12090CH09

Chapter Nine

RAY OPTICS AND OPTICAL INSTRUMENTS

9.1 Introduction

Nature has endowed the human eye (retina) with the sensitivity to detect electromagnetic waves within a small range of the electromagnetic spectrum. Electromagnetic radiation belonging to this region of the spectrum (wavelength of about 400 nm to 750 nm) is called light. It is mainly through light and the sense of vision that we know and interpret the world around us.

There are two things that we can intuitively mention about light from common experience. First, that it travels with enormous speed and second, that it travels in a straight line. It took some time for people to realise that the speed of light is finite and measurable. Its presently accepted value in vacuum is $c = 2.99792458 \times 10^8$ m s^{-1}. For many purposes, it suffices to take $c = 3 \times 10^8$ m s^{-1}. The speed of light in vacuum is the highest speed attainable in nature.

The intuitive notion that light travels in a straight line seems to contradict what we have learnt in Chapter 8, that light is an electromagnetic wave of wavelength belonging to the visible part of the spectrum. How to reconcile the two facts? The answer is that the wavelength of light is very small compared to the size of ordinary objects that we encounter commonly (generally of the order of a few cm or larger). In this situation, as you will learn in Chapter 10, a light wave can be considered to travel from one point to another, along a straight line joining them. The path is called a *ray* of light, and a bundle of such rays constitutes a *beam* of light.

In this chapter, we consider the phenomena of reflection, refraction and dispersion of light, using the ray picture of light. Using the basic laws of reflection and refraction, we shall study the image formation by plane and spherical reflecting and refracting surfaces. We then go on to describe the construction and working of some important optical instruments, including the human eye.

9.2 Reflection of Light by Spherical Mirrors

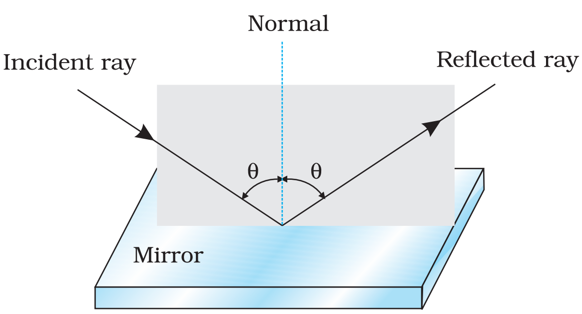

FIGURE 9.1 The incident ray, reflected ray and the normal to the reflecting surface lie in the same plane.

We are familiar with the laws of reflection. The angle of reflection (i.e., the angle between reflected ray and the normal to the reflecting surface or the mirror) equals the angle of incidence (angle between incident ray and the normal). Also that the incident ray, reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane (Fig. 9.1). These laws are valid at each point on any reflecting surface whether plane or curved. However, we shall restrict our discussion to the special case of curved surfaces, that is, spherical surfaces. The normal in this case is to be taken as normal to the tangent to surface at the point of incidence. That is, the normal is along the radius, the line joining the centre of curvature of the mirror to the point of incidence.

We have already studied that the geometric centre of a spherical mirror is called its pole while that of a spherical lens is called its optical centre. The line joining the pole and the centre of curvature of the spherical mirror is known as the *principal axis*. In the case of spherical lenses, the principal axis is the line joining the optical centre with its principal focus as you will see later.

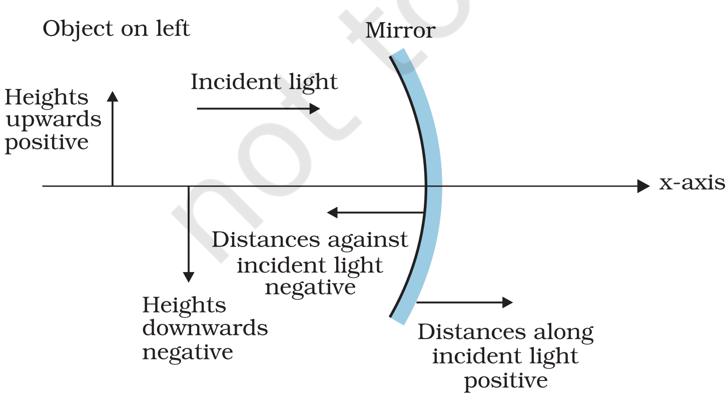

FIGURE 9.2 The Cartesian Sign Convention.

9.2.1 Sign convention

To derive the relevant formulae for reflection by spherical mirrors and refraction by spherical lenses, we must first adopt a sign convention for measuring distances. In this book, we shall follow the *Cartesian sign convention*. According to this convention, all distances are measured from the pole of the mirror or the optical centre of the lens. The distances measured in the same direction as the incident light are taken as positive and those measured in the direction opposite to the direction of incident light are taken as negative (Fig. 9.2). The heights measured upwards with respect to *x*-axis and normal to the principal axis (x-axis) of the mirror/lens are taken as positive (Fig. 9.2). The heights measured downwards are taken as negative.

With a common accepted convention, it turns out that a single formula for spherical mirrors and a single formula for spherical lenses can handle all different cases.

9.2.2 Focal length of spherical mirrors

Figure 9.3 shows what happens when a parallel beam of light is incident on (a) a concave mirror, and (b) a convex mirror. We assume that the rays are *paraxial*, i.e., they are incident at points close to the pole P of the mirror and make small angles with the principal axis. The reflected rays converge at a point F on the principal axis of a concave mirror [Fig. 9.3(a)]. For a convex mirror, the reflected rays appear to diverge from a point F on its principal axis [Fig. 9.3(b)]. The point F is called the *principal focus* of the mirror. If the parallel paraxial beam of light were incident, making some angle with the principal axis, the reflected rays would converge (or appear to diverge) from a point in a plane through F normal to the principal axis. This is called the *focal plane* of the mirror [Fig. 9.3(c)].

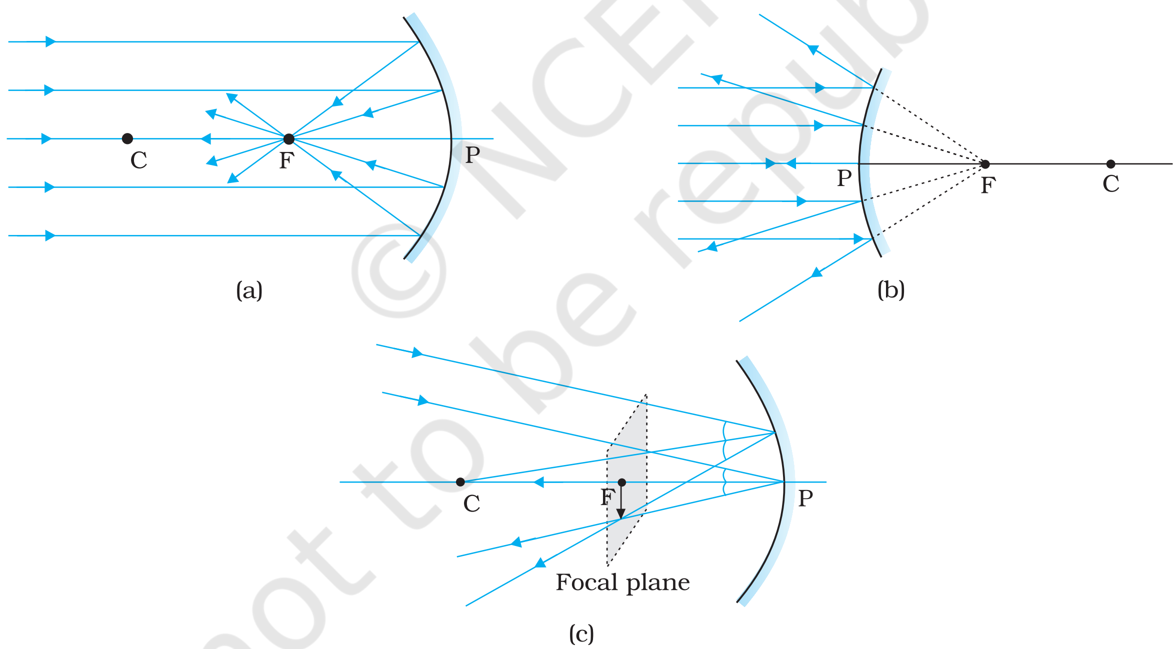

FIGURE 9.3 Focus of a concave and convex mirror.

The distance between the focus F and the pole P of the mirror is called the *focal length* of the mirror, denoted by f. We now show that $f = R/2$, where R is the radius of curvature of the mirror. The geometry of reflection of an incident ray is shown in Fig. 9.4.

Let C be the centre of curvature of the mirror. Consider a ray parallel to the principal axis striking the mirror at M. Then CM will be perpendicular to the mirror at M. Let θ be the angle of incidence, and MD

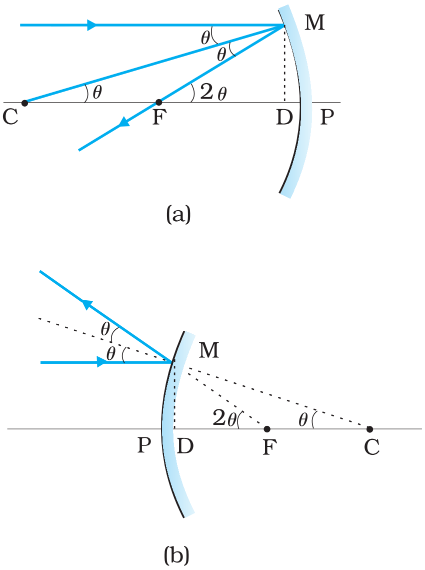

FIGURE 9.4 Geometry of reflection of an incident ray on (a) concave spherical mirror, and (b) convex spherical mirror.

be the perpendicular from M on the principal axis. Then,

$\angle MCP = \theta$ and $\angle MFP = 2\theta$

Now,

$$\tan\theta = \frac{MD}{CD} \text{ and } \tan 2\theta = \frac{MD}{FD} \qquad (9.1)$$

For small θ, which is true for paraxial rays, $\tan\theta \approx \theta$, $\tan 2\theta \approx 2\theta$. Therefore, Eq. (9.1) gives

$$\frac{MD}{FD} = 2\,\frac{MD}{CD}$$

$$\text{or, } FD = \frac{CD}{2} \qquad (9.2)$$

Now, for small θ, the point D is very close to the point P. Therefore, FD = f and CD = R. Equation (9.2) then gives

$$f = R/2 \qquad (9.3)$$

9.2.3 The mirror equation

If rays emanating from a point actually meet at another point after reflection and/or refraction, that point is called the *image* of the first point. The image is *real* if the rays actually converge to the point; it is *virtual* if the rays do not actually meet but appear to diverge from the point when produced backwards. An image is thus a point-to-point correspondence with the object established through reflection and/or refraction.

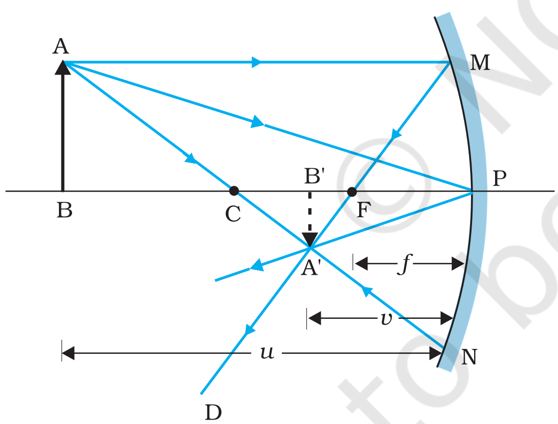

FIGURE 9.5 Ray diagram for image formation by a concave mirror.

In principle, we can take any two rays emanating from a point on an object, trace their paths, find their point of intersection and thus, obtain the image of the point due to reflection at a spherical mirror. In practice, however, it is convenient to choose any two of the following rays:

(i) The ray from the point which is parallel to the principal axis. The reflected ray goes through the focus of the mirror.

(ii) The ray passing through the centre of curvature of a concave mirror or appearing to pass through it for a convex mirror. The reflected ray simply retraces the path.

(iii) The ray passing through (or directed towards) the focus of the concave mirror or appearing to pass through (or directed towards) the focus of a convex mirror. The reflected ray is parallel to the principal axis.

(iv) The ray incident at any angle at the pole. The reflected ray follows laws of reflection.

Figure 9.5 shows the ray diagram considering three rays. It shows the image A′B′ (in this case, real) of an object AB formed by a concave mirror. It does not mean that only three rays emanate from the point A. An infinite number of rays emanate from any source, in all directions. Thus, point A′ is image point of A if every ray originating at point A and falling on the concave mirror after reflection passes through the point A′.

We now derive the mirror equation or the relation between the object distance (u), image distance (v) and the focal length (f).

From Fig. 9.5, the two right-angled triangles A′B′F and MPF are similar. (For paraxial rays, MP can be considered to be a straight line perpendicular to CP.) Therefore,

$$\frac{B'A'}{PM} = \frac{B'F}{FP}$$

$$\text{or} \quad \frac{B'A'}{BA} = \frac{B'F}{FP} \quad (\because PM = AB) \tag{9.4}$$

Since ∠ APB = ∠ A′PB′, the right angled triangles A′B′P and ABP are also similar. Therefore,

$$\frac{B'A'}{BA} = \frac{B'P}{BP} \tag{9.5}$$

Comparing Eqs. (9.4) and (9.5), we get

$$\frac{B'F}{FP} = \frac{B'P - FP}{FP} = \frac{B'P}{BP} \tag{9.6}$$

Equation (9.6) is a relation involving magnitude of distances. We now apply the sign convention. We note that light travels from the object to the mirror MPN. Hence this is taken as the positive direction. To reach the object AB, image A′B′ as well as the focus F from the pole P, we have to travel opposite to the direction of incident light. Hence, all the three will have negative signs. Thus,

B′ P = $-v$, FP = $-f$, BP = $-u$

Using these in Eq. (9.6), we get

$$\frac{-v+f}{-f} = \frac{-v}{-u}$$

$$\text{or} \quad \frac{v-f}{f} = \frac{v}{u}$$

$$\frac{v}{f} = 1 + \frac{v}{u}$$

Dividing it by v, we get

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \tag{9.7}$$

This relation is known as the *mirror equation*.

The size of the image relative to the size of the object is another important quantity to consider. We define linear *magnification* (m) as the ratio of the height of the image (h') to the height of the object (h):

$$m = \frac{h'}{h} \tag{9.8}$$

h and h' will be taken positive or negative in accordance with the accepted sign convention. In triangles A′B′P and ABP, we have,

$$\frac{B'A'}{BA} = \frac{B'P}{BP}$$

With the sign convention, this becomes