Power System Analysis, L1b

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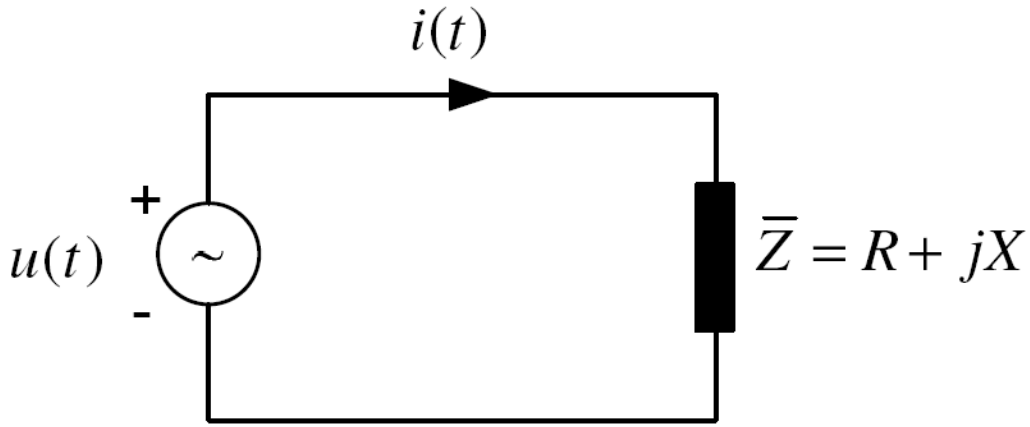
Professor in Electric Power Systems

Single-phase alternating voltage



$$u(t) = U_M \cos \omega t$$

$$i(t) = I_M \cos(\omega t - \phi)$$

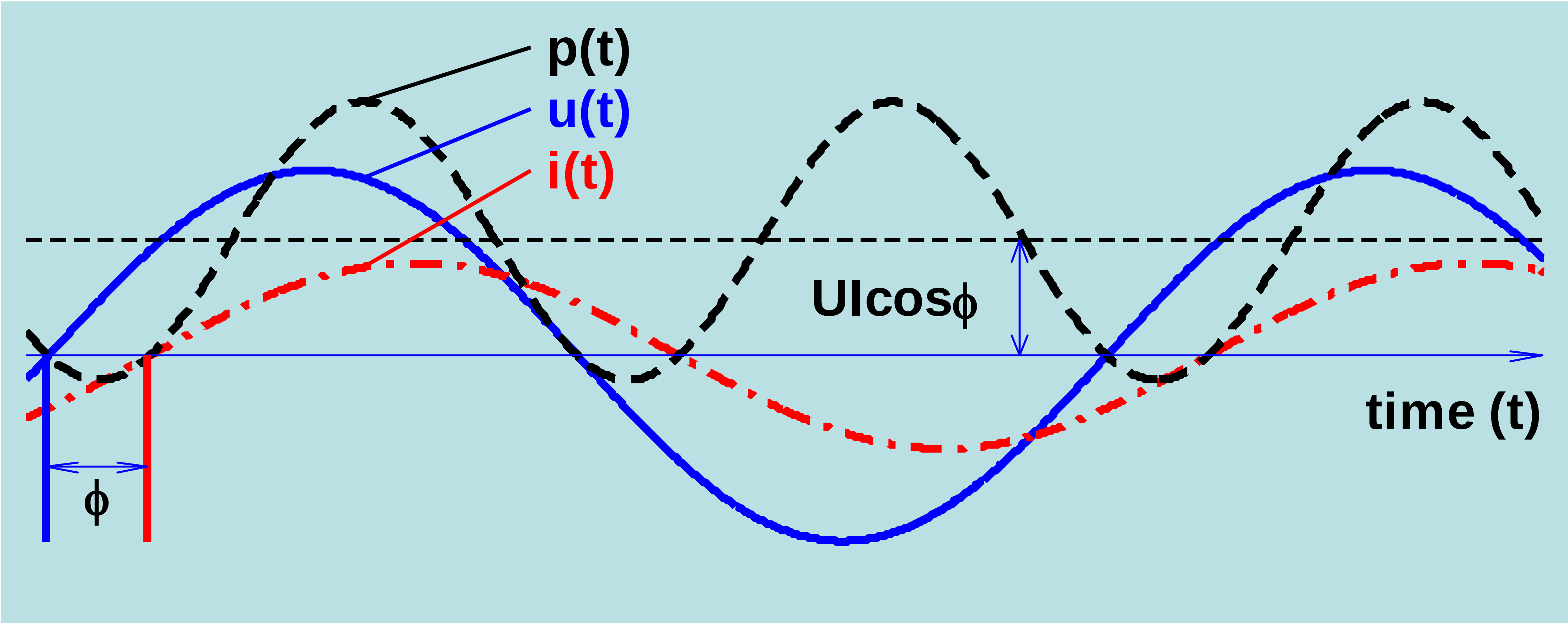
$$p(t) = u(t) \cdot i(t) = U_M I_M \cos \omega t \cos(\omega t - \phi) =$$

$$= U_M I_M \cos \omega t \left[\cos \omega t \cos \phi + \sin \omega t \sin \phi\right] =$$

$$= \frac{U_M}{\sqrt{2}} \frac{I_M}{\sqrt{2}} [(1 + \cos 2\omega t) \cos \phi + \sin 2\omega t \sin \phi] =$$

$$= P(1 + \cos 2\omega t) + Q \sin 2\omega t$$

u(t) = voltage, i(t) = current, p(t) = power



ϕ = angle between voltage and current

RMS-value of voltage and current

$$U = \sqrt{\frac{1}{T}\int_0^T u(t)^2 dt}$$

$$I = \sqrt{\frac{1}{T}\int_0^T i(t)^2 dt}$$

Sinusoidal voltage and current $\Rightarrow$

$$U = \sqrt{\frac{1}{T}\int_0^T U_M^2 \cos^2 \omega t} = U_M \sqrt{\frac{1}{T}\int_0^T \left(\frac{1}{2} + \frac{\cos 2\omega t}{2}\right)} = \frac{U_M}{\sqrt{2}}$$

$$I = \sqrt{\frac{1}{T}\int_0^T I_M^2 \cos^2(\omega t - \phi)} = \frac{I_M}{\sqrt{2}}$$

Complex power

$$\overline{U} = Ue^{j\arg(\overline{U})}$$

$$\overline{I} = Ie^{j\arg(\overline{I})}$$

The complex power is defined as

$$\overline{S} = Se^{j\arg(\overline{S})} = P + jQ = \overline{UI}^{*} = UIe^{j(\arg(\overline{U})-\arg(\overline{I}))}$$

With phase angles on voltage and current i.e. arg($\overline{U}$) = 0 and arg ($\overline{I}$) = $-\phi$

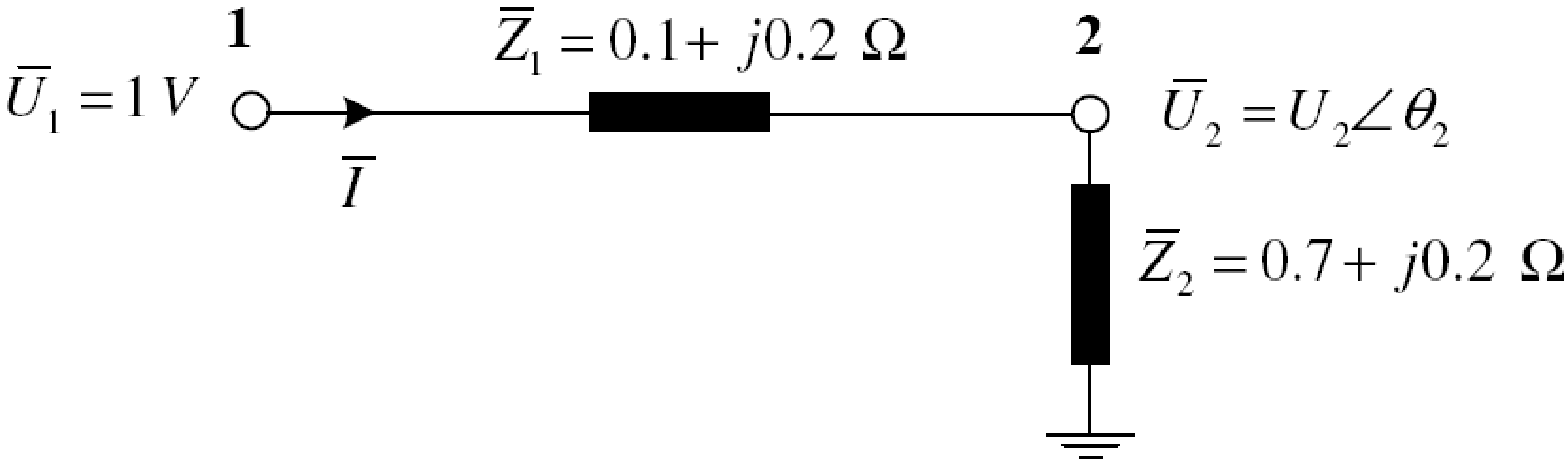
$$\overline{S} = P + jQ = \overline{U}\overline{I}^* = UIe^{j\phi} = UI(\cos\phi + j\sin\phi)$$

which implies that

$$P = S\cos\phi = UI\cos\phi$$

$$Q = S\sin\phi = UI\sin\phi$$

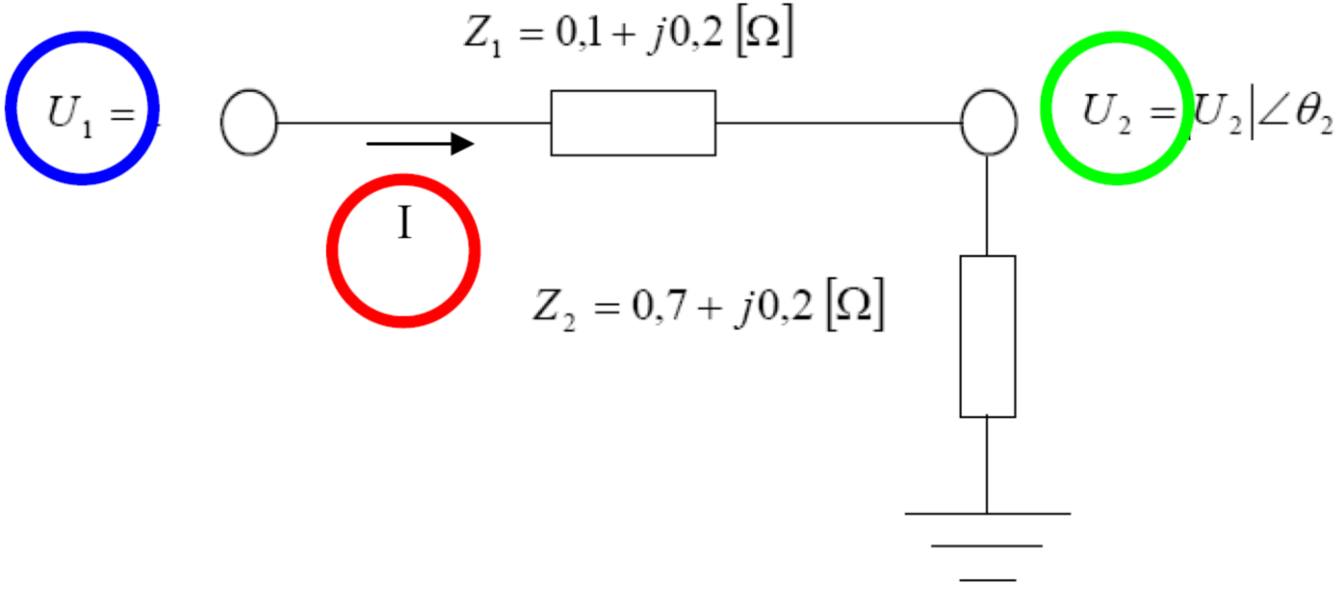
Example: Two series connected impedances are fed by a voltage having an RMS-value of 1 V according to the figure



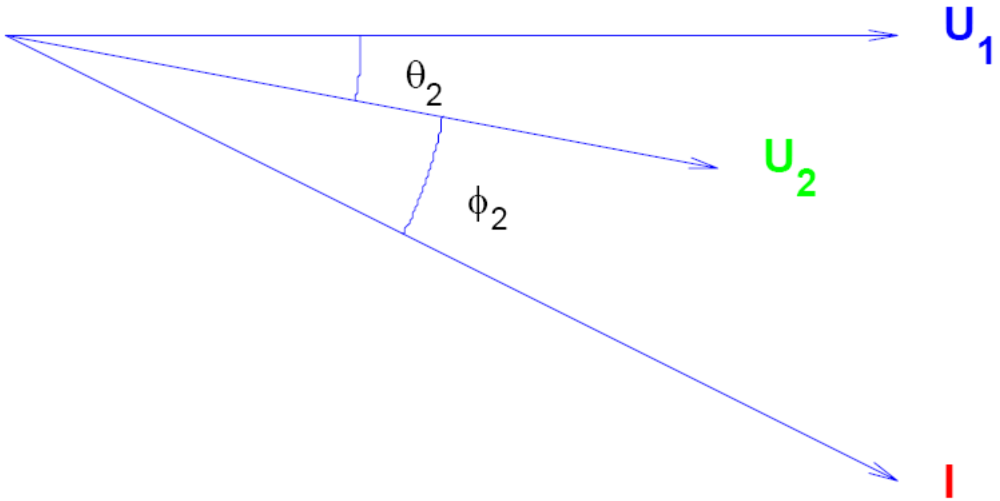
a) Calculate the power consumed by $\overline{Z}_2$ as well as the power factor ($\cos\varphi$) at bus 1 and 2 where φ_k is the phase angle between the voltage and the current at bus *k*.

b) Calculate U_2 when $\overline{Z}_2$ is capacitive : $\overline{Z}_2 = 0.7 - j0.5$.

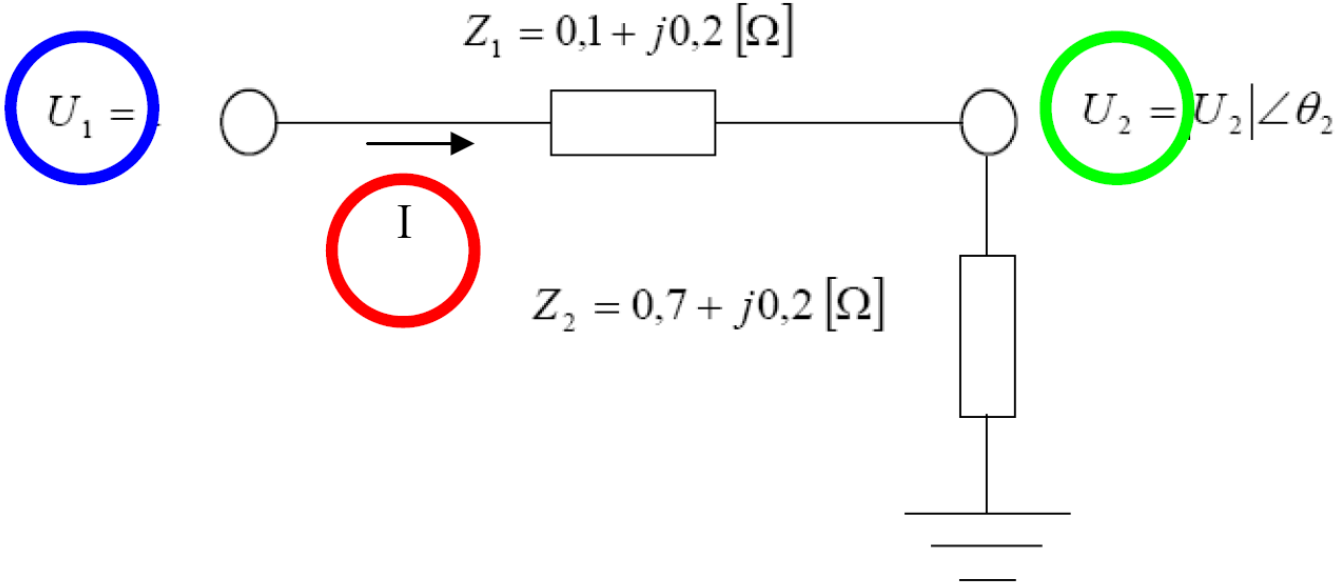
Example 2.3



Result: Complex axes

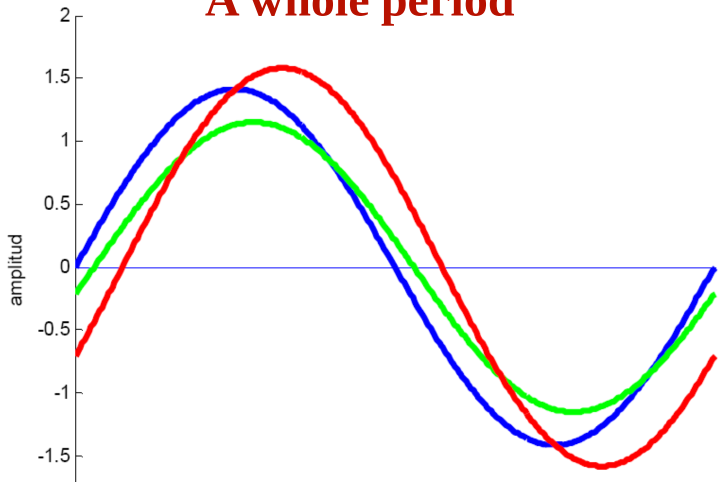


Example 2.3

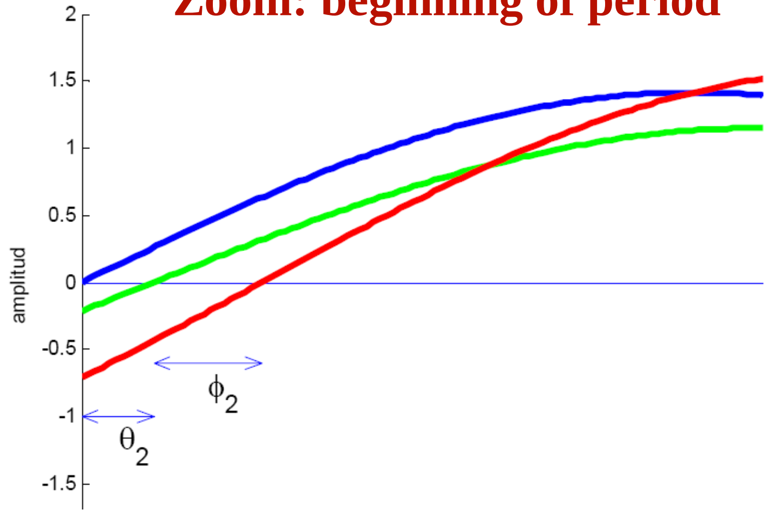


Result: Function of time

A whole period



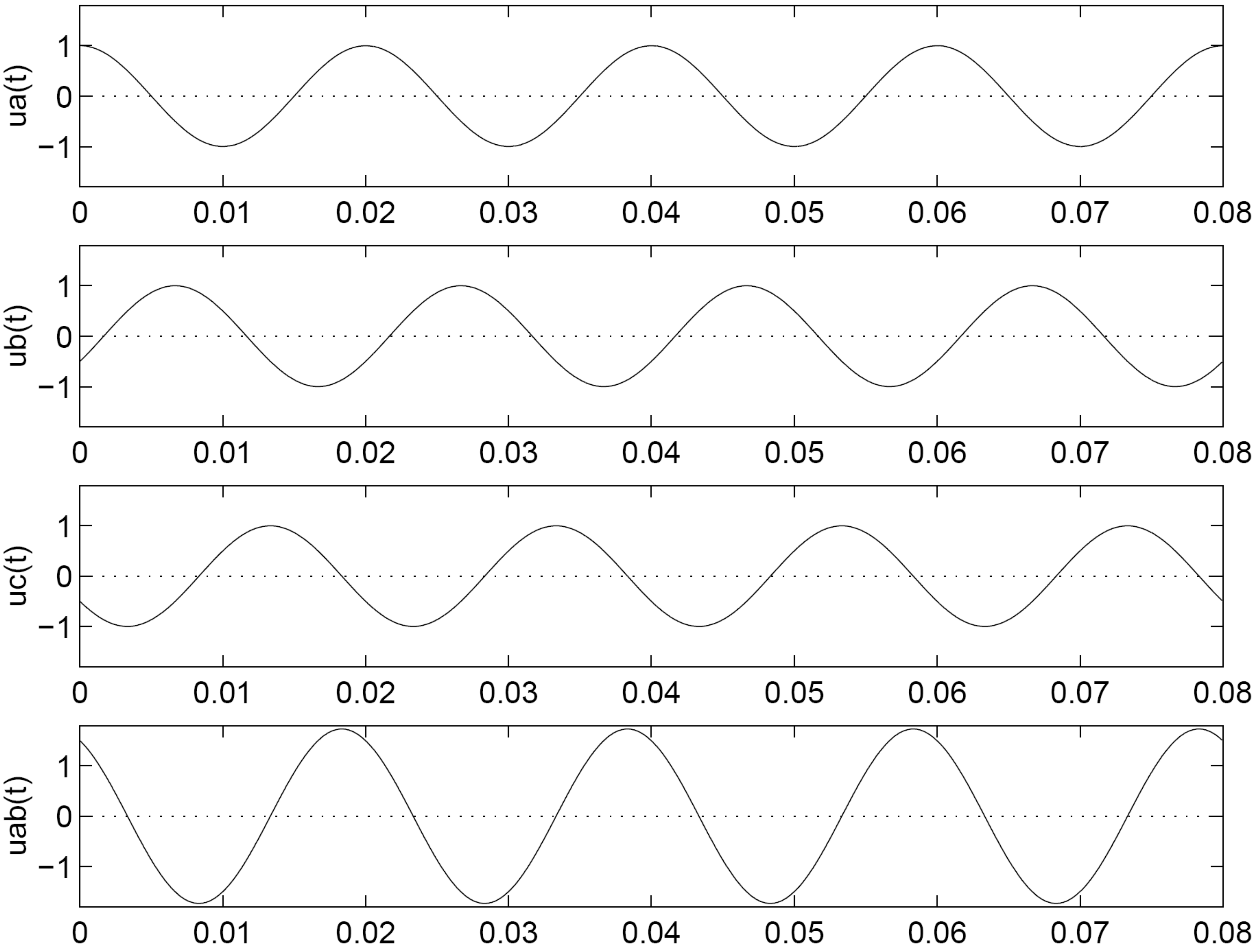
Zoom: beginning of period



Symmetrical three-phase alternating voltage

Time domain expressions

$$\begin{aligned}
u_a(t) &= U_M \cos \omega t \\
u_b(t) &= U_M \cos(\omega t - 120^\circ) \\
u_c(t) &= U_M \cos(\omega t + 120^\circ) \\
i_a(t) &= I_M \cos(\omega t - \phi) \\
i_b(t) &= I_M \cos(\omega t - 120^\circ - \phi) \\
i_c(t) &= I_M \cos(\omega t + 120^\circ - \phi) \\
p_3(t) &= p_a(t) + p_b(t) + p_c(t)
\end{aligned}$$



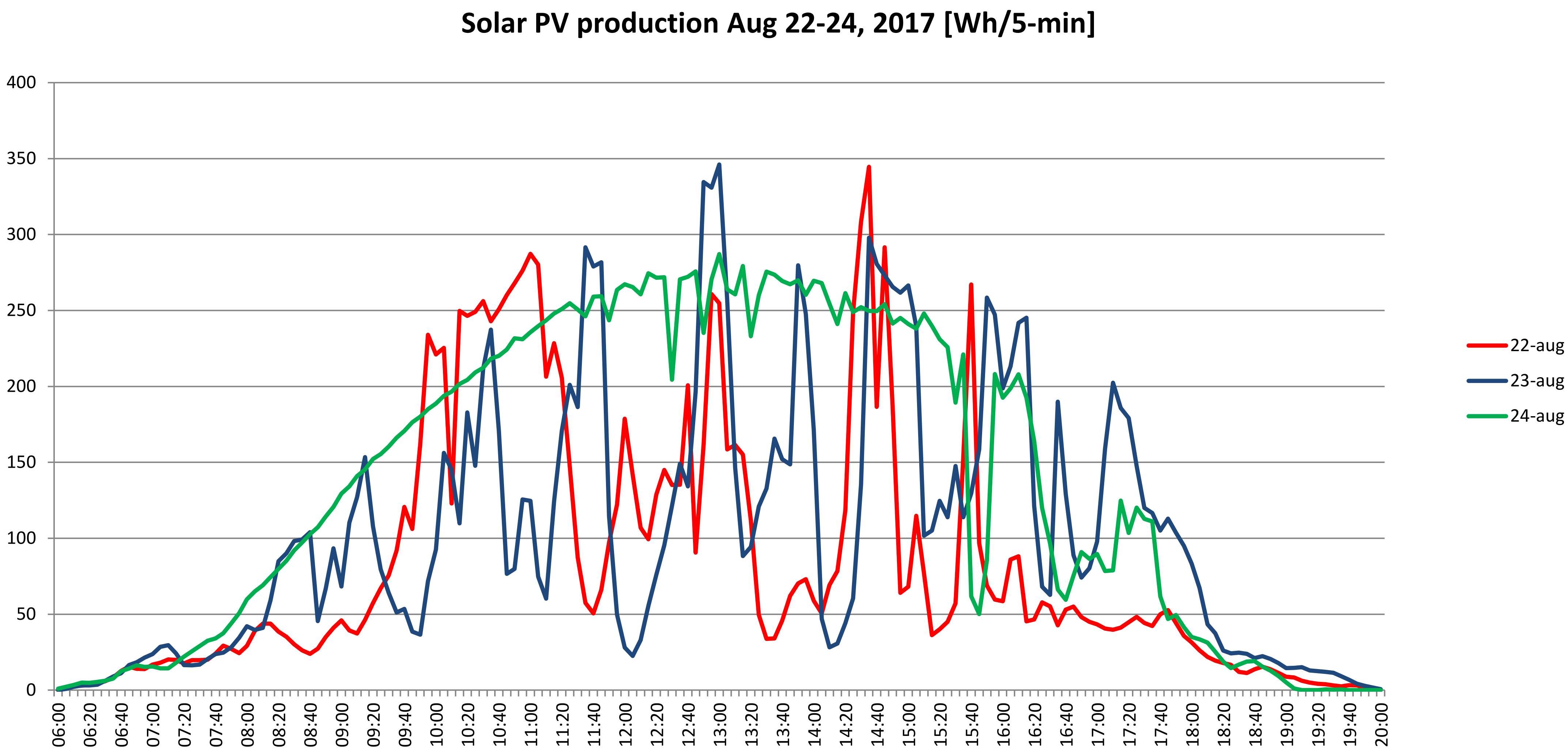
ua(t)
ub(t)
uc(t)
uab(t)
1
0
−1
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08

Lennart Söders solar PV: Ekerö / Ellevio

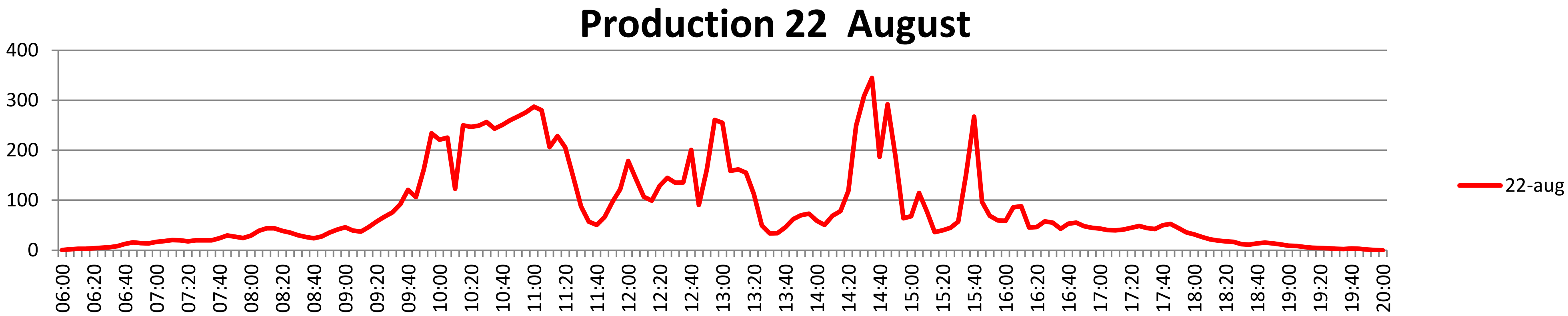
Converter from PV (DC) to grid (AC)

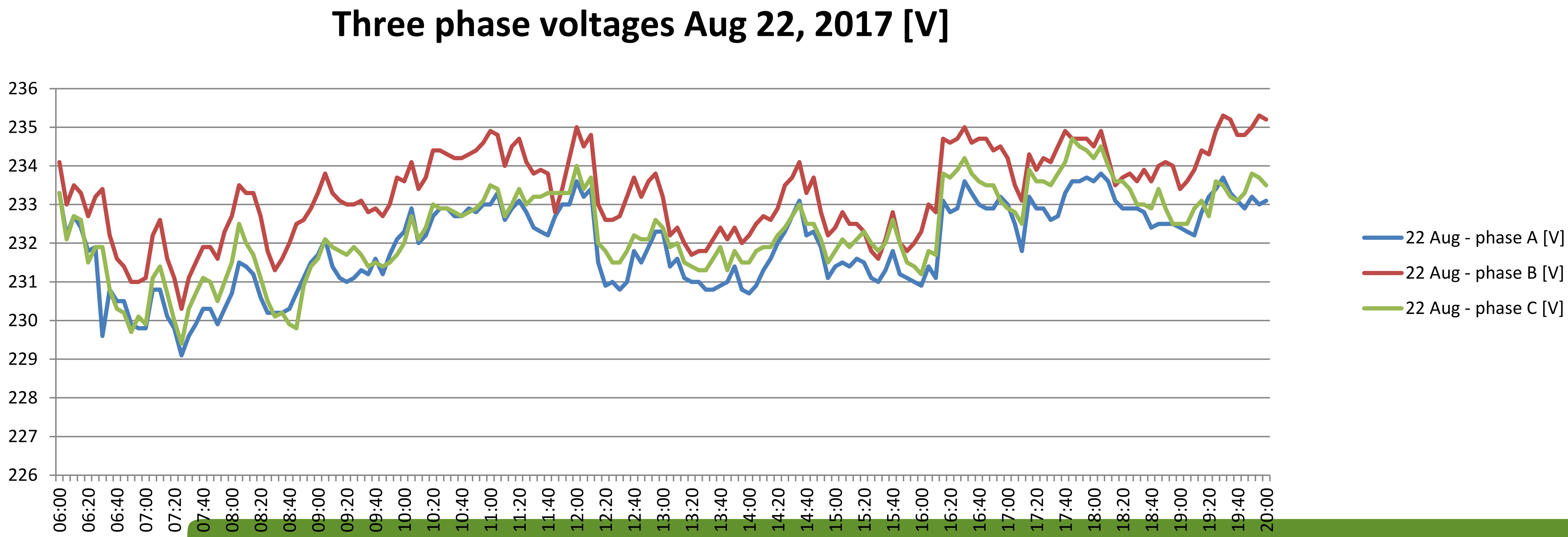
- 4,5 kW (PV-max and converter)
- Installed 17-19 July 2017
- Ca 4300 kWh/year ≈ yearly consumption
- 25 m^2

Production: 22-24 August, 2017

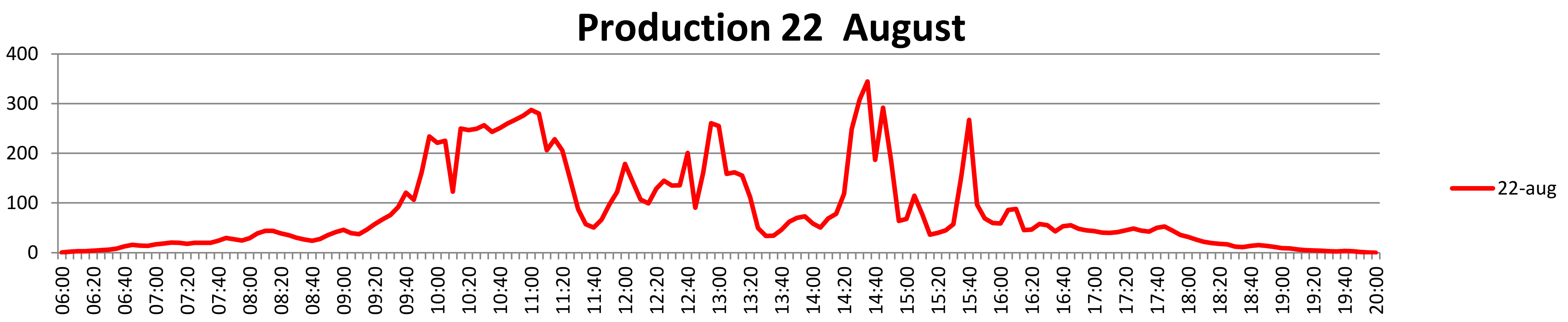


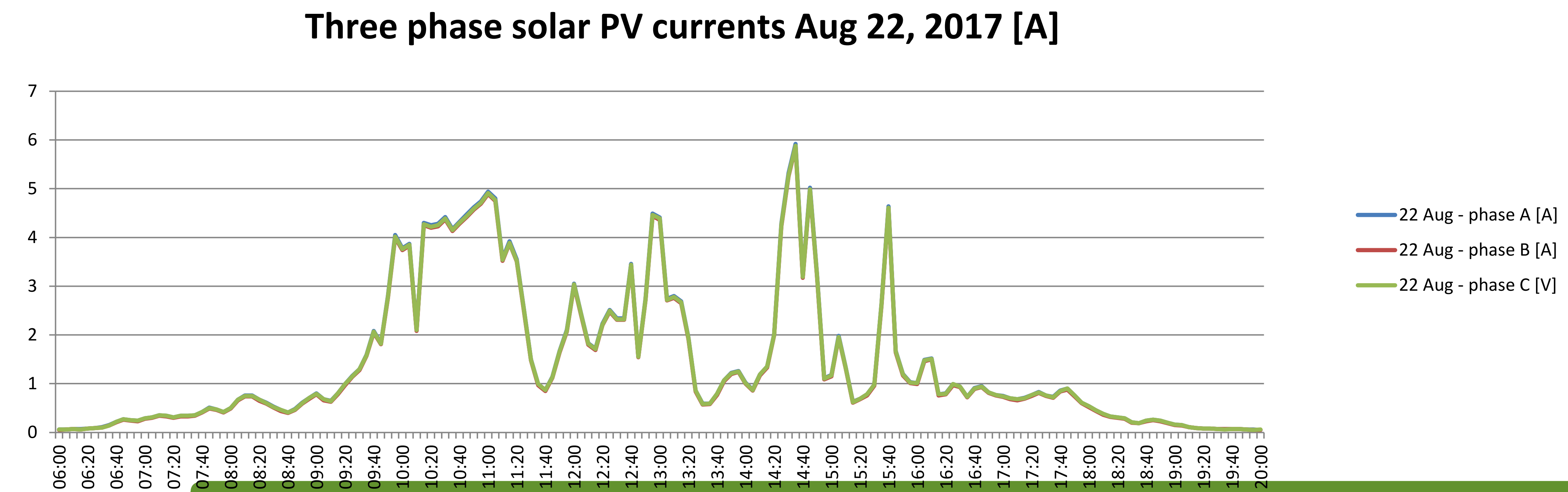
Three phase voltages: 22 August, 2017





Three phase currents: 22 August, 2017





Line-to-line voltage

$$
\begin{aligned}
u_{ab}(t) &= u_a(t) - u_b(t) = \\
&= U_M \cos \omega t - U_M \cos(\omega t - 120^\circ) = \\
&= \sqrt{3} U_M \cos(\omega t + 30^\circ)
\end{aligned}
$$

Three phase complex values

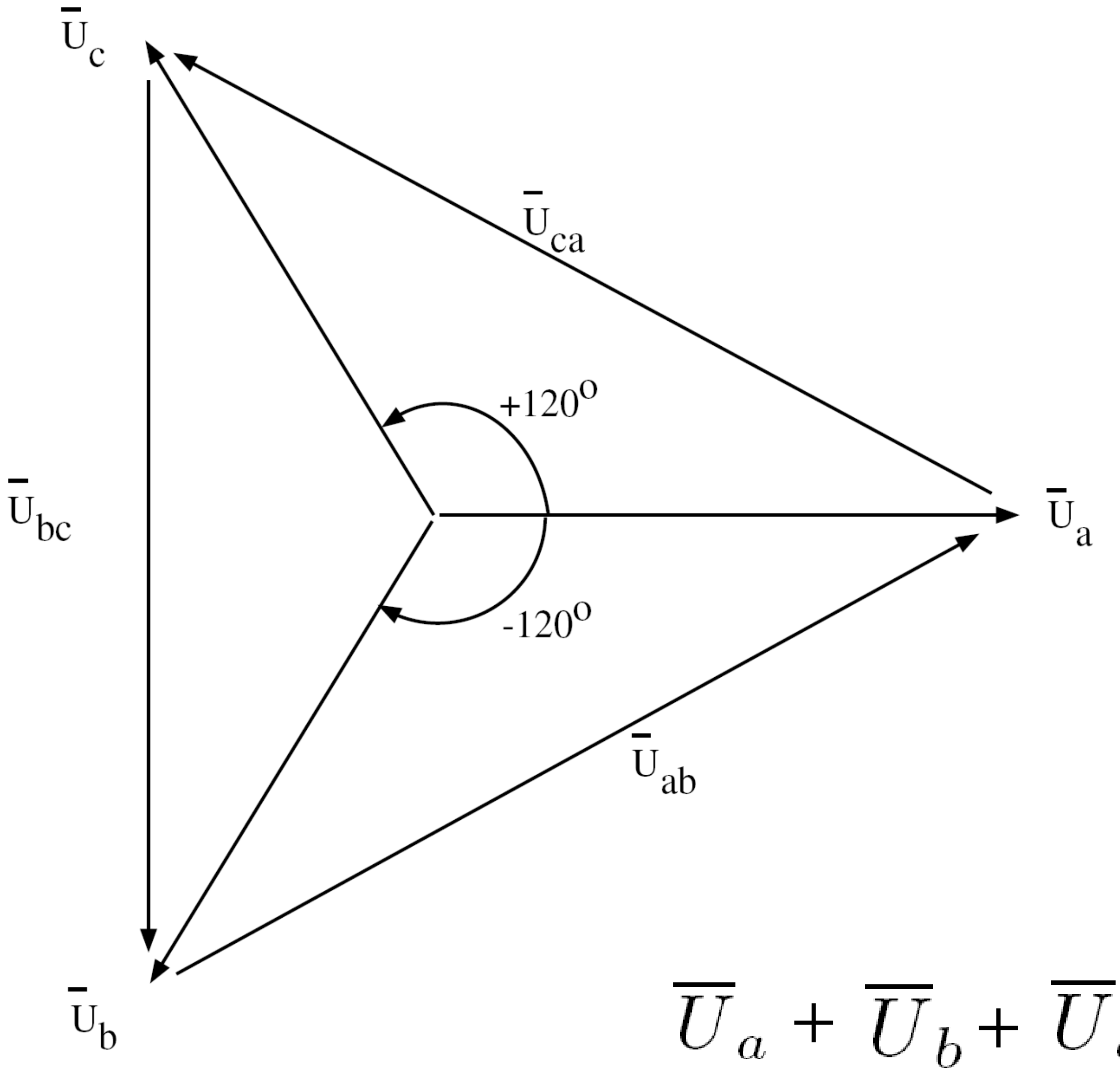
$$
\begin{aligned}
\overline{U}_a &= U_f \angle 0^\circ \\
\overline{U}_b &= U_f \angle -120^\circ \\
\overline{U}_c &= U_f \angle 120^\circ
\end{aligned}
$$

$$
\begin{aligned}
\overline{I}_a &= I \angle (0^\circ - \phi) \\
\overline{I}_b &= I \angle (-120^\circ - \phi) \\
\overline{I}_c &= I \angle (120^\circ - \phi)
\end{aligned}
$$

$$
\overline{S}_3 = \overline{U}_a \overline{I}_a^* + \overline{U}_b \overline{I}_b^* + \overline{U}_c \overline{I}_c^* =
$$

$$
= 3U_f I \cos\phi + j3U_f I \sin\phi
$$

Three phase complex values - 2



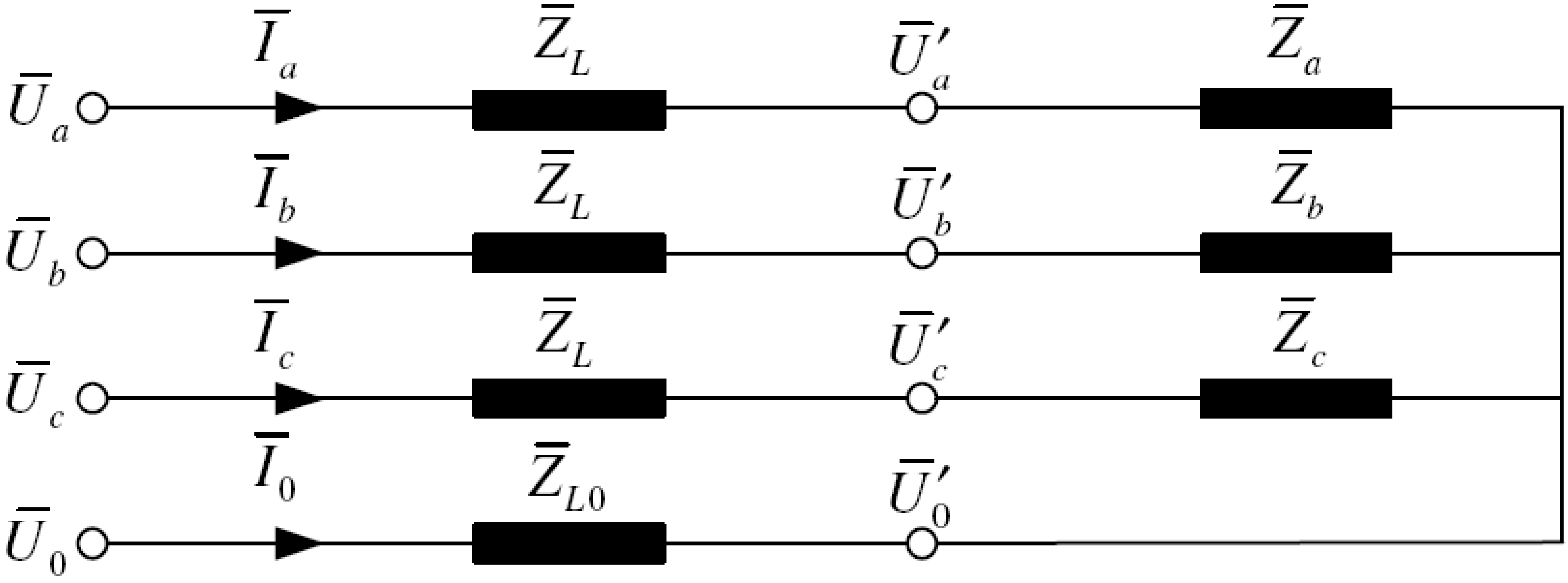
$$\overline{U}_a + \overline{U}_b + \overline{U}_c = 0$$

Symmetrical three-phase power

$$
\begin{aligned}
p_3(t) \quad &= \\
= \quad & p_a(t) + p_b(t) + p_c(t) = \\
&= u_a(t)i_a(t) + u_b(t)i_b(t) + u_c(t)i_c(t) = \\
= \quad & \frac{U_M}{\sqrt{2}}\frac{I_M}{\sqrt{2}}[(1+\cos 2\omega t)\cos\phi + \sin 2\omega t \sin\phi] + \\
+ \quad & \frac{U_M}{\sqrt{2}}\frac{I_M}{\sqrt{2}}[(1+\cos 2[\omega t - 120^\circ])\cos\phi + \\
&+ \sin 2[\omega t - 120^\circ]\sin\phi] + \\
+ \quad & \frac{U_M}{\sqrt{2}}\frac{I_M}{\sqrt{2}}[(1+\cos 2[\omega t + 120^\circ])\cos\phi + \\
&+ \sin 2[\omega t + 120^\circ]\sin\phi] =
\end{aligned}
$$

$$
\begin{aligned}
&= \quad 3\frac{U_M}{\sqrt{2}}\frac{I_M}{\sqrt{2}}\Big[\cos\phi + \\
&+ \underbrace{(\cos 2\omega t + \cos 2[\omega t - 120^\circ] + \cos 2[\omega t + 120^\circ])}_{=0} + \\
&+ \quad \underbrace{(\sin 2\omega t + \sin 2[\omega t - 120^\circ] + \sin 2[\omega t + 120^\circ])}_{=0}\Big] = \\
&= 3\frac{U_M}{\sqrt{2}}\frac{I_M}{\sqrt{2}}\cos\phi
\end{aligned}
$$

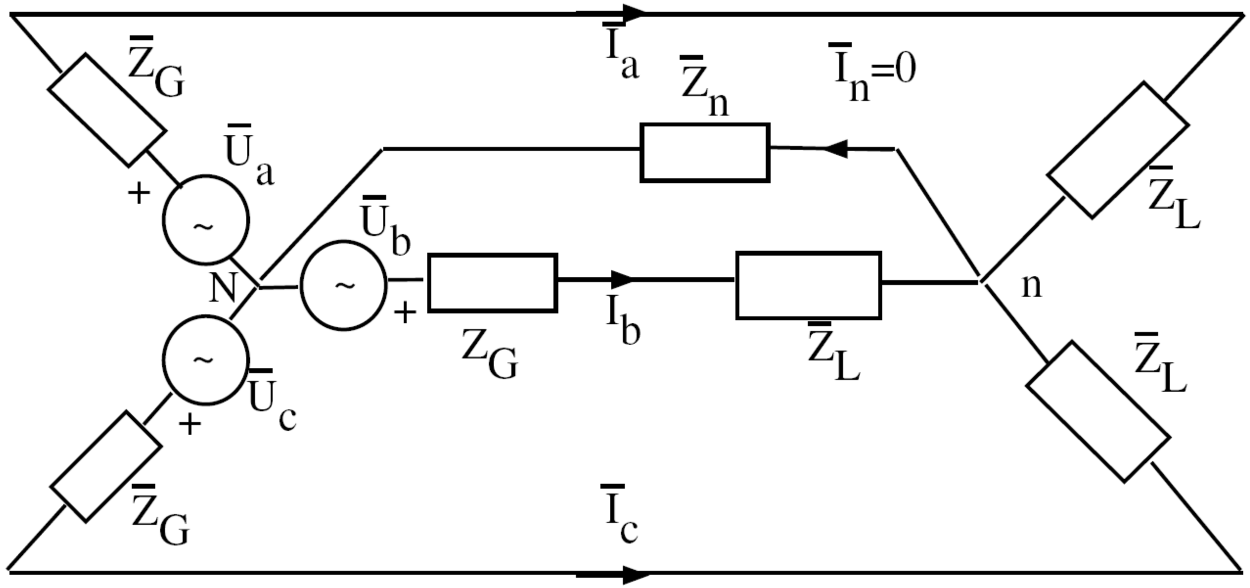
Example 2.4 The student Elektra lives in a house situated 2 km from a transformer having a completely symmetrical three-phase voltage ($U_a = 220V \angle 0°, U_b = 220V \angle -120°, U_c = 220V \angle 120°$). The house is connected to this transformer via a three-phase cable (EKKJ, 3x16 mm^2 + 16 mm^2). A cold day, Elektra switch on two electrical radiators to each phase, each radiator is rated 1000 W (at 220 V with cosφ = 0.995 lagging (inductive)). Assume that the cable can be modelled as four impedances connected in parallel ($z_L = 1.15 + j0.08\ \Omega$/phase,km, $z_{L0} = 1.15 + j0.015\ \Omega$ /km) and that the radiators also can be considered as impedances. Calculate the total thermal power given by the radiators.

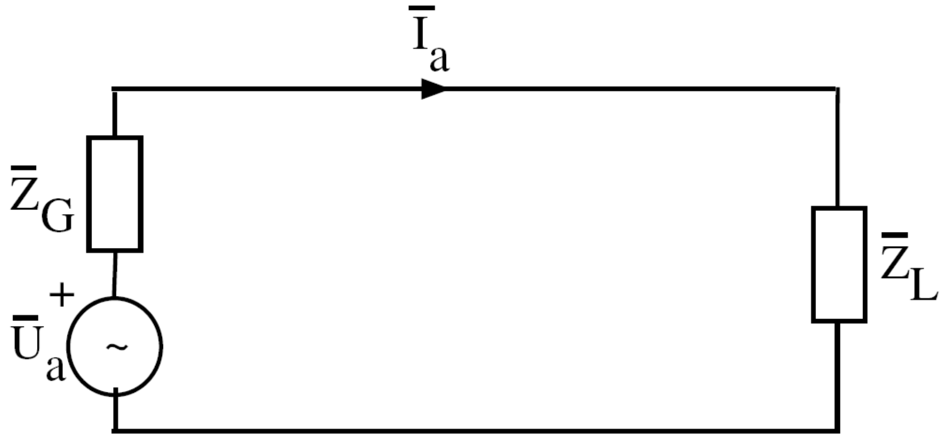


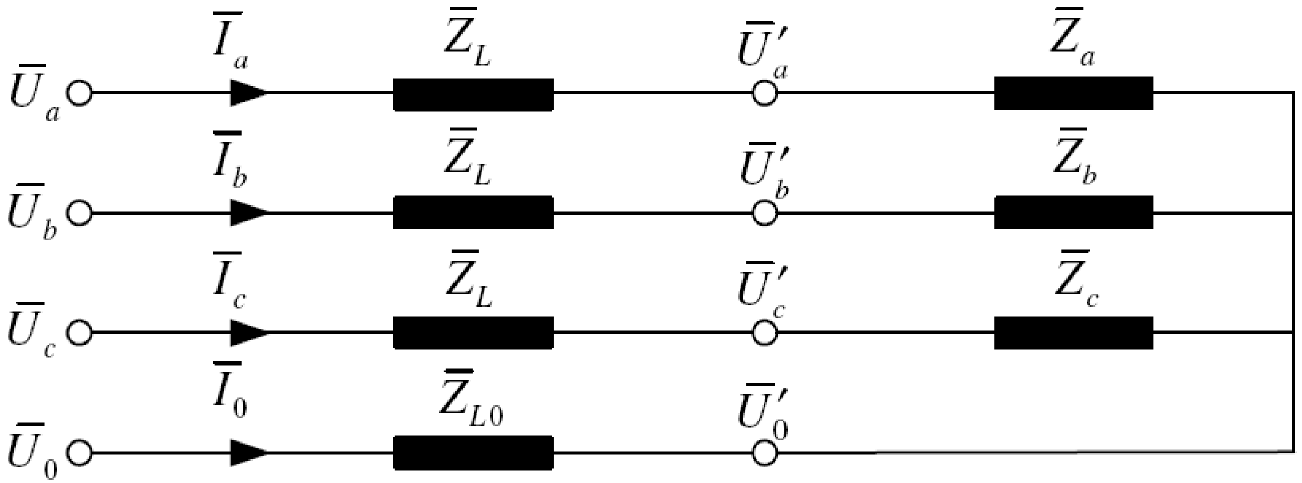
$\bar{I}_a$
$\bar{Z}_L$
$\bar{U}'_a$
$\bar{Z}_a$
$\bar{U}_a$
$\bar{I}_b$
$\bar{Z}_L$
$\bar{U}'_b$
$\bar{Z}_b$
$\bar{U}_b$
$\bar{I}_c$
$\bar{Z}_L$
$\bar{U}'_c$
$\bar{Z}_c$
$\bar{U}_c$
$\bar{I}_0$
$\bar{Z}_{L0}$
$\bar{U}'_0$
$\bar{U}_0$

Example 2.4

Symmetrical three phase systems







Example 2.4

The voltage at the radiators can be calculated as :

$$\overline{U}'_a = \overline{U}'_0 + \overline{I}_a \overline{Z}_a = 200.78\angle 0.15° \text{ V}$$

$$\overline{U}_b = 200.78\angle -119.85° \text{ V}$$

$$\overline{U}_c = 200.78\angle 120.15° \text{ V}$$

Finally, the power to the radiators can be calculated as

$$\overline{S}_{za} = \overline{Z}_a I_a^2 = 1666 + j167 \text{ VA}$$

$$\overline{S}_{zb} = \overline{Z}_b I_b^2 = 1666 + j167 \text{ VA}$$

$$\overline{S}_{zc} = \overline{Z}_a I_c^2 = 1666 + j167 \text{ VA}$$

Thus, the total consumed power is

$$\overline{S}_{za} + \overline{S}_{zb} + \overline{S}_{zc} = 4998 + j502 \text{ VA, i.e. the thermal power} = 4998 \text{ W}$$

Example 2.5 Use the data as in example 3.4 but with the difference that the student Elektra connects one 1000 W radiator (at 220 V with $\cos\varphi = 0.995$ lagging) to phase a, three radiators to phase b and two to phase c. Calculate the total thermal power given by the radiators, as well as the system losses.

Solution

$$\overline{U}_a = 220\angle 0^\circ \text{ V}, \overline{U}_b = 220\angle -120^\circ \text{ V}, \overline{U}_c$$

$$\overline{Z}_L = 2(1.15 + j0.08) = 2.3 + j0.16 \ \Omega$$

$$\overline{Z}_{L0} = 2(1.15 + j0.015) = 2.3 + j0.03 \ \Omega$$

$$P_a = 1000 \text{ W (vid 220 V, } \cos\phi = 0.995)$$

$$\sin\phi = \sqrt{1 - \cos^2\phi} = 0.0999$$

$$Q_a = S\sin\phi = \frac{P}{\cos\phi}\sin\phi = 100.4 \text{ VAr}$$

$$\overline{Z}_a = U^2/\overline{S}_a^* = U^2/(P_a - jQ_a) = 47.9 + j4.81 \ \Omega$$

$$\overline{Z}_b = \overline{Z}_a/3 = 15.97 + j1.60 \ \Omega$$

$$\overline{Z}_c = \overline{Z}_a/2 = 23.96 + j2.40 \ \Omega$$

$$\overline{U}'_0\left[\frac{1}{\overline{Z}_{L0}}+\frac{1}{\overline{Z}_L+\overline{Z}_a}+\frac{1}{\overline{Z}_L+\overline{Z}_b}+\frac{1}{\overline{Z}_L+\overline{Z}_c}\right]=\frac{\overline{U}_a}{\overline{Z}_L+\overline{Z}_a}+\frac{\overline{U}_b}{\overline{Z}_L+\overline{Z}_b}+\frac{\overline{U}_c}{\overline{Z}_L+\overline{Z}_c}$$

$$\Rightarrow \overline{U}'_0 = 12.08\angle-155.14^\circ\ \mathrm{V}$$

$$\Rightarrow \overline{I}_a = 4.58\angle-4.39^\circ\ \mathrm{A}, \overline{I}_b = 11.45\angle-123.62^\circ\ \mathrm{A}, \overline{I}_c = 8.31\angle111.28^\circ\ \mathrm{A}$$

$$\overline{U}'_a = \overline{U}'_0 + \overline{I}_a\overline{Z}_a = 209.45\angle0.02^\circ\ \mathrm{V}$$

$$\overline{U}'_b = \overline{U}'_0 + \overline{I}_b\overline{Z}_b = 193.60\angle-120.05^\circ\ \mathrm{V}$$

$$\overline{U}'_c = \overline{U}'_0 + \overline{I}_c\overline{Z}_c = 200.91\angle129.45^\circ\ \mathrm{V}$$

Observe that these voltages are not local phase voltages since they are calculated as $\overline{U}'_a - \overline{U}'_0$ etc. The power to the radiators can be calculated as :

$$\overline{S}_{za} = \overline{Z}_a I_a^2 = 1004 + j101 \text{ VA}$$

$$\overline{S}_{zb} = \overline{Z}_b I_b^2 = 2095 + j210 \text{ VA}$$

$$\overline{S}_{zc} = \overline{Z}_a I_c^2 = 1655 + j166 \text{ VA}$$

The total amount of power consumed is

$$\overline{S}_{za} + \overline{S}_{zb} + \overline{S}_{zc} = 4754 + j477 \text{ VA}$$

I.e. The thermal power is 4754 W

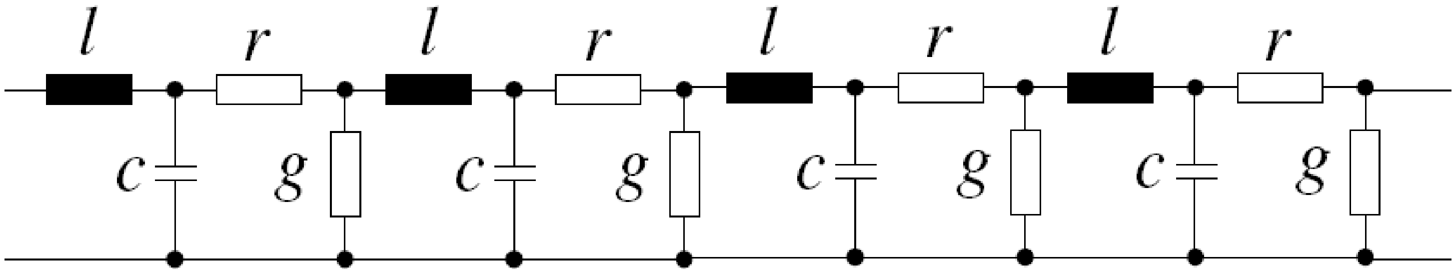
The total transmission losses are

$$\overline{Z}_L(I_a^2 + I_b^2 + I_c^2) + \overline{Z}_{L0}|\overline{I}_a + \overline{I}_b + \overline{I}_c|^2) = 572.1 + j36 \text{ VA}$$

I.e. 572.1 W

Transmission line - 1

One phase equivalent of a transmission line at symmetrical three phase transmission

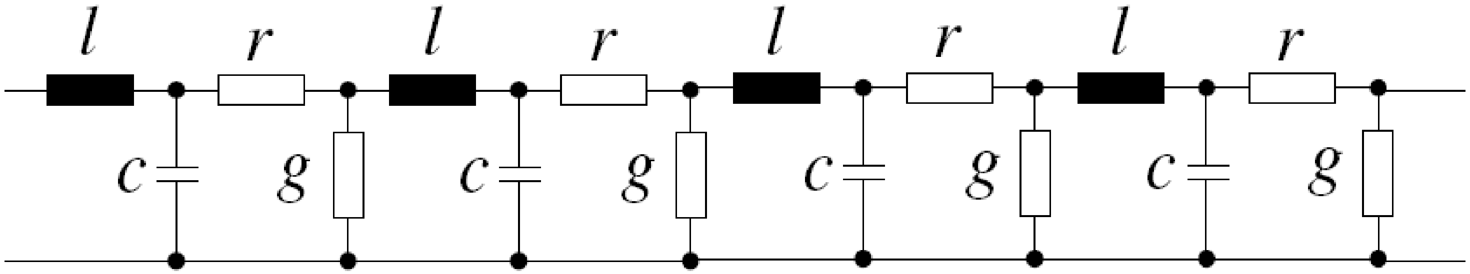


Resistance

The resistance of a conductor with the cross-section area A mm^2 and the resistivity ρ Ωmm^2/ km is

$$r = \frac{\rho}{A} \quad \Omega/\mathrm{km}$$

Shunt conductance



The shunt conductance *g* is neglected in this course.

Transmission line - 2

Inductance:

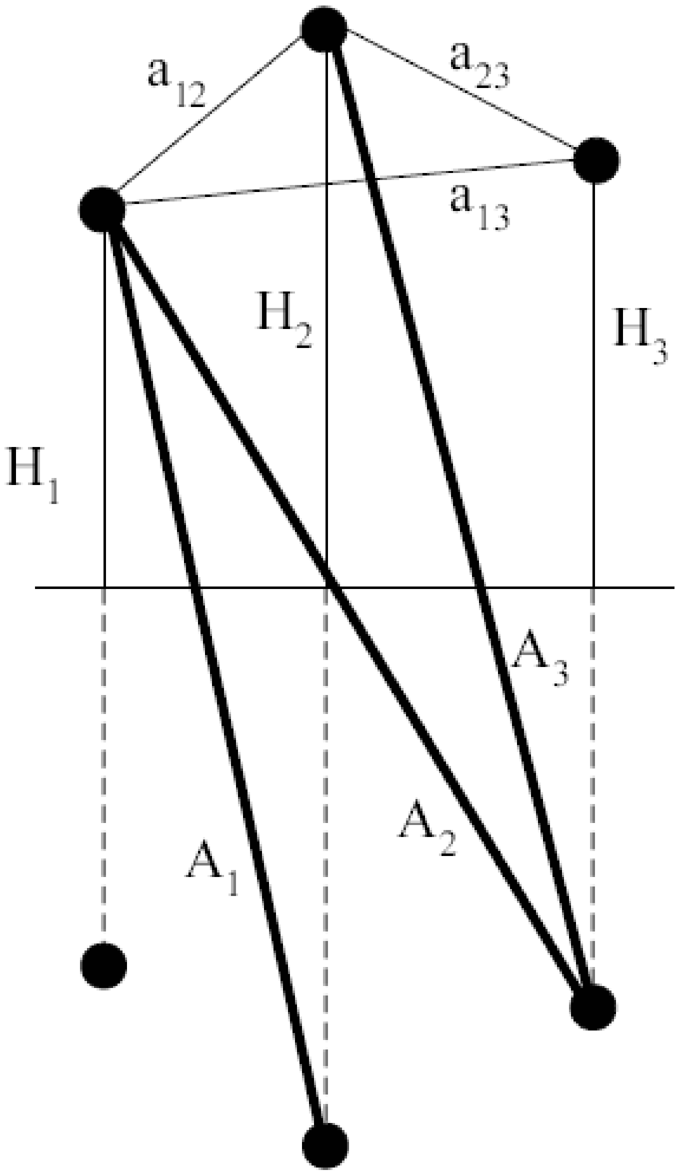
$$\ell = 2 \cdot 10^{-4} \left(\ln \frac{a}{d/2} + \frac{1}{4n} \right) \ \text{H/km,fas}$$

$a = \sqrt[3]{a_{12}a_{13}a_{23}}$ m, = Geometrical mean distance according to the figure

$d =$ Diameter of the conductor, m

$n =$ Number of conductors per phase

Transmission line - 2



Inductance:

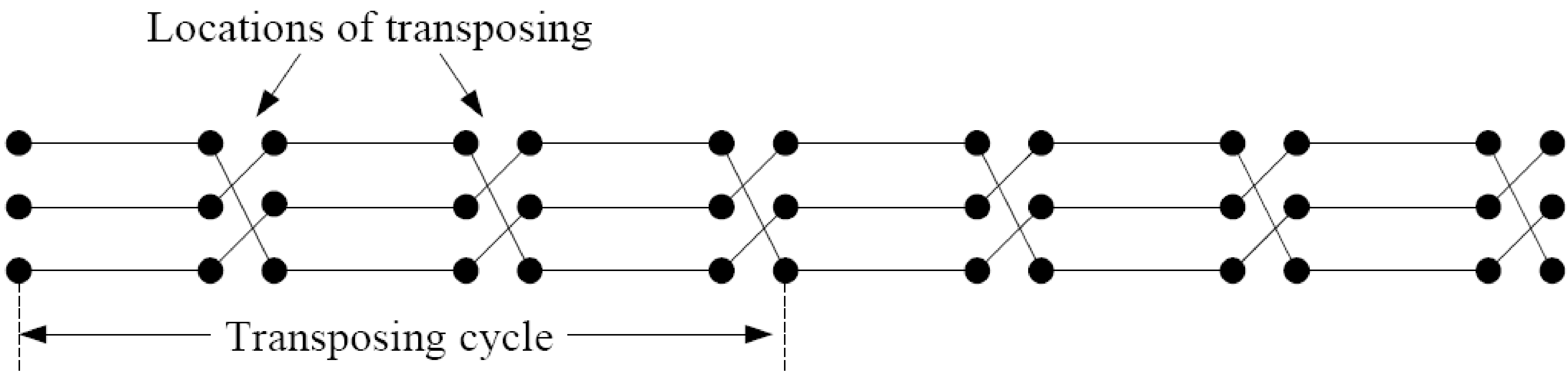
$$\ell = 2 \cdot 10^{-4} \left(\ln \frac{a}{d/2} + \frac{1}{4n} \right) \ \mathrm{H/km,fas}$$

$a = \sqrt[3]{a_{12}a_{13}a_{23}}$ m, = Geometrical mean distance according to the figure

$d =$ Diameter of the conductor, m

$n =$ Number of conductors per phase

Transposed three-phase overhead line

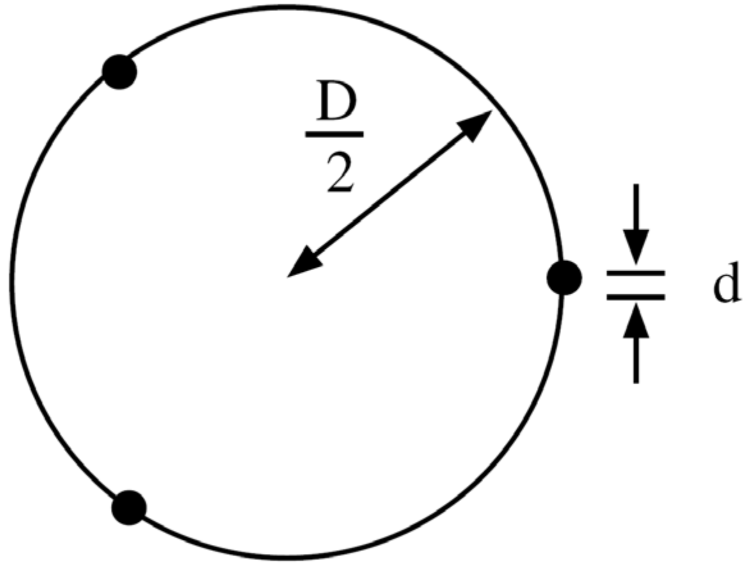


Transposed line

Alternative to transposed line

Transmission line - 3
Multiple conductor

Cross-section of a multiple conductor with three conductors per phase



Power lines

Transmission line - 4
Multiple conductor

$$(d/2)_{ekv} = \sqrt[n]{n(D/2)^{n-1} \cdot (d/2)}$$

where

n = number of conductors per phase

D/2 = radius in the circle formed by the conductors

Example 3.1

Determine the reactance of a 130 kV overhead line where the conductors arc located in a plane and the distance between two closely located conductors is 4 m. The conductor diameter is 20 mm. Repeat the calculations for a line with two conductors per phase, located 30 cm from one another.

Example 3.1 One conductor/phase

$$a_{12} = a_{23} = 4,\ a_{13} = 8$$

$$d/2 = 0.01 \text{ m}$$

$$a = \sqrt[3]{4 \cdot 4 \cdot 8} = 5.04$$

$$x = 2\pi \cdot 50 \cdot 2 \cdot 10^{-4} \left(\ln \frac{5.04}{0.01} + \frac{1}{4}\right) =$$

$$= 0.0628\,(\ln(504) + 0.25) =$$

$$= 0.41\ \Omega/\text{km}, \text{ phase}$$

Example 3.1 Multiple conductor

Multiple conductor (duplex)

$$(d/2)_{ekv} = \sqrt[2]{2(0.3/2)0.01} = 0.055$$

$$x = 0.0628\left(\ln\frac{5.04}{0.055} + \frac{1}{8}\right)$$

$$= 0.29\ \Omega/\text{km},$$ phase

The reactance is in this case reduced by $28\ \%$.

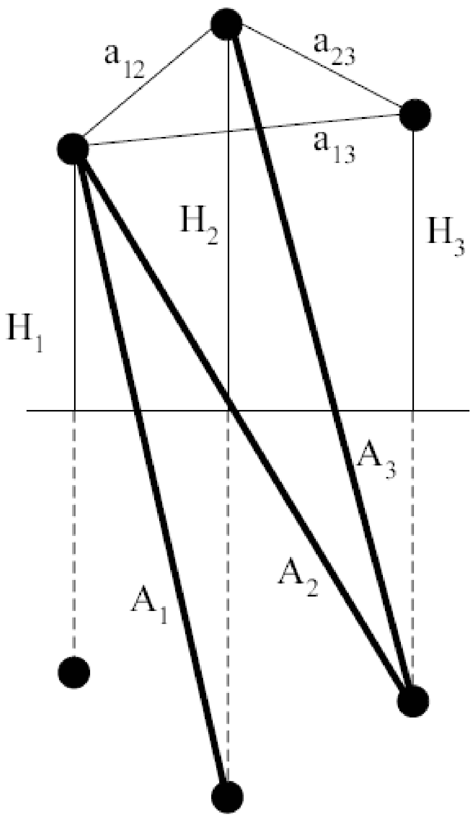
Transmission line - 5

Shunt capacitance

For a transposed overhead line and symmetrical conditions :

$$c = \frac{10^{-6}}{18 \ln\left(\frac{2H}{A} \cdot \frac{a}{(d/2)_{ekv}}\right)} \ \mathrm{F/km}, \text{ phase}$$

where



$$H = \sqrt[3]{H_1 H_2 H_3} =$$ Geometrical mean height for the conductors

$$A = \sqrt[3]{A_1 A_2 A_3} =$$ Geometrical mean distance between the conductors and their image conductors.

The shunt susceptance of a line is

$$b = 2\pi f \cdot c \quad \text{S/km, phase}$$

Common value : $3 \cdot 10^{-6}$ S/km,phase

Approximatively it can be shown that

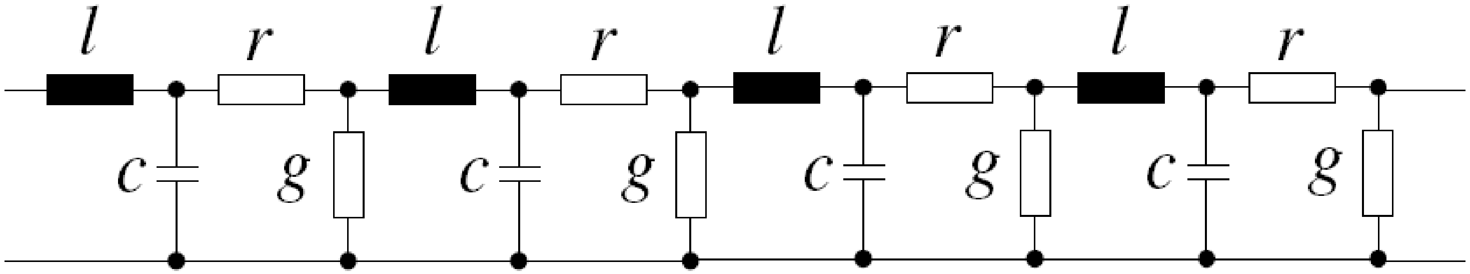
$$\ell \cdot c = 2 \cdot 10^{-4} \left(\ln \frac{a}{(d/2)_{ekv}} \right) \cdot \frac{10^{-6}}{18 \ln \left(\frac{a}{(d/2)_{ekv}} \right)} =$$

$$= \frac{1}{(3 \cdot 10^{-5})^2} \left(\frac{\mathrm{km}}{\mathrm{s}} \right)^{-2} = \frac{1}{v^2}$$

Where *v* = speed of light in vacuum in km/s.

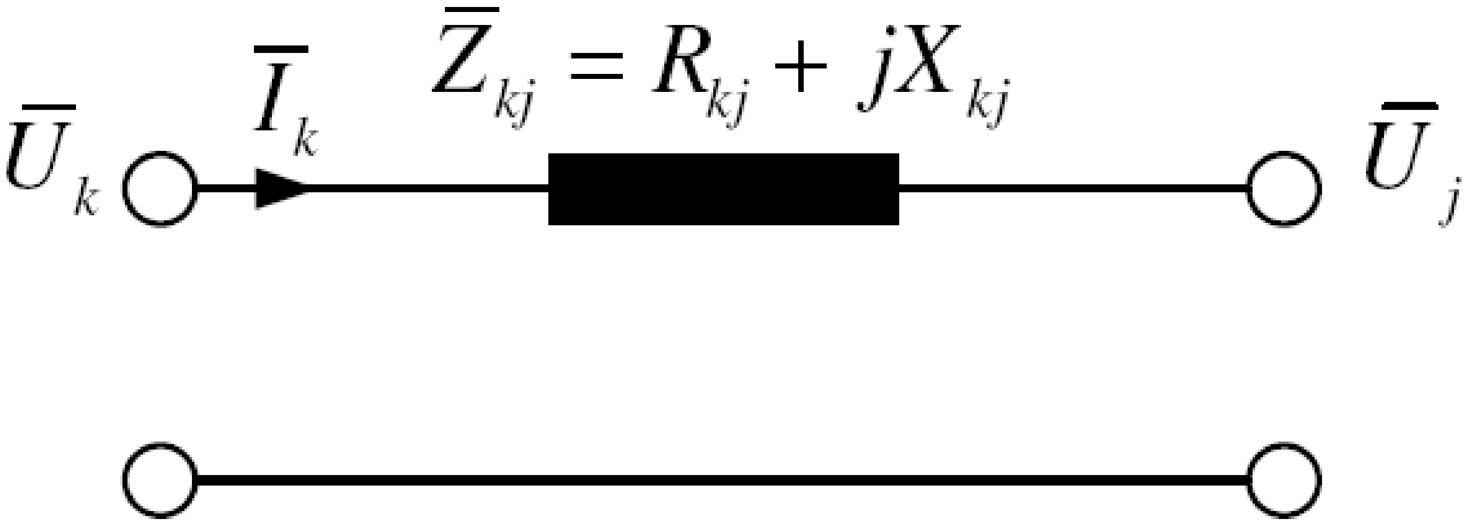
Transmission line

One phase equivalent of a transmission line.



Transmission line - 7

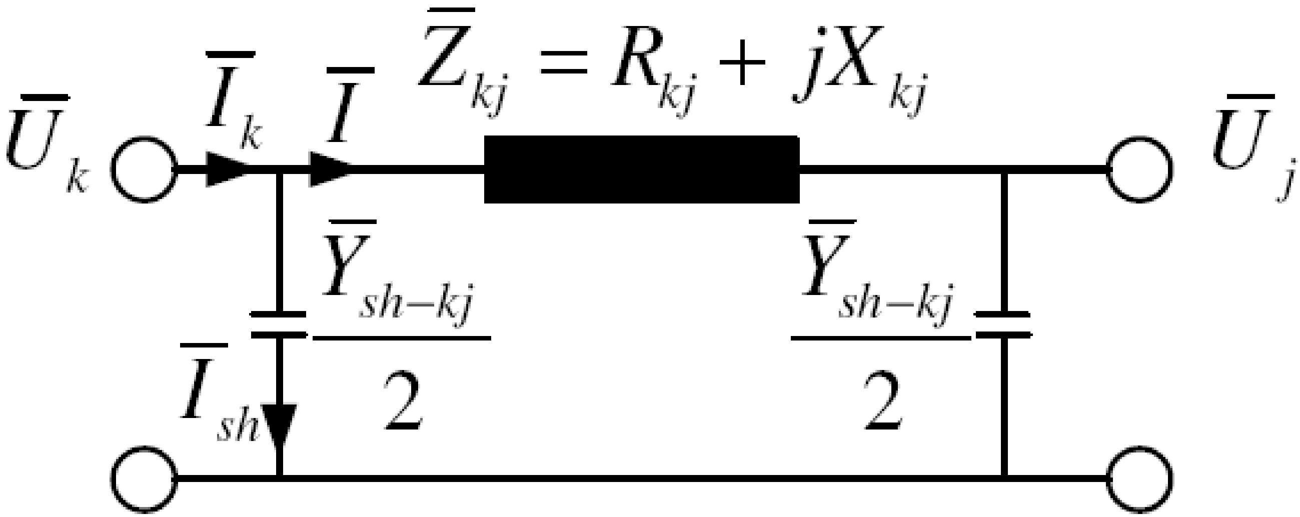
Model for a short line



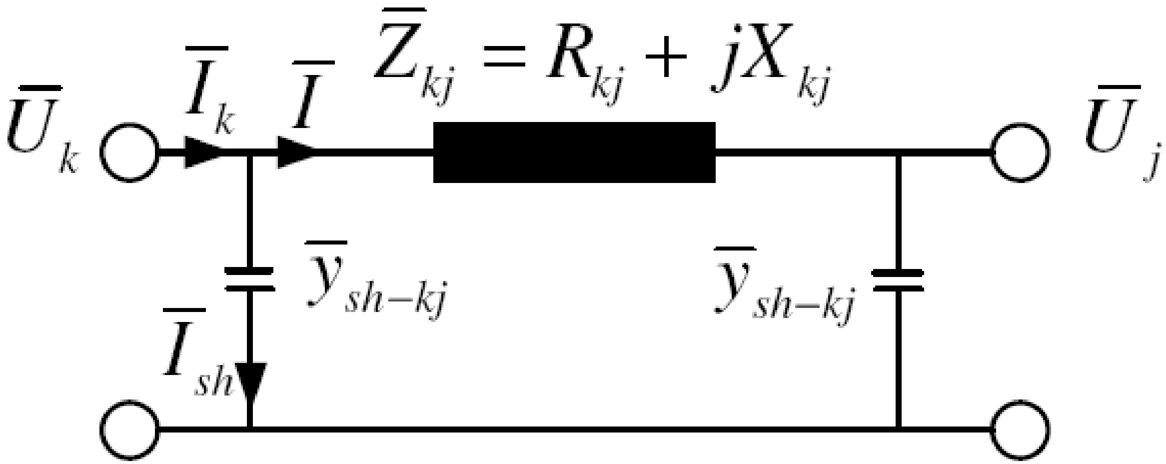
$$\overline{Z} = R + jX = (r + jx)s \quad \Omega/\text{phase}$$

Where *s* = length of line in km.

Models for a medium long line



or



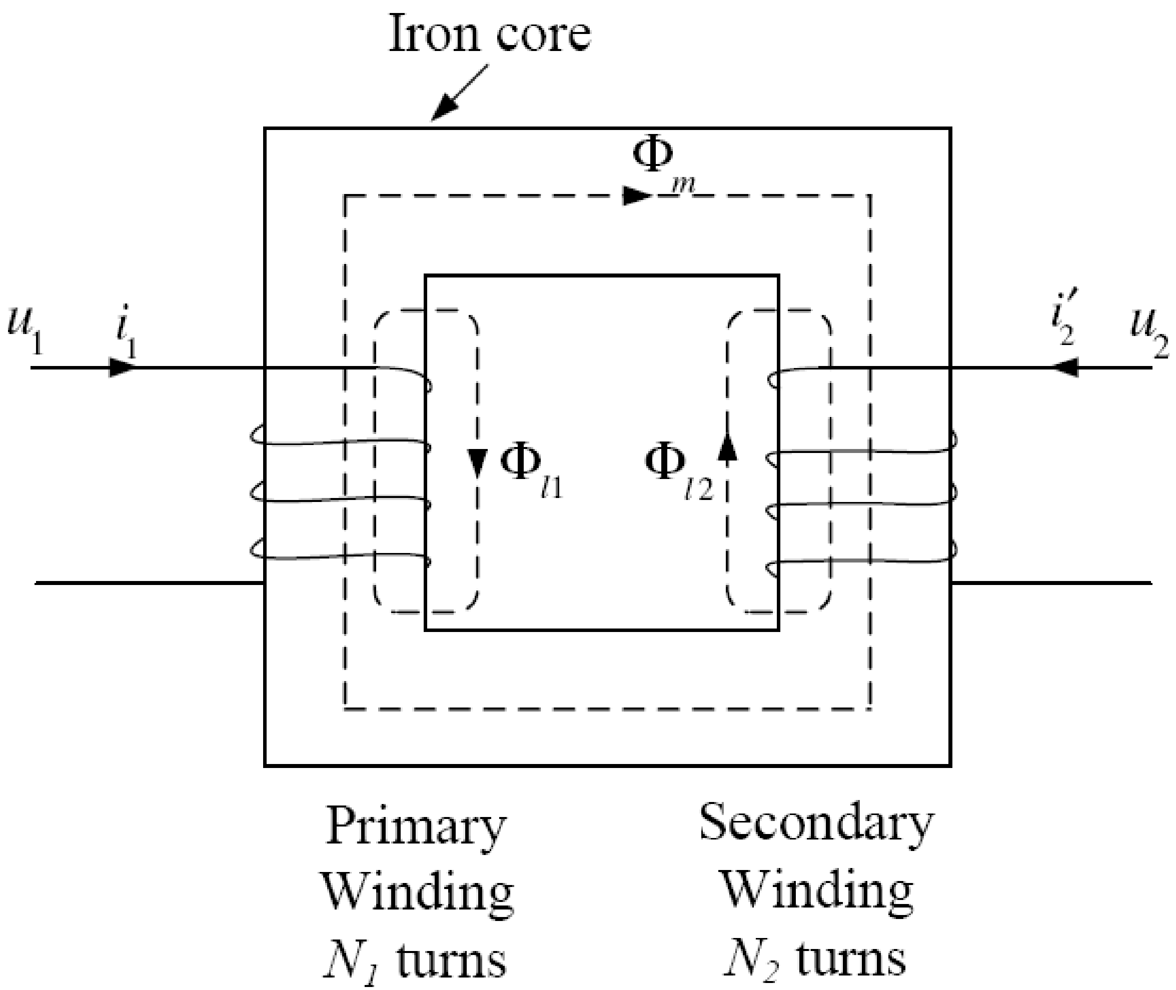
$$\boxed{\bar{Y} = jB = j\frac{bs}{2} \quad \mathrm{S/km}}$$

Right or wrong?

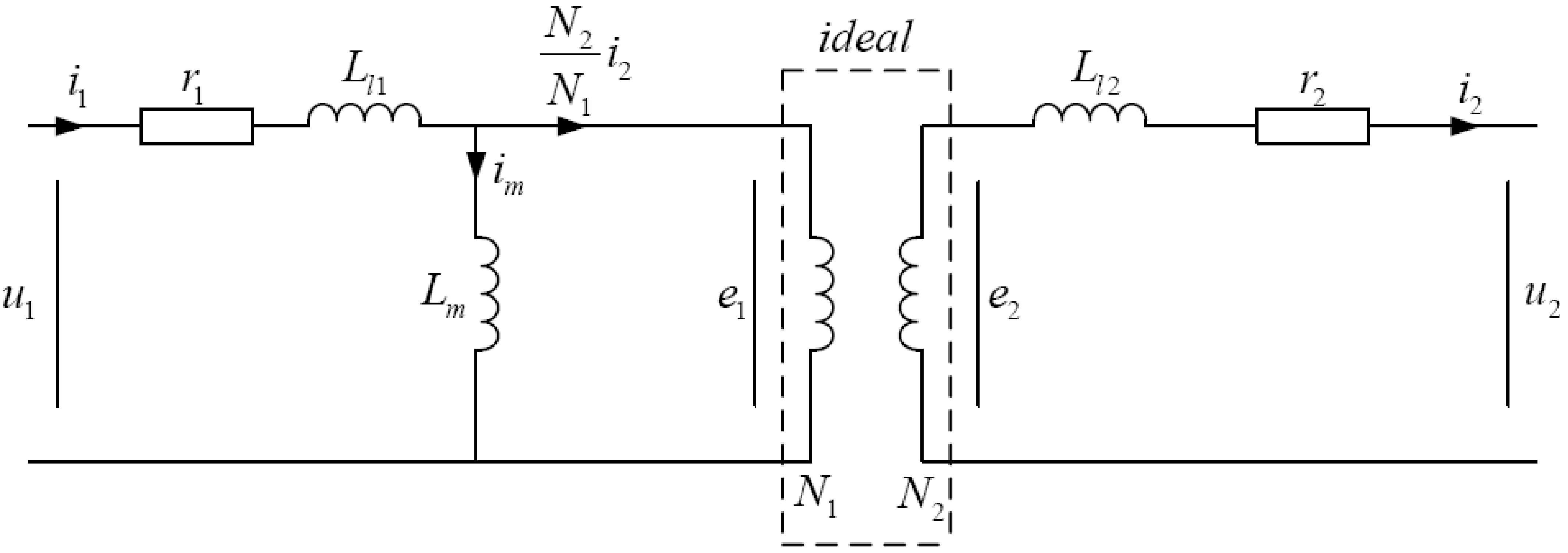
1. An overhead line has an inductance of ca 0,4 Ω/km,fas.
2. A cable has higher reactance/km compared the an overhead line with the same length.
3. At dubble radius on a power line conductor, the resistance is half as large.
4. If the overhead line has higher poles, then the reactance becomes lower.
5. Transpose of power lines makes the reactance to become the same per phase.

Single-phase transformer

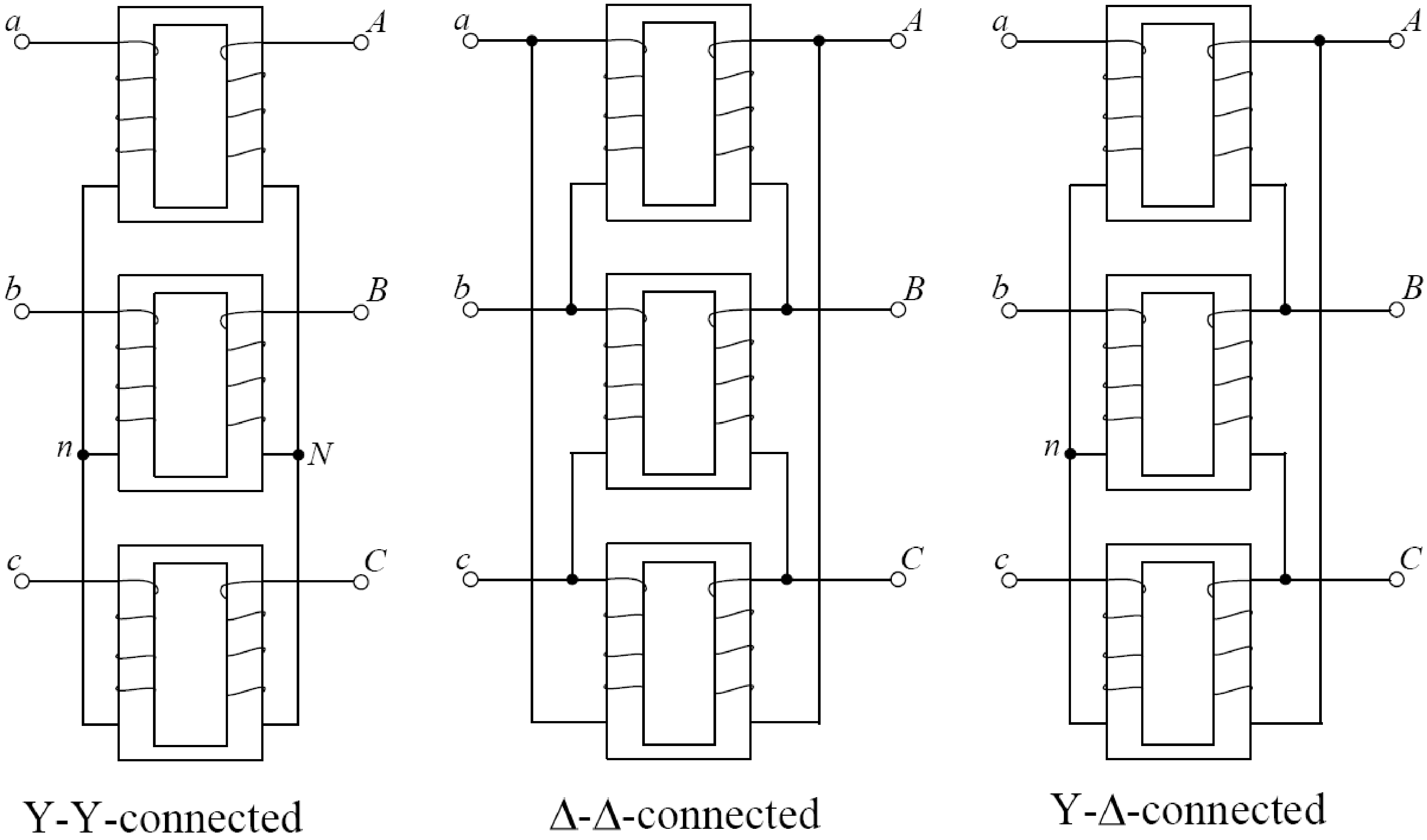
Principle design of a two winding transformer



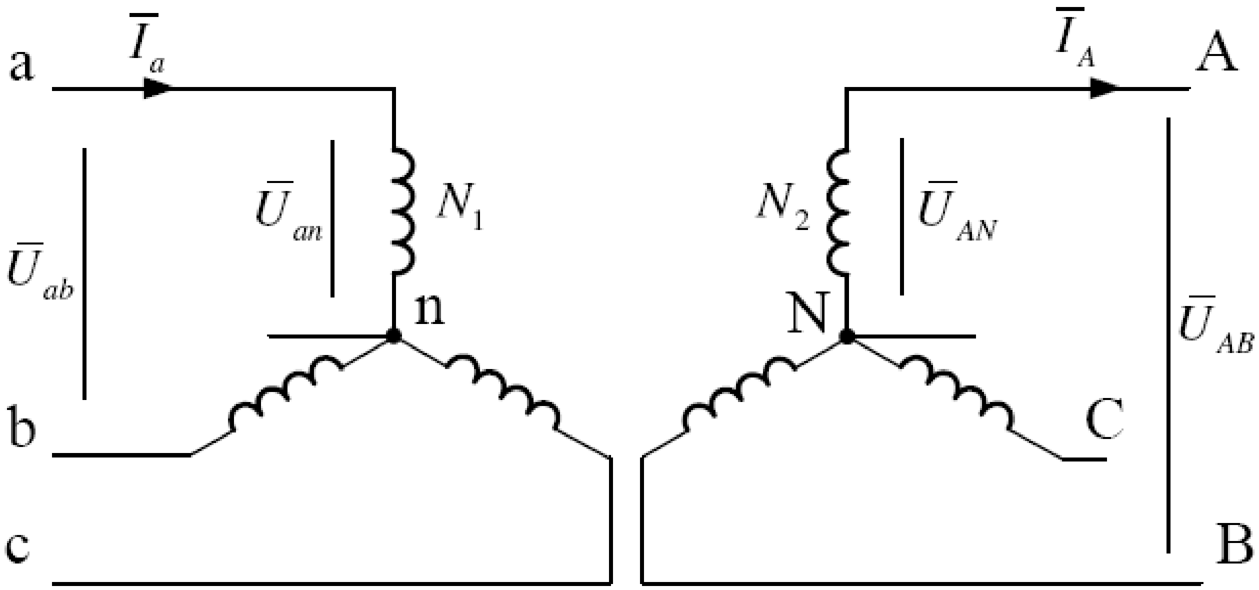
Equivalent diagram of the transformer



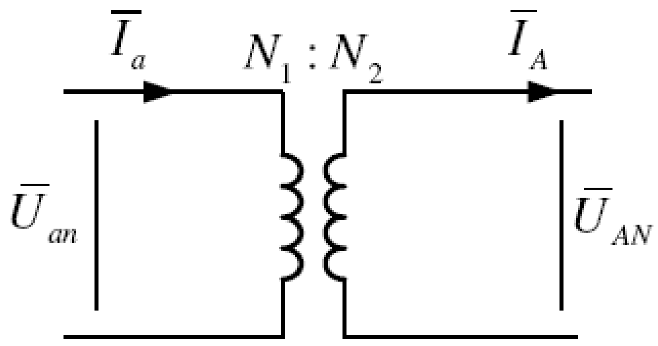
Three phase transformer - standard connections



Symmetrical three phase systems - 2



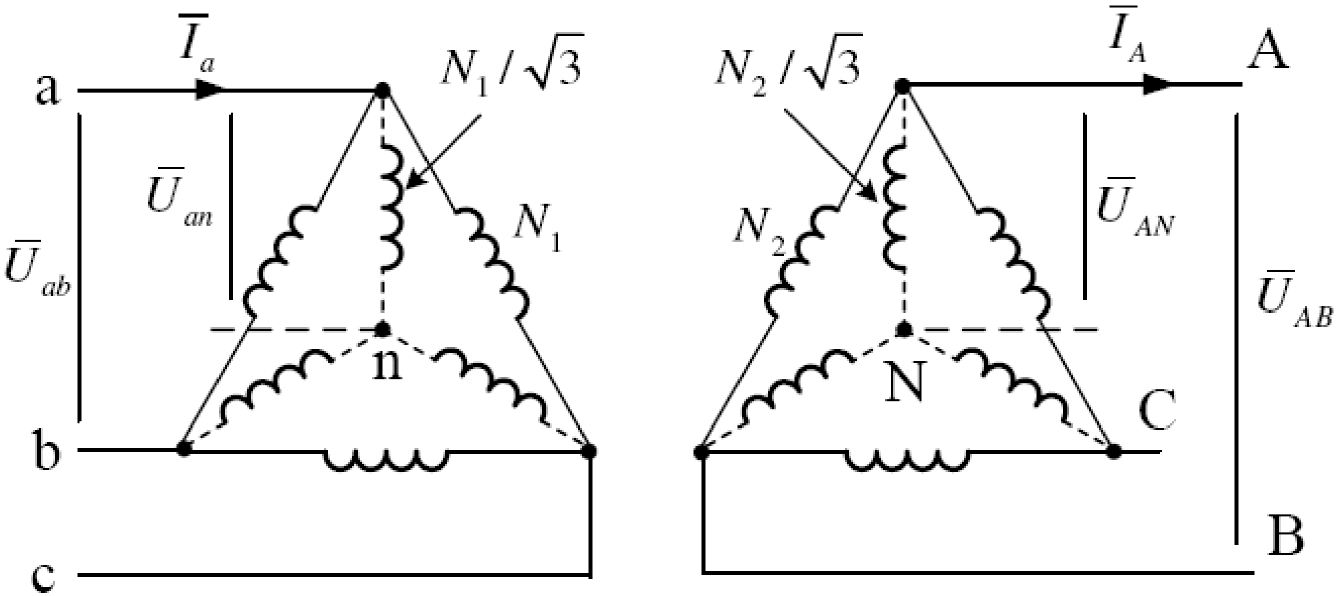
a)



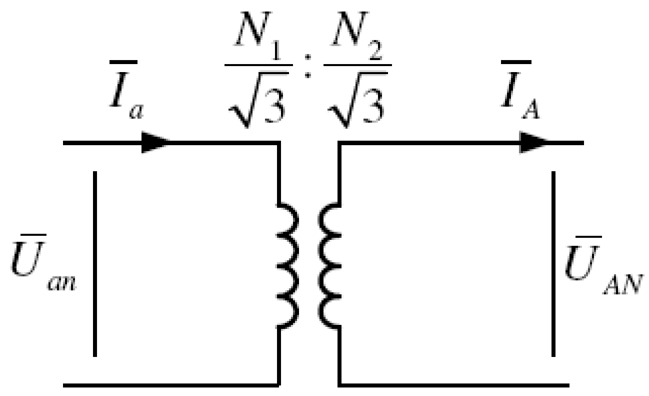
$$\frac{U_{an}}{U_{AN}} = \frac{N_1}{N_2} \; ; \; \frac{U_{ab}}{U_{AB}} = \frac{N_1}{N_2}$$

b)

Symmetrical three phase systems - 3



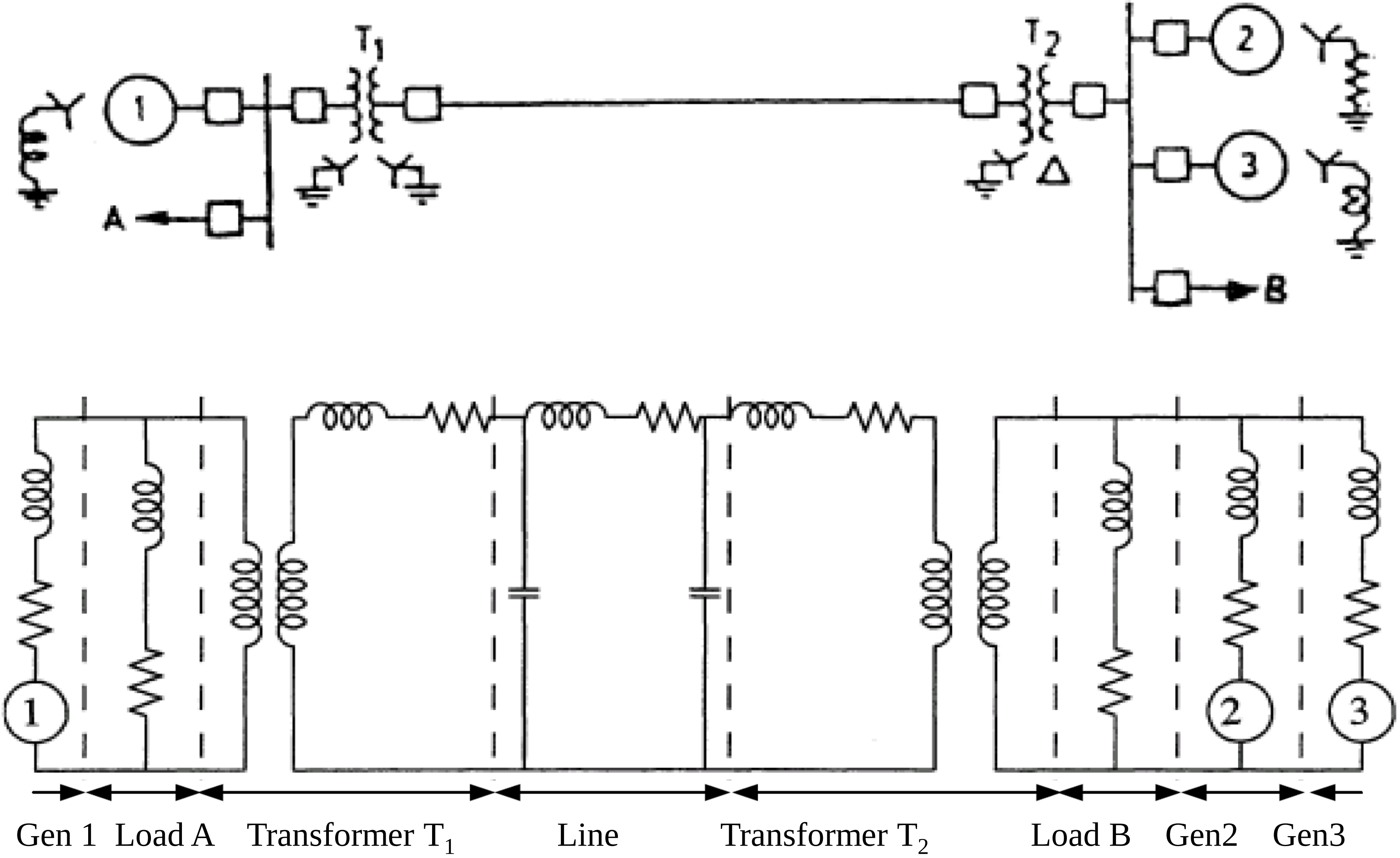
a)



$$\frac{U_{an}}{U_{AB}} = \frac{N_1/\sqrt{3}}{N_2/\sqrt{3}} \quad ; \quad \frac{U_{ab}}{U_{AB}} = \frac{N_1}{N_2}$$

b)

One-line diagram and impedance diagram



Per-unit (PU)-system - 1

The per-unit value of a certain quantity is defined as

$$\text{Per-unit value} = \frac{\text{true value}}{\text{base value of the quantity}}$$

By using:

U_b= phase-to-phase voltage = base voltage, kV and a base power,

S_b= three-phase base power, MVA the base current

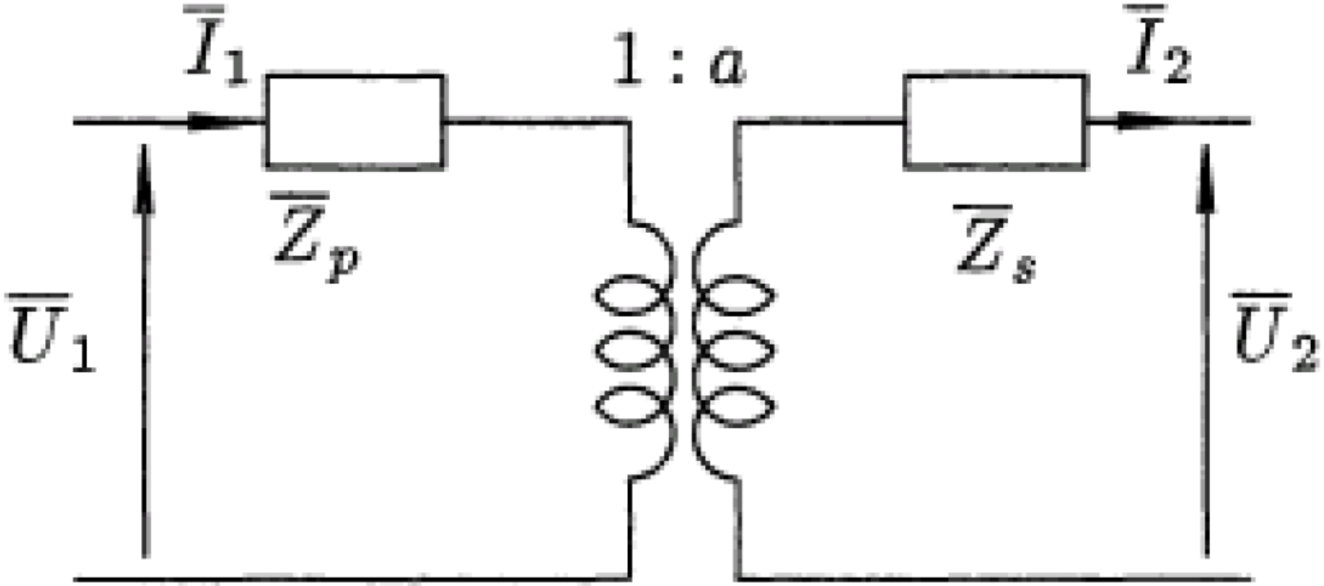
Per-unit (PU)-system - 2

$$I_b = \frac{S_b}{\sqrt{3}U_b} = \text{base current/phase, kA}$$

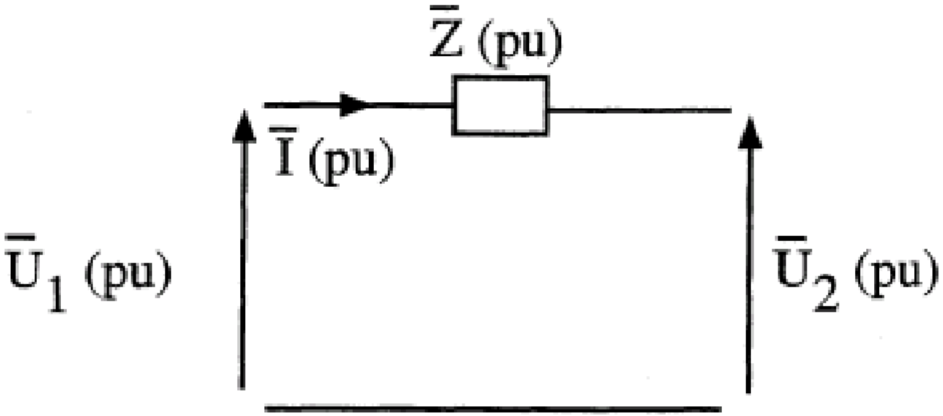
$$Z_b = \frac{U_b^2}{S_b} = \text{base impedance, } \Omega$$

can be calculated. In these expressions, the units kV and MVA have been assumed, which implies units kA and Ω. Of course, different combinations of units can be used e.g. V, VA, A, Ω or kV, kVA, A k Ω.

Per-unit (PU) system - 3



$$\frac{U_{1b}}{U_{2b}} = \frac{1}{a} \quad \Rightarrow$$



Why Per-unit ?

- You directly see e.g. the percentual voltage drop
- Simpler to make calculations for systems with several voltage levels
- Gives an estimation of the relative importance of different impedances
- Better numerical accuracy