Matrix Theory Assignment 2

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***Abstract*—This document contains the solution to problem No.66 from Lines and Planes**

1 Problem Statement

Matrices A and B will be inverse of each other only if

(A)AB=BA (B)AB=BA=0

(C)AB=0,BA=I (D)AB=BA=I

2 Solution

Consider a matrix A. We define matrix A as follows

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{2.0.1}$$

The inverse of A is A^{-1}. Let $B = A^{-1}$. Evaluate the inverse of A.

$$B = A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$AB = AA^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{ad-bc}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{ad-bc}\begin{pmatrix} ad-bc & -ab+ab \\ cd-cd & -bc+ad \end{pmatrix}$$

$$= \frac{1}{ad-bc}\begin{pmatrix} ad-bc & 0 \\ 0 & -bc+ad \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{ad-bc}(ad-bc) & 0 \\ 0 & \frac{1}{ad-bc}(-bc+ad) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$\implies AB = AA^{-1} = I \tag{2.0.2}$$

$$BA = A^{-1}A = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \frac{1}{ad-bc}\begin{pmatrix} da-bc & bd-bd \\ -ac+ac & -bc+ad \end{pmatrix}$$

$$= \frac{1}{ad-bc}\begin{pmatrix} ad-bc & 0 \\ 0 & -bc+ad \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{ad-bc}(ad-bc) & 0 \\ 0 & \frac{1}{ad-bc}(ad-bc) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$\implies BA = A^{-1}A = I \tag{2.0.3}$$

From Eq (2.0.2) and Eq (2.0.3),

$AB = BA = I$.

We can conclude that options A and D are correct

Python Code:

https://github.com/Hrithikraj2/MatrixTheory_EE5609/blob/master/Assignment_2/codes/A2_code1.py

https://github.com/Hrithikraj2/MatrixTheory_EE5609/blob/master/Assignment_2/codes/A2_code2.py

Latex codes:

https://github.com/Hrithikraj2/MatrixTheory_EE5609/blob/master/Assignment_2/latex/A2.tex