

Matrix Theory Assignment 1

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Abstract—This document contains the solution to problem No.66 from Lines and Planes

plex numbers and matrix multiplication,

1 PROBLEM STATEMENT

$$\text{Simplify } \mathbf{z} = \left(\begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} - \frac{2}{\begin{pmatrix} 2 \\ 1 \end{pmatrix}} \right) \frac{\begin{pmatrix} 3 \\ -4 \end{pmatrix}}{\begin{pmatrix} 5 \\ 1 \end{pmatrix}}.$$

2 SOLUTION

Inorder to simplify the above equation, we need to find the multiplicative inverse of the sub - matrices.

We, first define the multiplicative inverse of a matrix of a complex number. Let \mathbf{T}_a be the matrix of the complex number a , then \mathbf{b} is defined to be the multiplicative inverse of a if

$$\mathbf{T}_a \mathbf{T}_b = \mathbf{I} \quad (2.0.1)$$

$$\mathbf{b} = \mathbf{a}^{-1} = \begin{pmatrix} a_1 & -a_2 \\ a_2 & a_1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\|a\|^2} \begin{pmatrix} a_1 \\ -a_2 \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{z} = \left(\begin{pmatrix} 1 \\ -4 \end{pmatrix}^{-1} - 2 \begin{pmatrix} 2 \\ 1 \end{pmatrix}^{-1} \right) \begin{pmatrix} 3 \\ -4 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \end{pmatrix}^{-1} \quad (2.0.3)$$

Using Eq (2.0.2), equivalent matrices for the com-

$$\begin{aligned} &= \left(\begin{pmatrix} 1 & 4 \\ -4 & 1 \end{pmatrix}^{-1} - 2 \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}^{-1} \right) \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ 1 & 5 \end{pmatrix}^{-1} \\ &= \left(\frac{1}{1^2 + 4^2} \begin{pmatrix} 1 & -4 \\ 4 & 1 \end{pmatrix} - 2 \left(\frac{1}{2^2 + 1^2} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \right) \right) \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \\ &\quad \frac{1}{5^2 + 1^2} \begin{pmatrix} 5 & 1 \\ -1 & 5 \end{pmatrix} \\ &= \left(\frac{1}{1+16} \begin{pmatrix} 1 & -4 \\ 4 & 1 \end{pmatrix} - \frac{2}{4+1} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \right) \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \\ &\quad \frac{1}{25+1} \begin{pmatrix} 5 & 1 \\ -1 & 5 \end{pmatrix} \\ &= \left(\frac{1}{17} \begin{pmatrix} 1 & -4 \\ 4 & 1 \end{pmatrix} - \frac{2}{5} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \right) \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \frac{1}{26} \begin{pmatrix} 5 & 1 \\ -1 & 5 \end{pmatrix} \\ &= \left(\left(\frac{1}{17} \begin{pmatrix} 1 & -4 \\ 4 & 1 \end{pmatrix} \right) - \left(\frac{4}{5} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right) \right) \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \frac{1}{26} \begin{pmatrix} 5 & 1 \\ -1 & 5 \end{pmatrix} \\ &= \left(\frac{1}{17} - \frac{4}{5} \begin{pmatrix} -4 \\ 1 \end{pmatrix} - \frac{2}{5} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right) \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \frac{1}{26} \begin{pmatrix} 5 & 1 \\ -1 & 5 \end{pmatrix} \\ &= \left(\frac{-63}{85} \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \right) \frac{1}{26} \begin{pmatrix} 5 & 1 \\ -1 & 5 \end{pmatrix} \\ &= \frac{1}{85} \left(\begin{pmatrix} -63 & -54 \\ 54 & -63 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \right) \frac{1}{26} \begin{pmatrix} 5 & 1 \\ -1 & 5 \end{pmatrix} \\ &= \frac{1}{2210} \left(\begin{pmatrix} -63 & -54 \\ 54 & -63 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \right) \begin{pmatrix} 5 & 1 \\ -1 & 5 \end{pmatrix} \\ &= \frac{1}{2210} \begin{pmatrix} -189 + 216 & -162 - 252 \\ 162 + 252 & 216 - 189 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ -1 & 5 \end{pmatrix} \\ &= \frac{1}{2210} \begin{pmatrix} 27 & -414 \\ 414 & 27 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ -1 & 5 \end{pmatrix} \\ &= \frac{1}{2210} \begin{pmatrix} 27 & -414 \\ 414 & 27 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \frac{1}{2210} \begin{pmatrix} 135 + 414 & 27 - 2070 \\ 2070 - 27 & 414 + 135 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \frac{1}{2210} \begin{pmatrix} 549 & -2043 \\ 2043 & 549 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \frac{1}{2210} \begin{pmatrix} 549 \\ 2043 \end{pmatrix} \\ &\Rightarrow \mathbf{z} = \begin{pmatrix} \frac{549}{2210} \\ \frac{2043}{2210} \end{pmatrix} \quad (2.0.4) \end{aligned}$$

Python Code:

```
https://github.com/Hrithikraj2/  
MatrixTheory_EE5609/blob/master/  
Assignment_1/codes/A1_code1.py
```

```
https://github.com/Hrithikraj2/  
MatrixTheory_EE5609/blob/master/  
Assignment_1/codes/A1_code4.py
```

Latex codes:

```
https://github.com/Hrithikraj2/  
MatrixTheory_EE5609/blob/master/  
Assignment_1/latex/A1.tex
```