Assignment-6

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***Abstract*—This document contains solution of Problem Ramsey**(4.1.4)

Download latex-tikz codes from

https://github.com/Hrithikraj2/MatrixTheory_EE5609/blob/master/Assignment_6/A6.tex

1 Question

Trace the parabola

$$16x^2 + 24xy + 9y^2 - 5x - 10y + 1 = 0$$

2 Solution

Compare the given equation with the standard form

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.1}$$

Write the values Of V and u as follows

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} 16 & 12 \\ 12 & 9 \end{pmatrix} \quad \mathbf{u} = \begin{pmatrix} -\frac{5}{2} \\ -5 \end{pmatrix} \quad f = 1 \tag{2.0.2}$$

The characteristic equation of $\mathbf{V}$ is given as

$$\left|\lambda \mathbf{I} - \mathbf{V}\right| = 0 \tag{2.0.3}$$

$$\implies \begin{vmatrix} \lambda - 16 & -12 \\ -12 & \lambda - 9 \end{vmatrix} = 0 \tag{2.0.4}$$

$$\implies \lambda^2 - 25\lambda = 0 \tag{2.0.5}$$

The eigenvalues are the roots of the equation (2.0.5) are

$$\lambda_1 = 0, \quad \lambda_2 = 25 \tag{2.0.6}$$

The eigen vector $\mathbf{p}$ is defined as,

$$\mathbf{V}\mathbf{p} = \lambda \mathbf{p} \tag{2.0.7}$$

$$\implies (\lambda \mathbf{I} - \mathbf{V})\mathbf{p} = 0 \tag{2.0.8}$$

For $\lambda_1 = 0$

$$(\lambda_1 \mathbf{I} - \mathbf{V}) = \begin{pmatrix} -16 & -12 \\ -12 & -9 \end{pmatrix} \xleftrightarrow[R_2 \leftarrow R_2 - 3R_1]{R_1 \leftarrow \frac{1}{4}R_1} \begin{pmatrix} -4 & -3 \\ 0 & 0 \end{pmatrix} \tag{2.0.9}$$

$$\implies \mathbf{p_1} = \frac{1}{5}\begin{pmatrix} -3 \\ 4 \end{pmatrix} \tag{2.0.10}$$

For $\lambda_2 = 25$

$$(\lambda_2 \mathbf{I} - \mathbf{V}) = \begin{pmatrix} 9 & -12 \\ -12 & 16 \end{pmatrix} \xleftrightarrow[R_2 \leftarrow R_2 + 4R_1]{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 3 & -4 \\ 0 & 0 \end{pmatrix} \tag{2.0.11}$$

$$\implies \mathbf{p_2} = \frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix} \tag{2.0.12}$$

Use Eigenvalue decomposition, $\mathbf{P}^T\mathbf{V}\mathbf{P} = \mathbf{D}$, where

$$\mathbf{P} = \frac{1}{5}\begin{pmatrix} -3 & 4 \\ 4 & 3 \end{pmatrix} \tag{2.0.13}$$

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 25 \end{pmatrix} \tag{2.0.14}$$

Focal length of the parabola is given as

$$\text{focal length} = \left|\frac{2\eta}{\lambda_2}\right| \tag{2.0.15}$$

$$\eta = \mathbf{p}_1^T \mathbf{u} = -\frac{5}{2} \tag{2.0.16}$$

Substituting values from (2.0.16) and (2.0.6) in (2.0.15), we get

$$\text{focal length} = \frac{1}{5} \tag{2.0.17}$$

The standard equation of the parabola is given by

$$\mathbf{y}^T \mathbf{D} \mathbf{y} = -2\eta \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{y} \tag{2.0.18}$$

And the vertex $\mathbf{c}$ is given by

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \tag{2.0.19}$$

Substituting values from (2.0.2),(2.0.16),(2.0.10) in (2.0.19),

$$\begin{pmatrix} -4 & -7 \\ 16 & 12 \\ 12 & 9 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} \tag{2.0.20}$$

To find $\mathbf{c}$, performing row reduction on the augmented matrix as follows:

$$\begin{pmatrix} -4 & -7 & -1 \\ 16 & 12 & 4 \\ 12 & 9 & 3 \end{pmatrix} \xleftrightarrow[R_1 \leftarrow \frac{-1}{4} R_1]{R_3 \leftarrow R_3 - \frac{3}{4} R_2} \begin{pmatrix} 1 & \frac{7}{4} & \frac{1}{4} \\ 16 & 12 & 4 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.21}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 16 R_1} \begin{pmatrix} 1 & \frac{7}{4} & \frac{1}{4} \\ 0 & -16 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.22}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{-1}{16} R_2} \begin{pmatrix} 1 & \frac{7}{4} & \frac{1}{4} \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.23}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{7}{4} R_2} \begin{pmatrix} 1 & 0 & \frac{1}{4} \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.24}$$

Thus,

$$\mathbf{c} = \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix} \tag{2.0.25}$$

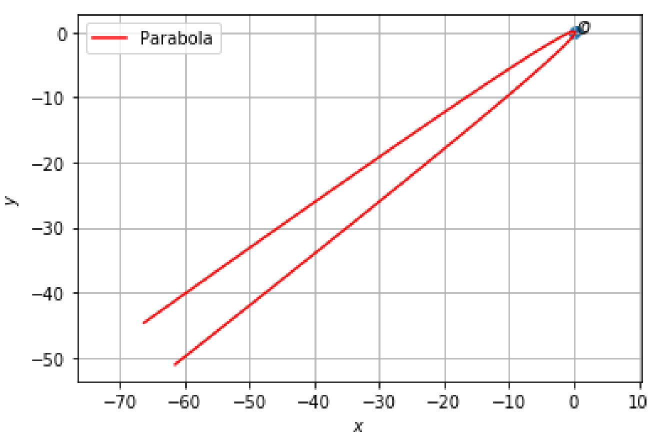

Fig. 1: Parabola with vertex c