# CONCEPTS OF GAME THEORY AND ITS VISUALIZATION

A SECOND-YEAR PROJECT REPORT

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE DEGREE OF B.Sc. IN COMPUTATIONAL MATHEMATICS

BY

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# CERTIFICATION

This project entitled "**CONCEPTS OF GAME THEORY AND ITS VISUALIZATION**" is carried out under my supervision for the specified entire period satisfactorily, and is hereby certified as work done by the following students

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I hereby declare that the candidate qualifies to submit this report of the Math Project (MATH-252) to the Department of Mathematics.

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# ABSTRACT

This project offers an in-depth exploration of game theory, emphasizing its mathematical underpinnings and applications in various fields. It systematically examines concepts such as Nash Equilibrium, zero-sum games, and mixed strategies, showcasing their mathematical significance. The study extends to a practical game simulation where mathematical strategies are applied, analyzing the decision-making processes and strategic outcomes. This practical application demonstrates the intricate relationship between theoretical game theory and its real-world implications, particularly in areas of economic and political strategy. The project underscores the critical role of mathematical analysis in understanding and predicting competitive behaviors, offering valuable insights into cooperative strategies for optimal outcomes in complex scenarios

**Keywords:** *Nash Equilibrium, zero-sum game, mixed strategies, Payoff matrix*

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# 1 Introduction

## 1.1 Introduction

Game theory, a multifaceted and intricate field of study, delves into the complexities of decision-making processes where outcomes are interdependent on the choices of multiple agents. This discipline finds its roots in the early 20th century, with significant advancements attributable to mathematicians such as John Von Neumann and economist Oskar Morgenstern. Their seminal work in the 1940s, particularly the publication of **"Theory of Games and Economic Behavior"**, laid the foundational principles of this field.

Game theory is the theory of independent and interdependent decision-making. It is concerned with decision-making in organizations where the outcome depends on the decisions of two or more autonomous players, one of which may be nature itself, and where no single decision-maker has full control over the outcomes. Games like chess and bridge fall within the ambit of game theory, but so do many social situations that are not commonly regarded as games in the everyday sense of the word [3].

Like other sciences, game theory consists of a collection of models. A model is an abstraction used to understand observations and experiences. What "understanding" entails is not clear-cut. Partly, at least, it entails our perceiving relationships between situations, isolating principles that apply to a range of problems, so that we can fit into our thinking new situations that we encounter. For example, we may fit our observation of the path taken by a lobbed tennis ball into a model that assumes the ball moves forward at a constant velocity and is pulled towards the ground by the constant force of "gravity". This model enhances our understanding because it fits well no matter how hard or in which direction the the ball is hit, and applies also to the paths taken by baseballs, cricket balls, and a wide variety of other missiles, launched in any direction [5].

## 1.2 History of Game Theory

### Early Beginnings

Game theory's history can be traced back to long before the formalization of the subject. Notably, Cardano's "Liber de ludo aleae" (Book on Games of Chance), written in the 16th century, introduced basic ideas about games of chance. In the 1650s, Pascal and Huygens further developed these ideas, laying foundational work for probability theory and strategic thinking in games. Charles Waldegrave's analysis of the card game "le Her" in 1713 provided one of the earliest examples of a minimax mixed strategy solution, a concept central to game theory (Wikipedia).

### Formative Developments

In 1913, Ernst Zermelo published a significant work on chess theory, and in 1838, Antoine Augustin Cournot made important contributions to the study of duopoly, both advancing the field of game theory in the 20th century. Frederik Zeuthen's use of Brouwer's fixed point theorem in 1938 to prove the existence of a winning strategy in mathematical models further contributed to the field's foundational concepts (Wikipedia).

### Establishment of Modern Game Theory

The establishment of modern game theory is primarily attributed to John von Neumann. His work on mixed-strategy equilibria in two-person zero-sum games and the influential "Theory of Games and Economic Behavior" (1944), co-authored with Oskar Morgenstern, set the foundation for both cooperative and non-cooperative games. This work was a leap forward, especially in its introduction of an axiomatic theory of expected utility, which became crucial for decision-making under uncertainty (Wikipedia).

### John Nash and Equilibrium Concept

John Nash's contribution to game theory was pivotal. Nash extended the concept of equilibrium, that is now known as the Nash Equilibrium. This concept, introduced in the early 1950s, revolutionized the understanding of non-cooperative games. It provided a way to predict the outcome of strategic interactions in which each participant considers the strategies of others while making their own decisions. Nash's work expanded the applicability of game theory beyond zero-sum games to more complex scenarios involving multiple players with potentially conflicting interests (Wikipedia).

#### Expansion and Application

In the latter half of the 20th century, game theory has extensive development and application across various fields. John Maynard Smith applied game theory to evolutionary biology in the 1970s. The 21st-century game theory has become a crucial tool in diverse fields, including economics, political science, biology, and computer science, underscoring its role in understanding strategic decision-making in a wide range of contexts (Wikipedia).

#### Philosophical and Political Implications

Game theory has significant implications in philosophy and political science. The work of Thomas Hobbes, especially in "Leviathan," showcases early game-theoretic reasoning in the analysis of human behavior and the state. Hobbes's insights, focusing on the strategic interactions and outcomes in human societies, have been foundational in modern political philosophy (Stanford Encyclopedia of Philosophy).

#### Recent Developments

Game theory continues to evolve with more sophisticated models addressing a wide array of strategic interactions. Its ability to provide insights into both cooperative and competitive scenarios in various disciplines highlights its versatility and importance.

## 1.3 Key Concepts Of Game Theory

Game theory, a branch of applied mathematics, provides a framework for analyzing situations in which the decisions of various entities (referred to as "players") are interdependent. This interdependence necessitates that each player takes into account the potential decisions or strategies of the other players when formulating their strategy. The goal of game theory is to determine the optimal decisions for these players, who may have similar, opposed, or mixed interests, and to predict the outcomes of these decisions. There are some of the terms in game theory which are described as below:

- **Players:** Players are the decision-makers in the game. They can be individuals, organizations, or even natural forces. Each player is considered rational and aims to maximize their outcome within the game's framework.
- **Outcomes:** An outcome in game theory is the result of all players making their strategic choices. It is assumed that players have consistent preferences among the possible outcomes.
- **Strategies:** A strategy for a player is a comprehensive plan for the entire game, delineating the player's actions in response to every possible decision point throughout the game.
- **Payoffs:** Payoffs represent the results or outcomes that players receive as a consequence of the strategies employed by themselves and others. These payoffs can be in various forms, such as profit, utility, or satisfaction, depending on the context of the game.

## 1.4 Nash Equilibrium

The Nash Equilibrium is a fundamental concept in game theory which was developed by American mathematician John Nash. It represents a state in a non-cooperative game where no player can improve their payoff by unilaterally changing their strategy, assuming other players' strategies remain constant. In essence, it's a situation of "no regrets," where each player's decision leaves them with no incentive to deviate, given the choices of others. Nash Equilibrium can happen multiple times in a game, indicating a stable state. At this point, players' strategies come together in a way that no one gains by changing their decision alone.

## 1.5 Zero Sum Games

Zero-sum games describe situations where one player's gain or loss is exactly balanced by the losses or gains of other players. In these games, the total benefit or loss remains constant, the distribution of this value becomes the central aspect of the game.

### Key Characteristics and Principles

Zero-sum games are characterized by a direct trade-off between the players' outcomes. They are often modeled as two-player scenarios where one player's gain is precisely the other's loss.

### Examples in Various Contexts

- **Board Games:** Chess and Go are classic examples of zero-sum games, where each move directly impacts the opponent's position.
- **Economic and Financial Scenarios:** In financial markets, particularly in derivatives and commodities trading, gains and losses among investors are often directly offsetting.
- **Political and Military Strategy:** The dynamics of zero-sum are prevalent in geopolitics and military strategies, where one nation's or group's gain can mean another's loss.

### Strategic Implications and Game Theory Concepts

In zero-sum games, players must develop strategies that maximize their own benefits while minimizing potential gains for their opponents. This involves a careful analysis of the opponent's potential moves and countermoves.

### Mathematical Representation and Analysis

**Payoff Matrix:** Payoff matrix is a crucial tool in game theory that represents potential payoffs for all players based on their chosen strategies.
**Saddle Point and Optimization:** The saddle point in a payoff matrix helps to identify the optimal strategies for players, aiming to balance between minimizing losses and maximizing gains.

**Limitations and Real-World Application Challenges**

While zero-sum games offer a clear framework for understanding competitive scenarios, they often oversimplify complex real-world interactions. These models typically involve only two players and assume strictly competitive dynamics, which may not accurately reflect more nuanced situations that include collaboration or involve more than two players.

# 2 Game of Strategy

## 2.1 Description of Game Strategy

A game of strategy is described by its set of rules. These rules specify clearly what each person called a player is allowed or required to do under all possible circumstances. The rule specifies the quantity of information, if any, that each player is entitled to receive. They also define the time the game ends, the amount each player receives, and the objective of each player[1].

It is convenient to classify games according to distinguish between games whose payoffs are zero-sum and those whose are not. If the players make payments only to each other, the game is said to be zero-sum.

To simplify the mathematical description of the game, we introduce the concept of **strategy**. We may think of a strategy of a player as a set of instructions for playing the game from the first move to the last.

Every pair of strategies, consisting of one of the strategies for each player determines a play of the game, which in turn determines a payoff to each player. Thus, we may consider a play of a game to consist of each player making one decision, namely the selection of strategy [1].

## 2.2 Determining the payoff matrix

Let us call two players Player 1 and Player 2, Suppose Player 1 has m strategies, which may be designated by the numbers,

$$i = 1, 2, 3, ..., m$$

On the other hand, Player 2 has n strategies, which may be designated by the numbers,

$$j = 1, 2, 3, ..., n$$

Then,for the first move, Player 1 chooses some strategy $i$. On the next move, Player 2, without being informed what choice Player 1 has made, chooses the strategy $j$.The two choices determine the play of the game and the payoff to the two players. Let $a_{ij}$ be the playoff to Player 1.

The game is thus determined by the payoff matrix of Player 1

$$\begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{bmatrix}$$

Here, each row of the matrix corresponds to a strategy available to Player 1. The $ij$ element of the matrix gives the payoff of Player1 if the chooses $i^{th}$ row and Player 2 selects $j^{th}$ column [1].

# 3 Description of Game of Strategy

This game is based on the scenario, where two players put one coin in each coin machine. The players do not know what other players are going to play. To talk about the strategy of this game, there are only two strategies: cooperate and cheat [4]. Co-operate is represented as 1 and cheat as 0. So, the strategies of each player can be represented in a set,

$$S = \{0, 1\}$$

Since we have 2 strategies for the game between 2 players, there are only 4 ways that a payoff will be received by players. So, the payoff matrix can be represented by,

| PLAYER1 \ PLAYER2 | Cheat | Co-operate |
|---|---|---|
| Cheat | 0 | 3 |
| Co-operate | -1 | 2 |

Figure 1: Payoff Matrix of Player1

The above payoff matrix can be described as:-

- If player1 and player2 both cheats(0),then both will receive 0 points
- If Player1 cheats(0) and Player2 co-operates(1) or vice versa, the player co-operating gets -1 points and the player who cheated receives 3 points.
- If both players cooperate then both players get 2 points each

## 3.1 Condition of Guaranteed payoff

In games, two players make decisions at the same time. Let Player 1 was forced to announce his decision before Player 2 makes his decision. Then,

1. For cheat(0), the Player2's best response is to cheat(+0)
2. For co-operate(1), the Player2's best response is to cheat(+3)

Here, Player 1 wishes to maximize his payoff and his best pure choice is to announce a cheat. He takes **maximin** strategy to maximize his minimum payoff. This guarantees a payoff of at least +0 to Player1, regardless of Player 2's strategy. Similarly, if Player 2

announces first then,

1. For cheat(0), the Player1's best response is to cheat(+0)
2. For co-operate(1), the Player1's best response is to cheat(+3)

Here, Player 2 wants to minimize the payoff of Player 1, and thus his best pure choice is to cheat. For this, Player 2 uses the **minimax** strategy to minimize the maximum payoff of Player 1. This guarantees a payoff of at most +0 to Player 1, regardless of Player 1's strategy.

In our scenario, we know the players have only two strategies **'Cheat'** and **'Cooperate'**; then the game is determined by $2 \times 2$ matrix $A = (a_{ij})$ where $i, j = 1, 2, \vdots$ and $a_{ij}$ is Player 1's payoff if he uses $i^{th}$ strategy and Player 2 uses his $j^{th}$ strategy.

Player 1's objective in the game is to make $a_{ij}$ as large as possible, whereas, Player 2's objective is the make $a_{ij}$ as small as possible. In terms of payoff to Player 1, we may refer to Player 1 as **maximizing player** and Player 2 as **minimizing player**.

Now, for any strategy $i$ which Player 1 may choose, he can be sure of getting at least

$$\min_{j \leq n} a_{ij},$$

where the minimum is taken over all of Player 2's strategies.

Player 1 is at liberty to choose $i$, therefore, he can make his choice in such a way as to ensure that he gets at least

$$\max_{i\leq m}\min_{j\leq n} a_{ij},$$

Also, for any strategy $j$ which Player 2 may choose, he can be sure that Player 1 gets no more than,

$$\max_{i\leq m} a_{ij},$$

Since Player 2 is also at the liberty to choose j, he can choose it in such a way that Player 1 will get at most

$$\min_{j\leq n}\max_{i\leq m} a_{ij},$$

Therefore, there exists a way for Player 1 to play so that Player 1 gets at least

$$\max_{i\leq m}\min_{j\leq n} a_{ij},$$

and there exists a way for Player 2 to play so that Player 1 gets no more than

$$\min_{j\leq n}\max_{i\leq m} a_{ij},$$

In general, these two quantities are different, but satisfy the relationship

$$\max_{i\leq m}\min_{j\leq n} a_{ij} \leq \min_{j\leq n}\max_{i\leq m} a_{ij}$$

We know, Given any $i$, then

$$\min_{j\leq n} a_{ij} \leq a_{ij}$$

Given any $j$, then

$$\max_{i \le m} a_{ij} \ge a_{ij}$$

hence we have,

$$\min_{j \le n} a_{ij} \le a_{ij} \le \max_{i \le m} a_{ij}$$

or,

$$\min_{j \le n} a_{ij} \le \max_{i \le m} a_{ij} \tag{1}$$

Since, the right-hand side of the preceding inequality (1) is independent of $i$, we have, by taking the maximum of both sides,

$$\max_{i \le m} \min_{j \le n} a_{ij} \le \max_{i \le m} a_{ij} \tag{2}$$

Now, the right-hand side of the preceding inequality (2) is independent of $i$, we have, by taking the minimum of both sides,

$$\max_{i \le m} \min_{j \le n} a_{ij} \le \min_{j \le n} \max_{i \le m} a_{ij} \tag{3}$$

Therefore,

$$\max_{i \le m} \min_{j \le n} a_{ij} \le a_{ij} \le \min_{j \le n} \max_{i \le m} a_{ij} \tag{4}$$

In the context of our game, the payoff matrix is given by,

$$\begin{bmatrix} 0 & 3 \\ -1 & 2 \end{bmatrix}$$

then,

min $a_{ij}$ = {0,-1} [lowest among rows of the matrix]

max min $a_{ij}$ = 0 [maximum between element of above set]

also,

max $a_{ij}$ = {0,3} [maximum among column of the matrix]

min max $a_{ij}$ = 0

Therefore, we can see that we get the same result by analytical method and by mathematical approach too.

## 3.2 Saddle Point

If the inequality

$$\max_{i \leq m} \min_{j \leq n} a_{ij} \leq \min_{j \leq n} \max_{i \leq m} a_{ij}$$

becomes equality, or that

$$\max_{i \leq m} \min_{j \leq n} a_{ij} = \min_{j \leq n} \max_{i \leq m} a_{ij} = v$$

Then, Player 1 can choose a strategy to get at least this common value, and Player 2 can keep Player 1 from getting more than $v$.

In this case, there are strategies $i^*$ and $j^*$ for the two players such that, for all $i$ and $j$,

$$a_{ij^*} \leq a_{i^*j^*} \leq a_{i^*j} \tag{5}$$

and,

$$a_{i^*j^*} = v$$

Thus, Player 1 cannot do better than to choose $i^*$ and Player 2 cannot do better than to choose $j^*$

We refer to $i^*$,$j^*$ as **optimal strategies** of Player 1 and Player 2.

The optimal strategies have the following properties:

1. If Player 1 chooses $i^*$, then no matter what strategy Player 2 chooses, Player1 can get at least $v$.
2. If Player 2 chooses $j^*$, then no matter what strategy Player 1 chooses, Player 1 gets at most $v$.

From (5) we also have,

$$max_{i \leq m} a_{ij^*} = min_{j \leq n} a_{i^*j} = v$$

Hence, a necessary and sufficient condition that a game has a saddle point is that there exists a member of the payoff matrix which is simultaneously the minimum of its row and the maximum of its column. It is also known as **Pure Nash Equilibrium**[1].

A game may have several saddle points but in the context of our game which has a payoff matrix,

$$\begin{bmatrix} 0 & 3 \\ -1 & 2 \end{bmatrix}$$

We only have one saddle point at location $i = 1$ and $j = 1$ that is $a_{11} = 0$ which is the lowest value of its row and maximum value of its column.

## 3.3 Mixed Strategies for Randomness

If we do not know the strategies of other players, we use a mixed strategy where the strategies are chosen at random probabilities. [2]

So, for the given payoff matrix,

$$\begin{bmatrix} 0 & 3 \\ -1 & 2 \end{bmatrix}$$

If Player 2 chooses column 1 or Cheat, then Player 1 will receive 0 with a probability of 0.5 or Player 1 will receive -1 with a probability of 0.5 respectively.
Similarly, If Player 2 chooses column 2 or Cooperate, then Player 1 will receive 3 with a probability of 0.5 or Player 1 will receive 2 with a probability of 0.5 respectively.
Since Player 2 aims to minimize Player 1's payoff. Player 2 chooses column 1. Thus, Player 1 can guarantee an expected payoff of at least -1.

## 3.4 Optimal mixed Strategies

Here for our game, the way to model the problem of finding Player 2's best strategy.
For each row $i$, let the decision variable $x_i$, be the probability of selecting row $i$.
If Player 2 chooses,

1. Column1(Cheat),expected value of Player 1 is,
   $P_1 = (0)x_1 + (-1)x_2$
   $= -x_2$

2. Row2(Cooperate), expected value of Player 1 is,
   $P_2 = (3)x_1 + (2)x_2$
   $= 3x_2 + 2x_2$

Thus, Player1's expected value is at least $min\{P_1, P_2\}$. Player 1 will assign the probabilities $x_1$ and $x_2$ in such a way as to maximize $min\{P_1, P_2\}$ to determine his best-mixed strategy.
Similarly, Player2 will assign the probabilities $y_1$ and $y_2$ in such a way to minimum $max\{P_1, P_2\}$ to determine his best-mixed strategy.
So, for any $x^*$ and $y^*$, we have such that, maximum $min\{P_1, P_2\} =$ minimum $max\{P_1, P_2\}$, where both Players get the maximum payoff possible in the situation. This condition is also known as **Nash Equilibrium** or **State of no regret**.

In this game, the payoff matrix is,

$$\begin{bmatrix} 0 & 3 \\ -1 & 2 \end{bmatrix}$$

Then,

1. $P_1 = -x_2$
2. $P_2 = 3x_1 + 2x_2$

Then, $min\{P_1, P_2\} = P_1$ as $0x_1 + -x_2 < 3x_1 + 2x_2$

as $x_1$ and $x_2$ are probabilities

And, maximum $min\{P_1, P_2\} = 0$

as $0 > -1$

Similarly, for columns, $max\{P_1, P_2\}$=$P_1$

as $0y_1 + 3y_2 > -y_1 + y2$

And, minimum $max\{P_1, P_2\} = 0$

as $0 < 3$

So,

The **Nash equilibrium** of our payoff matrix is at position $(x_1, y_1)$ that is $a_{11} = 0$.

Therefore, When both the players are unaware of the strategies of their fellow opponent, then, It is found that the **Nash Equilibrium** of our game is obtained if **both players choose to cheat.**. That means the choice of cheat strategy guarantees the payoff of 0 even if the opponents change their strategies. So, the cheat strategy is **State of no regret** in our game.

# 4 Game Mechanics

## 4.1 Game Visualization

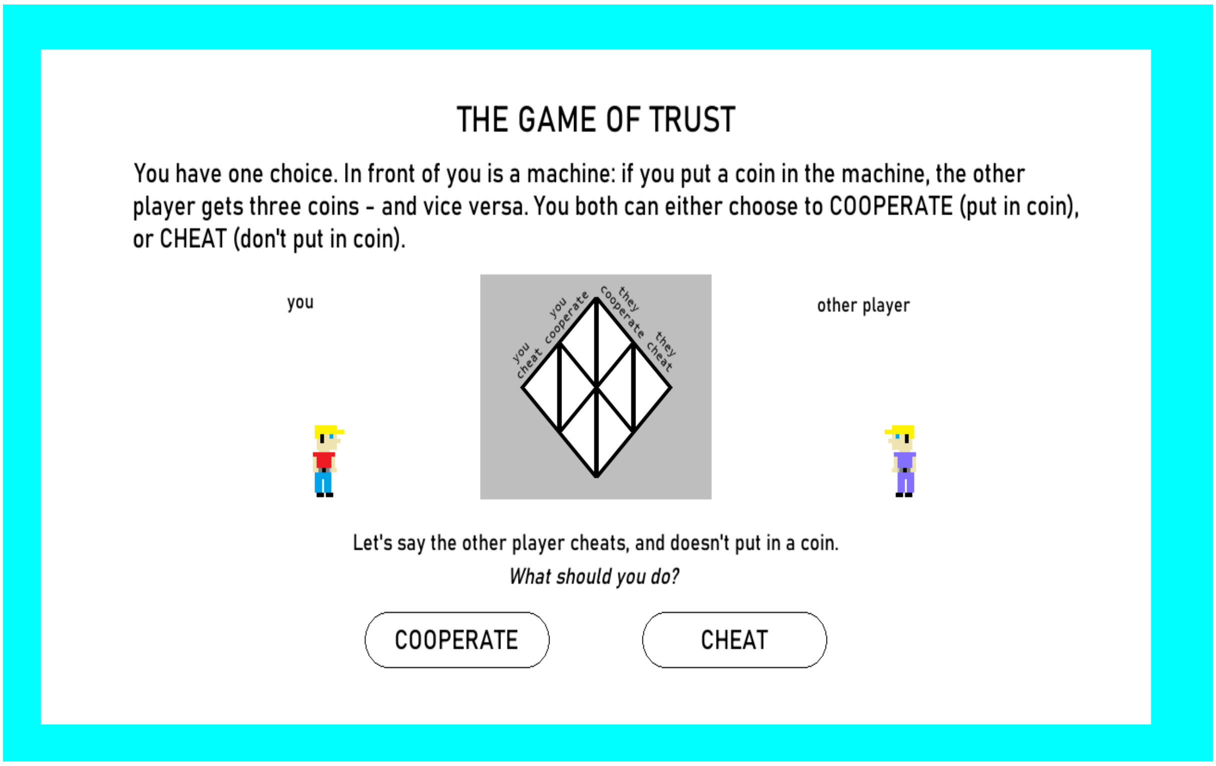


Figure 2: Demo of Game Visuals

## 4.2 Character Strategies in the Game

### Copycat

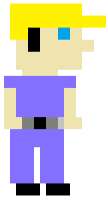

Figure 3: Copycat

- Mimics the opponent's last move. If no past move defaults to cooperating.

```
def copycat(player_response_list, *useless_stuff):
if len(player_response_list):
return player_response_list[-1]
else:
return 1
```

### All Cooperate

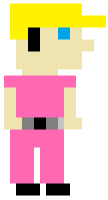

Figure 4: All Cooperate

- Always cooperate.

```
def all_cooperate(*useless_stuff):
return 1
```

**All Cheat**

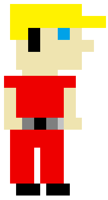

Figure 5: All Cheat

- Always cheat, regardless of the opponent's actions.

```
def all_cheat(*useless_stuff):
return 0
```

**Grudger**

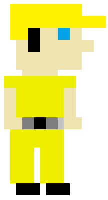

Figure 6: Grudger

- Starts by cooperating, but if the opponent ever cheats, it will cheat forever.

```
def grudger(player_response_list,*useless_stuff):
return int(not (0 in player_response_list))
```

**Detective**

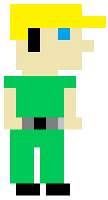

Figure 7: Detective

- Starts by cooperating and then plays the following sequence: Cooperate, Cheat, Cooperate, Cooperate. If the opponent ever cheats back, it will play like a copycat. If the opponent never cheats back, it will play like an all-cheat.

```
def detective(player_response_list,*useless_stuff):
l = len(player_response_list)
if l in [0,2,3]:
return 1
elif l==1:
return 0
else:
if 0 in player_response_list[2:]:
return player_response_list[l-1]
else:
return 0
```

**Copykitten**

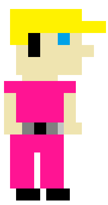

Figure 8: Copykitten

- Similar to copycat but forgives a single cheating move by the opponent.

```
def copykitten(player_response_list,*useless_stuff):
if len(player_response_list) < 2:
return 1
return (player_response_list[len(player_response_list)-1] or
player_response_list[len(player_response_list)-2])
```

**Simpleton**

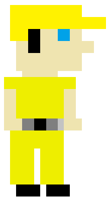

Figure 9: Simpleton

- Cooperates if both players did the same thing last time, else cheats.
- Requires knowledge of its past moves as well.

```
def simpleton(player_response_list,bot_response_list,*useless_stuff):
l = len(player_response_list)
if l:
return int(not (player_response_list[l-1]^bot_response_list[l-1])) %XOR
else:
return 1
```

# 5 Tournaments between the players

## 5.1 Introduction

Tournament process involving two competing bots. This tournament is a series of strategic games where each bot makes decisions based on predefined rules and the history of the opponent's moves. The objective is to analyze the strategies of the bots over multiple rounds to determine the most effective approach.

## 5.2 Tournament Process

The tournament involves a series of rounds where two bots, referred to as `bot1` and `bot2`, compete against each other. In each round, the bots decide whether to cooperate or compete, and points are awarded based on these decisions. The tournament aims to evaluate the strategies of each bot over a set number of rounds and record the outcomes for analysis.

## 5.3 Algorithm for the Tournament

The algorithm for conducting the tournament is as follows:

**Initialize:**<br>
Set initial scores of both bots to 0.<br>
Create empty lists to track the response history of each bot.<br>
**for** each round in the tournament **do**<br>
    Determine the move for each bot based on the other bot's response history.<br>
    Update the response history lists with the current round's moves.<br>
    **Score Calculation:**<br>
    **if** a bot cooperates (move is `True`) **then**<br>
        It gives away points (`givecoins`) and the other bot gains more points (`getcoins`).<br>
    **end if**<br>
    Update the scores of both bots based on their moves.<br>
**end for**<br>
**Result Recording:**

Check if the results file comma-separated values(CSV) exist.
Open the CSV file in append mode if it exists, otherwise in write mode.
If the file is new, write the header row.
Write the results of the current round, including moves and scores of both bots.
**Return Final Results:**
After all rounds are completed, return the final scores and the response histories of both bots.

### 5.4 Conclusion

The tournament provides valuable insights into the decision-making strategies of the bots. By analyzing the outcomes of multiple rounds, we can determine the effectiveness of different strategies in various scenarios.

## 6 Analysis of tournament

### 6.1 Analysis of scores between all the characters of the game

Data from the CSV file of the tournament have been extracted which stores the scores obtained by players when they play against each other. For the analysis, a series of bar plots are created to show the scores of each character with each other.
The analysis of bar charts shows that whenever both the players play as all cooperate, both players receive a maximum payoff. To understand, how in a maximum payoff situation, both players play as **All Cooperate**, we need to understand the characters of the players.
**The analysis of bar charts also shows that whenever the players play as all cheat, it guarantees the least payoff of 0 as calculated in section 3.4. Thus, the Nash Equilibrium derived from our analysis meets the mathematical criteria of the Nash Equilibrium as outlined in section 3.4 .**

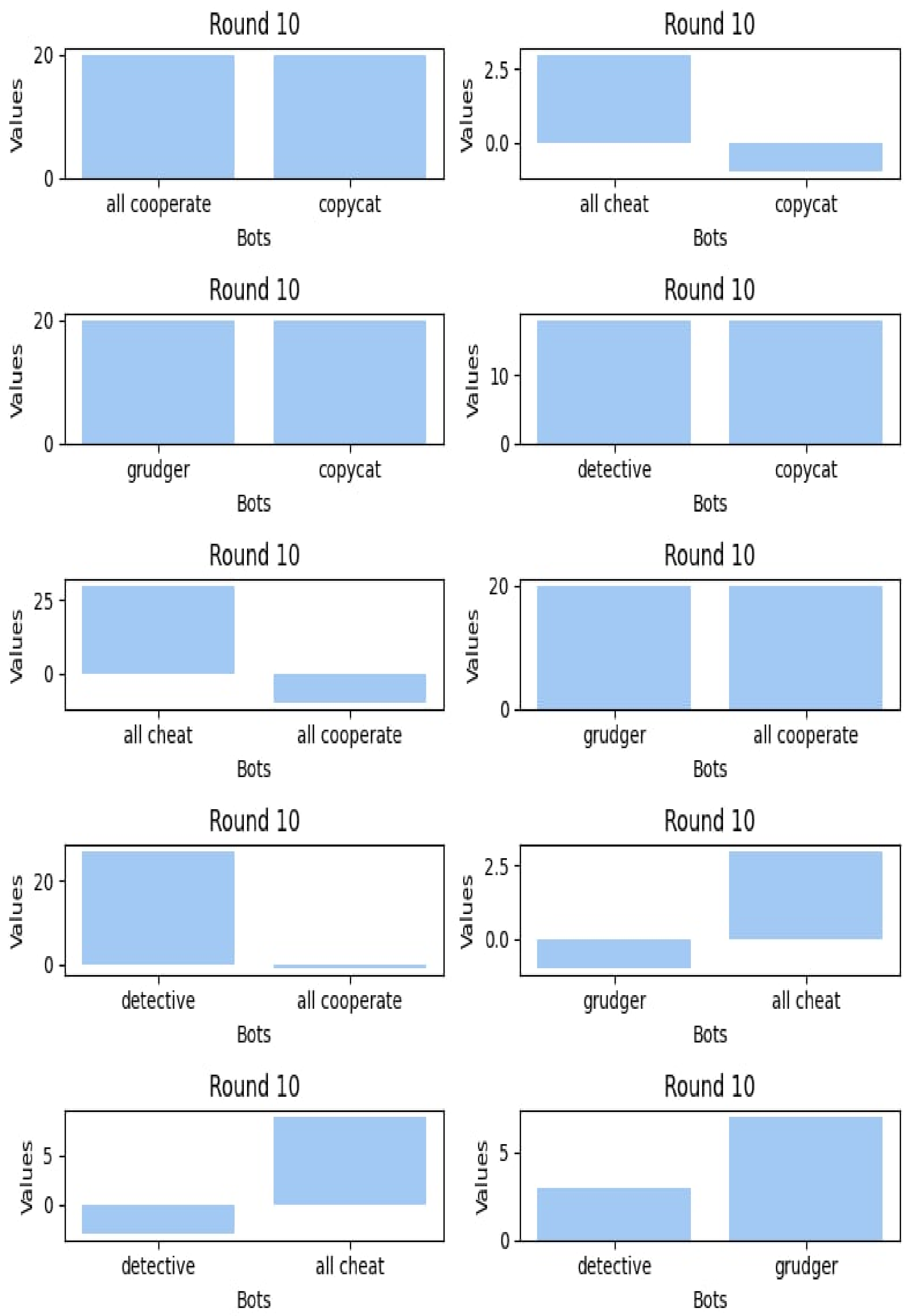


Figure 10: Analysis of tournament scores

# 7 Condition of mistakes

As cool as Copycat is, it has a huge, fatal weakness that hasn't been mentioned yet. To understand the problem, let's say two Copycats are playing against each other.

Normally, both will start with the strategy of cooperate. But, if one of the copycats makes a mistake unintentionally and if the other person doesn't think it was an accident, then by the nature of copycats they will spiral into an endless cycle of vengeance, that started over a single mistake, long ago [6].

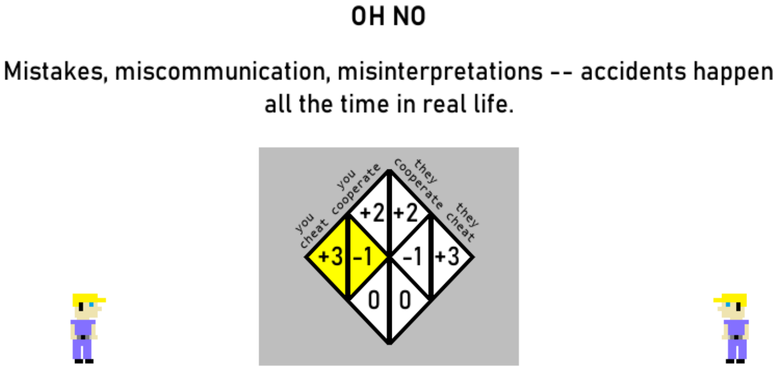


Figure 11: Mistake

This shows how miscommunication leads to a change of decision that may affect the long term.

# 8 Conclusions

In conclusion, this project offers a comprehensive exploration of game theory, highlighting its relevance in various real-world scenarios. We delved into concepts like Nash Equilibrium, saddle points, and mixed strategies, applying these to both economic and political contexts. The project's practical component, a game simulation, provided valuable insights into decision-making strategies and the effectiveness of different approaches in competitive environments. Ultimately, the findings underscore the significance of cooperative strategies in achieving optimal outcomes, bridging theoretical game theory with its practical applications. This integration of theory and practice not only enhances our understanding of strategic interactions but also illustrates the versatility and depth of game theory. Lastly, If there's one big takeaway from all of game theory, it's this: What the game is, defines what the players do. Our problem today isn't just that people are losing trust, it's that our environment acts against the evolution of trust.

That may seem cynical or naive that we're "merely" products of our environment but as game theory reminds us, we are each others' environment. In the short run, the game defines the players. But in the long run, it's us players who define the game.[4]

So, do what you can do, to create the conditions necessary to evolve trust. Build relationships. Find win-wins. Communicate clearly.

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