Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

1 Op-Amp RC Oscillator Circuits

1.0.1. Consider the quadrature-oscillator circuit given Fig. 1.0.1.1 without the limiter. Let the resistance R_f be equal to $\frac{2R}{1+\Delta}$ where $\Delta << 1$. Show that the poles of the characteristic equation are in the right-half s plane and given by s $\approx \frac{1}{CR}(\frac{\Delta}{4} \pm j)$

1.0.2. And the Equivalent circuit at the input of op-amp 2 is given in Fig 1.0.2.2

1.0.3. **Solution:** The Quadrature oscillator is based on the second integrator.
As an active filter, the loop is damped to locate the poles in the left half of the s-plane. In the quadrature oscillator, the op amp 1 is connected as an inverting miller integrator with a limiter in the feedback for controlling the amplitude. The op amp 2 is connected as a non-inverting integrator.
Draw the circuit without the limiter shown in Fig 1.0.3.3

1.0.4. Draw the equivalent control system representation for the circuit in Fig. 1.0.3.3 which is given in 1.0.4.4

1.0.5. From 1.0.3.3.
Use voltage division principle to write the expression of the fraction of its input voltage.

$$v_+ = v_- = \left(\frac{v_{o2}}{2R+2R}\right)(2R) = \frac{v_{o2}}{2} \tag{1.0.5.1}$$

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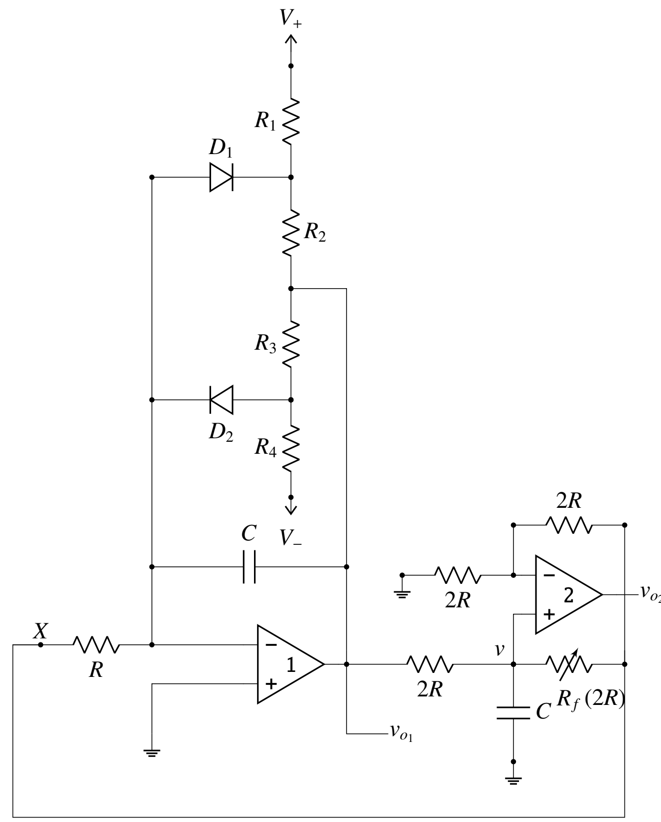

Fig. 1.0.1.1

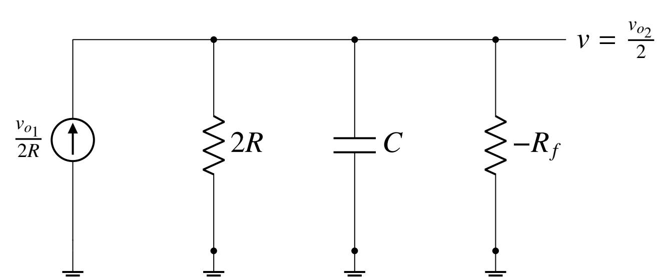

Fig. 1.0.2.2

1.0.6. Apply KCL at the non- inverting terminal of the op amp in Fig 1.0.3.3

$$\frac{\frac{v_{0_2}}{2} - v_{o_1}}{2R} + \frac{\frac{v_{0_2}}{2}}{\frac{1}{sC}} + \frac{\frac{v_{0_2}}{2} - v_{o_2}}{R_f} = 0 \tag{1.0.6.1}$$

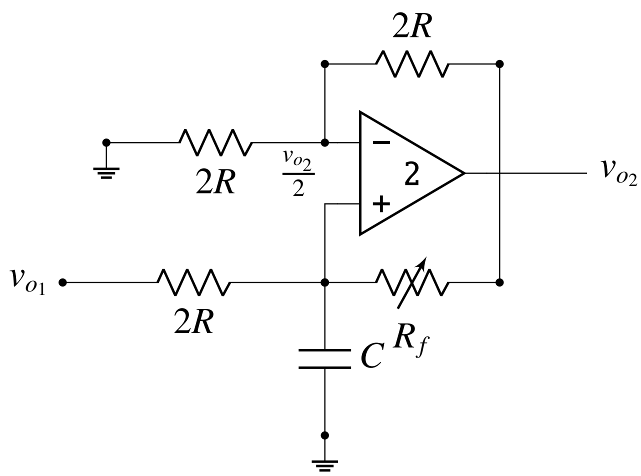

Fig. 1.0.3.3

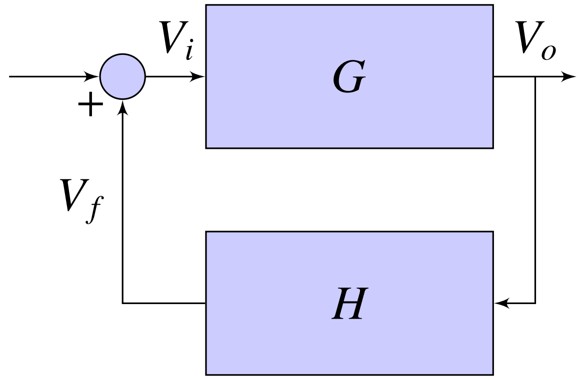

Fig. 1.0.4.4: Simplified equivalent block diagram

$$\frac{v_{o_2} - 2v_{o_1}}{4R} + sC\frac{v_{o_2}}{2} + \frac{v_{o_2} - 2v_{o_2}}{2R_f} = 0 \quad (1.0.6.2)$$

$$\frac{v_{o_2} - 2v_{o_1}}{4R} + sCv_{o_2} - \frac{v_{o_2}}{R_f} = 0 \quad (1.0.6.3)$$

1.0.7. Substitute $R_f = \frac{2R}{1+\Delta}$ and find the feedback factor H

$$\frac{v_{o_2} - 2v_{o_1}}{4R} + sCv_{o_2} - \frac{v_{o_2}}{2R}(1+\Delta) = 0 \quad (1.0.7.1)$$

$$v_{o_2}\left(\frac{1}{2R} + sC - \frac{1+\Delta}{2R}\right) = \frac{v_{o_1}}{R} \quad (1.0.7.2)$$

$$v_{o_2}(1 + 2sRC - 1 - \Delta) = 2v_{o_1} \quad (1.0.7.3)$$

Simplifying further,

$$\frac{v_{o_2}}{v_{o_1}} = \frac{1}{sRC - \frac{\Delta}{2}} \quad (1.0.7.4)$$

$$H = \frac{v_{o_2}}{v_{o_1}} = \frac{1}{sRC - \frac{\Delta}{2}} \quad (1.0.7.5)$$

1.0.8. Find the open loop gain

When we consider the circuit without the limiter and break the loop at X, The expression for open loop gain is

$$G = \frac{v_{o_2}}{v_x} = -\frac{1}{sCR} \quad (1.0.8.1)$$

The transfer function of the equivalent positive feedback circuit in Fig. 1.0.3.3 is

$$T = \frac{G}{1 - GH} \quad (1.0.8.2)$$

Therefore, loop gain is given by

$$L = GH \quad (1.0.8.3)$$

From (1.0.8.1) and (1.0.7.5)

$$L(s) = \frac{-1}{sCR}\frac{1}{sCR - \frac{\Delta}{2}} \quad (1.0.8.4)$$

$$L(s) = \frac{1}{-s^2C^2R^2 + \frac{sCR\Delta}{2}} \quad (1.0.8.5)$$

Consider the characteristic equation of the transfer function (1.0.8.2),

$$1 - L(s) = 0 \quad (1.0.8.6)$$

$$L(s) = 1 \quad (1.0.8.7)$$

$$-s^2C^2R^2 + \frac{sCR\Delta}{2} = 1 \quad (1.0.8.8)$$

$$\left(C^2R^2\right)s^2 + \left(-\frac{CR\Delta}{2}\right)s + 1 = 0 \quad (1.0.8.9)$$

1.0.9. Write the expression for roots of a general quadratic equation

$$s_p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (1.0.9.1)$$

Substitute $a = C^2R^2$, $b = -\frac{CR\Delta}{2}$, $c = 1$ in (1.0.9.1),

$$s_p = \frac{-\left(-\frac{CR\Delta}{2}\right) \pm \sqrt{\left(-\frac{CR\Delta}{2}\right)^2 - 4(C^2R^2)(1)}}{2C^2R^2} \quad (1.0.9.2)$$

$$= \frac{RC\left(\frac{\Delta}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 - 4}\right)}{2C^2R^2} \quad (1.0.9.3)$$

$$= \frac{\frac{\Delta}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 - 4}}{2RC} \quad (1.0.9.4)$$

$$= \frac{\frac{\Delta}{2} \pm 2j\sqrt{1 - \left(\frac{\Delta}{4}\right)^2}}{2RC} \quad (1.0.9.5)$$

As $\Delta << 1$,

$$\left(1 - \left(\frac{\Delta}{4}\right)^2\right)^{\frac{1}{2}} = 1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2 \quad (1.0.9.6)$$

$$s_p = \frac{\frac{\Delta}{2} \pm 2j\left(1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2\right)}{2RC} \quad (1.0.9.7)$$

$$s_p = \frac{\frac{\Delta}{2} \pm j\left(2 - \left(\frac{\Delta}{4}\right)^2\right)}{2RC} \quad (1.0.9.8)$$

From (1.0.9.8),

$$Re\left(s_p\right) > 0 \quad (1.0.9.9)$$

Hence, the poles of the characteristic equation are in the right half of the s plane. As $\Delta << 1$, higher order terms are neglected.

$$s_p = \frac{\frac{\Delta}{2} \pm 2j}{2RC} \quad (1.0.9.10)$$

$$s_p = \frac{\frac{\Delta}{4} \pm j}{RC} \quad (1.0.9.11)$$

1.0.10. Find the frequency for arbitrary R,C values as given in Table 1.0.10

Parameter	Value
$2R$	$2k\Omega$
C	$0.001F$
Δ	0.1
$R_f = \frac{2R}{1+\Delta}$	909

TABLE 1.0.10

The loop will oscillate at frequency ω_o, given by

$$\omega_o = \frac{1}{RC} \quad (1.0.10.1)$$

From Table 1.0.10,

$$\omega_o = 1rad/s \quad (1.0.10.2)$$

$$f = \frac{\omega_o}{2\pi} = 0.159Hz \quad (1.0.10.3)$$

Substituting (1.0.8.1) and (1.0.7.5) in (1.0.8.2),

$$T = \frac{-SCR + \frac{\Delta}{2}}{s^2C^2R^2 - \frac{sCR\Delta}{2} + 1} \quad (1.0.10.4)$$

Taking values from Table 1.0.10,

$$T = \frac{-s + 0.05}{s^2 - 0.05s + 1} \quad (1.0.10.5)$$

The following code plots the oscillating response of the system as given in Fig 1.0.10.5

```
codes/es17btech11009/es17btech11009_1_1.
    py
```

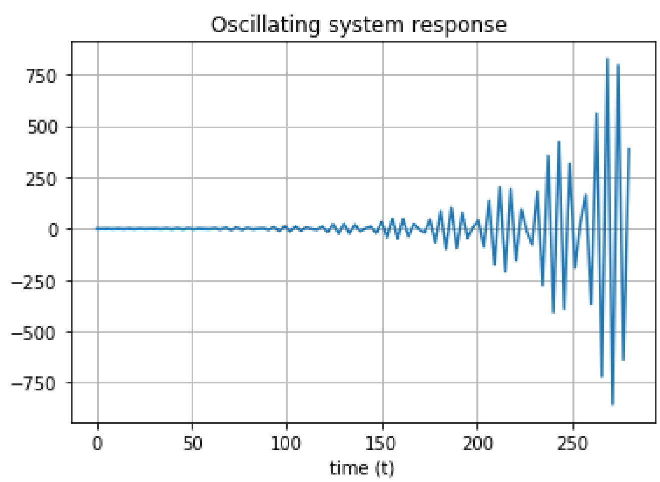

Fig. 1.0.10.5

1.0.11. Simulate the circuit Fig 1.0.3.3 using spice simulators and plot the generated output using python script.
Find the netlist for the simulated circuit here:

```
spice/es17btech11009.net
```

Python code used for generating the output:

```
codes/es17btech11009/es17btech11009_spice.
    py
```

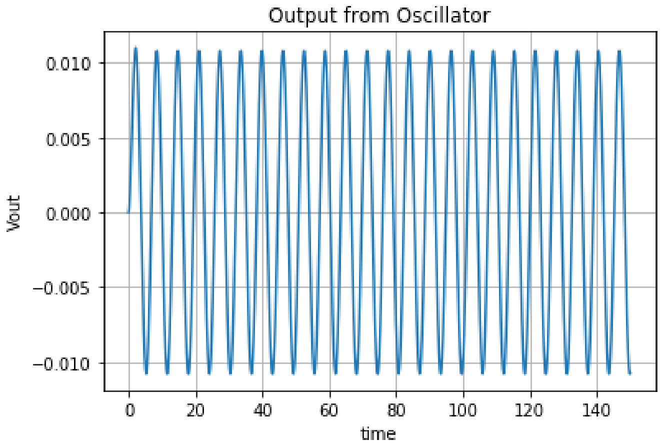

Fig. 1.0.11.6

From the Fig 1.0.11.6, Time period of oscillation

$$T = 6.28s \tag{1.0.11.1}$$

$$f = \frac{1}{T} = 0.159 \tag{1.0.11.2}$$

$$\omega_o = 2\pi f = 0.998 \tag{1.0.11.3}$$

Hence the frequency calculated from the formulae and the plot are approximately same.