Queue Theory Analysis Of Banking System In Nepal

Using TORA

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June 17, 2024

Context

In Kathmandu, Nepal, a Nepal Rastra Bank branch located in a busy commercial area experiences significant customer footfall daily. The bank offers various services such as cash deposits, withdrawals, account inquiries, and loan consultations. To manage customer service efficiently, the bank wants to analyze their service system to reduce wait times and improve customer satisfaction.

1. Theoretical Background
2. Analysis Goals
3. Queueing Analysis Using TORA

Queue Models

M/M/1 Queue Model

- Represents a single-server queue with exponential interarrival and service times.
- Suitable for continuous server operation without queue length limits.
- **Example:**
  - **Service Desk:** One service desk handling all transactions.
  - **Arrival Rate (λ):** 20 customers per hour.
  - **Service Rate (μ):** 25 customers per hour.
- **Assumptions:**
  - Customer arrivals follow a Poisson process.
  - Service times are exponentially distributed.
  - Single server with no limit on the queue length.

Queue Models (cont'd)

M/M/1/N Queue Model

- Represents a single-server queue with limited capacity N.
- Suitable for scenarios with restricted queue length.
- **Example:**
  - **Service Desk with Limited Space:** One service desk with a queue capacity of 10 customers.
  - **Arrival Rate (λ):** 20 customers per hour.
  - **Service Rate (μ):** 25 customers per hour.
  - **Queue Capacity (N):** 10 customers.
- **Assumptions:**
  - Customer arrivals follow a Poisson process.
  - Service times are exponentially distributed.
  - Single server with a queue capacity limit.

Analysis Goals: M/M/1

- Determine the average number of customers in the system (L).
- Calculate the average time a customer spends in the system (W).
- Evaluate the probability of the system being idle (P0).

Analysis Goals: M/M/1/N

- Determine the average number of customers in the system (L).
- Calculate the average time a customer spends in the system (W).
- Evaluate the probability of the system being full (Pn).

Queue Models

M/M/1 Queue Model

- **Arrival Rate (λ)**: The rate at which customers arrive at the service facility.
- **Service Rate (μ)**: The rate at which customers are served.
- **Traffic Intensity (ρ)**: $\rho = \frac{\lambda}{\mu}$
- **Average Number of Customers in the System (L_s)**: $L_s = \frac{\lambda}{\mu - \lambda}$
- **Average Time a Customer Spends in the System (W_s)**: $W_s = \frac{1}{\mu - \lambda}$

Queue Models (cont'd)

- **Average Number of Customers in the Queue (L_q)**: $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$
- **Average Time a Customer Spends Waiting in the Queue (W_q)**: $W_q = \frac{\lambda}{\mu(\mu - \lambda)}$
- **Probability of Zero Customers in the System (p0)**: $p_0 = 1 - \rho$
- **Probability of** n **Customers in the System (p_n)**: $p_n = (1 - \rho)\rho^n$

Queue Models (cont'd)

M/M/1/N Queue Model

- Similar to M/M/1 but with a finite queue capacity N.
- **Average Number of Customers in the System (L_s)**:

$$L_s = \frac{\rho(1-(N+1)\rho^N + N\rho^{N+1})}{(1-\rho)(1-\rho^{N+1})}$$

- **Average Number of Customers in the Queue (L_q)**:

$$L_q = L_s - \rho$$

- **Average Time a Customer Spends in the System (W_s)**:

$$W_s = \frac{L_s}{\lambda}$$

Queue Models (cont'd)

- **Average Time a Customer Spends Waiting in the Queue (W_q)**:

$$W_q = W_s - \frac{1}{\mu}$$

- **Probability of Exactly** n **Customers in the System (p_n)**:

$$P_n = \frac{(1-\rho)\rho^n}{1-\rho^{N+1}}$$

- **Probability of Maximum Capacity (P(N))**:

$$P(N) = \frac{\rho^N(1-\rho)}{1-\rho^{N+1}}$$

Scenario 1: M/M/1 Queue Analysis

Parameters:

- $\lambda = 22$ customers per hour
- $\mu = 27$ customers per hour

Scenario 1: M/M/1 Queue Analysis (cont'd)

Scenario 1:(M/M/1):(GD/infinity/infinity)

Lambda =	22.00000	Mu =	27.00000
L'da eff =	22.00000	Rho/c =	0.81481
Ls =	4.40000	Lq =	3.58519
Ws =	0.20000	Wq =	0.16296

n	Probability, pn	Cumulative, Pn	n	Probability, pn	Cumulative, Pn
0	0.18519	0.18519	24	0.00136	0.99402
1	0.15089	0.33608	25	0.00111	0.99513
2	0.12295	0.45903	26	0.00090	0.99603
3	0.10018	0.55921	27	0.00073	0.99677
4	0.08163	0.64083	28	0.00060	0.99737
5	0.06651	0.70735	29	0.00049	0.99785
6	0.05420	0.76154	30	0.00040	0.99825
7	0.04416	0.80570	31	0.00032	0.99857
8	0.03598	0.84168	32	0.00026	0.99884
9	0.02932	0.87100	33	0.00022	0.99905
10	0.02389	0.89489	34	0.00018	0.99923
11	0.01946	0.91435	35	0.00014	0.99937
12	0.01586	0.93021	36	0.00012	0.99949
13	0.01292	0.94314	37	0.00009	0.99958
14	0.01053	0.95367	38	0.00008	0.99966
15	0.00858	0.96225	39	0.00006	0.99972

Figure 1: Scenario 1: M/M/1 Queue Analysis

Scenario 1: M/M/1 Queue Analysis (cont'd)

Calculated Metrics:

- Traffic Intensity (ρ): 0.81481
- Average Number of Customers in the System (L_s): 4.40000
- Average Time a Customer Spends in the System (W_s): 0.20000 hours (12 minutes)
- Average Number of Customers in the Queue (L_q): 3.58519
- Average Time a Customer Spends Waiting in the Queue (W_q): 0.16296 hours (approximately 9.78 minutes)
- Probability of Zero Customers in the System (p0): 0.18519

Scenario 1: M/M/1 Queue Analysis (cont'd)

Probability Distribution:

- p_1: 0.15089 (15.089%)
- p_10: 0.02389 (2.389%)
- Cumulative P_10: 0.89492 (89.492%)
- p_20: 0.00604 (0.604%)
- Cumulative P_20: 0.99402 (99.402%)

Scenario 2: M/M/1/N Queue Analysis

Parameters:

- $\lambda = 22$ customers per hour
- $\mu = 27$ customers per hour
- $N = 50$ (queue capacity)

Scenario 2: M/M/1/N Queue Analysis (cont'd)

Scenario 2:(M/M/1):(GD/50/infinity)

Lambda =	22.00000	Mu =	27.00000
L'da eff =	22.00000	Rho/c =	0.81481
Ls =	4.39852	Lq =	3.58371
Ws =	0.19993	Wq =	0.16290

n	Probability, pn	Cumulative, Pn	n	Probability, pn	Cumulative, Pn
0	0.18519	0.18519	24	0.00136	0.99405
1	0.15090	0.33609	25	0.00111	0.99516
2	0.12295	0.45904	26	0.00090	0.99606
3	0.10018	0.55922	27	0.00073	0.99680
4	0.08163	0.64085	28	0.00060	0.99739
5	0.06651	0.70737	29	0.00049	0.99788
6	0.05420	0.76156	30	0.00040	0.99828
7	0.04416	0.80572	31	0.00032	0.99860
8	0.03598	0.84171	32	0.00026	0.99887
9	0.02932	0.87103	33	0.00022	0.99908
10	0.02389	0.89492	34	0.00018	0.99926
11	0.01947	0.91438	35	0.00014	0.99940
12	0.01586	0.93024	36	0.00012	0.99952
13	0.01292	0.94317	37	0.00009	0.99961
14	0.01053	0.95370	38	0.00008	0.99969
15	0.00858	0.96228	39	0.00006	0.99975

Scenario 2: M/M/1/N Queue Analysis (cont'd)

Calculated Metrics:

- Traffic Intensity (ρ): 0.81481
- Average Number of Customers in the System (L_s): 4.39852
- Average Time a Customer Spends in the System (W_s): 0.19993 hours (approximately 12 minutes)
- Average Number of Customers in the Queue (L_q): 3.58371
- Average Time a Customer Spends Waiting in the Queue (W_q): 0.16290 hours (approximately 9.77 minutes)
- Probability of Zero Customers in the System (p0): 0.18519

Scenario 2: M/M/1/N Queue Analysis (cont'd)

Probability Distribution:

- p_1: 0.15090 (15.090%)
- p_10: 0.02389 (2.389%)
- Cumulative P_10: 0.89492 (89.492%)
- p_20: 0.00604 (0.604%)
- Cumulative P_20: 0.99405 (99.405%)

Comparative Analysis

Comparative analysis

Scenario	c	Lambda	Mu	L'da eff	p0	Ls	Lq	Ws	Wq
1	1	22.00000	27.00000	22.00000	0.18519	4.40000	3.58519	0.20000	0.16296
2	1	22.00000	27.00000	22.00000	0.18519	4.39852	3.58371	0.19993	0.16290

Figure 3: Comparative Analysis of Scenarios

Comparative Analysis

Scenario 1:

- Traffic Intensity (ρ): 0.81481
- Average Number of Customers in the System (L_s): 4.40000
- Average Number of Customers in the Queue (L_q): 3.58519
- Average Time a Customer Spends in the System (W_s): 0.20000 hours (12 minutes)
- Average Time a Customer Spends Waiting in the Queue (W_q): 0.16296 hours (approximately 9.78 minutes)
- Probability of Zero Customers in the System (p0): 0.18519

Comparative Analysis

Scenario 2:

- Traffic Intensity (ρ): 0.81481
- Average Number of Customers in the System (L_s): 4.39852
- Average Number of Customers in the Queue (L_q): 3.58371
- Average Time a Customer Spends in the System (W_s): 0.19993 hours (approximately 12 minutes)
- Average Time a Customer Spends Waiting in the Queue (W_q): 0.16290 hours (approximately 9.77 minutes)
- Probability of Zero Customers in the System (p0): 0.18519

Conclusion

- Both the M/M/1 and M/M/1/N queue models demonstrate efficient performance for the given parameters.
- The traffic intensity (ρ) is approximately 0.81481 in both scenarios, indicating that the system is well-utilized but not overloaded.
- The average number of customers in the system (L_s) and in the queue (L_q) are very similar in both scenarios, with only minimal differences.
- The average time a customer spends in the system (W_s) and waiting in the queue (W_q) are also nearly identical, suggesting that queue capacity does not significantly impact these metrics under the given conditions.

Conclusion

- The probability of having zero customers in the system (p0) remains the same across both scenarios, reinforcing the efficiency of the service system.
- This analysis provides valuable insights for optimizing service operations and ensuring customer satisfaction with minimal wait times and system congestion.