Quadrature Oscillator

Hrithik Raj*

Consider the quadrature-oscillator circuit given Fig. 0 without the limiter. Let the resistance R_f be equal to $\frac{2R}{1+\Delta}$ where $\Delta << 1$. Show that the poles of the characteristic equation are in the right-half s plane and given by s $\approx \frac{1}{CR}(\frac{\Delta}{4} \pm j)$

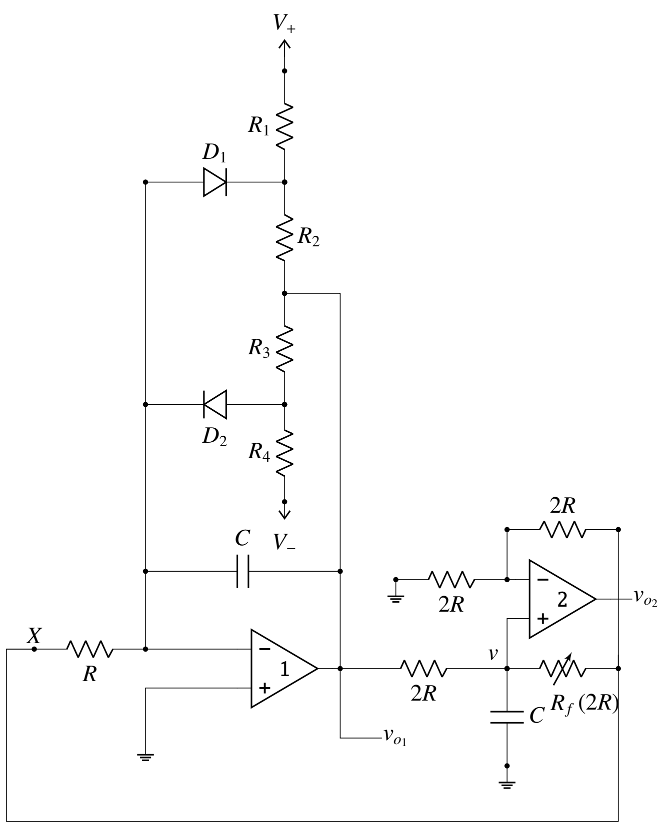

Fig. 0

1. Identify the the open loop gain and feedback components of the circuit.
   Solution: See Fig. 1.
2. Draw the block diagram and equivalent circuit for H.
   Solution: See Figs. 2.1 and 2.2.

$$H = \frac{v_f}{v_o} \tag{2.1}$$

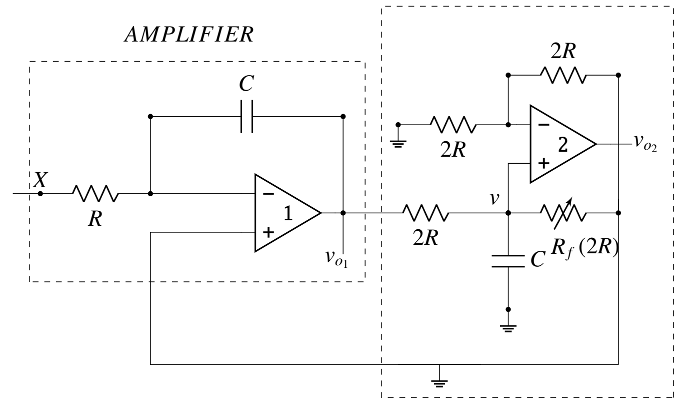

Fig. 1: Circuit without the limiter

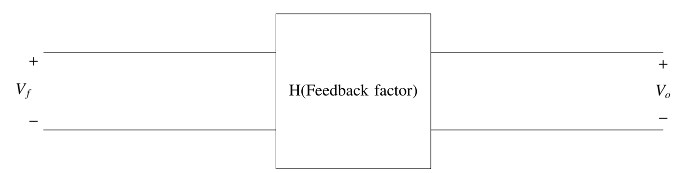

Fig. 2.1: Feedback Block diagram

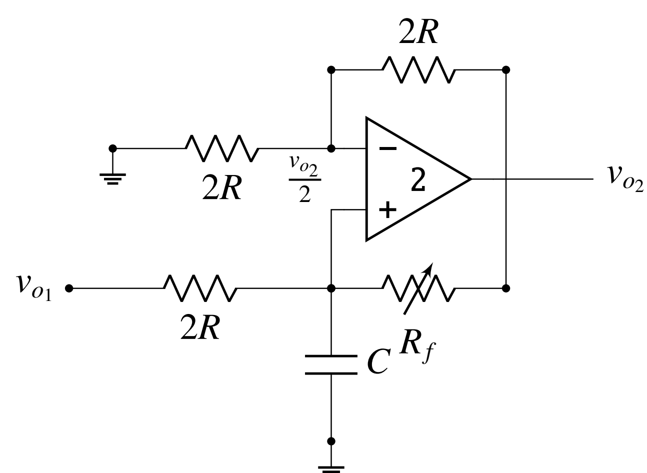

Fig. 2.2: Equivalent Feedback Circuit

3. Find H.
   Solution: In 2.2,

$$v_+ = v_- = \left(\frac{v_{o_2}}{2R + 2R}\right)(2R) = \frac{v_{o_2}}{2} \tag{3.1}$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India. All content in this manual is released under GNU GPL. Free and open source.

Using node analysis at the non- inverting terminal, and substituting

$$R_f = \frac{2R}{1+\Delta}, \tag{3.2}$$

$$\frac{\frac{v_{o_2}}{2} - v_{o_1}}{2R} + \frac{\frac{v_{o_2}}{2}}{\frac{1}{sC}} + \frac{\frac{v_{o_2}}{2} - v_{o_2}}{R_f} = 0 \tag{3.3}$$

$$\implies \frac{v_{o_2} - 2v_{o_1}}{4R} + sCv_{o_2} - \frac{v_{o_2}}{2R}(1+\Delta) = 0 \tag{3.4}$$

$$\text{or, } H = \frac{v_{o_2}}{v_{o_1}} = \frac{1}{sRC - \frac{\Delta}{2}} \tag{3.5}$$

after some algebra.

4. And the Equivalent circuit at the input of op-amp 2 is given in Fig 4.1

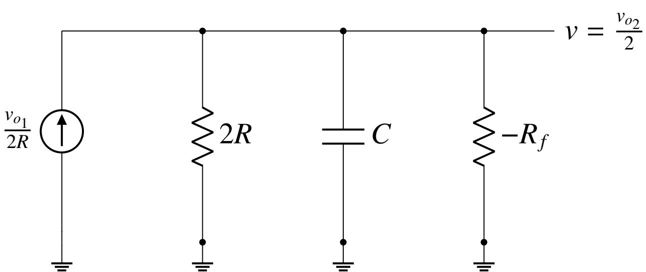

Fig. 4.1

5. **Solution:** The voltage v_{o_1} and 2R are replaced with the Norton equivalent composed of a current source $\frac{v_{o_1}}{2R}$ and parallel resistance 2R. The direction of current in R_f would be from output to input. Thus R_f gives rise to a negative input resistance $-R_f$ as indicated in equivalent circuit given in Fig 4.1
When we consider the circuit without the limiter and break the loop at X, The circuit looks as shown in Fig 1
6. Find the open loop gain.
Consider the general open loop block diagram as shown in Fig 6.1

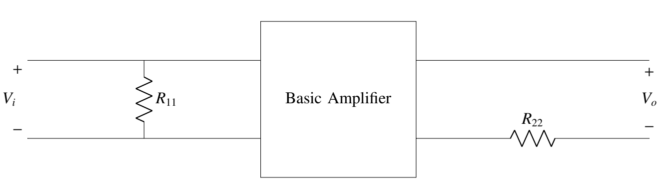

Fig. 6.1: Open Loop Block diagram

$$G = \frac{v_o}{v_i} \tag{6.1}$$

7. Find R_{11} and R_{22} from Fig 7.1

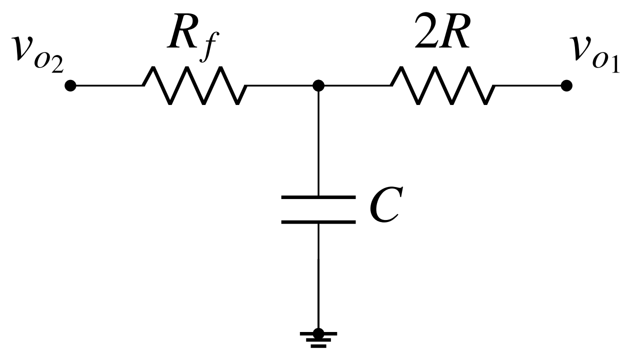

Fig. 7.1: From Feedback Circuit

Shorting v_{o_1} to ground,

$$R_{11} = R_f + \left(2R \parallel \frac{1}{sC}\right) \tag{7.1}$$

From (3.2),

$$R_{11} = \left(\frac{2R}{1+\Delta}\right) + \left(2R \parallel \frac{1}{sC}\right) \tag{7.2}$$

Shorting v_{o_2} to ground,

$$R_{22} = 2R + \left(R_f \parallel \frac{1}{sC}\right) \tag{7.3}$$

From (3.2),

$$R_{22} = 2R + \left(\left(\frac{2R}{1+\Delta}\right) \parallel \frac{1}{sC}\right) \tag{7.4}$$

8. Equivalent circuit diagram for Fig 6.1 is shown in 8.1

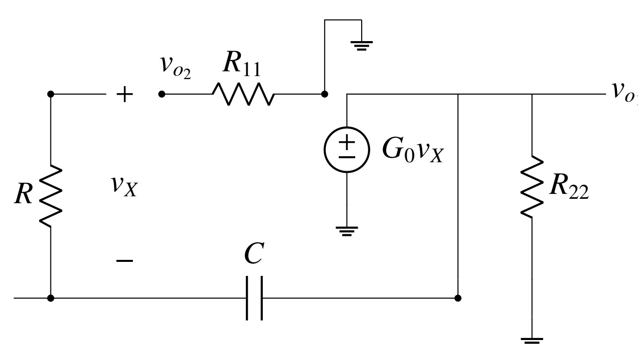

Fig. 8.1: Equivalent Circuit for open loop block diagram

where G_0 is the gain of the op-amp.

The expression for open loop gain is

$$G = \frac{v_{o_1}}{v_X} = -\frac{1}{sCR} \tag{8.1}$$

9. Draw the equivalent control system representation for the circuit in Fig. 1 which is given in 9.1

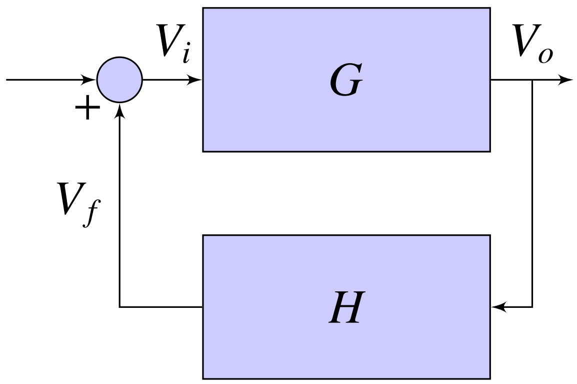

Fig. 9.1: Simplified equivalent block diagram

10. The Quadrature oscillator is based on the second integrator.
As an active filter, the loop is damped to locate the poles in the left half of the s-plane. In the quadrature oscillator, the op amp 1 is connected as an inverting miller integrator with a limiter in the feedback for controlling the amplitude. The op amp 2 is connected as a non-inverting integrator.
11. The transfer function of the equivalent positive feedback circuit in Fig. 2.2 is

$$T = \frac{G}{1 - GH} \tag{11.1}$$

Therefore, loop gain is given by

$$L = GH \tag{11.2}$$

From (8.1) and (3.5)

$$L(s) = \frac{-1}{sCR}\frac{1}{sCR - \frac{\Delta}{2}} \tag{11.3}$$

$$L(s) = \frac{1}{-s^2C^2R^2 + \frac{sCR\Delta}{2}} \tag{11.4}$$

Consider the characteristic equation of the transfer function (11.1),

$$1 - L(s) = 0 \tag{11.5}$$

$$L(s) = 1 \tag{11.6}$$

$$-s^2C^2R^2 + \frac{sCR\Delta}{2} = 1 \tag{11.7}$$

$$\left(C^2R^2\right)s^2 + \left(-\frac{CR\Delta}{2}\right)s + 1 = 0 \tag{11.8}$$

12. Write the expression for roots of a general quadratic equation

$$s_p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{12.1}$$

Substitute $a = C^2R^2$, $b = -\frac{CR\Delta}{2}$, $c = 1$ in (12.1),

$$s_p = \frac{-\left(-\frac{CR\Delta}{2}\right) \pm \sqrt{\left(-\frac{CR\Delta}{2}\right)^2 - 4\left(C^2R^2\right)(1)}}{2C^2R^2} \tag{12.2}$$

$$= \frac{RC\left(\frac{\Delta}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 - 4}\right)}{2C^2R^2} \tag{12.3}$$

$$= \frac{\frac{\Delta}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 - 4}}{2RC} \tag{12.4}$$

$$= \frac{\frac{\Delta}{2} \pm 2j\sqrt{1 - \left(\frac{\Delta}{4}\right)^2}}{2RC} \tag{12.5}$$

As $\Delta << 1$,

$$\left(1 - \left(\frac{\Delta}{4}\right)^2\right)^{\frac{1}{2}} = 1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2 \tag{12.6}$$

$$s_p = \frac{\frac{\Delta}{2} \pm 2j\left(1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2\right)}{2RC} \tag{12.7}$$

$$s_p = \frac{\frac{\Delta}{2} \pm j\left(2 - \left(\frac{\Delta}{4}\right)^2\right)}{2RC} \tag{12.8}$$

From (12.8),

$$Re\left(s_p\right) > 0 \tag{12.9}$$

Hence, the poles of the characteristic equation are in the right half of the s plane. As $\Delta << 1$, higher order terms are neglected.

$$s_p = \frac{\frac{\Delta}{2} \pm 2j}{2RC} \tag{12.10}$$

$$s_p = \frac{\frac{\Delta}{4} \pm j}{RC} \tag{12.11}$$

13. Find the frequency for arbitrary R,C values as given in Table 13

Parameter	**Value**
R	$1k\Omega$
C	$10\mu F$
Δ	0.1
$R_f = \frac{2R}{1+\Delta}$	1818.18

TABLE 13

The loop will oscillate at frequency ω_o, given by

$$\omega_o = \frac{1}{RC} \tag{13.1}$$

From Table 13,

$$\omega_o = 100rad/s \tag{13.2}$$

$$f = \frac{\omega_o}{2\pi} = 15.923Hz \tag{13.3}$$

From (11.1),

$$T = \frac{-SCR + \frac{\Delta}{2}}{s^2C^2R^2 - \frac{sCR\Delta}{2} + 1} \tag{13.4}$$

From Table 13,

$$T = \frac{-0.05s + 0.05}{0.0025s^2 - 0.0025s + 1} \tag{13.5}$$

The following code plots the step response of the system as shown in 13.1

```
codes/es17btech11009/es17btech11009_1_1.
    py
```

14. The following code plots the impulse response of the system as shown in 14.1

```
codes/es17btech11009/es17btech11009_imp.
    py
```

15. Simulate the circuit Fig 2.2 using spice simulators and plot the generated output using python script.

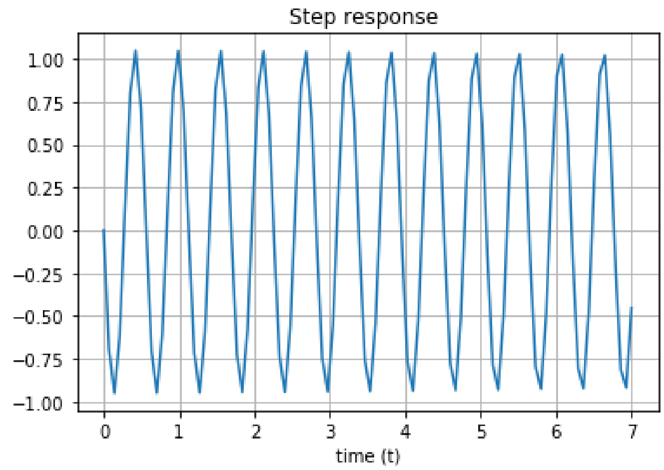

Fig. 13.1

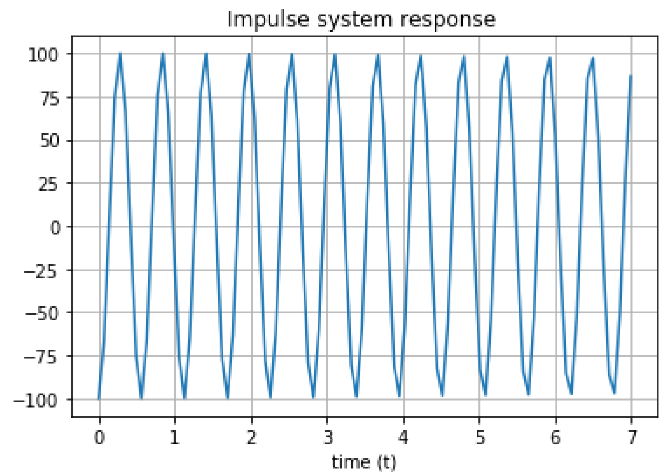

Fig. 14.1

Find the netlist for the simulated circuit here:

```
codes/es17btech11009/spice/es17btech11009.
    net
```

Python code used for generating the output:

```
codes/es17btech11009/es17btech11009_spice.
    py
```

16. Consider part of the spice simulation and the following code plots the part of the output as shown in Fig 16.1

```
codes/es17btech11009/
    es17btech11009_spice1.py
```

From the Fig 16.1, Time period of oscillation

$$T = 3.3801 - 3.317 \tag{16.1}$$

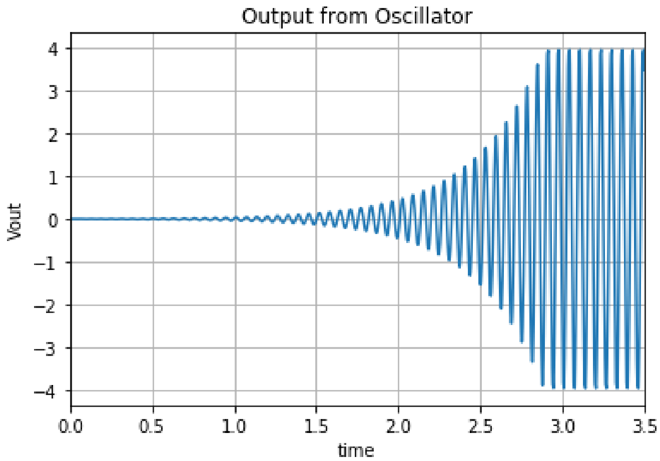

Fig. 15.1

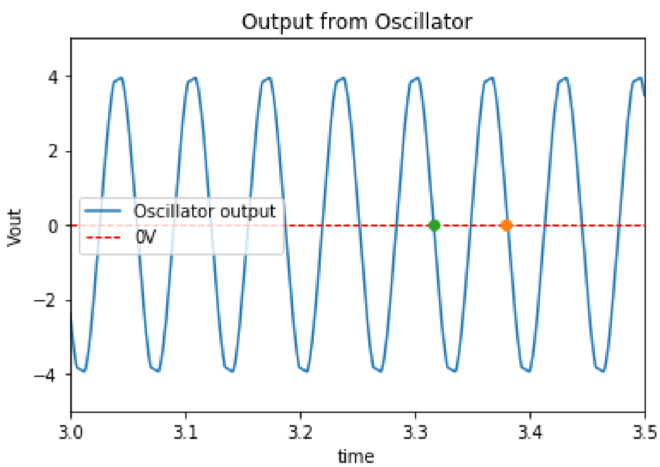

Fig. 16.1

$$f = \frac{1}{T} = 15.847Hz \tag{16.2}$$

$$\omega_o = 2\pi f = 99.5rad/s \tag{16.3}$$

Hence the frequency calculated from the formulae and the plot are approximately same.