

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

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1 MASON'S GAIN FORMULA

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3 SECOND ORDER SYSTEM

3.1 Damping

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4.3 Stability

5 STATE-SPACE MODEL

5.1 Controllability and Observability

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8 NYQUIST PLOT

8.1. Loop Transfer function of a feedback system is

$$G(s)H(s) = \frac{s+3}{s^2(s-3)} \quad (8.1.1)$$

Take the Nyquist contour in the clockwise direction.

Then the Nyquist plot of $G(s)H(s)$ encircles $-1 + j0$

- (A) Once in clockwise direction
- (B) Twice in clockwise direction
- (C) Once in anticlockwise direction
- (D) Twice in clockwise direction

Solution: Substituting $s = j\omega$ in (8.1.1),

$$G(j\omega)H(j\omega) = \frac{j\omega + 3}{(j\omega)^2(j\omega - 3)} \quad (8.1.2)$$

$$G(j\omega)H(j\omega) = \frac{j\omega + 3}{\omega^2(3 - j\omega)} \quad (8.1.3)$$

plot started in clockwise direction.

codes/es17btech11009.py

$$|G(j\omega)H(j\omega)| = \frac{(\sqrt{\omega^2 + 9})}{(\omega)^2 (\sqrt{\omega^2 + 9})} \quad (8.1.4)$$

$$|G(j\omega)H(j\omega)| = \frac{1}{(\omega)^2} \quad (8.1.5)$$

$$\angle G(j\omega)H(j\omega) = \tan^{-1}\left(\frac{\omega}{3}\right) - (\pi/2 + \pi/2 - \tan^{-1}\left(\frac{\omega}{3}\right)) \quad (8.1.6)$$

$$\angle G(j\omega)H(j\omega) = 2 \tan^{-1}\left(\frac{\omega}{3}\right) \quad (8.1.7)$$

8.2. Find $G(j\omega)H(j\omega)$ for the Nyquist plotting

Solution: From (8.1.5) and (8.1.7)

$$G(j\omega)H(j\omega) = \frac{1}{(\omega)^2} \angle 2 \tan^{-1}\left(\frac{\omega}{3}\right) \quad (8.2.1)$$

8.3. Nyquist plot and verify your result

Solution:

For the Nyquist plot,

We need to draw the polar plot by varying ω from 0 to ∞

$$\lim_{\omega \rightarrow \infty} G(j\omega)H(j\omega) = 0 \angle 180 \quad (8.3.1)$$

$$\lim_{\omega \rightarrow 0} G(j\omega)H(j\omega) = \infty \angle 0 \quad (8.3.2)$$

The Nyquist plot is as shown

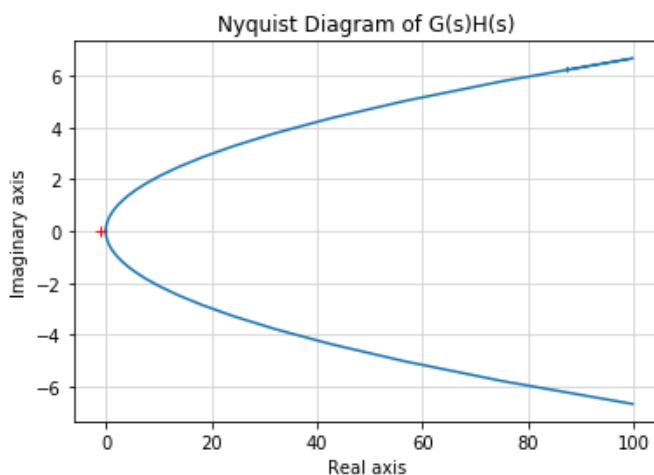


Fig. 8.3

Since there are two poles on the origin we get 2 infinite radius semicircles which start where the mirror image ends and terminate where the actual