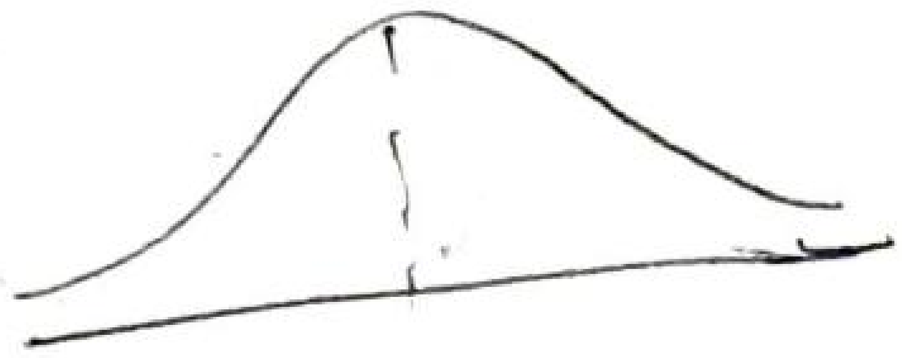
Student's T- distribution :-

* It is the probability distribution that is used to calculate population parameters when the sample size is small and when the population standard deviation is unknown.

$$\boxed{t = \frac{\bar{x} - \mu}{s/\sqrt{n}}}$$



where $\bar{x}$ is sample mean, μ = Population mean,

S → Sample std deviation

n → Sample size.

$$\boxed{S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}}$$

* Since S is the sample std dev, there will be uncertainities in the Value.

The uncertainity means if i take one sample, the S = 12 and then i take another sample, the S = 8. So different sample will give diff Std dev Values. So this is the uncertainity.

* So the degree of freedom comes into the role to avoid this uncertainity.

* Smaller the sample size, the more uncertain we are.

* Since the t ~~dis~~ test will deal with the small samples, there will be uncertainities.

t-test:- (for $n < 30$)

* A t-test is a statistical test that is used to compare the means of two groups. It is often used in hypothesis testing to determine whether a process or treatment actually has an effect on the population of interest or whether two groups are diff from each other.

* If the groups come from a single population then perform paired test

* If the groups come from two different population then perform two sample t test or independent t test

* If there is one group being compared against a standard value then it is one sample t test

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* The degree of freedom for 2 samples is

$$df = (n_A - 1) + (n_B - 1)$$

* ~~t-test~~ ~~t-value~~ ~~t-value~~ for 2 samples (t-test)

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

1) A tyre manufacturer claims that the avg life of a particular category of its tyre is 18000 km when used under normal driving conditions. A random sample of 16 tyres were tested. The mean and std dev of life of the tyres in the sample were 20000 km and 6000 km respectively. Assuming that the life of tyres is normally distributed, test the claim of the manufacture at 1% level of significance. Construct confidence interval also.

Ans:-

$\mu = 18000$ Km

$n = 16$

$\bar{x} = 20000$ km

$s = 6000$ km

$\alpha = 0.01$

① $H_0 : \mu = 18000$

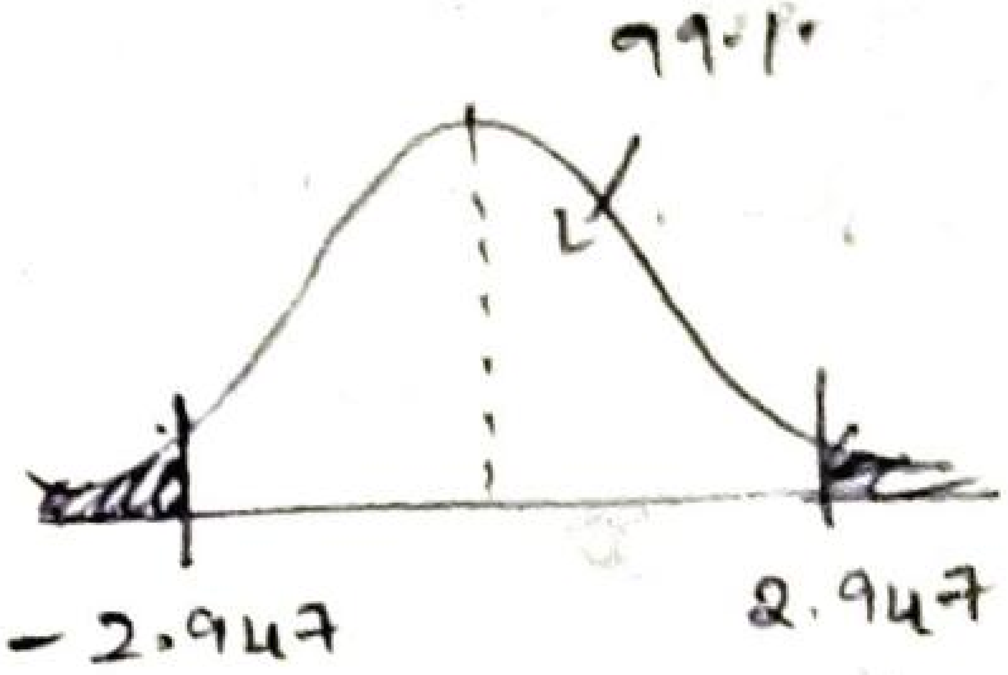
$H_1 : \mu \neq 18000$

② $\alpha = 0.01$

③ From t table

t value for $df = 15$ and two tailed $(\alpha = 0.01) = 2.947$

t value $= 2.947$



④ Test statistic

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{20000 - 18000}{6000/\sqrt{16}} = 1.33$$

t - test $= 1.33$

⑤ 1.33 lies b/w -2.947 to 2.947

So accept the ~~null~~ ~~Alternate~~ null hypothesis (H_0).

2) The means of two random samples of sizes 10 and 8 from two normal Population is 210.40 & 208.92. The sum of squares of deviation from their means is 26.94 and 24.50. Assuming population with equal variances, can we consider the normal populations have equal mean ($\alpha = 0.05$).

Ans:-

$\bar{x}_1 = 210.40$ $n_1 = 10$

$\bar{x}_2 = 208.92$ $n_2 = 8$

$\Sigma(x_1 - \bar{x}_1)^2 = 26.94$ $\Sigma(x_2 - \bar{x}_2)^2 = 24.50$

$\sigma_1^2 = \sigma_2^2$

$\alpha = 0.05$

1. $H_0 : \mu_1 = \mu_2$

   $H_1 : \mu_1 \neq \mu_2$

2. $\alpha = 0.05$

3. From t-table, t-value for

   $df = (n_1 - 1) + (n_2 - 1) = (10-1) + (8-1)$
   $= 16$

   t-value for $df = 16$ & $\alpha = 0.05$, two tailed

   $\boxed{\text{t-value} = 2.120}$

(4) test statistic

~~t= |x̄-μ|~~

~~t = t~~

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Since we dont have s_1, s_2, we should calculate it.

$$s_1 = \sqrt{\frac{\Sigma(x_1-\bar{x}_1)^2}{n_1-1}} = \sqrt{\frac{26.94}{10-1}} = \sqrt{\frac{26.94}{9}}$$

$$= \sqrt{2.993} = 1.73 \qquad \boxed{s_1 = 1.73}$$

$$s_2 = \sqrt{\frac{\Sigma(x_2-\bar{x}_2)^2}{n_2-1}} = \sqrt{\frac{24.50}{8-1}} = \sqrt{\frac{24.50}{7}}$$

$$= \sqrt{3.5} = 1.87$$

$$t = \frac{210.40 - 208.92}{\sqrt{\frac{(1.73)^2}{10} + \frac{(1.87)^2}{8}}} = \frac{1.48}{\sqrt{\frac{2.9929}{10} + \frac{3.4969}{8}}}$$

$$= \frac{1.48}{\sqrt{0.29929 + 0.43711}}$$

$$= \frac{1.48}{\sqrt{0.7364}}$$

$$= \frac{1.48}{0.85}$$

$$= 1.74$$

$$\boxed{t = 1.74}$$

(5) 1.74 lies between -2.120 to 2.120, therefore

accept the null hypothesis (H_0)

$$\mu_1 = \mu_2 \quad //$$