

PROBABILITY AND STATISTICS (PMA303)

Lecture-[30]

(Student's t-test- one sample mean)

Para. & Non-para., Hypothesis Testing: (Unit VI-VII)



Dr. Rajanish Rai

Assistant Professor

Department of Mathematics

Thapar Institute of Engineering and Technology, Patiala

~~Practical~~

Student's t-test - one sample mean

When sample size is small ($n < 30$), the student's t-statistic is given by

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}}$$

$$\text{Here, } S^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{(n-1)} \quad \text{--- (1)}$$

is called as unbiased sample variance and $(n-1)$ is called degree of freedom (df) associated with S^2 .

Whereas, for large sample ($n > 30$), the estimate provided by sample variance s^2 and normal test, i.e., Z-test applied

$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n} \quad \text{--- (2)}$$

① & ② \Rightarrow

$$S^2 = \left(\frac{n}{n-1}\right) s^2$$

$$\Rightarrow \boxed{S = s \sqrt{\frac{n}{n-1}}}$$

as $n \rightarrow \infty$ ($n > 30$)

$$s^2 \approx S^2$$

Degree of freedom -:

Consider 9 seven boxes and the task is to choose one box in 9 day.

Day-1	choices-7
Day-2	choices-6
Day-3	choices-5
Day-4	choices-4
Day-5	choices-3
Day-6	choices-2
Day-7	No choice

1	2	3	4
	5	6	7

If n sample $\Rightarrow (n-1)$ dof.

Test for single mean - Hypothesis testing -:

(i) Define the Hypothesis -:

Null Hypothesis: $H_0 : \mu = D$, where D is some specified task that you wish to test.

Alternative Hypothesis -:

One tailed test	Two-tailed test
$H_1 : \mu > D$ or $H_1 : \mu < D$	$H_1 : \mu \neq D$

② Test - Statistics -:

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}}$$

where,

$$S^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

\Rightarrow unbiased estimator of S.D.

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

\Rightarrow Biased estimator of S.D.

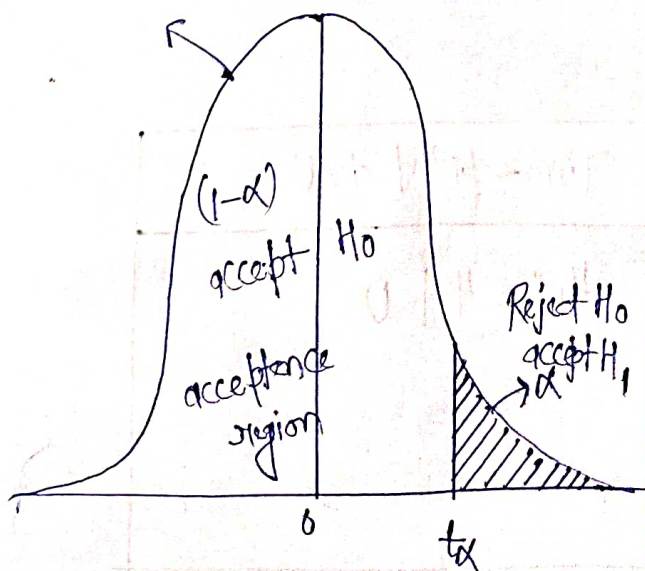
$$S = \sigma \sqrt{\frac{n}{n-1}}$$

③ Rejection region -:

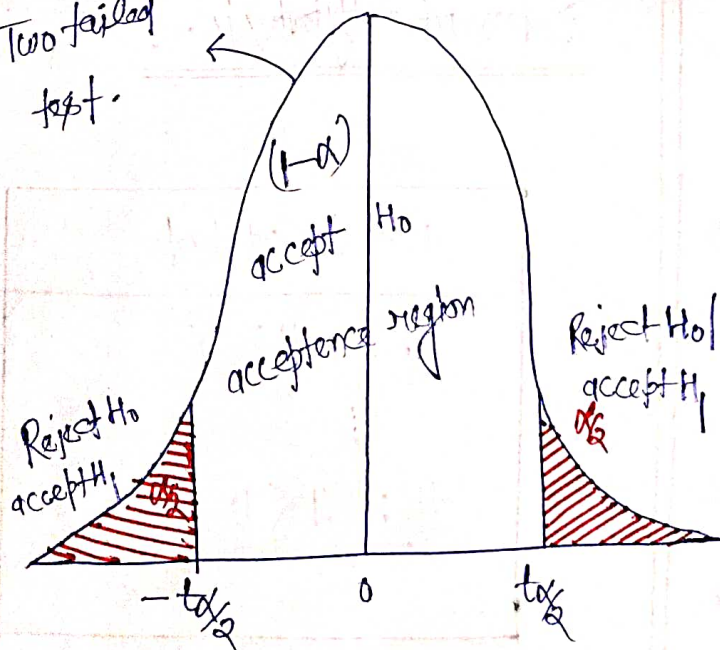
Let α is the level of significance,

One tailed test	Two tailed test
$t > t_{\alpha}$ or $t < -t_{\alpha}$	$t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$

one tailed test



Two tailed test.



or
p-value

One tailed test	Two tailed test
p value = $P(T > t)$ or $P(T < -t)$	p value = $P(T > t) + P(T < -t)$

The critical value of t , t_{α} , $t_{\alpha/2}$ are based on $(n-1)$ degree of freedom.

- ⑦ Conclusion:-
- Reject H_0 and conclude that H_1 is true.
 - Accept (do not reject) H_0 is true.

Note that:- For a t-distribution, with 5-degree of freedom, the value of t that has area 0.05 to the right (i.e., one tailed) is found in row 5 in the column marked $t_{0.05}$.

Example
 $n = 6$
 $Df = n - 1$
 $Df = 5$
 $t_5(0.05)$ or $t_{0.05}(5)$
 $t_{df}(\alpha)$ or $t_{\alpha}(df)$

$$t_5(0.05) = 2.015 \quad 1\text{-tailed}$$

$$t_5(0.10) = 2.015 \quad 2\text{-tailed}$$

Question:-

Ten cartons are taken at random from an automatic filling machine. The mean net weight of 10 cartons is 11.8 units and standard deviation is 0.15 unit. Does the sample mean differ significantly from the intended weight of 12 unit?

Solution:-

Given that

$$n = 10, \quad \bar{x} = 11.8$$

$$\sigma = 0.15 \rightarrow \text{biased estimator S.D.}$$

(i) Define the hypothesis:-

Null hypothesis: $H_0 \quad \mu = 12$

Alternative hypothesis: $H_1 \quad \mu \neq 12$ (a two tailed)

Since
$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n} \quad \text{and} \quad S^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

$$S^2 = \sigma^2 \left(\frac{n}{n-1} \right)$$

$$S^2 = (0.15)^2 \left(\frac{10}{10-1} \right) = 0.025$$

$$S = 0.15811$$

Hence t-statistic
$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{11.8 - 12}{0.15811/\sqrt{10}}$$

$$\Rightarrow \boxed{t = -4}$$

③ Rejection region -:

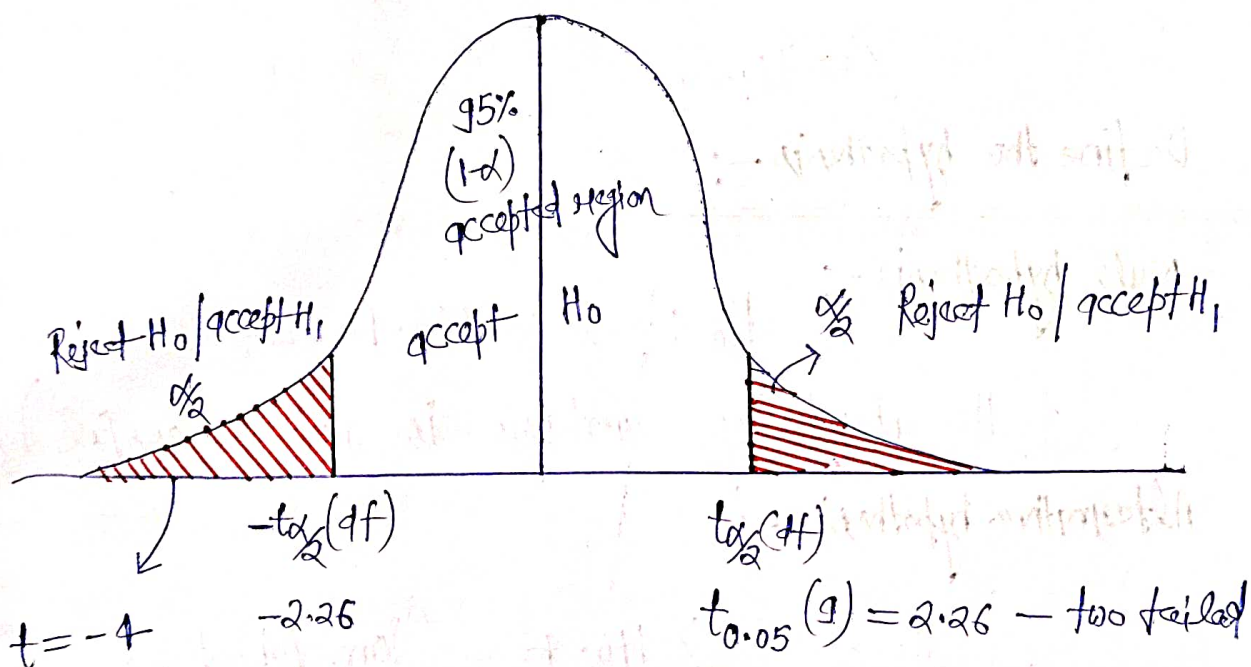
$$\text{Degree of freedom} = 10 - 1 = 9$$

The critical value at 9 degree of freedom with 5% level of significance is

$$t_9(0.05) = 2.26 \quad \text{for two-tailed test.}$$

or

$$t_{0.05}(9) = 2.26 \quad \text{for two-tailed test}$$



⑦ conclusion-: Since t -stat value (-4) lies in rejection region of H_0 , so it is highly significant.

Hence, H_0 is rejected at 5% level of significance and conclude that sample mean differs significantly from population mean of 12. #

Question:-

A mean weekly sales of the emty bar in 9 emty stores was 146.4 bars per store. After a advertising campaign the mean weekly sales in 22 stores for a typical week increases to 153.7 and showed a standard deviation of 17.2. \therefore Was the advertising campaign successful?

Solution:-

Given that

$$n = 22, \quad \bar{x} = 153.7,$$

$$s = 17.2.$$

(i) Define the hypothesis:-

Null hypothesis:- $H_0: \mu = 146.4$

(the advertising campaign is not successful).

Alternative hypothesis:-

$$H_1: \mu > 146.4 \quad (\text{one tailed}).$$

t-statistics:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}},$$

Since
$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n} \quad \text{and} \quad s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

Thus
$$s^2 = s^2 \left(\frac{n}{n-1} \right) = (17.2)^2 \left(\frac{22}{22-1} \right) = 309.9276.$$

Hence,
$$t = \frac{153.7 - 146.3}{\sqrt{309.9276} / \sqrt{22}} = 1.9716$$

$$t = 1.9716$$

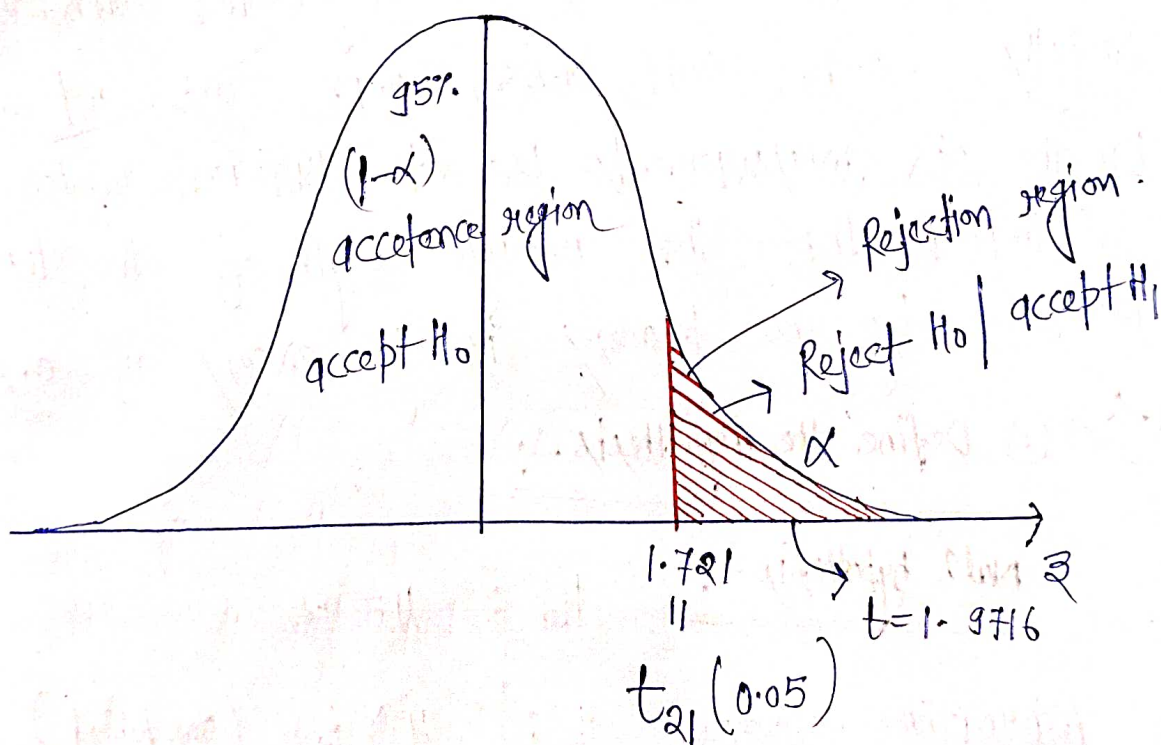
③ Rejection region :-

Degree of freedom = $22 - 1 = 21$

Critical value at 21 degree of freedom with 5% level of significance is

$t_{21}(0.05) = 1.721$ for one tailed.

or $t_{21}(0.10) = 1.721$ for two tailed.



④ Conclusion:-

Since the calculated t -test value (1.9716) lies in rejection region of H_0 , so it is highly significant.

Hence, H_0 is rejected at 5% level of significance and conclude that advertising campaigns was successful in promoting sales.

Question-③ A new process for producing synthetic diamonds can be operated at a profitable level only if the average weight of the diamonds is greater than 0.5 Karat. To evaluate the probability of the process, six diamonds are generated with recorded weights, 0.46, 0.61, 0.52, 0.48, 0.57 and 0.59 Karat. Do the six measurements present sufficient evidence to indicate that the average weight of the diamonds produced by the process is in excess of 0.5 Karat.

Solution:- ① Define the Hypothesis:-

Null hypothesis:- $H_0 : \mu = 0.5$

Alternative Hypothesis $H_1 : \mu > 0.5$ (one tailed).

② T-Statistic:-

Compute $\bar{x} = \frac{\sum_{i=1}^n x_i}{n} = \frac{0.46 + 0.61 + 0.52 + 0.48 + 0.57 + 0.51}{6}$

$\bar{x} = 0.53$

$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = 0.0559$

Thus, $t = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{0.53 - 0.50}{0.0559/\sqrt{6}} = 1.32$

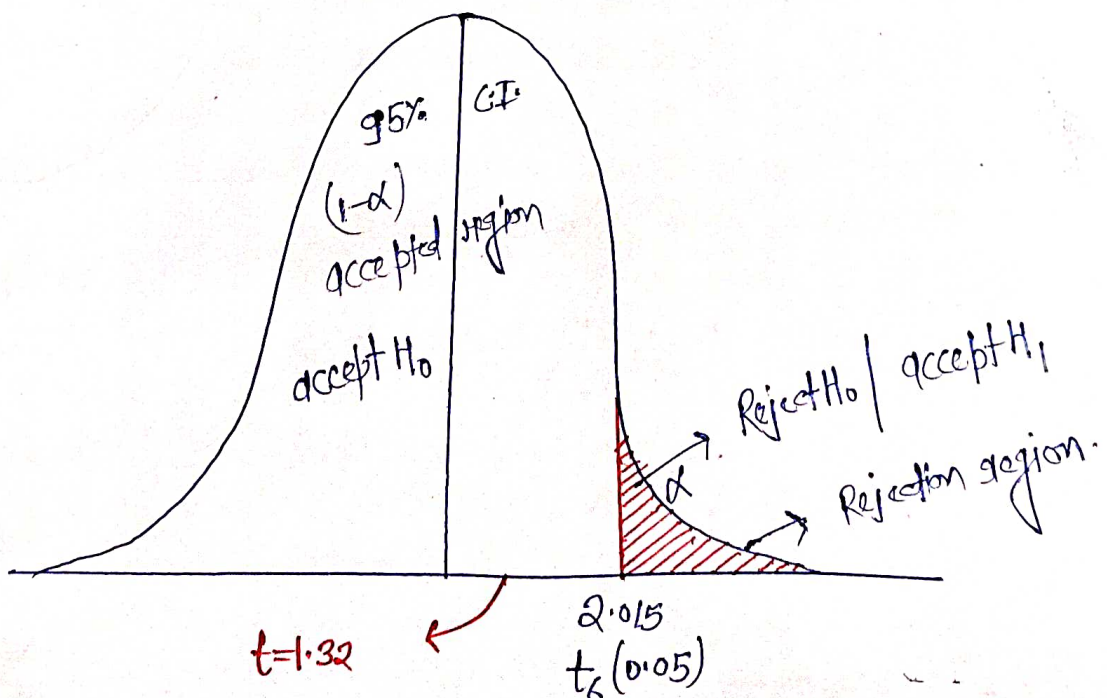
$t = 1.32$

③ Rejection region:-

Degree of freedom = $6 - 1 = 5$

Critical value at 5 degree of freedom with 5% level of significance is

$t_5(0.05) = 2.015$ for one tailed.



④ Conclusion :-

Since the calculated t -statistics value (1.32) lies in acceptance region of H_0 .

Hence H_0 can not be rejected at 5% level of significance and conclude that the data Do Not present sufficient evidence to indicate that the mean diamond weight exceeds.

