Assignment-5

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Abstract—**This document contains solution of Problem Ramsey**(4.1.4)

Download latex-tikz codes from

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https://github.com/Hrithikraj2/
    MatrixTheory_EE5609/blob/master/
    Assignment_5/A5.tex
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1 Question

The line

$$\begin{pmatrix} -m & 1 \end{pmatrix} \mathbf{x} = 1 \tag{1.0.1}$$

is a tangent to the curve $y^2 = 4x$. Find the value of m

2 Solution

Compare $y^2 = 4x$ to the general equation

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.1}$$

a=b=e=0, d=-2, c=1, f=0

$$\mathbf{V} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{2.0.3}$$

$$\left|V\right| = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 \tag{2.0.4}$$

$\implies$ The curve is a parabola. Since

$$\mathbf{V p_1} = 0 \tag{2.0.5}$$

$$\implies \mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.6}$$

The slope of the given line is m. The direction vector $\mathbf{m}$ would be

$$\mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{2.0.7}$$

$$\mathbf{m}^T \mathbf{n} = 0 \tag{2.0.8}$$

$$\mathbf{n} = \begin{pmatrix} m \\ -1 \end{pmatrix} \tag{2.0.9}$$

Equation for point of contact for the parabola is

$$\begin{pmatrix} \mathbf{u}^T + \kappa \mathbf{n}^T \\ \mathbf{V} \end{pmatrix} \mathbf{q} = \begin{pmatrix} -f \\ \kappa \mathbf{n} - \mathbf{u} \end{pmatrix} \tag{2.0.10}$$

$$where, \kappa = \frac{\mathbf{p}_1^T \mathbf{u}}{\mathbf{p}_1^T \mathbf{n}} = -\frac{2}{m} \tag{2.0.11}$$

Substitute the values to get

$$\begin{pmatrix} -4 & \frac{2}{m} \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} = \begin{pmatrix} 0 \\ 0 \\ \frac{2}{m} \end{pmatrix} \tag{2.0.12}$$

Solving for $\mathbf{q}$ by removing the zero row and represent Eq (2.0.12) as augmented matrix and convert it into echelon form

$$\implies \begin{pmatrix} -4 & \frac{2}{m} & 0 \\ 0 & 1 & \frac{2}{m} \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{-R_1}{4}} \begin{pmatrix} 1 & -\frac{1}{2m} & 0 \\ 0 & 1 & \frac{2}{m} \end{pmatrix} \tag{2.0.13}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + \frac{1}{2m} R_2} \begin{pmatrix} 1 & 0 & \frac{1}{m^2} \\ 0 & 1 & \frac{2}{m} \end{pmatrix} \tag{2.0.14}$$

$$\tag{2.0.15}$$

The point of contact is $\begin{pmatrix} \frac{1}{m^2} \\ \frac{2}{m} \end{pmatrix}$. Substitute this in Eq (1.0.1)

$$\begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{m^2} \\ \frac{2}{m} \end{pmatrix} = 1 \tag{2.0.16}$$

$$-\frac{1}{m} + \frac{2}{m} = 1 \tag{2.0.17}$$

$$\frac{1}{m} = 1 \tag{2.0.18}$$

$$\implies m = 1 \tag{2.0.19}$$

Python Code to verify the result,

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https://github.com/Hrithikraj2/
    MatrixTheory_EE5609/blob/master/
    Assignment_5/A5.py
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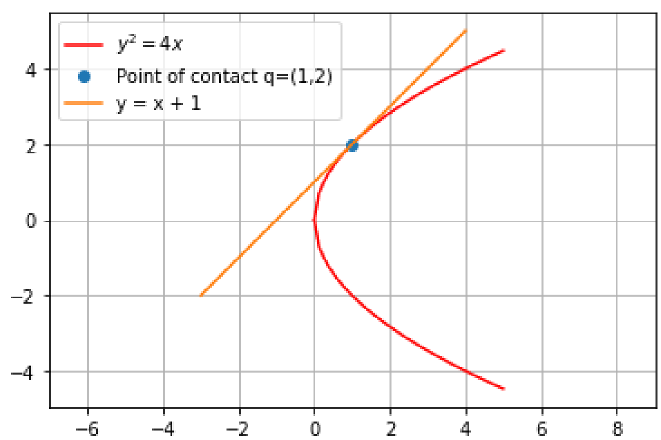

Fig. 0