**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: Option B 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44. 🡪 TRUE
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees. 🡪 TRUE
4. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: As we know that if X ∼N (μ1, σ1^2), and Y ∼N (μ2, σ2^2) are two independent

Variables then X + Y ∼N (μ1 + μ2, σ1^2 + σ2^2), and X − Y ∼N (μ1 − μ2, σ1^2 + σ2^2).

Similarly, if Z = aX + bY, where X and Y are as defined above, i.e., Z is linear combination of X and Y, then Z ∼N (aμ1 + bμ2, a^2σ1^2 + b^2σ2^2).

Therefore, in the question

2X1~ N (2u,4 σ^2) and

X1+X2 ~ N (μ + μ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4μ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: Option D 48.5,151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: 95% of the **probability lies**between 1.96 **standard deviations**of the **mean.**

(12 – 1.96\*5, 12 + 1.96\*5)

($2.2M, $22.8M)

Thus, Range = (Rs.99M, Rs.1026M)

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: Fifth percentile is calculated as : P(Z <= (p-12)/5) = 0.05

From p values of z score table we get (p-12)/5 = -1.644

P = 12 – 8.22 = 3.78

Therefore at $3.78M dollars or Rs. 170.1M amount 5th percentile of profit lies.

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: First **division**of**company**, thus have **larger probability**of making a loss in a given year.