Matrix Theory Assignment 1

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***Abstract*—This document contains the solution to problem No.66 from Lines and Planes**

1 Problem Statement

$$\text{Simplify } \mathbf{z} = \left(\frac{1}{\begin{pmatrix}1\\-4\end{pmatrix}} - \frac{2}{\begin{pmatrix}2\\1\end{pmatrix}}\right)\frac{\begin{pmatrix}3\\-4\end{pmatrix}}{\begin{pmatrix}5\\1\end{pmatrix}}.$$

2 Solution

Inorder to simplify the above equation, we need to find the multiplicative inverse of the sub - matrices.

We, first define the multiplicative inverse of a matrix of a complex number. Let $\mathbf{T_a}$ be the matrix of the complex number a, then b is defined to be the multiplicative inverse of a if

$$\mathbf{T_a T_b} = \mathbf{I} \tag{2.0.1}$$

$$\mathbf{b} = \mathbf{a}^{-1} = \begin{pmatrix}a_1 & -a_2\\ a_2 & a_1\end{pmatrix}^{-1}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.2}$$

$$= \frac{1}{\|a\|^2}\begin{pmatrix}a_1\\-a_2\end{pmatrix} \tag{2.0.3}$$

$$\mathbf{z} = \left(\begin{pmatrix}1\\-4\end{pmatrix}^{-1} - 2\begin{pmatrix}2\\1\end{pmatrix}^{-1}\right)\begin{pmatrix}3\\-4\end{pmatrix}\begin{pmatrix}5\\1\end{pmatrix}^{-1} \tag{2.0.4}$$

Using Eq (2.0.3),

$$= \left(\frac{1}{17}\begin{pmatrix}1\\4\end{pmatrix} - \frac{2}{5}\begin{pmatrix}2\\-1\end{pmatrix}\right)\begin{pmatrix}3\\-4\end{pmatrix}\frac{1}{26}\begin{pmatrix}5\\1\end{pmatrix} \tag{2.0.5}$$

$$= \left(\begin{pmatrix}\frac{1}{17}\\ \frac{4}{17}\end{pmatrix} - \begin{pmatrix}\frac{4}{5}\\ \frac{-2}{5}\end{pmatrix}\right)\begin{pmatrix}3\\-4\end{pmatrix}\begin{pmatrix}\frac{5}{26}\\ \frac{-1}{26}\end{pmatrix} \tag{2.0.6}$$

$$= \left(\begin{pmatrix}0.058\\0.235\end{pmatrix} - \begin{pmatrix}0.8\\-0.4\end{pmatrix}\right)\begin{pmatrix}3\\-4\end{pmatrix}\begin{pmatrix}0.192\\-0.038\end{pmatrix} \tag{2.0.7}$$

$$= \begin{pmatrix}0.058-0.8\\0.235+0.4\end{pmatrix}\begin{pmatrix}3\\-4\end{pmatrix}\begin{pmatrix}0.192\\-0.038\end{pmatrix} \tag{2.0.8}$$

$$= \begin{pmatrix}-0.742\\0.635\end{pmatrix}\begin{pmatrix}3\\-4\end{pmatrix}\begin{pmatrix}0.192\\-0.038\end{pmatrix} \tag{2.0.9}$$

Using equivalent matrices for the complex numbers,

$$= \left(\begin{pmatrix}-0.742 & -0.635\\ 0.635 & -0.742\end{pmatrix}\begin{pmatrix}3 & 4\\ -4 & 3\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right)\begin{pmatrix}0.192\\-0.038\end{pmatrix} \tag{2.0.10}$$

Matrix multiplication gives,

$$= \begin{pmatrix}-2.226+2.54 & -2.968-1.905\\ 1.905+2.968 & 2.54-2.226\end{pmatrix}\begin{pmatrix}0.192\\-0.038\end{pmatrix} \tag{2.0.11}$$

$$= \begin{pmatrix}0.314 & -4.873\\ 4.873 & 0.314\end{pmatrix}\begin{pmatrix}0.192\\-0.038\end{pmatrix} \tag{2.0.12}$$

$$= \begin{pmatrix}0.314\\4.873\end{pmatrix}\begin{pmatrix}0.192\\-0.038\end{pmatrix} \tag{2.0.13}$$

Again using equivalent and multiplying,

$$= \begin{pmatrix}0.314 & -4.873\\ 4.873 & 0.314\end{pmatrix}\begin{pmatrix}0.192 & 0.038\\ -0.038 & 0.192\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.14}$$

$$= \begin{pmatrix}0.245 & -0.9236\\ 0.9236 & 0.245\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.15}$$

$$\implies \mathbf{z} = \begin{pmatrix}0.245\\0.9236\end{pmatrix} \tag{2.0.16}$$

Python Code:

https://github.com/Hrithikraj2/MatrixTheory_EE5609/blob/master/Assignment_1/codes/A1_code1.py

https://github.com/Hrithikraj2/MatrixTheory_EE5609/blob/master/Assignment_1/codes/A1_code4.py

Latex codes:

https://github.com/Hrithikraj2/
MatrixTheory_EE5609/blob/master/
Assignment_1/latex/A1.tex