Matrix Theory Assignment 1

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Abstract—This document contains the solution to problem No.66 from Lines and Planes

1 PROBLEM STATEMENT

Simplify $\mathbf{z} = \left(\dfrac{1}{\begin{pmatrix}1\\-4\end{pmatrix}} - \dfrac{2}{\begin{pmatrix}2\\1\end{pmatrix}} \right) \dfrac{\begin{pmatrix}3\\-4\end{pmatrix}}{\begin{pmatrix}5\\1\end{pmatrix}}$.

2 SOLUTION

Inorder to simplify the above equation, we need to find the multiplicative inverse of the sub - matrices.

We, first define the multiplicative inverse of a matrix of a complex number. Let $\mathbf{T_a}$ be the matrix of the complex number a, then b is defined to be the multiplicative inverse of a if

$$\mathbf{T_a T_b} = \mathbf{I} \quad (2.0.1)$$

$$\mathbf{b} = \mathbf{a}^{-1} = \begin{pmatrix}a_1 & -a_2\\a_2 & a_1\end{pmatrix}^{-1}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\|a\|^2}\begin{pmatrix}a_1\\-a_2\end{pmatrix} \quad (2.0.2)$$

$$\mathbf{z} = \left(\begin{pmatrix}1\\-4\end{pmatrix}^{-1} - 2\begin{pmatrix}2\\1\end{pmatrix}^{-1} \right)\begin{pmatrix}3\\-4\end{pmatrix}\begin{pmatrix}5\\1\end{pmatrix}^{-1} \quad (2.0.3)$$

Using Eq (2.0.2), equivalent matrices for the complex numbers and matrix multiplication,

$$= \left(\frac{1}{17}\begin{pmatrix}1\\4\end{pmatrix} - \frac{2}{5}\begin{pmatrix}2\\-1\end{pmatrix} \right)\begin{pmatrix}3\\-4\end{pmatrix}\frac{1}{26}\begin{pmatrix}5\\-1\end{pmatrix}$$

$$= \left(\begin{pmatrix}\frac{1}{17}\\\frac{4}{17}\end{pmatrix} - \begin{pmatrix}\frac{4}{5}\\-\frac{2}{5}\end{pmatrix} \right)\begin{pmatrix}3\\-4\end{pmatrix}\frac{1}{26}\begin{pmatrix}5\\-1\end{pmatrix}$$

$$= \begin{pmatrix}\frac{1}{17} - \frac{4}{5}\\\frac{4}{17} + \frac{2}{5}\end{pmatrix}\begin{pmatrix}3\\-4\end{pmatrix}\begin{pmatrix}\frac{5}{26}\\\frac{-1}{26}\end{pmatrix}$$

$$= \begin{pmatrix}-\frac{63}{85}\\\frac{54}{85}\end{pmatrix}\begin{pmatrix}3\\-4\end{pmatrix}\begin{pmatrix}\frac{5}{26}\\-\frac{1}{26}\end{pmatrix}$$

$$= \frac{1}{85}\left(\begin{pmatrix}-63 & -54\\54 & -63\end{pmatrix}\begin{pmatrix}3 & 4\\-4 & 3\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \right)\frac{1}{26}\begin{pmatrix}5\\-1\end{pmatrix}$$

$$= \frac{1}{2210}\left(\begin{pmatrix}-63 & -54\\54 & -63\end{pmatrix}\begin{pmatrix}3 & 4\\-4 & 3\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \right)\begin{pmatrix}5\\-1\end{pmatrix}$$

$$= \frac{1}{2210}\begin{pmatrix}-189+216 & -162-252\\162+252 & 216-189\end{pmatrix}\begin{pmatrix}5\\-1\end{pmatrix}$$

$$= \frac{1}{2210}\begin{pmatrix}27 & -414\\414 & 27\end{pmatrix}\begin{pmatrix}5\\-1\end{pmatrix}$$

$$= \frac{1}{2210}\begin{pmatrix}27\\414\end{pmatrix}\begin{pmatrix}5\\-1\end{pmatrix}$$

$$= \frac{1}{2210}\begin{pmatrix}27 & -414\\414 & 27\end{pmatrix}\begin{pmatrix}5 & 1\\-1 & 5\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \frac{1}{2210}\begin{pmatrix}549 & -2043\\2043 & 549\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \frac{1}{2210}\begin{pmatrix}549\\2043\end{pmatrix}$$

$$\implies \mathbf{z} = \begin{pmatrix}\frac{549}{2210}\\\frac{2043}{2210}\end{pmatrix} \quad (2.0.4)$$

Python Code:

https://github.com/Hrithikraj2/
   MatrixTheory_EE5609/blob/master/
   Assignment_1/codes/A1_code1.py

https://github.com/Hrithikraj2/
   MatrixTheory_EE5609/blob/master/
   Assignment_1/codes/A1_code4.py

Latex codes:

https://github.com/Hrithikraj2/
MatrixTheory_EE5609/blob/master/
Assignment_1/latex/A1.tex