

# Matrix Theory Assignment 1

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**Abstract**—This document contains the solution to problem No.66 from Lines and Planes

## 1 PROBLEM STATEMENT

$$\text{Simplify } \mathbf{z} = \left( \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} - \frac{2}{\begin{pmatrix} 2 \\ 1 \end{pmatrix}} \right) \frac{\begin{pmatrix} 3 \\ -4 \\ 5 \\ 1 \end{pmatrix}}{\begin{pmatrix} 5 \\ 1 \end{pmatrix}}.$$

## 2 SOLUTION

Inorder to simplify the above equation, we need to find the multiplicative inverse of the sub - matrices.

We, first define the multiplicative inverse of a matrix of a complex number. Let  $\mathbf{T}_a$  be the matrix of the complex number  $a$ , then  $b$  is defined to be the multiplicative inverse of  $a$  if

$$\mathbf{T}_a \mathbf{T}_b = \mathbf{I} \quad (2.0.1)$$

$$\mathbf{b} = \mathbf{a}^{-1} = \begin{pmatrix} a_1 & -a_2 \\ a_2 & a_1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.2)$$

$$= \frac{1}{\|a\|^2} \begin{pmatrix} a_1 \\ -a_2 \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{z} = \left( \begin{pmatrix} 1 \\ -4 \end{pmatrix}^{-1} - 2 \begin{pmatrix} 2 \\ 1 \end{pmatrix}^{-1} \right) \begin{pmatrix} 3 \\ -4 \end{pmatrix} \frac{\begin{pmatrix} 5 \\ 1 \end{pmatrix}}{\begin{pmatrix} 5 \\ 1 \end{pmatrix}}^{-1} \quad (2.0.4)$$

Using Eq (2.0.3),

$$= \left( \frac{1}{17} \begin{pmatrix} 1 \\ 4 \end{pmatrix} - \frac{2}{5} \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right) \begin{pmatrix} 3 \\ -4 \end{pmatrix} \frac{1}{26} \begin{pmatrix} 5 \\ -1 \end{pmatrix} \quad (2.0.5)$$

$$= \left( \left( \frac{1}{17} \right) - \left( \frac{4}{5} \right) \right) \begin{pmatrix} 3 \\ -4 \end{pmatrix} \frac{1}{26} \begin{pmatrix} 5 \\ -1 \end{pmatrix} \quad (2.0.6)$$

$$= \left( \frac{1}{17} - \frac{4}{5} \right) \begin{pmatrix} 3 \\ -4 \end{pmatrix} \left( \frac{5}{26} \right) \quad (2.0.7)$$

$$= \begin{pmatrix} -\frac{63}{85} \\ \frac{54}{85} \end{pmatrix} \begin{pmatrix} 3 \\ -4 \end{pmatrix} \begin{pmatrix} \frac{5}{26} \\ -\frac{1}{26} \end{pmatrix} \quad (2.0.8)$$

Using equivalent matrices for the complex numbers,

$$= \frac{1}{85} \left( \begin{pmatrix} -63 & -54 \\ 54 & -63 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right) \frac{1}{26} \begin{pmatrix} 5 \\ -1 \end{pmatrix} \quad (2.0.9)$$

$$= \frac{1}{2210} \left( \begin{pmatrix} -63 & -54 \\ 54 & -63 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right) \begin{pmatrix} 5 \\ -1 \end{pmatrix} \quad (2.0.10)$$

Matrix multiplication gives,

$$= \frac{1}{2210} \begin{pmatrix} -189 + 216 & -162 - 252 \\ 162 + 252 & 216 - 189 \end{pmatrix} \begin{pmatrix} 5 \\ -1 \end{pmatrix} \quad (2.0.11)$$

$$= \frac{1}{2210} \begin{pmatrix} 27 & -414 \\ 414 & 27 \end{pmatrix} \begin{pmatrix} 5 \\ -1 \end{pmatrix} \quad (2.0.12)$$

$$= \frac{1}{2210} \begin{pmatrix} 27 \\ 414 \end{pmatrix} \begin{pmatrix} 5 \\ -1 \end{pmatrix} \quad (2.0.13)$$

Again using equivalent and multiplying,

$$= \frac{1}{2210} \begin{pmatrix} 27 & -414 \\ 414 & 27 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.14)$$

$$= \frac{1}{2210} \begin{pmatrix} 549 & -2043 \\ 2043 & 549 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.15)$$

$$= \frac{1}{2210} \begin{pmatrix} 549 \\ 2043 \end{pmatrix} \quad (2.0.16)$$

$$\Rightarrow \mathbf{z} = \begin{pmatrix} \frac{549}{2210} \\ \frac{2043}{2210} \end{pmatrix} \quad (2.0.17)$$

### Python Code:

```
https://github.com/Hrithikraj2/
MatrixTheory_EE5609/blob/master/
Assignment_1/codes/A1_code1.py
```

```
https://github.com/Hrithikraj2/
MatrixTheory_EE5609/blob/master/
Assignment_1/codes/A1_code4.py
```

### Latex codes:

[https://github.com/Hrithikraj2/  
MatrixTheory\\_EE5609/blob/master/  
Assignment\\_1/latex/A1.tex](https://github.com/Hrithikraj2/MatrixTheory_EE5609/blob/master/Assignment_1/latex/A1.tex)