Control Systems

HRITHIK RAJ
ES17BTECH11009

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Question

Loop transfer function of a feedback system is G(s)H(s) = $\dfrac{s+3}{s^2(s-3)}$.

Take the Nyquist contour in the clockwise direction.

Then the Nyquist plot of G(s)H(s) encircles -1 + j0

(A) Once in clockwise direction

(B) Twice in clockwise direction

(C) Once in anticlockwise direction

(D) Twice in clockwise direction

Solution

Given GH $= \dfrac{s+3}{s^2(s-3)}$.

Taking the magnitude of the above function

$$\mid GH \mid = \frac{\sqrt(\omega^2+3^2)}{\omega^2(\sqrt(\omega^2+3^2))} = \frac{1}{\omega^2} - - - - (1)$$

Now considering the phase, we get

$$\angle GH = [\arctan\frac{\omega}{3}] - [\pi + \pi - \arctan\frac{\omega}{3}] = 2\arctan\frac{\omega}{3} - - - - (2)$$

Cont...

Equations (1) and (2) would give us

$$\text{GH} = \frac{1}{\omega^2}\angle 2\arctan\frac{\omega}{3}$$

For the Nyquist plot,

We need to draw the polar plot by varying

$\omega from 0 to \infty$

Cont...

At $\omega = 0$, $GH = \infty\angle 0$

At $\omega = 3$, $GH = \frac{1}{9}\angle 90$

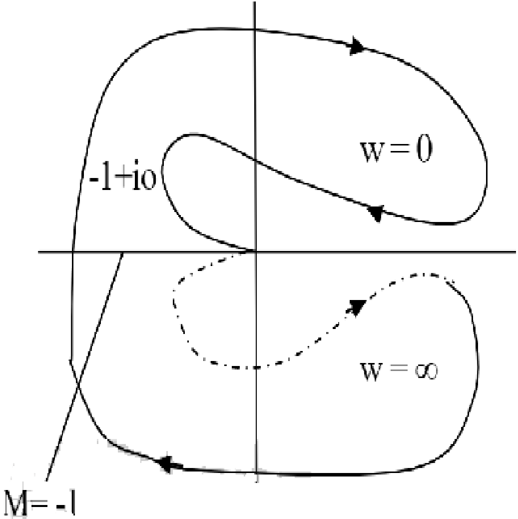
At $\omega = \infty$, $GH = 0\angle 180$

So we plot stating from 0 to 180 degrees,

Since there are 2 poles on the origin we get 2 infinite radius semicircles which start where the mirror image ends and terminate where the actual plot started in clockwise direction.

Nyquist plot

So the plot would look like



Cont..

So the Nyquist plot of $G(s)H(s)$ encircles $-1 + j0$ once in clockwise direction

The correct option is (A)