Matrix Theory Assignment 3

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Abstract—**This document contains the solution to problem No.66 from Lines and Planes**

1 Problem Statement

In ΔABC, AD is the perpendicular bisector of BC. Show that ΔABC is an isosceles triangle in which AB = AC.

2 Solution

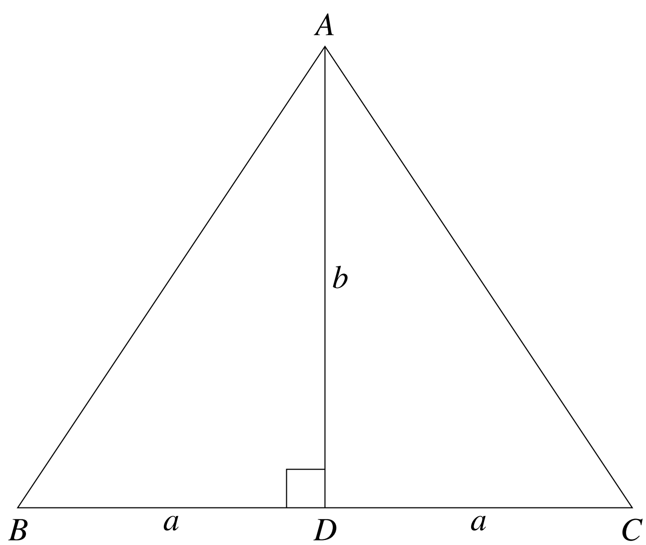

Fig. 0

Consider the above ΔABC. Given that AD is the perpendicular bisector of BC.So,let BD = DC = a and ∠ADB = ∠ADC = 90. Let AD=b.

$$\begin{aligned} \mathbf{B}-\mathbf{D} &= \mathbf{D}-\mathbf{C} \\ \|\mathbf{B}-\mathbf{D}\| &= \|\mathbf{D}-\mathbf{C}\| = a \\ \|\mathbf{D}-\mathbf{A}\| &= b \end{aligned} \tag{2.0.1}$$

Find the length of AB,

$$\begin{aligned} &(\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{A}) \\ &= (\mathbf{B}-\mathbf{D}+\mathbf{D}-\mathbf{A})^T(\mathbf{B}-\mathbf{D}+\mathbf{D}-\mathbf{A}) \\ &= [(\mathbf{B}-\mathbf{D})^T+(\mathbf{D}-\mathbf{A})^T][(\mathbf{B}-\mathbf{D})+(\mathbf{D}-\mathbf{A})] \\ &= (\mathbf{B}-\mathbf{D})^T(\mathbf{B}-\mathbf{D})+(\mathbf{B}-\mathbf{D})^T(\mathbf{D}-\mathbf{A})+ \\ &(\mathbf{D}-\mathbf{A})^T(\mathbf{B}-\mathbf{D})+(\mathbf{D}-\mathbf{A})^T(\mathbf{D}-\mathbf{A}) \end{aligned} \tag{2.0.2}$$

Since BC is the perpendicular bisector of AD the inner product is zero

$$(\mathbf{B}-\mathbf{D})^T(\mathbf{D}-\mathbf{A}) = (\mathbf{D}-\mathbf{A})^T(\mathbf{B}-\mathbf{D}) = 0 \tag{2.0.3}$$

which gives

$$\begin{aligned} &(\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{A}) \\ &= (\mathbf{B}-\mathbf{D})^T(\mathbf{B}-\mathbf{D})+(\mathbf{D}-\mathbf{A})^T(\mathbf{D}-\mathbf{A}) \\ &\implies \|\mathbf{B}-\mathbf{A}\|^2 = \|\mathbf{B}-\mathbf{D}\|^2+\|\mathbf{D}-\mathbf{A}\|^2 \\ &\implies \|\mathbf{B}-\mathbf{A}\| = \sqrt{a^2+b^2} \end{aligned} \tag{2.0.4}$$

Similarily, find the length of CA

$$\begin{aligned} &(\mathbf{C}-\mathbf{A})^T(\mathbf{C}-\mathbf{A}) \\ &= (\mathbf{C}-\mathbf{D}+\mathbf{D}-\mathbf{A})^T(\mathbf{C}-\mathbf{D}+\mathbf{D}-\mathbf{A}) \\ &= [(\mathbf{C}-\mathbf{D})^T+(\mathbf{D}-\mathbf{A})^T][(\mathbf{C}-\mathbf{D})+(\mathbf{D}-\mathbf{A})] \\ &= (\mathbf{C}-\mathbf{D})^T(\mathbf{C}-\mathbf{D})+(\mathbf{C}-\mathbf{D})^T(\mathbf{D}-\mathbf{A})+ \\ &(\mathbf{D}-\mathbf{A})^T(\mathbf{C}-\mathbf{D})+(\mathbf{D}-\mathbf{A})^T(\mathbf{D}-\mathbf{A}) \end{aligned} \tag{2.0.5}$$

Again the inner product of lines BC and DA is zero.

$$\begin{aligned} &(\mathbf{C}-\mathbf{A})^T(\mathbf{C}-\mathbf{A}) \\ &= (\mathbf{C}-\mathbf{D})^T(\mathbf{C}-\mathbf{D})+(\mathbf{D}-\mathbf{A})^T(\mathbf{D}-\mathbf{A}) \\ &\implies \|\mathbf{C}-\mathbf{A}\|^2 = \|\mathbf{C}-\mathbf{D}\|^2+\|\mathbf{D}-\mathbf{A}\|^2 \\ &\implies \|\mathbf{C}-\mathbf{A}\| = \sqrt{a^2+b^2} \end{aligned} \tag{2.0.6}$$

From Eq (2.0.4) and Eq (2.0.6),

$$\|\mathbf{B}-\mathbf{A}\| = \|\mathbf{C}-\mathbf{A}\| \tag{2.0.7}$$

Since the lengths AB and AC are equal, We can conclude that the ΔABC is an isosceles triangle with AB = AC.

Latex codes:

https://github.com/Hrithikraj2/MatrixTheory_EE5609/tree/master/Assignment_3/latex