# Matrix Theory Assignment 1

Hrithik Raj

***Abstract*—This document contains the solution to problem No.66 from Lines and Planes**

## 1 Problem Statement

$$\text{Simplify } \mathbf{z} = \left(\frac{1}{\begin{pmatrix}1\\-4\end{pmatrix}} - \frac{2}{\begin{pmatrix}2\\1\end{pmatrix}}\right)\frac{\begin{pmatrix}3\\-4\end{pmatrix}}{\begin{pmatrix}5\\1\end{pmatrix}}.$$

## 2 Solution

Inorder to simplify the above equation, we need to find the multiplicative inverse of the sub - matrices.

We, first define the multiplicative inverse of a matrix of a complex number. Let $\mathbf{T_a}$ be the matrix of the complex number a, then b is defined to be the multiplicative inverse of a if

$$\mathbf{T_a T_b} = \mathbf{I} \tag{2.0.1}$$

$$\mathbf{b} = \mathbf{a}^{-1} = \begin{pmatrix}a_1 & -a_2\\ a_2 & a_1\end{pmatrix}^{-1}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\|a\|^2}\begin{pmatrix}a_1\\-a_2\end{pmatrix} \tag{2.0.2}$$

$$\mathbf{z} = \left(\begin{pmatrix}1\\-4\end{pmatrix}^{-1} - 2\begin{pmatrix}2\\1\end{pmatrix}^{-1}\right)\begin{pmatrix}3\\-4\end{pmatrix}\begin{pmatrix}5\\1\end{pmatrix}^{-1} \tag{2.0.3}$$

Using Eq (2.0.2), equivalent matrices for the complex numbers and matrix multiplication,

$$= \left(\begin{pmatrix}1 & 4\\-4 & 1\end{pmatrix}^{-1} - 2\begin{pmatrix}2 & -1\\1 & 2\end{pmatrix}^{-1}\right)\begin{pmatrix}3 & 4\\-4 & 3\end{pmatrix}\begin{pmatrix}5 & -1\\1 & 5\end{pmatrix}^{-1}$$

$$= \left(\frac{1}{1^2+4^2}\begin{pmatrix}1 & -4\\4 & 1\end{pmatrix} - 2\left(\frac{1}{2^2+1^2}\right)\begin{pmatrix}2 & 1\\-1 & 2\end{pmatrix}\right)\begin{pmatrix}3 & 4\\-4 & 3\end{pmatrix}\frac{1}{5^2+1^2}\begin{pmatrix}5 & 1\\-1 & 5\end{pmatrix}$$

$$= \left(\frac{1}{1+16}\begin{pmatrix}1 & -4\\4 & 1\end{pmatrix} - \frac{2}{4+1}\begin{pmatrix}2 & 1\\-1 & 2\end{pmatrix}\right)\begin{pmatrix}3 & 4\\-4 & 3\end{pmatrix}\frac{1}{25+1}\begin{pmatrix}5 & 1\\-1 & 5\end{pmatrix}$$

$$= \left(\frac{1}{17}\begin{pmatrix}1 & -4\\4 & 1\end{pmatrix} - \frac{2}{5}\begin{pmatrix}2 & 1\\-1 & 2\end{pmatrix}\right)\begin{pmatrix}3 & 4\\-4 & 3\end{pmatrix}\frac{1}{26}\begin{pmatrix}5 & 1\\-1 & 5\end{pmatrix}$$

$$= \left(\begin{pmatrix}\frac{1}{17} & \frac{-4}{17}\\ \frac{4}{17} & \frac{1}{17}\end{pmatrix} - \begin{pmatrix}\frac{4}{5} & \frac{2}{5}\\ \frac{-2}{5} & \frac{4}{5}\end{pmatrix}\right)\begin{pmatrix}3 & 4\\-4 & 3\end{pmatrix}\frac{1}{26}\begin{pmatrix}5 & 1\\-1 & 5\end{pmatrix}$$

$$= \begin{pmatrix}\frac{1}{17}-\frac{4}{5} & \frac{-4}{17}-\frac{2}{5}\\ \frac{4}{17}+\frac{2}{5} & \frac{1}{17}-\frac{4}{5}\end{pmatrix}\begin{pmatrix}3 & 4\\-4 & 3\end{pmatrix}\frac{1}{26}\begin{pmatrix}5 & 1\\-1 & 5\end{pmatrix}$$

$$= \begin{pmatrix}\frac{-63}{85} & \frac{-54}{85}\\ \frac{54}{85} & \frac{-63}{85}\end{pmatrix}\begin{pmatrix}3 & 4\\-4 & 3\end{pmatrix}\frac{1}{26}\begin{pmatrix}5 & 1\\-1 & 5\end{pmatrix}$$

$$= \frac{1}{85}\left(\begin{pmatrix}-63 & -54\\54 & -63\end{pmatrix}\begin{pmatrix}3 & 4\\-4 & 3\end{pmatrix}\right)\frac{1}{26}\begin{pmatrix}5 & 1\\-1 & 5\end{pmatrix}$$

$$= \frac{1}{2210}\left(\begin{pmatrix}-63 & -54\\54 & -63\end{pmatrix}\begin{pmatrix}3 & 4\\-4 & 3\end{pmatrix}\right)\begin{pmatrix}5 & 1\\-1 & 5\end{pmatrix}$$

$$= \frac{1}{2210}\begin{pmatrix}-189+216 & -162-252\\162+252 & 216-189\end{pmatrix}\begin{pmatrix}5 & 1\\-1 & 5\end{pmatrix}$$

$$= \frac{1}{2210}\begin{pmatrix}27 & -414\\414 & 27\end{pmatrix}\begin{pmatrix}5 & 1\\-1 & 5\end{pmatrix}$$

$$= \frac{1}{2210}\begin{pmatrix}27 & -414\\414 & 27\end{pmatrix}\begin{pmatrix}5 & 1\\-1 & 5\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \frac{1}{2210}\begin{pmatrix}135+414 & 27-2070\\2070-27 & 414+135\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \frac{1}{2210}\begin{pmatrix}549 & -2043\\2043 & 549\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \frac{1}{2210}\begin{pmatrix}549\\2043\end{pmatrix}$$

$$= \mathbf{z} = \begin{pmatrix}\frac{549}{2210}\\ \frac{2043}{2210}\end{pmatrix} \tag{2.0.4}$$

**Python Code:**

https://github.com/Hrithikraj2/MatrixTheory_EE5609/blob/master/Assignment_1/codes/A1_code1.py

https://github.com/Hrithikraj2/MatrixTheory_EE5609/blob/master/Assignment_1/codes/A1_code4.py

**Latex codes:**

https://github.com/Hrithikraj2/MatrixTheory_EE5609/blob/master/Assignment_1/latex/A1.tex