

Matrix Theory Assignment 3

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Abstract—This document contains the solution to problem No.66 from Lines and Planes

1 PROBLEM STATEMENT

In $\triangle ABC$, AD is the perpendicular bisector of BC. Show that $\triangle ABC$ is an isosceles triangle in which $AB = AC$.

2 SOLUTION

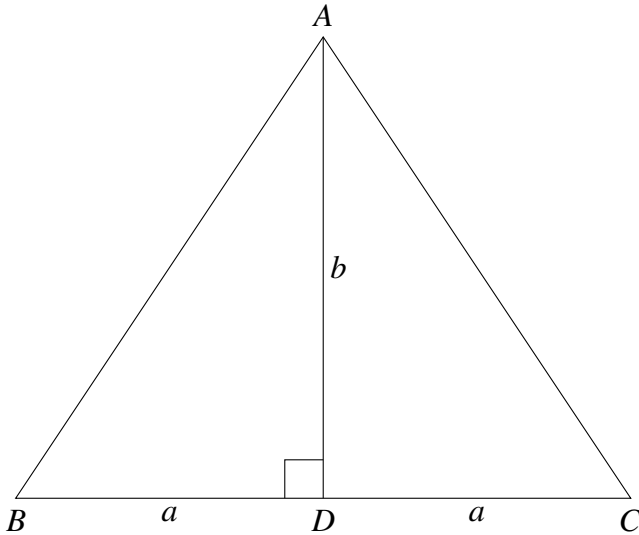


Fig. 0

Consider the above $\triangle ABC$. Given that AD is the perpendicular bisector of BC. So, let $BD = DC = a$ and $\angle ADB = \angle ADC = 90^\circ$. Let $AD = b$.

$$\mathbf{B} - \mathbf{D} = \mathbf{D} - \mathbf{C}$$

$$\begin{aligned} \|\mathbf{B} - \mathbf{D}\| &= \|\mathbf{D} - \mathbf{C}\| = a \\ \|\mathbf{D} - \mathbf{A}\| &= b \end{aligned} \quad (2.0.1)$$

Find the length of AB,

$$\begin{aligned} &(\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{A}) \\ &= (\mathbf{B} - \mathbf{D} + \mathbf{D} - \mathbf{A})^T (\mathbf{B} - \mathbf{D} + \mathbf{D} - \mathbf{A}) \\ &= [(\mathbf{B} - \mathbf{D})^T + (\mathbf{D} - \mathbf{A})^T][(\mathbf{B} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})] \\ &= (\mathbf{B} - \mathbf{D})^T (\mathbf{B} - \mathbf{D}) + (\mathbf{B} - \mathbf{D})^T (\mathbf{D} - \mathbf{A}) + \\ &\quad (\mathbf{D} - \mathbf{A})^T (\mathbf{B} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})^T (\mathbf{D} - \mathbf{A}) \end{aligned} \quad (2.0.2)$$

Since BC is the perpendicular bisector of AD the inner product is zero

$$(\mathbf{B} - \mathbf{D})^T (\mathbf{D} - \mathbf{A}) = (\mathbf{D} - \mathbf{A})^T (\mathbf{B} - \mathbf{D}) = 0 \quad (2.0.3)$$

which gives

$$\begin{aligned} &(\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{A}) \\ &= (\mathbf{B} - \mathbf{D})^T (\mathbf{B} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})^T (\mathbf{D} - \mathbf{A}) \\ &\Rightarrow \|\mathbf{B} - \mathbf{A}\|^2 = \|\mathbf{B} - \mathbf{D}\|^2 + \|\mathbf{D} - \mathbf{A}\|^2 \\ &\Rightarrow \|\mathbf{B} - \mathbf{A}\| = \sqrt{a^2 + b^2} \end{aligned} \quad (2.0.4)$$

Similarly, find the length of CA

$$\begin{aligned} &(\mathbf{C} - \mathbf{A})^T (\mathbf{C} - \mathbf{A}) \\ &= (\mathbf{C} - \mathbf{D} + \mathbf{D} - \mathbf{A})^T (\mathbf{C} - \mathbf{D} + \mathbf{D} - \mathbf{A}) \\ &= [(\mathbf{C} - \mathbf{D})^T + (\mathbf{D} - \mathbf{A})^T][(\mathbf{C} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})] \\ &= (\mathbf{C} - \mathbf{D})^T (\mathbf{C} - \mathbf{D}) + (\mathbf{C} - \mathbf{D})^T (\mathbf{D} - \mathbf{A}) + \\ &\quad (\mathbf{D} - \mathbf{A})^T (\mathbf{C} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})^T (\mathbf{D} - \mathbf{A}) \end{aligned} \quad (2.0.5)$$

Again the inner product of lines BC and DA is zero.

$$\begin{aligned} &(\mathbf{C} - \mathbf{A})^T (\mathbf{C} - \mathbf{A}) \\ &= (\mathbf{C} - \mathbf{D})^T (\mathbf{C} - \mathbf{D}) + (\mathbf{D} - \mathbf{A})^T (\mathbf{D} - \mathbf{A}) \\ &\Rightarrow \|\mathbf{C} - \mathbf{A}\|^2 = \|\mathbf{C} - \mathbf{D}\|^2 + \|\mathbf{D} - \mathbf{A}\|^2 \\ &\Rightarrow \|\mathbf{C} - \mathbf{A}\| = \sqrt{a^2 + b^2} \end{aligned} \quad (2.0.6)$$

From Eq (2.0.4) and Eq (2.0.6),

$$\|\mathbf{B} - \mathbf{A}\| = \|\mathbf{C} - \mathbf{A}\| \quad (2.0.7)$$

Since the lengths AB and AC are equal, We can conclude that the $\triangle ABC$ is an isosceles triangle with $AB = AC$.

Latex codes:

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https://github.com/Hrithikraj2/  
MatrixTheory_EE5609/blob/master/  
Assignment_3/latex
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