

## Kernal density estimation:- (KDE)

\* KDE is a non parametric way to estimate the probability density function of a random variable.

\* Non parametric way means the distribution will not follow any specified parameter such as mean, median etc..., simply defined as distribution - free way.

\* KDE is a fundamental data smoothing problem where inferences abt the population were made based on a finite data sample.

### what is Kernal ?

\* Kernal of PDFs or PMFs is the form of PDFs ~~and~~ and PMFs in which any factors that are not functions of any of the variables in the domain are omitted.

\* Simply it means ~~that~~ in PDFs the factors which doesn't involved the domain that is the variable will removed because it will create unnecessary PDFs.

\* Kernal is a weighing function.



PDFs of normal distribution =  $\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Its associated kernel is

$$p(x|\mu, \sigma^2) \propto e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

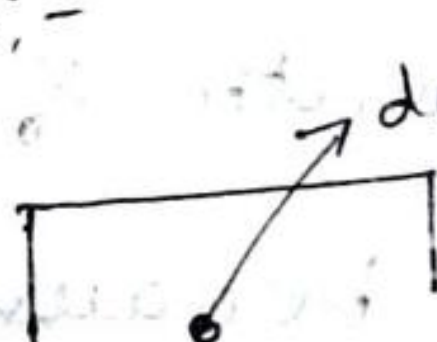
Here  $\frac{1}{\sqrt{2\pi\sigma^2}}$  is removed ~~because~~ even though it has  $\sigma^2$ , it ~~is not~~ because it is not the function of  $x$ .

\* A kernel is a non-negative real valued integrable function  $K$ .

& ~~It~~  $K$  is symmetric  $K(x) = K(-x)$

Some Kernel functions:-

$$K(u) = 1/2$$



Box car

$$K(u) = (1 - |u|)$$



Triangular

$$K(u) = \frac{3}{4} (1 - u^2)$$



Epanechnikov

$$K(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}$$



Gaussian

and many more



## Graphical Explanation with respect to data

\* Suppose there are 5 data points in the dataset (Just for example)

\* On every data point  $x_i$ , we place a Kernel function  $K$

\* The Kernel density estimate is

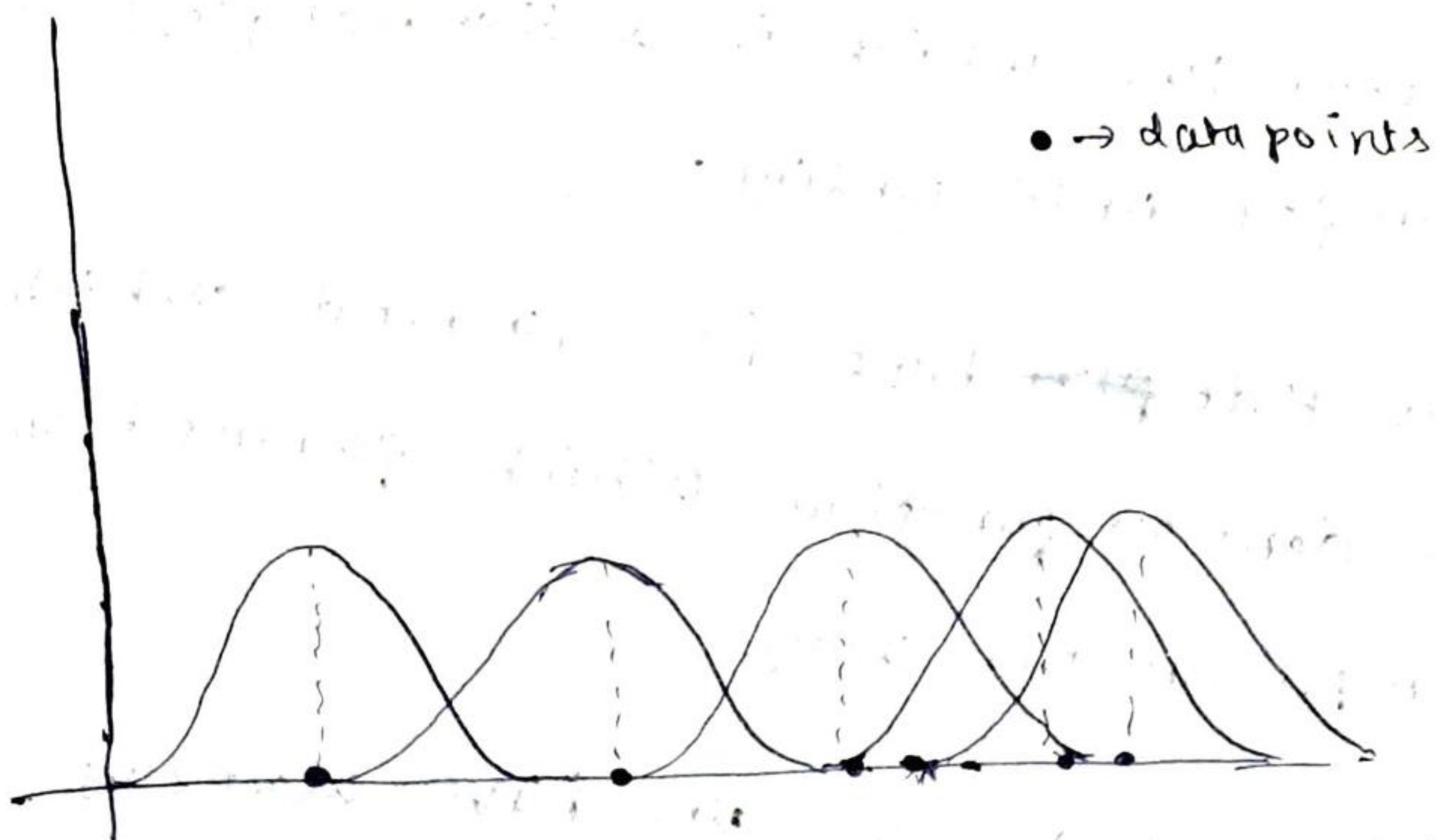
$$\hat{f}(x) = \frac{1}{N} \sum_{i=1}^N K(x - x_i)$$

\* Since there are more Kernel functions like Boxcar, Gaussian, triangular etc..., the choosing of the Kernel is not crucial.

\* But choosing of bandwidth of Kernel is more important and crucial because it determines the shape and smoothness of the Kde.

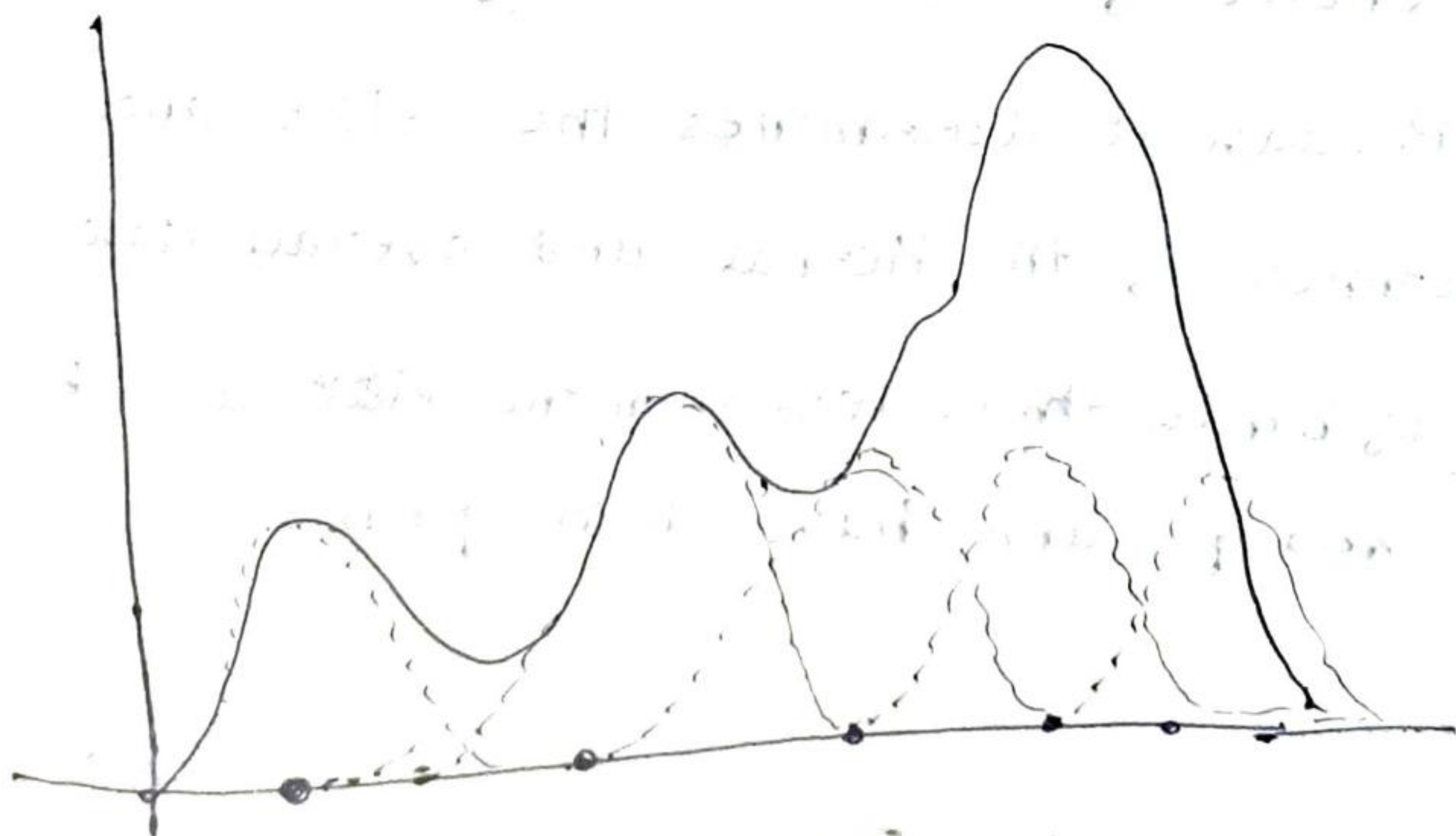
① ~~But~~ But we don't know exactly for ~~reason~~ what reason choosing of Kernel is not crucial.





Since I have said for example 5 data points I have centered a Gaussian Kernel function for each data point. The kernel have some bandwidth  $h$ .

Now ~~also~~ all the kernels should be joined are added up to form a kernel density estimate



The dark blue line is a KDE plot for a feature.



\* dotted line which is a Kernel, just shown for understanding.

\* The ~~Kde plot~~ KDE plot is formed which is a density function which is formed in a non parametric way.

\* This kind of plot we have already seen in distplot (Histogram + KDE plot).

\* Simple if we join the edges or centres of Histogram we will get KDE plot.

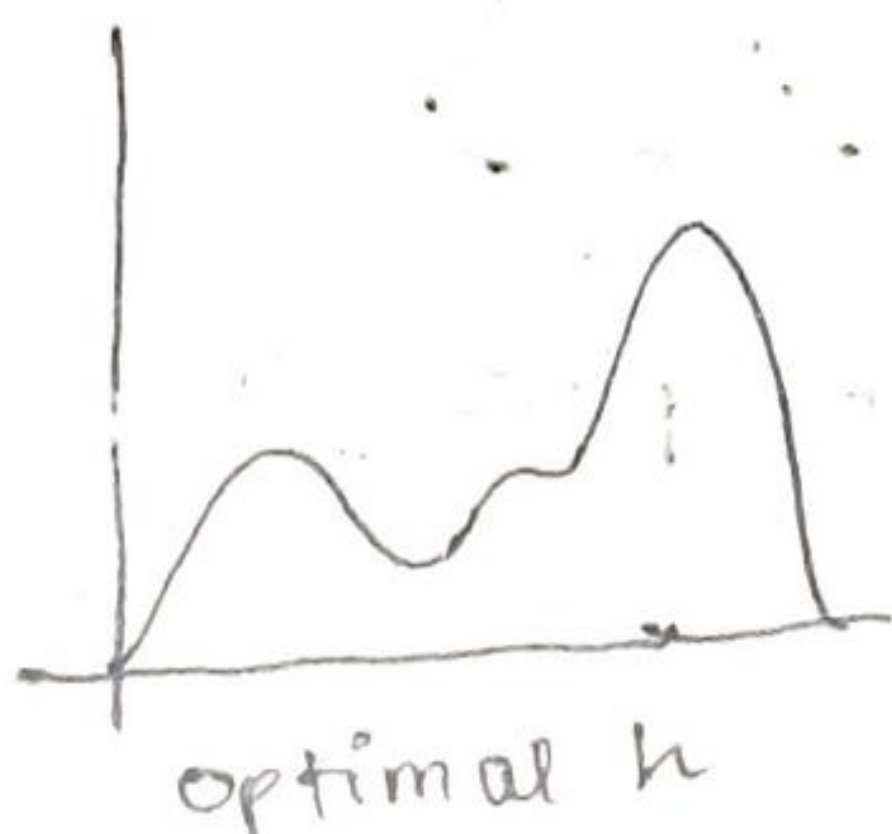
\* It is mainly known for its smoothness.

Choice of bandwidth of Kernels :-

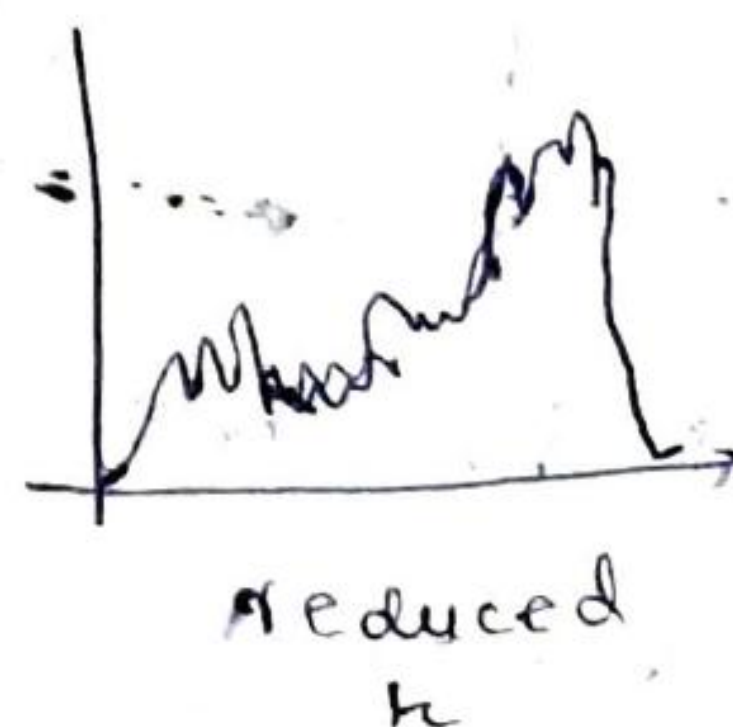
\* choice of bandwidth is very crucial point.

\* Because it determines the size and smoothness of the Kernel and overall KDE.

\* If bandwidth is less then the KDE will be more sharp and have more peaks.



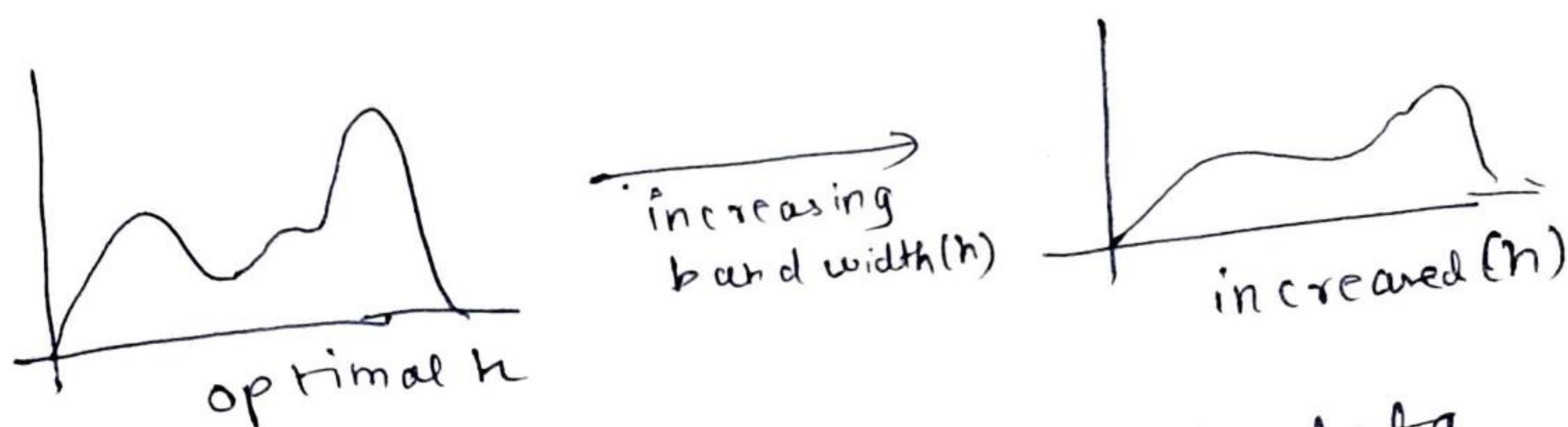
→  
reducing  
the bandwidth  
(h)





\* If I reduce the bandwidth it ~~may~~ will increase variance of KDE plot.

\* If I increase the bandwidth it will smooth the plot



\* If I increase the bandwidth the data may lose its significance because due to the ~~high~~ increased bandwidth the data ~~may~~ may lose its modality since its peaks are smoothening.

\* The optimal bandwidth  $h$  can be computed using Silverman's thumb rule. ~~It~~

\* It assumes the data is normally distributed.