Load Flow Calculations (LFC)

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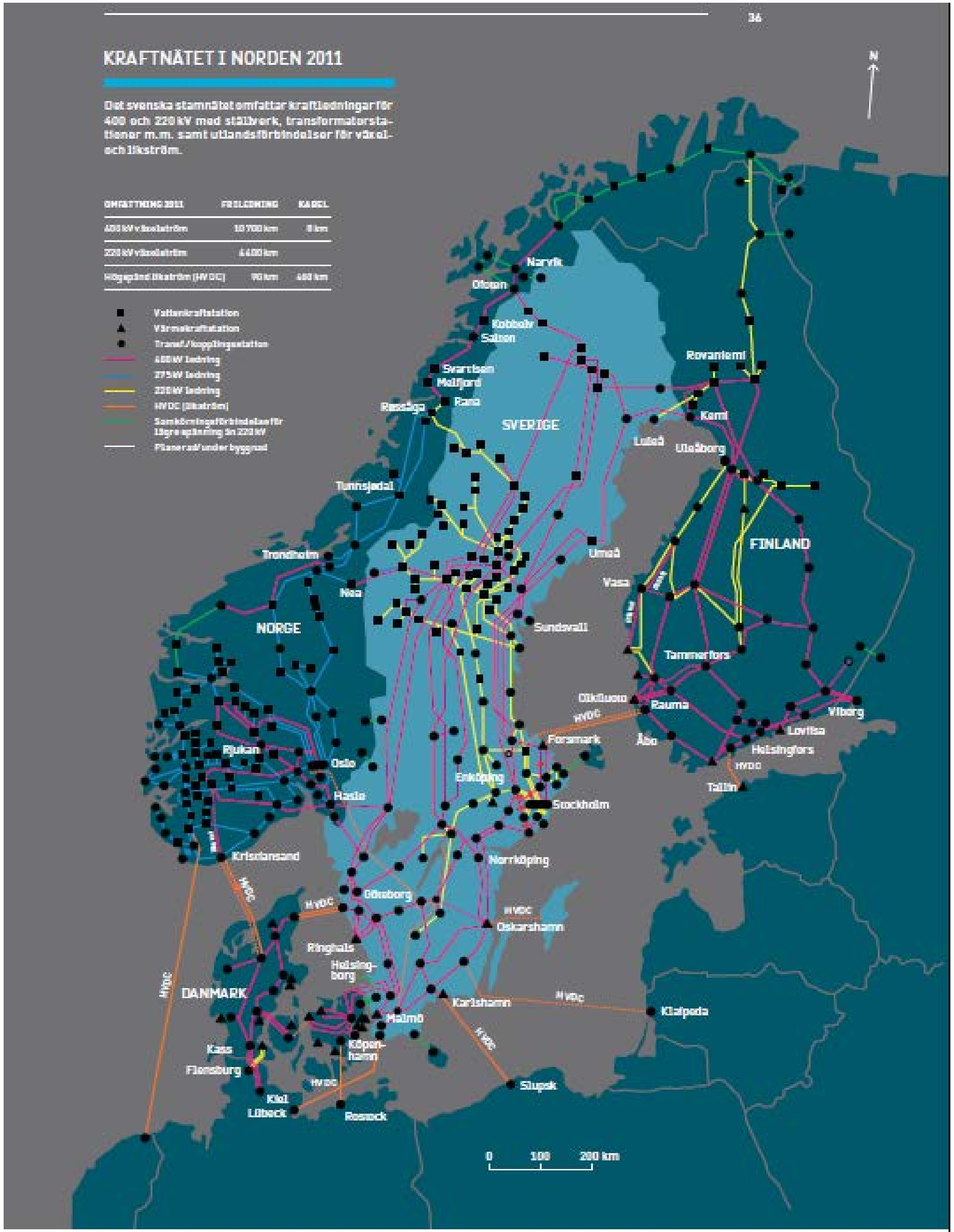
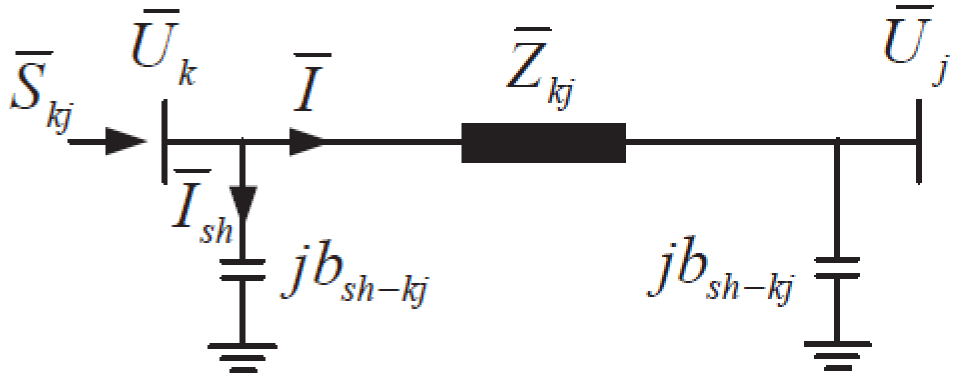

KRAFTNÄTET I NORDEN 2011
Det svenska stamnätet omfattar kraftledningar för 400 och 220 kV med ställverk, transformatorstationer m.m. samt utlandsförbindelser för växel- och likström.
OMFATTNING 2011 FRILEDNING KABEL
400 kV växelström 10 700 km 8 km
220 kV växelström 4 400 km
Högspänd likström (HVDC) 90 km 460 km
Vattenkraftstation
Värmekraftstation
Transf./kopplingsstation
400 kV ledning
275 kV ledning
220 kV ledning
HVDC (likström)
Samkörningsförbindelser för lägre spänning än 220 kV
Planerad/under byggnad
N
SVERIGE
NORGE
FINLAND
DANMARK
Narvik
Ofoten
Kobbelv
Salten
Svartisen
Melfjord
Rana
Røssåga
Tunnsjødal
Trondheim
Nea
Rovaniemi
Kemi
Luleå
Uleåborg
Umeå
Vasa
Sundsvall
Tammerfors
Olkiluoto
Rauma
Viborg
Lovisa
Helsingfors
Åbo
Forsmark
Tallin
Rjukan
Oslo
Hasle
Enköping
Stockholm
Kristiansand
Norrköping
Göteborg
Oskarshamn
Ringhals
Helsingborg
Karlshamn
Klaipeda
Malmö
Köpenhamn
Kass
Flensburg
Kiel
Lübeck
Rostock
Slupsk
HVDC
0 100 200 km

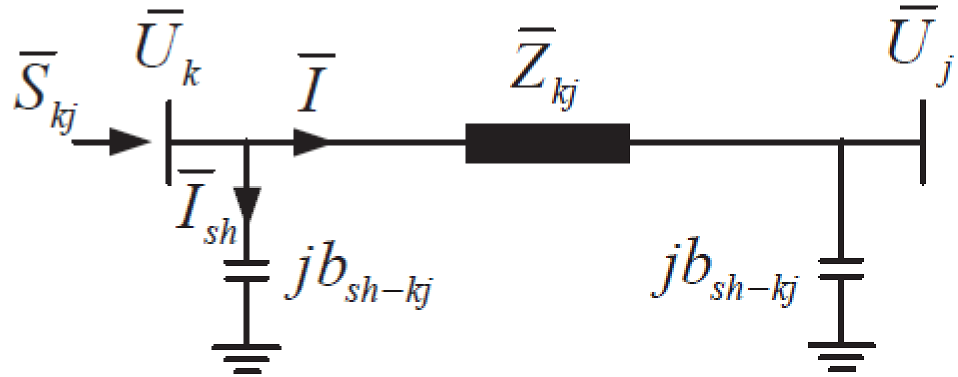
Power flow in a line



$$\overline{U}_k = U_k e^{j\theta_k} \quad , \quad \overline{U}_j = U_j e^{j\theta_j}$$

$$\overline{Z}_{kj} = R_{kj} + jX_{kj} \quad , \quad Z_{kj} = \sqrt{R_{kj}^2 + X_{kj}^2}$$

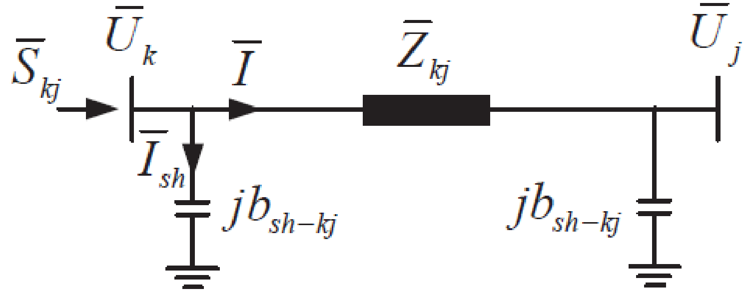
$$\theta_{kj} = \theta_k - \theta_j$$



$$\overline{S}_{kj} = \overline{U}_k \left(\overline{I}_{sh}^* + \overline{I}^*\right) = \overline{U}_k \left(\left(j\, b_{sh-kj}\, \overline{U}_k\right)^* + \frac{\overline{U}_k^* - \overline{U}_j^*}{\overline{Z}_{kj}^*} \right) =$$

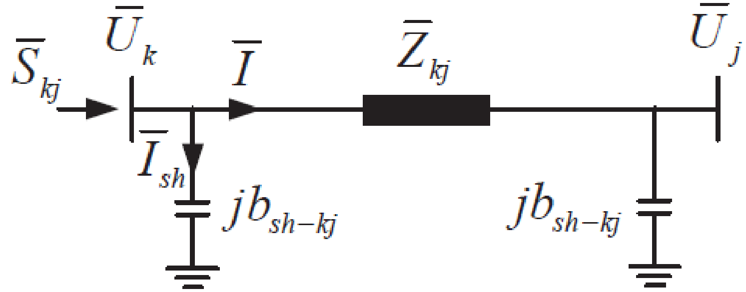
$$= -j\, b_{sh-kj}\, U_k^2 + \frac{U_k^2}{R_{kj} - jX_{kj}} - \frac{U_k U_j}{R_{kj} - jX_{kj}} e^{j(\theta_k - \theta_j)} =$$

$$= -j\, b_{sh-kj}\, U_k^2 + \frac{U_k^2}{Z_{kj}^2}(R_{kj} + jX_{kj}) - \frac{U_k U_j}{Z_{kj}^2}(R_{kj} + jX_{kj})\left(\cos\theta_{kj} + j\sin\theta_{kj}\right)$$



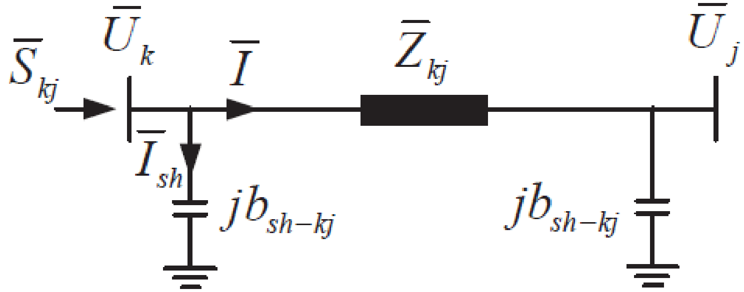
$$\overline{S}_{kj} = \overline{U}_k \left(\overline{I}_{sh}^* + \overline{I}^* \right) = P_{kj} + j\, Q_{kj}$$

$$\begin{aligned} P_{kj} &= \frac{R_{kj}}{Z_{kj}^2} U_k^2 + \frac{U_k\, U_j}{Z_{kj}^2} \left(X_{kj} \sin\theta_{kj} - R_{kj} \cos\theta_{kj} \right) \\ &= \frac{R_{kj}}{Z_{kj}^2} U_k^2 + \frac{U_k\, U_j}{Z_{kj}} \sin\left(\theta_{kj} - \arctan\left(\frac{R_{kj}}{X_{kj}} \right) \right) \end{aligned}$$



$$\overline{S}_{kj} = \overline{U}_k \left(\overline{I}_{sh}^{*} + \overline{I}^{*} \right) = P_{kj} + j\,Q_{kj}$$

$$\begin{aligned} Q_{kj} &= -b_{sh-kj}\,U_k^2 + \frac{X_{kj}}{Z_{kj}^2}\,U_k^2 - \frac{U_k U_j}{Z_{kj}^2}\left(R \sin\theta_{kj} + X_{kj}\cos\theta_{kj}\right) \\ &= \left(-b_{sh-kj} + \frac{X_{kj}}{Z_{kj}^2}\right) U_k^2 - \frac{U_k\,U_j}{Z_{kj}}\,\cos\left(\theta_{kj} - \arctan\left(\frac{R_{kj}}{X_{kj}}\right)\right) \end{aligned}$$



$$\overline{U}_1 = 225\angle 0^\circ \ kV$$

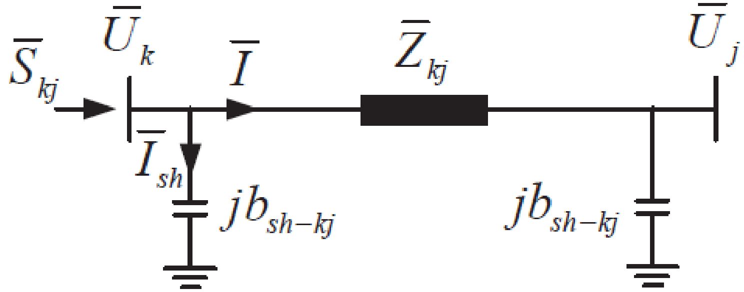
$$\overline{U}_2 = 213.08\angle -3.572^\circ \ kV.$$

The line has a length of 100 km

$$x = 0.4 \ \Omega/km, \ r = 0.04 \ \Omega/km$$

$$b_c = 3 \times 10^{-6} \ S/km.$$

$$P_{12} = \textbf{??} \qquad Q_{12} = \textbf{??}$$



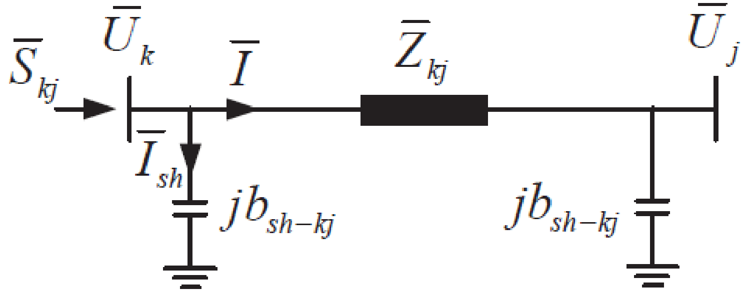
$$S_{base} = 100 \text{ MVA and } U_{base} = 225 \text{ kV}$$

$$Z_{base} = U_{base}^2 / S_{base} = 506.25\ \Omega$$

$$U_1 = 225/U_{base} = 1.0 \text{ pu}$$

$$U_2 = 213.08/U_{base} = 0.9470 \text{ pu}$$

$$\theta_{12} = 0\text{-}(\text{-}3.572) = 3.572°$$

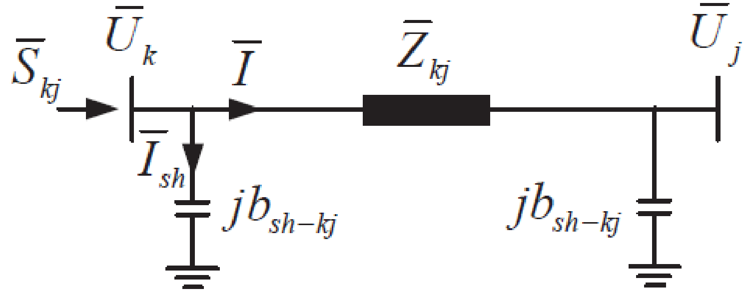


$$R_{12} = 0.04 \cdot 100/Z_{base} = 0.0079 \text{ pu}$$

$$X_{12} = 0.4 \cdot 100/Z_{base} = 0.0790 \text{ pu}$$

$$b_{sh-12} = 3 \times 10^{-6} \cdot 100 \cdot Z_{base}/2$$
$$= 0.0759 \text{ pu}$$

$$Z_{12} = \sqrt{R_{12}^2 + X_{12}^2} = 0.0794 \text{ pu}$$



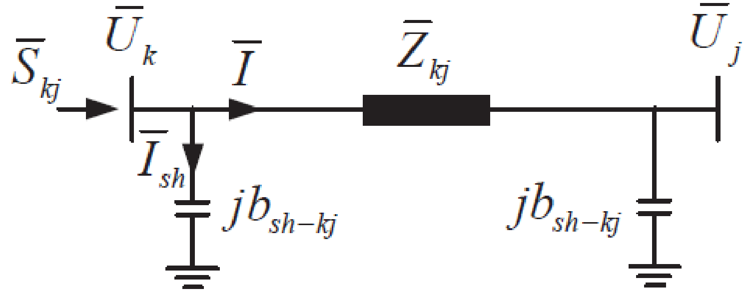
$$P_{kj} = \frac{R_{kj}}{Z_{kj}^2} U_k^2 + \frac{U_k U_j}{Z_{kj}} \sin\left(\theta_{kj} - \arctan\left(\frac{R_{kj}}{X_{kj}}\right)\right)$$

$$= \quad 0.8081 \text{ pu}$$

$$Q_{kj} =$$

$$\left(-b_{sh-kj} + \frac{X_{kj}}{Z_{kj}^2}\right) U_k^2 - \frac{U_k U_j}{Z_{kj}} \cos\left(\theta_{kj} - \arctan\left(\frac{R_{kj}}{X_{kj}}\right)\right)$$

$$= \quad 0.5373 \text{ pu}$$



expressed in nominal values

$$
\begin{aligned}
P_{12} &= 0.8081 \cdot S_{base} = 80.81 \text{ MW} \\
Q_{12} &= 0.5373 \cdot S_{base} = 53.73 \text{ MVAr}
\end{aligned}
$$

For a high voltage overhead line (U > 70 kV)

$$R_{kj} \ll X_{kj} \qquad \text{(i.e. } R_{kj} \approx 0\text{)}$$

$$P_{kj} = \frac{R_{kj}}{Z_{kj}^2} U_k^2 + \frac{U_k U_j}{Z_{kj}} \sin\left(\theta_{kj} - \arctan\left(\frac{R_{kj}}{X_{kj}}\right)\right)$$

$$P_{kj} \approx \frac{U_k U_j}{X_{kj}} \sin\theta_{kj}$$

the sign of θ_{kj} determines the direction of

the active power flow on the line

$$R_{kj} \approx 0 \qquad \cos\theta_{kj} \approx 1$$ Lightly loaded

$$Q_{kj} =$$

$$= \left(-b_{sh-kj} + \frac{X_{kj}}{Z_{kj}^2}\right) U_k^2 - \frac{U_k U_j}{Z_{kj}} \cos\left(\theta_{kj} - \arctan\left(\frac{R_{kj}}{X_{kj}}\right)\right)$$

$$Q_{kj} \approx -b_{sh-kj} U_k^2 + \frac{U_k(U_k - U_j)}{X_{kj}}$$

Small voltage difference implies reactive power generation by the line.

$$P_{12} \approx \frac{1.0 \cdot 0.9470}{0.0790} \sin 3.572^\circ = 0.7468 \text{ pu} \Rightarrow 74.68 \text{ MW}$$

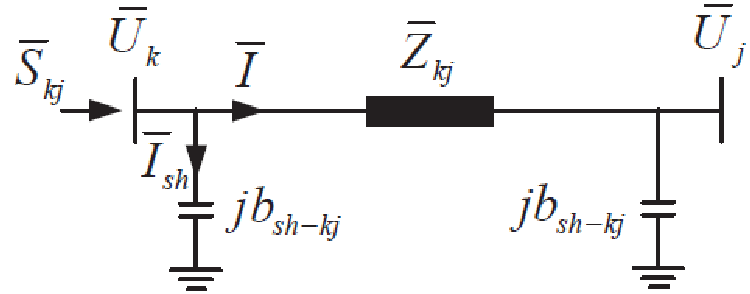
$$Q_{12} \approx -0.0759 \cdot 1.0^2 + \frac{1.0(1.0 - 0.9470)}{0.0790} = 0.5948 \text{ pu} \Rightarrow 59.48 \text{ MVAr}$$

have correct direction of the power flow

P_{12} 8 % too low

Q_{12} 11 % too large

Line losses



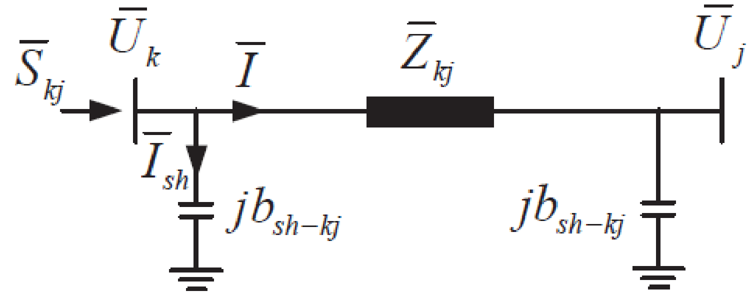
$$P_f = 3R_{kj} I^2$$

$$I^2 = Ie^{j\gamma} Ie^{-j\gamma} = \overline{I}\,\overline{I}^* = \frac{\overline{S}^*}{\sqrt{3}\overline{U}^*} \frac{\overline{S}}{\sqrt{3}\overline{U}} =$$

$$= \frac{S^2}{3U^2} = \frac{P^2 + Q^2}{3U^2}$$

$$\boxed{P_f = R_{kj} \frac{P_{kj}^2 + (Q_{kj} + b_{sh-kj} U_k^2)^2}{U_k^2}}$$

Line losses

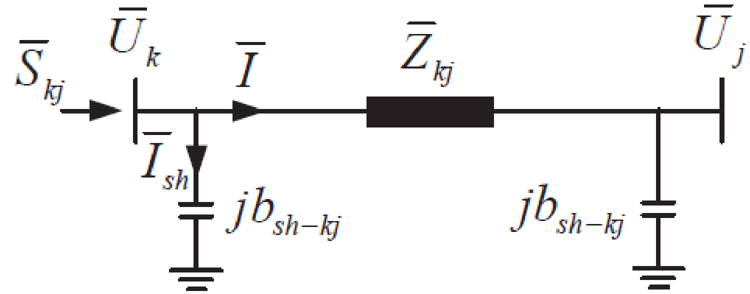


$$\begin{aligned} P_f(\mathrm{MW}) &= R_{12}\frac{P_{12}^2 + (Q_{12} + b_{sh-12}U_1^2)^2}{U_1^2}S_{base} = \\ &= 0.0079\frac{0.8081^2 + (0.5373 + 0.0759 \cdot 1.0^2)^2}{1.0^2}100 = 0.81 \text{ MW} \end{aligned}$$

$$\begin{aligned} P_f(\mathrm{MW}) &= R_{12}\frac{P_{21}^2 + (Q_{21} + b_{sh-12}U_2^2)^2}{U_2^2}S_{base} = \\ &= 0.0079\frac{(-0.80)^2 + (-0.60 + 0.0759 \cdot 0.9470^2)^2}{0.9470^2}100 = 0.81 \text{ MW} \end{aligned}$$

$$P_f(\mathrm{MW}) = [P_{12} + P_{21}]S_{base} = [0.8081 + (-0.80)]100 = 0.81 \text{ MW}$$

Line losses

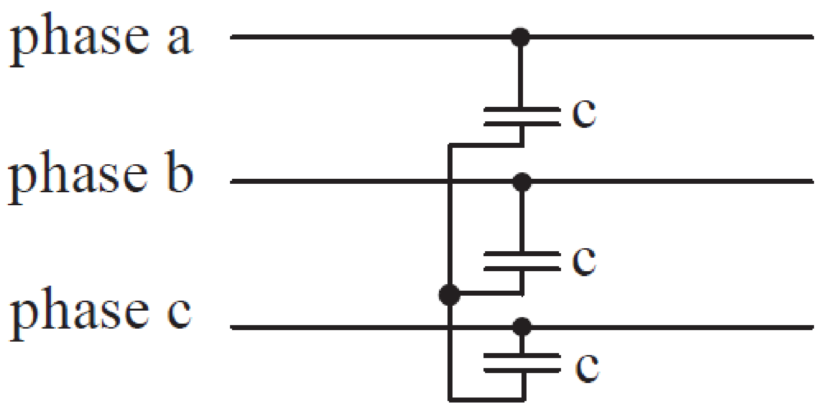


$$P_f = R_{kj}\frac{P_{kj}^2 + (Q_{kj} + b_{sh-kj}\,U_k^2)^2}{U_k^2}$$

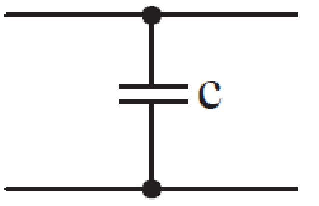
the losses

will increase if reactive power is transmitted over the line.

Shunt capacitors and shunt reactors



Three-phase connection



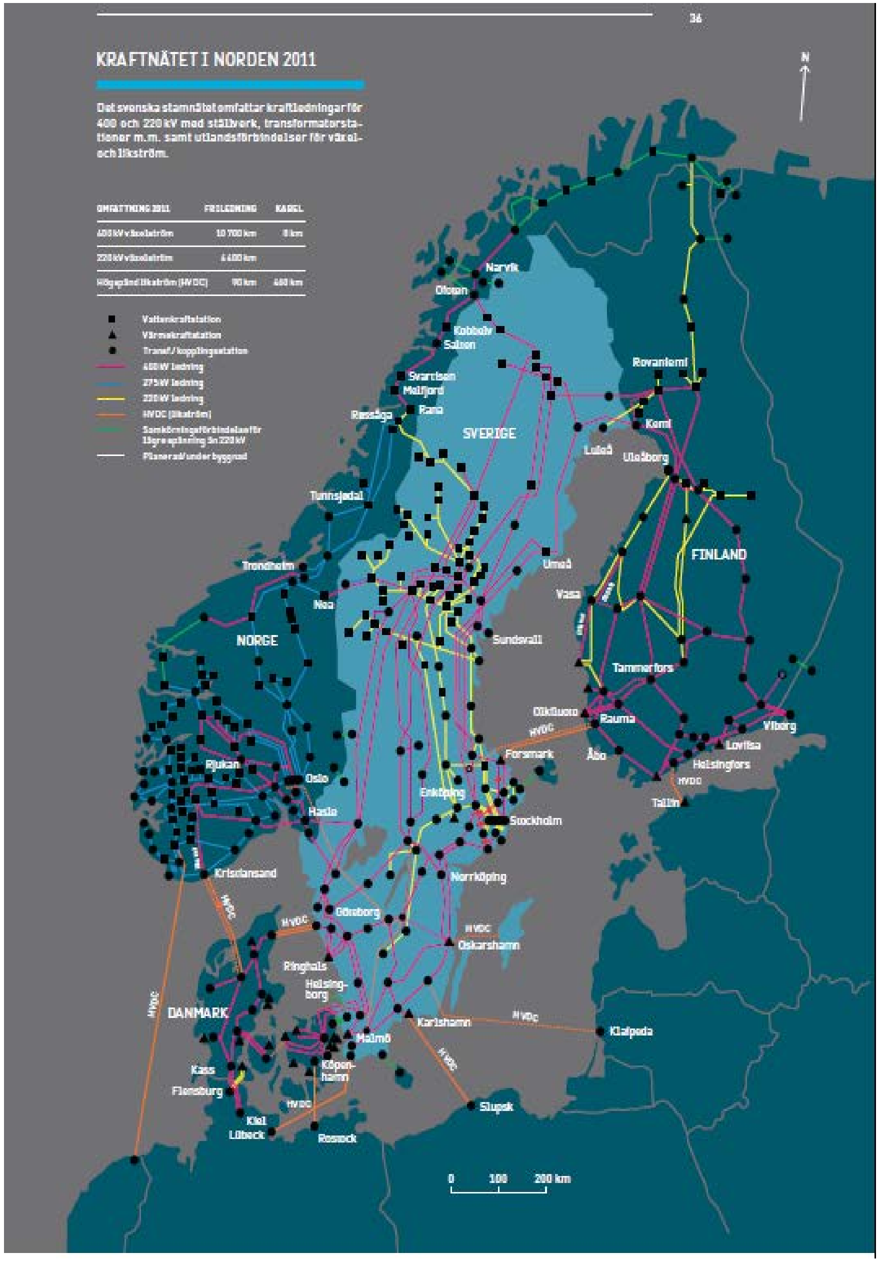
Single-phase equivalent

Injected reactive power

$$Q_{sh} = B_{sh} U^2 = 2\pi f c\, U^2$$

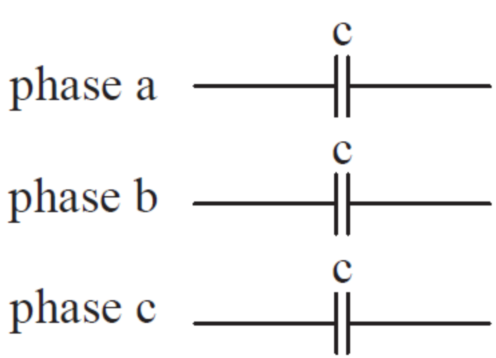
$$Q_{kj} \approx$$

$$-b_{sh-kj}\, U_k^2 + \frac{U_k(U_k - U_j)}{X_{kj}}$$

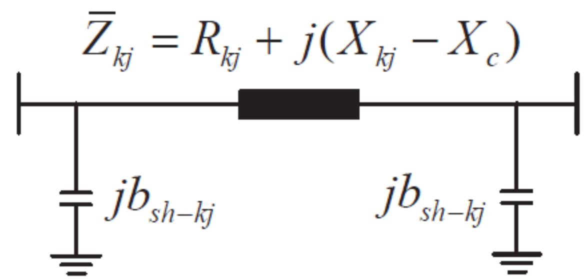


Series capacitors

$$P_{kj-max} \approx \max_{\theta_{kj}} \frac{U_k U_j}{X_{kj}} \sin\theta_{kj} = \frac{U_k U_j}{X_{kj}}$$



Three-phase connection



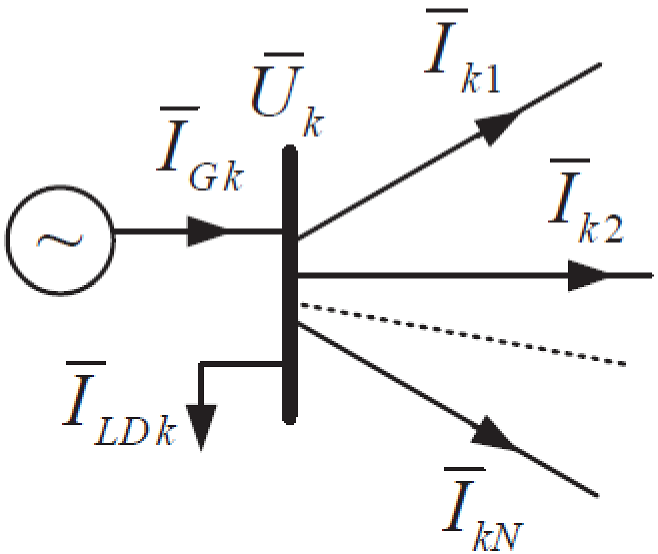
Single-phase equivalent

$$\boxed{P_{kj-max} \approx \frac{U_k U_j}{X_{kj} - X_c}}$$

reduce the voltage drop along the line

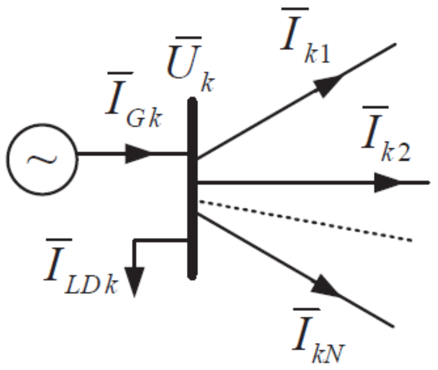
Non-linear power flow equations

a balanced power system with N buses.



$$\boxed{\overline{I}_{Gk} - \overline{I}_{LDk} = \sum_{j=1}^{N} \overline{I}_{kj}}$$

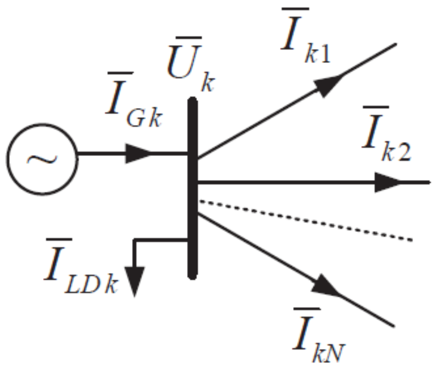
$$\boxed{\overline{U}_k \overline{I}^*_{Gk} - \overline{U}_k \overline{I}^*_{LDk} = \sum_{j=1}^{N} \overline{U}_k \overline{I}^*_{kj}}$$



$$\boxed{\overline{I}_{Gk} - \overline{I}_{LDk} = \sum_{j=1}^{N} \overline{I}_{kj}}$$

$$\boxed{\overline{U}_k \overline{I}^*_{Gk} - \overline{U}_k \overline{I}^*_{LDk} = \sum_{j=1}^{N} \overline{U}_k \overline{I}^*_{kj}}$$

$$\overline{S}_{Gk} - \overline{S}_{LDk} = \sum_{j=1}^{N} \overline{S}_{kj}$$



$$\overline{S}_{Gk} - \overline{S}_{LDk} = \sum_{j=1}^{N} \overline{S}_{kj}$$

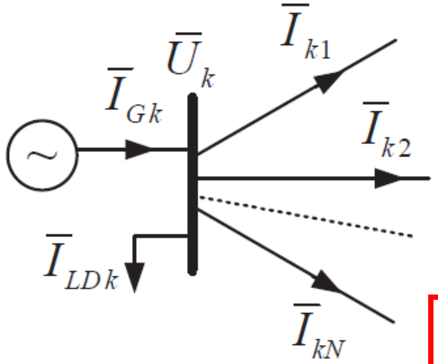
$\overline{S}_{Gk} = P_{Gk} + jQ_{Gk}$ is the generated complex at bus k, complex power

$\overline{S}_{LDk} = P_{LDk} + jQ_{LDk}$ is the consumed complex power at bus k, complex power

$\overline{S}_{kj} = P_{kj} + jQ_{kj}$ is the complex power flow from bus k to bus j.

$$\boxed{\left(P_{Gk} + jQ_{Gk}\right) - \left(P_{LDk} + jQ_{LDk}\right) = \sum_{j=1}^{N} \left(P_{kj} + jQ_{kj}\right)}$$

$$\boxed{\left(P_{Gk} - P_{LDk}\right) + j\left(Q_{Gk} - Q_{LDk}\right) = \sum_{j=1}^{N} P_{kj} + j\sum_{j=1}^{N} Q_{kj}}$$

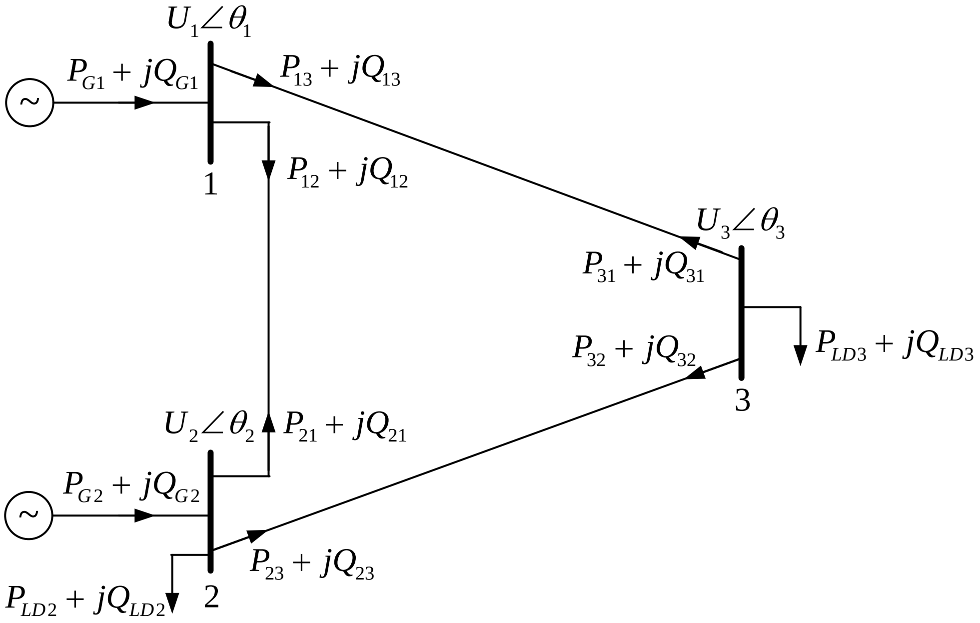


$$(P_{Gk} - P_{LDk}) + j(Q_{Gk} - Q_{LDk}) = \sum_{j=1}^{N} P_{kj} + j\sum_{j=1}^{N} Q_{kj}$$

$$P_{GDk} = P_{Gk} - P_{LDk} = \sum_{j=1}^{N} P_{kj}$$

$$Q_{GDk} = Q_{Gk} - Q_{LDk} = \sum_{j=1}^{N} Q_{kj}$$

P_{GDk} and Q_{GDk} : the net generation of active and reactive power at bus k



$$N = 3$$

$$P_{GDk} = P_{Gk} - P_{LDk} = \sum_{j=1}^{N} P_{kj}$$

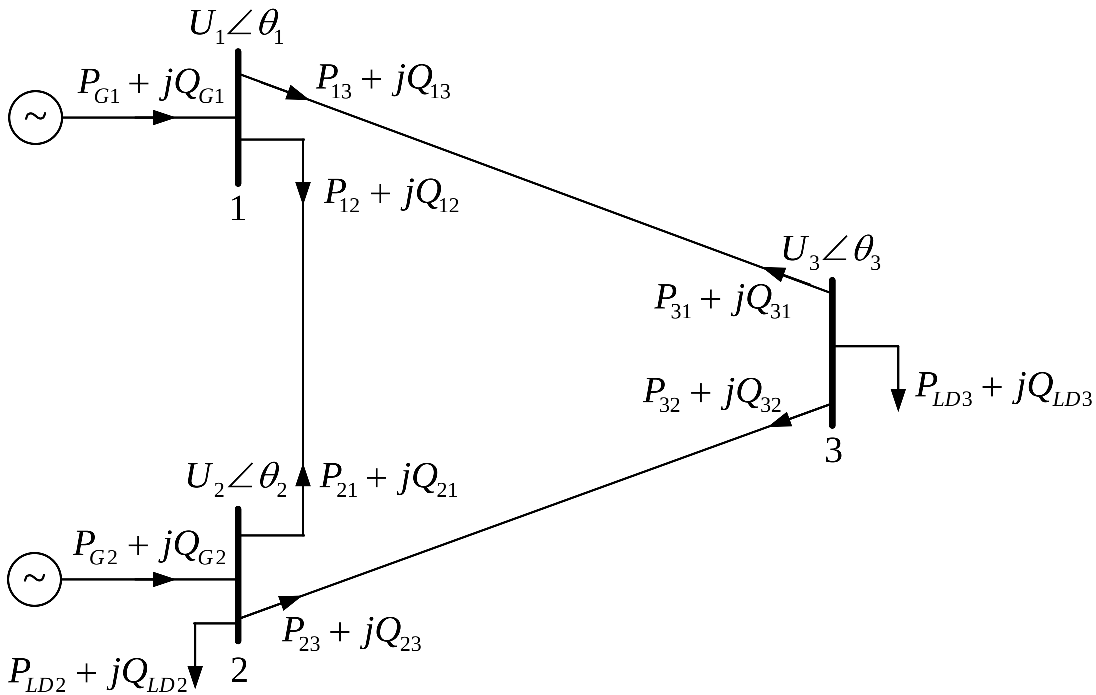
$$Q_{GDk} = Q_{Gk} - Q_{LDk} = \sum_{j=1}^{N} Q_{kj}$$

$$\begin{aligned}
P_{G1} &= P_{12} + P_{13} \\
Q_{G1} &= Q_{12} + Q_{13} \\
P_{G2} - P_{LD2} &= P_{21} + P_{23} \\
Q_{G2} - Q_{LD2} &= Q_{21} + Q_{23} \\
-P_{LD3} &= P_{31} + P_{32} \\
-Q_{LD3} &= Q_{31} + Q_{32}
\end{aligned}$$

$$
\begin{aligned}
P_{G1} &= P_{12} + P_{13} \\
Q_{G1} &= Q_{12} + Q_{13} \\
P_{G2} - P_{LD2} &= P_{21} + P_{23} \\
Q_{G2} - Q_{LD2} &= Q_{21} + Q_{23} \\
-P_{LD3} &= P_{31} + P_{32} \\
-Q_{LD3} &= Q_{31} + Q_{32}
\end{aligned}
$$

$$
P_{kj} = \frac{R_{kj}}{Z_{kj}^2} U_k^2 + \frac{U_k U_j}{Z_{kj}^2} \left(X_{kj} \sin\theta_{kj} - R_{kj} \cos\theta_{kj} \right)
$$

$$
Q_{kj} = -b_{sh-kj} U_k^2 + \frac{X_{kj}}{Z_{kj}^2} U_k^2 - \frac{U_k U_j}{Z_{kj}^2} \left(R \sin\theta_{kj} + X_{kj} \cos\theta_{kj} \right)
$$



$$N = 3$$

$$\boxed{\begin{array}{rcl} P_{G1} & = & P_{12} + P_{13} \\ Q_{G1} & = & Q_{12} + Q_{13} \\ P_{G2} - P_{LD2} & = & P_{21} + P_{23} \\ Q_{G2} - Q_{LD2} & = & Q_{21} + Q_{23} \\ -P_{LD3} & = & P_{31} + P_{32} \\ -Q_{LD3} & = & Q_{31} + Q_{32} \end{array}}$$

$N \cdot 2$ equations

four variables are of interest:

P_{GDk}, Q_{GDk}, U_k and θ_k.

$N \cdot 4 = 3 \cdot 4 = 12$ number of variables

Bus types

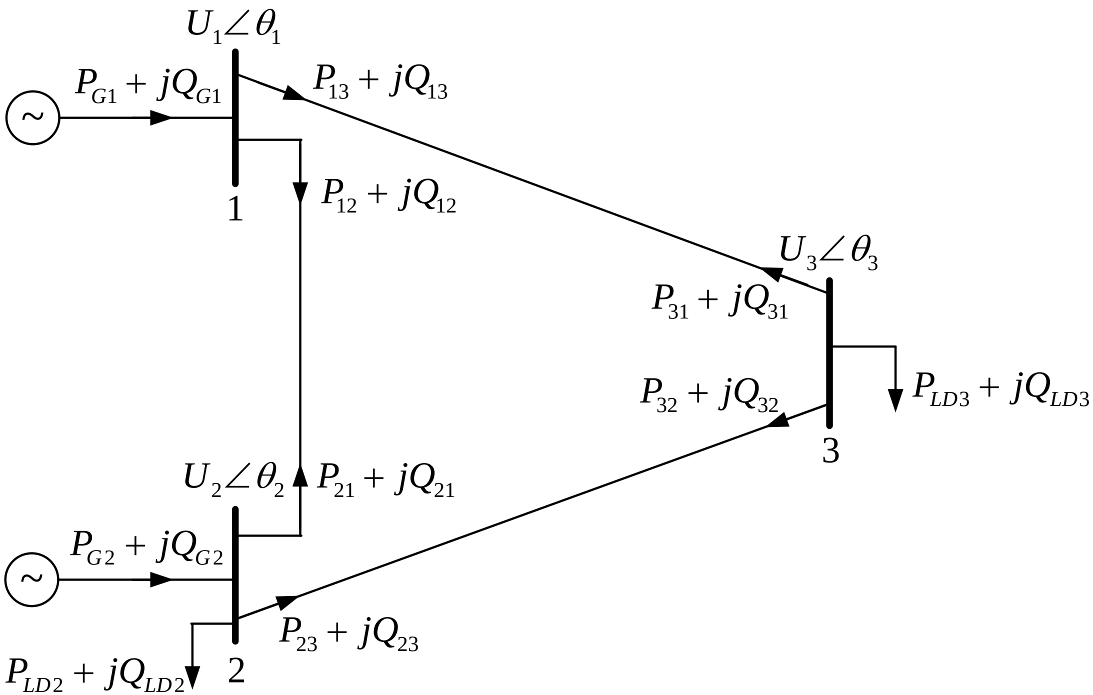
$N \cdot 2$ equations

$N \cdot 4$ variables

$$\boxed{\begin{aligned} P_{GDk} = P_{Gk} - P_{LDk} &= \sum_{j=1}^{N} P_{kj} \\ Q_{GDk} = Q_{Gk} - Q_{LDk} &= \sum_{j=1}^{N} Q_{kj} \end{aligned}}$$

Bus model	Number	Known quantities	Unknown quantities
$U\theta$-bus, Slack bus	1	U, θ	P_{GD}, Q_{GD}
PU-bus, Generator bus	M	P_{GD}, U	Q_{GD}, θ
PQ-bus, Load bus	N-M-1	P_{GD}, Q_{GD}	U, θ

Aim: to solve for unknown U, θ



bus 1 slack bus

bus 2 PU-bus

bus 3 PQ-bus

$$\cancel{P_{GD1}(\text{unknown}) = P_{12}(U_1, \theta_1, U_2, \theta_2) + P_{13}(U_1, \theta_1, U_3, \theta_3)}$$

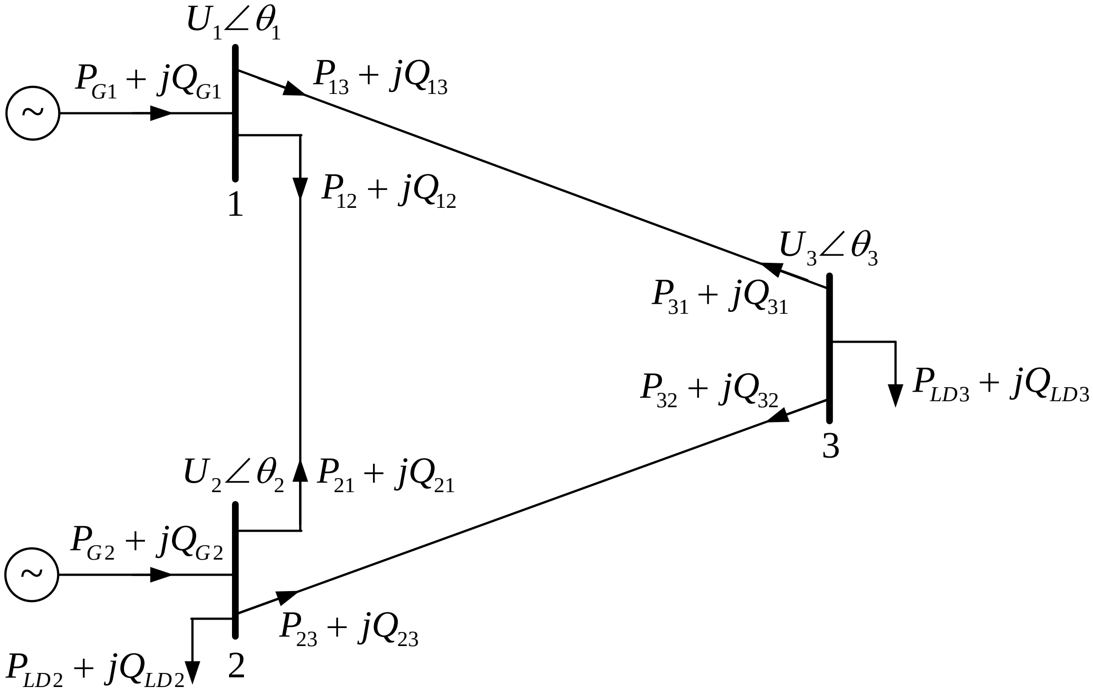
$$\cancel{Q_{G1}(\text{unknown}) = Q_{12}(U_1, \theta_1, U_2, \theta_2) + Q_{13}(U_1, \theta_1, U_3, \theta_3)}$$

$$P_{GD2} = P_{21}(U_1, \theta_1, U_2, \theta_2) + P_{23}(U_2, \theta_2, U_3, \theta_3)$$

$$\cancel{Q_{GD2}(\text{unknown}) = Q_{21}(U_1, \theta_1, U_2, \theta_2) + Q_{23}(U_2, \theta_2, U_3, \theta_3)}$$

$$P_{GD3} = P_{31}(U_1, \theta_1, U_3, \theta_3) + P_{32}(U_2, \theta_2, U_3, \theta_3)$$

$$Q_{GD3} = Q_{31}(U_1, \theta_1, U_3, \theta_3) + Q_{32}(U_2, \theta_2, U_3, \theta_3)$$



bus 1 slack bus

bus 2 PU-bus

bus 3 PQ-bus

$$
\begin{aligned}
P_{GD2} &= P_{21}(U_1,\theta_1,U_2,\theta_2) + P_{23}(U_2,\theta_2,U_3,\theta_3) \\
P_{GD3} &= P_{31}(U_1,\theta_1,U_3,\theta_3) + P_{32}(U_2,\theta_2,U_3,\theta_3) \\
Q_{GD3} &= Q_{31}(U_1,\theta_1,U_3,\theta_3) + Q_{32}(U_2,\theta_2,U_3,\theta_3)
\end{aligned}
$$

Bus model	Number	Known quantities	Unknown quantities
$U\theta$-bus, Slack bus	1	U, θ	P_{GD}, Q_{GD}
PU-bus, Generator bus	M	P_{GD}, U	Q_{GD}, θ
PQ-bus, Load bus	N-M-1	P_{GD}, Q_{GD}	U, θ

Bus model	Number	Balance equations		Unknown quantities	
		$P_{GDk} = \sum P_{kj}$	$Q_{GDk} = \sum Q_{kj}$	U_k	θ_k
Slack bus	1	0	0	0	0
PU-bus	M	M	0	0	M
PQ-bus	N-M-1	N-M-1	N-M-1	N-M-1	N-M-1
Total	N	2N-M-2		2N-M-2	

Newton-Raphson method

$$\boxed{\begin{aligned} P_{GDk} = P_{Gk} - P_{LDk} &= \sum_{j=1}^{N} P_{kj} \\ Q_{GDk} = Q_{Gk} - Q_{LDk} &= \sum_{j=1}^{N} Q_{kj} \end{aligned}}$$

$$\boxed{\begin{aligned} g_1(x_1, x_2, \cdots, x_n) &= f_1(x_1, x_2, \cdots, x_n) - b_1 = 0 \\ g_2(x_1, x_2, \cdots, x_n) &= f_2(x_1, x_2, \cdots, x_n) - b_2 = 0 \\ &\vdots \\ g_n(x_1, x_2, \cdots, x_n) &= f_n(x_1, x_2, \cdots, x_n) - b_n = 0 \end{aligned}}$$

$$\boxed{\begin{aligned} g_1(x_1, x_2, \cdots, x_n) &= f_1(x_1, x_2, \cdots, x_n) - b_1 = 0 \\ g_2(x_1, x_2, \cdots, x_n) &= f_2(x_1, x_2, \cdots, x_n) - b_2 = 0 \\ &\vdots \\ g_n(x_1, x_2, \cdots, x_n) &= f_n(x_1, x_2, \cdots, x_n) - b_n = 0 \end{aligned}}$$

$$g(x) = f(x) - b = 0$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad , \quad g(x) = \begin{bmatrix} g_1(x) \\ g_2(x) \\ \vdots \\ g_n(x) \end{bmatrix} \quad , \quad f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix} \quad , \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$g(x) = f(x) - b = 0$$

From an initial estimate (or guess) $x^{(0)}$, a sequence of gradually better estimates $x^{(1)}, x^{(2)}, x^{(3)}, \cdots$ will be made that hopefully will converge to the solution x^*

$$g(x) = f(x) - b = 0$$

Let x^* be the solution, i.e. $g(x^*) = 0$

and $x^{(i)}$ be an estimate of x^*

$\Delta x^{(i)} = x^* - x^{(i)}$

$$g(x^*) = g(x^{(i)} + \Delta x^{(i)}) = 0$$

$$g(x^*) = g(x^{(i)} + \Delta x^{(i)}) = 0$$

Taylor's series expansion

$$g(x^{(i)} + \Delta x^{(i)}) = g(x^{(i)}) + JAC^{(x^{(i)})} \Delta x^{(i)} = 0$$

$$JAC^{(x^{(i)})} = \left[\frac{\partial g(x)}{\partial x}\right]_{x=x^{(i)}} = \begin{bmatrix} \frac{\partial g_1(x)}{\partial x_1} & \cdots & \frac{\partial g_1(x)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_n(x)}{\partial x_1} & \cdots & \frac{\partial g_n(x)}{\partial x_n} \end{bmatrix}_{x=x^{(i)}}$$

$$g(x) = f(x) - b = 0$$

$$JAC^{(x^{(i)})} = \left[\frac{\partial g(x)}{\partial x}\right]_{x=x^{(i)}} = \left[\frac{\partial f(x)}{\partial x}\right]_{x=x^{(i)}}$$

$$= \begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \cdots & \frac{\partial f_1(x)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n(x)}{\partial x_1} & \cdots & \frac{\partial f_n(x)}{\partial x_n} \end{bmatrix}_{x=x^{(i)}}$$

$$g(x^{(i)} + \Delta x^{(i)}) = g(x^{(i)}) + JAC^{(x^{(i)})}\Delta x^{(i)} = 0$$

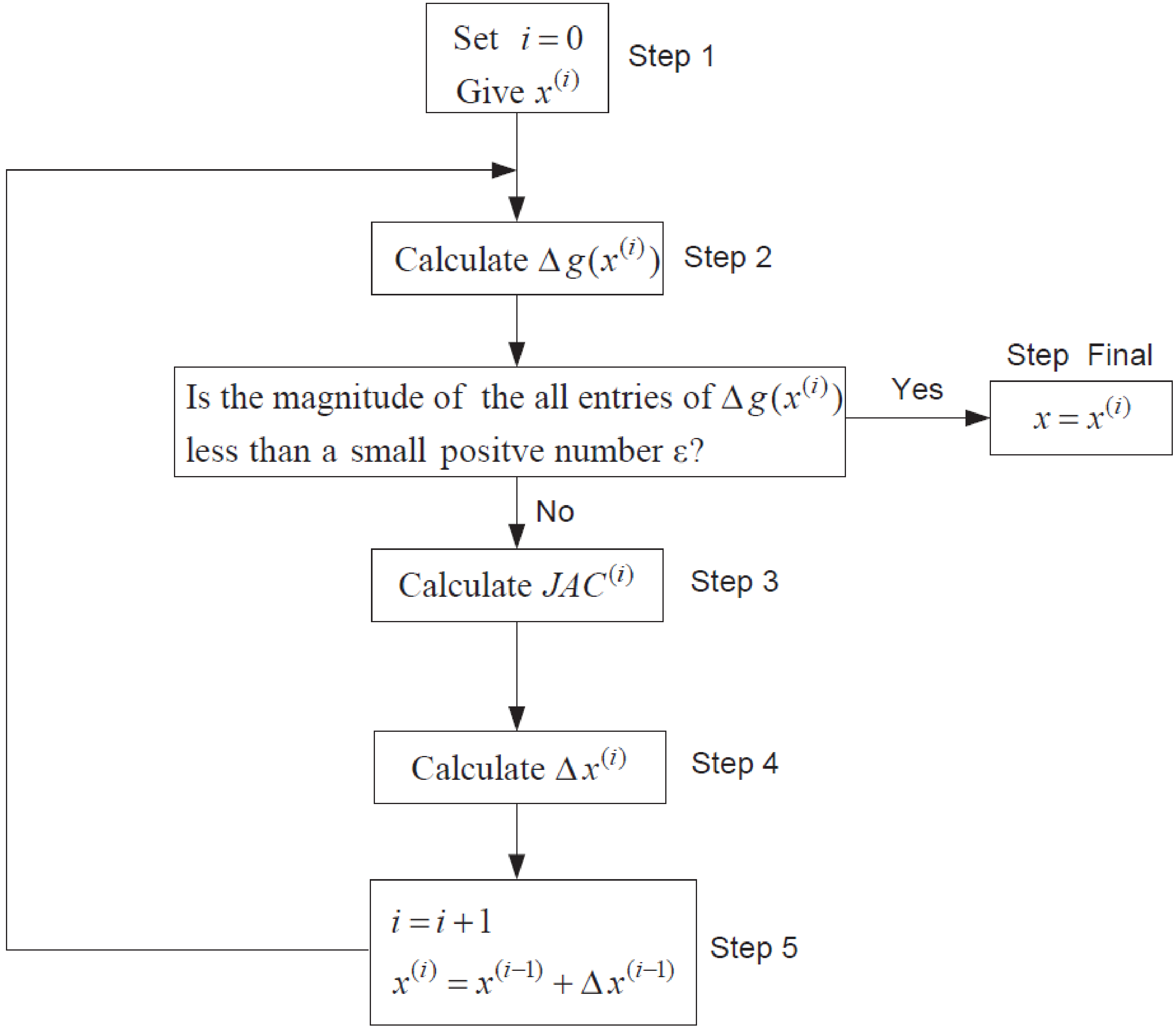
$$\boxed{\begin{aligned} JAC^{(x^{(i)})}\Delta x^{(i)} &= 0 - g(x^{(i)}) = \Delta g(x^{(i)}) \\ \Delta x^{(i)} &= \left[JAC^{(x^{(i)})}\right]^{-1} \Delta g(x^{(i)}) \end{aligned}}$$

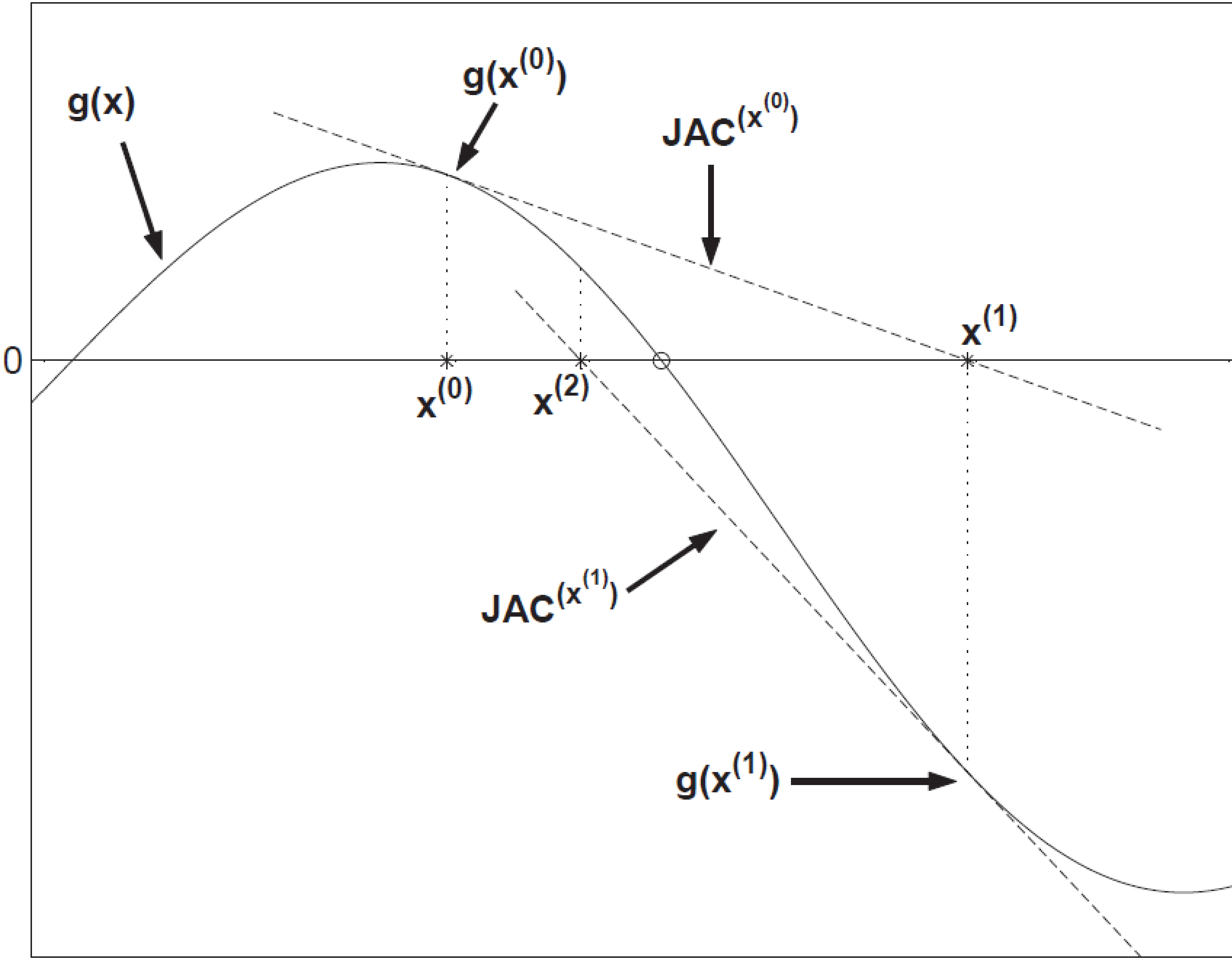
$$\boxed{\Delta g(x^{(i)}) = b - f(x^{(i)})}$$

$$\Delta x^{(i)} = \begin{bmatrix} \Delta x_1^{(i)} \\ \vdots \\ \Delta x_n^{(i)} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \cdots & \frac{\partial f_1(x)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n(x)}{\partial x_1} & \cdots & \frac{\partial f_n(x)}{\partial x_n} \end{bmatrix}_{x=x^{(i)}}^{-1} \begin{bmatrix} b_1 - f_1(x_1^{(i)}, \cdots, x_n^{(i)}) \\ \vdots \\ b_n - f_n(x_1^{(i)}, \cdots, x_n^{(i)}) \end{bmatrix}$$

$$\boxed{\begin{aligned} i &= i + 1 \\ x^{(i)} &= x^{(i-1)} + \Delta x^{(i-1)} \end{aligned}}$$

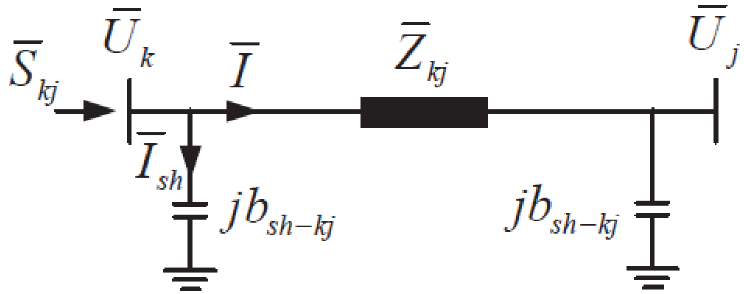
$$\boxed{\Delta g(x^{(i)}) = b - f(x^{(i)})}$$


Set $i = 0$
Give $x^{(i)}$
Step 1
Calculate $\Delta g(x^{(i)})$
Step 2
Is the magnitude of the all entries of $\Delta g(x^{(i)})$ less than a small positve number ε?
Yes
Step Final
$x = x^{(i)}$
No
Calculate $JAC^{(i)}$
Step 3
Calculate $\Delta x^{(i)}$
Step 4
$i = i + 1$
$x^{(i)} = x^{(i-1)} + \Delta x^{(i-1)}$
Step 5



g(x)
g(x(0))
JAC(x(0))
0
x(0)
x(2)
x(1)
JAC(x(1))
g(x(1))

Application to power systems

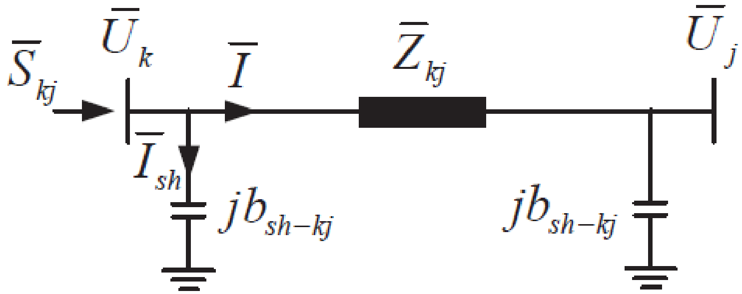


$$g_{kj} + j\,b_{kj} = \frac{1}{\overline{Z}_{kj}} = \frac{1}{R_{kj} + j\,X_{kj}} = \frac{R_{kj}}{Z_{kj}^2} + j\,\frac{-X_{kj}}{Z_{kj}^2}$$

$$g_{kj} = \frac{R_{kj}}{Z_{kj}^2}$$

$$b_{kj} = -\frac{X_{kj}}{Z_{kj}^2}$$

Application to power systems

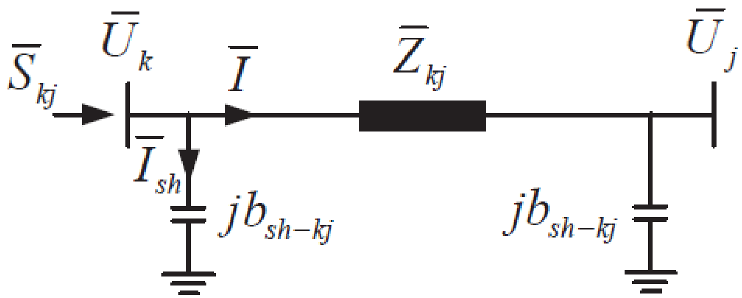


$$g_{kj} = \frac{R_{kj}}{Z_{kj}^2}$$

$$b_{kj} = -\frac{X_{kj}}{Z_{kj}^2}$$

$$P_{kj} = \frac{R_{kj}}{Z_{kj}^2} U_k^2 + \frac{U_k U_j}{Z_{kj}^2} (X_{kj} \sin\theta_{kj} - R_{kj} \cos\theta_{kj})$$

$$= g_{kj} U_k^2 - U_k U_j [g_{kj} \cos(\theta_{kj}) + b_{kj} \sin(\theta_{kj})]$$

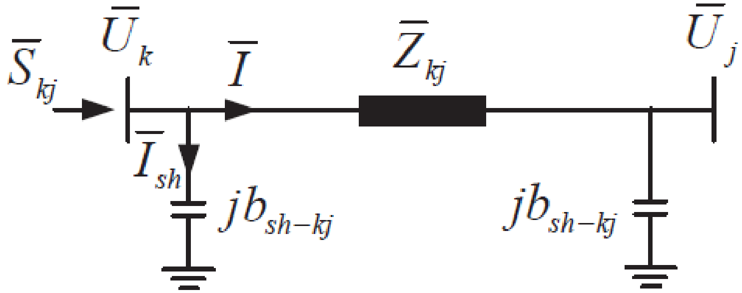


$$g_{kj} = \frac{R_{kj}}{Z_{kj}^2}$$

$$b_{kj} = -\frac{X_{kj}}{Z_{kj}^2}$$

$$Q_{kj} = -b_{sh-kj}\,U_k^2 + \frac{X_{kj}}{Z_{kj}^2}\,U_k^2 - \frac{U_k U_j}{Z_{kj}^2}\,(R\sin\theta_{kj} + X_{kj}\cos\theta_{kj})$$

$$= U_k^2(-b_{sh-kj} - b_{kj}) - U_k\,U_j\,[g_{kj}\sin(\theta_{kj}) - b_{kj}\cos(\theta_{kj})]$$



$$g_{kj} = \frac{R_{kj}}{Z_{kj}^2}$$

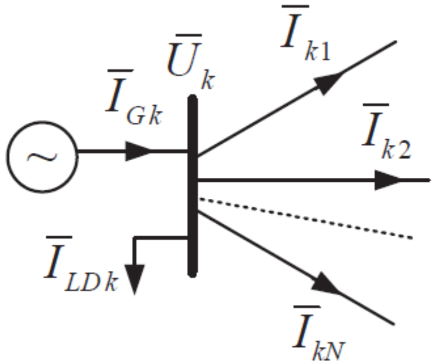
$$b_{kj} = -\frac{X_{kj}}{Z_{kj}^2}$$

$$P_{kj} = g_{kj} U_k^2 - U_k U_j \left[g_{kj} \cos(\theta_{kj}) + b_{kj} \sin(\theta_{kj}) \right]$$

$$Q_{kj} = U_k^2 (-b_{sh-kj} - b_{kj}) - U_k U_j \left[g_{kj} \sin(\theta_{kj}) - b_{kj} \cos(\theta_{kj}) \right]$$

the losses in the line

$$P_{lkj} = P_{kj} + P_{jk}$$



the admittance matrix

$$Y_{N\times N} = G + jB$$

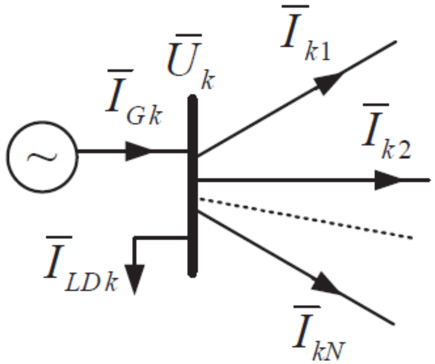
the injected current into bus k

$$\overline{I}_k = \sum_{j=1}^{N} \overline{Y}_{kj}\,\overline{U}_j$$

The injected complex power into bus k

$$\overline{S}_k = \overline{U}_k\,\overline{I}_k^* = \overline{U}_k \sum_{j=1}^{N} \overline{Y}_{kj}^*\overline{U}_j^* =$$

$$= U_k \sum_{j=1}^{N} (G_{kj} - jB_{kj})\,U_j(\cos(\theta_{kj}) + j\sin(\theta_{kj}))$$



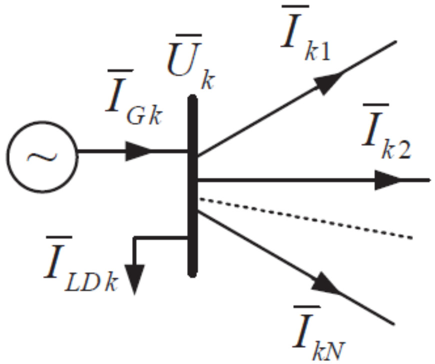
$$\overline{S}_k = \overline{U}_k \, \overline{I}_k^* = P_k + j\, Q_k$$

$$P_k = U_k \sum_{j=1}^{N} U_j \left[G_{kj} \cos(\theta_{kj}) + B_{kj} \sin(\theta_{kj})\right]$$

$$Q_k = U_k \sum_{j=1}^{N} U_j \left[G_{kj} \sin(\theta_{kj}) - B_{kj} \cos(\theta_{kj})\right]$$

Note that $G_{kj} = -g_{kj}$ and $B_{kj} = -b_{kj}$ for $k \neq j$

$$\begin{aligned} P_{kj} &= g_{kj} U_k^2 - U_k U_j \left[g_{kj} \cos(\theta_{kj}) + b_{kj} \sin(\theta_{kj})\right] \\ Q_{kj} &= U_k^2(-b_{sh-kj} - b_{kj}) - U_k U_j \left[g_{kj} \sin(\theta_{kj}) - b_{kj} \cos(\theta_{kj})\right] \end{aligned}$$



$$P_k = U_k \sum_{j=1}^{N} U_j \left[G_{kj} \cos(\theta_{kj}) + B_{kj} \sin(\theta_{kj})\right]$$

$$Q_k = U_k \sum_{j=1}^{N} U_j \left[G_{kj} \sin(\theta_{kj}) - B_{kj} \cos(\theta_{kj})\right]$$

$$P_k = \sum_{j=1}^{N} P_{kj}$$

$$Q_k = \sum_{j=1}^{N} Q_{kj}$$

$$\boxed{\begin{aligned} P_{GDk} = P_{Gk} - P_{LDk} &= \sum_{j=1}^{N} P_{kj} \\ Q_{GDk} = Q_{Gk} - Q_{LDk} &= \sum_{j=1}^{N} Q_{kj} \end{aligned}}$$

$$\boxed{\begin{aligned} P_k - P_{GDk} &= 0 \\ Q_k - Q_{GDk} &= 0 \end{aligned}}$$

$$\boxed{\begin{aligned} P_k - P_{GDk} &= 0 \\ Q_k - Q_{GDk} &= 0 \end{aligned}}$$

$$\boxed{g(x) = f(x) - b = 0}$$

$$\boxed{\begin{aligned} P_k &= U_k \sum_{j=1}^{N} U_j \left[G_{kj} \cos(\theta_{kj}) + B_{kj} \sin(\theta_{kj})\right] \\ Q_k &= U_k \sum_{j=1}^{N} U_j \left[G_{kj} \sin(\theta_{kj}) - B_{kj} \cos(\theta_{kj})\right] \end{aligned}}$$

$$x = \begin{bmatrix} \theta \\ \\ U \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_N \\ U_1 \\ \vdots \\ U_N \end{bmatrix}, \; f(\theta, U) = \begin{bmatrix} f_P(\theta, U) \\ \\ f_Q(\theta, U) \end{bmatrix} = \begin{bmatrix} P_1 \\ \vdots \\ P_N \\ Q_1 \\ \vdots \\ Q_N \end{bmatrix}, \; b = \begin{bmatrix} b_P \\ \\ b_Q \end{bmatrix} = \begin{bmatrix} P_{GD1} \\ \vdots \\ P_{GDN} \\ Q_{GD1} \\ \vdots \\ Q_{GDN} \end{bmatrix}$$

$$\boxed{\Delta g(x^{(i)}) = b - f(x^{(i)})}$$

$$\Delta P_k = \; P_{GDk} - P_k \quad k \neq \text{slack bus}$$
$$\Delta Q_k = \; Q_{GDk} - Q_k \quad k \neq \text{slack bus and PU-bus}$$

$$\boxed{\begin{aligned} JAC^{(x^{(i)})}\Delta x^{(i)} &= 0 - g(x^{(i)}) = \Delta g(x^{(i)}) \\ \Delta x^{(i)} &= \left[JAC^{(x^{(i)})}\right]^{-1}\Delta g(x^{(i)}) \end{aligned}}$$

$$JAC = \begin{bmatrix} \frac{\partial f_P(\theta,U)}{\partial \theta} & \frac{\partial f_P(\theta,U)}{\partial U} \\ \\ \frac{\partial f_Q(\theta,U)}{\partial \theta} & \frac{\partial f_Q(\theta,U)}{\partial U} \end{bmatrix} = \begin{bmatrix} H & N' \\ \\ J & L' \end{bmatrix}$$

$$\begin{array}{llll} H & \text{is an} & (N-1)\times(N-1) & \text{matrix} \\ N' & \text{is an} & (N-1)\times(N-M-1) & \text{matrix} \\ J & \text{is an} & (N-M-1)\times(N-1) & \text{matrix} \\ L' & \text{is an} & (N-M-1)\times(N-M-1) & \text{matrix} \end{array}$$

$$JAC = \begin{bmatrix} \frac{\partial f_P(\theta,U)}{\partial \theta} & \frac{\partial f_P(\theta,U)}{\partial U} \\ \frac{\partial f_Q(\theta,U)}{\partial \theta} & \frac{\partial f_Q(\theta,U)}{\partial U} \end{bmatrix} = \begin{bmatrix} H & N' \\ J & L' \end{bmatrix}$$

$H_{kj} = \frac{\partial P_k}{\partial \theta_j}$ $k \neq$ slack bus $j \neq$ slack bus

$N'_{kj} = \frac{\partial P_k}{\partial U_j}$ $k \neq$ slack bus $j \neq$ slack bus and PU-bus

$J_{kj} = \frac{\partial Q_k}{\partial \theta_j}$ $k \neq$ slack bus and PU-bus $j \neq$ slack bus

$L'_{kj} = \frac{\partial Q_k}{\partial U_j}$ $k \neq$ slack bus and PU-bus $j \neq$ slack bus and PU-bus

$$\boxed{\begin{aligned} JAC^{(x^{(i)})}\Delta x^{(i)} &= 0 - g(x^{(i)}) = \Delta g(x^{(i)}) \\ \Delta x^{(i)} &= \left[JAC^{(x^{(i)})}\right]^{-1}\Delta g(x^{(i)}) \end{aligned}}$$

$$\begin{bmatrix} H & N' \\ J & L' \end{bmatrix}\begin{bmatrix} \Delta\theta \\ \Delta U \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

$$\boxed{\begin{bmatrix} H & N \\ J & L \end{bmatrix}\begin{bmatrix} \Delta\theta \\ \frac{\Delta U}{U} \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix}\begin{bmatrix} \Delta\theta \\ \Delta U' \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}}$$

$$
\begin{aligned}
JAC^{(x^{(i)})}\Delta x^{(i)} &= 0 - g(x^{(i)}) = \Delta g(x^{(i)}) \\
\Delta x^{(i)} &= \left[JAC^{(x^{(i)})}\right]^{-1}\Delta g(x^{(i)})
\end{aligned}
$$

$$
\begin{bmatrix} H & N \\ J & L \end{bmatrix}\begin{bmatrix} \Delta\theta \\ \frac{\Delta U}{U} \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix}\begin{bmatrix} \Delta\theta \\ \Delta U' \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}
$$

$$
\begin{bmatrix} \Delta\theta \\ \frac{\Delta U}{U} \end{bmatrix} = \begin{bmatrix} \Delta\theta \\ \Delta U' \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix}^{-1}\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}
$$

$$\boxed{\begin{bmatrix} \Delta\theta \\ \frac{\Delta U}{U} \end{bmatrix} = \begin{bmatrix} \Delta\theta \\ \Delta U' \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}}$$

$$\boxed{\begin{aligned} i &= i + 1 \\ x^{(i)} &= x^{(i-1)} + \Delta x^{(i-1)} \end{aligned}}$$

$$\boxed{\begin{array}{ll} \theta_k = \theta_k + \Delta\theta_k & k \neq \text{slack bus} \\ U_k = U_k\left(1 + \Delta U'_k\right) & k \neq \text{slack bus and PU-bus} \end{array}}$$

$$\begin{bmatrix} H & N \\ J & L \end{bmatrix}$$

$$\boxed{\begin{aligned} P_k &= U_k \sum_{j=1}^{N} U_j \left[G_{kj} \cos(\theta_{kj}) + B_{kj} \sin(\theta_{kj})\right] \\ Q_k &= U_k \sum_{j=1}^{N} U_j \left[G_{kj} \sin(\theta_{kj}) - B_{kj} \cos(\theta_{kj})\right] \end{aligned}}$$

for $k \neq j$

$$H_{kj} = \frac{\partial P_k}{\partial \theta_j} = U_k\, U_j \left[G_{kj} \sin(\theta_{kj}) - B_{kj} \cos(\theta_{kj})\right]$$

$$N_{kj} = U_j\, N'_{kj} = U_j \frac{\partial P_k}{\partial U_j} = U_k\, U_j \left[G_{kj} \cos(\theta_{kj}) + B_{kj} \sin(\theta_{kj})\right]$$

$$J_{kj} = \frac{\partial Q_k}{\partial \theta_j} = -U_k\, U_j \left[G_{kj} \cos(\theta_{kj}) + B_{kj} \sin(\theta_{kj})\right]$$

$$L_{kj} = U_j\, L'_{kj} = U_j \frac{\partial Q_k}{\partial U_j} = U_k\, U_j \left[G_{kj} \sin(\theta_{kj}) - B_{kj} \cos(\theta_{kj})\right]$$

$$\begin{bmatrix} H & N \\ J & L \end{bmatrix}$$

$$\boxed{\begin{aligned} P_k &= U_k \sum_{j=1}^{N} U_j \left[G_{kj} \cos(\theta_{kj}) + B_{kj} \sin(\theta_{kj})\right] \\ Q_k &= U_k \sum_{j=1}^{N} U_j \left[G_{kj} \sin(\theta_{kj}) - B_{kj} \cos(\theta_{kj})\right] \end{aligned}}$$

for $k = j$

$$H_{kk} = \frac{\partial P_k}{\partial \theta_k} = -Q_k - B_{kk} U_k^2$$

$$N_{kk} = U_k \frac{\partial P_k}{\partial U_k} = P_k + G_{kk} U_k^2$$

$$J_{kk} = \frac{\partial Q_k}{\partial \theta_k} = P_k - G_{kk} U_k^2$$

$$L_{kj} = U_k \frac{\partial Q_k}{\partial U_k} = Q_k - B_{kk} U_k^2$$

Newton-Raphson method for solving power flow equations

- **Step 1**

  1a) Read bus and line data. Identify slack bus (i.e. Uθ-bus), PU-buses and PQ-buses.

  1b) Build the Y-matrix and calculate the net productions, i.e. $P_{GD} = P_G - P_{LD}$ and $Q_{GD} = Q_G - Q_{LD}$.

  1c) Give the initial estimate of the unknown variables, i.e. U for PQ-buses and θ for PU- and PQ-buses. It is very common to set $U = U_{slack}$ and $\theta = \theta_{slack}$. However, the flat initial estimate may also be applied, i.e. $U = 1$ and $\theta = 0$.

  1d) Go to **Step 2**.

- **Step 2**

2a) Calculate the injected power into each bus by equation (7.43).

2b) Calculate the difference between the net production and the injected power for each bus, i.e. ΔP and ΔQ by equation (7.46).

2c) Is the magnitude of all entries of $[\Delta P \quad \Delta Q]^T$ less than a specified small positive constant ϵ ?

  * If yes, go to **Step Final**.
  * if no, go to **Step 3**.

- **Step 3**

  3a) Calculate the jacobian by equations (7.50) and (7.51).

  3b) Go to **Step 4**.

- **Step 4**

  4a) Calculate $[\Delta\theta \quad \Delta U']^T$ by equation (7.52).

  4b) Go to **Step 5**.

- **Step 5**

  5a) Update U and θ by equation (7.53).

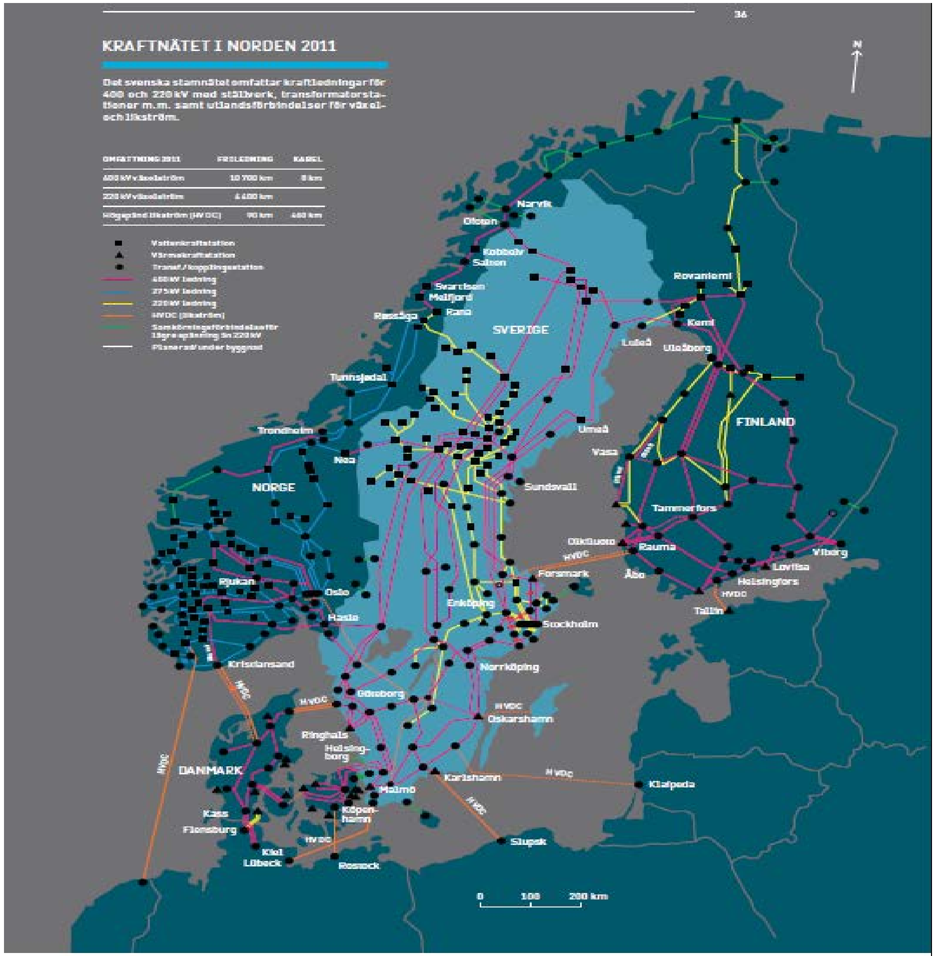
  5b) Go till **Step 2**.

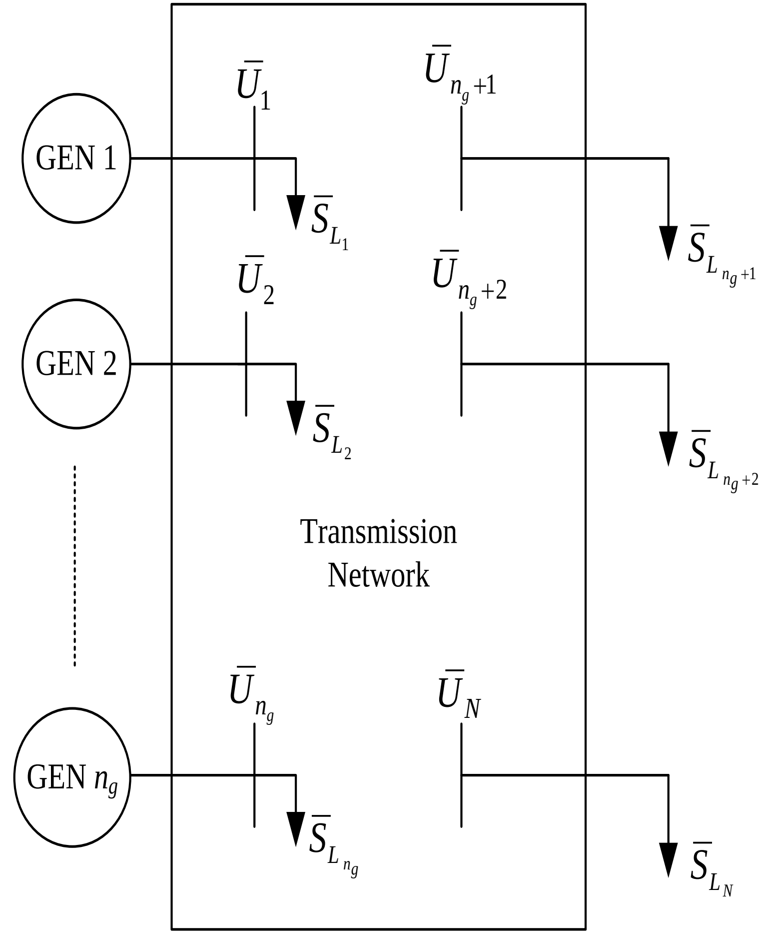
- **Step Final**
  - Calculate the generated powers, i.e. P_G (MW) and Q_G (MVAr) in the slack bus, and Q_G (MVAr) in the PU-buses by using equation (7.44).
  - Calculate the power flows (MW, MVAr) by using equations (7.38) and (7.39).
  - Calculate active power losses (MW) by using equation (7.41).
  - Give all the voltage magnitudes (kV) and the voltage phase angles (degrees).
  - Print out the results.

$$\boxed{\begin{aligned} P_k - P_{GDk} &= 0 \\ Q_k - Q_{GDk} &= 0 \end{aligned}}$$

$$\boxed{\begin{aligned} P_{kj} &= g_{kj} U_k^2 - U_k U_j \left[g_{kj}\cos(\theta_{kj}) + b_{kj}\sin(\theta_{kj})\right] \\ Q_{kj} &= U_k^2(-b_{sh-kj} - b_{kj}) - U_k U_j \left[g_{kj}\sin(\theta_{kj}) - b_{kj}\cos(\theta_{kj})\right] \end{aligned}}$$

$$\boxed{P_{lkj} = P_{kj} + P_{jk}}$$





$$P_k - P_{GDk} = 0$$
$$Q_k - Q_{GDk} = 0$$

$$P_k = U_k \sum_{j=1}^{N} U_j \left[G_{kj} \cos(\theta_{kj}) + B_{kj} \sin(\theta_{kj})\right]$$

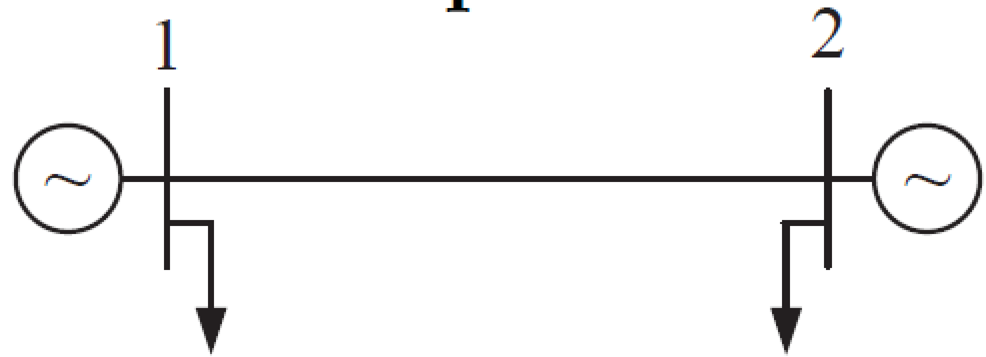
$$Q_k = U_k \sum_{j=1}^{N} U_j \left[G_{kj} \sin(\theta_{kj}) - B_{kj} \cos(\theta_{kj})\right]$$

$S_{base} = 100\ MVA$

$U_{base} = 220\ kV$

$\varepsilon = 10^{-6}$

Example 7.13



The following data (all in pu) is known:

- **Line between Bus 1 and Bus 2:** *short line*, $\bar{Z}_{12} = 0.02 + j\,0.2$
- **Bus 1:** *slack bus*, $U_1 = 1$, $\theta_1 = 0$, $P_{LD1} = 0.2$, $Q_{LD1} = 0.02$
- **Bus 2:** *PQ-bus*, $P_{G2} = 1$, $Q_{G2} = 0.405255$, $P_{LD2} = 2$, $Q_{LD2} = 0.2$

By applying Newton-Raphson method, calculate θ_2, P_{G1}, Q_{G1} *and the active power losses.*

Step 1

1a) bus 1 is a slack bus , bus 2 is a PQ-bus , $U_1 = 1$ and $\theta_1 = 0$.

1b)

$$
\begin{aligned}
\mathbf{Y} &= \begin{bmatrix} G_{11} + j\,B_{11} & G_{12} + j\,B_{12} \\ \\ G_{21} + j\,B_{21} & G_{22} + j\,B_{22} \end{bmatrix} = \mathbf{G} + j\,\mathbf{B} \\
P_{GD2} &= P_{G2} - P_{LD2} \\
Q_{GD2} &= Q_{G2} - Q_{LD2}
\end{aligned}
$$

1c)

As an initial value , let $\theta_2 = 0$ and $U_2 = 1$.

Step 2

2a)

$$
\begin{aligned}
P_2 &= U_2\,U_1\,[G_{21}\,\cos(\theta_2-\theta_1)+B_{21}\,\sin(\theta_2-\theta_1)]+U_2^2\,G_{22}\\
Q_2 &= U_2\,U_1\,[G_{21}\,\sin(\theta_2-\theta_1)-B_{21}\,\cos(\theta_2-\theta_1)]-U_2^2\,B_{22}
\end{aligned}
$$

2b)

$$
\begin{aligned}
\Delta P &= \Delta P_2 = P_{GD2}-P_2\\
\Delta Q &= \Delta Q_2 = Q_{GD2}-Q_2
\end{aligned}
$$

$$\varepsilon = 10^{-6}$$

As long as $|\Delta P_2| > \varepsilon$ **and** $|\Delta Q_2| > \varepsilon$, perform **Step 3**

Step 3

$$\begin{aligned}
H &= -Q_2 - B_{22}U_2^2 \\
N &= P_2 + G_{22}U_2^2 \\
J &= P_2 - G_{22}U_2^2 \\
L &= Q_2 - B_{22}U_2^2 \\
JAC &= \begin{bmatrix} H & N \\ J & L \end{bmatrix} \\
\begin{bmatrix} \Delta\theta_2 \\ \Delta U_2' \end{bmatrix} &= JAC^{-1} \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}
\end{aligned}$$

Steps 4-5

$$
\begin{aligned}
\theta_2 &= \theta_2 + \Delta\theta_2 \\
U_2 &= U_2\,(1 + \Delta U_2') \\
P_2 &= U_2\,U_1\,[G_{21}\,\cos(\theta_2 - \theta_1) + B_{21}\,\sin(\theta_2 - \theta_1)] + U_2^2\,G_{22} \\
Q_2 &= U_2\,U_1\,[G_{21}\,\sin(\theta_2 - \theta_1) - B_{21}\,\cos(\theta_2 - \theta_1)] - U_2^2\,B_{22} \\
\Delta P &= \Delta P_2 = P_{GD2} - P_2 \\
\Delta Q &= \Delta Q_2 = Q_{GD2} - Q_2
\end{aligned}
$$

check if $|\Delta P_2| < \varepsilon$ **and** $|\Delta Q_2| < \varepsilon$

Step Final

$$P_1 = U_1 U_2 [G_{12} \cos(\theta_1 - \theta_2) + B_{12} \sin(\theta_1 - \theta_2)] + U_1^2 G_{11}$$

$$Q_1 = U_1 U_2 [G_{12} \sin(\theta_1 - \theta_2) - B_{12} \cos(\theta_1 - \theta_2)] - U_1^2 B_{11}$$

$$P_{G1} = (P_1 + P_{LD1}) * S_{base}$$

$$Q_{G1} = (Q_1 + Q_{LD1}) * S_{base}$$

$$g = -G \quad , \quad b = -B \quad \text{and} \quad b_{sh-12} = 0$$

$$P_{12} = \left(g_{12} U_1^2 - U_1 U_2 [g_{12} \cos(\theta_1 - \theta_2) + b_{12} \sin(\theta_1 - \theta_2)]\right) * S_{base}$$

$$P_{21} = \left(g_{21} U_2^2 - U_2 U_1 [g_{21} \cos(\theta_2 - \theta_1) + b_{21} \sin(\theta_2 - \theta_1)]\right) * S_{base}$$

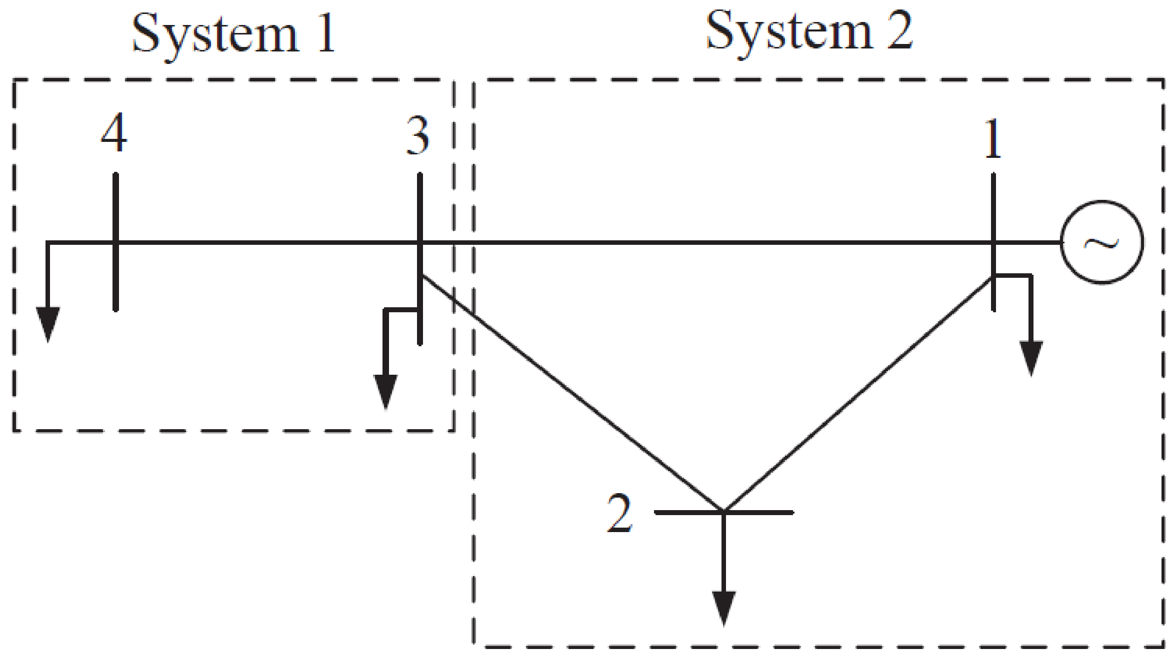
$$P_{Loss}^{tot} = P_{12} + P_{21}$$

$$= (P_{G1} + P_{G2}) - (P_{LD1} + P_{LD2})$$

Example 7.14

$S_{base} = 100\ MVA$

$U_{base} = 220\ kV$



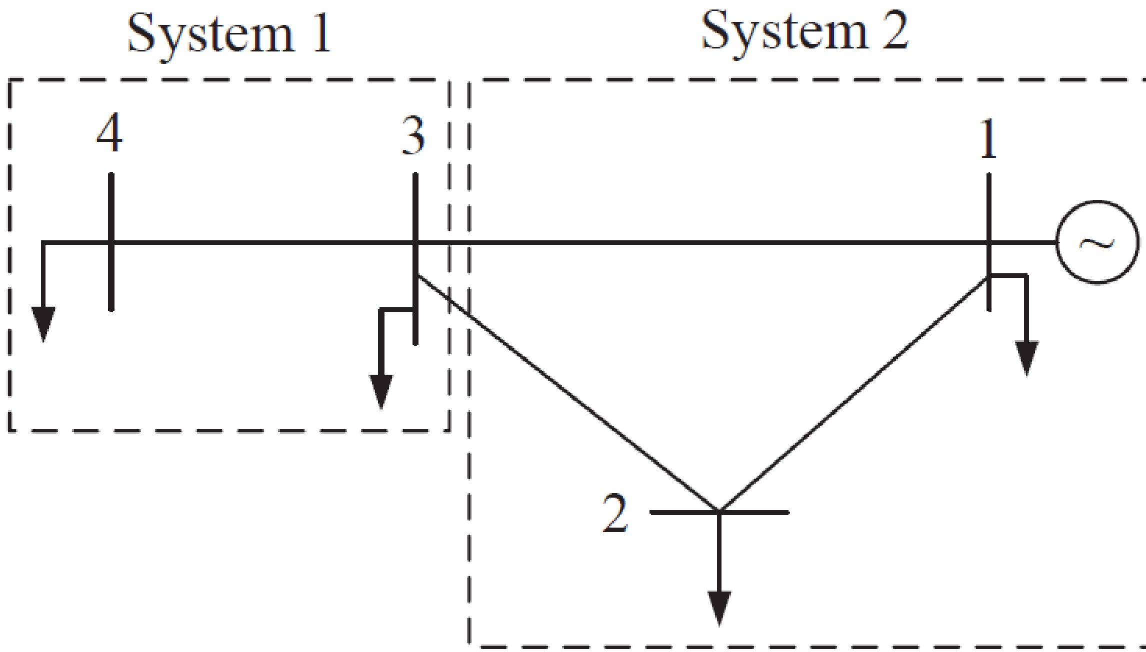
The system data (in MW, MVAr, kV, Ω and S) is given as follows:

- **Line between Bus 1 and Bus 2:** $\bar{Z}_{12} = 5 + j\,65\ ,\ b_{sh-12} = 0.0002$
- **Line between Bus 1 and Bus 3:** $\bar{Z}_{13} = 4 + j\,60\ ,\ b_{sh-13} = 0.0002$
- **Line between Bus 2 and Bus 3:** $\bar{Z}_{12} = 5 + j\,68\ ,\ b_{sh-12} = 0.0002$
- **Line between Bus 3 and Bus 4:** $\bar{Z}_{34} = 3 + j\,30$, *short line*

Example 7.14

$S_{base} = 100\ MVA$

$U_{base} = 220\ kV$

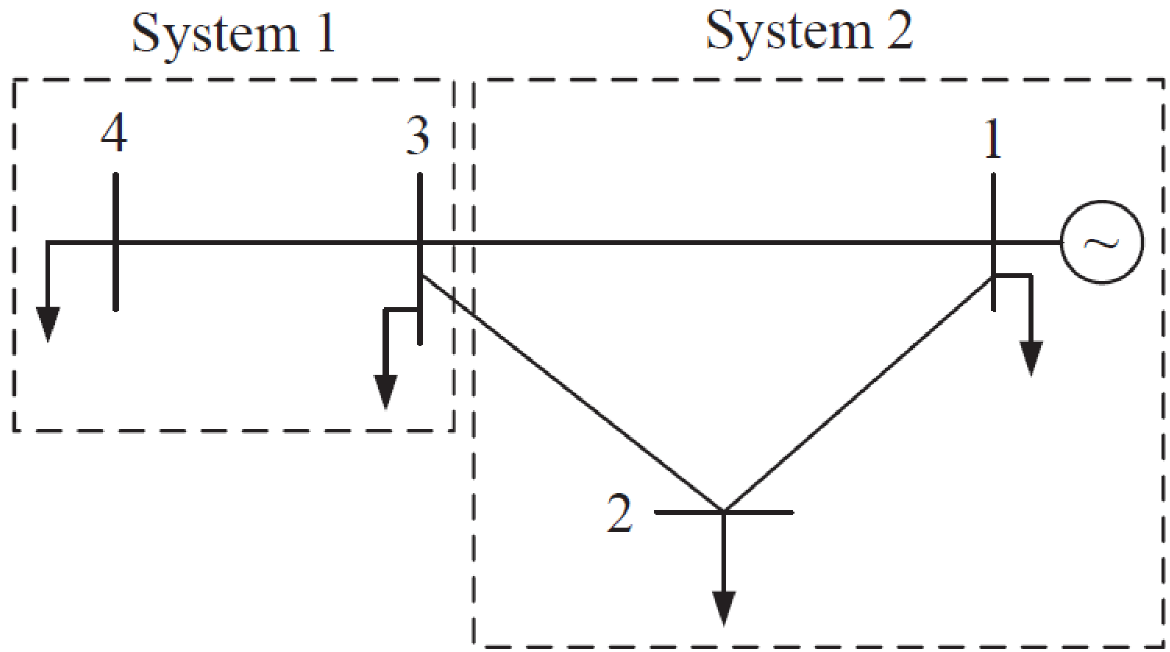


- **Bus 1:** $U_1 = 220,\ \theta_1 = 0,\ P_{LD1} = 10,\ Q_{LD1} = 2$
- **Bus 2:** $P_{LD2} = 90,\ Q_{LD2} = 10$
- **Bus 3:** $P_{LD3} = 80,\ Q_{LD3} = 10$
- **Bus 4:** $P_{LD4} = 50,\ Q_{LD4} = 10$

Example 7.14

$S_{base} = 100\ MVA$

$U_{base} = 220\ kV$



Use "fsolve" function in MATLAB, and find

a) *the unknown voltage magnitudes and voltage phase angles,*

b) *the generated active and reactive powers at the slack bus, and the generated reactive powers at PU-buses (if any),*

c) *the total active power losses, and the losses in System1 and System 2,*

d) *the changes (in % compared to the obtained results in task* **c)***) of power losses in both systems, for an active load increased at bus 2 with* 30 MW, *i.e.* $P_{LD2}^{new} = 120\ MW$.

e) *Let* $P_{LD2} = 90\ MW$. *Re-do task* **d)** *for a reactive load increased at bus 3 with* 10 $MVAr$, *i.e.* $Q_{LD3}^{new} = 20\ MVAr$.

- **Bus 1:** $U_1 = 220,\ \theta_1 = 0,\ P_{LD1} = 10,\ Q_{LD1} = 2$
- **Bus 2:** $P_{LD2} = 90,\ Q_{LD2} = 10$
- **Bus 3:** $P_{LD3} = 80,\ Q_{LD3} = 10$
- **Bus 4:** $P_{LD4} = 50,\ Q_{LD4} = 10$

Bus types

Bus 1

Bus 2

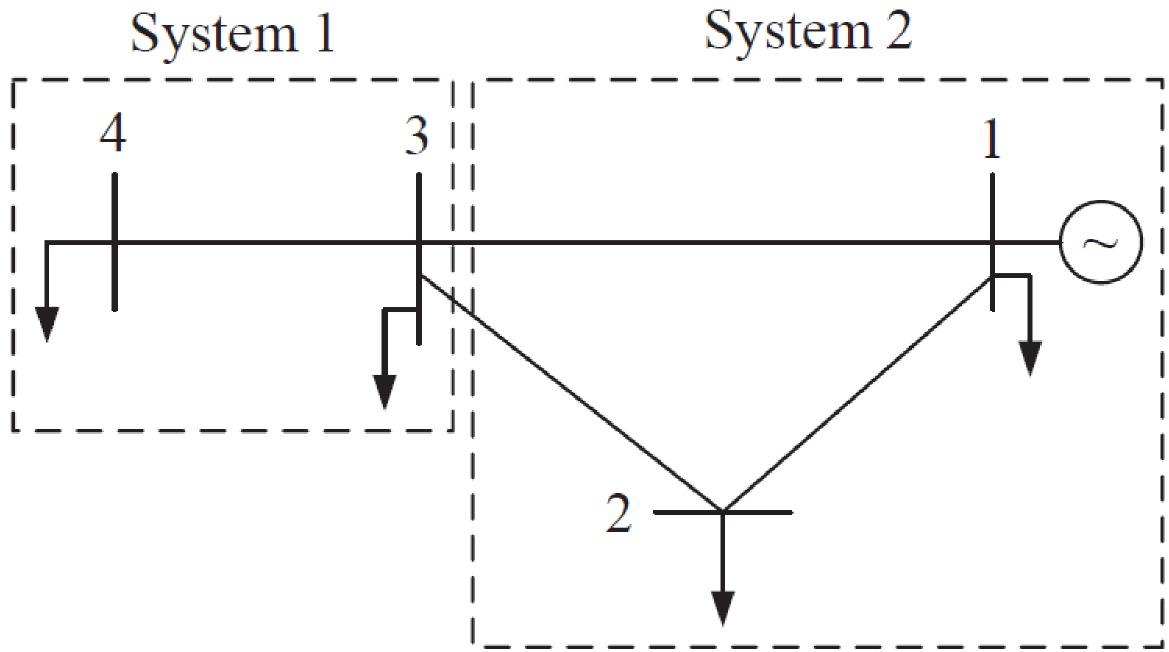
Bus 3

Bus 4

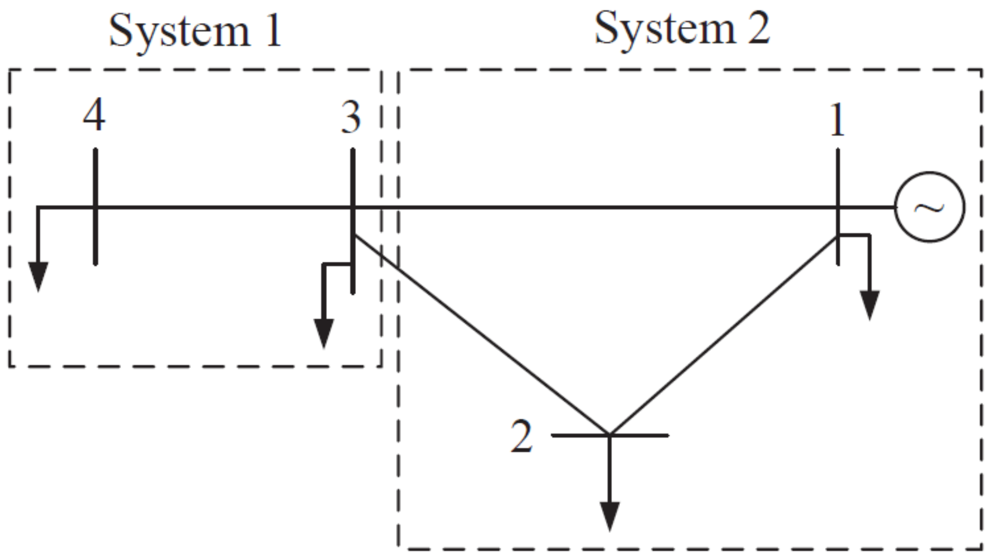
- **Line between Bus 1 and Bus 2:** $\bar{Z}_{12} = 5 + j\,65\ ,\ b_{sh-12} = 0.0002$
- **Line between Bus 1 and Bus 3:** $\bar{Z}_{13} = 4 + j\,60\ ,\ b_{sh-13} = 0.0002$
- **Line between Bus 2 and Bus 3:** $\bar{Z}_{12} = 5 + j\,68\ ,\ b_{sh-12} = 0.0002$
- **Line between Bus 3 and Bus 4:** $\bar{Z}_{34} = 3 + j\,30$, *short line*

First, all expressed in (pu)

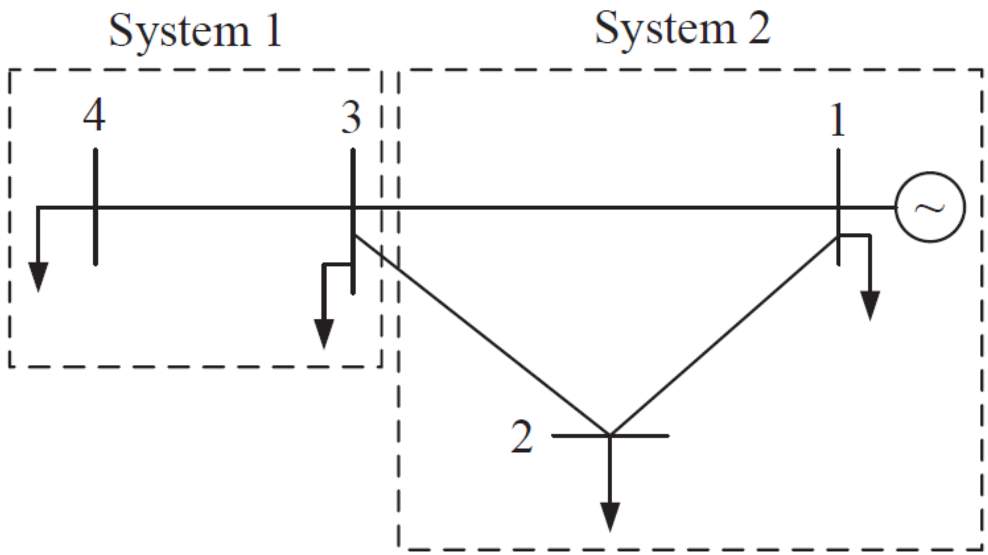
$$Y = G + jB$$



$$Y = \begin{bmatrix} \bar{y}_{11} & \bar{y}_{12} & \bar{y}_{13} & 0 \\ \bar{y}_{21} & \bar{y}_{22} & \bar{y}_{23} & 0 \\ \bar{y}_{31} & \bar{y}_{32} & \bar{y}_{33} & \bar{y}_{34} \\ 0 & 0 & \bar{y}_{43} & \bar{y}_{44} \end{bmatrix}$$



- Write the mismatch equations which will be used for power flow calculations.
- Give the unknown variables and their initial values.



$$
\begin{aligned}
x &= [\theta_2\ \theta_3\ \theta_4\ U_2\ U_3\ U_4] \\
0 &= g_1(x) = P_2 - P_{GD2} \quad , \quad \text{(active power mismatch at bus 2, see equation (7.44))} \\
0 &= g_2(x) = P_3 - P_{GD3} \quad , \quad \text{(active power mismatch at bus 3)} \\
0 &= g_3(x) = P_4 - P_{GD4} \quad , \quad \text{(active power mismatch at bus 4)} \\
0 &= g_4(x) = Q_2 - Q_{GD2} \quad , \quad \text{(reactive power mismatch at bus 2)} \\
0 &= g_5(x) = Q_3 - Q_{GD3} \quad , \quad \text{(reactive power mismatch at bus 3)} \\
0 &= g_6(x) = Q_4 - Q_{GD4} \quad , \quad \text{(reactive power mismatch at bus 4)}
\end{aligned}
$$

```
% To run Load Flow (LF), two MATLAB-files are used, namely
% (run_LF.m) and (solve_lf.m)

% Start of file (run_LF.m)

clear%
clear global
%%%%%%%%%%%%%%%%%%%%%%%%%%
tole=1e-9; deg=180/pi; rad=1/deg ;
%%%%%%%%%%%%%%
% Base values
%%%%%%%%%%%%%%
Sbase=100; Ubase=220; Zb=Ubase^2/Sbase;
```

```
%%%%%%%%%%%%
% Bus data
%%%%%%%%%%%%
% Number of buses
nbus=4;
%Bus 1, slack bus
U1=220/Ubase; theta1=0*rad; PLD1=10/Sbase; QLD1=2/Sbase;
%Bus 2, PQ-bus
PG2=0/Sbase; QG2=0/Sbase; PLD2=90/Sbase; QLD2=10/Sbase;
%Bus 3, PQ-bus
PG3=0/Sbase; QG3=0/Sbase; PLD3=80/Sbase; QLD3=10/Sbase;
%Bus 4, PQ-bus
PG4=0/Sbase; QG4=0/Sbase; PLD4=50/Sbase; QLD4=10/Sbase;
```

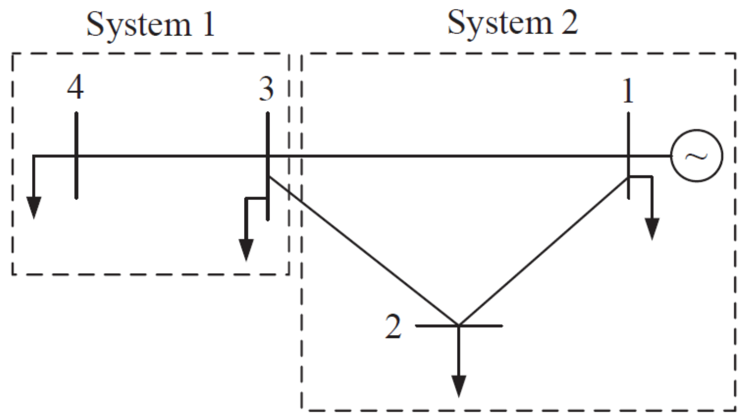
```
%%%%%%%%%%%%
% Line data
%%%%%%%%%%%%
Z12=(5+j*65)/Zb;bsh12=0.0002*Zb;%
Z13=(4+j*60)/Zb;bsh13=0.0002*Zb;%
Z23=(5+j*68)/Zb;bsh23=0.0002*Zb;%
Z34=(3+j*30)/Zb;bsh34=0;
%%%%%%%%%%%%%%
% YBUS matrix
%%%%%%%%%%%%%%
y11=1/Z12+1/Z13+j*bsh12+j*bsh13; y12=-1/Z12; y13=-1/Z13; y14=0;%
y21=-1/Z12; y22=1/Z12+1/Z23+j*bsh12+j*bsh23; y23=-1/Z23; y24=0;%
y31=-1/Z13; y32=-1/Z23; y33=1/Z13+1/Z23+1/Z34+j*bsh13+j*bsh23; y34=-1/Z34;%
y41=0; y42=0; y43=-1/Z34; y44=1/Z34;%
YBUS=[ ];% Define YBUS
G=real(YBUS); B=imag(YBUS);
```

$$Y = \begin{bmatrix} \bar{y}_{11} & \bar{y}_{12} & \bar{y}_{13} & 0 \\ \bar{y}_{21} & \bar{y}_{22} & \bar{y}_{23} & 0 \\ \bar{y}_{31} & \bar{y}_{32} & \bar{y}_{33} & \bar{y}_{34} \\ 0 & 0 & \bar{y}_{43} & \bar{y}_{44} \end{bmatrix}$$

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% PGD for PU- and PQ-buses
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
PGD2=PG2-PLD2; % for bus 2 (PQ-bus)
PGD3=PG3-PLD3; % for bus 3 (PQ-bus)
PGD4=PG4-PLD4; % for bus 4 (PQ-bus)
PGD=[PGD2 ; PGD3 ; PGD4];
%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Define the column vector QGD for PQ-buses
%%%%%%%%%%%%%%%%%%%%%%%%%%


QGD=[ ];
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Use fsolve function in MATLAB to run load flow
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Unknown variables [theta2 theta3 theta4 U2 U3 U4]';
% Define the initial values of the unknown variables
X0=[0 0 0 1 1 1]'; % Flat initial values
s_z=size(X0);
nx=s_z(1,1); % number of unknown variables
% The function below is used for fsolve (type "help fsolve" in MATLAB)
options_solve=optimset('Display','off','TolX',tole,'TolFun',tole);

% Parameters used for fsolve
PAR=[nx ; nbus ; U1 ; theta1];% U1 and theta1 are known (slack bus).
[X_X]=fsolve('solve_lf',X0,options_solve,G,B,PGD,QGD,PAR);

% Solved variables X_X=[theta2 theta3 theta4 U2 U3 U4]';
ANG=[theta1 X_X(1) X_X(2) X_X(3)]';% Voltage phase angles
VOLT=[U1 X_X(4) X_X(5) X_X(6)]'; % Voltage magnitudes
```

```
ANG_deg=ANG*deg;% in degrees
VOLT_kV=VOLT*Ubase; % in kV

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% The generated active and reactive power at slack bus and PU-buses
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
g=-G;b=-B;
% At slack bus, bus 1
P12=g(1,2)*VOLT(1)^2-...
    VOLT(1)*VOLT(2)*(g(1,2)*cos(ANG(1)-ANG(2))+b(1,2)*sin(ANG(1)-ANG(2)));
P13=g(1,3)*VOLT(1)^2-...
    VOLT(1)*VOLT(3)*(g(1,3)*cos(ANG(1)-ANG(3))+b(1,3)*sin(ANG(1)-ANG(3)));
Q12=(-bsh12-b(1,2))*VOLT(1)^2-...
    VOLT(1)*VOLT(2)*(g(1,2)*sin(ANG(1)-ANG(2))-b(1,2)*cos(ANG(1)-ANG(2)));
Q13=(-bsh13-b(1,3))*VOLT(1)^2-...
    VOLT(1)*VOLT(3)*(g(1,3)*sin(ANG(1)-ANG(3))-b(1,3)*cos(ANG(1)-ANG(3)));%

PG1=P12+P13+PLD1;%
QG1=Q12+Q13+QLD1;%
% You may also use equation (7.43) (Pk and Qk) to find PG1 and QG1
PG1= ; % based on equation (7.43)
QG1= ; % based on equation (7.43)

PG1_MW=PG1*Sbase;%
QG1_MVAr=QG1*Sbase;
```




```
%%%%%%%%%
% Losses
%%%%%%%%%
PLoss_tot=(PG1+PG2+PG3+PG4)-(PLD1+PLD2+PLD3+PLD4);%
PLoss_tot_MW=PLoss_tot*Sbase;


PLoss_Sys1=  ;% Find the power losses in System 1 (pu)
PLoss_Sys1_MW=PLoss_Sys1*Sbase;

PLoss_Sys2_MW= ; % Find the power losses in System 2 (MW)

% End of file (run_LF.m)
```

```
%%%%%%%%%%%%%%
% Second file
%%%%%%%%%%%%%%
% Start of file (solve_lf.m)
% This function solves g(x)=0 for x.

function [g_x]=solve_lf(X,G,B,PGD,QGD,PAR);

nx=PAR(1); nbus=PAR(2); U1=PAR(3); theta1=PAR(4);

PGD2=PGD(1); PGD3=PGD(2); PGD4=PGD(3); QGD2=QGD(1);
QGD3=QGD(2); QGD4=QGD(3);

theta2=X(1); theta3=X(2); theta4=X(3);

U2=X(4); U3=X(5); U4=X(6);
```

```
ANG=[theta1 theta2 theta3 theta4]' ;  VOLT=[U1 U2 U3 U4]';

% We have nx unknown variables, therefore the
% size of g(x) is nx by 1.

g_x=zeros(nx,1);

%Based on equation (7.43), find Pk and Qk
P2=  ;
P3=  ;
P4=  ;
Q2=  ;
Q3=  ;
Q4=  ;
```

```
% Active power mismatch (PU- and PQ-buses)
% Bus 2
g_x(1)=P2-PGD2;
% Bus 3
g_x(2)=P3-PGD3;
% Bus 4
g_x(3)=P4-PGD4;
% Reactive power mismatch (PQ-buses)
% Bus 2
g_x(4)=Q2-QGD2;
% Bus 3
g_x(5)=Q3-QGD3;
% Bus 4
g_x(6)=Q4-QGD4;

% End of file (solve_lf.m)
```

a) *the unknown voltage magnitudes and voltage phase angles,*

b) *the generated active and reactive powers at the slack bus, and the generated reactive powers at PU-buses (if any),*

c) *the total active power losses, and the losses in System1 and System 2,*

d) *the changes (in % compared to the obtained results in task* **c)***) of power losses in both systems, for an active load increased at bus 2 with* 30 *MW, i.e.* $P_{LD2}^{new} = 120$ *MW.*

e) *Let* $P_{LD2} = 90$ *MW. Re-do task* **d)** *for a reactive load increased at bus 3 with* 10 *MVAr, i.e.* $Q_{LD3}^{new} = 20$ *MVAr.*

a)
$$U_1 = 1.0000 \times U_{base} = 220.0000 \text{ kV}, \ \theta_1 = 0°,$$
$$U_2 = 0.9864 \times U_{base} = 216.9990 \text{ kV}, \ \theta_2 = -7.8846°,$$
$$U_3 = 0.9794 \times U_{base} = 215.4704 \text{ kV}, \ \theta_3 = -8.7252°,$$
$$U_4 = 0.9693 \times U_{base} = 213.2499 \text{ kV}, \ \theta_4 = -10.5585°,$$

b) Slack bus (bus 1): $P_{G1} = 232.4938$ MW, $Q_{G1} = 9.6185$ MVAr

c) $P_{Loss}^{\text{Sys1}} = 0.1715$ MW , $P_{Loss}^{\text{Sys2}} = 2.3222$ MW , $P_{Loss}^{\text{tot}} = 2.4937$ MW

a) *the unknown voltage magnitudes and voltage phase angles,*

b) *the generated active and reactive powers at the slack bus, and the generated reactive powers at PU-buses (if any),*

c) *the total active power losses, and the losses in System1 and System 2,*

d) *the changes (in % compared to the obtained results in task* **c)***) of power losses in both systems, for an active load increased at bus 2 with* 30 *MW, i.e.* $P_{LD2}^{new} = 120$ *MW.*

e) *Let* $P_{LD2} = 90$ *MW. Re-do task* **d)** *for a reactive load increased at bus 3 with* 10 *MVAr, i.e.* $Q_{LD3}^{new} = 20$ *MVAr.*

d)

$$P_{Loss}^{\mathrm{Sys1}} = 0.1729 \text{ MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\mathrm{Sys1}} = 0.8163\,\%$$

$$P_{Loss}^{\mathrm{Sys2}} = 3.0236 \text{ MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\mathrm{Sys2}} = 30.2041\,\%$$

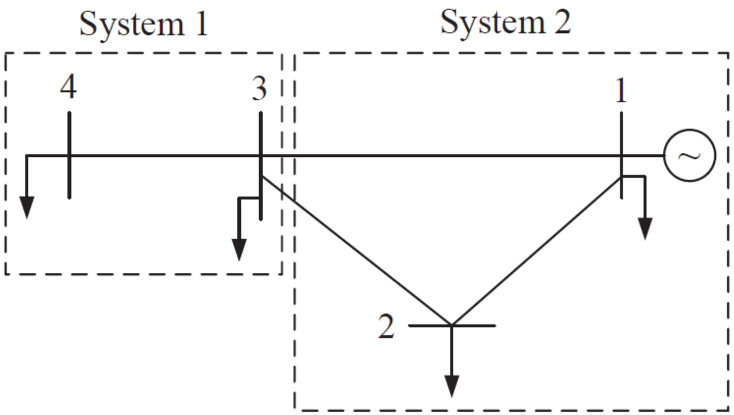
$$P_{Loss}^{\mathrm{tot}} = 3.1965 \text{ MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\mathrm{tot}} = 28.1830\,\%$$

e)

$$P_{Loss}^{\mathrm{Sys1}} = 0.1749 \text{ MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\mathrm{Sys1}} = 1.9825\,\%$$

$$P_{Loss}^{\mathrm{Sys2}} = 2.3629 \text{ MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\mathrm{Sys2}} = 1.7526\,\%$$

$$P_{Loss}^{\mathrm{tot}} = 2.5378 \text{ MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\mathrm{tot}} = 1.7685\,\%$$



Bus types

Bus 1

Bus 2

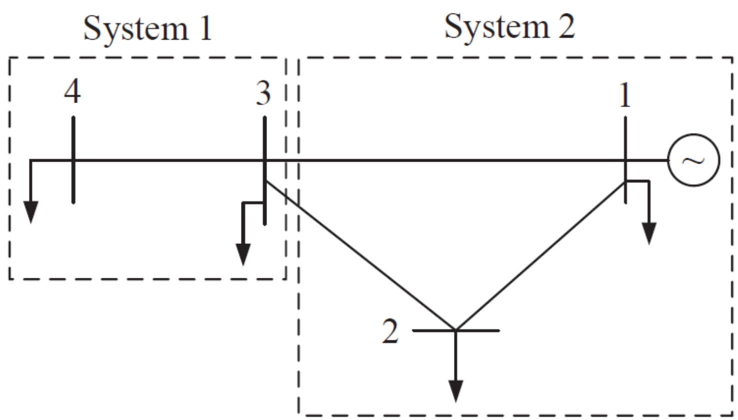
Bus 3

Bus 4

Example 7.15 *Consider again the power system in Example 7.14. The System 1 operator is interested in the results of the power flow calculations when installing a controllable shunt capacitor at bus 3 to keep the voltage at its rated (or nominal) value, i.e. $U_3 = 220\ kV$. Re-do the tasks in Example 7.14, and also find the size of the shunt capacitor B_{sh} in S.*

What should be changed here?

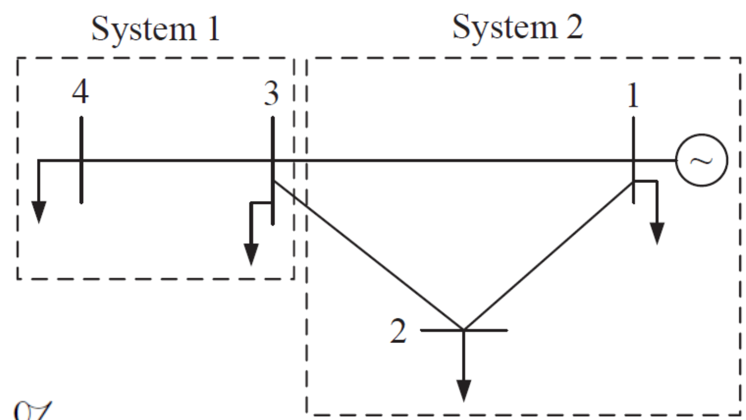
$$
\begin{aligned}
x &= [\theta_2\ \theta_3\ \theta_4\ U_2\ \cancel{U_3}\ U_4] \\
0 &= g_1(x) = P_2 - P_{GD2} \quad , \quad \text{(active power mismatch at bus 2, see equation (7.44))} \\
0 &= g_2(x) = P_3 - P_{GD3} \quad , \quad \text{(active power mismatch at bus 3)} \\
0 &= g_3(x) = P_4 - P_{GD4} \quad , \quad \text{(active power mismatch at bus 4)} \\
0 &= g_4(x) = Q_2 - Q_{GD2} \quad , \quad \text{(reactive power mismatch at bus 2)} \\
\cancel{0} &\cancel{= g_5(x) = Q_3 - Q_{GD3}} \quad , \quad \text{(reactive power mismatch at bus 3)} \\
0 &= g_6(x) = Q_4 - Q_{GD4} \quad , \quad \text{(reactive power mismatch at bus 4)}
\end{aligned}
$$



a) $U_1 = 1.0000 \times U_{base} = 220.0000\ \text{kV},\ \theta_1 = 0°,$
$U_2 = 0.9968 \times U_{base} = 219.2882\ \text{kV},\ \theta_2 = -7.8192°,$
$U_3 = 1.0000 \times U_{base} = 220.0000\ \text{kV},\ \theta_3 = -8.6473°,$
$U_4 = 0.9901 \times U_{base} = 217.8306\ \text{kV},\ \theta_4 = -10.4051°,$

b) Slack bus (bus 1): $P_{G1} = 232.4490$ MW, $Q_{G1} = -14.7469$ MVAr
PU-buses (bus 3): $Q_{G3} = 22.5772$ MVAr and $B_{sh} = 0.00046647$ S

c) $P_{Loss}^{\text{Sys1}} = 0.1644$ MW , $P_{Loss}^{\text{Sys2}} = 2.2846$ MW , $P_{Loss}^{\text{tot}} = 2.4490$ MW

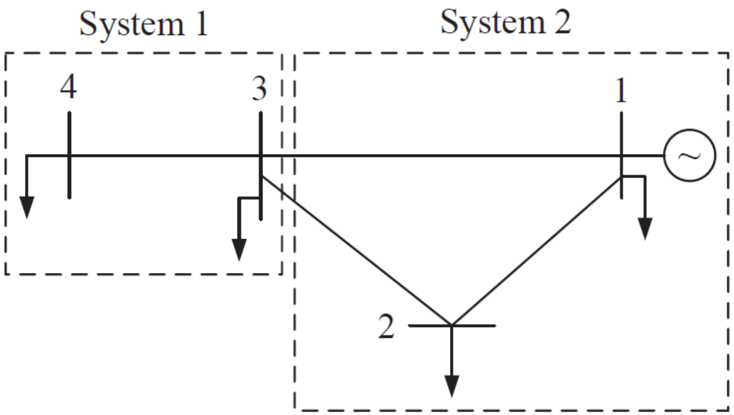


$$P_{LD2}^{new} = 120\ MW.$$

d)
$$\begin{aligned}
P_{Loss}^{\mathrm{Sys1}} &= 0.1644 \text{ MW} &\Rightarrow\quad \Delta P_{Loss}^{\mathrm{Sys1}} &= 0\,\% \\
P_{Loss}^{\mathrm{Sys2}} &= 2.9622 \text{ MW} &\Rightarrow\quad \Delta P_{Loss}^{\mathrm{Sys2}} &= 29.6595\,\% \\
P_{Loss}^{\mathrm{tot}} &= 3.1266 \text{ MW} &\Rightarrow\quad \Delta P_{Loss}^{\mathrm{tot}} &= 27.6684\,\%
\end{aligned}$$

$$Q_{LD3}^{new} = 20\ MVAr.$$

e)
$$\begin{aligned}
P_{Loss}^{\mathrm{Sys1}} &= 0.1644 \text{ MW} &\Rightarrow\quad \Delta P_{Loss}^{\mathrm{Sys1}} &= 0\,\% \\
P_{Loss}^{\mathrm{Sys2}} &= 2.2846 \text{ MW} &\Rightarrow\quad \Delta P_{Loss}^{\mathrm{Sys2}} &= 0\,\% \\
P_{Loss}^{\mathrm{tot}} &= 2.4490 \text{ MW} &\Rightarrow\quad \Delta P_{Loss}^{\mathrm{tot}} &= 0\,\%
\end{aligned}$$



Bus types

Bus 1

Bus 2

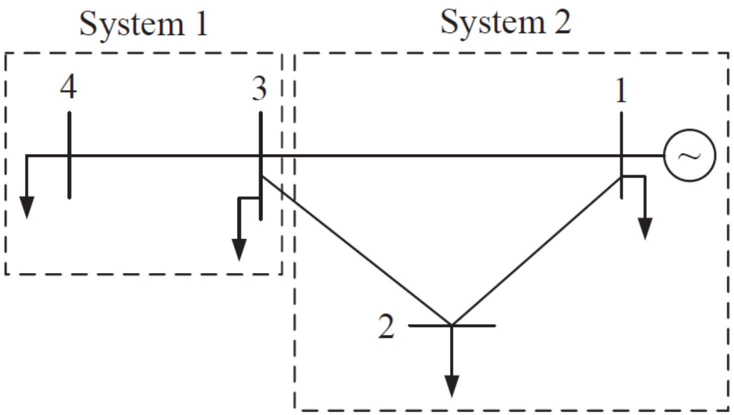
Bus 3

Bus 4

Example 7.16 *Consider the power system described in Example 7.15. Now, both system operators are interested in the results of the power flow calculations when the generator at bus 1 has a fixed generation with* P_{G1} *and* Q_{G1} *obtained in Example 7.15, and a new generator is installed at bus 4 to be a slack bus with* U_4 *and* θ_4 *obtained in Example 7.15. Re-do the tasks in Example 7.15.*

What should be changed here?

$$
\begin{aligned}
x &= [\theta_2 \; \theta_3 \; \theta_4 \; U_2 \; \cancel{U_3} \; U_4] \\
0 &= g_1(x) = P_2 - P_{GD2} \\
0 &= g_2(x) = P_3 - P_{GD3} \\
0 &= g_3(x) = P_4 - P_{GD4} \\
0 &= g_4(x) = Q_2 - Q_{GD2} \\
&\cancel{0 = g_5(x) = Q_3 - Q_{GD3}} \\
0 &= g_6(x) = Q_4 - Q_{GD4}
\end{aligned}
$$



a) $U_1 = 1.0000 \times U_{base} = 220.0000$ kV, $\theta_1 = 0°$,
$U_2 = 0.9968 \times U_{base} = 219.2882$ kV, $\theta_2 = -7.8192°$,
$U_3 = 1.0000 \times U_{base} = 220.0000$ kV, $\theta_3 = -8.6473°$,
$U_4 = 0.9901 \times U_{base} = 217.8306$ kV, $\theta_4 = -10.4051°$,

a) $U_1 = 1.0000 \times U_{base} = 220.0000$ kV, $\theta_1 = 0°$,
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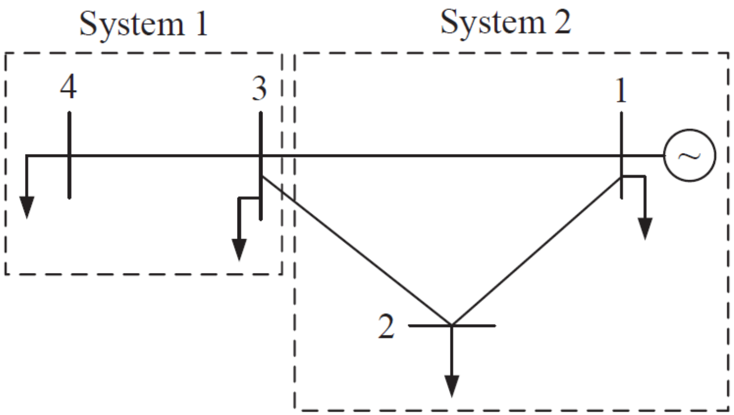
$U_4 = 0.9901 \times U_{base} = 217.8306$ kV, $\theta_4 = -10.4051°$,

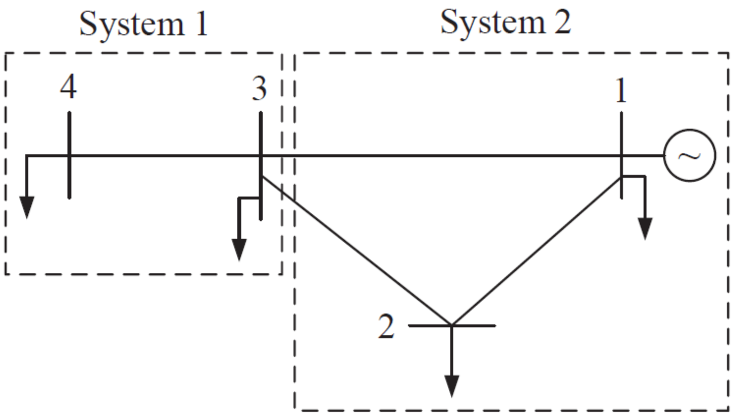


b) Slack bus (bus 1): $P_{G1} = 232.4490$ MW, $Q_{G1} = -14.7469$ MVAr
PU-buses (bus 3): $Q_{G3} = 22.5772$ MVAr and $B_{sh} = 0.00046647$ S

b) Slack bus (bus 4): $P_{G4} = 0$ $\quad Q_{G4} = 0$

PU-buses (bus 3): $Q_{G3} = 22.5772$ MVAr

$B_{sh} = 0.00046647$ S

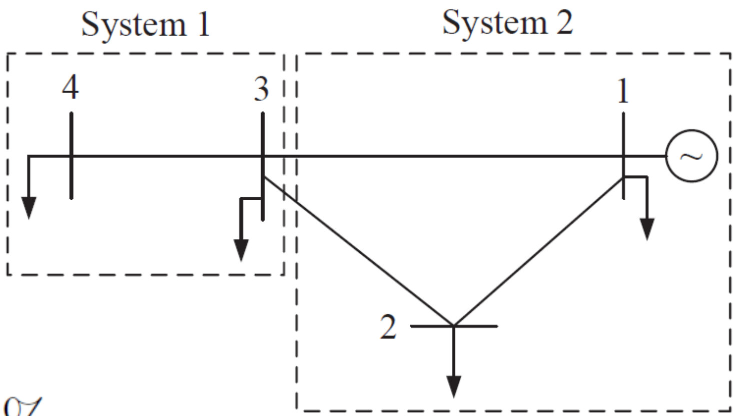


c) $P_{Loss}^{\mathrm{Sys1}} = 0.1644\ \mathrm{MW}$, $P_{Loss}^{\mathrm{Sys2}} = 2.2846\ \mathrm{MW}$, $P_{Loss}^{\mathrm{tot}} = 2.4490\ \mathrm{MW}$

c) $P_{Loss}^{\mathrm{Sys1}} = 0.1644\ \mathrm{MW}$

$P_{Loss}^{\mathrm{Sys2}} = 2.2846\ \mathrm{MW}$

$P_{Loss}^{\mathrm{tot}} = 2.4490\ \mathrm{MW}$

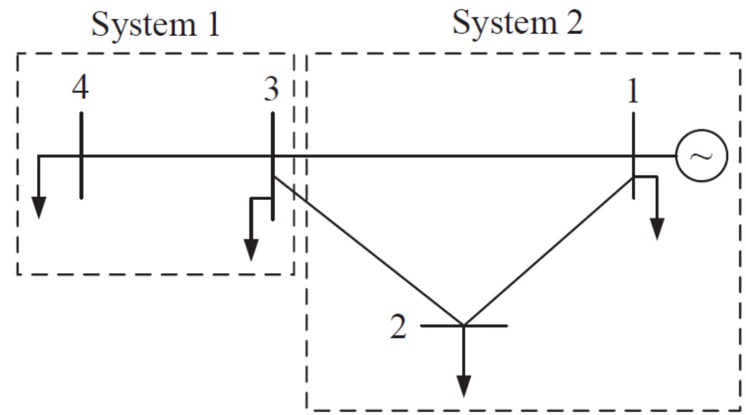


$$P_{LD2}^{new} = 120\ MW.$$

d)
$$P_{Loss}^{\text{Sys1}} = 0.1644\ \text{MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\text{Sys1}} = 0\,\%$$
$$P_{Loss}^{\text{Sys2}} = 2.9622\ \text{MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\text{Sys2}} = 29.6595\,\%$$
$$P_{Loss}^{\text{tot}} = 3.1266\ \text{MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\text{tot}} = 27.6684\,\%$$

d)
$$P_{Loss}^{\text{Sys1}} = 0.0373\ \text{MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\text{Sys1}} = -77.3114\,\%$$
$$P_{Loss}^{\text{Sys2}} = 2.3232\ \text{MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\text{Sys2}} = 1.6896\,\%$$
$$P_{Loss}^{\text{tot}} = 2.3605\ \text{MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\text{tot}} = -3.6137\,\%$$

$\boxed{P_{G4} = 29.9115}$ $\boxed{Q_{G4} = \text{-}3.6331}$



$$Q_{LD3}^{new} = 20\ MVAr.$$

e) $P_{Loss}^{\mathrm{Sys1}} = 0.1644\ \mathrm{MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\mathrm{Sys1}} = 0\,\%$

$P_{Loss}^{\mathrm{Sys2}} = 2.2846\ \mathrm{MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\mathrm{Sys2}} = 0\,\%$

$P_{Loss}^{\mathrm{tot}} = 2.4490\ \mathrm{MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\mathrm{tot}} = 0\,\%$

e) $P_{Loss}^{\mathrm{Sys1}} = 0.1644\ \mathrm{MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\mathrm{Sys1}} = 0\,\%$

$P_{Loss}^{\mathrm{Sys2}} = 2.2846\ \mathrm{MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\mathrm{Sys2}} = 0\,\%$

$P_{Loss}^{\mathrm{tot}} = 2.4490\ \mathrm{MW} \quad \Rightarrow \quad \Delta P_{Loss}^{\mathrm{tot}} = 0\,\%$