

Assignment-6

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Abstract—This document contains solution of Problem Ramsey(4.1.4)

Download latex-tikz codes from

https://github.com/Hrithikraj2/MatrixTheory_EE5609/blob/master/Assignment_6/A6.tex

1 QUESTION

Trace the parabola

$$16x^2 + 24xy + 9y^2 - 5x - 10y + 1 = 0$$

2 SOLUTION

Compare the given equation with the standard form

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.1)$$

Write the values Of V and u as follows

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} 16 & 12 \\ 12 & 9 \end{pmatrix} \quad \mathbf{u} = \begin{pmatrix} -\frac{5}{2} \\ -5 \end{pmatrix} \quad f = 1 \quad (2.0.2)$$

The characteristic equation of \mathbf{V} is given as

$$|\lambda \mathbf{I} - \mathbf{V}| = 0 \quad (2.0.3)$$

$$\Rightarrow \begin{vmatrix} \lambda - 16 & -12 \\ -12 & \lambda - 9 \end{vmatrix} = 0 \quad (2.0.4)$$

$$\Rightarrow \lambda^2 - 25\lambda = 0 \quad (2.0.5)$$

The eigenvalues are the roots of the equation (2.0.5) are

$$\lambda_1 = 0, \quad \lambda_2 = 25 \quad (2.0.6)$$

The eigen vector \mathbf{p} is defined as,

$$\mathbf{V}\mathbf{p} = \lambda\mathbf{p} \quad (2.0.7)$$

$$\Rightarrow (\lambda \mathbf{I} - \mathbf{V})\mathbf{p} = 0 \quad (2.0.8)$$

For $\lambda_1 = 0$

$$(\lambda_1 \mathbf{I} - \mathbf{V}) = \begin{pmatrix} -16 & -12 \\ -12 & -9 \end{pmatrix} \xrightarrow[R_2 \leftarrow R_2 - 3R_1]{R_1 \leftarrow \frac{1}{4}R_1} \begin{pmatrix} -4 & -3 \\ 0 & 0 \end{pmatrix} \quad (2.0.9)$$

$$\Rightarrow \mathbf{p}_1 = \frac{1}{5} \begin{pmatrix} -3 \\ 4 \end{pmatrix} \quad (2.0.10)$$

For $\lambda_2 = 25$

$$(\lambda_2 \mathbf{I} - \mathbf{V}) = \begin{pmatrix} 9 & -12 \\ -12 & 16 \end{pmatrix} \xrightarrow[R_2 \leftarrow R_2 + 4R_1]{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 3 & -4 \\ 0 & 0 \end{pmatrix} \quad (2.0.11)$$

$$\Rightarrow \mathbf{p}_2 = \frac{1}{5} \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad (2.0.12)$$

Use Eigenvalue decomposition, $\mathbf{P}^T \mathbf{V} \mathbf{P} = \mathbf{D}$, where

$$\mathbf{P} = \frac{1}{5} \begin{pmatrix} -3 & 4 \\ 4 & 3 \end{pmatrix} \quad (2.0.13)$$

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 25 \end{pmatrix} \quad (2.0.14)$$

Focal length of the parabola is given as

$$\text{focal length} = \left| \frac{2\eta}{\lambda_2} \right| \quad (2.0.15)$$

$$\eta = \mathbf{p}_1^T \mathbf{u} = -\frac{5}{2} \quad (2.0.16)$$

Substituting values from (2.0.16) and (2.0.6) in (2.0.15), we get

$$\text{focal length} = \frac{1}{5} \quad (2.0.17)$$

The standard equation of the parabola is given by

$$\mathbf{y}^T \mathbf{D} \mathbf{y} = -2\eta \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{y} \quad (2.0.18)$$

And the vertex \mathbf{c} is given by

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \quad (2.0.19)$$

Substituting values from (2.0.2),(2.0.16),(2.0.10) in (2.0.19),

$$\begin{pmatrix} -1 & -7 \\ 16 & 12 \\ 12 & 9 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} \quad (2.0.20)$$

To find \mathbf{c} , performing row reduction on the augmented matrix as follows:

$$\begin{pmatrix} -1 & -7 & -1 \\ 16 & 12 & 4 \\ 12 & 9 & 3 \end{pmatrix} \xleftrightarrow[R_1 \leftarrow -R_1]{R_3 \leftarrow R_3 - \frac{3}{4}R_2} \begin{pmatrix} 1 & 7 & 1 \\ 16 & 12 & 4 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.21)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 16R_1} \begin{pmatrix} 1 & 7 & 1 \\ 0 & -100 & -12 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.22)$$

$$\xleftrightarrow{R_2 \leftarrow -\frac{1}{100}R_2} \begin{pmatrix} 1 & 7 & 1 \\ 0 & 1 & \frac{3}{25} \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.23)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 7R_2} \begin{pmatrix} 1 & 0 & \frac{4}{25} \\ 0 & 1 & \frac{3}{25} \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.24)$$

Thus,

$$\mathbf{c} = \begin{pmatrix} \frac{4}{25} \\ \frac{3}{25} \\ \frac{3}{25} \end{pmatrix} \quad (2.0.25)$$

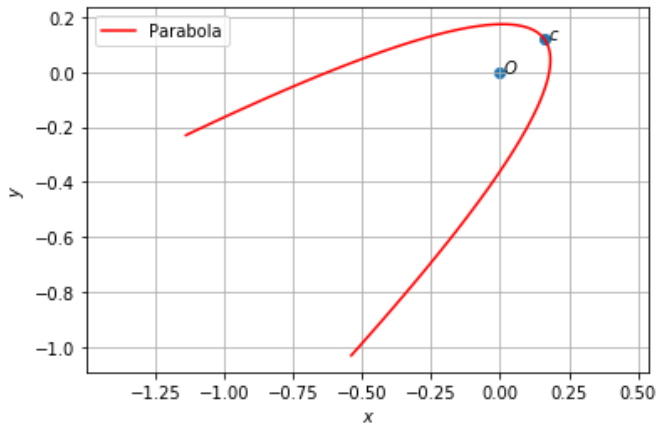


Fig. 1: Parabola with vertex \mathbf{c}