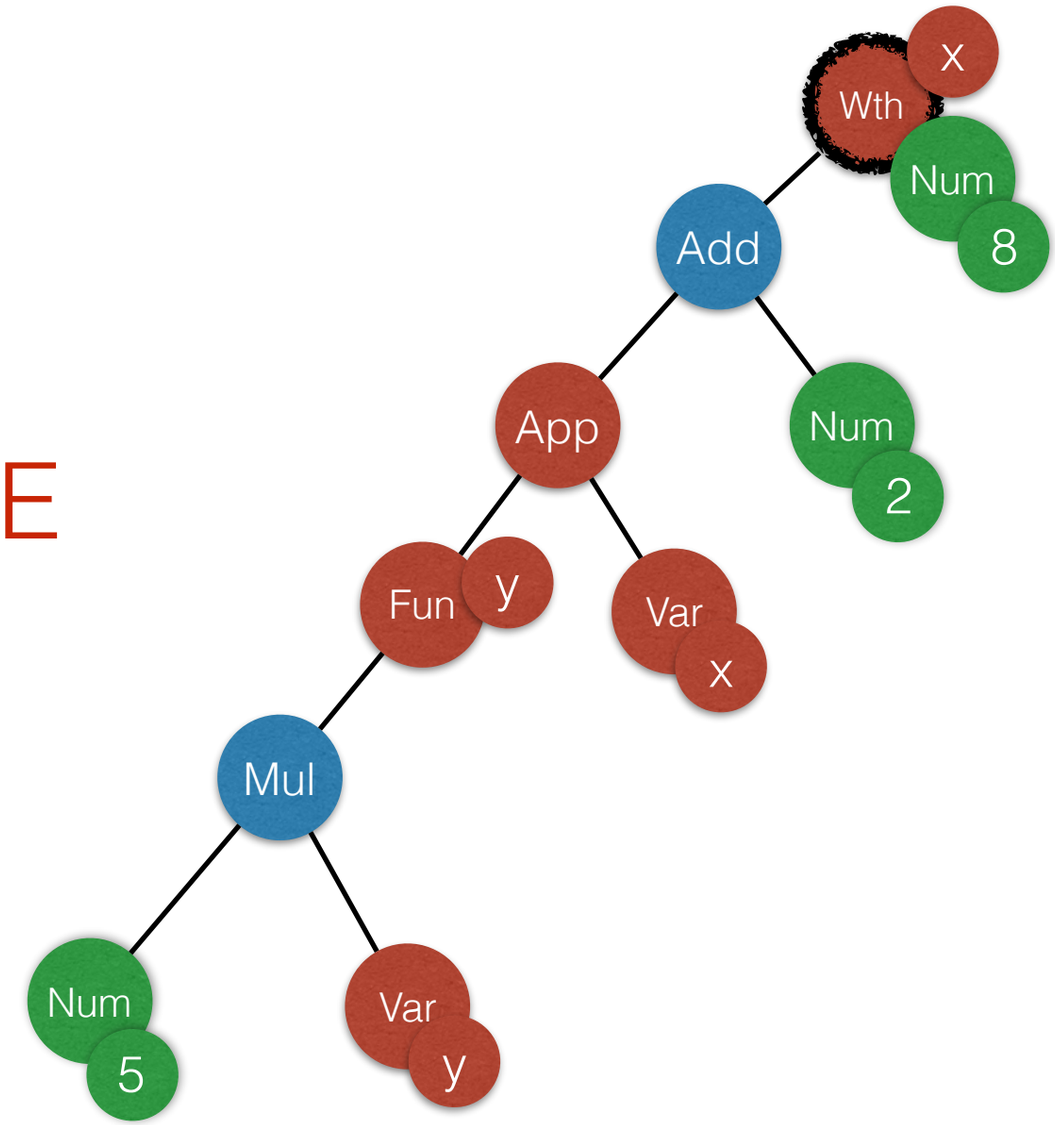


(empty)

Environment-based: carries along an environment

7

FAE

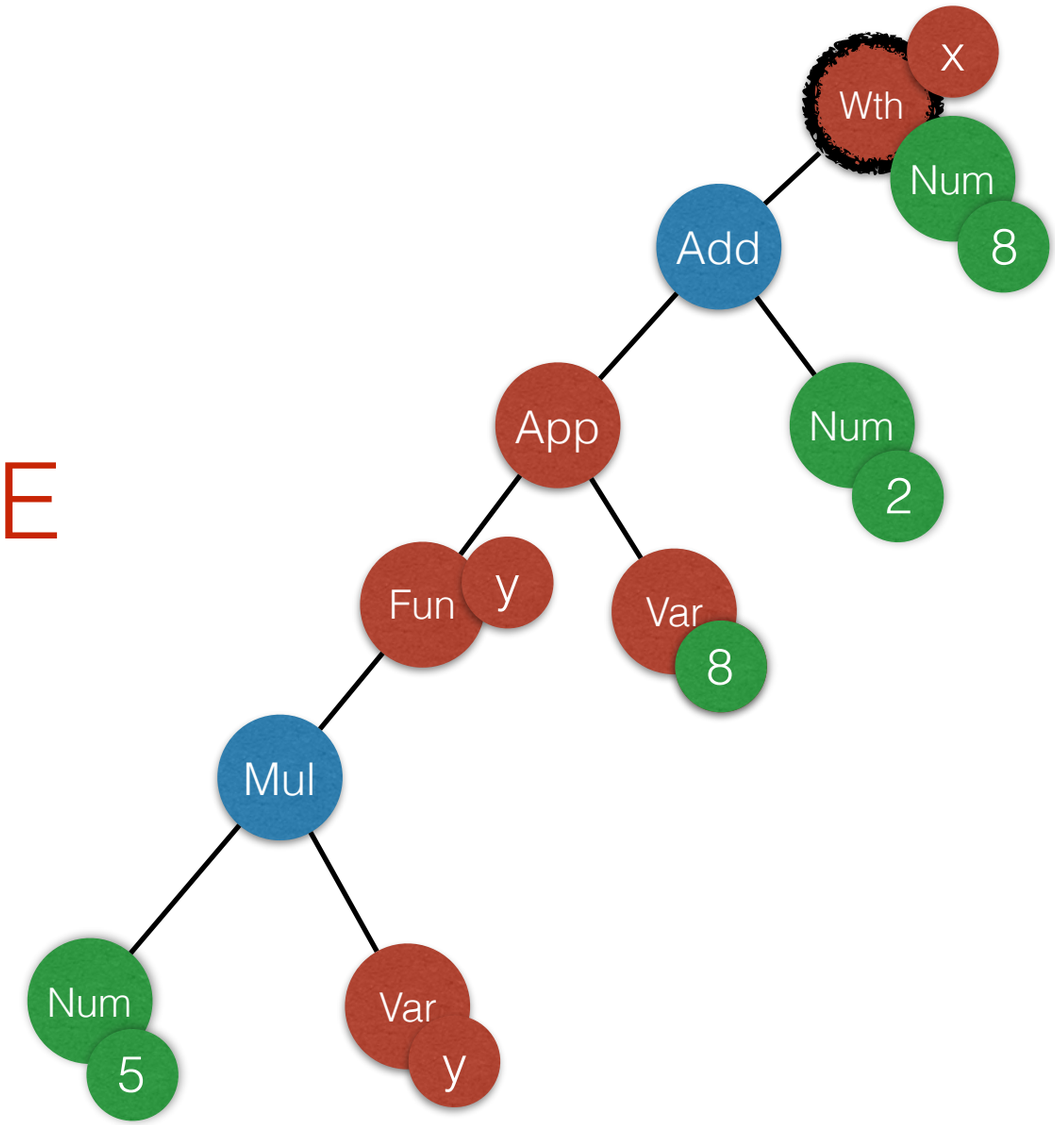


x —> 8

Environment-based: carries along an environment

7

FAE

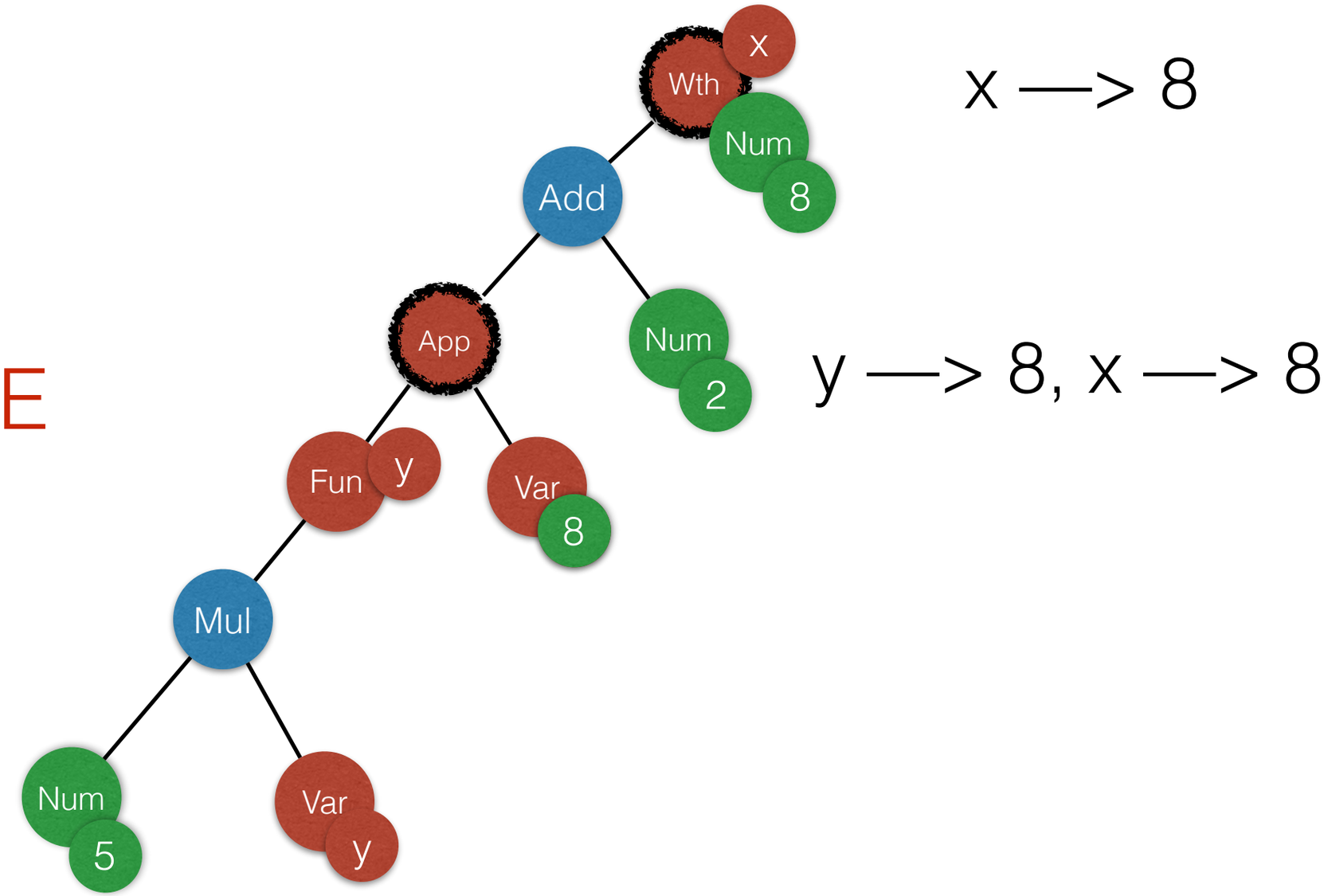


x —> 8

Environment-based: carries along an environment

7

FAE



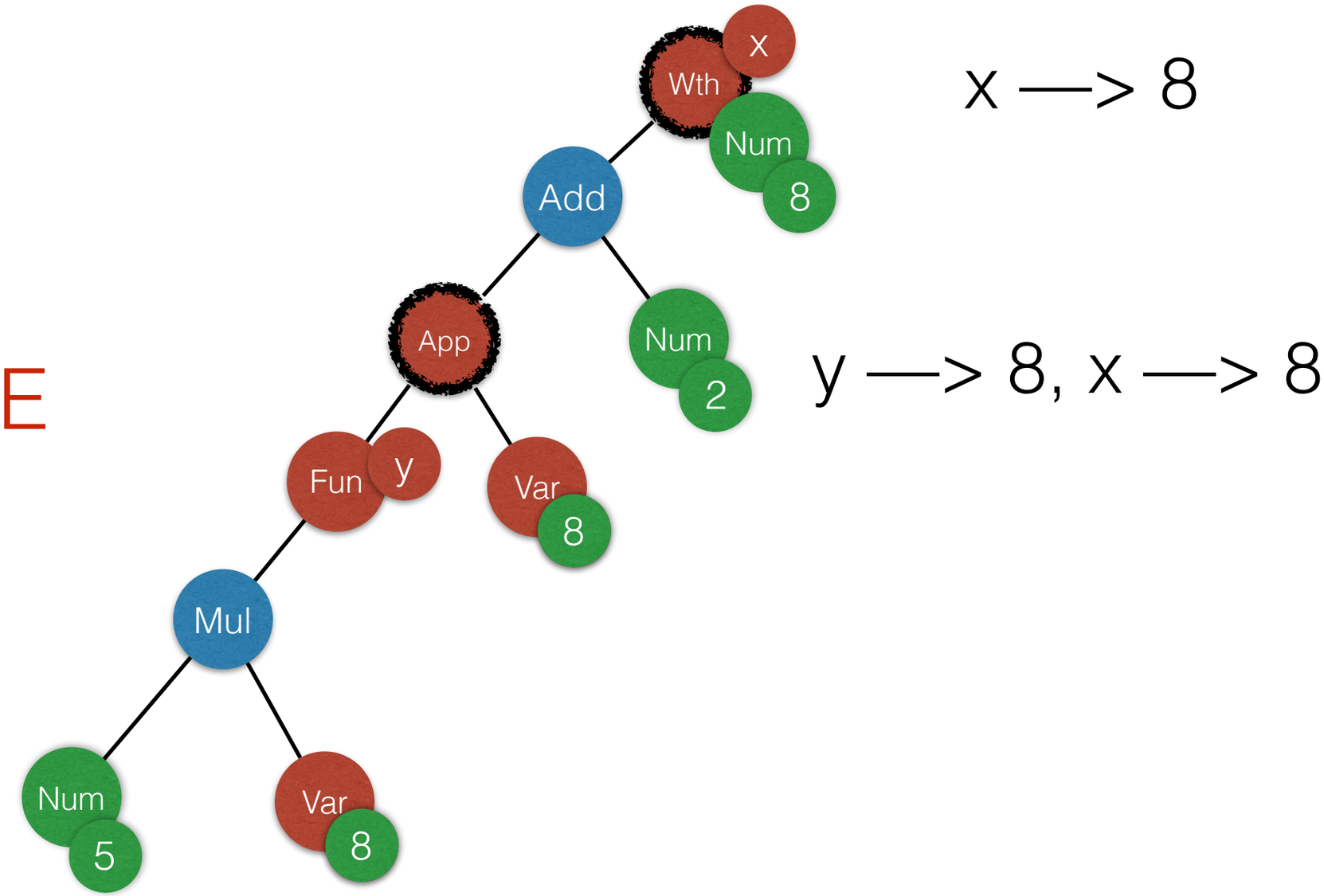
x —> 8

y —> 8, x —> 8

Environment-based: carries along an environment

7

FAE



x —> 8

y —> 8, x —> 8

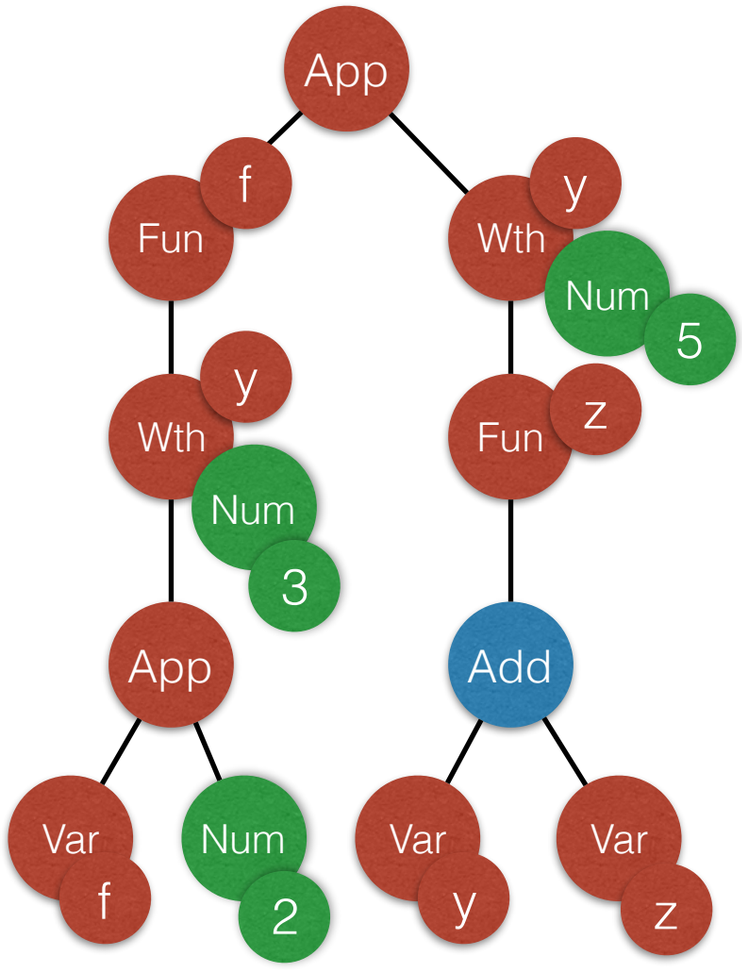
Environment-based: carries along an environment

7 FAE

Note: Values can now also be functions, not just numbers!

But how exactly should they be represented?

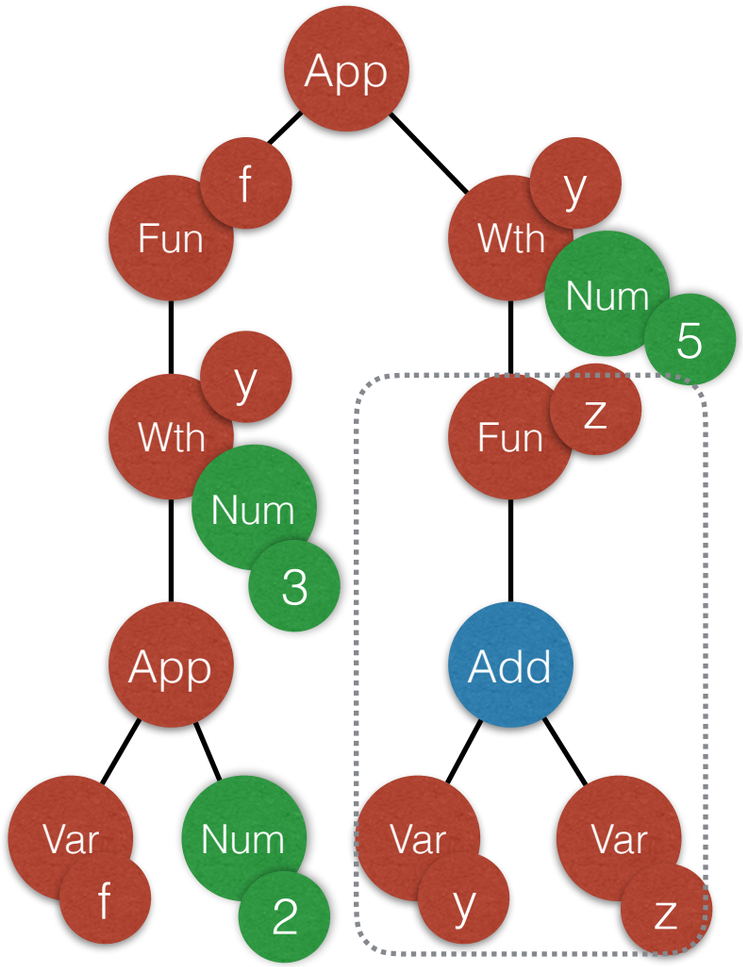
7 FAE



Note: Values can now also be functions, not just numbers!

But how exactly should they be represented as values?

7 FAE



First idea:
value = term

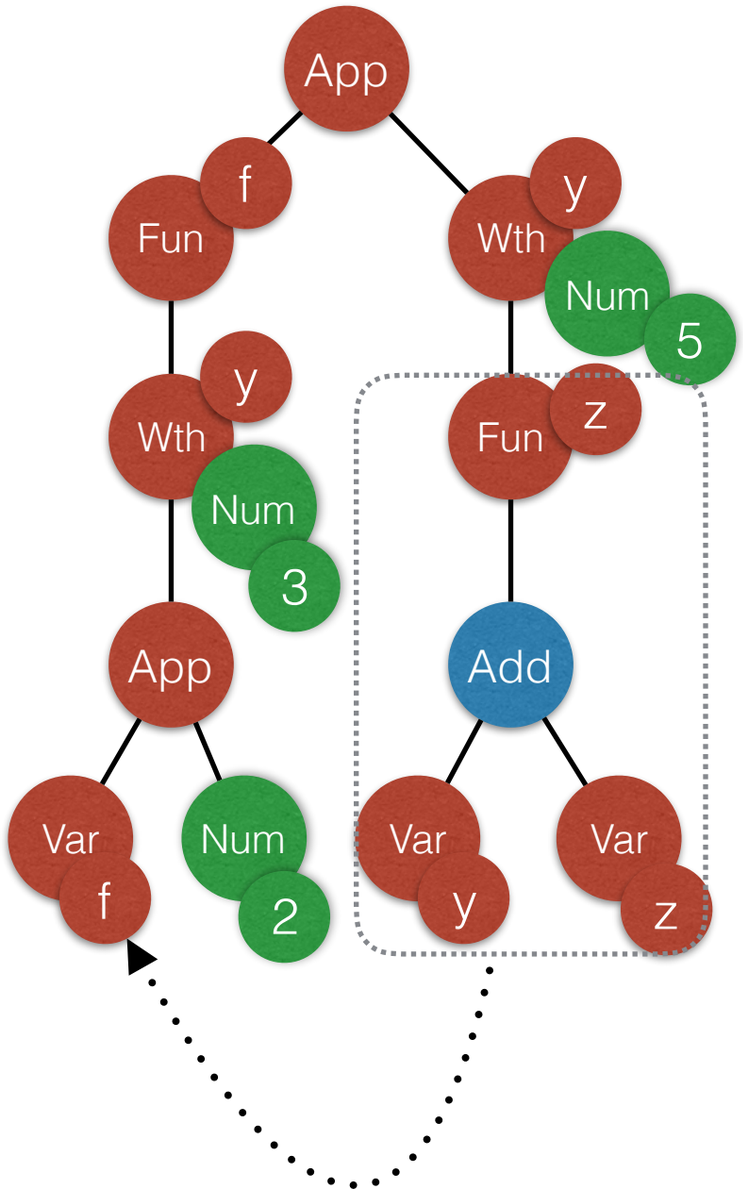
Note: Values can now also be functions, not just numbers!

But how exactly should they be represented as values?

7 FAE

Problem: we then have an environment assignment:

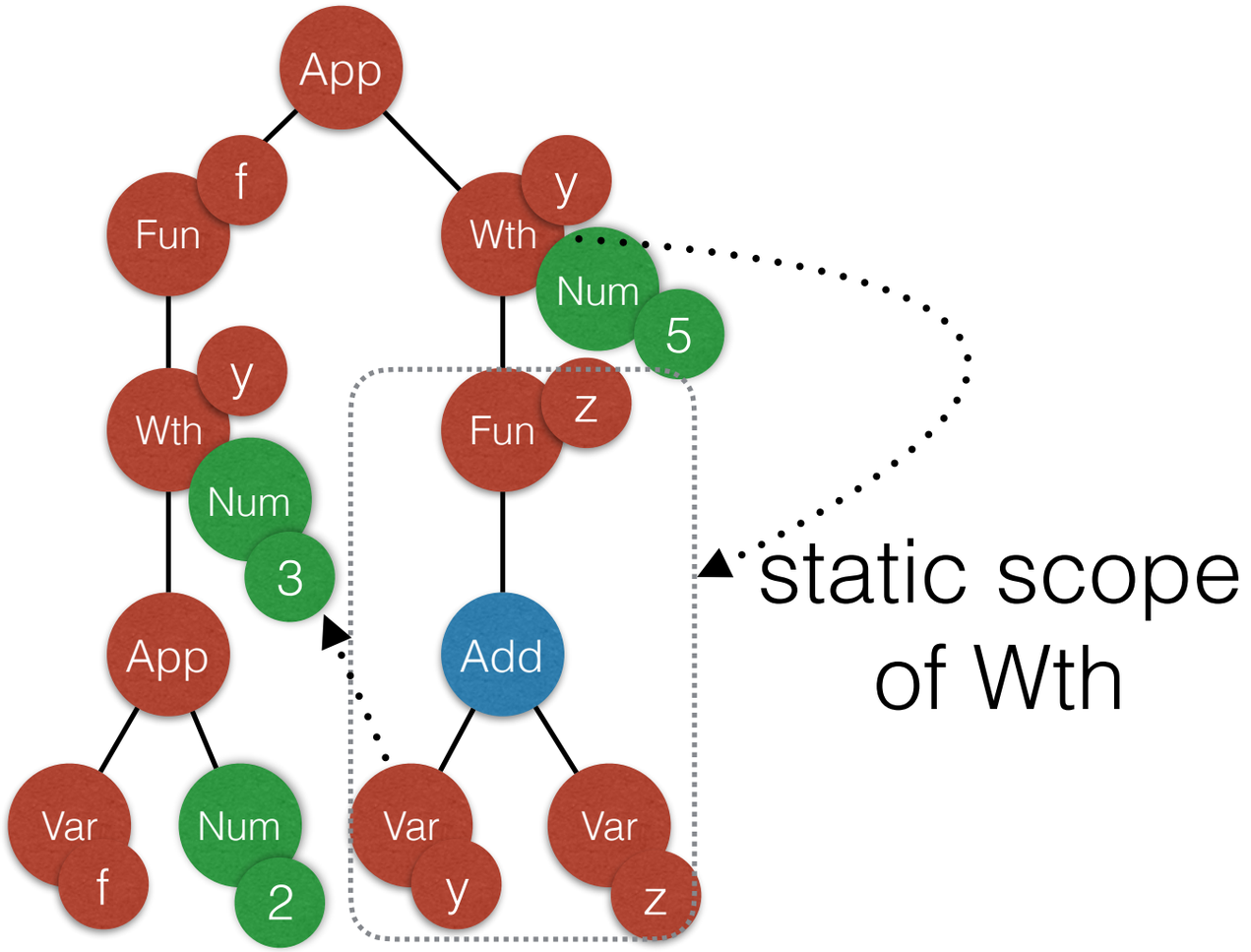
f —> Fun(z, Add(y, z))



So this term gets inserted in place of f.

7

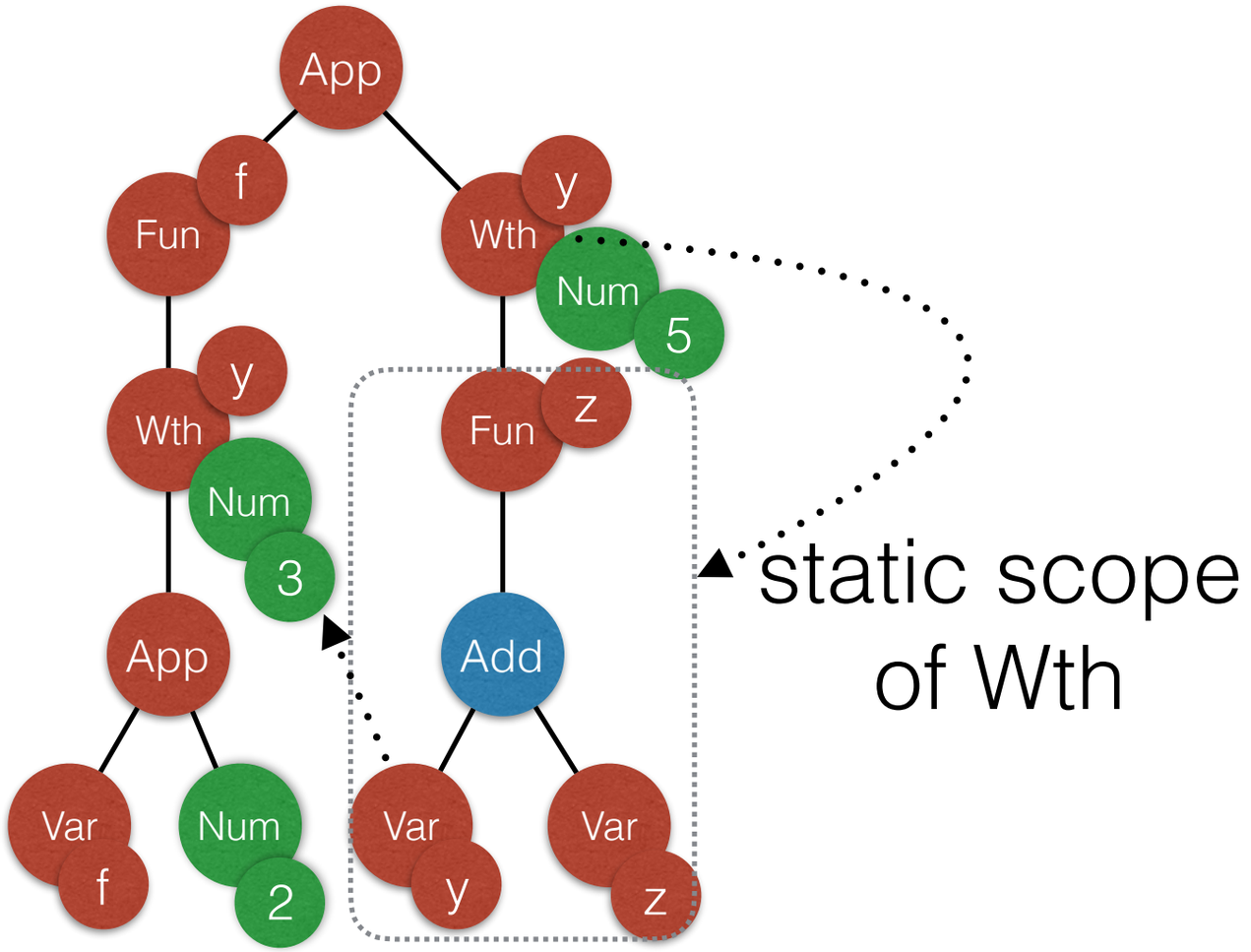
FAE



In effect, y is bound outside of the static scope.

=> We have a violation of static scoping!

7 FAE



Underlying problem:

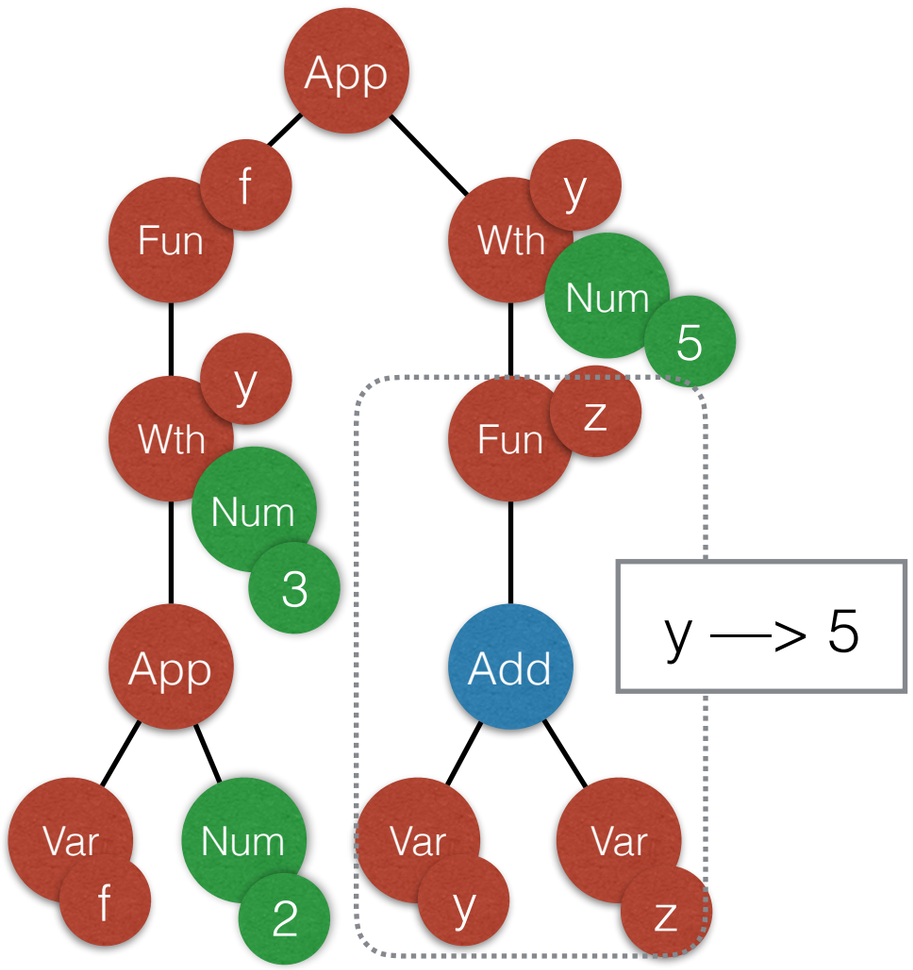
The function term alone does not reflect **all** the knowledge of the evaluation.
Thus it does not really qualify as a value.

The remaining knowledge is in the environment!
With this in mind, the solution is rather straightforward:

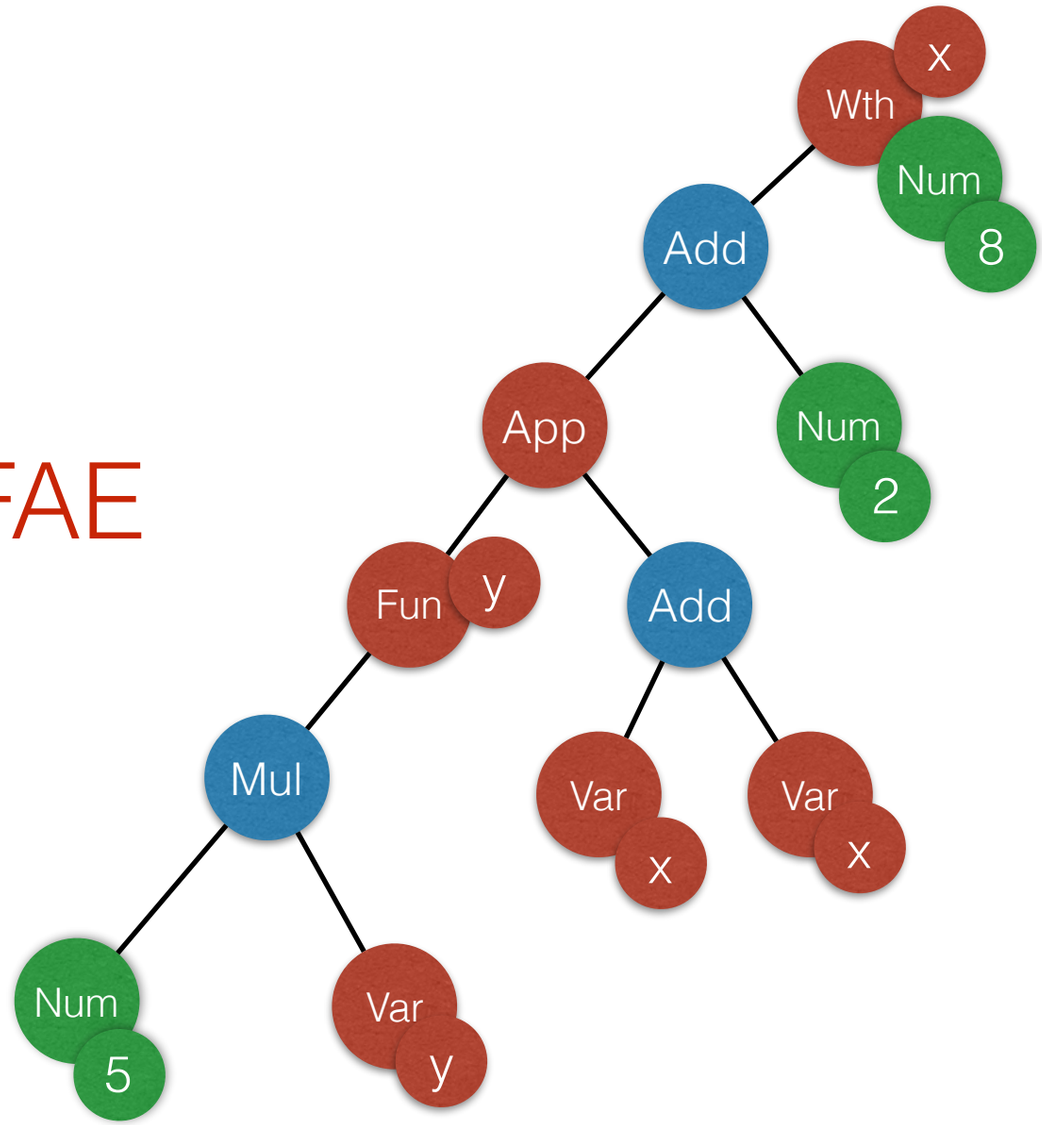
7 FAE

Solution: value = term together with environment at the time the term is evaluated

f —>
Closure(
Fun(z, Add(y, z)),
(y —> 5))



8 LCFAE

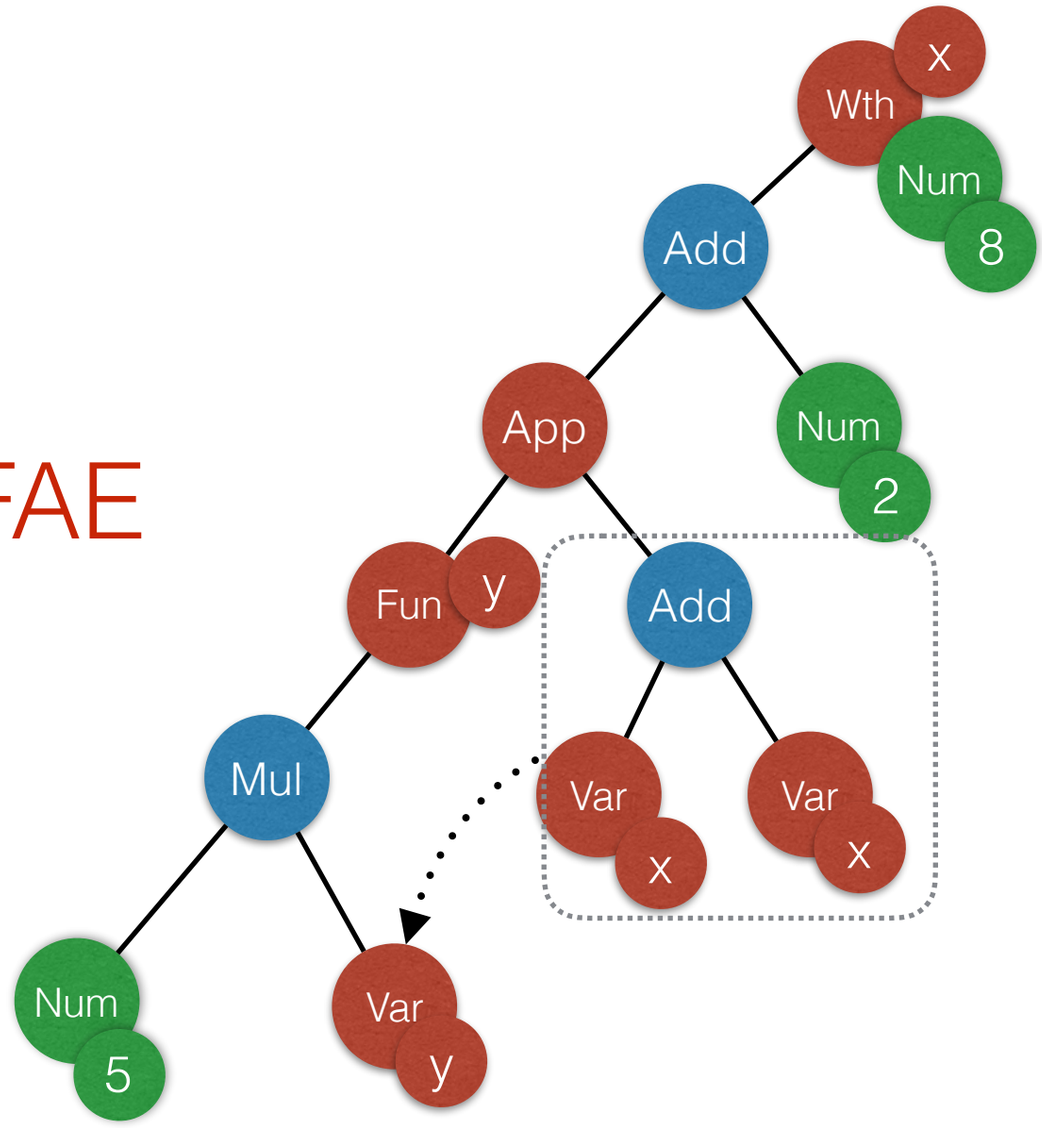


Fun

Same node types, but the Fun node
is now evaluated differently, namely **call-by-name**

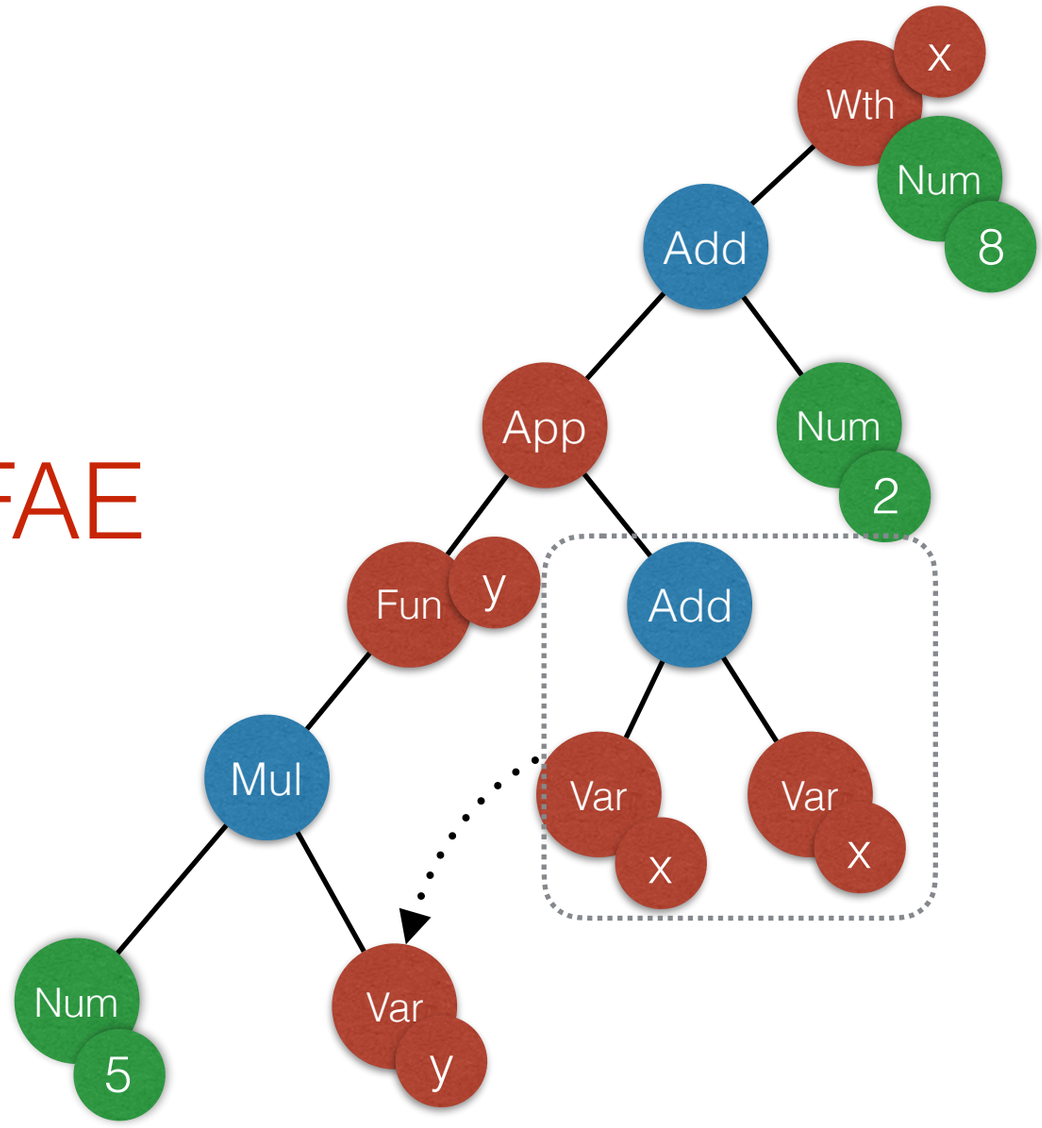
(or alternatively **call-by-need**, which is the same with caching)

8 LCFAE



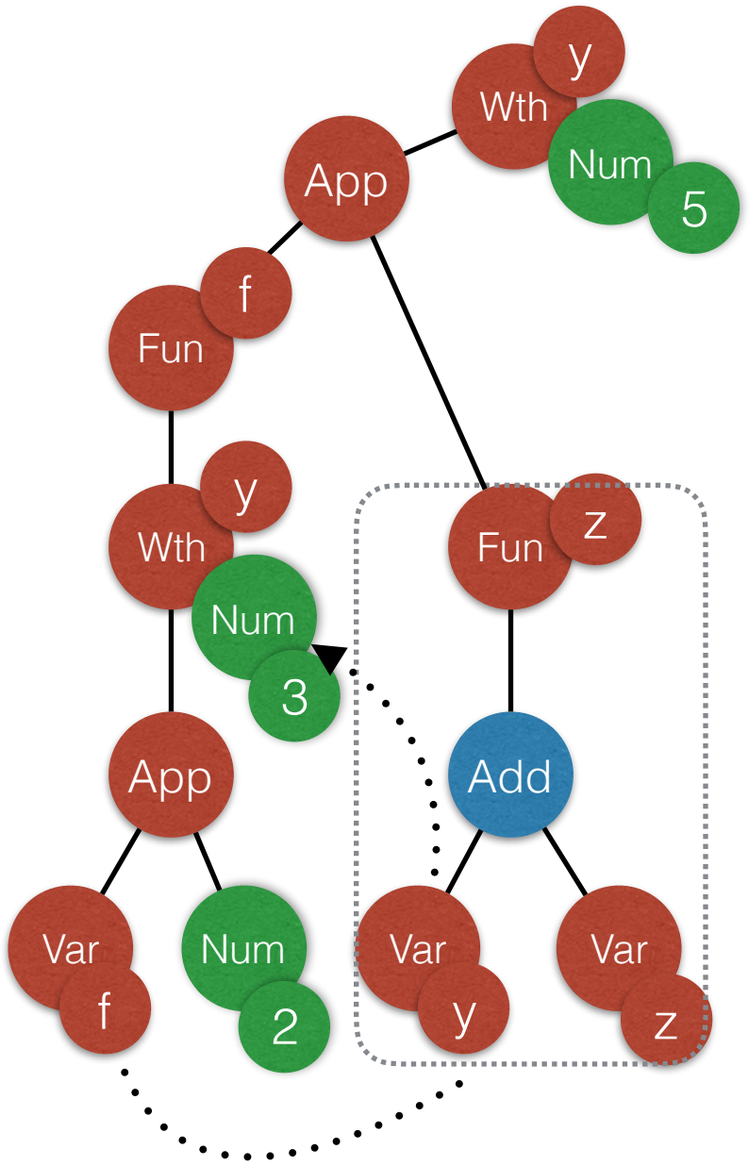
substitution-based interpreter:
the argument is not evaluated, but copied entirely

8 LCFAE



What does this mean for environment-based interpretation?

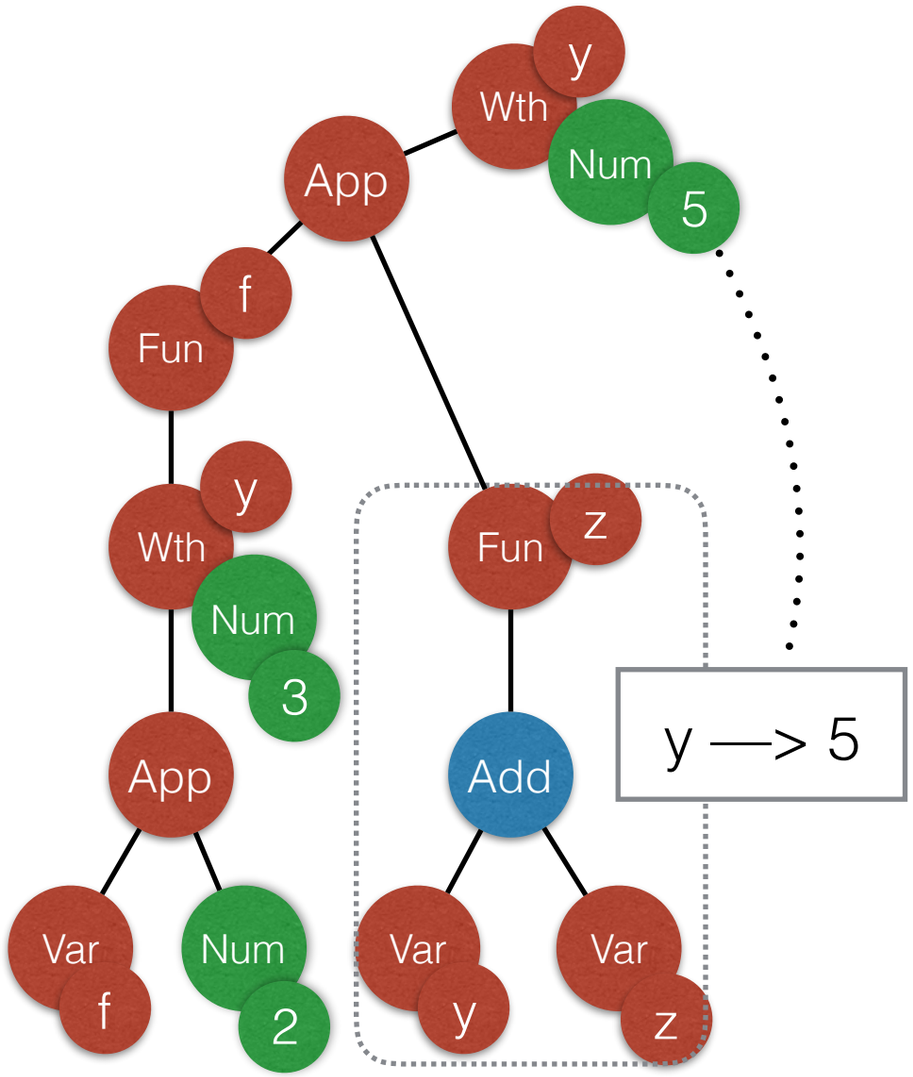
8 LCFAE



We can't just use *values* for the right-hand sides, because we don't evaluate unless necessary.

But when we just use terms, we run into a similar problem as with first-class functions under call-by-value, violating static scoping!

8 LCFAE



Again, the underlying problem is
that we ”threw away” the environment!

8 LCFAE

Solution: rhs = term together with environment at the time the term is copied

f —>
Thunk(
Fun(z, Add(y, z)),
(y —> Num(5)))