

# Control Systems

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**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

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## 1 MASON'S GAIN FORMULA

### 2 BODE PLOT

#### 2.1 Introduction

#### 2.2 Example

## 3 SECOND ORDER SYSTEM

#### 3.1 Damping

#### 3.2 Example

## 4 ROUTH HURWITZ CRITERION

#### 4.1 Routh Array

#### 4.2 Marginal Stability

#### 4.3 Stability

## 5 STATE-SPACE MODEL

#### 5.1 Controllability and Observability

#### 5.2 Second Order System

## 6 NYQUIST PLOT

## 7 COMPENSATORS

## 8 NYQUIST PLOT

8.1. Loop Transfer function of a feedback system is

$$G(s)H(s) = \frac{s+3}{s^2(s-3)} \quad (8.1.1)$$

Take the Nyquist contour in the clockwise direction.

Then the Nyquist plot of  $G(s)H(s)$  encircles  $-1 + j0$

- (A) Once in clockwise direction
- (B) Twice in clockwise direction
- (C) Once in anticlockwise direction
- (D) Twice in clockwise direction

**Solution:** Substituting  $s = j\omega$  in (8.1.1),

$$G(j\omega)H(j\omega) = \frac{j\omega + 3}{(j\omega)^2(j\omega - 3)} \quad (8.1.2)$$

$$G(j\omega)H(j\omega) = \frac{j\omega + 3}{\omega^2(3 - j\omega)} \quad (8.1.3)$$

codes/es17btech11009.py

$$|G(j\omega)H(j\omega)| = \frac{(\sqrt{\omega^2 + 9})}{(\omega)^2 (\sqrt{\omega^2 + 9})} \quad (8.1.4)$$

$$|G(j\omega)H(j\omega)| = \frac{1}{(\omega)^2} \quad (8.1.5)$$

Above code gives us the Nyquist plot  
The Nyquist plot of  $G(s)H(s)$  encircles  $-1 + j0$  once in clockwise direction

$$\angle G(j\omega)H(j\omega) = \tan^{-1}\left(\frac{\omega}{3}\right) - (\pi/2 + \pi/2 - \tan^{-1}\left(\frac{\omega}{3}\right)) \quad (8.1.6)$$

$$\angle G(j\omega)H(j\omega) = 2 \tan^{-1}\left(\frac{\omega}{3}\right) \quad (8.1.7)$$

8.2. Find  $G(j\omega)H(j\omega)$  for the Nyquist plotting

**Solution:** From (8.1.5) and (8.1.7)

$$G(j\omega)H(j\omega) = \frac{1}{(\omega)^2} \angle 2 \tan^{-1}\left(\frac{\omega}{3}\right) \quad (8.2.1)$$

8.3. Nyquist plot and verify your result

**Solution:** For the Nyquist plot,  
We need to draw the polar plot by varying  $\omega$  from 0 to  $\infty$

$$\lim_{\omega \rightarrow \infty} G(j\omega)H(j\omega) = 0 \angle 180 \quad (8.3.1)$$

$$\lim_{\omega \rightarrow 0} G(j\omega)H(j\omega) = \infty \angle 0 \quad (8.3.2)$$

The Nyquist plot is as shown

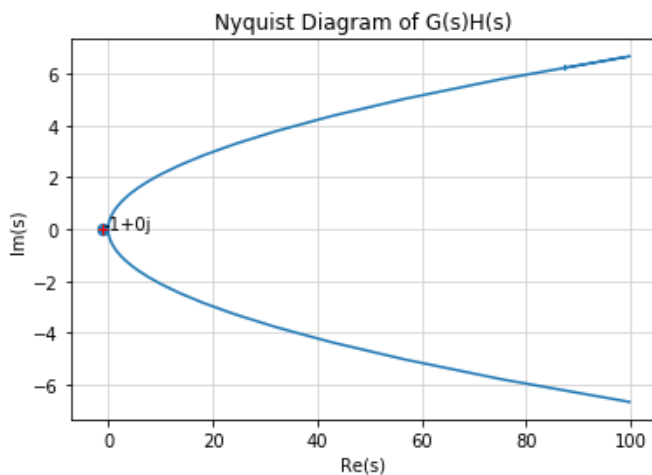


Fig. 8.3

Since there are two poles on the origin we get 2 infinite radius semicircles which start where the mirror image ends and terminate where the actual plot started in clockwise direction.