Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```


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8.1. The open loop transfer function of a system is

$$G(s) = \frac{2}{(s+1)(s+2)} \tag{8.1.1}$$

Find its magnitude and phase response.
Solution: Substituting $s = \jmath\omega$ in (8.1.1),

$$G(\jmath\omega) = \frac{1}{(j\omega+1)(j\omega+2)} \tag{8.1.2}$$

$$\implies \left|G(\jmath\omega)\right| = \frac{2}{\left(\sqrt{\omega^2+1}\right)\left(\sqrt{\omega^2+4}\right)} \tag{8.1.3}$$

$$\angle G(\jmath\omega) = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) \tag{8.1.4}$$

8.2. Find ω for which the gain of (8.1.1) first becomes 1.
Solution: From (8.1.3)

$$\left|G(\jmath\omega)\right| = 1 \quad (8.2.1)$$

$$\implies \frac{2}{\left(\sqrt{\omega^2+1}\right)\left(\sqrt{\omega^2+4}\right)} = 1 \quad (8.2.2)$$

$$\implies \omega_{gc} = 0 \quad (8.2.3)$$

which is the desired frequency.

8.3. Find $\angle G(\jmath\omega_{gc}) + 180°$. This is known as the *phase margin*(PM)
Solution: From (8.1.4),

$$\angle G(\jmath\omega) = 0° \implies PM = 180° \quad (8.3.1)$$

8.4. Verify your result by plotting the gain and phase plots of $G(\jmath\omega)$.
Solution: The following code plots Fig. 8.4

```
codes/ee18btech11017.py
```

The Phase plot is as shown,

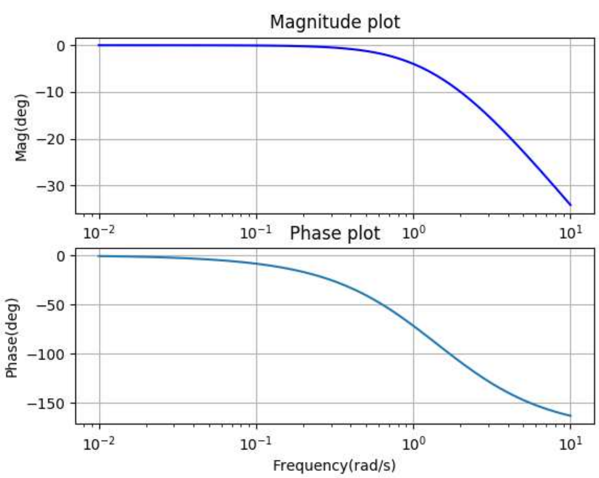

Fig. 8.4

8.5. A positive phase margin for the open loop system indicates a stable closed loop system. (8.3.1) indicates that $G(s)$ with unity feedback is stable. Show that the roots of $1+G(s)$ lie in the left half plane proving closed loop stability.
Solution: Let the closed loop transfer function

$$T(s) = \frac{G(s)}{1+G(s)} \quad (8.5.1)$$

Then

$$1 + G(s) = 0 \quad (8.5.2)$$

$$\implies s^2 + 3s + 4 = 0 \quad (8.5.3)$$

$$\text{or } s = -1.5 + 1.3j, -1.5 - 1.3j \quad (8.5.4)$$

Since the roots are in the left half plane, the system is stable.

8.6. Instead of unity feedback, consider a system with

$$H(s) = \frac{50}{s+1} \quad (8.6.1)$$

Compute the open loop phase margin for this system.
Solution:

$$\because G(s)H(s) = \frac{100}{(s+1)^2(s+2)}, \quad (8.6.2)$$

the magnitude and phase are

$$\left|G(\jmath\omega)H(\jmath\omega)\right| = \frac{10^2}{\sqrt{(\omega^2+1)^2}\sqrt{\omega^2+4}} \quad (8.6.3)$$

$$\angle G(\jmath\omega)H(\jmath\omega) = -\tan^{-1}\frac{\omega}{2} - 2\tan^{-1}(\omega) \quad (8.6.4)$$

The gain crossover frequency is given by

$$\frac{10^2}{\sqrt{\omega_{gc}^2+4}\sqrt{(\omega_{gc}^2+1^2)^2}} = 1 \quad (8.6.5)$$

$$(8.6.6)$$

$$\omega_{gc}^6 + 6\omega_{gc}^4 + 9\omega_{gc}^2 - 9996 = 0 \quad (8.6.7)$$

$$\implies \omega_{gc} = 4.42 \quad (8.6.8)$$

From (8.6.4) and (8.6.8), the phase margin is

$$PM = 180° - 2\tan^{-1}(\omega_{gc}) - \tan^{-1}\left(\frac{\omega_{gc}}{2}\right) \quad (8.6.9)$$

$$\implies P.M = -40.15° \quad (8.6.10)$$

8.7. Verify your result through the magnitude and phase plot.
Solution: The following code plots Fig. 8.7

```
codes/ee18btech11017_2.py
```

8.8. Since the PM in (8.6.10) is negative, the closed loop system is unstable . Verify this using the Routh-Hurwitz criterion.

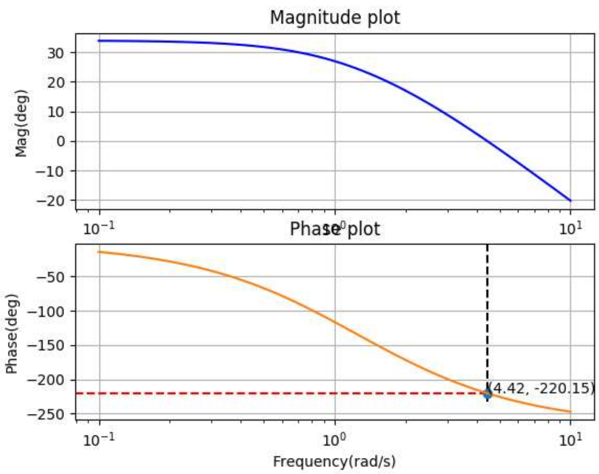

Fig. 8.7

Solution: The characteristic equation is

$$1 + G(s)H(s) = 0 \tag{8.8.1}$$

$$\implies s^3 + 4s^2 + 5s + 102 = 0 \tag{8.8.2}$$

Constructing the routh array for (8.8.2),

$$\begin{vmatrix} s^3 \\ s^2 \\ s \end{vmatrix} \begin{vmatrix} 1 & 5 & 0 \\ 4 & 102 & 0 \\ -20.5 & 0 & 0 \end{vmatrix} \tag{8.8.3}$$

$$\begin{vmatrix} s^3 \\ s^2 \\ s \\ s^0 \end{vmatrix} \begin{vmatrix} 1 & 5 & 0 \\ 4 & 102 & 0 \\ -20.5 & 0 & 0 \\ 102 & 0 & 0 \end{vmatrix} \tag{8.8.4}$$

$\because$ there are two sign changes in the first column of the routh array, two poles lie on right half of s-plane. Therefore,the system is unstable.

9 Gain Margin