Power Systems Analysis, L3
2018

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Reciprocity theorem

Assume that a voltage source is connected to a terminal *a* in a linear reciprocal networkand is giving rise to a current at terminal *b*. According to the reciprocity theorem, the voltage source will cause the same current at *a* if it is connected to *b*.

Twoport theory

$$\begin{bmatrix} \overline{U}_a \\ \overline{I}_a \end{bmatrix} = \begin{bmatrix} \overline{A} & \overline{B} \\ \overline{C} & \overline{D} \end{bmatrix} \begin{bmatrix} \overline{U}_b \\ \overline{I}_b \end{bmatrix}$$

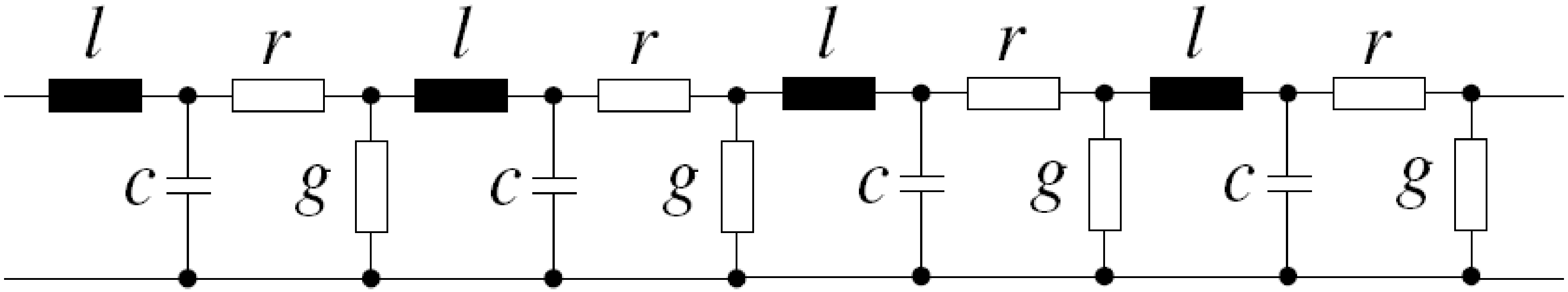
A reciprocal twoport $\Rightarrow$

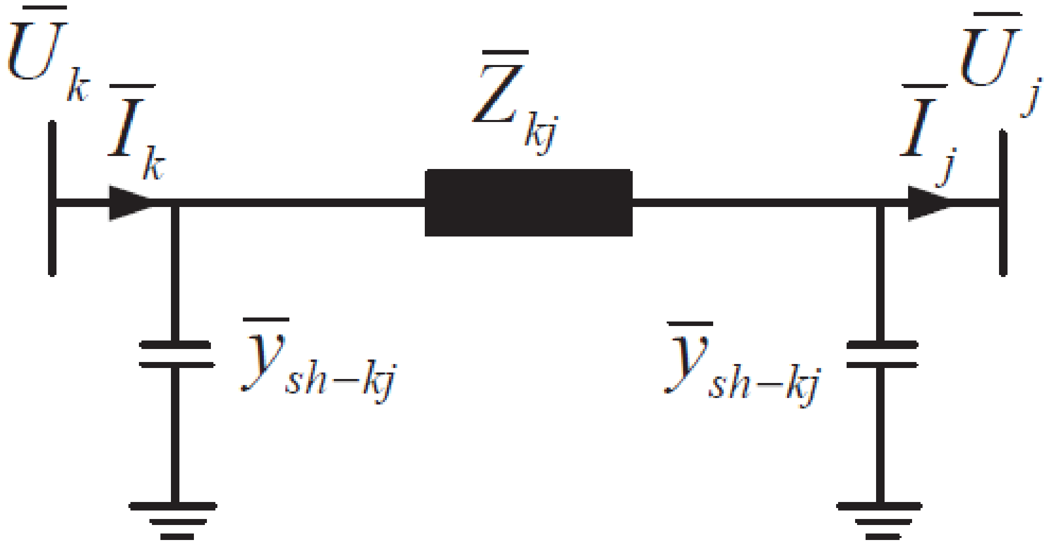
$$\overline{A} \cdot \overline{D} - \overline{B} \cdot \overline{C} = 1$$

Symmetrical twoports $\Rightarrow$

$$\overline{A} = \overline{D}$$

Model of symmetrical transmission line





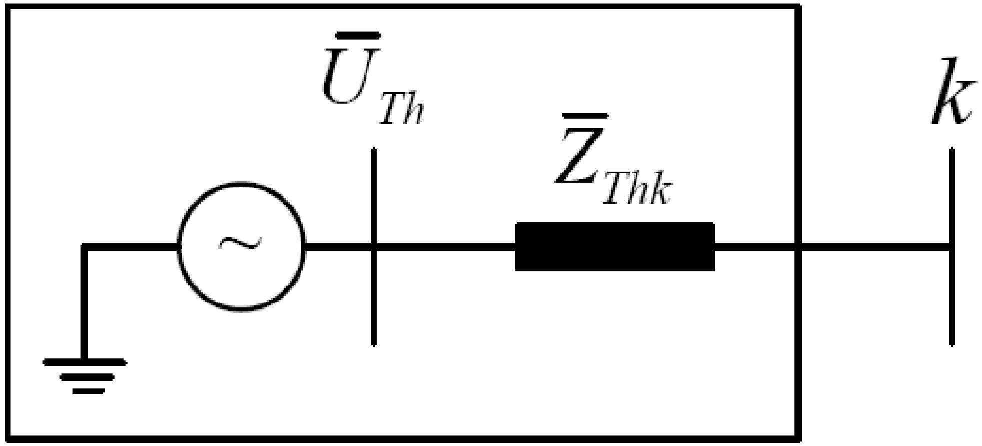
Two port parameters for the line :

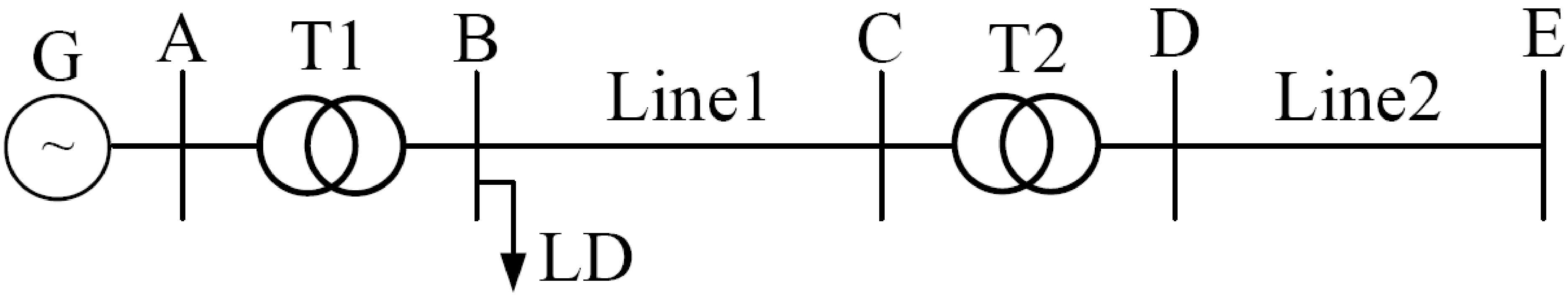
$$\begin{bmatrix} \overline{U}_k \\ \overline{I}_k \end{bmatrix} = \begin{bmatrix} \overbrace{1+\overline{y}_{sh-kj}\cdot\overline{Z}_{kj}}^{\overline{A}} & \overbrace{\overline{Z}_{kj}}^{\overline{B}} \\ \underbrace{\overline{y}_{sh-kj}(2+\overline{y}_{sh-kj}\cdot\overline{Z}_{kj})}_{\overline{C}} & \underbrace{1+\overline{y}_{sh-kj}\cdot\overline{Z}_{kj}}_{\overline{D}} \end{bmatrix} \begin{bmatrix} \overline{U}_j \\ \overline{I}_j \end{bmatrix}$$

Two-port parameters for a transformer and a short line $\overline{Y}_{sh-kj} = 0$

$$\begin{bmatrix} \overline{U}_k \\ \overline{I}_k \end{bmatrix} = \begin{bmatrix} 1 & \overline{Z}_{kj} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \overline{U}_j \\ \overline{I}_j \end{bmatrix}$$

Connection to a network

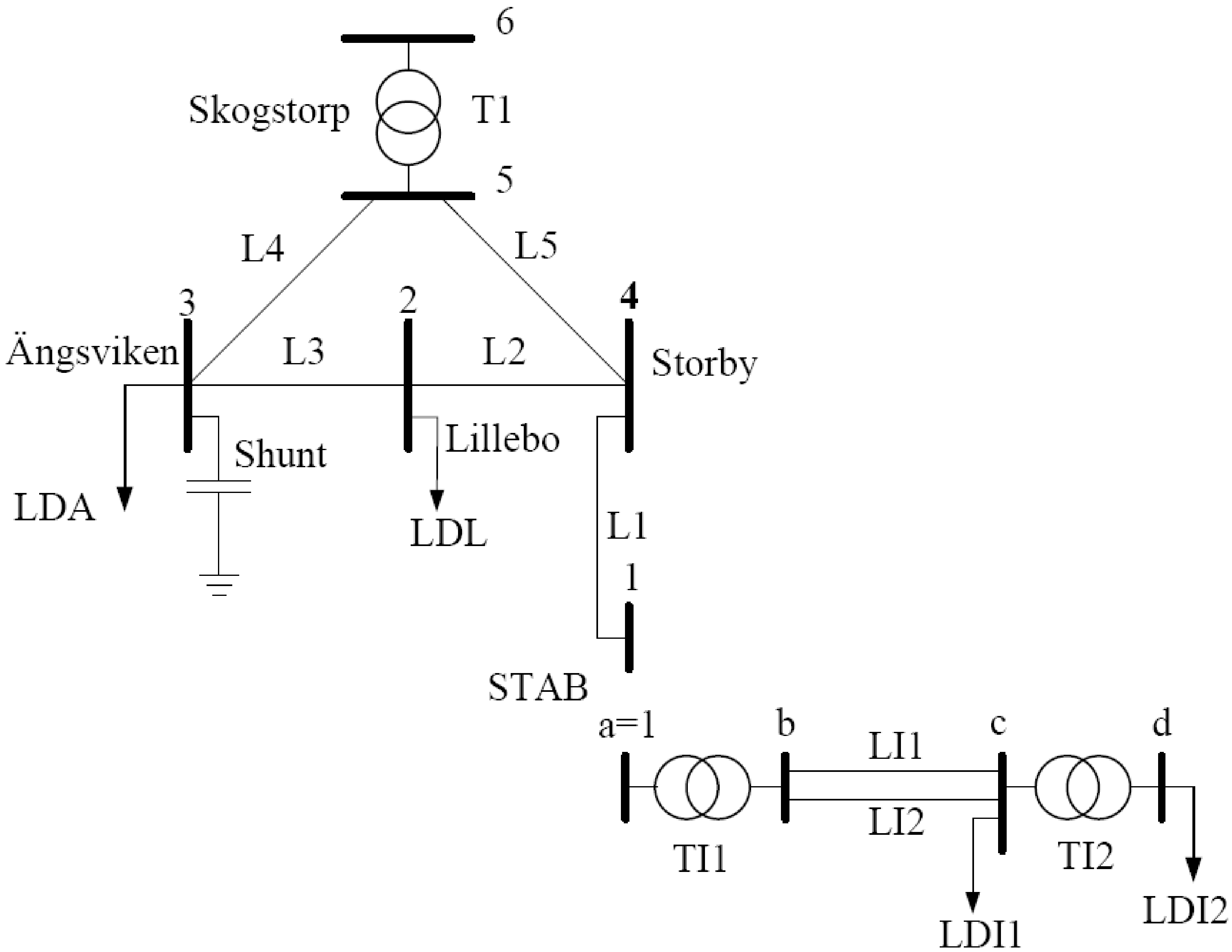




Example 6.1

At a bus with a short circuit capacity of 500 MVA and $\cos\varphi_k = 0$, inductive, an impedance load of 4 MW, $\cos\varphi_{LD}=0.8$ at nominal voltage, is connected. Calculate the change in bus voltage when the load is connected.

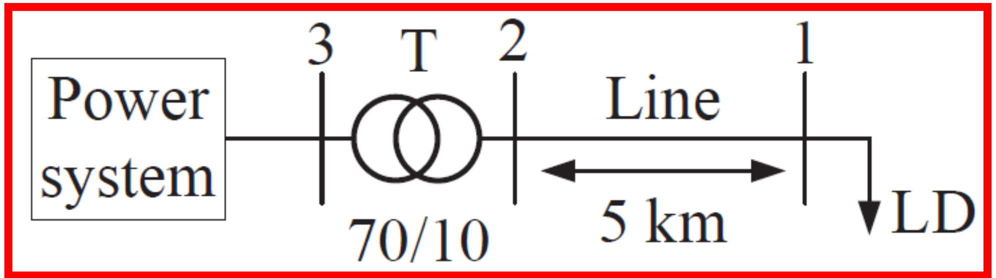
Home Exam S1-S3

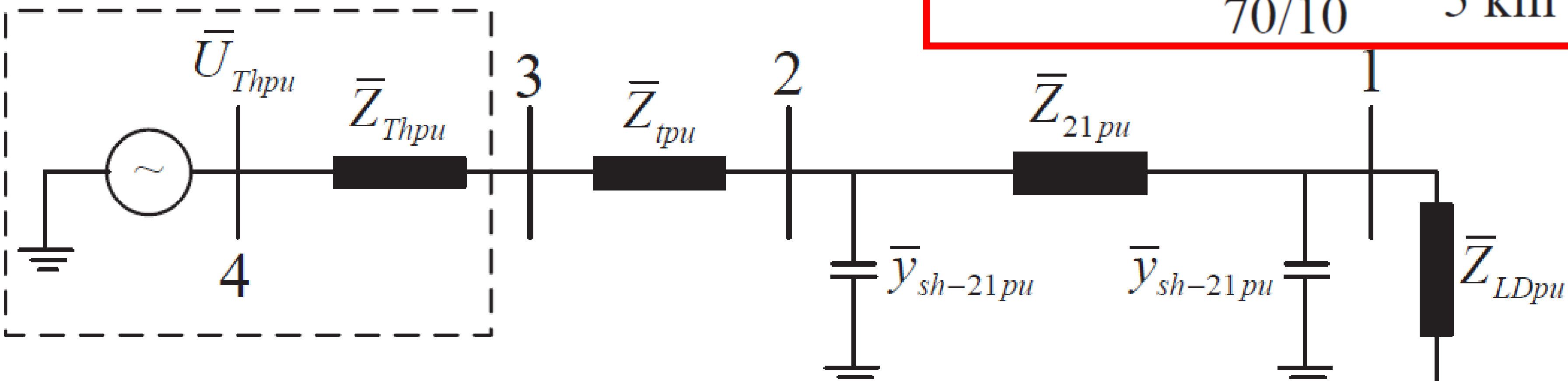


Home Exam S1 (Sept-18)

This assignment is dealing with the electrical network of a factory **<u>fed from</u>** a 40 kV meshed grid. The system data can be found in the Excel file.

Example 6.2





Thévenin equivalent of the power system

Transformer

Line

Industry

Home Exam S1 (Sept-18), Part 1

a. Based on the given data, draw the single-line diagram of the electrical network of the factory. **(5%)**

b. As seen from node **a**, make the two-port model and give the values of the elements of the two-port matrix in pu. **(10%)**

c. Give the value of the impedance of the entire system in pu. **(10%)**

d. Assume that the transformer "TI1" is fed with nominal voltage. Find the voltage at node **e** in kV. **(20%)**

e. Find the consumed active power of the load at node **e** in MW, and also its power factor. **(20%)**

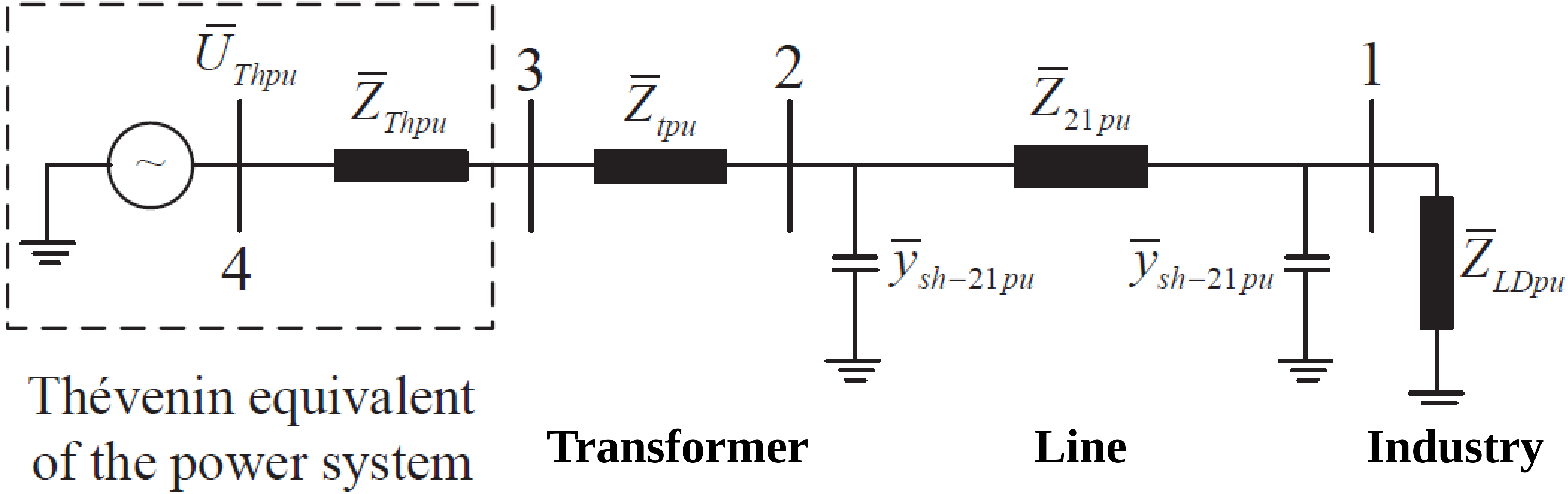
f. Find the total losses in the factory in kW. **(20%)**

g. Plot the normalized active power consumption at node **e** (based on the obtained active power in task d) when the voltage at node **a** varies from 0.8 to 1.2 pu. **(15%)**

Example 6.2

A small industry is fed by a transformer (5 MVA, 70/10 kV, x = 4%) which is located at a distance of 5 km. The electric power demand of the industry is 400 kW at cosφ=0.8, lagging, at a voltage of 10 kV. The industry can be modeled as an impedance load. The 10 kV line has an series impedance of 0.9+j0.3 Ω/phase, km and a shunt admittance of $j3\cdot10^{-6}$ S/phase, km. Assume that the line is modeled by the II-equivalent. Calculate the voltage level at the industry as well as the power fed by the transformer into the line. When the industry is not connected, a short circuit current of 0.3 kA side of the transformer when a three-phase short circuit is applied at nominal voltage.

Example 6.2

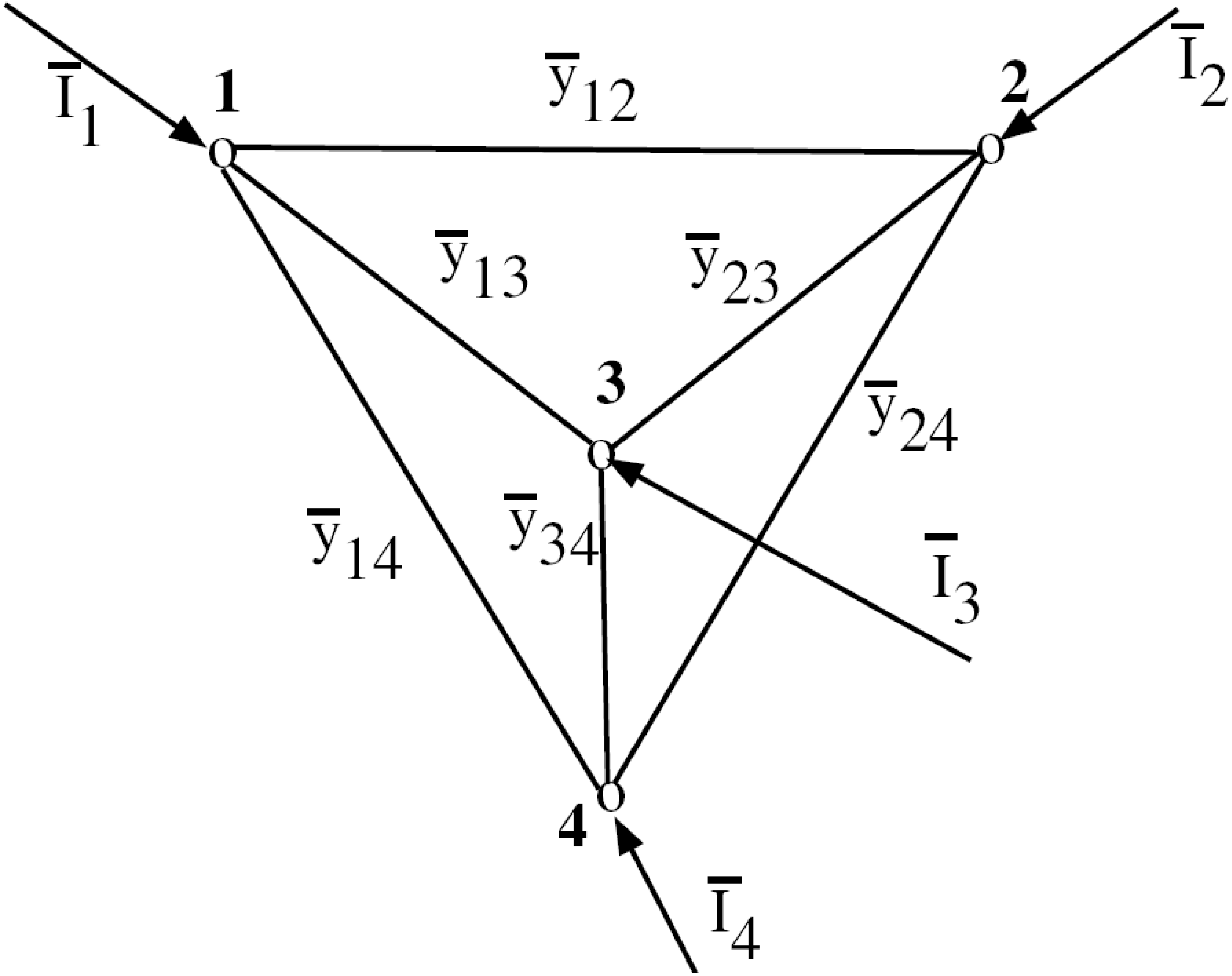


$\bar{U}_{Thpu}$
$\bar{Z}_{Thpu}$
3
$\bar{Z}_{tpu}$
2
$\bar{Z}_{21pu}$
1
4
$\bar{y}_{sh-21pu}$
$\bar{y}_{sh-21pu}$
$\bar{Z}_{LDpu}$
Thévenin equivalent of the power system
Transformer
Line
Industry

```
clear

deg=180/pi;
rad=1/deg;

%--- Example 6.2
% Choose the base values
Sb=0.5; Ub10=10; Ib10=Sb/Ub10/sqrt(3);Zb10=Ub10^2/Sb;
Ub70=70; Ib70=Sb/Ub70/sqrt(3);

%Calculate the per-unit values of the Thevenin equivalent of the system
UTh=70*exp(j*0*rad);
Isc=0.3*exp(j*-90*rad);
UThpu =UTh/Ub70;
Iscpu =Isc/Ib70;
ZThpu =UThpu/Iscpu;

% Calculate the per-unit values of the transformer
Zt=j*4/100;Snt=5;
Ztpu=Zt*Sb/Snt;

%Calculate the per-unit values of the line
Z21pu=5*(0.9+j*0.3)/Zb10;
ysh21pu=5*(j*3*1E-6)*Zb10/2;

%Calculate the per-unit values of the industry impedance
cosphi=0.8;sinphi=sqrt(1-cosphi^2);
Un=Ub10;PLD=0.4;absSLD=PLD/cosphi;
SLD=absSLD*(cosphi+j*sinphi);
ZLDpu=Un^2/conj(SLD)/Zb10
```

```matlab
% The twoport of the system
AL=1+ysh21pu*Z21pu;
BL=Z21pu;
CL=ysh21pu*(2+ysh21pu*Z21pu);
DL=AL;

F_L=[AL BL ; CL DL];
F_Th_tr=[1 ZThpu+Ztpu ; 0 1];
F_tot=F_Th_tr*F_L;

%The impedance of the entire system
Ztotpu=(F_tot(1,1)*ZLDpu+F_tot(1,2))/(F_tot(2,1)*ZLDpu+F_tot(2,2))
I4pu = UThpu/Ztotpu;

%The power fed by the transformer into the line
U2pu_I2pu=inv(F_Th_tr)*[UThpu;I4pu];
S2=U2pu_I2pu(1,1)*conj(U2pu_I2pu(2,1))*Sb

%The voltage at the industry
U1pu_I1pu=inv(F_tot)*[UThpu;I4pu];
U1=abs(U1pu_I1pu(1,1))*Ub10,
```

Admittance matrix - 1

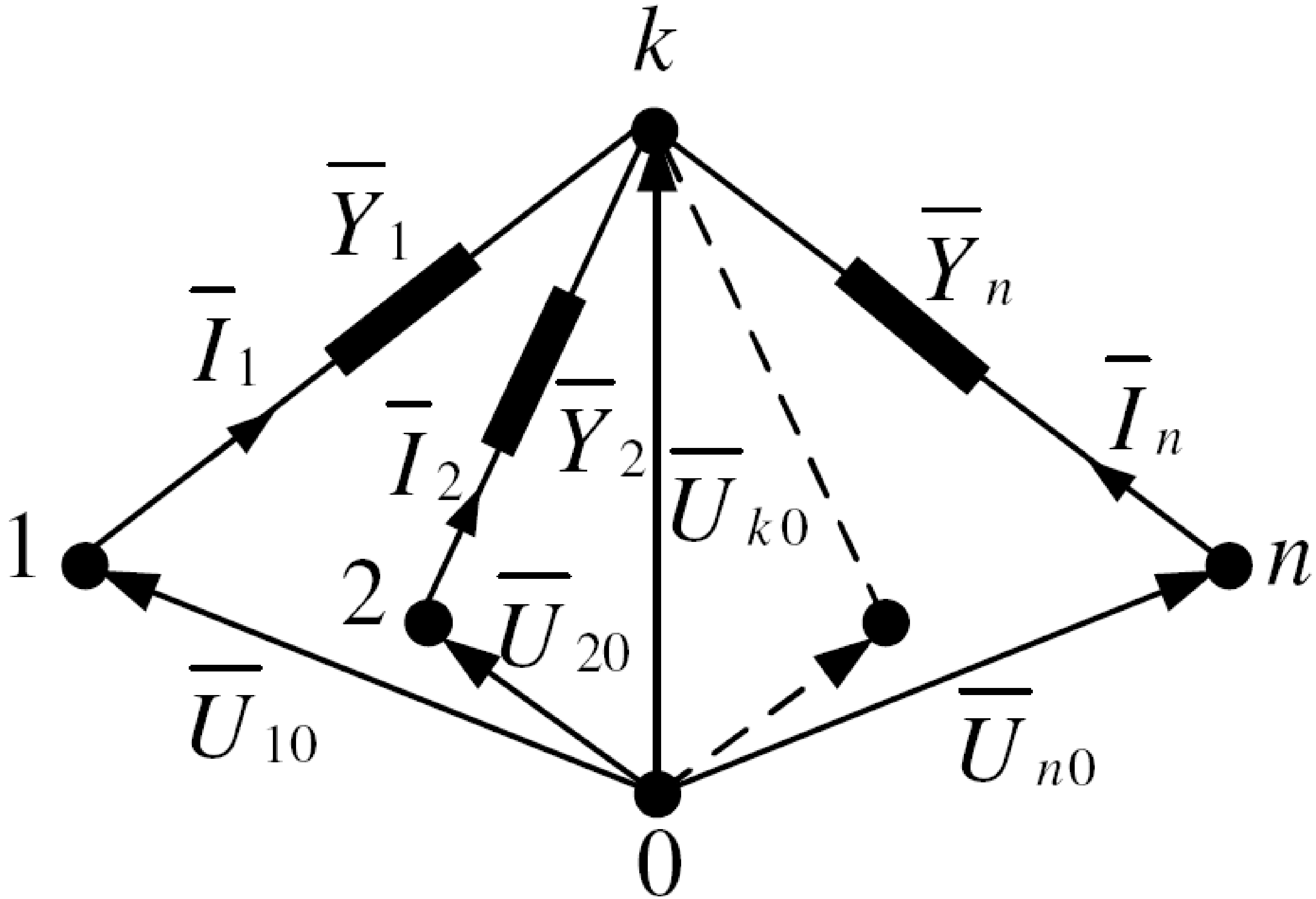


Admittance matrix - 2

For the Y-bus matrix the following properties are valid:

- It can be uniquely determined from a given admittance network.
- The diagonal element Y_{kk} = the sum of all admittances connected to bus k.
- Non-diagonal element $Y_{ik} = -y_{ik}$ where y_{ik} is the admittance between bus *i* and bus *k*.
- This gives that the matrix is symmetric, i.e. $Y_{ik} = Y_{ki}$ (one exception is when the network includes phase shifting transformers).
- It is singular since $I_1+I_2+I_3+I_4=0$

Millman's teorem - 1



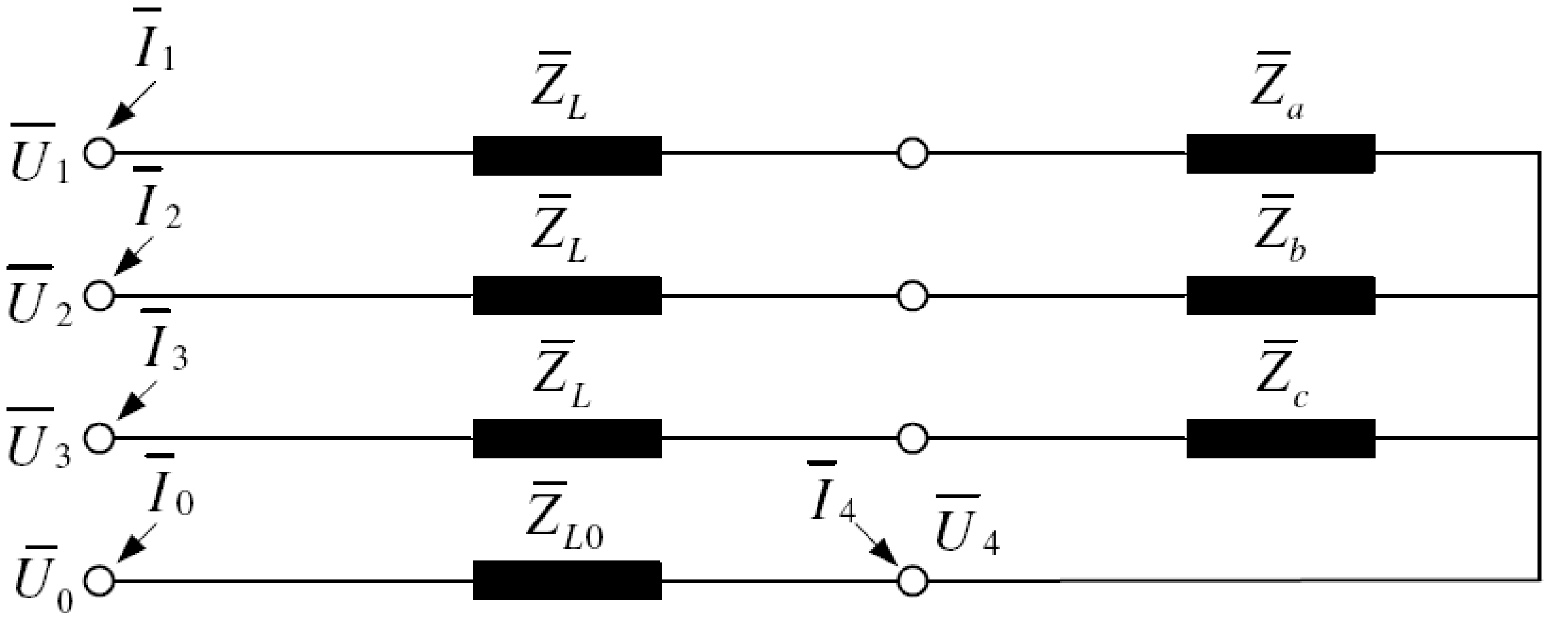
Millman's teorem - 2

The Y-bus matrix for this network can be formed as

$$\begin{bmatrix} \overline{I}_1 \\ \overline{I}_2 \\ \vdots \\ \overline{I}_n \\ \overline{I}_k \end{bmatrix} = \begin{bmatrix} \overline{Y}_1 & 0 & \dots & 0 & -\overline{Y}_1 \\ 0 & \overline{Y}_2 & \dots & 0 & -\overline{Y}_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \overline{Y}_n & -\overline{Y}_n \\ -\overline{Y}_1 & -\overline{Y}_2 & \dots & -\overline{Y}_n & (\overline{Y}_1 + \overline{Y}_2 + \dots \overline{Y}_n) \end{bmatrix} \begin{bmatrix} \overline{U}_{10} \\ \overline{U}_{20} \\ \vdots \\ \overline{U}_{n0} \\ \overline{U}_{k0} \end{bmatrix}$$

Example 5.2

The student Elektra lives in a house situated 2 km from a transformer having a completely symmetrical three-phase voltage (Ua = 220 V $\angle$ 0°, U_b = 220 V $\angle$ –120°, U_c = 220 V $\angle$120°). The house is connected to this transformer via a three-phase cable (EKKJ, 3·16 mm^2 + 16 mm^2). A cold day, Elektra switches on one 1000 W radiator (at 220 V with cosϕ = 0.995 lagging) to phase a, three radiators to phase b and two to phase c. Assume that the cable can be modelled as four impedances connected in parallel (z_{L0} = 1.15 + j0.08 Ω/phase, km, z_{L0} = 1.15 + j0.015 Ω/km) and that the radiators also can be considered as impedances. Calculate the total thermal power given by the radiators.



$$\overline{Z}_L = 2.3 + j0.16\ \Omega \quad \overline{Z}_{L0} = 2.3 + j0.03\ \Omega$$

Z_a = 47,9+j4.81 Ω, Z_b = 15,97+j1.60 Ω,

Z_c = 23,96+j2.40 Ω