Power System Analysis, L4-L5
2018

Lennart Söder
Professor in Electric Power Systems

Superposition theorem

Generally for an admittance network

$$\mathbf{I} = \mathbf{Y}\mathbf{U}$$

Superposition implies that

$$\mathbf{I} = \begin{bmatrix} \overline{I}_1 \\ \overline{I}_2 \\ \vdots \\ \overline{I}_n \end{bmatrix} = \mathbf{Y} \begin{bmatrix} \overline{U}_1 \\ \overline{U}_2 \\ \vdots \\ \overline{U}_n \end{bmatrix} =$$

$$= \mathbf{Y} \begin{bmatrix} \overline{U}_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \mathbf{Y} \begin{bmatrix} 0 \\ \overline{U}_2 \\ \vdots \\ 0 \end{bmatrix} + \ldots + \mathbf{Y} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \overline{U}_n \end{bmatrix}$$

Reciprocity theorem

Assume that a voltage source is connected to a terminal *a* in a linear reciprocal network and causes a current at terminal *b*. According to the reciprocity theorem, the voltage source will cause the same current at *a* if it is connected to *b*. The Y-bus matrix (and by that also the Z-bus matrix) and symmetrical matrices for a reciprocal electric network.

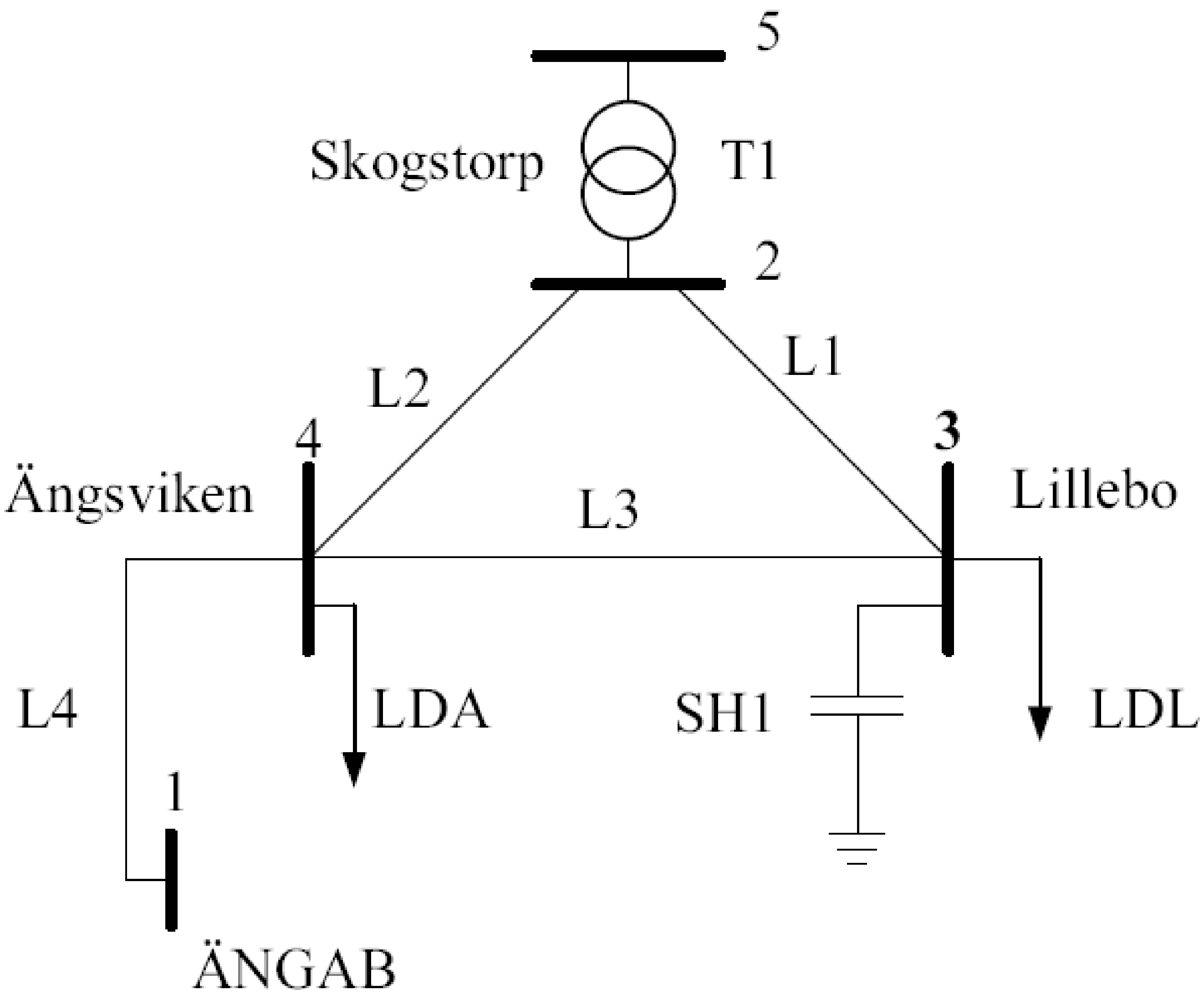
Assume that an electric network with n buses can be described by a symmetric Y-bus matrix, i.e.

$$\begin{bmatrix} \overline{I}_1 \\ \overline{I}_2 \\ \vdots \\ \overline{I}_n \end{bmatrix} = \mathbf{I} = \mathbf{YU} = \begin{bmatrix} \overline{Y}_{11} & \overline{Y}_{12} & \dots & \overline{Y}_{1n} \\ \overline{Y}_{21} & \overline{Y}_{22} & \dots & \overline{Y}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{Y}_{n1} & \overline{Y}_{n2} & \dots & \overline{Y}_{nn} \end{bmatrix} \begin{bmatrix} \overline{U}_1 \\ \overline{U}_2 \\ \vdots \\ \overline{U}_n \end{bmatrix}$$

Thévenin-Helmholtz' theorem - 1

- Assume that a linear electric network has two terminals *a* and *b*. When looking into the system from these two terminals, the rest of the system can be expressed as a voltage source U_T in series with an impedance Z_T. The voltage U_T has the same amplitude as the voltage between the terminals *a* and *b*, whereas the impedance Z_T is the impedance between *a* and *b* assuming that all voltage sources are short-circuited and all current sources are disconnected.

Home exam S2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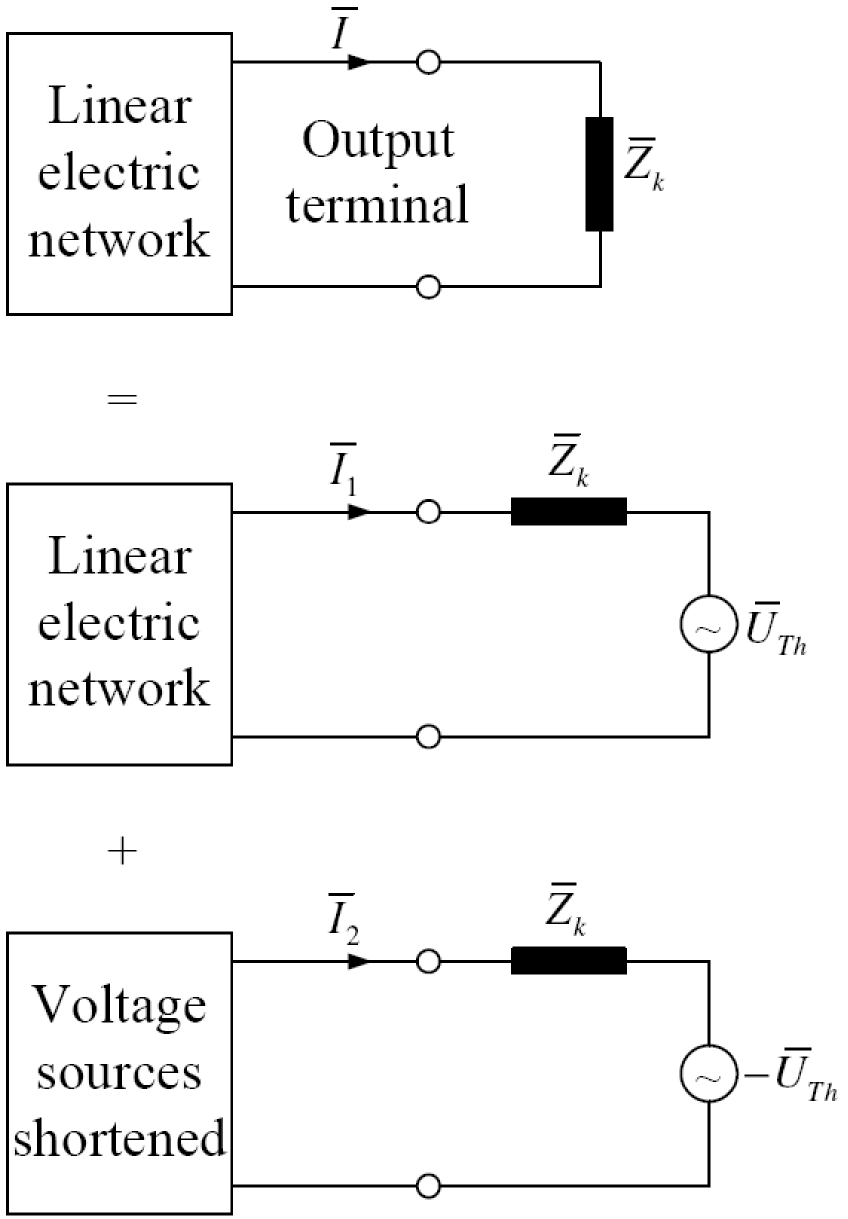
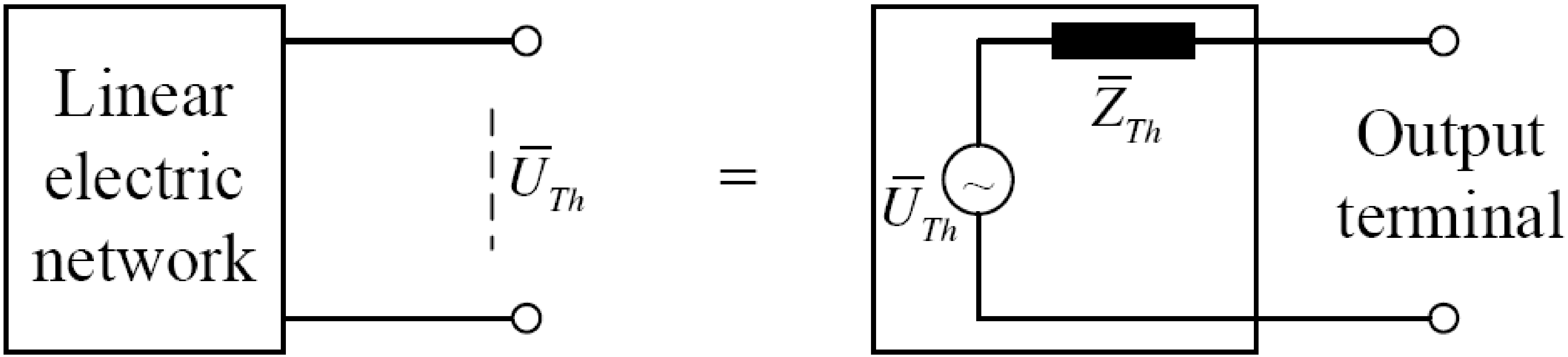
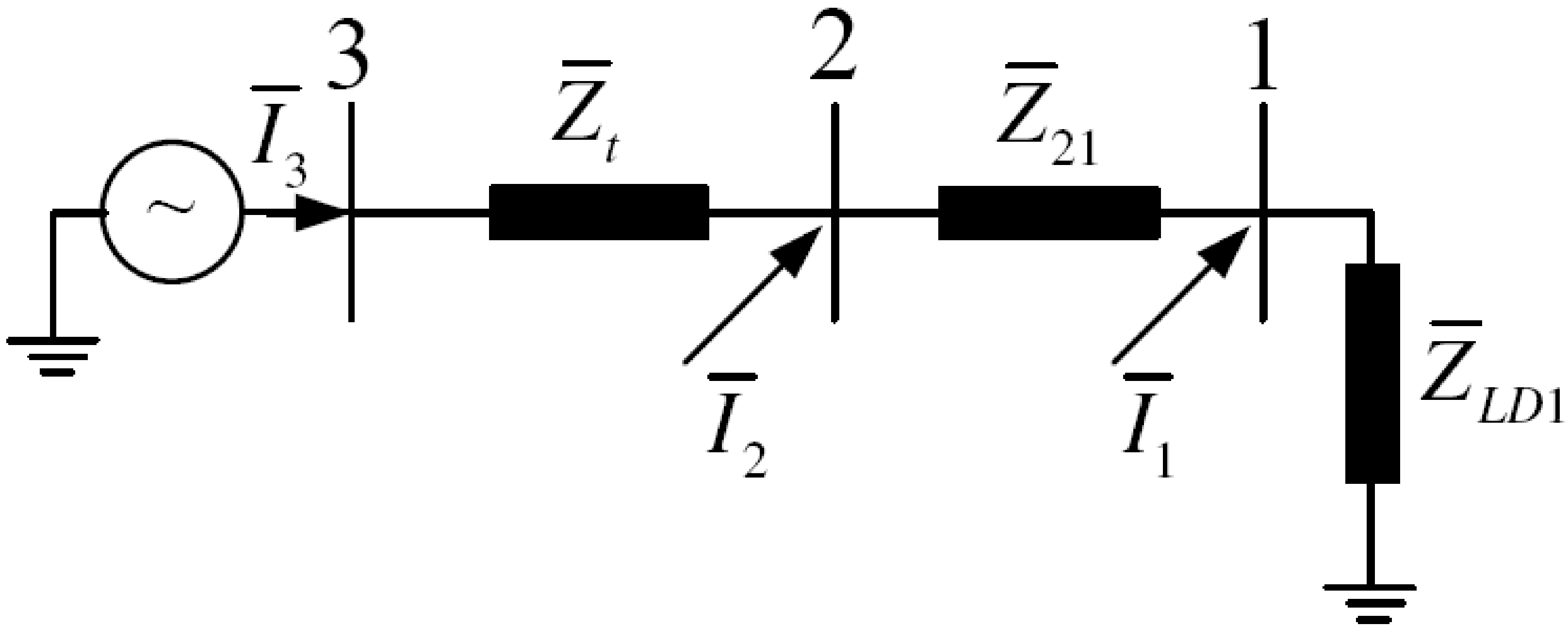
Thévenin-Helmholtz’ theorem - 2



Thévenin-Helmholtz' theorem - 3



General method of calculations of symmetrical three-phase system with one voltage source and impedance loads.



General method… - 2

$$\begin{bmatrix} \overline{I}_1 \\ \overline{I}_2 \\ \overline{I}_3 \end{bmatrix} = \mathbf{I} = \mathbf{YU} = \begin{bmatrix} \frac{1}{\overline{\overline{Z}}_{LD1}} + \frac{1}{\overline{\overline{Z}}_{21}} & -\frac{1}{\overline{\overline{Z}}_{21}} & 0 \\ -\frac{1}{\overline{\overline{Z}}_{21}} & \frac{1}{\overline{\overline{Z}}_{21}} + \frac{1}{\overline{\overline{Z}}_{t}} & -\frac{1}{\overline{\overline{Z}}_{t}} \\ 0 & -\frac{1}{\overline{\overline{Z}}_{t}} & \frac{1}{\overline{\overline{Z}}_{t}} \end{bmatrix} \begin{bmatrix} \overline{U}_1 \\ \overline{U}_2 \\ \overline{U}_3 \end{bmatrix}$$

$$\mathbf{U} = \mathbf{Y}^{-1}\mathbf{I} = \mathbf{ZI}$$

$$\begin{aligned} \overline{U}_3 &= Z(3,3) \cdot \overline{I}_3 \\ &\Rightarrow \overline{I}_3 = \overline{U}_3 / Z(3,3) \end{aligned}$$

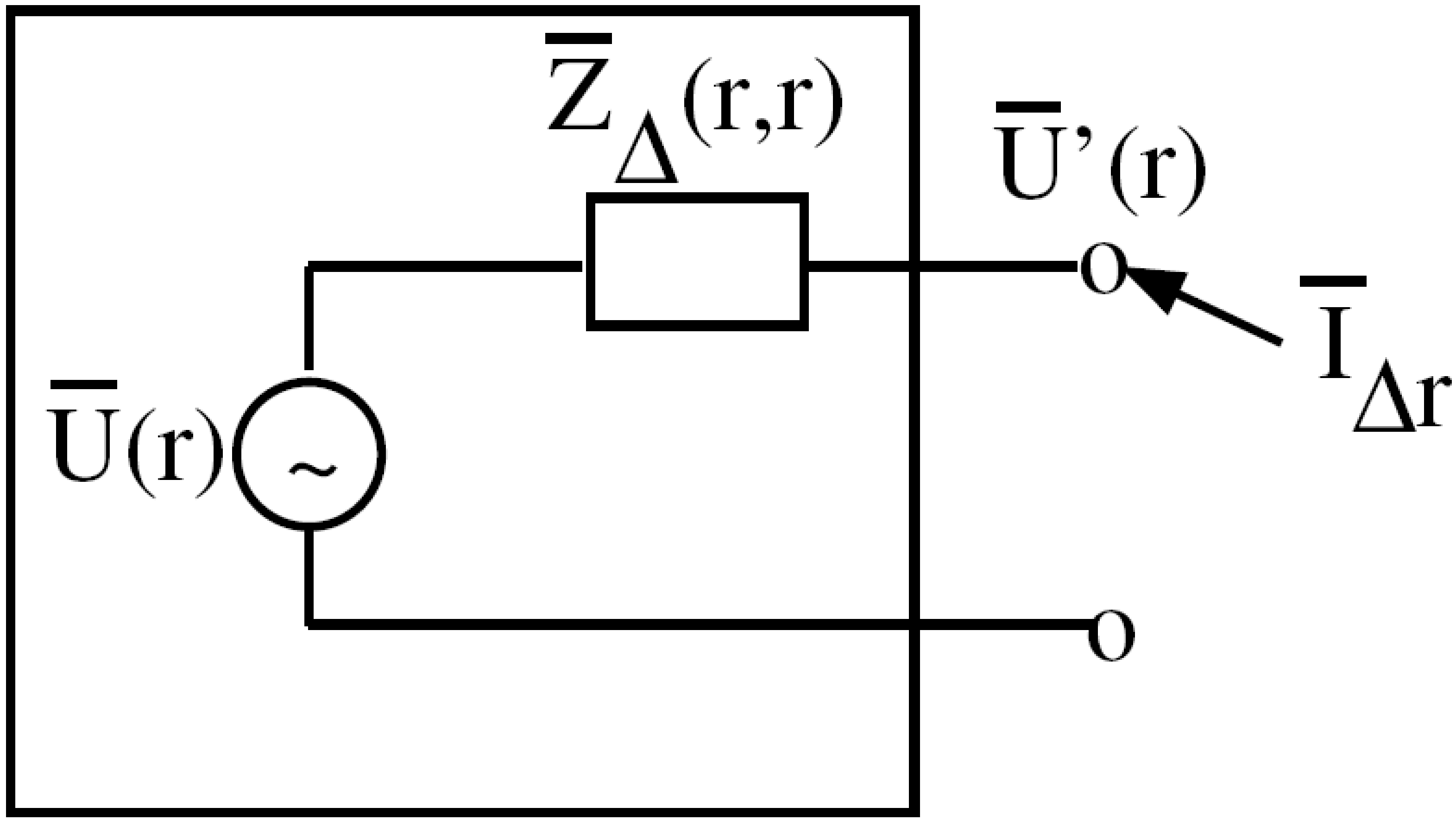
General method… - 3

$$\begin{bmatrix} \overline{I}_{\Delta 1} \\ \overline{I}_{\Delta 2} \end{bmatrix} = \mathbf{I}_{\boldsymbol{\Delta}} = \mathbf{Y}_{\boldsymbol{\Delta}}\mathbf{U}_{\boldsymbol{\Delta}} = \begin{bmatrix} \frac{1}{\overline{\overline{Z}}_{LD1}} + \frac{1}{\overline{\overline{Z}}_{21}} & -\frac{1}{\overline{\overline{Z}}_{21}} \\ -\frac{1}{\overline{\overline{Z}}_{21}} & \frac{1}{\overline{\overline{Z}}_{21}} + \frac{1}{\overline{\overline{Z}}_{t}} \end{bmatrix} \begin{bmatrix} \overline{U}_{\Delta 1} \\ \overline{U}_{\Delta 2} \end{bmatrix}$$

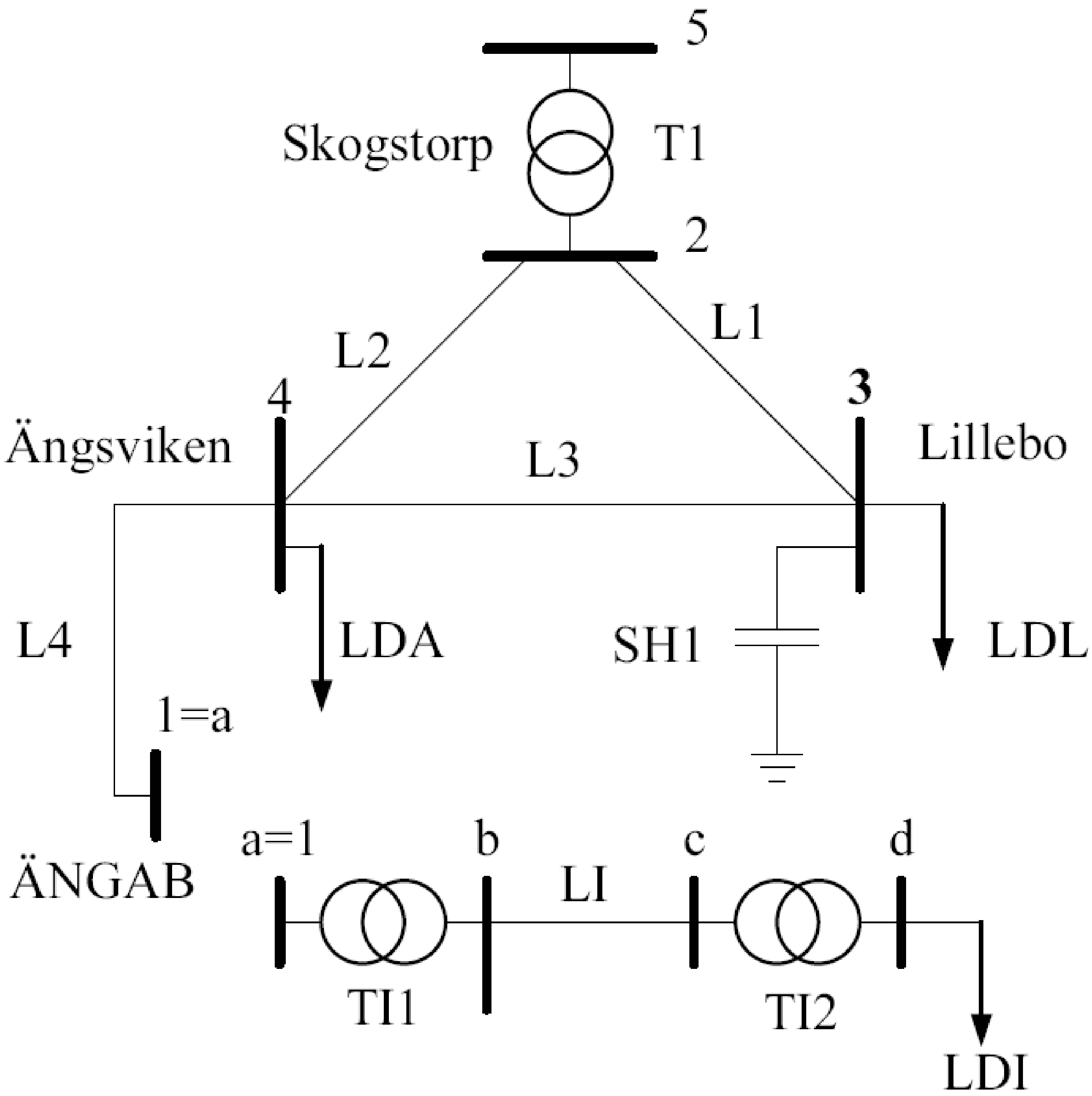
$$\mathbf{U}_{\boldsymbol{\Delta}} = \mathbf{Y}_{\boldsymbol{\Delta}}^{-1}\mathbf{I}_{\boldsymbol{\Delta}} = \mathbf{Z}_{\boldsymbol{\Delta}}\mathbf{I}_{\boldsymbol{\Delta}}$$

$$\overline{U}_{\Delta 2} = Z_{\Delta}(2,2)\overline{I}_{\Delta 2}$$

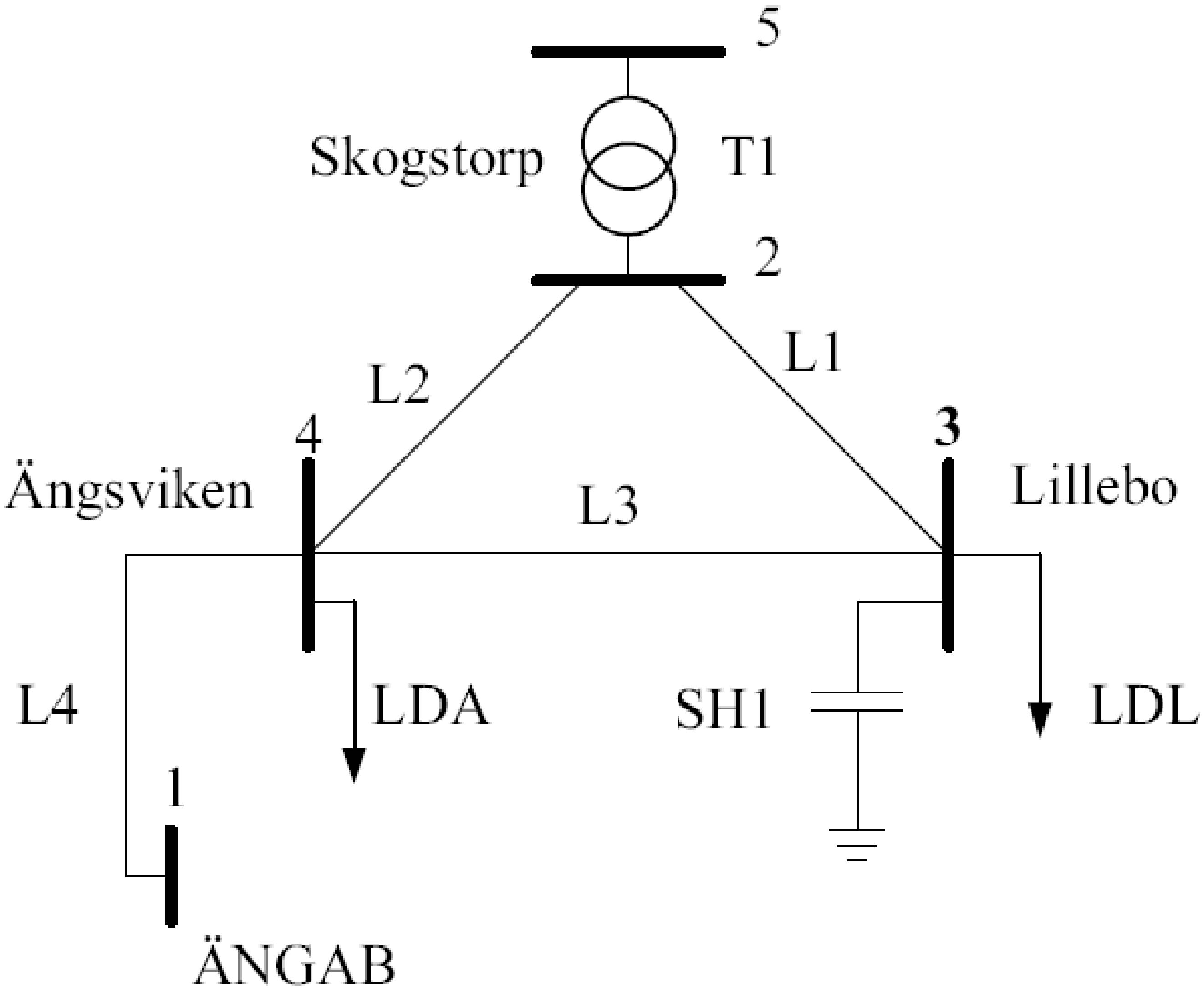
General method… - 3



Home exam S1-S3



Home exam S2



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Assignment S2 (7p)

This assignment is dealing with the electrical network of a meshed grid fed from a 70 kV power system. This meshed grid supplies power to the factory in assignment 1 when they are connected. The node **a** in assignment S1 can be considered as a bus in the meshed system. This bus is given in the excel file.

For tasks a) to h), it is assumed that the factory in assignment S1 is not connected to the meshed grid.

a. Based on the given data, draw the single-line diagram of this meshed system including the Thévenin equivalent of the power system. **(10%)**

b. Build the Y-bus matrix of the system and give the numerical values of the elements of the Y-bus matrix in pu. **(15%)**

c. Find the injected power into the meshed grid from the secondary side of the transformer T1 in MVA, and compare the result with the rating of the transformer. **(10%)**

Assignment S2 (7p)

d. Find the total losses in the mesh grid in MW. **(10%)**

e. Based on the diagram, as seen from the given bus (indicated in your excel file) find the Thévenin equivalent of the system and give the value of Thévenin voltage and impedance in pu. **(10%)**

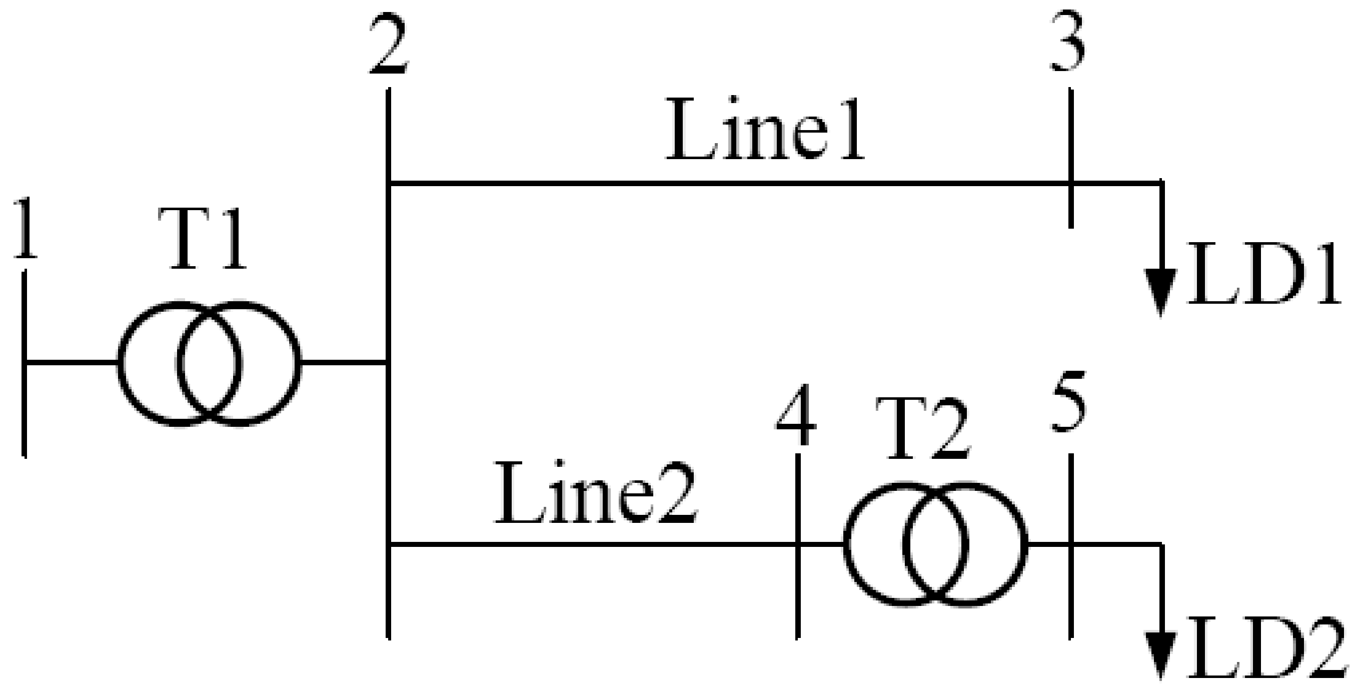
f. Assumed that a solid three-phase short circuit is applied to the given bus in task e. Find the short circuit current in kA and the three-phase short circuit power in MVA. **(10%)**

g. Find the injected power into the meshed grid in MVA during the three-phase short circuit at the given bus and compare the result with the rating of the transformer. **(15%)**

h. Find the total losses in the mesh grid in MW during the three-phase short circuit at the given bus. **(10%)**

i. Let the system in assignment S1 be connected to the given bus in task e, i.e. node **a** in assignment S1 is the given bus in task e. Using the Thévenin equivalent in task e, find the voltage at node **e**, in kV, when the load at this node is reduced to 50%. **(10%)**

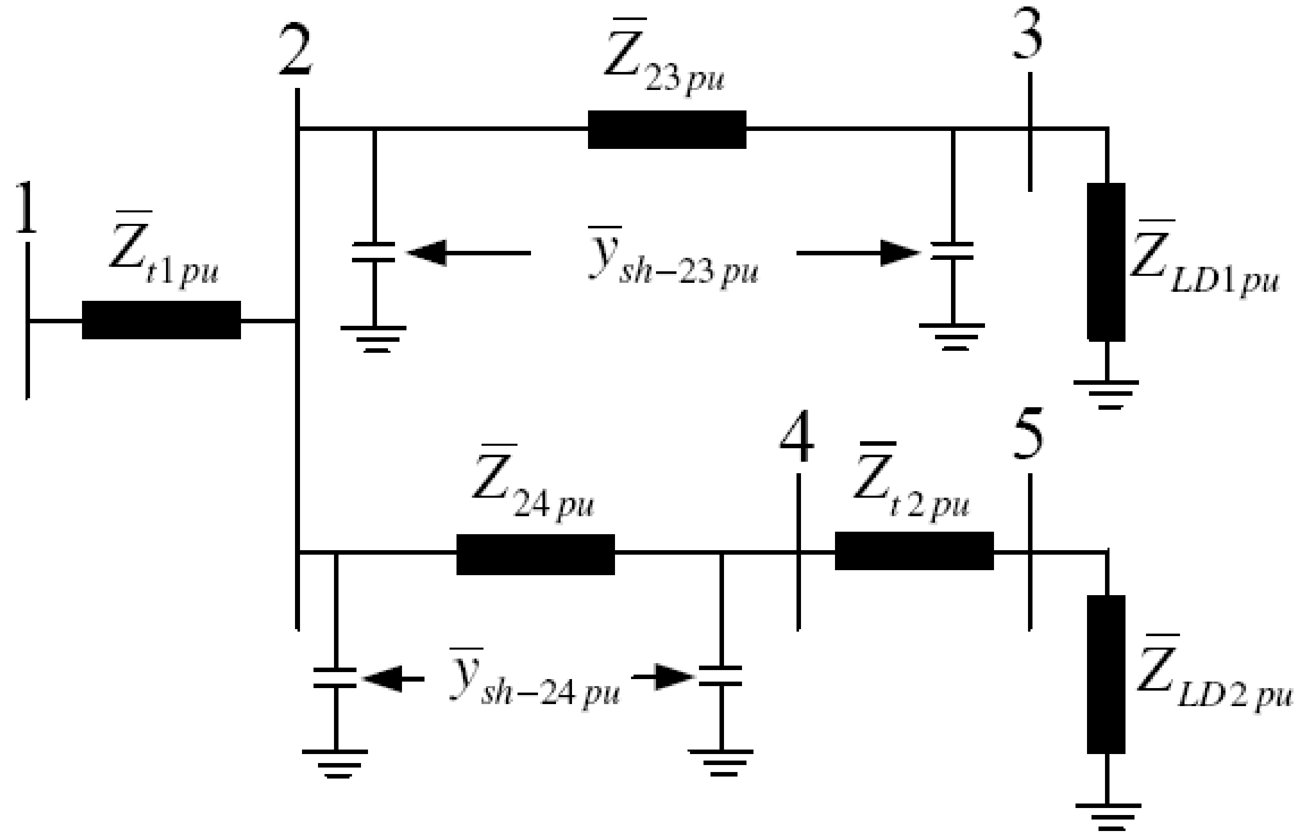
Example 6.3 In the figure, an internal network of an industry is given. The energy is delivered by an infinite bus with a nominal
voltage at bus **1**. The energy is transmitted via transformer *T1*, line *L2* and transformer *T2* to the load *LD2*.



- Transformer T_1 : 800 kVA, 70/10 kV, x = 7 %
- Transformer T_2 : 300 kVA, 0.4/10 kV, x = 8 %
- Line L_1 : r = 0.17 Ω/km, wL = 0.3 Ω /km, wC = 3.2×10^{-6} S/km, l = 2 km
- Line L_2 : r = 0:17 Ω /km, wL = 0.3 Ω /km, wC = 3.2×10^{-6} S/km, l = 1 km
- Load LD1 : Impedance characteristic, 500 kW, cosϕ = 0.80, inductive at 10 kV
- Load LD2 : Impedance characteristic , 200 kW, cosϕ = 0.95, inductive at 400 V

Example 6.3

Assume that lines can be modeled by using the Π-equivalent. Calculate the efficiency of the internal network as well as the short circuit current that is obtained at a solid three-phase short circuit at bus 4.



Example 6.3

First change all data to p.u.!

$$
\mathbf{Y} = \begin{bmatrix}
\frac{1}{\overline{Z}_{t1pu}} & -\frac{1}{\overline{Z}_{t1pu}} & 0 & 0 & 0 \\
-\frac{1}{\overline{Z}_{t1pu}} & \overline{Y}_{22} & -\frac{1}{\overline{Z}_{23pu}} & -\frac{1}{\overline{Z}_{24pu}} & 0 \\
0 & -\frac{1}{\overline{Z}_{23pu}} & \overline{Y}_{33} & 0 & 0 \\
0 & -\frac{1}{\overline{Z}_{24pu}} & 0 & \overline{Y}_{44} & -\frac{1}{\overline{Z}_{t2pu}} \\
0 & 0 & 0 & -\frac{1}{\overline{Z}_{t2pu}} & \frac{1}{\overline{Z}_{t2pu}} + \frac{1}{\overline{Z}_{LD2pu}}
\end{bmatrix}
$$

```
%Y-BUS
Y22=1/Zt1pu+1/Z23pu+ysh23pu+1/Z24pu+ysh24pu;
Y33=1/Z23pu+ysh23pu+1/ZLD1pu;
Y44=1/Z24pu+ysh24pu+1/Zt2pu;

Ybus=[ 1/Zt1pu    -1/Zt1pu       0          0           0;
       -1/Zt1pu      Y22      -1/Z23pu   -1/Z24pu        0;
          0        -1/Z23pu     Y33         0            0;
          0        -1/Z24pu      0         Y44       -1/Zt2pu;
          0           0          0       -1/Zt2pu  1/Zt2pu+1/ZLD2pu];
Zbus=inv(Ybus);
```

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c. Find the injected power into the meshed grid from the secondary side of the transformer T1 in MVA, and compare the result with the rating of the transformer. **(10%)**

Assignment S2 (7p)

d. Find the total losses in the mesh grid in MW. **(10%)**

e. Based on the diagram, as seen from the given bus (indicated in your excel file) find the Thévenin equivalent of the system and give the value of Thévenin voltage and impedance in pu. **(10%)**

f. Assumed that a solid three-phase short circuit is applied to the given bus in task e. Find the short circuit current in kA and the three-phase short circuit power in MVA. **(10%)**

g. Find the injected power into the meshed grid in MVA during the three-phase short circuit at the given bus and compare the result with the rating of the transformer. **(15%)**

h. Find the total losses in the mesh grid in MW during the three-phase short circuit at the given bus. **(10%)**

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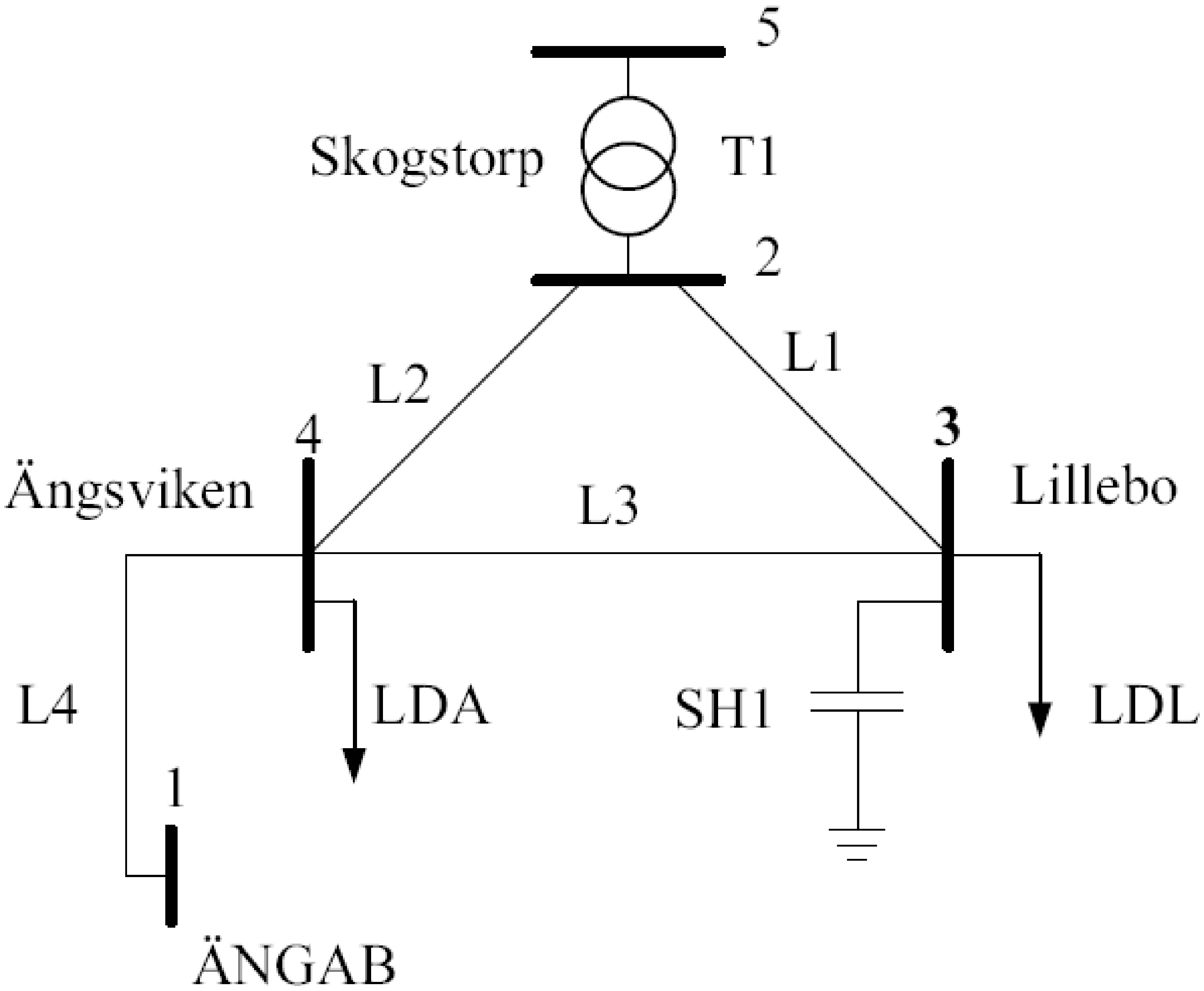
Home exam S2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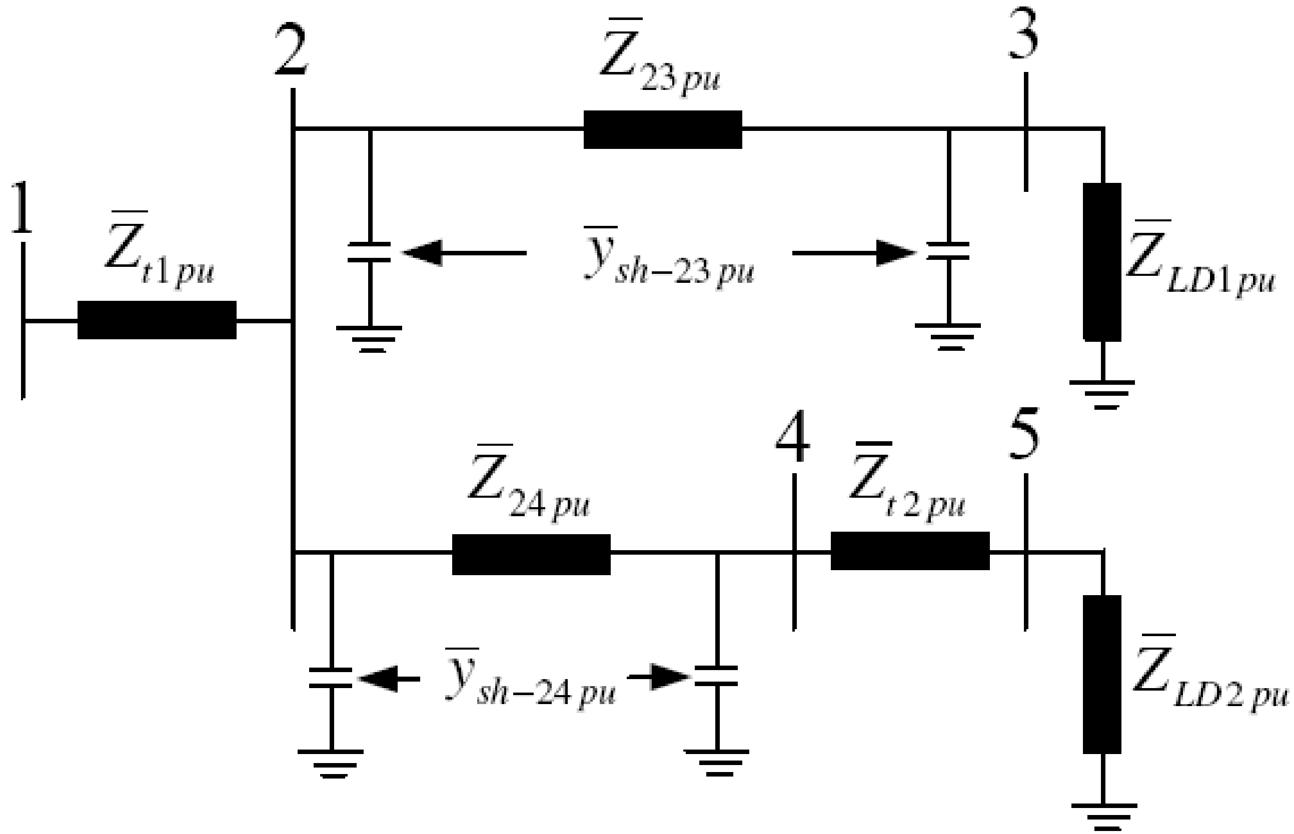

```
%Calculate the efficiency
I1=U1/Zbus(1,1);
U2=Zbus(2,1)*I1;
U3=Zbus(3,1)*I1;
U4=Zbus(4,1)*I1;
U5=Zbus(5,1)*I1;

S1=U1*conj(I1)*Sb;
IZ23=(U2-U3)/Z23pu;
IZ24=(U2-U4)/Z24pu;
PfLine1=real(Z23pu)*abs(IZ23)^2*Sb;
PfLine2=real(Z24pu)*abs(IZ24)^2*Sb;

eta=(real(S1)-PfLine1-PfLine2)/real(S1);
```

Example 6.3

$$\mathbf{Z} = \begin{bmatrix} 0.510+j0.375 & 0.510+j0.331 & 0.508+j0.329 & 0.510+j0.331 & 0.516+j0.298 \\ 0.510+j0.331 & 0.510+j0.331 & 0.508+j0.329 & 0.510+j0.331 & 0.516+j0.298 \\ 0.508+j0.329 & 0.508+j0.329 & 0.509+j0.330 & 0.508+j0.329 & 0.515+j0.296 \\ 0.510+j0.331 & 0.510+j0.331 & 0.508+j0.329 & 0.510+j0.332 & 0.517+j0.299 \\ 0.516+j0.298 & 0.516+j0.298 & 0.515+j0.296 & 0.517+j0.299 & 0.529+j0.397 \end{bmatrix}$$

$$\mathbf{Z}_{\boldsymbol{\Delta}} = \begin{bmatrix} 0.0024+j0.0420 & 0.0025+j0.0418 & 0.0025+j0.0419 & 0.0046+j0.0410 \\ 0.0025+j0.0418 & 0.0043+j0.0446 & 0.0025+j0.0418 & 0.0046+j0.0408 \\ 0.0025+j0.0419 & 0.0025+j0.0418 & 0.0033+j0.0434 & 0.0055+j0.0424 \\ 0.0046+j0.0410 & 0.0046+j0.0408 & 0.0055+j0.0424 & 0.0144+j0.1719 \end{bmatrix}$$