# Assignment-8

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***Abstract*—This document contains solution of Problem Ramsey**(4.1.4)

Download latex-tikz codes from

https://github.com/Hrithikraj2/MatrixTheory_EE5609/blob/master/Assignment_8/A8.tex

## 1 Problem

If A is an $m \times n$ matrix, B is a $n \times m$ matrix and $n < m$, then Prove that AB is not invertible.

## 2 Solution

### 2.1 Part 1 : Proof

Let us represent A and B as follows

$$A = (a_{ij}) \in \mathfrak{R}^{m \times n} \tag{2.1.1}$$

$$B = (b_{ij}) \in \mathfrak{R}^{n \times m} \tag{2.1.2}$$

Let C = AB which would be an $m \times m$ matrix.

$$C = (c_{ij}) \in \mathfrak{R}^{m \times m} \tag{2.1.3}$$

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} \tag{2.1.4}$$

for j=1,2,....,m. Let

$$\mathbf{c}_j = C_{ej} \in \mathfrak{R}^m \tag{2.1.5}$$

where $(e_1, e_2, ......, e_m)$ stands for the canonical basis of $\mathfrak{R}^m$. Then, we have

$$c_j = \begin{pmatrix} \sum_{k=1}^{n} a_{1k} b_{kj} \\ . \\ . \\ . \\ \sum_{k=1}^{n} a_{mk} b_{kj} \end{pmatrix} \tag{2.1.6}$$

$$= \sum_{k=1}^{n} b_{kj} \begin{pmatrix} a_{1k} \\ . \\ . \\ .a_{mk} \end{pmatrix} \tag{2.1.7}$$

$$= \sum_{k=1}^{n} b_{kj} \mathbf{a}_k \tag{2.1.8}$$

with

$$\mathbf{a}_j = \begin{pmatrix} a_{1j} \\ . \\ . \\ a_{mj} \end{pmatrix} \in \mathfrak{R}^n \tag{2.1.9}$$

That gives us,

$$|C| = |\mathbf{c}_1, ....., \mathbf{c}_m| \tag{2.1.10}$$

$$= \sum_{k_1=1}^{n} \sum_{k_2=1}^{n} .. \sum_{k_m=1}^{n} b_{k_1,1}, b_{k_2,2}, .., b_{k_m,m} \left|\mathbf{a}_{k_1}, .., \mathbf{a}_{k_m}\right| \tag{2.1.11}$$

Since there are exactly n vectors $\mathbf{a}_j$ and the determinant has $m > n$ entries, then we have

$$\left|\mathbf{a}_{k_1}, .., \mathbf{a}_{k_m}\right| = 0 \tag{2.1.12}$$

$$\forall k_1, ...k_m \in (1, 2, .., n) \tag{2.1.13}$$

Because atleast two entries must be equal. Hence

$$|C| = 0 \tag{2.1.14}$$

Since $|AB| = 0$. Therefore, AB is not invertible.

### 2.2 Part 2:Example

Since we need to satisfy the condition $n < m$ Let A be a $2 \times 1$ matrix and B be a $1 \times 2$ matrix. Let

$$A = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.2.1}$$

$$B = \begin{pmatrix} c & d \end{pmatrix} \tag{2.2.2}$$

Then, AB would be

$$AB = \begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} c & d \end{pmatrix} = \begin{pmatrix} ac & ad \\ bc & bd \end{pmatrix} \tag{2.2.3}$$

$$|AB| = acbd - bcad = 0 \tag{2.2.4}$$

Hence Proved.