Matrix Theory Assignment 3

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***Abstract*—This document contains the solution to problem No.66 from Lines and Planes**

1 Problem Statement

In ΔABC, AD is the perpendicular bisector of BC. Show that ΔABC is an isosceles triangle in which AB = AC.

2 Solution

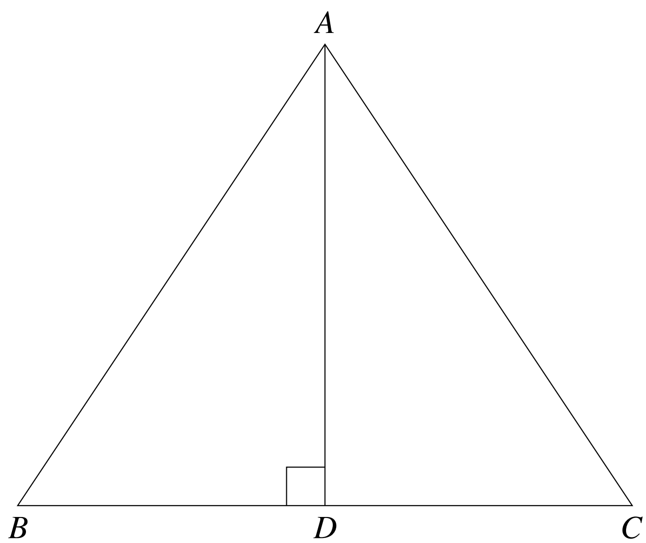

Fig. 0

Consider the above ΔABC. Given that AD is the perpendicular bisector of BC. So, BD = DC and $\angle$ADB = $\angle$ADC = 90.

$$||\mathbf{B} - \mathbf{D}|| = ||\mathbf{D} - \mathbf{C}|| \tag{2.0.1}$$

Find the length of AB,

$$\begin{aligned}
&(\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{A}) \\
&= (\mathbf{B} - \mathbf{D} + \mathbf{D} - \mathbf{A})^T (\mathbf{B} - \mathbf{D} + \mathbf{D} - \mathbf{A}) \\
&= [(\mathbf{B} - \mathbf{D})^T + (\mathbf{D} - \mathbf{A})^T][(\mathbf{B} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})] \\
&= (\mathbf{B} - \mathbf{D})^T (\mathbf{B} - \mathbf{D}) + (\mathbf{B} - \mathbf{D})^T (\mathbf{D} - \mathbf{A}) + \\
&\quad (\mathbf{D} - \mathbf{A})^T (\mathbf{B} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})^T (\mathbf{D} - \mathbf{A})
\end{aligned} \tag{2.0.2}$$

Since BC is the perpendicular bisector of AD the inner product is zero

$$(\mathbf{B} - \mathbf{D})^T (\mathbf{D} - \mathbf{A}) = (\mathbf{D} - \mathbf{A})^T (\mathbf{B} - \mathbf{D}) = 0 \tag{2.0.3}$$

which gives

$$\begin{aligned}
&(\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{A}) \\
&= (\mathbf{B} - \mathbf{D})^T (\mathbf{B} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})^T (\mathbf{D} - \mathbf{A}) \\
&\implies ||\mathbf{B} - \mathbf{A}||^2 = ||\mathbf{B} - \mathbf{D}||^2 + ||\mathbf{D} - \mathbf{A}||^2 \\
&\implies ||\mathbf{B} - \mathbf{A}|| = \sqrt{||\mathbf{B} - \mathbf{D}||^2 + ||\mathbf{D} - \mathbf{A}||^2}
\end{aligned} \tag{2.0.4}$$

Similarily, find the length of CA

$$\begin{aligned}
&(\mathbf{C} - \mathbf{A})^T (\mathbf{C} - \mathbf{A}) \\
&= (\mathbf{C} - \mathbf{D} + \mathbf{D} - \mathbf{A})^T (\mathbf{C} - \mathbf{D} + \mathbf{D} - \mathbf{A}) \\
&= [(\mathbf{C} - \mathbf{D})^T + (\mathbf{D} - \mathbf{A})^T][(\mathbf{C} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})] \\
&= (\mathbf{C} - \mathbf{D})^T (\mathbf{C} - \mathbf{D}) + (\mathbf{C} - \mathbf{D})^T (\mathbf{D} - \mathbf{A}) + \\
&\quad (\mathbf{D} - \mathbf{A})^T (\mathbf{C} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})^T (\mathbf{D} - \mathbf{A})
\end{aligned} \tag{2.0.5}$$

Again the inner product of lines BC and DA is zero.

$$\begin{aligned}
&(\mathbf{C} - \mathbf{A})^T (\mathbf{C} - \mathbf{A}) \\
&= (\mathbf{C} - \mathbf{D})^T (\mathbf{C} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})^T (\mathbf{D} - \mathbf{A}) \\
&\implies ||\mathbf{C} - \mathbf{A}||^2 = ||\mathbf{C} - \mathbf{D}||^2 + ||\mathbf{D} - \mathbf{A}||^2 \\
&\implies ||\mathbf{C} - \mathbf{A}|| = \sqrt{||\mathbf{C} - \mathbf{D}||^2 + ||\mathbf{D} - \mathbf{A}||^2}
\end{aligned} \tag{2.0.6}$$

From Eq (2.0.1)

$$\implies ||\mathbf{C} - \mathbf{A}|| = \sqrt{||\mathbf{B} - \mathbf{D}||^2 + ||\mathbf{D} - \mathbf{A}||^2} \tag{2.0.7}$$

Eq (2.0.4) and Eq (2.0.7)

$$||\mathbf{B} - \mathbf{A}|| = ||\mathbf{C} - \mathbf{A}|| \tag{2.0.8}$$

Since the lengths AB and AC are equal, We can conclude that the ΔABC is an isosceles triangle with AB = AC.

Latex codes:

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https://github.com/Hrithikraj2/
    MatrixTheory_EE5609/tree/master/
    Assignment_3/latex
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