Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

1 Op-Amp RC Oscillator Circuits

1.0.1. Consider the quadrature-oscillator circuit given Fig. 1.0.1.1 without the limiter. Let the resistance R_f be equal to $\frac{2R}{1+\Delta}$ where $\Delta << 1$. Show that the poles of the characteristic equation are in the right-half s plane and given by s $\approx \frac{1}{CR}(\frac{\Delta}{4} \pm j)$

1.0.2. And the Equivalent circuit at the input of op-amp 2 is given in Fig 1.0.2.2

1.0.3. **Solution:** The voltage v_{o_1} and 2R are replaced with the Norton equivalent composed of a current source $\frac{v_{o_1}}{2R}$ and parallel resistance 2R. The direction of current in R_f would be from output to input. Thus R_f gives rise to a negative input resistance $-R_f$ as indicated in equivalent circuit given in Fig 1.0.2.2
When we consider the circuit without the limiter and break the loop at X, The circuit looks as shown in Fig 1.0.3.3

1.0.4. Find the open loop gain.
Consider the general open loop block diagram as shown in Fig 1.0.4.4

$$G = \frac{v_o}{v_i} \tag{1.0.4.1}$$

1.0.5. Equivalent circuit diagram for Fig 1.0.4.4 is shown in 1.0.5.5
The expression for open loop gain is

$$G = \frac{v_{o_1}}{v_x} = -\frac{1}{sCR} \tag{1.0.5.1}$$

1.0.6. Consider the general block diagram for Feedback network in Fig 1.0.6.6

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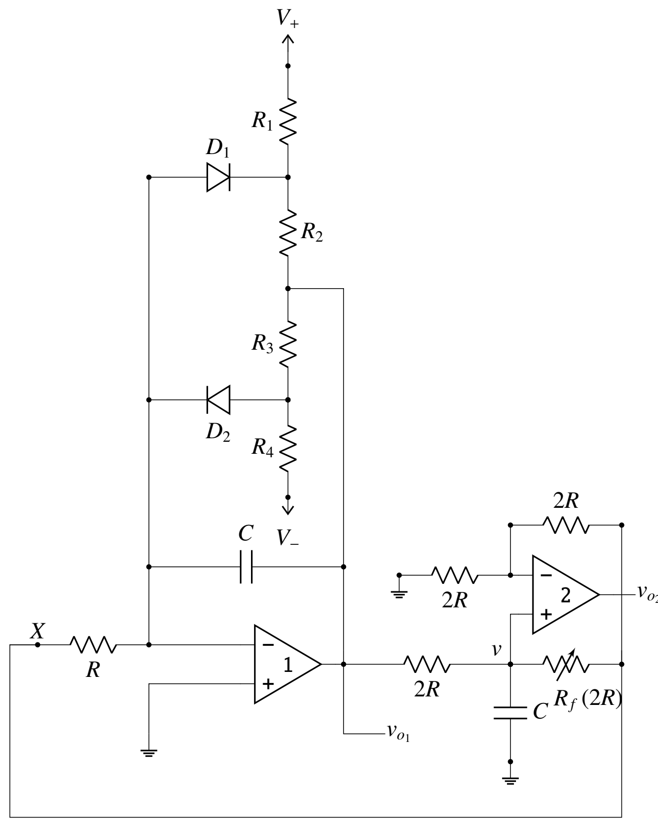

Fig. 1.0.1.1

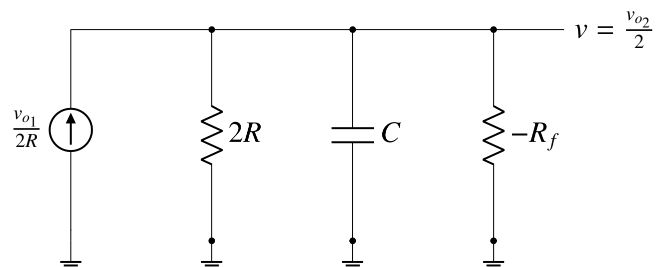

Fig. 1.0.2.2

$$H = \frac{v_f}{v_o} \tag{1.0.6.1}$$

1.0.7. The equivalent Circuit is shown in Fig 1.0.7.7

$$H = \frac{v_{o_2}}{v_{o_1}} \tag{1.0.7.1}$$

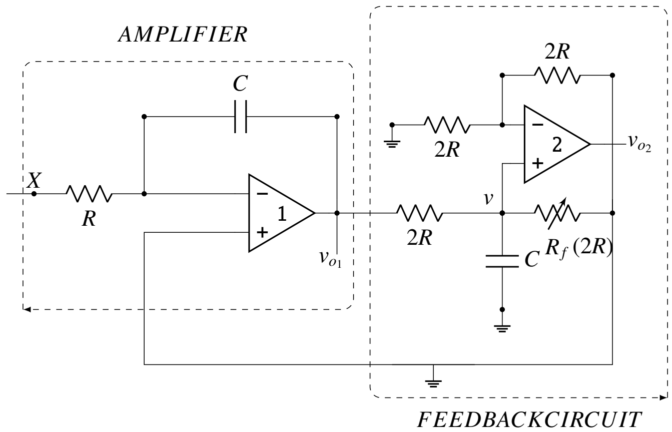

Fig. 1.0.3.3: Circuit without the limiter

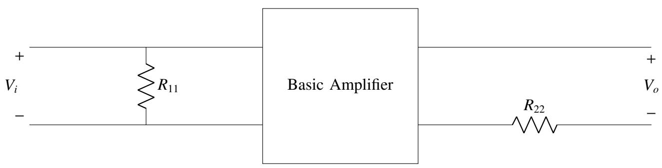

Fig. 1.0.4.4: Open Loop Block diagram

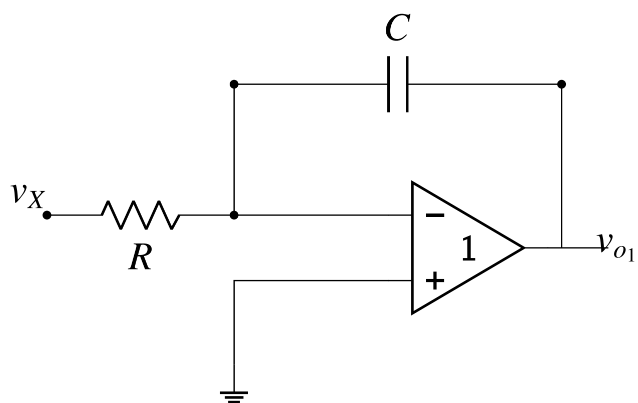

Fig. 1.0.5.5: Equivalent Circuit for open loop block diagram

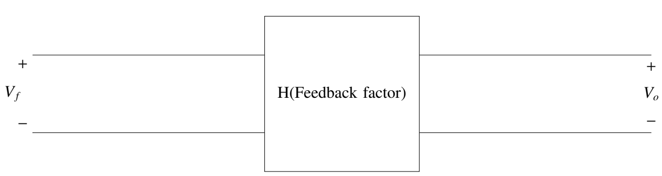

Fig. 1.0.6.6: Feedback Block diagram

1.0.8. Draw the equivalent control system representation for the circuit in Fig. 1.0.3.3 which is given in 1.0.8.8

1.0.9. The Quadrature oscillator is based on the second integrator.
As an active filter, the loop is damped to locate the poles in the left half of the s-plane. In the quadrature oscillator, the op amp 1 is connected as an inverting miller integrator with a limiter in the feedback for controlling the amplitude. The op amp 2 is connected as a non-inverting integrator.

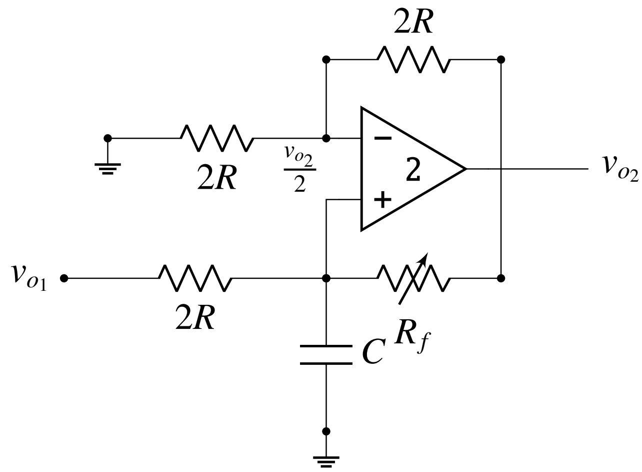

Fig. 1.0.7.7: Equivalent Feedback Circuit

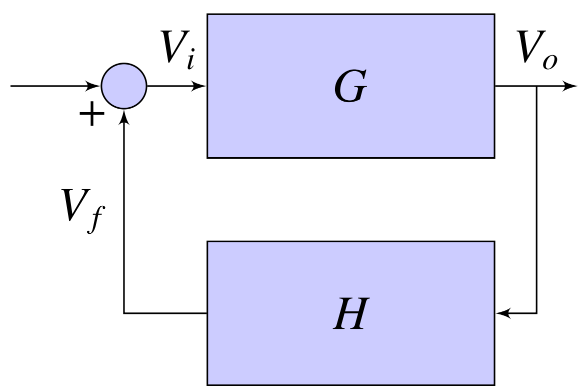

Fig. 1.0.8.8: Simplified equivalent block diagram

1.0.10. In 1.0.7.7.
Use voltage division principle to write the expression of the fraction of its input voltage.

$$v_+ = v_- = \left(\frac{v_{o_2}}{2R + 2R}\right)(2R) = \frac{v_{o_2}}{2} \quad (1.0.10.1)$$

1.0.11. Apply KCL at the non- inverting terminal of the op amp in Fig 1.0.7.7

$$\frac{\frac{v_{0_2}}{2} - v_{o_1}}{2R} + \frac{\frac{v_{0_2}}{2}}{\frac{1}{sC}} + \frac{\frac{v_{0_2}}{2} - v_{o_2}}{R_f} = 0 \quad (1.0.11.1)$$

$$\frac{v_{o_2}-2v_{o_1}}{4R}+sC\frac{v_{o_2}}{2}+\frac{v_{o_2}-2v_{o_2}}{2R_f}=0 \tag{1.0.11.2}$$

$$\frac{v_{o_2}-2v_{o_1}}{4R}+sCv_{o_2}-\frac{v_{o_2}}{R_f}=0 \tag{1.0.11.3}$$

1.0.12. Substitute $R_f = \frac{2R}{1+\Delta}$ and find the feedback factor H

$$\frac{v_{o_2}-2v_{o_1}}{4R}+sCv_{o_2}-\frac{v_{o_2}}{2R}(1+\Delta)=0 \tag{1.0.12.1}$$

$$v_{o_2}\left(\frac{1}{2R}+sC-\frac{1+\Delta}{2R}\right)=\frac{v_{o_1}}{R} \tag{1.0.12.2}$$

$$v_{o_2}(1+2sRC-1-\Delta)=2v_{o_1} \tag{1.0.12.3}$$

Simplifying further,

$$\frac{v_{o_2}}{v_{o_1}}=\frac{1}{sRC-\frac{\Delta}{2}} \tag{1.0.12.4}$$

$$H=\frac{v_{o_2}}{v_{o_1}}=\frac{1}{sRC-\frac{\Delta}{2}} \tag{1.0.12.5}$$

1.0.13. The transfer function of the equivalent positive feedback circuit in Fig. 1.0.7.7 is

$$T=\frac{G}{1-GH} \tag{1.0.13.1}$$

Therefore, loop gain is given by

$$L=GH \tag{1.0.13.2}$$

From (1.0.5.1) and (1.0.12.5)

$$L(s)=\frac{-1}{sCR}\frac{1}{sCR-\frac{\Delta}{2}} \tag{1.0.13.3}$$

$$L(s)=\frac{1}{-s^2C^2R^2+\frac{sCR\Delta}{2}} \tag{1.0.13.4}$$

Consider the characteristic equation of the transfer function (1.0.13.1),

$$1-L(s)=0 \tag{1.0.13.5}$$

$$L(s)=1 \tag{1.0.13.6}$$

$$-s^2C^2R^2+\frac{sCR\Delta}{2}=1 \tag{1.0.13.7}$$

$$\left(C^2R^2\right)s^2+\left(-\frac{CR\Delta}{2}\right)s+1=0 \tag{1.0.13.8}$$

1.0.14. Write the expression for roots of a general quadratic equation

$$s_p=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \tag{1.0.14.1}$$

Substitute $a = C^2R^2$, $b = -\frac{CR\Delta}{2}$, $c = 1$ in (1.0.14.1),

$$s_p=\frac{-\left(-\frac{CR\Delta}{2}\right)\pm\sqrt{\left(-\frac{CR\Delta}{2}\right)^2-4\left(C^2R^2\right)(1)}}{2C^2R^2} \tag{1.0.14.2}$$

$$=\frac{RC\left(\frac{\Delta}{2}\pm\sqrt{\left(\frac{\Delta}{2}\right)^2-4}\right)}{2C^2R^2} \tag{1.0.14.3}$$

$$=\frac{\frac{\Delta}{2}\pm\sqrt{\left(\frac{\Delta}{2}\right)^2-4}}{2RC} \tag{1.0.14.4}$$

$$=\frac{\frac{\Delta}{2}\pm2j\sqrt{1-\left(\frac{\Delta}{4}\right)^2}}{2RC} \tag{1.0.14.5}$$

As $\Delta << 1$,

$$\left(1-\left(\frac{\Delta}{4}\right)^2\right)^{\frac{1}{2}}=1-\frac{1}{2}\left(\frac{\Delta}{4}\right)^2 \tag{1.0.14.6}$$

$$s_p=\frac{\frac{\Delta}{2}\pm2j\left(1-\frac{1}{2}\left(\frac{\Delta}{4}\right)^2\right)}{2RC} \tag{1.0.14.7}$$

$$s_p=\frac{\frac{\Delta}{2}\pm j\left(2-\left(\frac{\Delta}{4}\right)^2\right)}{2RC} \tag{1.0.14.8}$$

From (1.0.14.8),

$$Re\left(s_p\right)>0 \tag{1.0.14.9}$$

Hence, the poles of the characteristic equation are in the right half of the s plane. As $\Delta << 1$, higher order terms are neglected.

$$s_p=\frac{\frac{\Delta}{2}\pm2j}{2RC} \tag{1.0.14.10}$$

$$s_p=\frac{\frac{\Delta}{4}\pm j}{RC} \tag{1.0.14.11}$$

1.0.15. Find the frequency for arbitrary R,C values as given in Table 1.0.15

Parameter	**Value**
R	$5k\Omega$
C	$10\mu F$
Δ	0.1
$R_f = \frac{2R}{1+\Delta}$	9090.9

TABLE 1.0.15

The loop will oscillate at frequency ω_o, given by

$$\omega_o = \frac{1}{RC} \tag{1.0.15.1}$$

From Table 1.0.15,

$$\omega_o = 20rad/s \tag{1.0.15.2}$$

$$f = \frac{\omega_o}{2\pi} = 3.184Hz \tag{1.0.15.3}$$

From (1.0.13.1),

$$T = \frac{-SCR + \frac{\Delta}{2}}{s^2C^2R^2 - \frac{sCR\Delta}{2} + 1} \tag{1.0.15.4}$$

From Table 1.0.15,

$$T = \frac{-0.05s + 0.05}{0.0025s^2 - 0.0025s + 1} \tag{1.0.15.5}$$

The following code plots the oscillating response of the system as shown in 1.0.15.9

```
codes/es17btech11009/es17btech11009_1_1.
    py
```

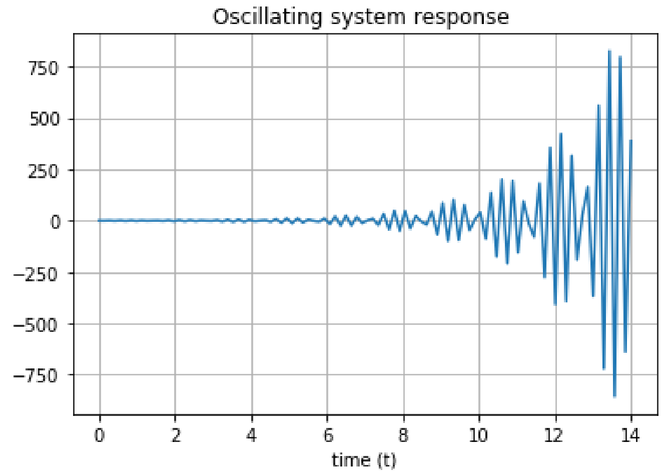

Fig. 1.0.15.9

1.0.16. Simulate the circuit Fig 1.0.7.7 using spice simulators and plot the generated output using python script.
Find the netlist for the simulated circuit here:

```
spice/es17btech11009.net
```

Python code used for generating the output:

```
codes/es17btech11009/es17btech11009_spice.
    py
```

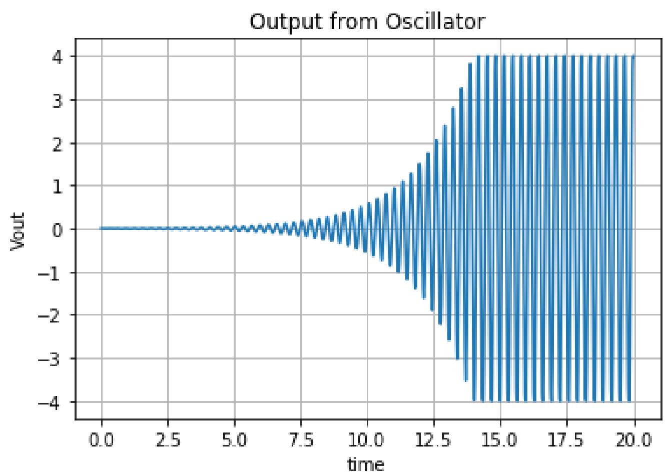

Fig. 1.0.16.10

1.0.17. Consider part of the spice simulation and the following code plots the part of the output as shown in Fig 1.0.17.11

```
codes/es17btech11009/
    es17btech11009_spice1.py
```

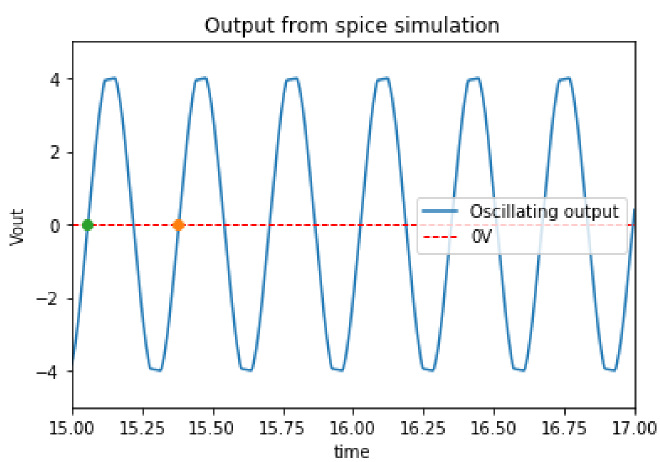

Fig. 1.0.17.11

From the Fig 1.0.16.10, Time period of oscillation

$$T = 15.3738 - 15.0569 \tag{1.0.17.1}$$

$$f = \frac{1}{T} = 3.155Hz \tag{1.0.17.2}$$

$$\omega_o = 2\pi f = 19.8rad/s \tag{1.0.17.3}$$

Hence the frequency calculated from the formulae and the plot are approximately same.