Symmetrical components

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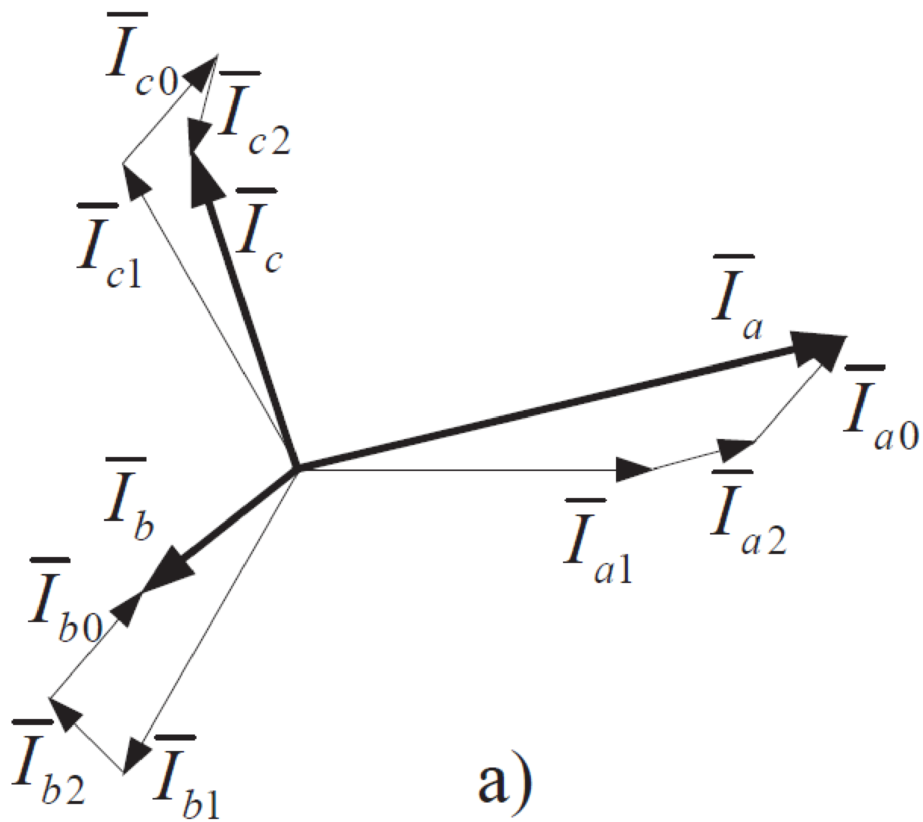
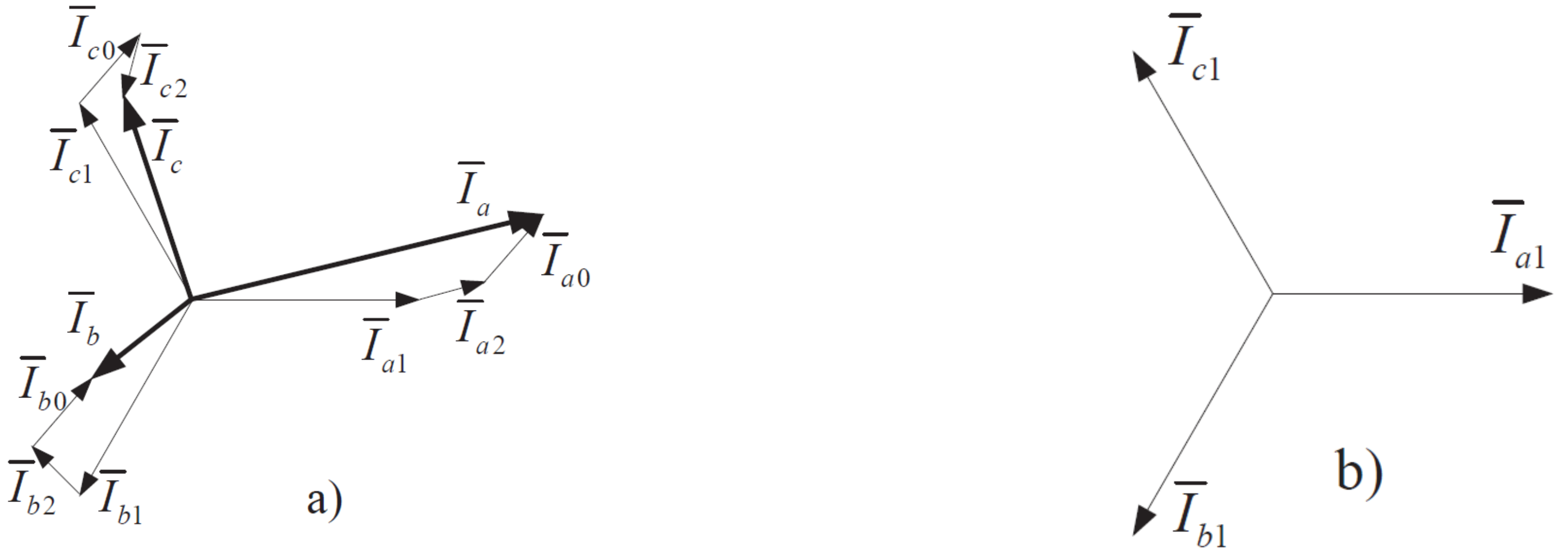
A set of three unbalanced phasors in a three-phase system can be resolved into three balanced systems of phasors (also known as symmetrical components).

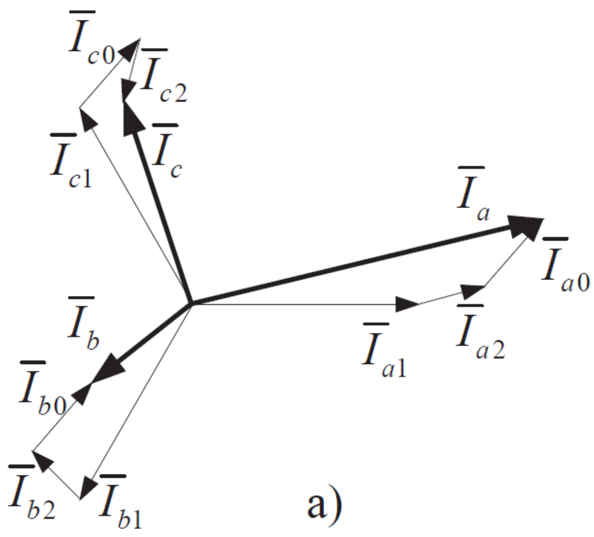


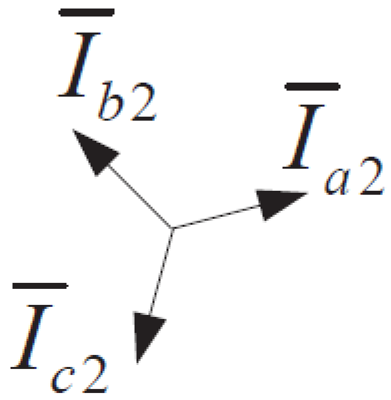
a)



Positive-sequence components (symbolized with 1):

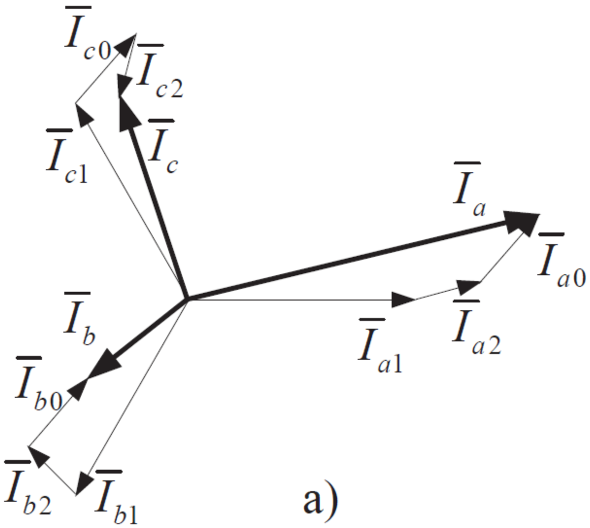
which consist of a balanced system of three phasors equal in magnitude, and displaced from each other by 120° in phase. They have the same phase sequence as the original phasors.



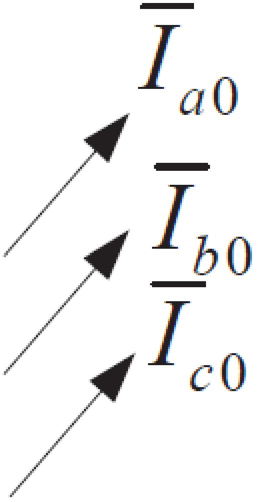


Negative-sequence components (symbolized with 2):

which consist of a balanced system of three phasors equal in magnitude, and displaced from each other by 120° in phase. They have the phase sequence opposite to that of the original phasors.

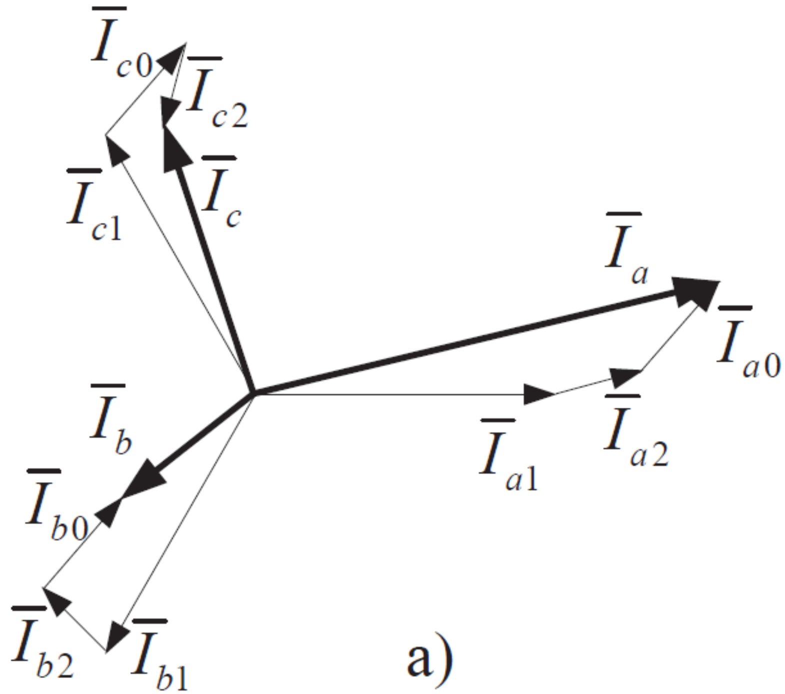


a)



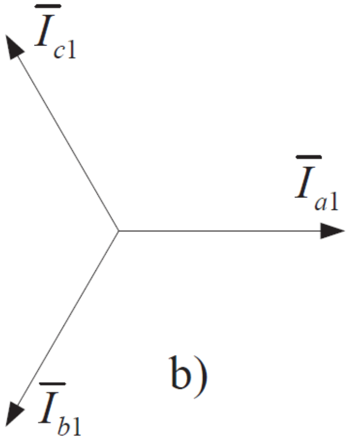
Zero-sequence components (symbolized with 0):

which consist of a balanced system of three phasors equal in magnitude and phase.



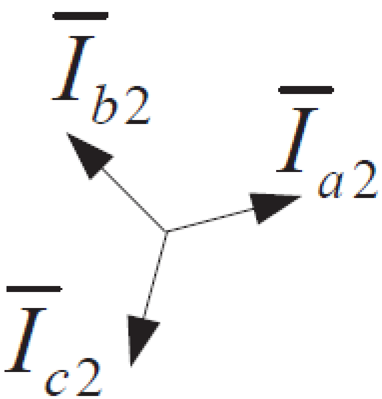
a)

$$
\begin{aligned}
\overline{I}_a &= \overline{I}_{a1} + \overline{I}_{a2} + \overline{I}_{a0} \\
\overline{I}_b &= \overline{I}_{b1} + \overline{I}_{b2} + \overline{I}_{b0} \\
\overline{I}_c &= \overline{I}_{c1} + \overline{I}_{c2} + \overline{I}_{c0}
\end{aligned}
$$

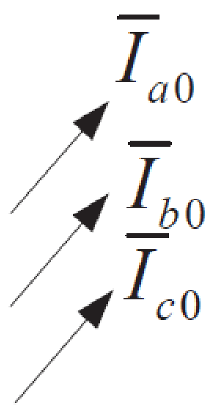


b)

$$\overline{I}_{b1} = \overline{I}_{a1} e^{-j120^\circ}$$
$$\overline{I}_{c1} = \overline{I}_{a1} e^{j120^\circ}$$



$$\overline{I}_{b2} = \overline{I}_{a2} e^{j120^\circ}$$
$$\overline{I}_{c2} = \overline{I}_{a2} e^{-j120^\circ}$$



$$\overline{I}_{a0} = \overline{I}_{b0} = \overline{I}_{c0}$$

$$\begin{aligned}\overline{I}_{b1} &= \overline{I}_{a1}e^{-j120^\circ} & \overline{I}_{b2} &= \overline{I}_{a2}e^{j120^\circ}\\ \overline{I}_{c1} &= \overline{I}_{a1}e^{j120^\circ} & \overline{I}_{c2} &= \overline{I}_{a2}e^{-j120^\circ}\end{aligned}$$

$$\overline{I}_{a0} = \overline{I}_{b0} = \overline{I}_{c0}$$

$$\boxed{\begin{aligned}\overline{I}_a &= \overline{I}_{a1} + \overline{I}_{a2} + \overline{I}_{a0}\\ \overline{I}_b &= \overline{I}_{b1} + \overline{I}_{b2} + \overline{I}_{b0}\\ \overline{I}_c &= \overline{I}_{c1} + \overline{I}_{c2} + \overline{I}_{c0}\end{aligned}}$$

$$\boxed{\begin{aligned}\overline{I}_a &= \overline{I}_{a1} + \overline{I}_{a2} + \overline{I}_{a0}\\ \overline{I}_b &= \alpha^2\overline{I}_{a1} + \alpha\overline{I}_{a2} + \overline{I}_{a0}\\ \overline{I}_c &= \alpha\overline{I}_{a1} + \alpha^2\overline{I}_{a2} + \overline{I}_{a0}\end{aligned}}$$

$$\boxed{\begin{aligned} \overline{I}_a &= \overline{I}_{a1} + \overline{I}_{a2} + \overline{I}_{a0} \\ \overline{I}_b &= \alpha^2 \overline{I}_{a1} + \alpha \overline{I}_{a2} + \overline{I}_{a0} \\ \overline{I}_c &= \alpha \overline{I}_{a1} + \alpha^2 \overline{I}_{a2} + \overline{I}_{a0} \end{aligned}}$$

$$\alpha = e^{j120^\circ} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$\alpha^2 = e^{j240^\circ} = e^{-j120^\circ} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$$\alpha = e^{j120^\circ} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$\alpha^2 = e^{j240^\circ} = e^{-j120^\circ} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$$\begin{aligned} \alpha^3 &= 1 \\ 1 + \alpha + \alpha^2 &= 0 \\ \alpha^* &= \alpha^2 \\ (\alpha^2)^* &= \alpha \end{aligned}$$

$$\boxed{\begin{aligned} \bar{I}_a &= \bar{I}_{a1} + \bar{I}_{a2} + \bar{I}_{a0} \\ \bar{I}_b &= \alpha^2 \bar{I}_{a1} + \alpha \bar{I}_{a2} + \bar{I}_{a0} \\ \bar{I}_c &= \alpha \bar{I}_{a1} + \alpha^2 \bar{I}_{a2} + \bar{I}_{a0} \end{aligned}}$$

$$\boxed{\mathbf{I_{ph}} = \mathbf{T I_s}}$$

$$\mathbf{I_{ph}} = \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix}$$

$$\mathbf{I_s} = \begin{bmatrix} \bar{I}_{a1} \\ \bar{I}_{a2} \\ \bar{I}_{a0} \end{bmatrix} = \begin{bmatrix} \bar{I}_{-1} \\ \bar{I}_{-2} \\ \bar{I}_{-0} \end{bmatrix}$$

$$\mathbf{I_s} = \mathbf{T}^{-1}\mathbf{I_{ph}} \qquad \mathbf{T}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix}$$

$$\boxed{\mathbf{I_{ph}} = \mathbf{T}\mathbf{I_s}} \qquad \mathbf{T} = \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix}$$

$$\mathbf{U_s} = \mathbf{T}^{-1}\mathbf{U_{ph}}$$

$$\mathbf{U_{ph}} = \mathbf{T}\mathbf{U_s}$$

Example 8.1

Calculate the symmetrical components for the following symmetrical voltages

$$\mathbf{U_{ph}} = \begin{bmatrix} \overline{U}_a \\ \overline{U}_b \\ \overline{U}_c \end{bmatrix} = \begin{bmatrix} 277\angle 0^\circ \\ 277\angle -120^\circ \\ 277\angle +120^\circ \end{bmatrix} \text{ V}$$

$$
\begin{bmatrix} \overline{U}_{-1} \\ \overline{U}_{-2} \\ \overline{U}_{-0} \end{bmatrix} = \mathbf{U_s} = \mathbf{T}^{-1}\mathbf{U_{ph}}
$$

$$
= \frac{1}{3} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \overline{U}_a \\ \overline{U}_b \\ \overline{U}_c \end{bmatrix}
$$

$$
= \begin{bmatrix} 277\angle 0^\circ \\ 0 \\ 0 \end{bmatrix}
$$

$$\begin{bmatrix} \overline{U}_{-1} \\ \overline{U}_{-2} \\ \overline{U}_{-0} \end{bmatrix} = \mathbf{U_s} = \mathbf{T}^{-1}\mathbf{U_{ph}}$$

$$= \begin{bmatrix} 277\angle 0^\circ \\ 0 \\ 0 \end{bmatrix}$$

A symmetric three-phase system with a phase sequence of abca only gives rise to a positive-sequence voltage, having the same amplitude and angle as the voltage in phase a.

Example 8.2

For a Y0-connected three-phase load with a neutral-ground conductor, phase b is at one occasion disconnected. The load currents at that occasion are:

$$\mathbf{I_{ph}} = \begin{bmatrix} \overline{I}_a \\ \overline{I}_b \\ \overline{I}_c \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 0 \\ 10\angle +120° \end{bmatrix} \quad A$$

Calculate the symmetrical components of the load currents as well as the current in the neutral-ground conductor, $\overline{I}_n$.

$$\mathbf{I_s} = \mathbf{T^{-1} I_{ph}} \qquad \mathbf{T^{-1}} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \overline{I}_{-1} \\ \overline{I}_{-2} \\ \overline{I}_{-0} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \cdot 10\angle 0^\circ + 1\angle 120^\circ \cdot 0 + 1\angle 240^\circ \cdot 10\angle + 120^\circ \\ 1 \cdot 10\angle 0^\circ + 1\angle 240^\circ \cdot 0 + 1\angle 120^\circ \cdot 10\angle + 120^\circ \\ 1 \cdot 10\angle 0^\circ + 1 \cdot 0 + 1 \cdot 10\angle + 120^\circ \end{bmatrix} =$$

$$= \begin{bmatrix} 6.667\angle 0^\circ \\ 3.333\angle - 60^\circ \\ 3.333\angle 60^\circ \end{bmatrix}$$

$$\begin{aligned} \overline{I}_n &= \overline{I}_a + \overline{I}_b + \overline{I}_c = \\ &= 10\angle 0^\circ + 0 + 10\angle + 120^\circ \\ &= 10\angle 60^\circ = 3\overline{I}_{-0} \end{aligned}$$

For an unbalanced system, the current in the neutral-ground conductor (if any) is three times as large as the zero-sequence current.

$$\overline{I}_n \;=\; \overline{I}_a + \overline{I}_b + \overline{I}_c = 3\overline{I}_{-0}$$

Power calculations under unbalanced conditions

the three-phase complex power

$$\overline{S} = P + jQ = \overline{U}_a \overline{I}_a^* + \overline{U}_b \overline{I}_b^* + \overline{U}_c \overline{I}_c^* =$$

$$= \mathbf{U}_{\mathbf{ph}}^{\mathbf{t}} \mathbf{I}_{\mathbf{ph}}^{*}$$

converted to symmetrical components,

$$\overline{S} = \mathbf{U_{ph}^{t}} \mathbf{I_{ph}^{*}} = (\mathbf{T}\mathbf{U_{s}})^{\mathbf{t}} (\mathbf{T}\mathbf{I_{s}})^{*} =$$

$$= \mathbf{U_{s}^{t}} \mathbf{T^{t}} \mathbf{T^{*}} \mathbf{I_{s}^{*}}$$

$$\mathbf{T^{t}T^{*}} = \begin{bmatrix} 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & \alpha & 1 \end{bmatrix} = 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\overline{S} = \mathbf{U_{ph}^{t}} \mathbf{I_{ph}^{*}} = 3\, \mathbf{U_{s}^{t}} \mathbf{I_{s}^{*}}$$

$$= 3\, \overline{U}_{-1}\, \overline{I}_{-1}^{*} + 3\, \overline{U}_{-2}\, \overline{I}_{-2}^{*} + 3\, \overline{U}_{-0}\, \overline{I}_{-0}^{*}$$

$$\overline{S} = \mathbf{U}^{\mathbf{t}}_{\mathbf{ph}} \mathbf{I}^{*}_{\mathbf{ph}} \; = \; 3\,\mathbf{U}^{\mathbf{t}}_{\mathbf{s}}\,\mathbf{I}^{*}_{\mathbf{s}}$$

$$\mathbf{U}^{\mathbf{t}}_{\mathbf{ph}} \mathbf{I}^{*}_{\mathbf{ph}} \; \neq \; \mathbf{U}^{\mathbf{t}}_{\mathbf{s}}\mathbf{I}^{*}_{\mathbf{s}}$$

the transformation is not power invariant

Power invariant transformation

Power in the original coordinate system

$$\overline{S}_{abc} = \overline{U}_a \overline{I}_a^* + \overline{U}_b \overline{I}_b^* + \overline{U}_c \overline{I}_c^* = \mathbf{U}_{\mathbf{abc}}^{\mathbf{t}} \mathbf{I}_{\mathbf{abc}}^*$$

Transformed to a new coordinate system

$$\overline{S}_{ABC} = \overline{U}_A \overline{I}_A^* + \overline{U}_B \overline{I}_B^* + \overline{U}_C \overline{I}_C^* = \mathbf{U}_{\mathbf{ABC}}^{\mathbf{t}} \mathbf{I}_{\mathbf{ABC}}^*$$

By a transformation matrix

$$\mathbf{T}$$

$$\mathbf{U}_{\mathbf{abc}}^{\mathbf{t}}\mathbf{I}_{\mathbf{abc}}^{*} = (\mathbf{T}\mathbf{U}_{\mathbf{ABC}})^{\mathbf{t}}(\mathbf{T}\mathbf{I}_{\mathbf{ABC}})^{*} =$$

$$= \mathbf{U}_{\mathbf{ABC}}^{\mathbf{t}}\mathbf{T}^{\mathbf{t}}\mathbf{T}^{*}\mathbf{I}_{\mathbf{ABC}}^{*}$$

$$= \mathbf{U}_{\mathbf{ABC}}^{\mathbf{t}}\mathbf{I}_{\mathbf{ABC}}^{*}$$

$$\mathbf{T}^{\mathbf{t}}\mathbf{T}^{*} = \left((\mathbf{T}^{*})^{\mathbf{t}}\,\mathbf{T}\right)^{\mathbf{t}} = \underline{\mathbf{1}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}^{-\mathbf{1}} = (\mathbf{T}^{*})^{\mathbf{t}}$$

$$\overline{S} = \mathbf{U}^{\mathbf{t}}_{\mathbf{ph}} \mathbf{I}^{*}_{\mathbf{ph}} = 3\,\mathbf{U}^{\mathbf{t}}_{\mathbf{s}} \mathbf{I}^{*}_{\mathbf{s}}$$

$$\mathbf{U}^{\mathbf{t}}_{\mathbf{ph}} \mathbf{I}^{*}_{\mathbf{ph}} \neq \mathbf{U}^{\mathbf{t}}_{\mathbf{s}} \mathbf{I}^{*}_{\mathbf{s}}$$

the transformation is not power invariant

$$S_{base} = \sqrt{3} \cdot U_{base} \cdot I_{base};$$

$$\overline{S}_{pu} = \frac{3\,\mathbf{U_s^t}\,\mathbf{I_s^*}}{\sqrt{3} \cdot U_{base} \cdot I_{base}} =$$

$$= \frac{\sqrt{3}(\sqrt{3}\mathbf{U_s})^t \cdot \mathbf{I_s^*}}{\sqrt{3} \cdot U_{base} \cdot I_{base}} =$$

$$= \overline{U}_{pu-1}\,\overline{I}^*_{pu-1} + \overline{U}_{pu-2}\,\overline{I}^*_{pu-2} + \overline{U}_{pu-0}\,\overline{I}^*_{pu-0}$$

$$\overline{S}_{pu} = \overline{U}_{pu-1}\,\overline{I}^{*}_{pu-1} + \overline{U}_{pu-2}\,\overline{I}^{*}_{pu-2} + \overline{U}_{pu-0}\,\overline{I}^{*}_{pu-0}$$

The total complex power (in pu) in an unbalanced system can be expressed by the sum of the symmetrical components of power.

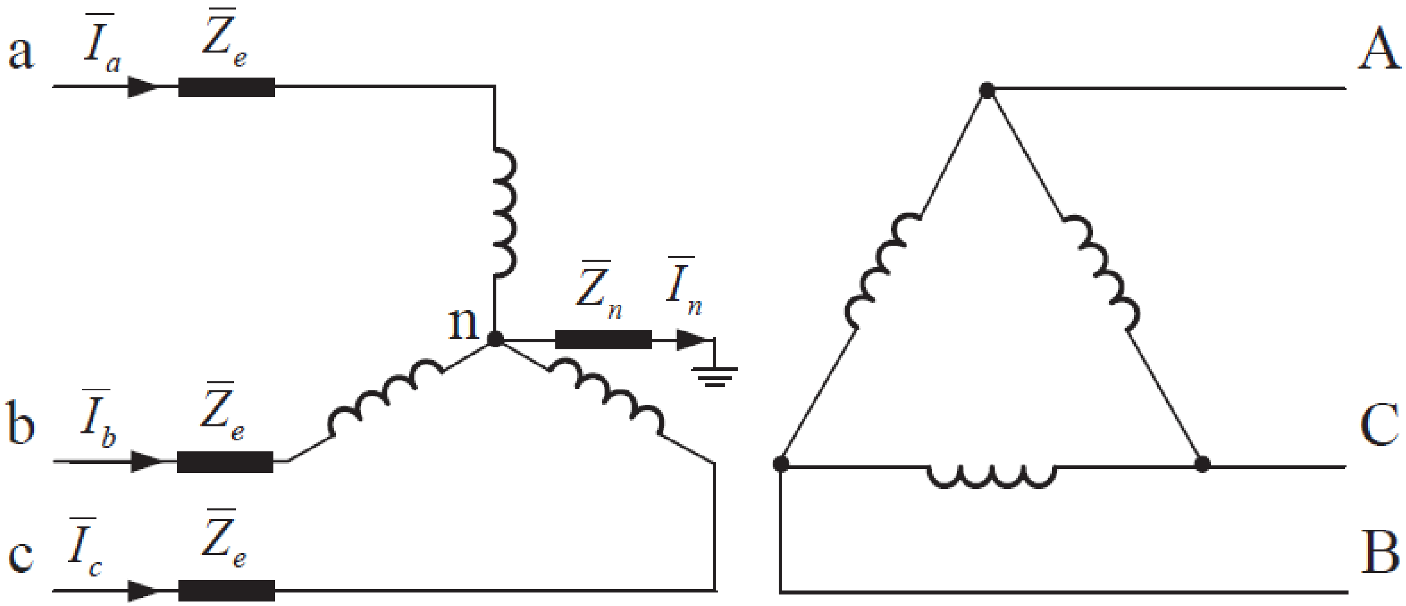
The total complex power in (VA) is then given by

$$\overline{S} = \overline{S}_{pu}\,S_{base}$$

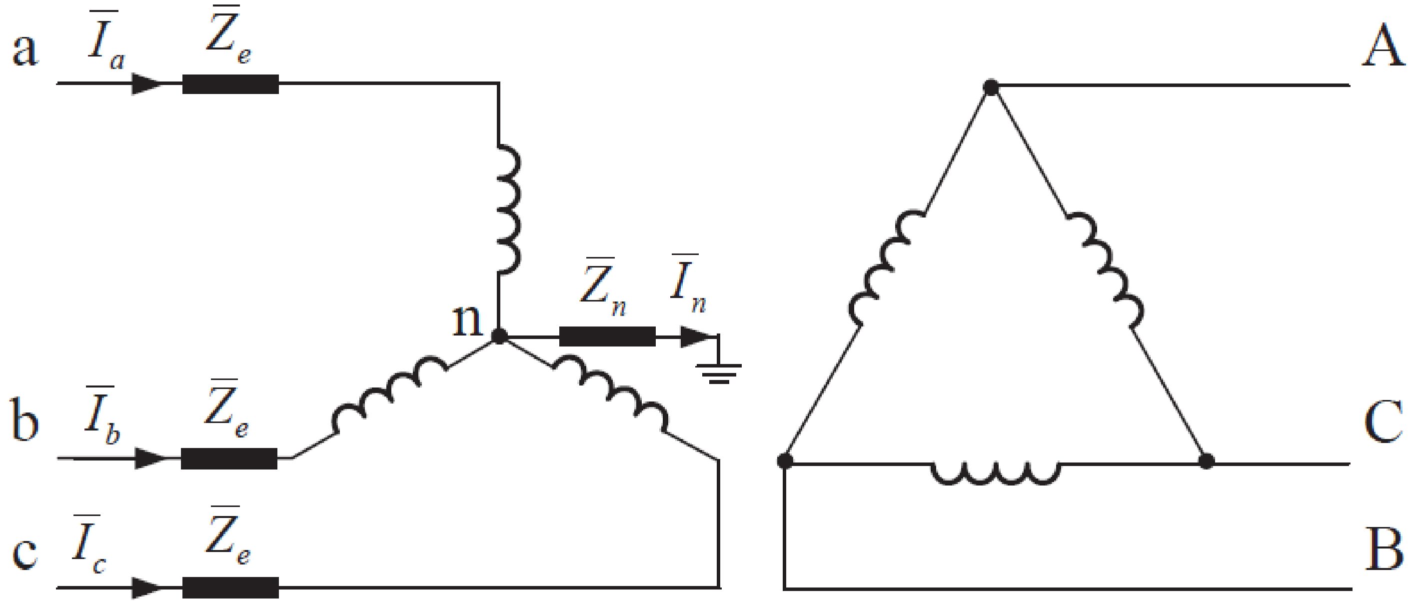
$$= \overline{U}_{pu-1}\,\overline{I}^{*}_{pu-1}\,S_{base} + \overline{U}_{pu-2}\,\overline{I}^{*}_{pu-2}\,S_{base} + \overline{U}_{pu-0}\,\overline{I}^{*}_{pu-0}\,S_{base}$$

Sequence circuits of power system components

Transformers

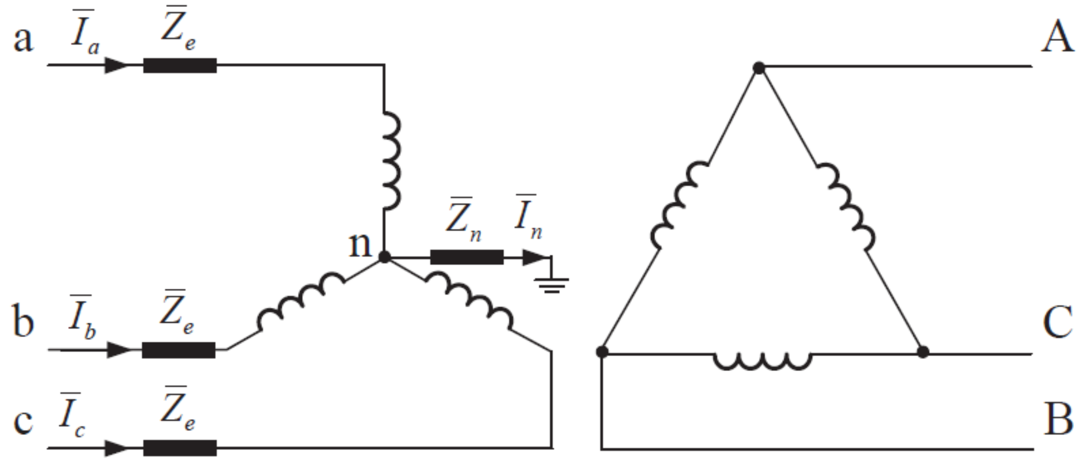


Y0-Δ connected transformer with neutral point grounded through an impedance $\overline{Z}_n$.



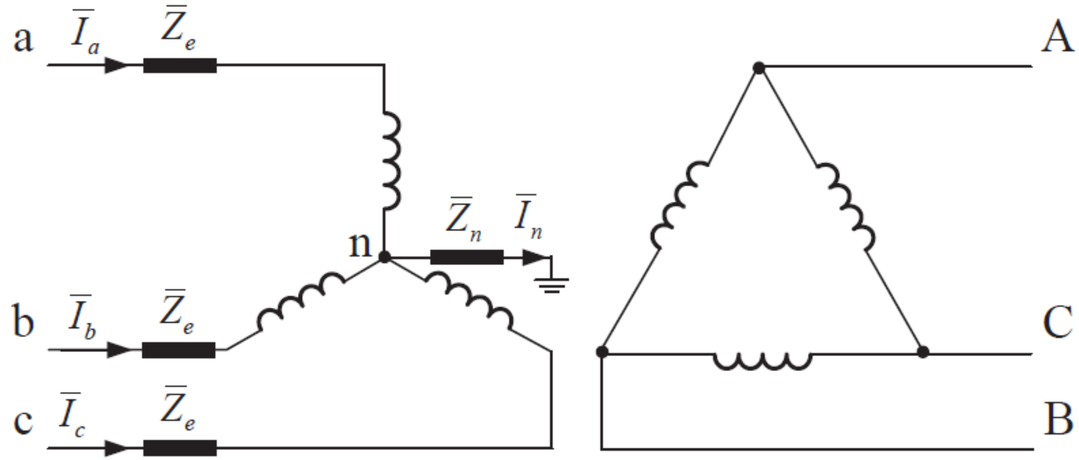
$\overline{Z}_e$ represents the equivalent impedance of each phase and consists of both leakage reactance of the primary and secondary windings as well as the resistance of the windings

the windings are considered as ideal windings.



$$
\begin{aligned}
\Delta\overline{U}_a &= \overline{I}_a\overline{Z}_e + \overline{I}_n\overline{Z}_n \\
\Delta\overline{U}_b &= \overline{I}_b\overline{Z}_e + \overline{I}_n\overline{Z}_n \\
\Delta\overline{U}_c &= \overline{I}_c\overline{Z}_e + \overline{I}_n\overline{Z}_n
\end{aligned}
$$

$$\overline{I}_n = \overline{I}_a + \overline{I}_b + \overline{I}_c$$



$$\mathbf{\Delta U_{ph}} = \begin{bmatrix} \Delta\overline{U}_a \\ \Delta\overline{U}_b \\ \Delta\overline{U}_c \end{bmatrix} = \begin{bmatrix} \overline{Z}_e + \overline{Z}_n & \overline{Z}_n & \overline{Z}_n \\ \overline{Z}_n & \overline{Z}_e + \overline{Z}_n & \overline{Z}_n \\ \overline{Z}_n & \overline{Z}_n & \overline{Z}_e + \overline{Z}_n \end{bmatrix} \begin{bmatrix} \overline{I}_a \\ \overline{I}_b \\ \overline{I}_c \end{bmatrix}$$

$$= \mathbf{Z_{tr} I_{ph}}$$

$$\mathbf{U_{ph}} = \mathbf{T}\mathbf{U_s} = \mathbf{Z_{ph}}\mathbf{T}\mathbf{I_s} = \mathbf{Z_{ph}}\mathbf{I_{ph}}$$

$$\mathbf{U_s} = \mathbf{T}^{-1}\mathbf{Z_{ph}}\mathbf{T}\mathbf{I_s} = \mathbf{Z_s}\mathbf{I_s}$$

$$\mathbf{T}^{-1}\mathbf{Z_{tr}}\mathbf{T}$$

$$\mathbf{T}^{-1}\begin{bmatrix} \overline{Z}_e + \overline{Z}_n & \overline{Z}_n & \overline{Z}_n \\ \overline{Z}_n & \overline{Z}_e + \overline{Z}_n & \overline{Z}_n \\ \overline{Z}_n & \overline{Z}_n & \overline{Z}_e + \overline{Z}_n \end{bmatrix}\mathbf{T}$$

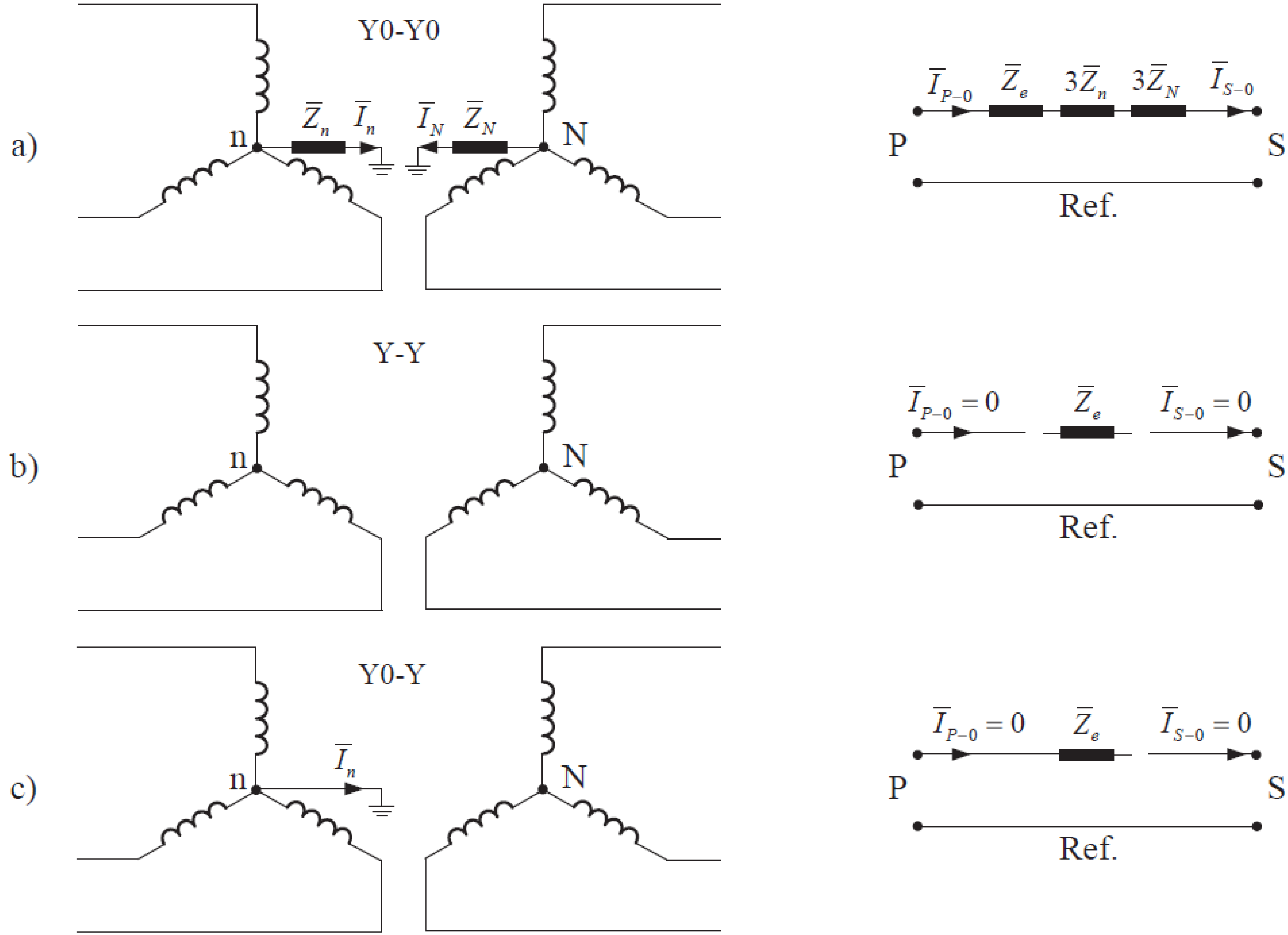
$$\mathbf{Z_{trs}} = \mathbf{T}^{-1}\mathbf{Z_{tr}}\mathbf{T} = \begin{bmatrix} \overline{Z}_e & 0 & 0 \\ 0 & \overline{Z}_e & 0 \\ 0 & 0 & \overline{Z}_e + 3\overline{Z}_n \end{bmatrix}$$

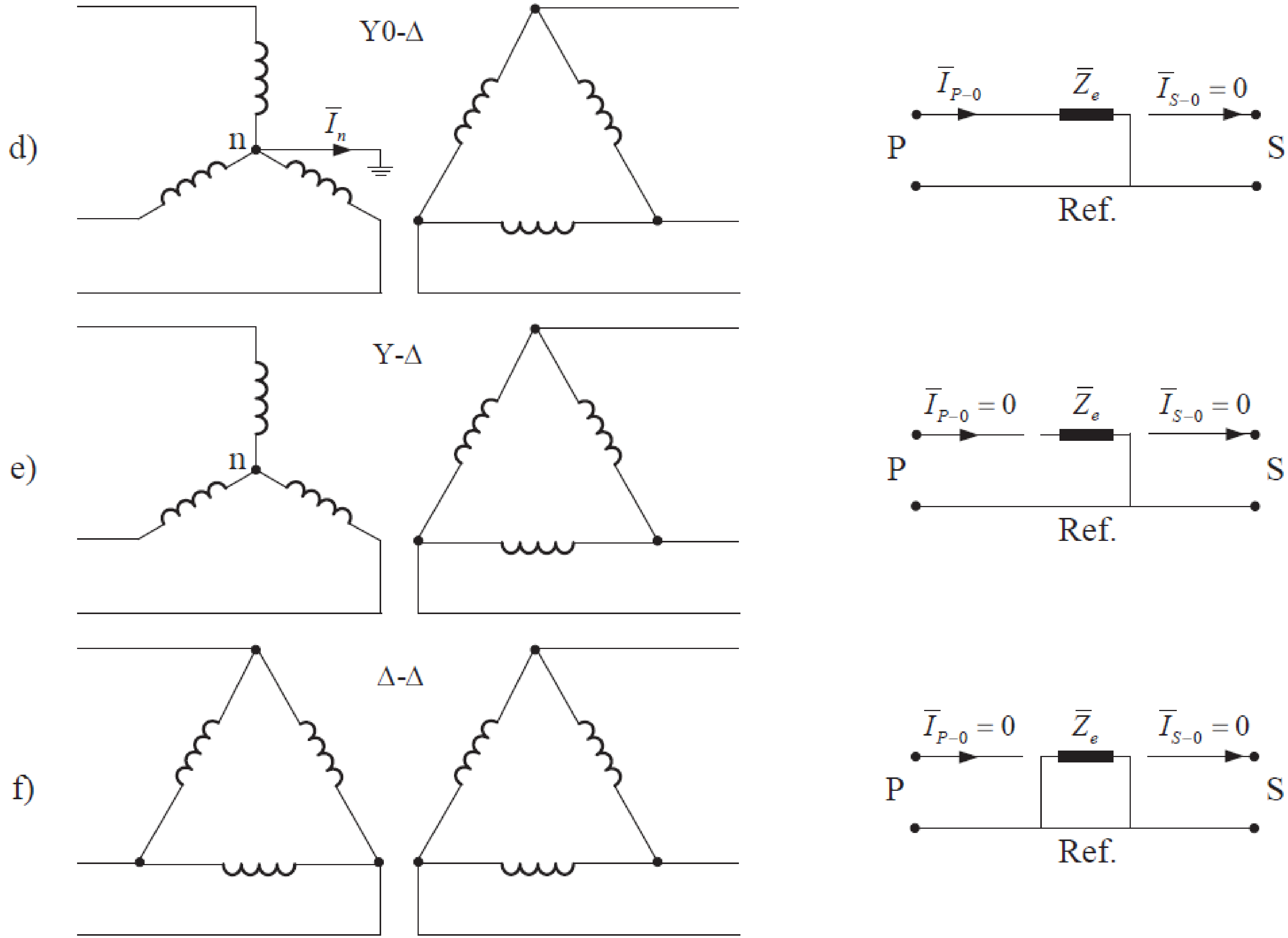
$$\begin{aligned} \overline{Z}_{t-1} &= \overline{Z}_e = \text{ positive-sequence impedance} \\ \overline{Z}_{t-2} &= \overline{Z}_e = \text{ negative-sequence impedance} \\ \overline{Z}_{t-0} &= \overline{Z}_e + 3\overline{Z}_n = \text{ zero-sequence impedance} \end{aligned}$$

$$\overline{Z}_{t-1} = \overline{Z}_{t-2}$$

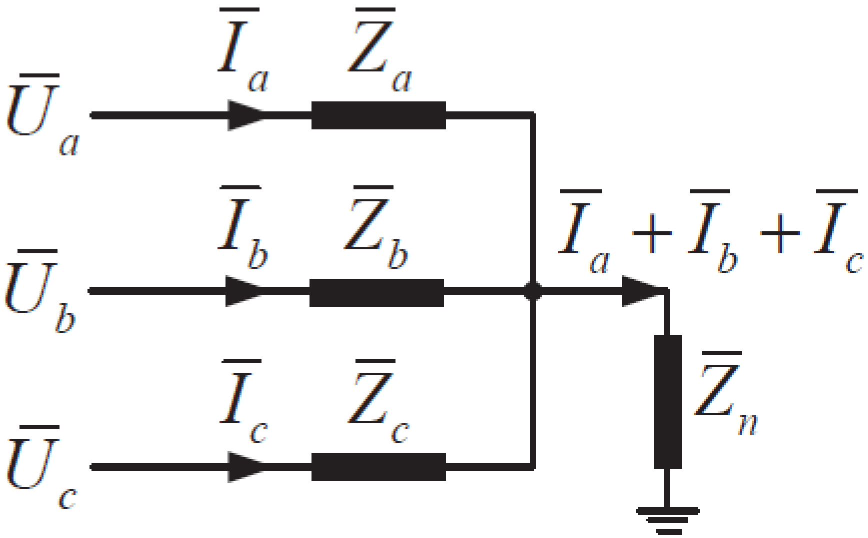
Winding connection

Zero-sequence equivalent circuit




d)
Y0-Δ
n
$\overline{I}_n$
$\overline{I}_{P-0}$
$\overline{Z}_e$
$\overline{I}_{S-0} = 0$
P
S
Ref.
e)
Y-Δ
n
$\overline{I}_{P-0} = 0$
$\overline{Z}_e$
$\overline{I}_{S-0} = 0$
P
S
Ref.
f)
Δ-Δ
$\overline{I}_{P-0} = 0$
$\overline{Z}_e$
$\overline{I}_{S-0} = 0$
P
S
Ref.

Impedance loads



Y0-connected load

$$\begin{bmatrix} \overline{U}_a \\ \overline{U}_b \\ \overline{U}_c \end{bmatrix} = \begin{bmatrix} \overline{Z}_a + \overline{Z}_n & \overline{Z}_n & \overline{Z}_n \\ \overline{Z}_n & \overline{Z}_b + \overline{Z}_n & \overline{Z}_n \\ \overline{Z}_n & \overline{Z}_n & \overline{Z}_c + \overline{Z}_n \end{bmatrix} \begin{bmatrix} \overline{I}_a \\ \overline{I}_b \\ \overline{I}_c \end{bmatrix}$$

$$\mathbf{U}_{\mathbf{Ph}} = \mathbf{Z}_{\mathbf{LDPh}} \mathbf{I}_{\mathbf{Ph}}$$

$$
\begin{aligned}
\mathbf{U_{ph}} &= \mathbf{T U_s} = \mathbf{Z_{LDph} I_{ph}} = \mathbf{Z_{LDph} T I_s} \\
&\Rightarrow \\
\mathbf{U_s} &= \mathbf{T^{-1} Z_{LDph} T I_s} = \mathbf{Z_{LDs} I_s}
\end{aligned}
$$

$$
\mathbf{Z_{LDs}} \equiv \mathbf{T^{-1} Z_{LDph} T} =
$$

$$
= \frac{1}{3}\begin{bmatrix} \overline{Z}_a + \overline{Z}_b + \overline{Z}_c & \overline{Z}_a + \alpha^2\overline{Z}_b + \alpha\overline{Z}_c & \overline{Z}_a + \alpha\overline{Z}_b + \alpha^2\overline{Z}_c \\ \overline{Z}_a + \alpha\overline{Z}_b + \alpha^2\overline{Z}_c & \overline{Z}_a + \overline{Z}_b + \overline{Z}_c & \overline{Z}_a + \alpha^2\overline{Z}_b + \alpha\overline{Z}_c \\ \overline{Z}_a + \alpha^2\overline{Z}_b + \alpha\overline{Z}_c & \overline{Z}_a + \alpha\overline{Z}_b + \alpha^2\overline{Z}_c & \overline{Z}_a + \overline{Z}_b + \overline{Z}_c + 9\overline{Z}_n \end{bmatrix}
$$

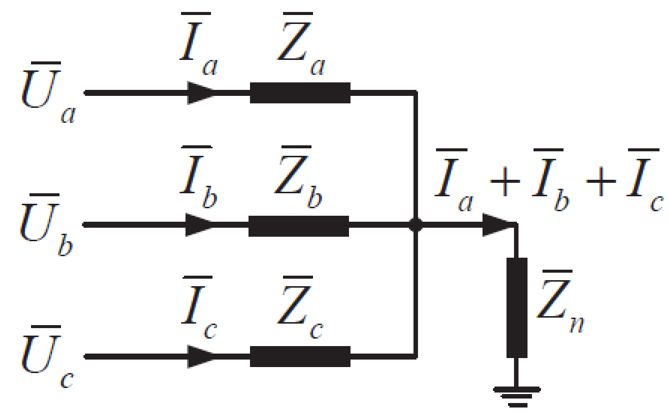
$$
\overline{Z}_a = \overline{Z}_b = \overline{Z}_c
$$

$$
\mathbf{Z_{LDs}} = \begin{bmatrix} \overline{Z}_a & 0 & 0 \\ 0 & \overline{Z}_a & 0 \\ 0 & 0 & \overline{Z}_a + 3\overline{Z}_n \end{bmatrix}
$$

$$\overline{Z}_a = \overline{Z}_b = \overline{Z}_c \qquad \mathbf{Z_{LDs}} = \begin{bmatrix} \overline{Z}_a & 0 & 0 \\ 0 & \overline{Z}_a & 0 \\ 0 & 0 & \overline{Z}_a + 3\overline{Z}_n \end{bmatrix}$$

$$\overline{Z}_{LD-1} = \overline{Z}_{LD-2} = \overline{Z}_a$$

$$\overline{Z}_{LD-0} = \overline{Z}_a + 3\overline{Z}_n.$$



$$\overline{Z}_n = 0 \text{ then } \overline{Z}_{LD-1} = \overline{Z}_{LD-2} = \overline{Z}_{LD-0}$$

Y-connected load $\qquad \overline{Z}_n = \infty = \overline{Z}_{LD-0}$

Δ-Y transformed

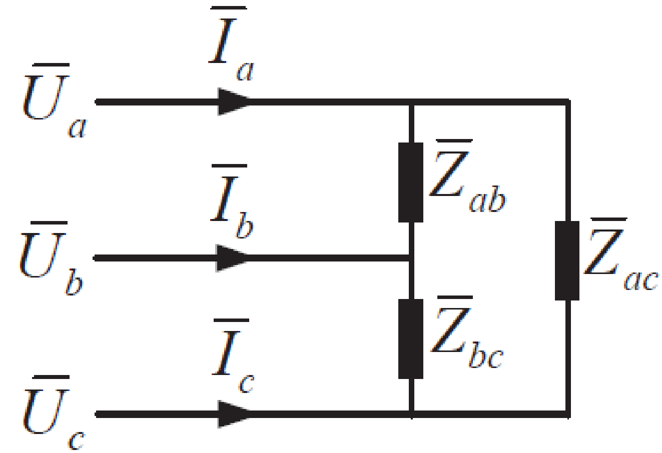
$$\overline{Z}_a = \frac{\overline{Z}_{ab}\overline{Z}_{ac}}{\overline{Z}_{ab}+\overline{Z}_{ac}+\overline{Z}_{bc}}$$

$$\overline{Z}_b = \frac{\overline{Z}_{ab}\overline{Z}_{bc}}{\overline{Z}_{ab}+\overline{Z}_{ac}+\overline{Z}_{bc}}$$

$$\overline{Z}_c = \frac{\overline{Z}_{ac}\overline{Z}_{bc}}{\overline{Z}_{ab}+\overline{Z}_{ac}+\overline{Z}_{bc}}$$

$$\overline{Z}_n = \infty$$

Δ-connected load

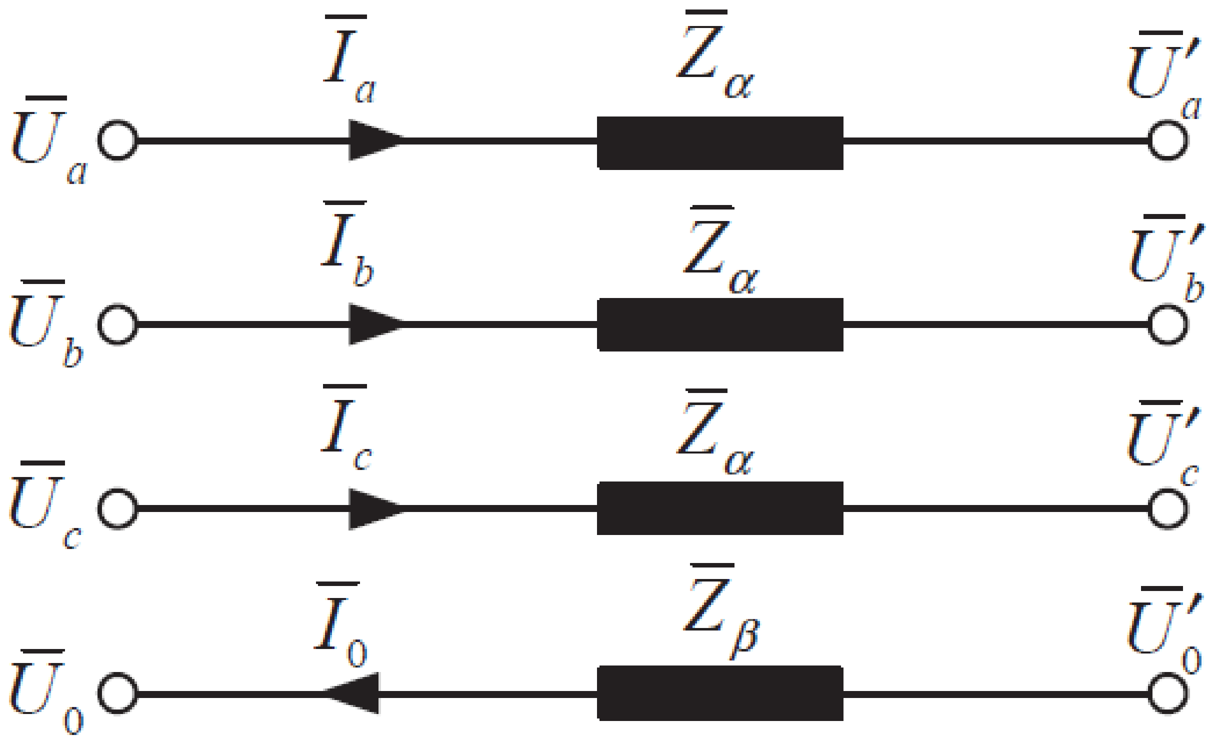


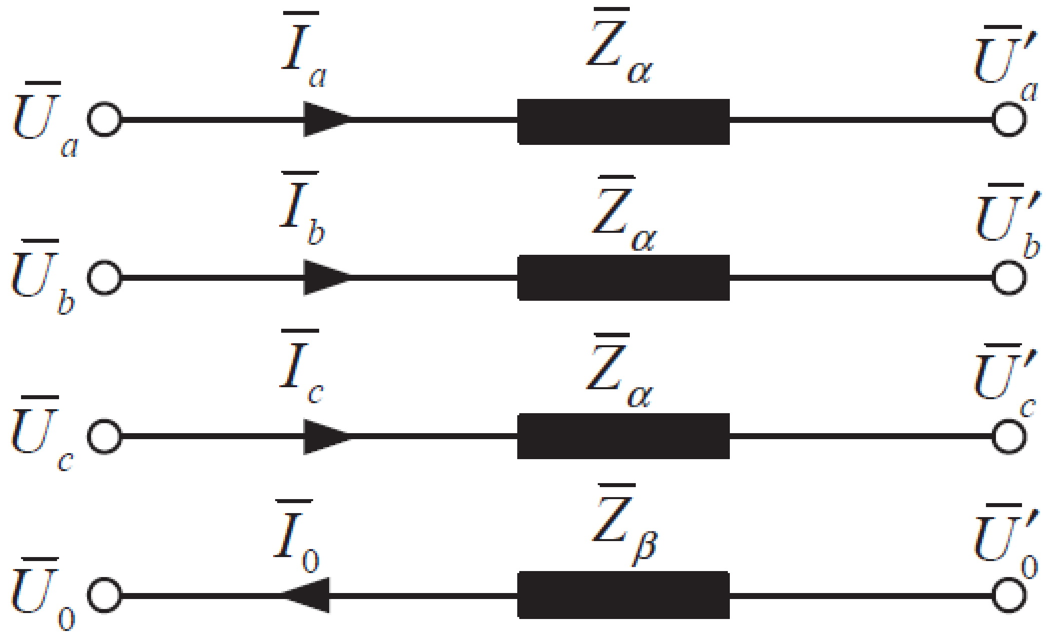
$$\overline{Z}_{ab} = \overline{Z}_{bc} = \overline{Z}_{ac}$$

$$\overline{Z}_{LD-1} = \overline{Z}_{ab}/3$$
$$\overline{Z}_{LD-2} = \overline{Z}_{ab}/3$$
$$\overline{Z}_{LD-0} = \infty$$

Equivalent diagram of the series impedance of a transmission line

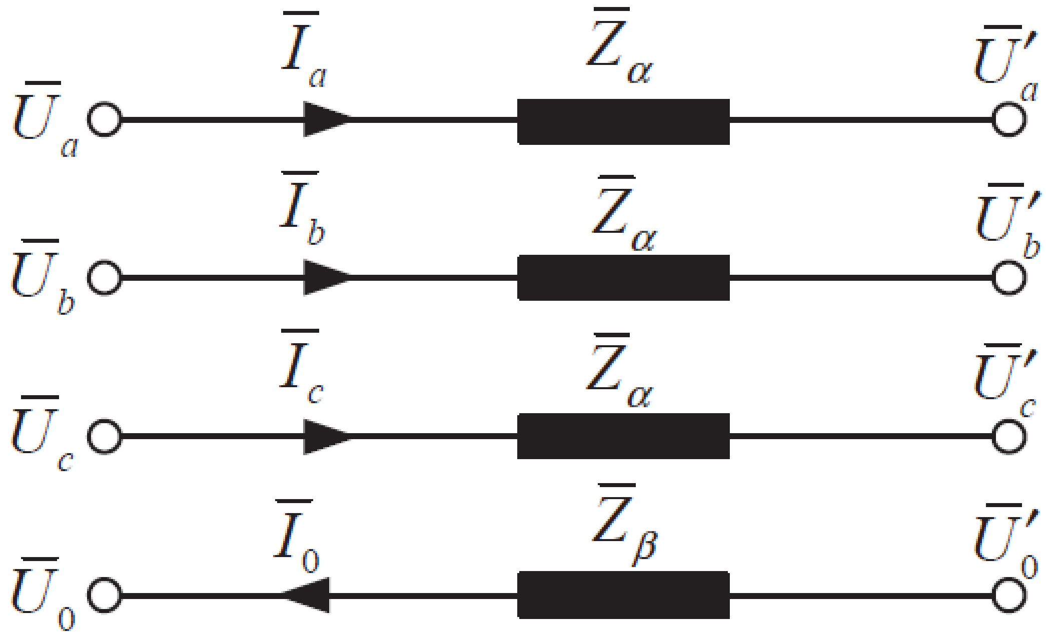
Equivalent circuit of a three-phase line





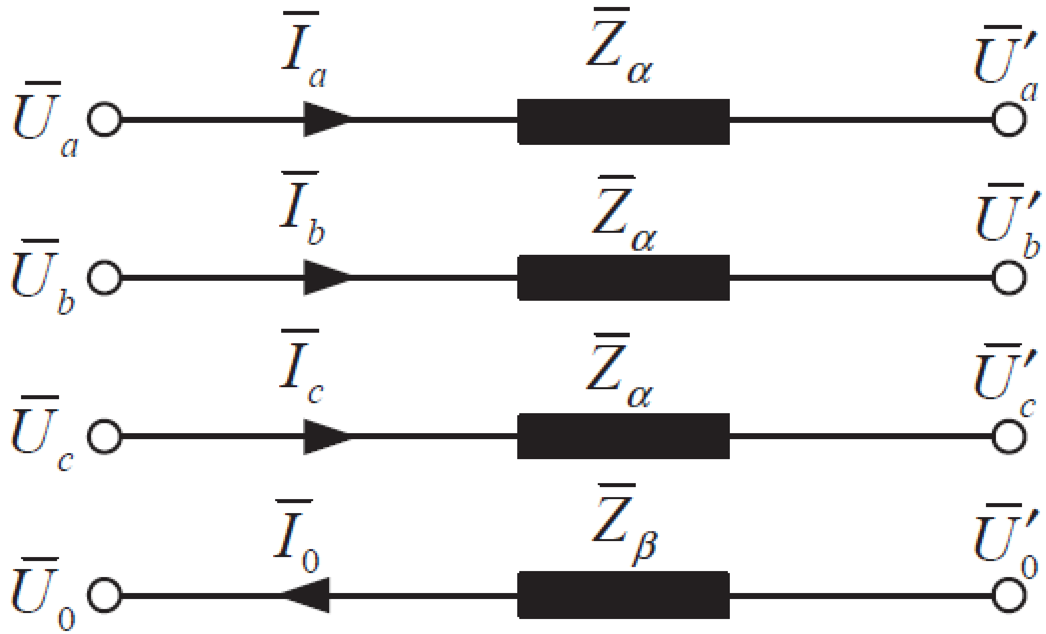
$$\overline{I}_0 = \overline{I}_a + \overline{I}_b + \overline{I}_c$$

$$\begin{aligned}
\overline{U}'_a - \overline{U}'_0 &= \overline{U}_a - \overline{U}_0 - \overline{I}_a \cdot \overline{Z}_\alpha - (\overline{I}_a + \overline{I}_b + \overline{I}_c)\overline{Z}_\beta \\
\overline{U}'_b - \overline{U}'_0 &= \overline{U}_b - \overline{U}_0 - \overline{I}_b \cdot \overline{Z}_\alpha - (\overline{I}_a + \overline{I}_b + \overline{I}_c)\overline{Z}_\beta \\
\overline{U}'_c - \overline{U}'_0 &= \overline{U}_c - \overline{U}_0 - \overline{I}_c \cdot \overline{Z}_\alpha - (\overline{I}_a + \overline{I}_b + \overline{I}_c)\overline{Z}_\beta
\end{aligned}$$



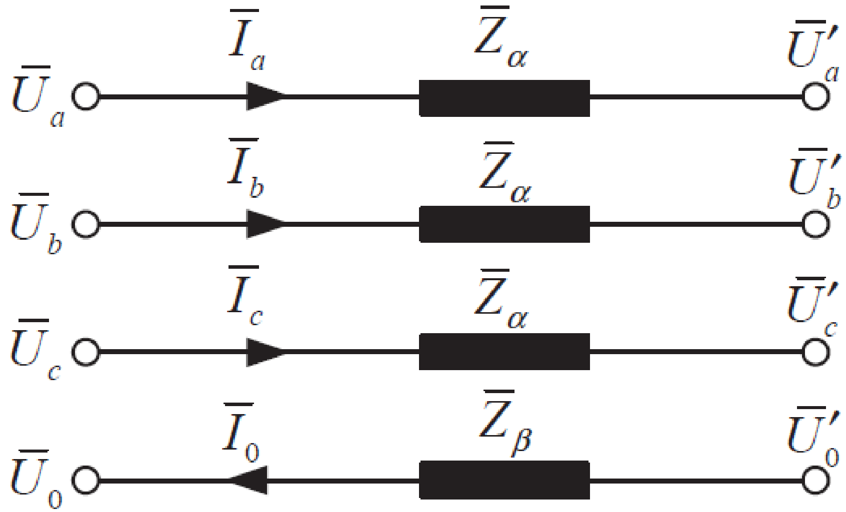
$$\begin{bmatrix} \overline{U}'_a - \overline{U}'_0 \\ \overline{U}'_b - \overline{U}'_0 \\ \overline{U}'_c - \overline{U}'_0 \end{bmatrix} = \begin{bmatrix} \overline{U}_a - \overline{U}_0 \\ \overline{U}_b - \overline{U}_0 \\ \overline{U}_c - \overline{U}_0 \end{bmatrix} - \begin{bmatrix} \overline{Z}_\alpha + \overline{Z}_\beta & \overline{Z}_\beta & \overline{Z}_\beta \\ \overline{Z}_\beta & \overline{Z}_\alpha + \overline{Z}_\beta & \overline{Z}_\beta \\ \overline{Z}_\beta & \overline{Z}_\beta & \overline{Z}_\alpha + \overline{Z}_\beta \end{bmatrix} \begin{bmatrix} \overline{I}_a \\ \overline{I}_b \\ \overline{I}_c \end{bmatrix}$$

$$\mathbf{U}'_{\mathbf{ph}} = \mathbf{U}_{\mathbf{ph}} - \mathbf{Z}_{\alpha\beta}\mathbf{I}_{\mathbf{ph}}$$



$$\mathbf{U}'_{\mathbf{ph}} = \mathbf{U}_{\mathbf{ph}} - \mathbf{Z}_{\alpha\beta}\mathbf{I}_{\mathbf{ph}}$$

$$\mathbf{Z}_{\mathbf{s}\alpha\beta} = \mathbf{T}^{-1}\mathbf{Z}_{\alpha\beta}\mathbf{T} = \begin{bmatrix} \overline{Z}_\alpha & 0 & 0 \\ 0 & \overline{Z}_\alpha & 0 \\ 0 & 0 & \overline{Z}_\alpha + 3\overline{Z}_\beta \end{bmatrix}$$

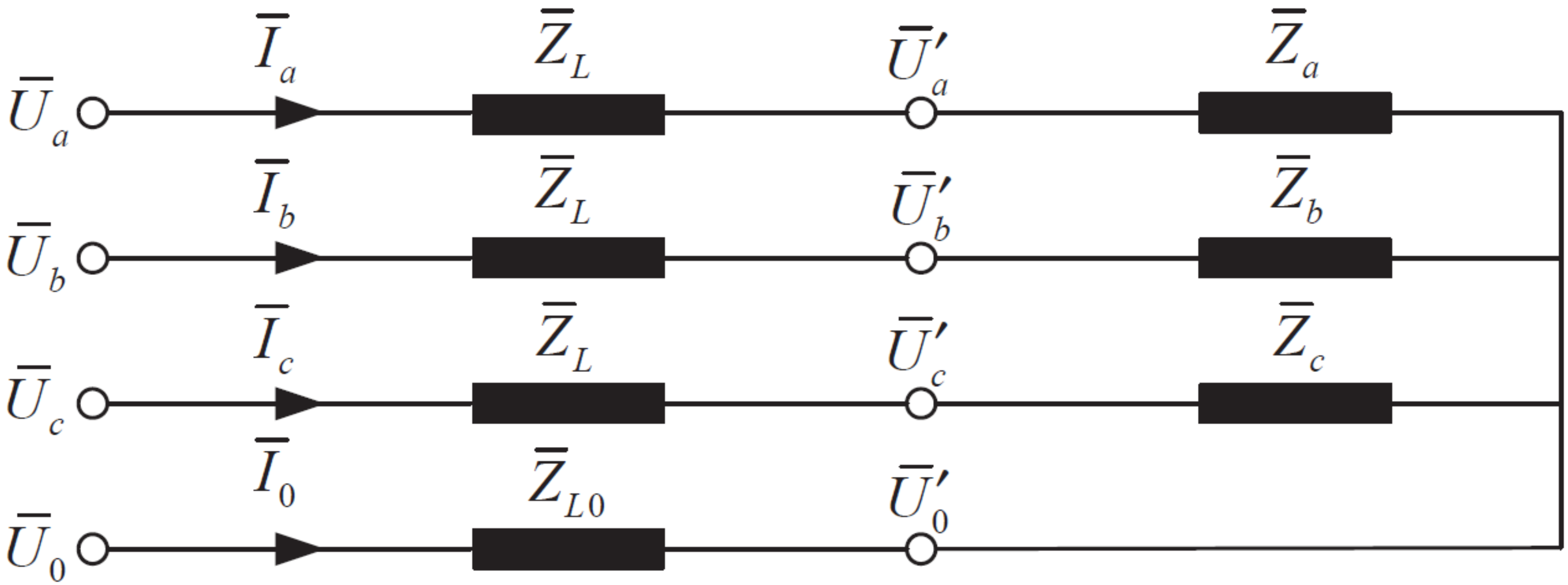


Knowing the Symmetrical components of the line

$$\mathbf{Z_{s\alpha\beta}} = \begin{bmatrix} \overline{Z}_\alpha & 0 & 0 \\ 0 & \overline{Z}_\alpha & 0 \\ 0 & 0 & \overline{Z}_\alpha + 3\overline{Z}_\beta \end{bmatrix}$$

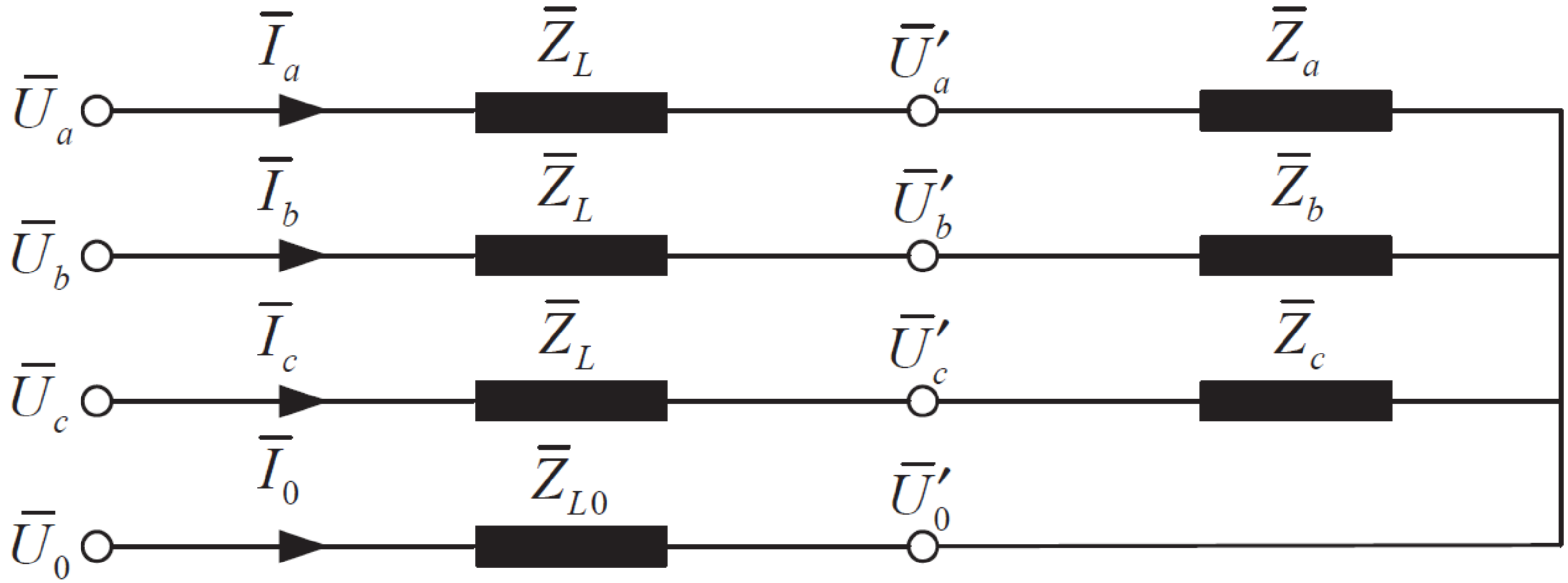
$$\begin{aligned} \overline{Z}_\alpha &= \overline{Z}_{-1} \\ \overline{Z}_\beta &= \frac{\overline{Z}_{-0} - \overline{Z}_{-1}}{3} \end{aligned}$$

Example 8.3 *Solve Example 2.5 by using symmetrical components.*



$$\overline{Z}_L = 2.3 + j0.16\ \Omega$$

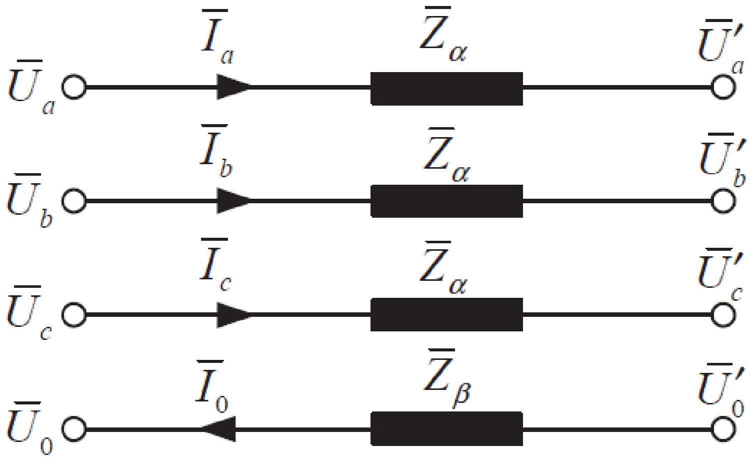
$$\overline{Z}_{L0} = 2.3 + j0.03\ \Omega$$



$$\overline{Z}_a = 47.9 + j4.81\ \Omega \quad \overline{Z}_b = 15.97 + j1.60\ \Omega$$

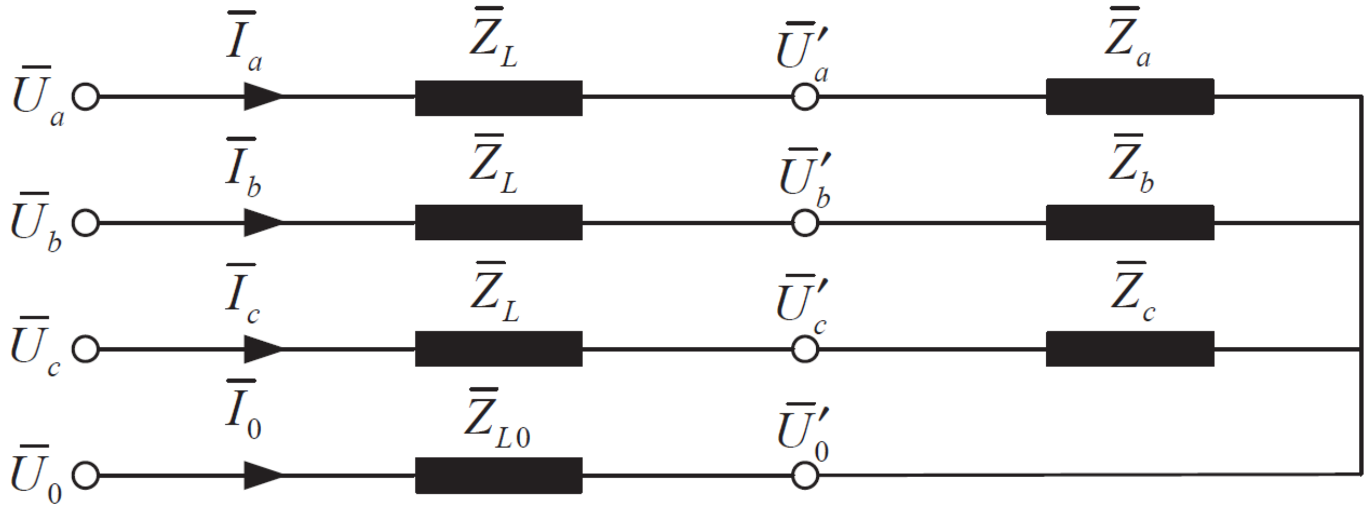
$$\overline{Z}_c = 23.96 + j2.40\ \Omega.$$

Calculate the total thermal power given by the radiators



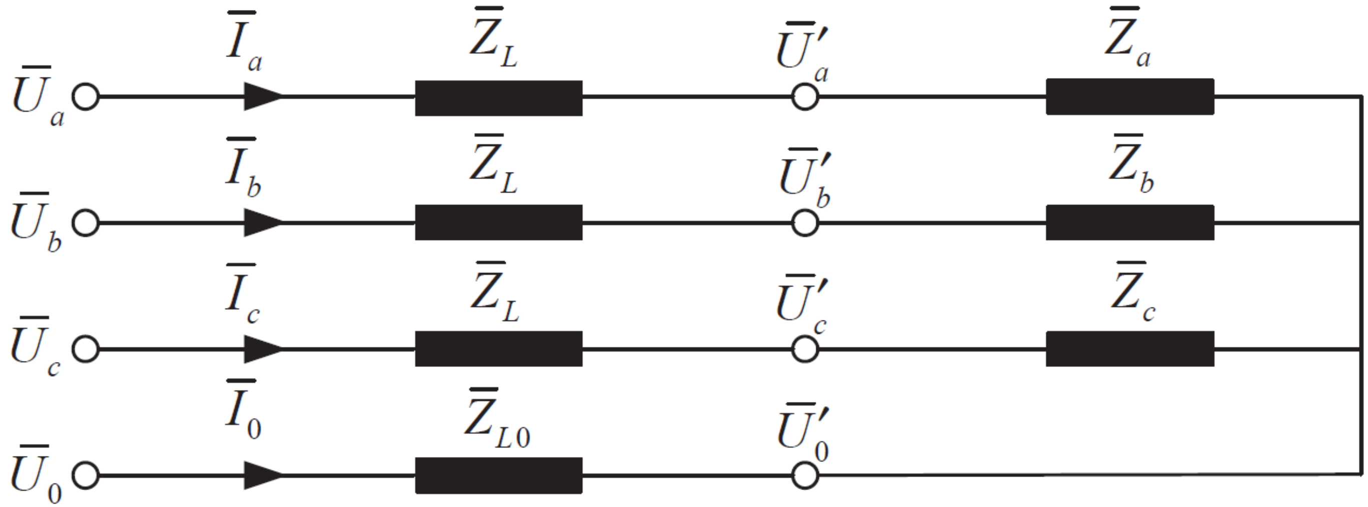
$$\mathbf{Z}_{\mathbf{s}\alpha\beta} = \begin{bmatrix} \overline{Z}_\alpha & 0 & 0 \\ 0 & \overline{Z}_\alpha & 0 \\ 0 & 0 & \overline{Z}_\alpha + 3\overline{Z}_\beta \end{bmatrix}$$

$$\begin{aligned} \overline{Z}_{-1} = \overline{Z}_{-2} &= \overline{Z}_L = 2.3 + j0.16\ \Omega \\ \overline{Z}_{-0} &= \overline{Z}_L + 3\overline{Z}_{L0} = 9.2 + j0.25\ \Omega \end{aligned}$$



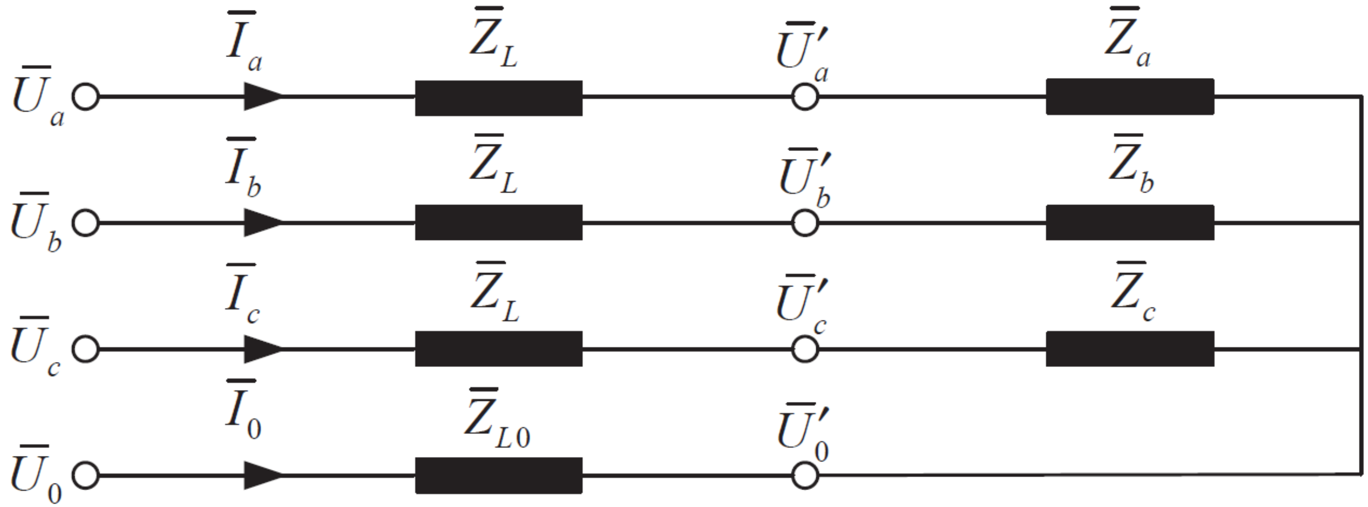
$$\mathbf{Z_s} = \begin{bmatrix} \overline{Z}_{-1} & 0 & 0 \\ 0 & \overline{Z}_{-2} & 0 \\ 0 & 0 & \overline{Z}_{-0} \end{bmatrix} =$$

$$\begin{bmatrix} 2.3 + j0.16 & 0 & 0 \\ 0 & 2.3 + j0.16 & 0 \\ 0 & 0 & 9.2 + j0.25 \end{bmatrix}$$



$$
\mathbf{Z_{LDs}} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} \overline{Z}_a & 0 & 0 \\ 0 & \overline{Z}_b & 0 \\ 0 & 0 & \overline{Z}_c \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix} =
$$

$$
= \begin{bmatrix} 29.28 + j2.94 & 9.09 + j3.24 & 9.55 - j1.37 \\ 9.55 - j1.37 & 29.28 + j2.94 & 9.09 + j3.24 \\ 9.09 + j3.24 & 9.55 - j1.37 & 29.28 + j2.94 \end{bmatrix} \Omega
$$



$$\mathbf{U_s} = \mathbf{T}^{-1}\mathbf{U_{ph}} = \begin{bmatrix} 220\angle 0^\circ \\ 0 \\ 0 \end{bmatrix} \text{ V}$$

$$\mathbf{U_s} = (\mathbf{Z_s} + \mathbf{Z_{LDs}})\mathbf{I_s}$$

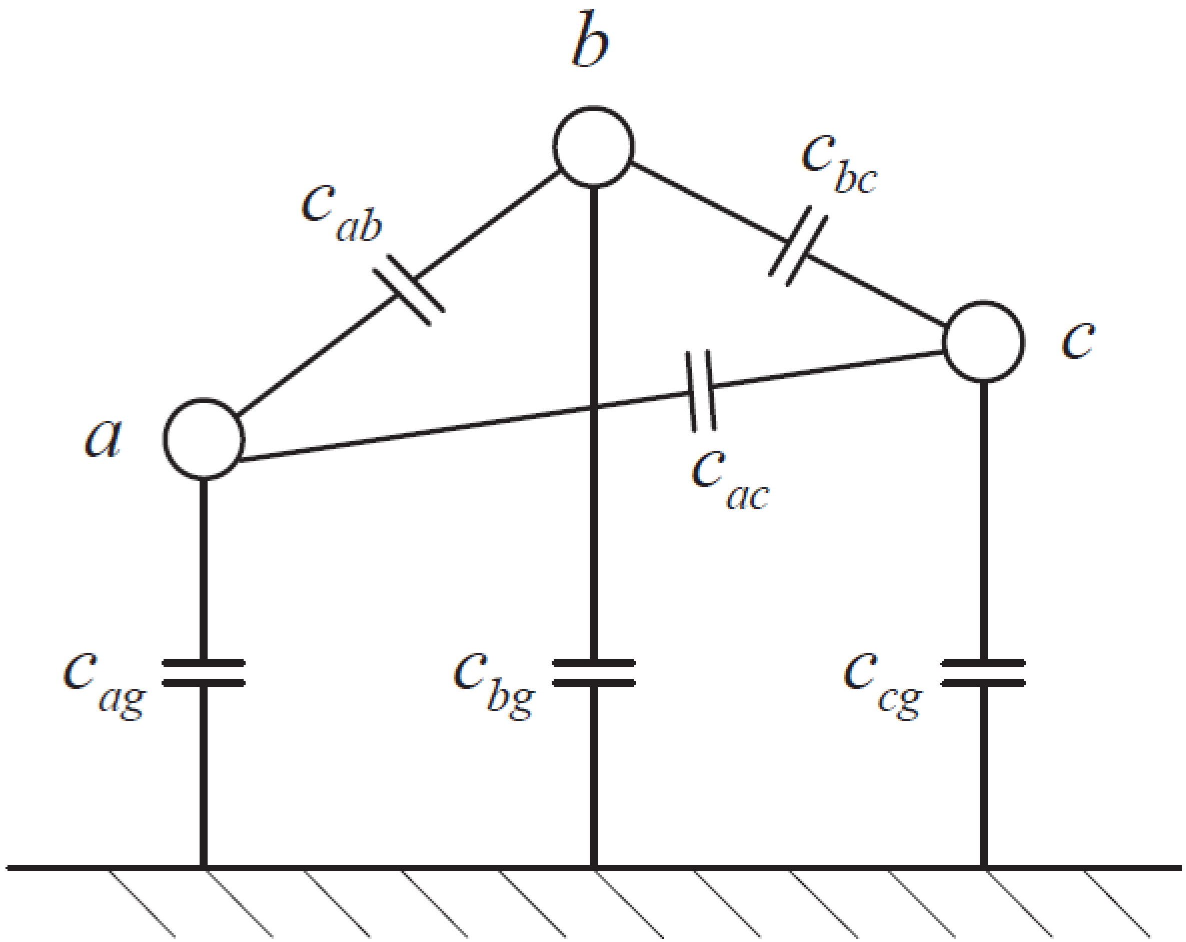
$$\mathbf{I_s} = (\mathbf{Z_s} + \mathbf{Z_{LDs}})^{-1}\mathbf{U_s} = \begin{bmatrix} 8.11\angle - 5.51^\circ \\ 2.22\angle 149.09^\circ \\ 1.75\angle - 155.89^\circ \end{bmatrix} \text{ A}$$

$$\mathbf{U_{LDs}} = \mathbf{Z_{LDs}}\mathbf{I_s} = \begin{bmatrix} 201.32\angle - 0.14^\circ \\ 5.13\angle - 26.93^\circ \\ 16.10\angle 25.67^\circ \end{bmatrix} \text{ V}$$

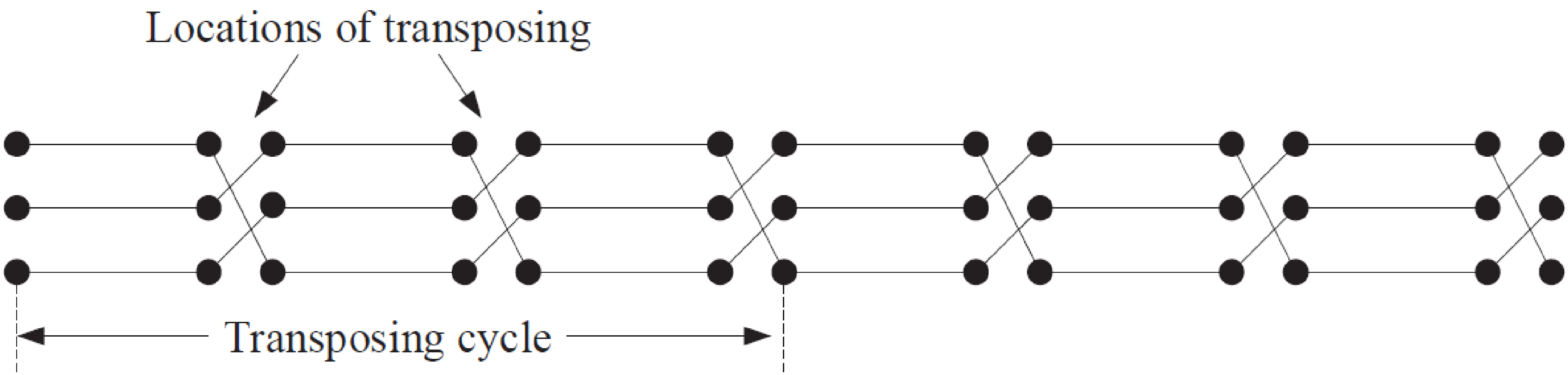
$$\overline{S} = 3\mathbf{U_{LDs}^t}\mathbf{I_s^*} = 4754 + j477 \text{ VA}$$

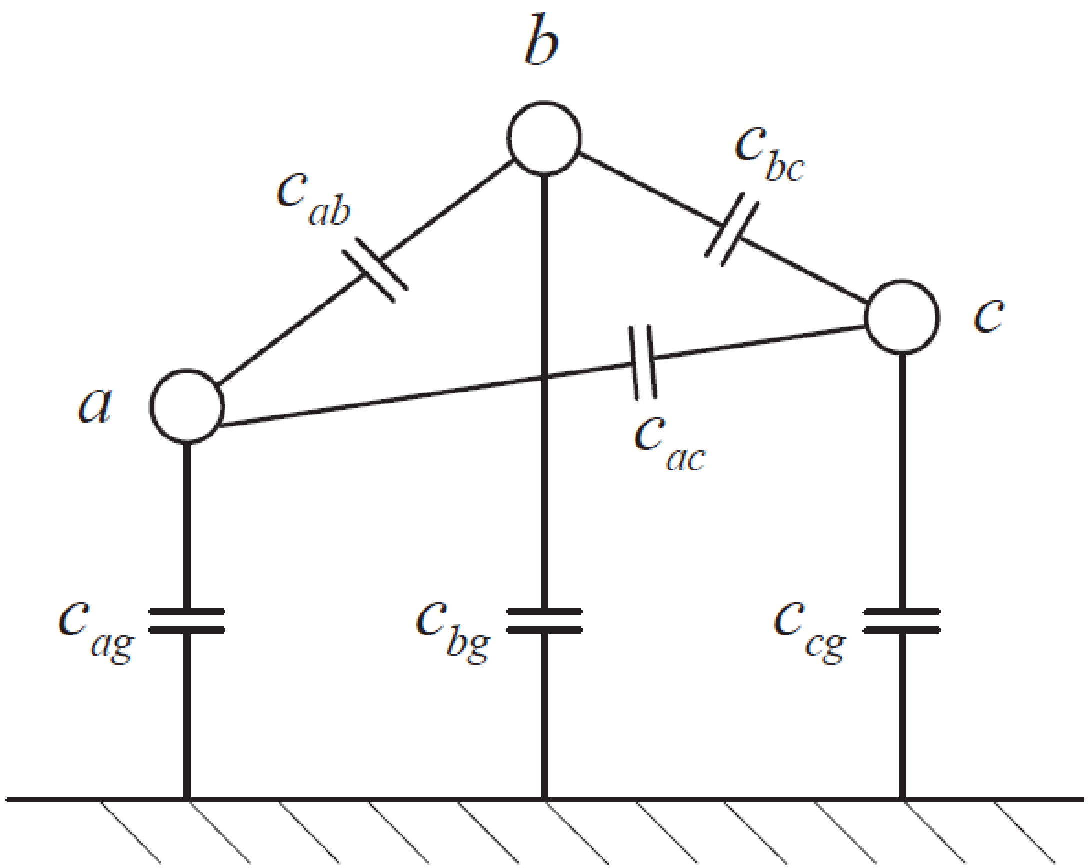
the thermal power is 4754 W.

Shunt capacitance of a three-phase line



Capacitances of a three-phase overhead line without earth wires

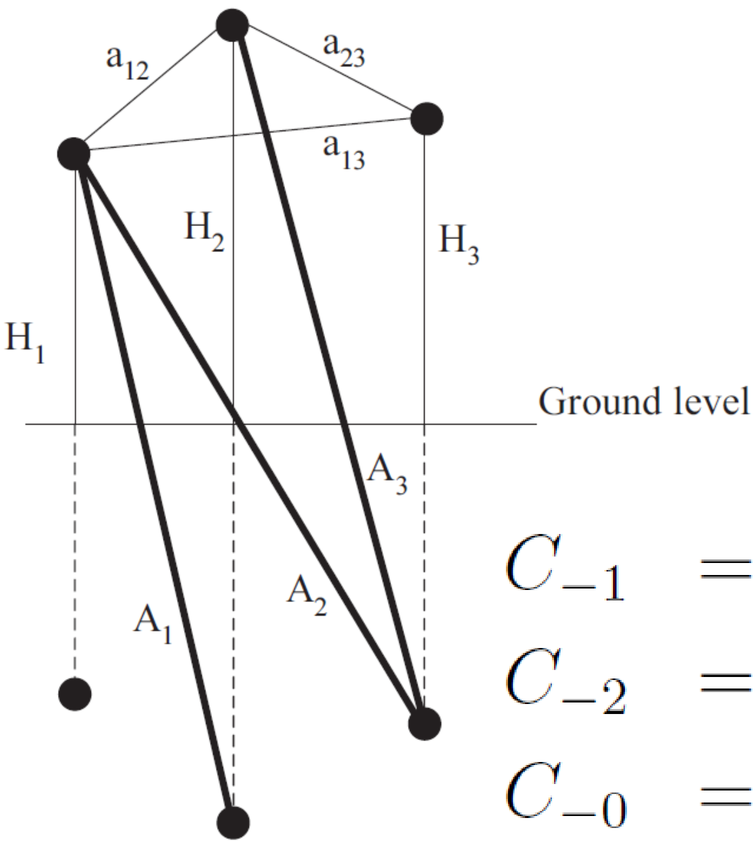




$$c_{ab} = c_{bc} = c_{ac}$$

$$c_{ag} = c_{bg} = c_{cg}$$

$$C_{-1} = C_{-2} = \frac{2\pi\epsilon_0}{\ln\left[\frac{2Ha}{Ar_{eq}}\right]} \quad \mathrm{F/m}$$



$$C_{-0} = \frac{2\pi\epsilon_0}{\ln\left[\frac{2HA^2}{r_{eq}a^2}\right]} \quad \mathrm{F/m}$$

C_{-1} = positive-sequence capacitance

C_{-2} = negative-sequence capacitance

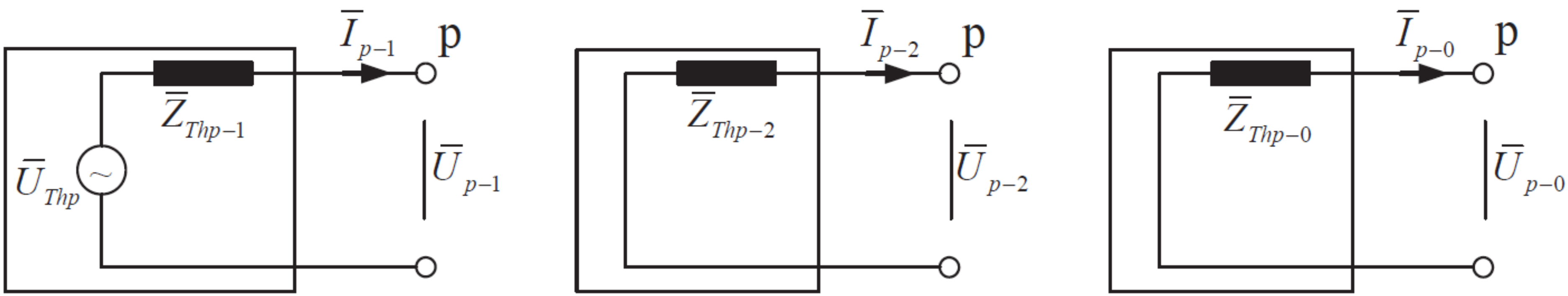
C_{-0} = zero-sequence capacitance

Geometrical quantities of a line

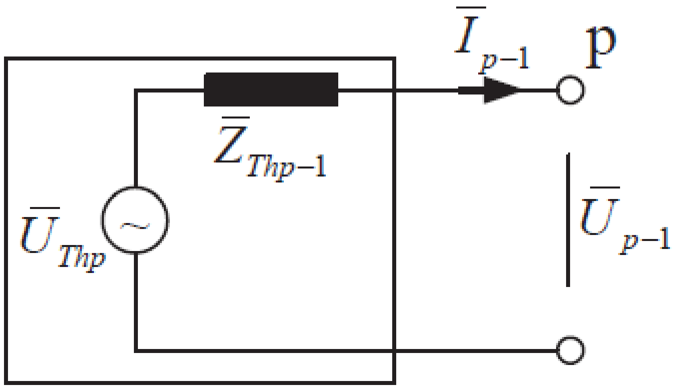
$\boxed{\Delta\text{-Y-transformation}}$

Analysis of unbalanced three-phase systems

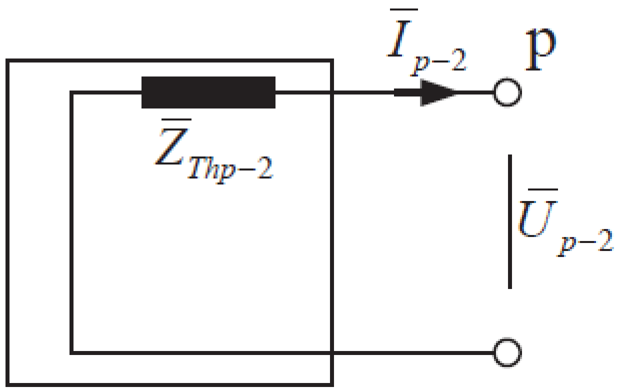
A **balanced** power system as seen from a selected point **p** can be described by three de-coupled single-line sequence systems



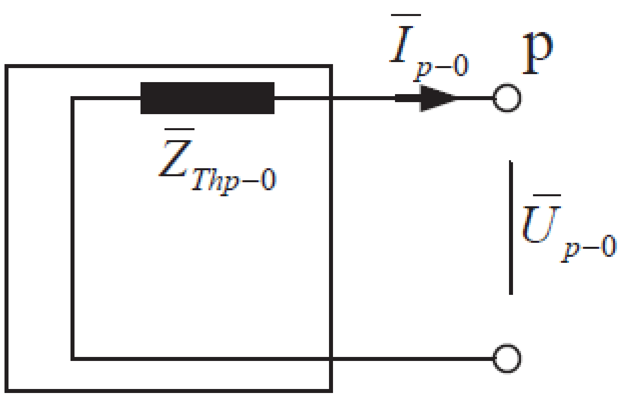
a) Positive-sequence b) Negative-sequence c) Zero-sequence



$$\overline{U}_{p-1} = \overline{U}_{Thp} - \overline{Z}_{Thp-1}\,\overline{I}_{p-1}$$

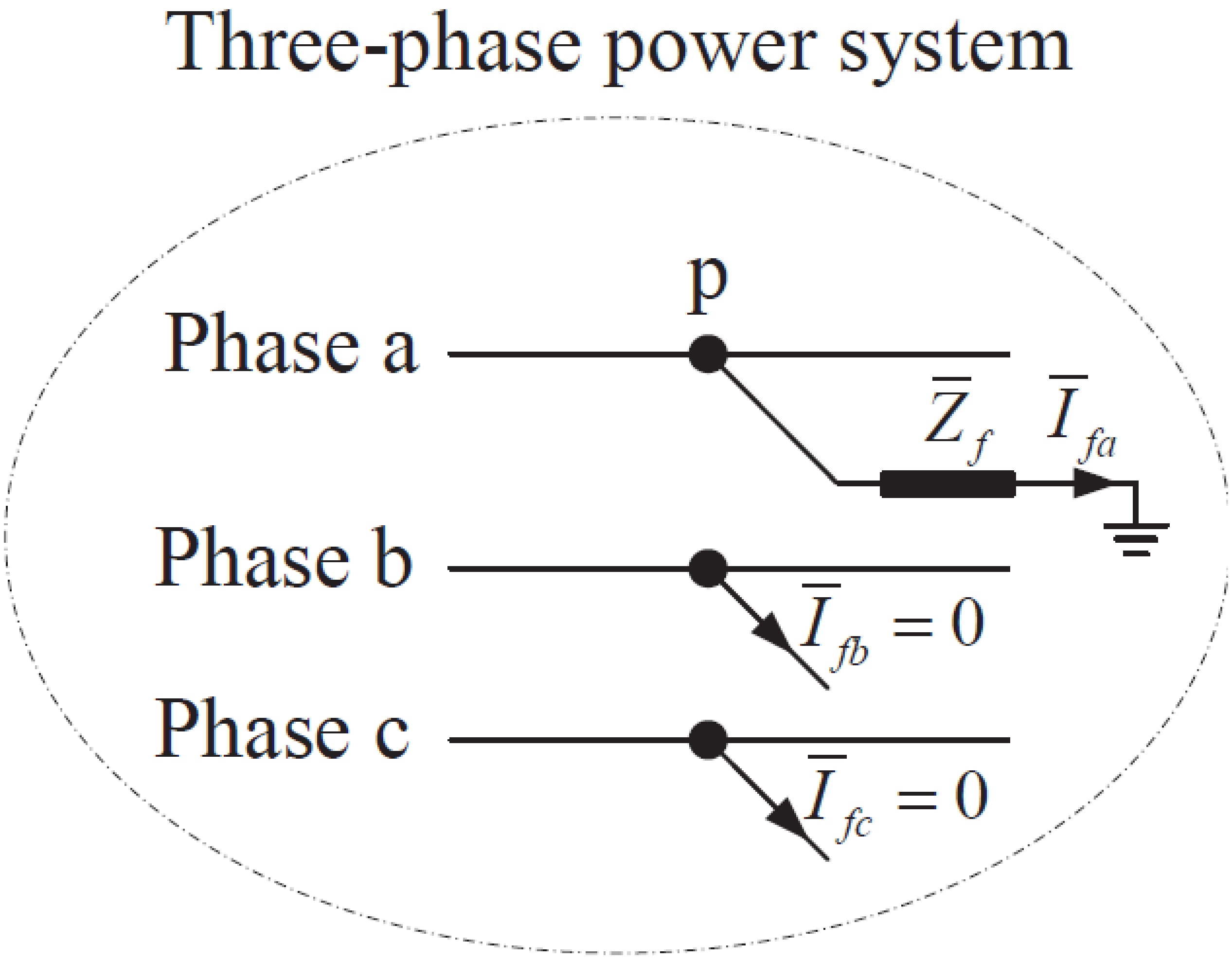


$$\overline{U}_{p-2} = 0 - \overline{Z}_{Thp-2}\,\overline{I}_{p-2}$$

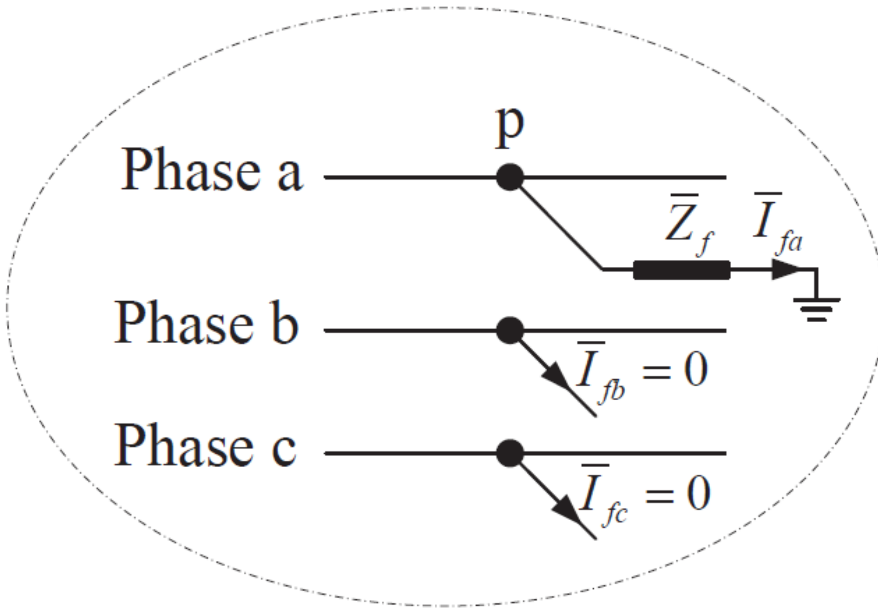


$$\overline{U}_{p-0} = 0 - \overline{Z}_{Thp-0}\,\overline{I}_{p-0}$$

Single line-to-ground fault



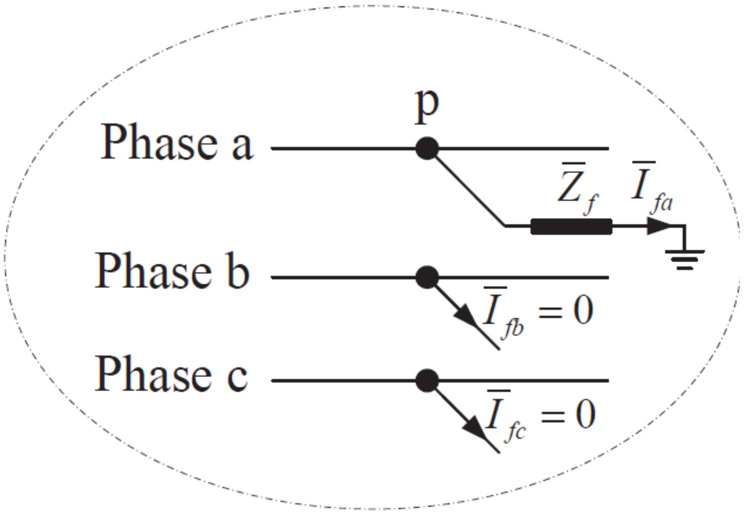
Three-phase power system



$$\mathbf{I_s} = \begin{bmatrix} \overline{I}_{p-1} \\ \overline{I}_{p-2} \\ \overline{I}_{p-0} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} \overline{I}_{fa} \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} \overline{I}_{fa} \\ \overline{I}_{fa} \\ \overline{I}_{fa} \end{bmatrix}$$

$$\Rightarrow \quad \overline{I}_{p-1} = \overline{I}_{p-2} = \overline{I}_{p-0} = \frac{1}{3}\overline{I}_{fa}$$

Three-phase power system



$$\boxed{\begin{aligned} \overline{U}_{p-1} &= \overline{U}_{Thp} - \overline{Z}_{Thp-1}\,\overline{I}_{p-1} \\ \overline{U}_{p-2} &= 0 - \overline{Z}_{Thp-2}\,\overline{I}_{p-2} \\ \overline{U}_{p-0} &= 0 - \overline{Z}_{Thp-0}\,\overline{I}_{p-0} \end{aligned}}$$

$$\boxed{\overline{I}_{p-1} = \overline{I}_{p-2} = \overline{I}_{p-0} = \frac{1}{3}\overline{I}_{fa}}$$

$$\boxed{\overline{U}_{pa} = \overline{Z}_f \overline{I}_{fa}}$$

$$\overline{U}_{pa} = \overline{U}_{p-1} + \overline{U}_{p-2} + \overline{U}_{p-0} = \overline{Z}_f \overline{I}_{fa} = 3\overline{Z}_f \overline{I}_{p-1} \quad \Rightarrow$$

$$\overline{U}_{pa} = \overline{U}_{Thp} - \overline{Z}_{Thp-1}\,\overline{I}_{p-1} - \overline{Z}_{Thp-2}\,\overline{I}_{p-1} - \overline{Z}_{Thp-0}\,\overline{I}_{p-1} =$$

$$= 3\overline{Z}_f \overline{I}_{p-1} \quad \text{pu}$$

$$\boxed{\overline{I}_{fa} = 3\,\overline{I}_{p-1} = \frac{3\,\overline{U}_{Thp}}{\overline{Z}_{Thp-1} + \overline{Z}_{Thp-2} + \overline{Z}_{Thp-0} + 3\,\overline{Z}_f} \quad \text{pu}}$$

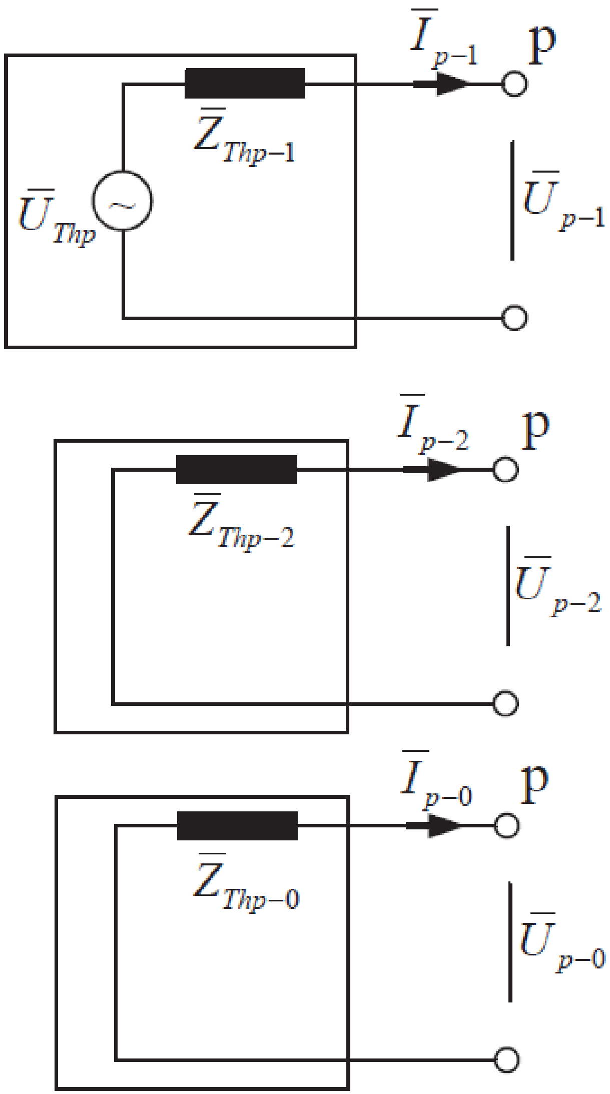
$$\boxed{\overline{I}_{fa} = 3\,\overline{I}_{p-1} = \frac{3\,\overline{U}_{Thp}}{\overline{Z}_{Thp-1} + \overline{Z}_{Thp-2} + \overline{Z}_{Thp-0} + 3\,\overline{Z}_{f}} \quad \text{pu}}$$

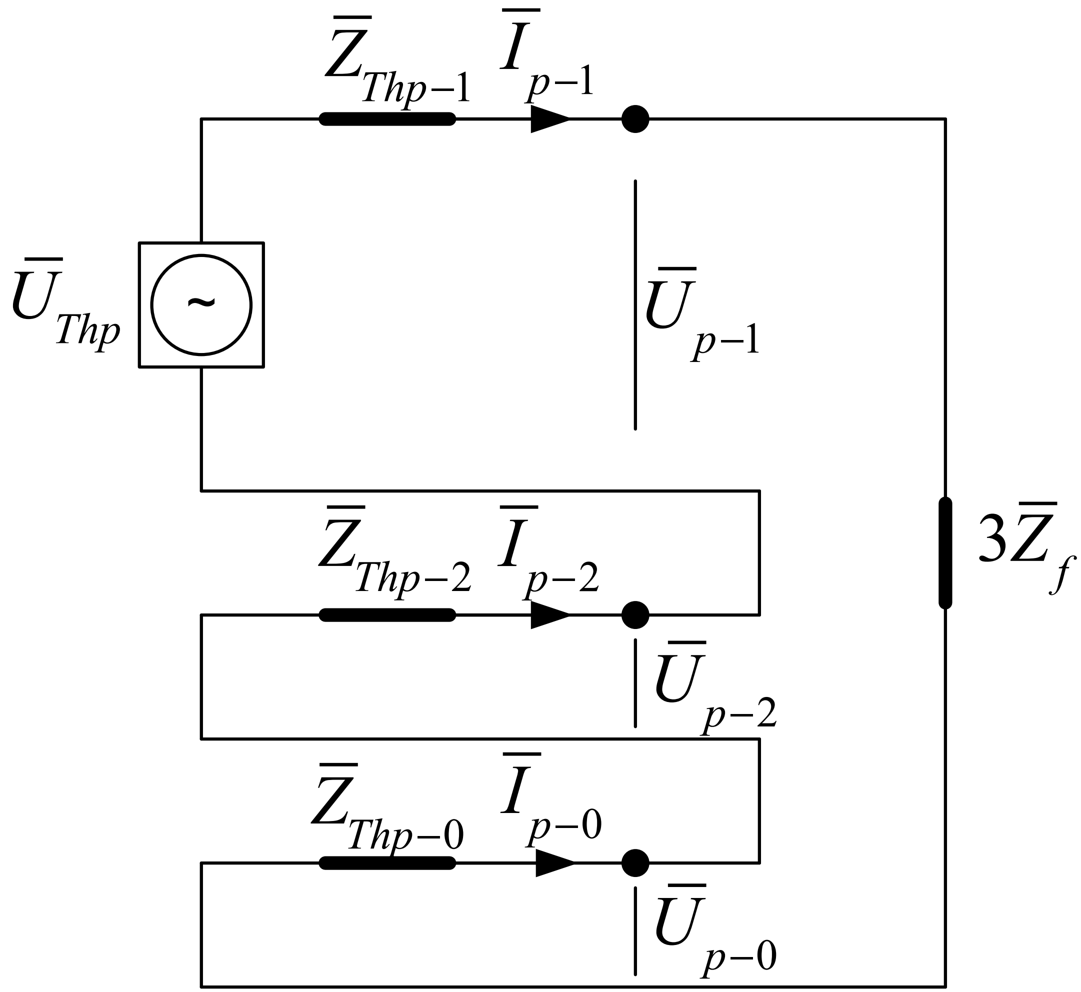
$$\overline{U}_{pa} = \frac{\overline{U}_{Thp}}{\sqrt{3}} - \overline{Z}_{Thp-1}\,\overline{I}_{p-1} - \overline{Z}_{Thp-2}\,\overline{I}_{p-1} - \overline{Z}_{Thp-0}\,\overline{I}_{p-1}$$

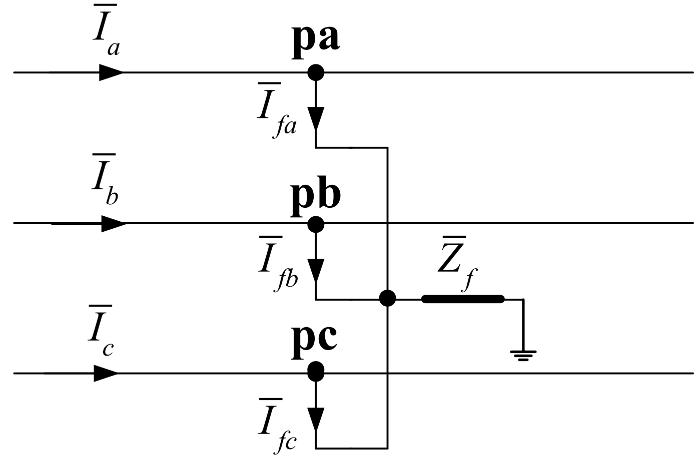
$$= 3\overline{Z}_{f}\overline{I}_{p-1} \quad \text{kV}$$

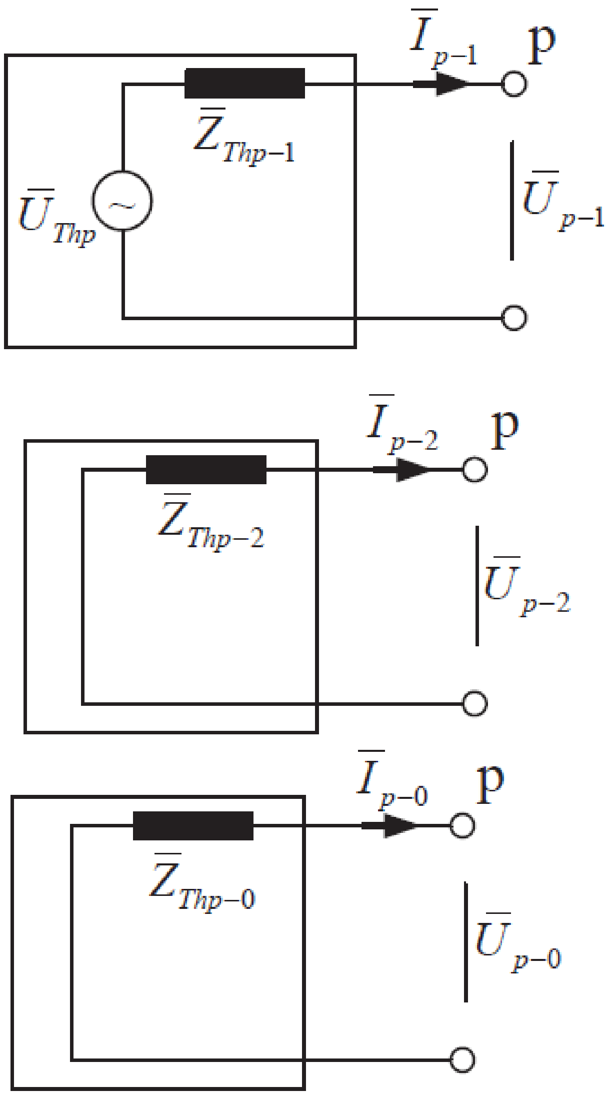
$$\boxed{\overline{I}_{fa} = 3\,\overline{I}_{p-1} = \frac{\frac{3\,\overline{U}_{Thp}}{\sqrt{3}}}{\overline{Z}_{Thp-1} + \overline{Z}_{Thp-2} + \overline{Z}_{Thp-0} + 3\,\overline{Z}_{f}} \quad \text{kA}}$$

$$\overline{I}_{p-1} = \overline{I}_{p-2} = \overline{I}_{p-0} = \frac{\overline{U}_{Thp}}{\overline{Z}_{Thp-1} + \overline{Z}_{Thp-2} + \overline{Z}_{Thp-0} + 3\,\overline{Z}_f}$$









$$\begin{bmatrix} \bar{I}_{p-1} \\ \bar{I}_{p-2} \\ \bar{I}_{p-0} \end{bmatrix} = T^{-1} \begin{bmatrix} \bar{I}_{fa} \\ \bar{I}_{fb} \\ \bar{I}_{fc} \end{bmatrix} = \begin{bmatrix} \bar{I}_{fa} \\ 0 \\ 0 \end{bmatrix}$$

Example 8.4 At a 400 kV bus, a solid three-phase short circuit occurs, giving a fault current of 20 kA per phase. If a solid single line-to-ground fault occurs at the same bus, the fault current will be 15 kA in the faulted phase. The Thévenin impedances in the positive- and negative-sequence systems at the bus can be assumed to be purely reactive and equal. Also the zero-sequence impedance can be assumed to be purely reactive. Calculate the Thévenin equivalents for the positive-, negative- and zero-sequences at the fault.

$$\overline{Z}_f = 0 \qquad\qquad \overline{Z}_{Th-1} = \overline{Z}_{Th-2}$$

$$\overline{I}_{sc_{3\Phi}} = -j20 \text{ kA}$$

$$\overline{Z}_{Th-1} = \overline{Z}_{Th-2} = \frac{\overline{U}_{Th}}{\sqrt{3}\,\overline{I}_{sc_{3\Phi}}} = \frac{400}{\sqrt{3}\cdot(-j20)} = j11.55 \quad \Omega$$

$$\overline{I}_{sc_{1\Phi}} = -j15 \text{ kA}$$

$$\boxed{\overline{I}_{fa} = 3\,\overline{I}_{p-1} = \frac{\frac{3\,\overline{U}_{Thp}}{\sqrt{3}}}{\overline{Z}_{Thp-1} + \overline{Z}_{Thp-2} + \overline{Z}_{Thp-0} + 3\,\overline{Z}_f} \quad \text{kA}}$$

$$\overline{Z}_{Th-0} = \frac{3\,\overline{U}_{Th}}{\sqrt{3}\,\overline{I}_{sc_{1\Phi}}} - \overline{Z}_{Th-1} - \overline{Z}_{Th-2} - 3\,\overline{Z}_f = j23.09 \quad \Omega$$

Analysis of a linear three-phase system with one unbalanced load

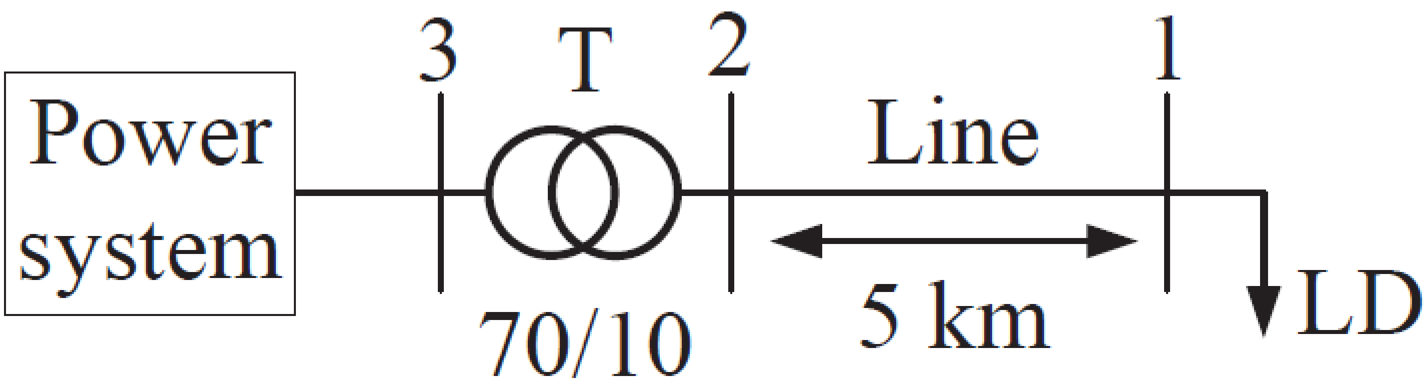
1. Draw the impedance diagrams of the positive-, negative and zero-sequence systems, for the entire network excluding the unbalanced load.
2. Find the Thévenin equivalents of the positive-, negative- and zero-sequence systems as seen from the point the unsymmetrical load is located.
3. Calculate the positive-, negative- and zero-sequence currents through the unbalanced load.

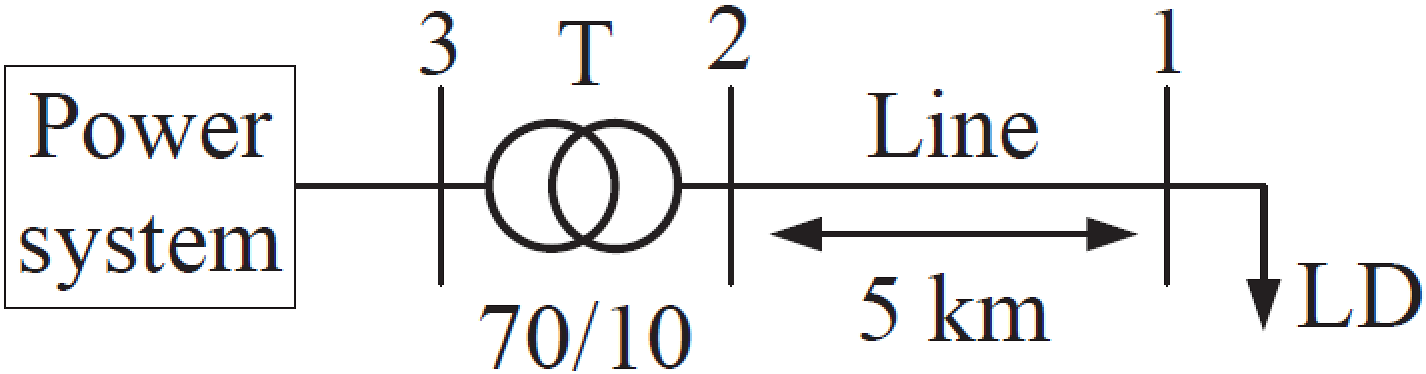
Analysis of a linear three-phase system with one unbalanced load

4. Calculate the positive-, negative- and zero-components of quantities that are of interest.
5. Transform those symmetrical components to the phase quantities that are asked for.

Example 8.5 *Consider again the system described in Example 6.2.*

Example 6.2 *As shown in Figure 6.12, a small industry (LD) is fed by a power system via a transformer (5 MVA, 70/10, $x = 4$ %) which is located at a distance of 5 km. The electric power demand of the industry is 400 kW at cosϕ=0.8, lagging, at a voltage of 10 kV. The industry can be modeled as an impedance load. The 10 kV line has an series impedance of 0.9+j0.3 Ω/km and a shunt admittance of $j3 \times 10^{-6}$ S/km. Assume that the line is modeled by the π-equivalent. When the transformer is disconnected from bus 3, the voltage at this bus is 70 kV, and a three-phase short circuit applied to this bus results in a pure inductive short circuit current of 0.3 kA.*



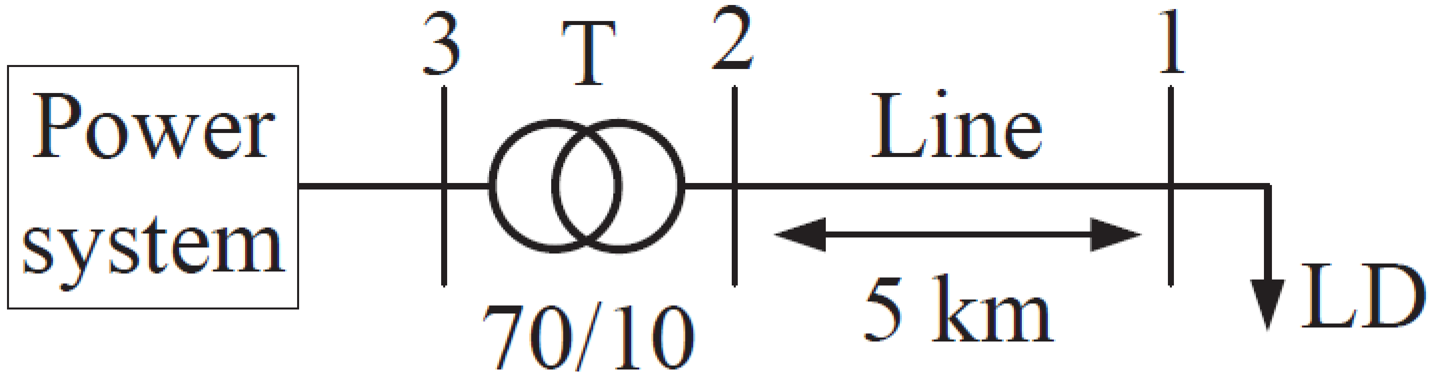


$$S_{base} = 0.5 \text{ MVA},\ U_{base10} = 10 \text{ kV} \Rightarrow$$

$$I_{base10} = S_{base}/\sqrt{3}U_{base10} = 0.0289 \text{ kA}$$

$$Z_{base10} = U_{base10}^2/S_{base} = 200\ \Omega$$

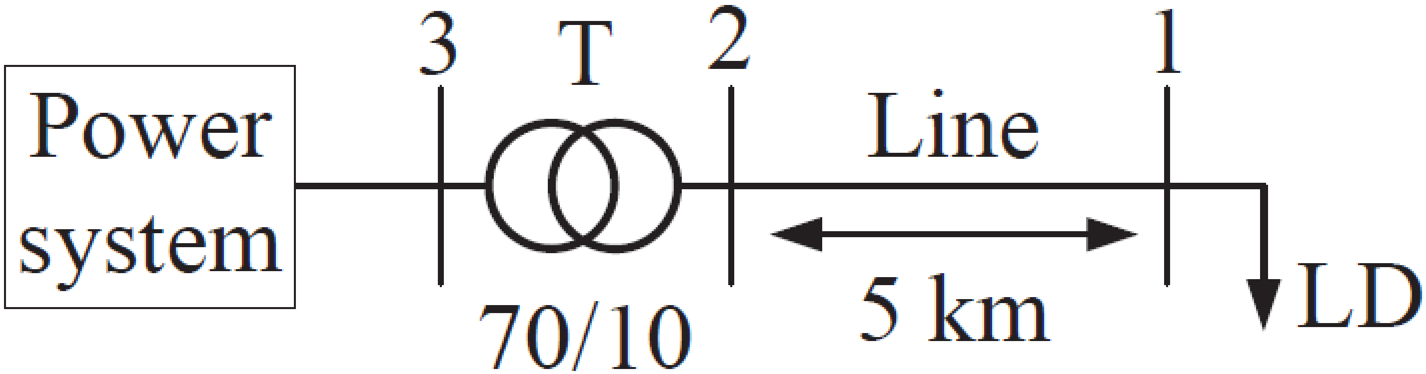
$$U_{base70} = 70 \text{ kV} \Rightarrow I_{base70} = S_{base}/\sqrt{3}U_{base70} = 0.0041 \text{ kA}$$



$$\overline{U}_{Thpu} = \frac{\overline{U}_{Th}}{U_{base70}} = \frac{70\angle 0}{70} = 1\angle 0^\circ = 1$$

$$\overline{I}_{scpu} = \frac{\overline{I}_{sc}}{I_{base70}} = \frac{0.3\angle -90^\circ}{0.00412} = 72.8155\angle -90^\circ$$

$$\overline{Z}_{Thpu} = \frac{\overline{U}_{Thpu}}{\overline{I}_{scpu}} = j0.0137$$

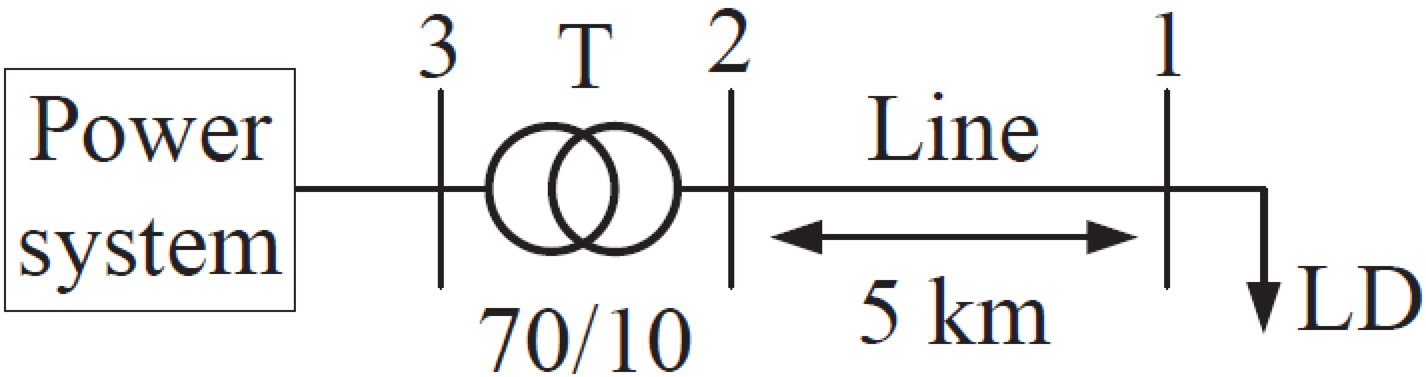


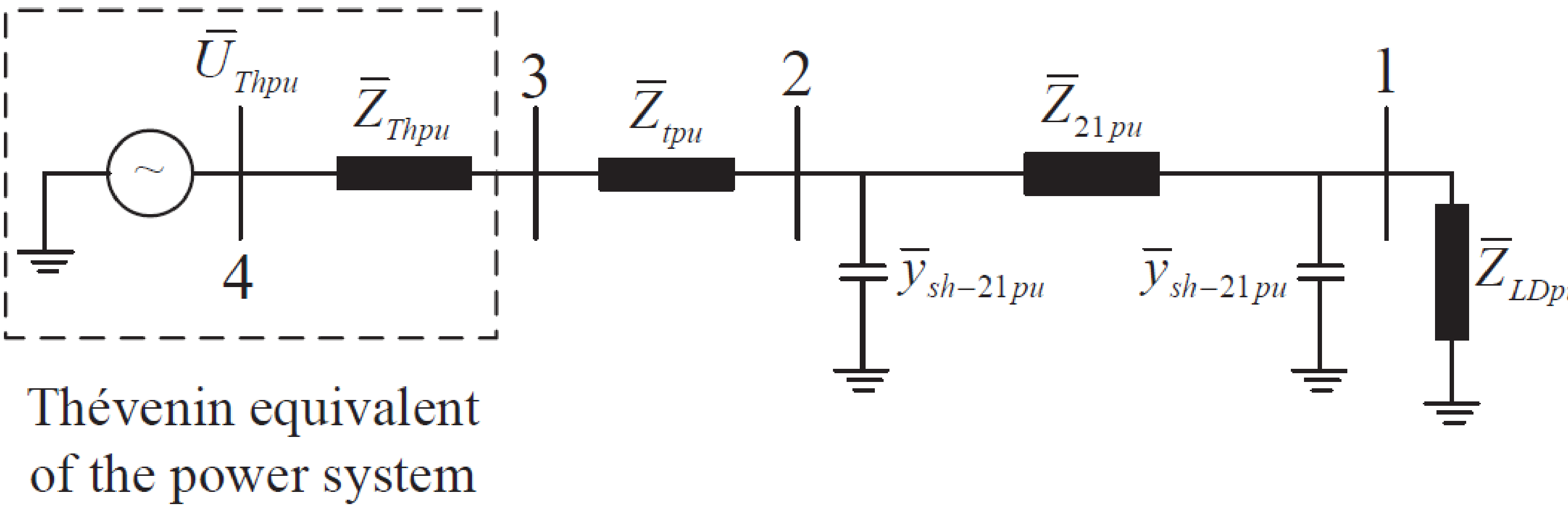
$$\overline{Z}_{tpu} = \frac{\overline{Z}_{t\%}}{100}\frac{Z_{tbase10}}{Z_{base10}} = \frac{\overline{Z}_{t\%}}{100}\frac{U_{2n}^2}{S_{nt}}\frac{S_{base}}{U_{base10}^2} = \frac{j4}{100}\frac{10^2}{5}\frac{0.5}{10^2} = \frac{j4}{100}\frac{0.5}{5} = j0.004$$

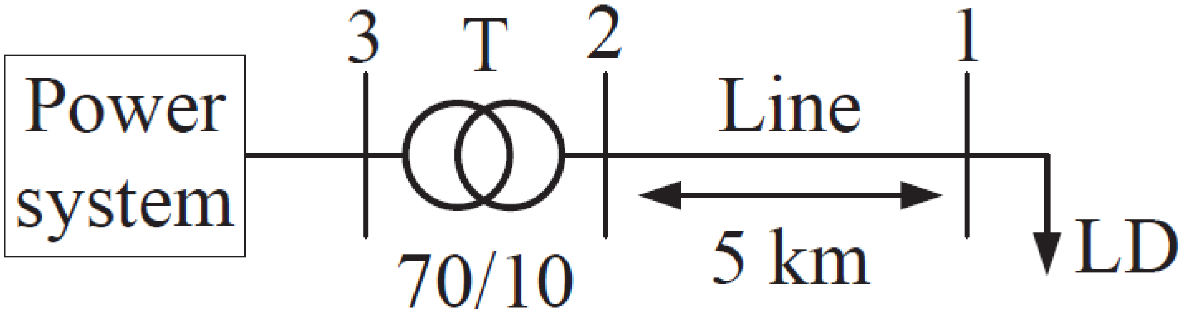
$$\overline{Z}_{21pu} = \frac{5\cdot(0.9+j0.3)}{Z_{base10}} = 0.0225 + j0.0075$$

$$\overline{y}_{sh-21pu} = \frac{\overline{Y}_{sh-21pu}}{2} = \frac{5\cdot(j3\times 10^{-6})}{2} Z_{base10} = \frac{j0.003}{2}$$

$$\overline{Z}_{LDpu} = \frac{U_n^2}{\overline{S}_{LD}^*}\frac{1}{Z_{base10}} = \frac{10^2}{\frac{0.4}{0.8}}(0.8+j0.6)\frac{1}{200} = 0.8 + j0.6$$

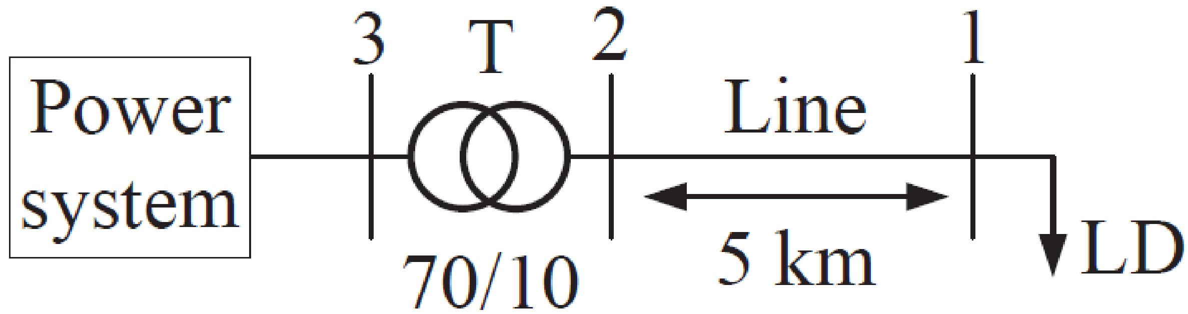

Power system
3
T
2
1
Line
70/10
5 km
LD


$\bar{U}_{Thpu}$
$\bar{Z}_{Thpu}$
3
$\bar{Z}_{tpu}$
2
$\bar{Z}_{21pu}$
1
4
$\bar{y}_{sh-21pu}$
$\bar{y}_{sh-21pu}$
$\bar{Z}_{LDpu}$
Thévenin equivalent
of the power system



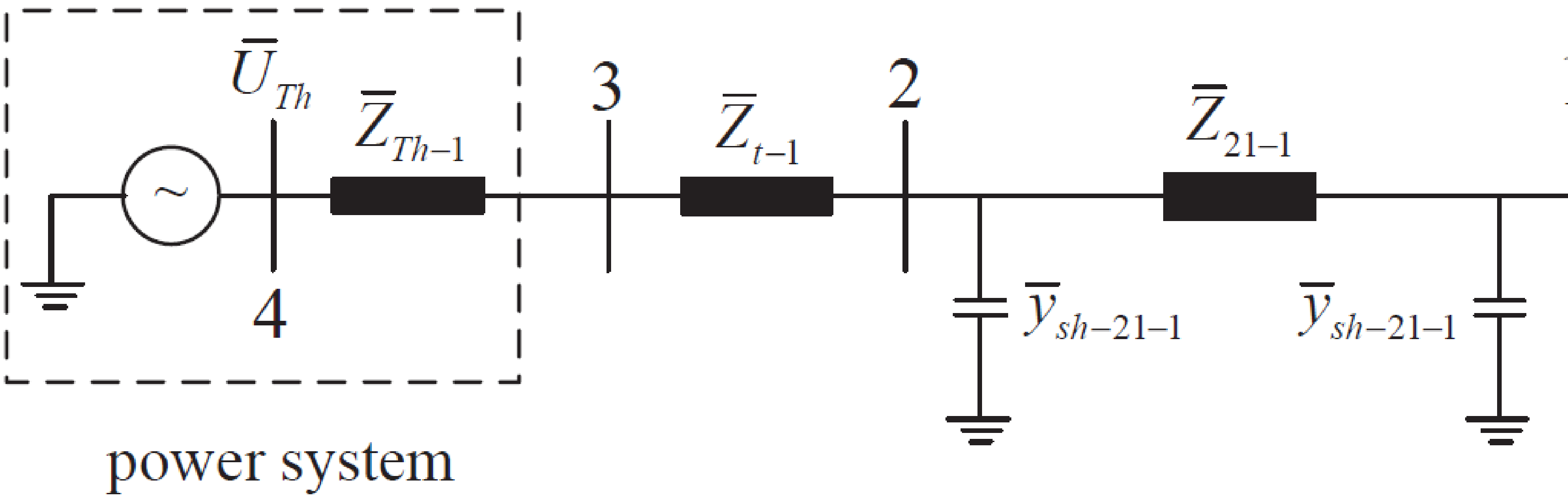
additional data

- *Transformer is Δ-Y0 connected with Y0 on the 10 kV-side, and* $\overline{Z}_n = 0$.
- *The zero-sequence impedance of the line is 3 times the positive-sequence impedance, i.e.* $\overline{Z}_{21-0} = 3\,\overline{Z}_{21-1}$.
- *The zero-sequence shunt admittance of the line is 0.5 times the positive-sequence shunt admittance, i.e.* $\overline{y}_{sh-21-0} = 0.5\,\overline{y}_{sh-21-1}$.
- *When the transformer is disconnected from bus 3, a solid (i.e.* $\overline{Z}_f = 0$*) single line-to-ground applied to this bus results in a pure inductive fault current of 0.2 kA.*
- *The positive- and negative-sequence Thévenin impedances of the power system are identical, i.e.* $\overline{Z}_{Th-1} = \overline{Z}_{Th-2}$.
- *The load is Y0-connected with* $\overline{Z}_n = 0$. *Furthermore, half of the normal load connected to phase a is disconnected while the other phases are loaded as normal,* **i.e. it is an unsymmetrical load**.

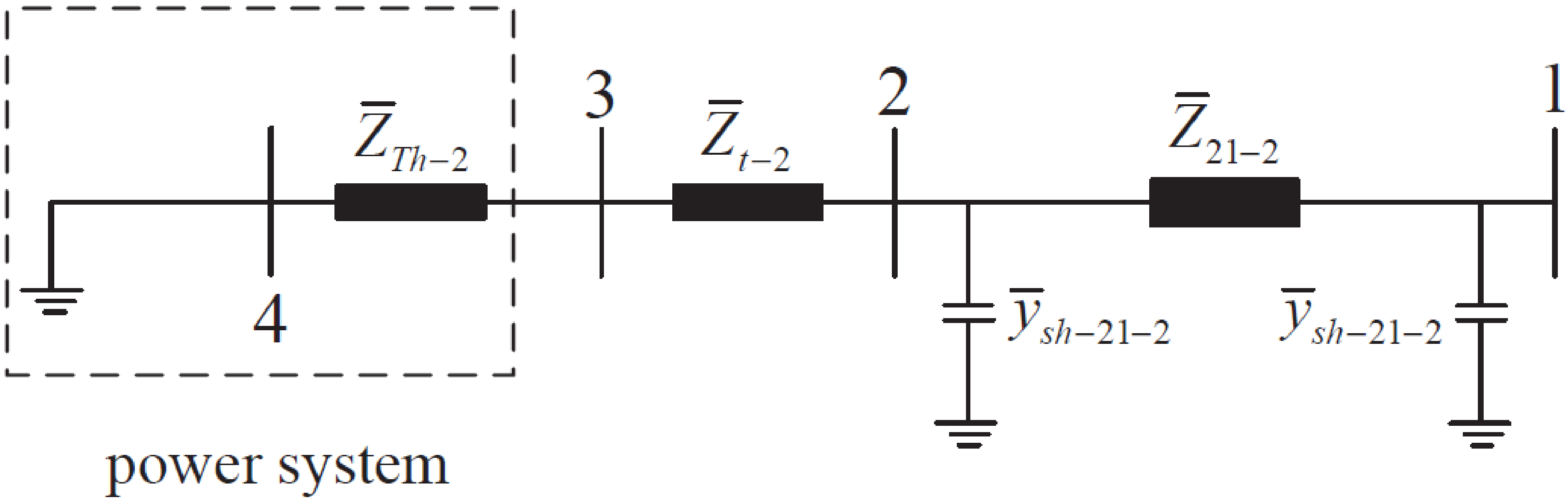


Calculate the voltage at the industry as well as the power fed by the transformer into the line.

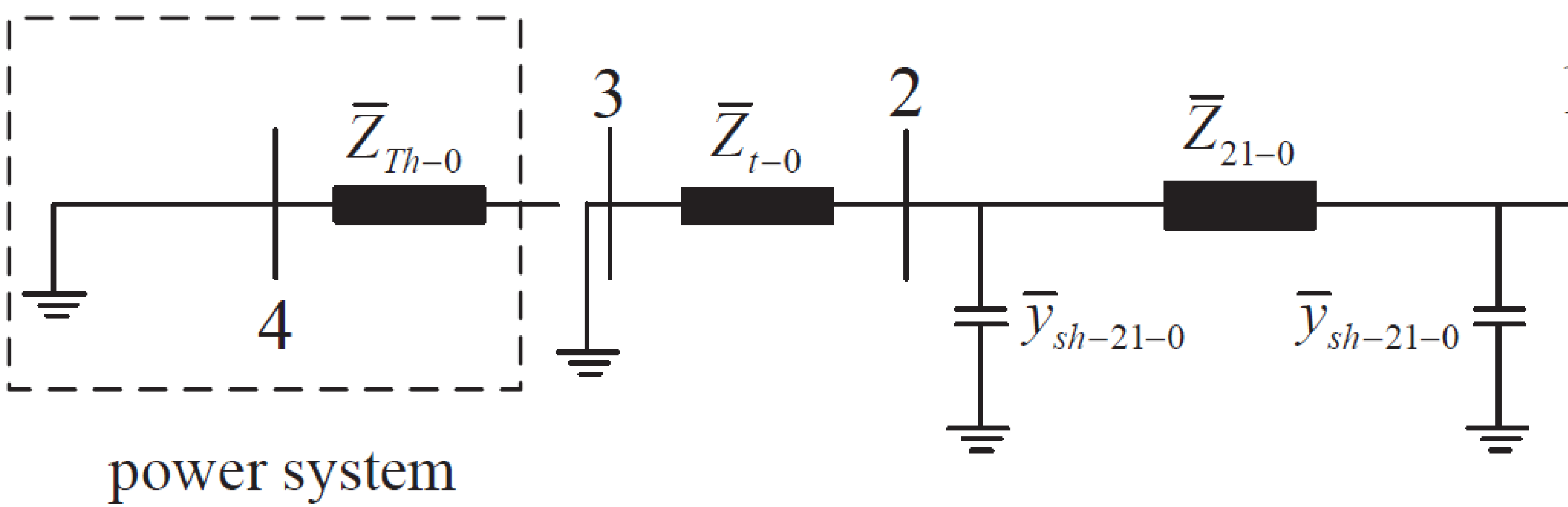
1. Start with the building of the impedance diagram of the positive-, negative- and zero-sequence for the whole system excluding the unsymmetrical load.



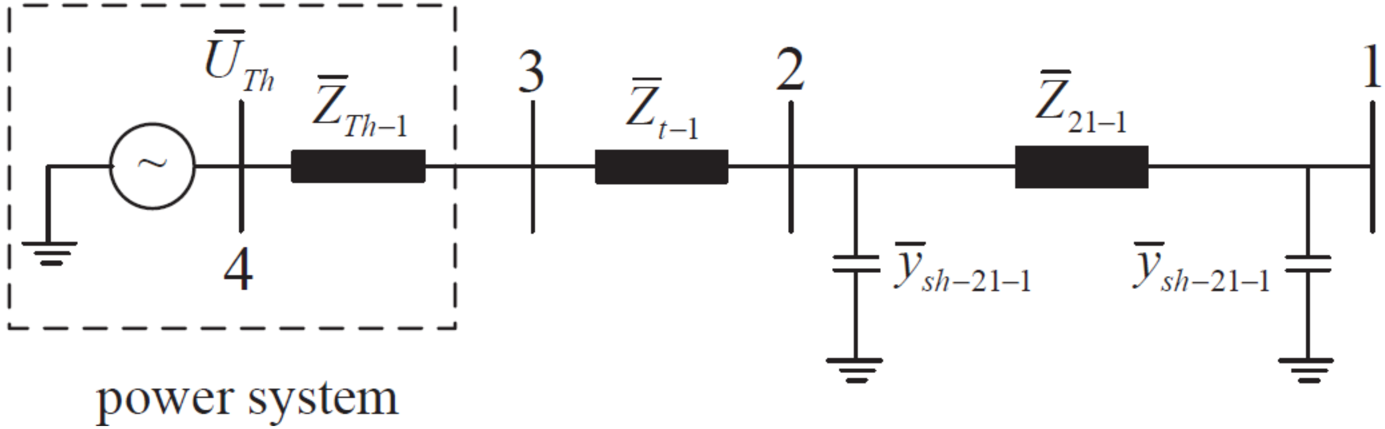
a) Positive-sequence system



b) Negative-sequence system



c) Zero-sequence system



$$\overline{U}_{Th} = \overline{U}_{Thpu} = 1\angle 0^\circ$$

$$\overline{Z}_{Th-1} = \overline{Z}_{Th-2} = \overline{Z}_{Thpu} = j\,0.0137$$

$$\overline{Z}_{t-1} = \overline{Z}_{t-2} = \overline{Z}_{tpu} = j\,0.004$$

$$\overline{Z}_{21-1} = \overline{Z}_{21-2} = \overline{Z}_{21pu} = 0.0225 + j\,0.0075$$

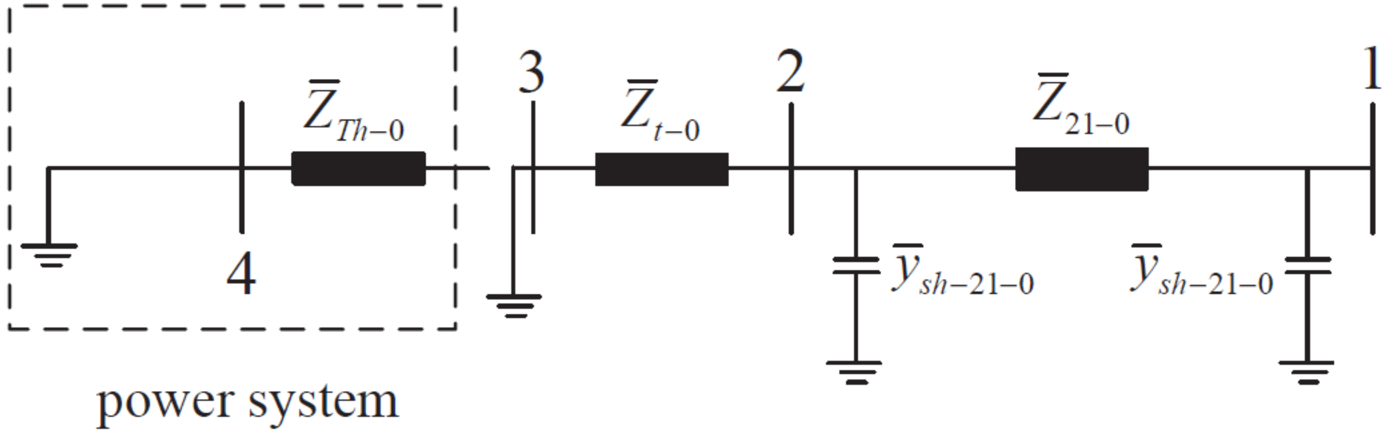
$$\overline{y}_{sh-21-1} = \overline{y}_{sh-21-2} = \overline{y}_{sh-21pu} = \frac{j\,0.003}{2}$$

$$\overline{A}_{L-1} = \overline{A}_{L-2} = \overline{A}_{L} \quad , \quad \overline{B}_{L-1} = \overline{B}_{L-2} = \overline{B}_{L}$$

$$\overline{C}_{L-1} = \overline{C}_{L-2} = \overline{C}_{L} \quad , \quad \overline{D}_{L-1} = \overline{D}_{L-2} = \overline{D}_{L}$$

$$\overline{A}_{-1} = \overline{A}_{-2} = \overline{A} \quad , \quad \overline{B}_{-1} = \overline{B}_{-2} = \overline{B}$$

$$\overline{C}_{-1} = \overline{C}_{-2} = \overline{C} \quad , \quad \overline{D}_{-1} = \overline{D}_{-2} = \overline{D}$$



$$\overline{I}_{sc1\Phi} = \frac{0.2\angle -90^\circ}{I_{b70}} = \frac{0.2\angle -90^\circ}{0.00412} = 48.5437\angle -90^\circ$$

$$\overline{Z}_{Th-0} = \frac{3\,\overline{U}_{Th}}{\overline{I}_{sc1\Phi}} - 2\,\overline{Z}_{Th-1} - 0 = j\,0.0344$$

$$\overline{Z}_{t-0} = \overline{Z}_{t-1} = j\,0.004 \quad , \quad \text{since } \overline{Z}_n = 0$$

$$\overline{Z}_{21-0} = 3\,\overline{Z}_{21-1} = 0.0675 + j\,0.0225$$

$$\overline{y}_{sh-21-0} = 0.5\,\overline{y}_{sh-21-1} = \frac{j\,0.003}{4}$$

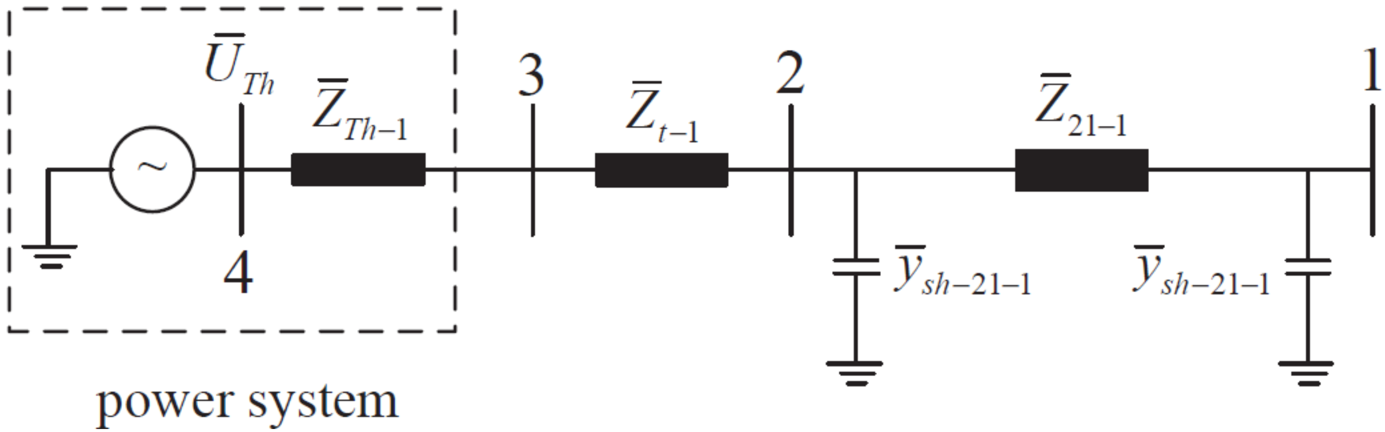
$$\overline{A}_{L-0} = 1 + \overline{y}_{sh-21-0} \cdot \overline{Z}_{21-0} = 1.0000 + j\,0.0001$$

$$\overline{B}_{L-0} = \overline{Z}_{21-0} = 0.0675 + j\,0.0225$$

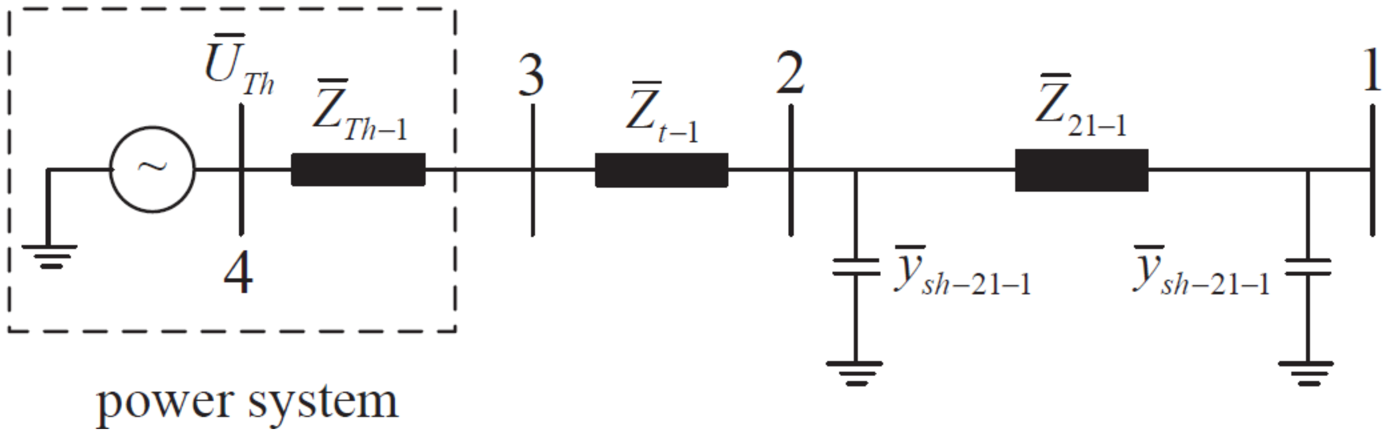
$$\overline{C}_{L-0} = \overline{y}_{sh-21-0}(2 + \overline{y}_{sh-21-0} \cdot \overline{Z}_{21-0}) = 0.0000 + j\,0.0015$$

$$\overline{D}_{L-0} = \overline{A}_{L-0} = 1.0000 + j\,0.0001$$

2. Find the Thévenin equivalents of the positive-, negative- and zero-sequence systems as seen from the point the unsymmetrical load is located.



$$\begin{bmatrix} \overline{U}_{Th} \\ \overline{I}_{bus4-1} \end{bmatrix} = \begin{bmatrix} \overline{A}_{-1} & \overline{B}_{-1} \\ \overline{C}_{-1} & \overline{D}_{-1} \end{bmatrix} \begin{bmatrix} \overline{U}_{bus1-1} \\ \overline{I}_{bus1-1} \end{bmatrix} =$$

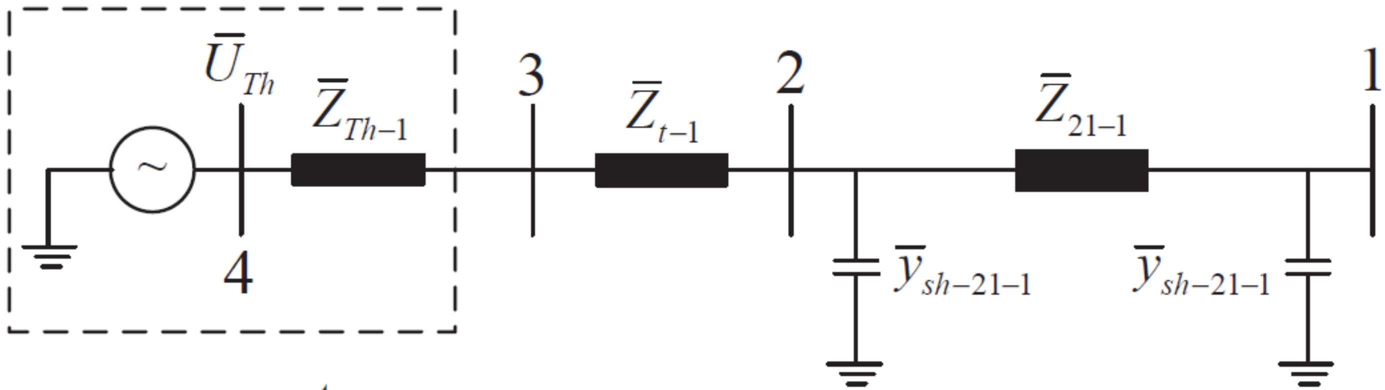


$$\begin{bmatrix} \overline{U}_{Th} \\ \overline{I}_{bus4-1} \end{bmatrix} = \begin{bmatrix} \overline{A}_{-1} & \overline{B}_{-1} \\ \overline{C}_{-1} & \overline{D}_{-1} \end{bmatrix} \begin{bmatrix} \overline{U}_{bus1-1} \\ \overline{I}_{bus1-1} \end{bmatrix} =$$

$\overline{U}_{Thbus1-1}$ is obtained by setting $\overline{I}_{bus1-1} = 0$

$$\overline{U}_{Th} = \overline{A}_{-1}\,\overline{U}_{Thbus1-1} + \overline{B}_{-1} \cdot 0 \quad \Rightarrow$$

$$\overline{U}_{Thbus1-1} = \frac{\overline{U}_{Th}}{\overline{A}_{-1}} = 1.0001\angle - 0.0019^{\circ}$$

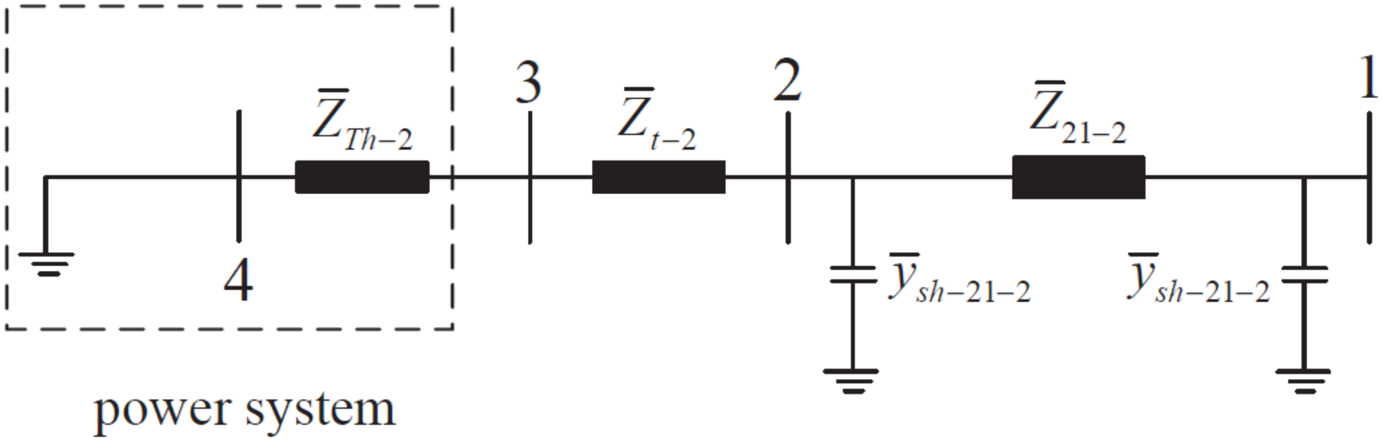


$$\begin{bmatrix} \overline{U}_{Th} \\ \overline{I}_{bus4-1} \end{bmatrix} = \begin{bmatrix} \overline{A}_{-1} & \overline{B}_{-1} \\ \overline{C}_{-1} & \overline{D}_{-1} \end{bmatrix} \begin{bmatrix} \overline{U}_{bus1-1} \\ \overline{I}_{bus1-1} \end{bmatrix} =$$

$\overline{Z}_{Thbus1-1}$ is obtained by setting $\overline{U}_{Th} = 0$

$$0 = \overline{A}_{-1}\,\overline{U}_{bus1-1} + \overline{B}_{-1}\,\overline{I}_{bus1-1} \quad \Rightarrow$$

$$\overline{Z}_{Thbus1-1} = -\frac{\overline{U}_{bus1-1}}{\overline{I}_{bus1-1}} = \frac{\overline{B}_{-1}}{\overline{A}_{-1}} = 0.0225 + j0.0252$$

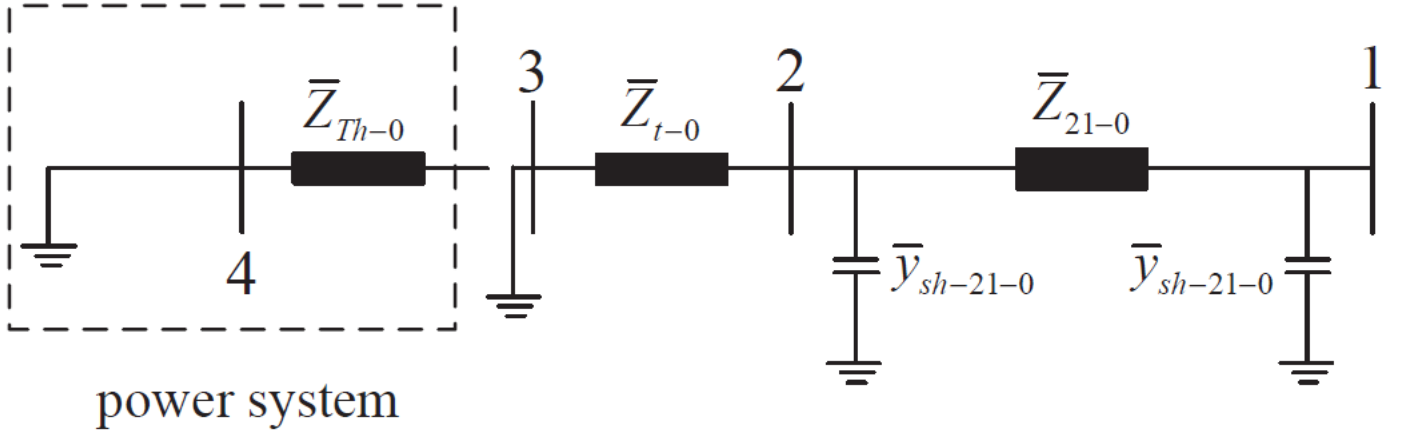


$\overline{Z}_{Thbus1-1}$ is obtained by setting $\overline{U}_{Th} = 0$

$\overline{Z}_{t-1} = \overline{Z}_{t-2}$, $\overline{Z}_{12-1} = \overline{Z}_{12-2}$ and $\overline{y}_{sh-21-1} = \overline{y}_{sh-21-2}$

$\overline{A}_{-1} = \overline{A}_{-2}$ and $\overline{B}_{-1} = \overline{B}_{-2}$

$$\boxed{\overline{Z}_{Thbus1-2} = -\frac{\overline{U}_{bus1-2}}{\overline{I}_{bus1-2}} = \frac{\overline{B}_{-2}}{\overline{A}_{-2}} = \frac{\overline{B}_{-1}}{\overline{A}_{-1}} = \overline{Z}_{Thbus1-1}}$$



$$\begin{bmatrix} \overline{U}_{bus3-0} \\ \overline{I}_{bus3-0} \end{bmatrix} = \begin{bmatrix} 1 & \overline{Z}_{t-0} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \overline{A}_{L-0} & \overline{B}_{L-0} \\ \overline{C}_{L-0} & \overline{D}_{L-0} \end{bmatrix} \begin{bmatrix} \overline{U}_{bus1-0} \\ \overline{I}_{bus1-0} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ \overline{I}_{bus3-0} \end{bmatrix} = \begin{bmatrix} \overline{A}_{-0} & \overline{B}_{-0} \\ \overline{C}_{-0} & \overline{D}_{-0} \end{bmatrix} \begin{bmatrix} \overline{U}_{bus1-0} \\ \overline{I}_{bus1-0} \end{bmatrix}$$

$$0 = \overline{A}_{-0} \overline{U}_{bus1-0} + \overline{B}_{-0} \overline{I}_{bus1-0} \quad \Rightarrow$$

$$\overline{Z}_{Thbus1-0} = -\frac{\overline{U}_{bus1-0}}{\overline{I}_{bus1-0}} = \frac{\overline{B}_{-0}}{\overline{A}_{-0}} = 0.0675 + j0.0265$$

3. Calculate the positive-, negative- and zero-sequence currents through the unbalanced load.

$\overline{Z}_{LD}$ is known. $\overline{Z}_{LDa} = 2\overline{Z}_{LD}$

$$\mathbf{Z_{LDs}} = \mathbf{T}^{-1}\,\mathbf{Z_{LDPh}}\,\mathbf{T} = \mathbf{T}^{-1}\begin{bmatrix} 2\overline{Z}_{LD} & 0 & 0 \\ 0 & \overline{Z}_{LD} & 0 \\ 0 & 0 & \overline{Z}_{LD} \end{bmatrix}\mathbf{T}$$

$$\mathbf{U_{Th}} = \begin{bmatrix} \overline{U}_{Thbus1} \\ 0 \\ 0 \end{bmatrix} = (\mathbf{Z_s} + \mathbf{Z_{LDs}})\,\mathbf{I_s}$$

$$\mathbf{U_{Th}} = \begin{bmatrix} \overline{U}_{Thbus1} \\ 0 \\ 0 \end{bmatrix} = (\mathbf{Z_s} + \mathbf{Z_{LDs}})\,\mathbf{I_s}$$

$$\mathbf{Z_s} = \begin{bmatrix} \overline{Z}_{Thbus1-1} & 0 & 0 \\ 0 & \overline{Z}_{Thbus1-2} & 0 \\ 0 & 0 & \overline{Z}_{Thbus1-0} \end{bmatrix}$$

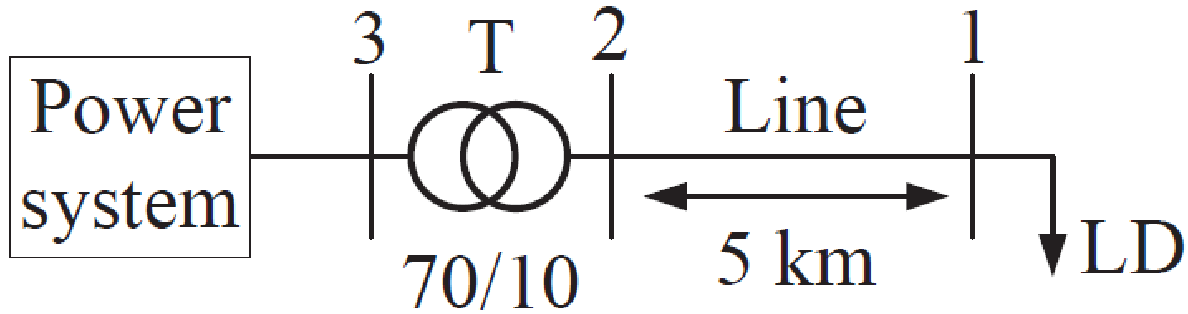
$$\mathbf{I_s} = \begin{bmatrix} \overline{I}_{bus1-1} \\ \overline{I}_{bus1-2} \\ \overline{I}_{bus1-0} \end{bmatrix} =$$

$$= (\mathbf{Z_s} + \mathbf{Z_{LDs}})^{-1}\,\mathbf{U_{Th}} = \begin{bmatrix} 0.8084\angle - 37.1616^\circ \\ 0.1596\angle 142.3433^\circ \\ 0.1541\angle 143.7484^\circ \end{bmatrix}$$

4. Calculate the positive-, negative- and zero-components of quantities that are of interest.
5. Transform those symmetrical components to the phase quantities that are asked for.

$$\mathbf{U_{bus1s}} = \begin{bmatrix} \overline{U}_{bus1-1} \\ \overline{U}_{bus1-2} \\ \overline{U}_{bus1-0} \end{bmatrix} = \mathbf{Z_{LDs}}\,\mathbf{I_s} = \begin{bmatrix} 0.9733\angle-0.3126° \\ 0.0054\angle10.6340° \\ 0.0112\angle-14.8199° \end{bmatrix}$$

$$\begin{bmatrix} \overline{U}_{bus1a} \\ \overline{U}_{bus1b} \\ \overline{U}_{bus1c} \end{bmatrix} = \mathbf{T}\,\mathbf{U_{bus1s}} \cdot \frac{U_{base10}}{\sqrt{3}} = \begin{bmatrix} 5.7122\angle-0.4154° \\ 5.5918\angle-119.9774° \\ 5.5535\angle119.4555° \end{bmatrix}$$



$$\begin{bmatrix} \overline{U}_{bus2-1} \\ \overline{I}_{bus2-1} \end{bmatrix} = \begin{bmatrix} \overline{A}_{L-1} & \overline{B}_{L-1} \\ \overline{C}_{L-1} & \overline{D}_{L-1} \end{bmatrix} \begin{bmatrix} \overline{U}_{bus1-1} \\ \overline{I}_{bus1-1} \end{bmatrix} = \begin{bmatrix} 0.9915\angle -0.6607^\circ \\ 0.8066\angle -36.9937^\circ \end{bmatrix}$$

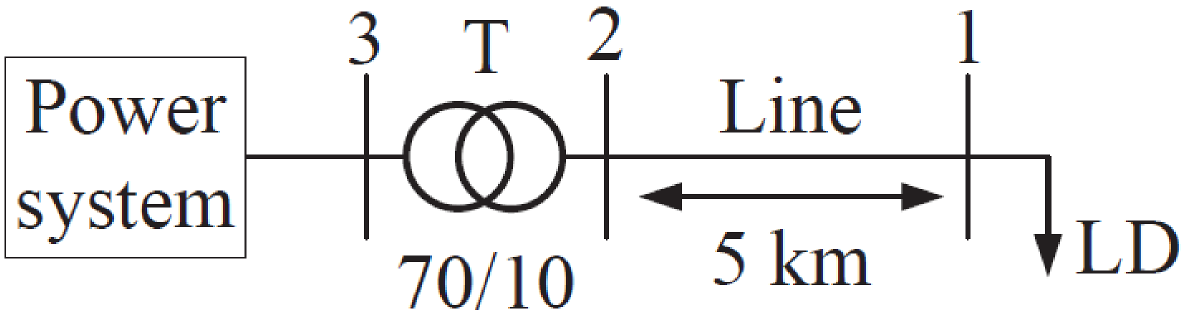
$$\begin{bmatrix} \overline{U}_{bus2-2} \\ \overline{I}_{bus2-2} \end{bmatrix} = \begin{bmatrix} \overline{A}_{L-2} & \overline{B}_{L-2} \\ \overline{C}_{L-2} & \overline{D}_{L-2} \end{bmatrix} \begin{bmatrix} \overline{U}_{bus1-2} \\ \overline{I}_{bus1-2} \end{bmatrix} = \begin{bmatrix} 0.0028\angle 52.3414^\circ \\ 0.1596\angle 142.3414^\circ \end{bmatrix}$$

$$\begin{bmatrix} \overline{U}_{bus2-0} \\ \overline{I}_{bus2-0} \end{bmatrix} = \begin{bmatrix} \overline{A}_{L-0} & \overline{B}_{L-0} \\ \overline{C}_{L-0} & \overline{D}_{L-0} \end{bmatrix} \begin{bmatrix} \overline{U}_{bus1-0} \\ \overline{I}_{bus1-0} \end{bmatrix} = \begin{bmatrix} 0.0006\angle 53.7455^\circ \\ 0.1541\angle 143.7455^\circ \end{bmatrix}$$

$$\overline{S}_{bus2-1} = \overline{U}_{bus2-1}\,\overline{I}^*_{bus2-1}\,S_{base} = 0.3221 + j\,0.2369 \text{ MVA}$$

$$\overline{S}_{bus2-2} = \overline{U}_{bus2-2}\,\overline{I}^*_{bus2-2}\,S_{base} = 0 - j\,0.0002 \text{ MVA}$$

$$\overline{S}_{bus2-0} = \overline{U}_{bus2-0}\,\overline{I}^*_{bus2-0}\,S_{base} = 0 - j\,0.00005 \text{ MVA}$$



$$\overline{S}_{bus2-1} = \overline{U}_{bus2-1}\,\overline{I}^{*}_{bus2-1}\,S_{base} = 0.3221 + j\,0.2369 \text{ MVA}$$

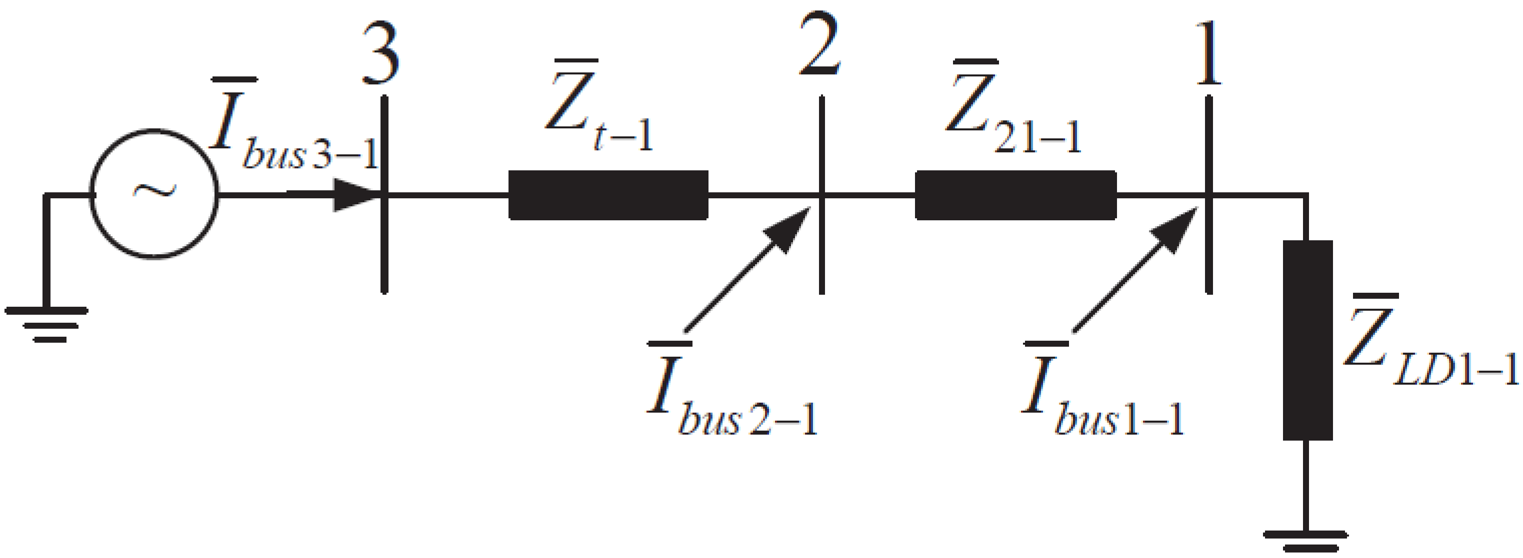
$$\overline{S}_{bus2-2} = \overline{U}_{bus2-2}\,\overline{I}^{*}_{bus2-2}\,S_{base} = 0 - j\,0.0002 \text{ MVA}$$

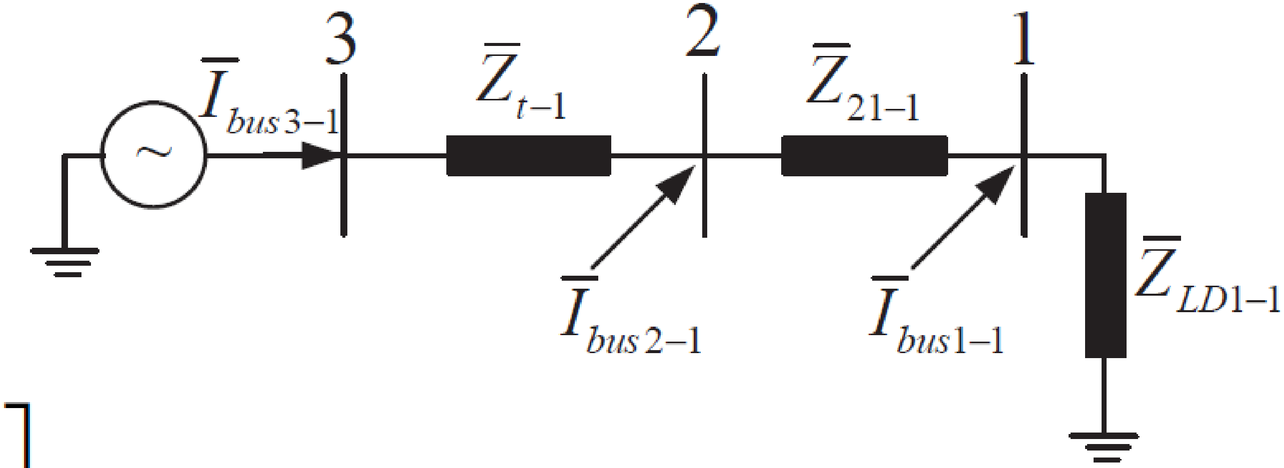
$$\overline{S}_{bus2-0} = \overline{U}_{bus2-0}\,\overline{I}^{*}_{bus2-0}\,S_{base} = 0 - j\,0.00005 \text{ MVA}$$

$$\overline{S} = \overline{S}_{bus2-1} + \overline{S}_{bus2-2} + \overline{S}_{bus2-0} = 0.3221 + j0.2366 \text{ MVA}$$

A general method for analysis of linear three-phase systems with one unbalanced load

For a balanced system, all system quantities and components can be represented only by their positive-sequence components.

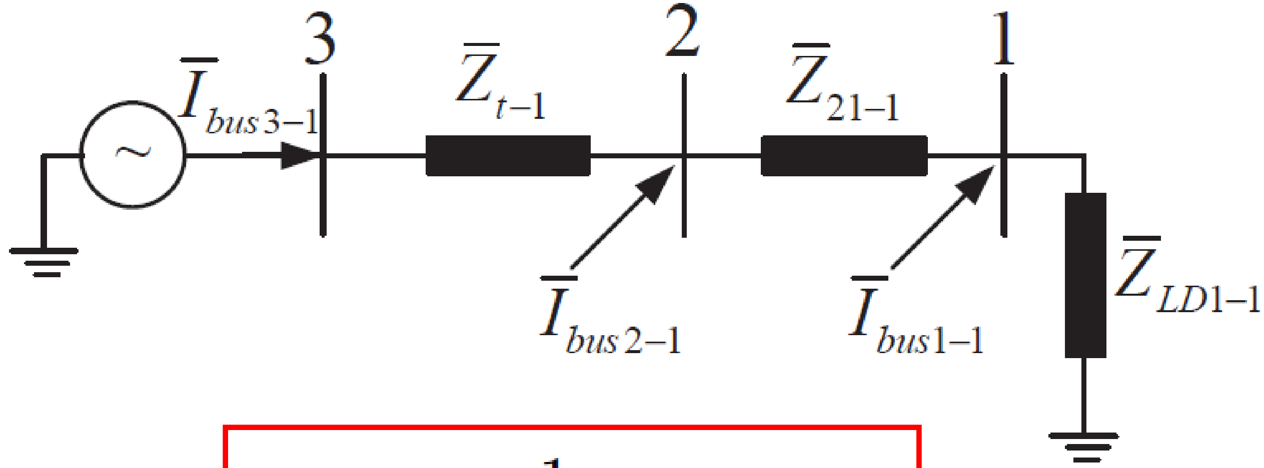




$$\begin{bmatrix} \overline{I}_{bus1-1} \\ \overline{I}_{bus2-1} \\ \overline{I}_{bus3-1} \end{bmatrix} = \mathbf{I}_1 = \mathbf{Y}_1\,\mathbf{U}_1 =$$

$$= \begin{bmatrix} \frac{1}{\overline{Z}_{LD1-1}} + \frac{1}{\overline{Z}_{12-1}} & -\frac{1}{\overline{Z}_{12-1}} & 0 \\ -\frac{1}{\overline{Z}_{21-1}} & \frac{1}{\overline{Z}_{21-1}} + \frac{1}{\overline{Z}_{t-1}} & -\frac{1}{\overline{Z}_{t-1}} \\ 0 & -\frac{1}{\overline{Z}_{t-1}} & \frac{1}{\overline{Z}_{t-1}} \end{bmatrix} \begin{bmatrix} \overline{U}_{bus1-1} \\ \overline{U}_{bus2-1} \\ \overline{U}_{bus3-1} \end{bmatrix}$$

$$\boxed{\mathbf{U}_1 = \mathbf{Y}_1^{-1}\,\mathbf{I}_1 = \mathbf{Z}_1\,\mathbf{I}_1}$$

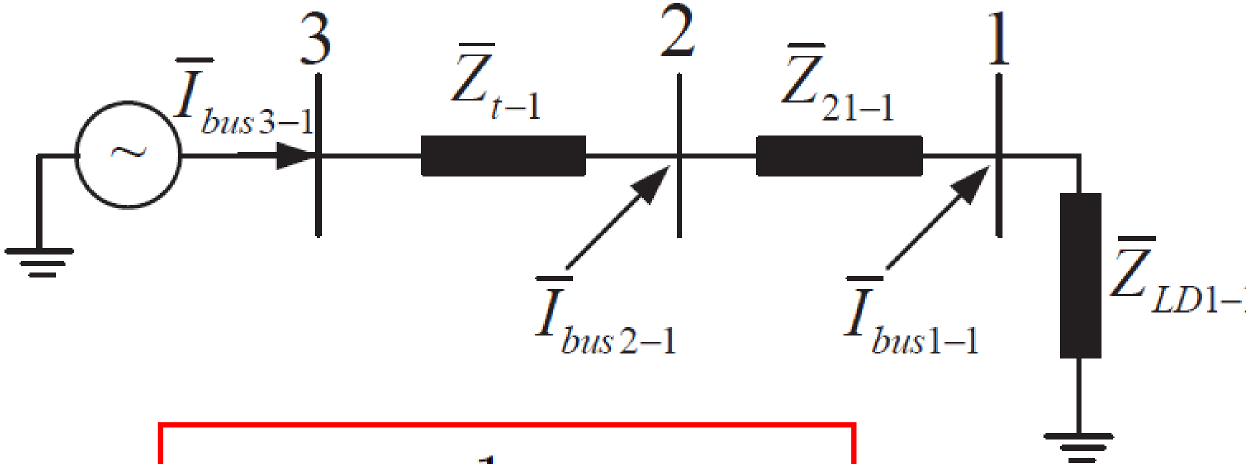


$$\boxed{\mathbf{U_1} = \mathbf{Y_1^{-1}}\,\mathbf{I_1} = \mathbf{Z_1}\,\mathbf{I_1}}$$

$$\overline{I}_{bus1-1} = \overline{I}_{bus2-1} = 0$$

$$\overline{U}_{bus3-1} = \mathbf{Z_1}(3,3) \cdot \overline{I}_{bus3-1} \quad \Rightarrow$$

$$\overline{I}_{bus3-1} = \frac{\overline{U}_{bus3-1}}{\mathbf{Z_1}(3,3)}$$



$$\boxed{\mathbf{U_1} = \mathbf{Y}_1^{-1}\,\mathbf{I}_1 = \mathbf{Z}_1\,\mathbf{I}_1}$$

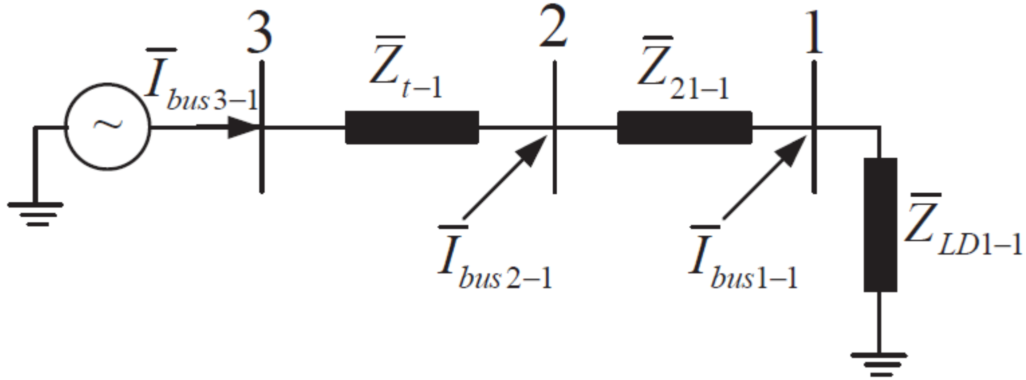
$$\overline{I}_{bus3-1} = \frac{\overline{U}_{bus3-1}}{\mathbf{Z_1}(3,3)}$$

$$\begin{aligned}
\overline{U}_{bus1-1} &= \mathbf{Z_1}(1,3)\cdot\overline{I}_{bus3-1} \\
\overline{U}_{bus2-1} &= \mathbf{Z_1}(2,3)\cdot\overline{I}_{bus3-1}
\end{aligned}$$

Assume a system with a voltage source at bus ***i***. The current at bus i and the voltage at another bus ***r*** can then be calculated as

$$\overline{I}_{busi-1} = \frac{\overline{U}_{busi-1}}{\mathbf{Z_1}(i,i)}$$

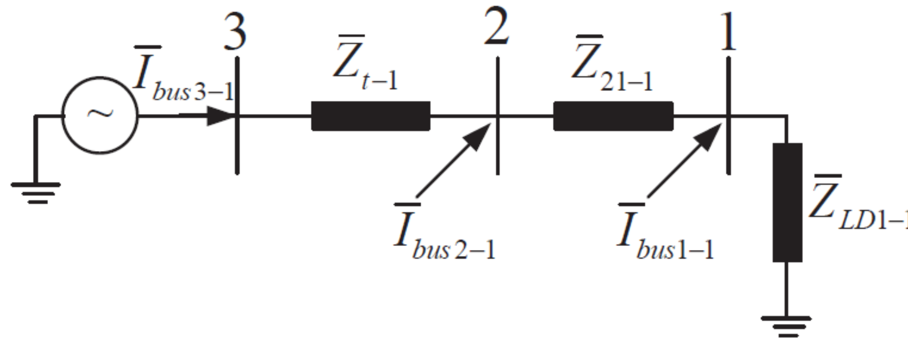
$$\overline{U}_{busr-1} = \overline{U}_{Thbusr} = \mathbf{Z_1}(r,i) \cdot \overline{I}_{busi-1}$$



Now assume that an unsymmetrical impedance load is connected to bus 2. Then the actual voltages can be obtained by using the theorem of superposition, i.e. as the sum of the voltages before the connection of the load and with the voltage change obtained by the load connection.

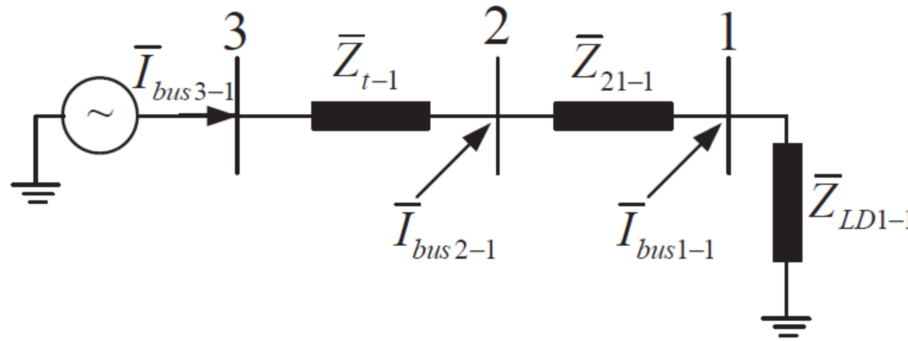
$$\boxed{\begin{aligned}\mathbf{U}'_1 &= \mathbf{U}_{\mathbf{pre1}} + \mathbf{U}_{\mathbf{\Delta 1}}\\ \mathbf{U}'_2 &= \mathbf{U}_{\mathbf{pre2}} + \mathbf{U}_{\mathbf{\Delta 2}}\\ \mathbf{U}'_0 &= \mathbf{U}_{\mathbf{pre0}} + \mathbf{U}_{\mathbf{\Delta 0}}\end{aligned}}$$

$$\boxed{\begin{aligned}\mathbf{U}'_1 &= \mathbf{U}_{\mathbf{pre1}} + \mathbf{U}_{\Delta 1}\\ \mathbf{U}'_2 &= \mathbf{U}_{\mathbf{pre2}} + \mathbf{U}_{\Delta 2}\\ \mathbf{U}'_0 &= \mathbf{U}_{\mathbf{pre0}} + \mathbf{U}_{\Delta 0}\end{aligned}}$$



$\mathbf{U}'_1$, $\mathbf{U}'_2$, $\mathbf{U}'_0$ are vectors containing the positive-, negative-, and zero-sequence voltages, respectively, at all buses (excluding the bus connected to the voltage source) due to the connection of the unsymmetrical load.

$$\boxed{\begin{aligned} \mathbf{U}'_1 &= \mathbf{U}_{\mathrm{pre1}} + \mathbf{U}_{\Delta 1} \\ \mathbf{U}'_2 &= \mathbf{U}_{\mathrm{pre2}} + \mathbf{U}_{\Delta 2} \\ \mathbf{U}'_0 &= \mathbf{U}_{\mathrm{pre0}} + \mathbf{U}_{\Delta 0} \end{aligned}}$$

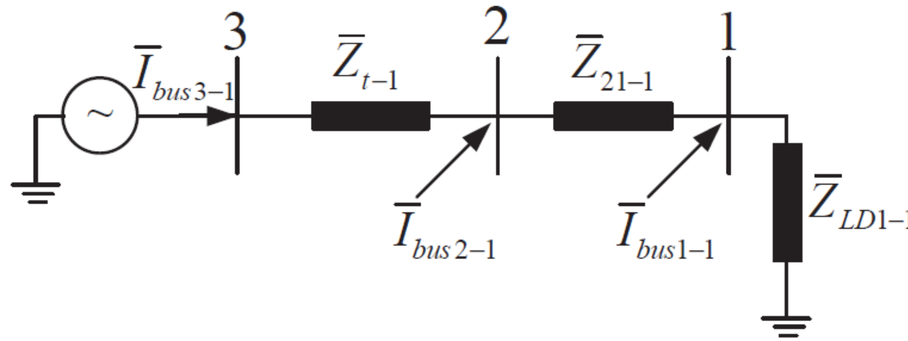


$\mathbf{U}_{\mathrm{pre1}}$ $\mathbf{U}_{\mathrm{pre2}}$ $\mathbf{U}_{\mathrm{pre0}}$ are vectors containing the positive-, negative-, and zero-sequence voltages, respectively, at all buses (excluding the bus connected to the voltage source) prior to the connection of the unsymmetrical load.

$$U_{pre2} = ??$$

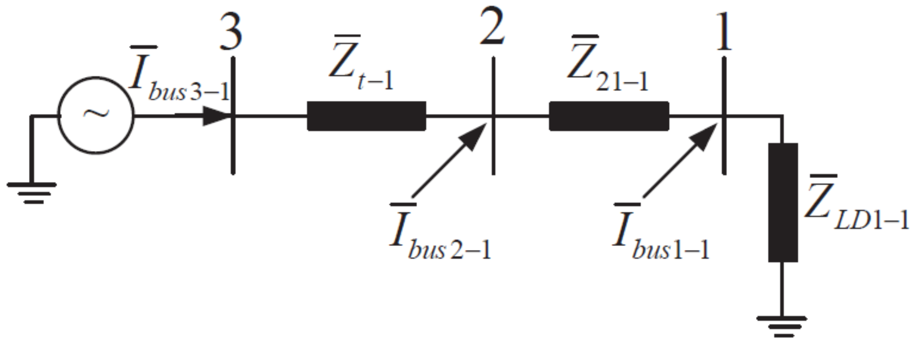
$$U_{pre0} = ??$$

$$\begin{aligned} \mathbf{U}'_1 &= \mathbf{U}_{\text{pre1}} + \mathbf{U}_{\Delta 1} \\ \mathbf{U}'_2 &= \mathbf{U}_{\text{pre2}} + \mathbf{U}_{\Delta 2} \\ \mathbf{U}'_0 &= \mathbf{U}_{\text{pre0}} + \mathbf{U}_{\Delta 0} \end{aligned}$$



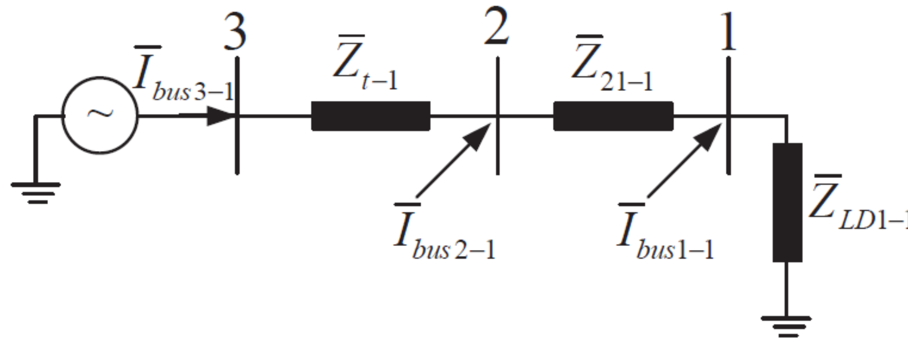
$\mathbf{U}_{\Delta 1}$, $\mathbf{U}_{\Delta 2}$, $\mathbf{U}_{\Delta 0}$ are vectors containing the changes in the positive-, negative-, and zero-sequence voltages, respectively, at all buses (excluding the bus connected to the voltage source) due to the connection of the unsymmetrical load.

$$
\boxed{\begin{aligned}
\mathbf{U}_1' &= \mathbf{U}_{\mathbf{pre1}} + \mathbf{U}_{\boldsymbol{\Delta}\mathbf{1}} \\
\mathbf{U}_2' &= \mathbf{U}_{\mathbf{pre2}} + \mathbf{U}_{\boldsymbol{\Delta}\mathbf{2}} \\
\mathbf{U}_0' &= \mathbf{U}_{\mathbf{pre0}} + \mathbf{U}_{\boldsymbol{\Delta}\mathbf{0}}
\end{aligned}}
$$



$$
\boxed{\begin{aligned}
\mathbf{U}_1' &= \mathbf{U}_{\mathbf{pre1}} + \mathbf{Z}_{\boldsymbol{\Delta}\mathbf{1}}\,\mathbf{I}_{\boldsymbol{\Delta}\mathbf{1}} \\
\mathbf{U}_2' &= 0 + \mathbf{Z}_{\boldsymbol{\Delta}\mathbf{2}}\,\mathbf{I}_{\boldsymbol{\Delta}\mathbf{2}} \\
\mathbf{U}_0' &= 0 + \mathbf{Z}_{\boldsymbol{\Delta}\mathbf{0}}\,\mathbf{I}_{\boldsymbol{\Delta}\mathbf{0}}
\end{aligned}}
$$

$$
\begin{aligned}
\mathbf{U}_1' &= \mathbf{U}_{\mathbf{pre1}} + \mathbf{Z}_{\Delta 1}\,\mathbf{I}_{\Delta 1} \\
\mathbf{U}_2' &= \mathbf{0} + \mathbf{Z}_{\Delta 2}\,\mathbf{I}_{\Delta 2} \\
\mathbf{U}_0' &= \mathbf{0} + \mathbf{Z}_{\Delta 0}\,\mathbf{I}_{\Delta 0}
\end{aligned}
$$

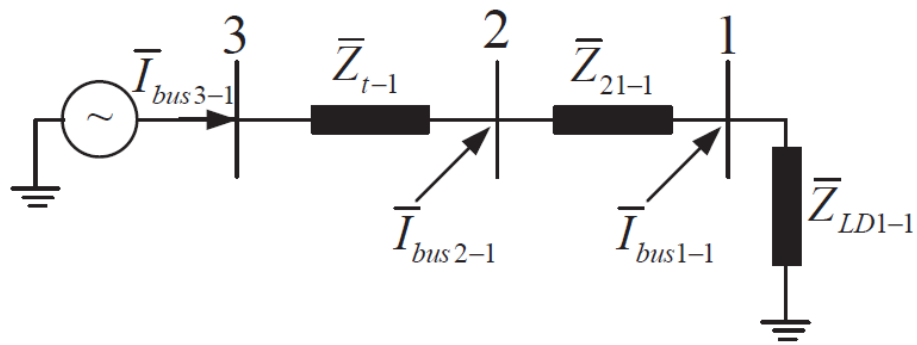


$\mathbf{Z}_{\Delta 1}$ is the Z-bus matrix of the positive-sequence system with the shortened voltage source.

$\mathbf{Z}_{\Delta 2}$ is the Z-bus matrix of the negative-sequence system.

$\mathbf{Z}_{\Delta 0}$ is the Z-bus matrix of the zero-sequence system.

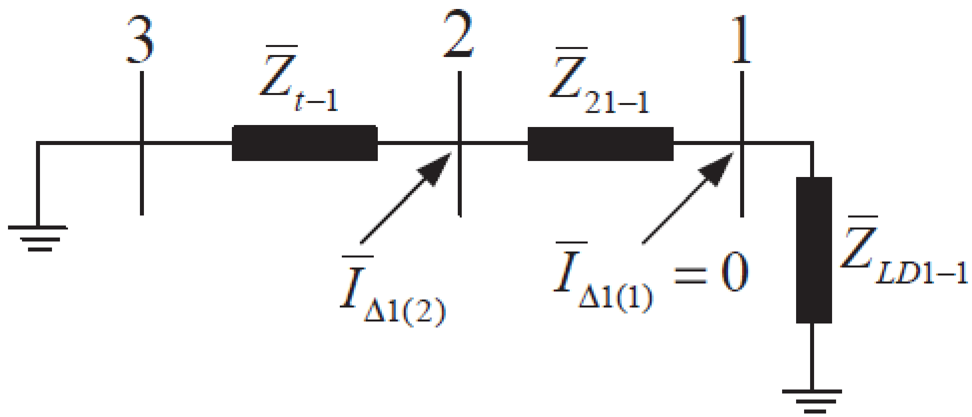
$$
\begin{aligned}
\mathbf{U}_1' &= \mathbf{U}_{\mathbf{pre1}} + \mathbf{Z}_{\Delta 1}\,\mathbf{I}_{\Delta 1} \\
\mathbf{U}_2' &= \mathbf{0} + \mathbf{Z}_{\Delta 2}\,\mathbf{I}_{\Delta 2} \\
\mathbf{U}_0' &= \mathbf{0} + \mathbf{Z}_{\Delta 0}\,\mathbf{I}_{\Delta 0}
\end{aligned}
$$



$\mathbf{I}_{\Delta 1}$, $\mathbf{I}_{\Delta 2}$, $\mathbf{I}_{\Delta 0}$ are vectors containing the injected positive-, negative- and zero-sequence current changes into the buses, respectively.

In this example only

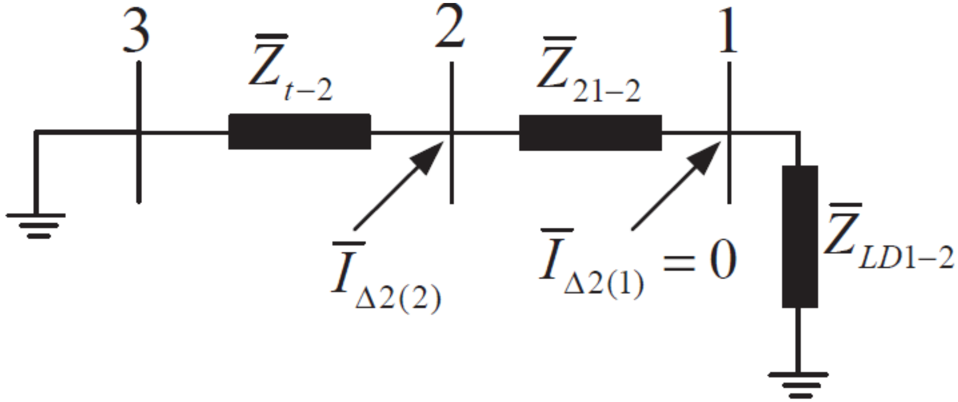
$$
\mathbf{I}_{\Delta 1}(2) \neq 0
$$
$$
\mathbf{I}_{\Delta 2}(2) \neq 0
$$
$$
\mathbf{I}_{\Delta 0}(2) \neq 0
$$



$$\mathbf{Y}_{\mathbf{\Delta 1}} = \begin{bmatrix} \frac{1}{\overline{\overline{Z}}_{LD1-1}} + \frac{1}{\overline{\overline{Z}}_{21-1}} & -\frac{1}{\overline{\overline{Z}}_{21-1}} \\ -\frac{1}{\overline{\overline{Z}}_{21-1}} & \frac{1}{\overline{\overline{Z}}_{21-1}} + \frac{1}{\overline{\overline{Z}}_{t-1}} \end{bmatrix}$$

$$\mathbf{Y}_{\mathbf{\Delta 1}} = \mathbf{Y}_{\mathbf{1}}(1:2, 1:2)$$

$$\mathbf{Z}_{\mathbf{\Delta 1}} = \mathbf{Y}_{\mathbf{\Delta 1}}^{-1}$$

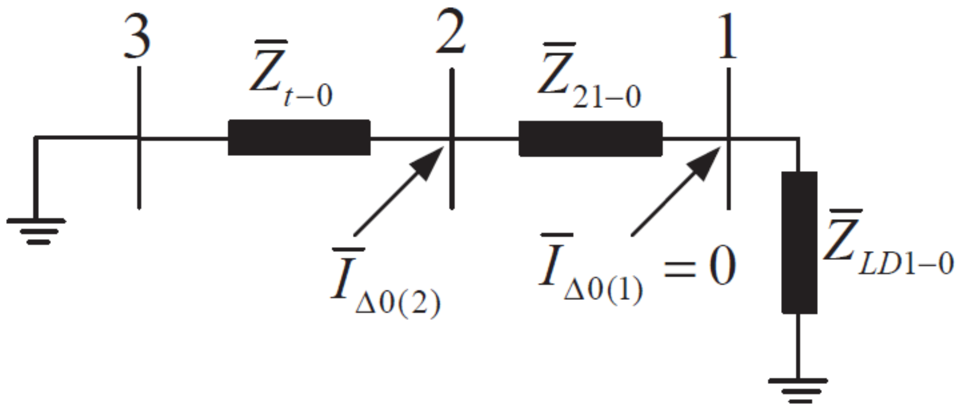


Negative-sequence

$$\mathbf{Y}_{\mathbf{\Delta 2}} = \begin{bmatrix} \frac{1}{\overline{Z}_{LD1-2}} + \frac{1}{\overline{Z}_{21-2}} & -\frac{1}{\overline{Z}_{21-2}} \\ -\frac{1}{\overline{Z}_{21-2}} & \frac{1}{\overline{Z}_{21-2}} + \frac{1}{\overline{Z}_{T-2}} \end{bmatrix}$$

$$\mathbf{Y}_{\mathbf{\Delta 2}} = \mathbf{Y}_{\mathbf{\Delta 1}} \qquad \mathbf{Z}_{\mathbf{\Delta 2}} = \mathbf{Z}_{\mathbf{\Delta 1}} = \mathbf{Y}_{\mathbf{\Delta 1}}^{-1}$$

for a system that is only composed of lines, transformers symmetrical impedance loads, since their positive- and negative-sequence components are identical.



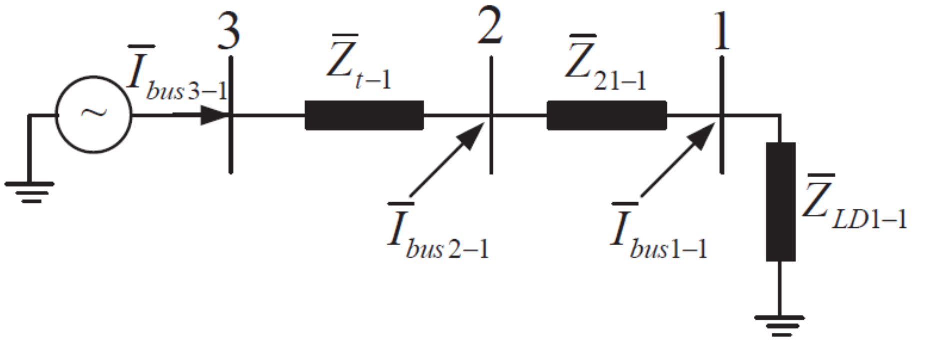
Zero-sequence

$$\mathbf{Y}_{\mathbf{\Delta 0}} = \begin{bmatrix} \frac{1}{\overline{Z}_{LD1-0}} + \frac{1}{\overline{Z}_{21-0}} & -\frac{1}{\overline{Z}_{21-0}} \\ -\frac{1}{\overline{Z}_{21-0}} & \frac{1}{\overline{Z}_{21-0}} + \frac{1}{\overline{Z}_{t-0}} \end{bmatrix}$$

$$\mathbf{Z}_{\mathbf{\Delta 0}} = \mathbf{Y}_{\mathbf{\Delta 0}}^{-1}$$

where, Y0-connected load and Y0-Y0 transformer are assumed.

$$\boxed{\begin{array}{rcl} \mathbf{U}'_1 & = & \mathbf{U_{pre1}} + \mathbf{Z_{\Delta 1}}\,\mathbf{I_{\Delta 1}} \\ \mathbf{U}'_2 & = & \mathbf{0} \quad + \mathbf{Z_{\Delta 2}}\,\mathbf{I_{\Delta 2}} \\ \mathbf{U}'_0 & = & \mathbf{0} \quad + \mathbf{Z_{\Delta 0}}\,\mathbf{I_{\Delta 0}} \end{array}}$$



$$\overline{U}'_{bus2-1} = \mathbf{U}'_1(2) =$$

$$= \overline{U}_{bus2-1} + \mathbf{Z_{\Delta 1}}(2,2)\,\mathbf{I_{\Delta 1}}(2)$$

$$\overline{U}'_{bus2-2} = \mathbf{U}'_2(2) = 0 + \mathbf{Z_{\Delta 2}}(2,2)\,\mathbf{I_{\Delta 2}}(2)$$

$$\overline{U}'_{bus2-0} = \mathbf{U}'_0(2) = 0 + \mathbf{Z_{\Delta 0}}(2,2)\,\mathbf{I_{\Delta 0}}(2)$$

$$\begin{aligned}\mathbf{U}'_1 &= \mathbf{U}_{\mathbf{pre1}} + \mathbf{Z}_{\boldsymbol{\Delta}\mathbf{1}}\,\mathbf{I}_{\boldsymbol{\Delta}\mathbf{1}}\\ \mathbf{U}'_2 &= \mathbf{0} + \mathbf{Z}_{\boldsymbol{\Delta}\mathbf{2}}\,\mathbf{I}_{\boldsymbol{\Delta}\mathbf{2}}\\ \mathbf{U}'_0 &= \mathbf{0} + \mathbf{Z}_{\boldsymbol{\Delta}\mathbf{0}}\,\mathbf{I}_{\boldsymbol{\Delta}\mathbf{0}}\end{aligned}$$

The unsymmetrical load is connected to bus *r*.

$$\mathbf{U}'_{\mathbf{s}}(r) = \begin{bmatrix} \overline{U}'_{busr-1} \\ \overline{U}'_{busr-2} \\ \overline{U}'_{busr-0} \end{bmatrix} =$$

$$I_{\Delta}(r) = ??$$

$$= \mathbf{U}_{\mathbf{pre}}(r) + \mathbf{Z}_{\boldsymbol{\Delta}}(r,r)\,\mathbf{I}_{\boldsymbol{\Delta}}(r) =$$

$$= \begin{bmatrix} \overline{U}_{Thbusr} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{Z}_{\boldsymbol{\Delta}\mathbf{1}}(r,r) & 0 & 0 \\ 0 & \mathbf{Z}_{\boldsymbol{\Delta}\mathbf{2}}(r,r) & 0 \\ 0 & 0 & \mathbf{Z}_{\boldsymbol{\Delta}\mathbf{0}}(r,r) \end{bmatrix} \begin{bmatrix} \overline{I}_{\Delta 1}(r) \\ \overline{I}_{\Delta 2}(r) \\ \overline{I}_{\Delta 0}(r) \end{bmatrix}$$

Y0-connected unsymmetrical load at bus *r*

$$\mathbf{U_{LDbusrp_h}} = \begin{bmatrix} \overline{U}_{LDbusra} \\ \overline{U}_{LDbusrb} \\ \overline{U}_{LDbusrc} \end{bmatrix} =$$

$$= \begin{bmatrix} \overline{Z}_{LDbusra} & 0 & 0 \\ 0 & \overline{Z}_{LDbusrb} & 0 \\ 0 & 0 & \overline{Z}_{LDbusrc} \end{bmatrix} \begin{bmatrix} \overline{I}_{LDbusra} \\ \overline{I}_{LDbusrb} \\ \overline{I}_{LDbusrc} \end{bmatrix} =$$

$$= \mathbf{Z_{LDbusrp_h}} \, \mathbf{I_{LDbusrp_h}}$$

$$\mathbf{U}'_{\mathbf{s}}(r) = \begin{bmatrix} \overline{U}'_{busr-1} \\ \overline{U}'_{busr-2} \\ \overline{U}'_{busr-0} \end{bmatrix} = \underbrace{\mathbf{T}^{-1}\,\mathbf{Z}_{\mathbf{LDbusrph}}\,\mathbf{T}}_{=\mathbf{Z}_{\mathbf{LDbusrs}}}\,\mathbf{I}_{\mathbf{LDbusrs}}$$

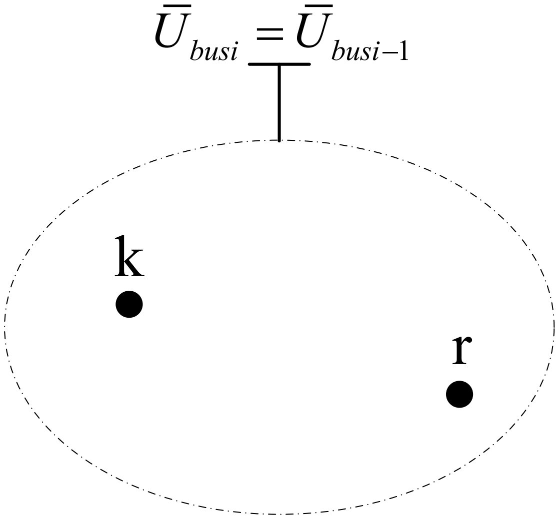
$$\boxed{= -\mathbf{Z}_{\mathbf{LDbusrs}}\,\mathbf{I}_{\mathbf{\Delta}}(r)}$$

$$\boxed{I_{\Delta}(r) = ??}$$

$$\boxed{= \mathbf{U}_{\mathbf{pre}}(r) + \mathbf{Z}_{\mathbf{\Delta}}(r,r)\,\mathbf{I}_{\mathbf{\Delta}}(r)}$$

$$\boxed{\mathbf{I}_{\mathbf{\Delta}}(r) = -\left[\mathbf{Z}_{\mathbf{\Delta}}(r,r) + \mathbf{Z}_{\mathbf{LDbusrs}}\right]^{-1}\mathbf{U}_{\mathbf{pre}}(r)}$$

$$\boxed{\begin{aligned} \mathbf{U}'_{\mathbf{1}}(k) &= \mathbf{U}_{\mathbf{pre1}}(k) + \mathbf{Z}_{\mathbf{\Delta 1}}(k,r)\,\mathbf{I}_{\mathbf{\Delta 1}}(r) \\ \mathbf{U}'_{\mathbf{2}}(k) &= \mathbf{Z}_{\mathbf{\Delta 2}}(k,r)\,\mathbf{I}_{\mathbf{\Delta 2}}(r) \\ \mathbf{U}'_{\mathbf{0}}(k) &= \mathbf{Z}_{\mathbf{\Delta 0}}(k,r)\,\mathbf{I}_{\mathbf{\Delta 0}}(r) \end{aligned}}$$



Three-phase power system

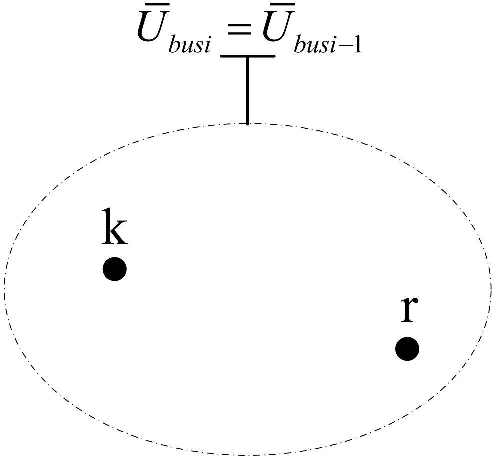
$$\mathbf{U_1} = \mathbf{Y_1^{-1}}\,\mathbf{I_1} = \mathbf{Z_1}\,\mathbf{I_1}$$

$$\overline{I}_{busi-1} = \frac{\overline{U}_{busi-1}}{\mathbf{Z_1}(i,i)}$$

$$\overline{U}_{busr-1} = \overline{U}_{Thbusr} = \mathbf{Z_1}(r,i) \cdot \overline{I}_{busi-1}$$

$$\mathbf{U'_s}(r) = \begin{bmatrix} \overline{U}'_{busr-1} \\ \overline{U}'_{busr-2} \\ \overline{U}'_{busr-0} \end{bmatrix} = \mathbf{U_{pre}}(r) + \mathbf{Z_\Delta}(r,r)\,\mathbf{I_\Delta}(r) =$$

$$\equiv \begin{bmatrix} \overline{U}_{Thbusr} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{Z_{\Delta 1}}(r,r) & 0 & 0 \\ 0 & \mathbf{Z_{\Delta 2}}(r,r) & 0 \\ 0 & 0 & \mathbf{Z_{\Delta 0}}(r,r) \end{bmatrix} \begin{bmatrix} \overline{I}_{\Delta 1}(r) \\ \overline{I}_{\Delta 2}(r) \\ \overline{I}_{\Delta 0}(r) \end{bmatrix}$$



Three-phase power system

$$\mathbf{U}'_{\mathbf{s}}(r) = \begin{bmatrix} \overline{U}'_{busr-1} \\ \overline{U}'_{busr-2} \\ \overline{U}'_{busr-0} \end{bmatrix} = \mathbf{U}_{\mathbf{pre}}(r) + \mathbf{Z}_{\mathbf{\Delta}}(r,r)\,\mathbf{I}_{\mathbf{\Delta}}(r) =$$

$$\equiv \begin{bmatrix} \overline{U}_{Thbusr} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{Z}_{\mathbf{\Delta 1}}(r,r) & 0 & 0 \\ 0 & \mathbf{Z}_{\mathbf{\Delta 2}}(r,r) & 0 \\ 0 & 0 & \mathbf{Z}_{\mathbf{\Delta 0}}(r,r) \end{bmatrix} \begin{bmatrix} \overline{I}_{\Delta 1}(r) \\ \overline{I}_{\Delta 2}(r) \\ \overline{I}_{\Delta 0}(r) \end{bmatrix}$$

$$\mathbf{Z}_{\mathbf{\Delta 1}} = \mathbf{Y}_{\mathbf{\Delta 1}}^{-1}$$
$$\mathbf{Z}_{\mathbf{\Delta 2}} = \mathbf{Y}_{\mathbf{\Delta 2}}^{-1}$$
$$\mathbf{Z}_{\mathbf{\Delta 0}} = \mathbf{Y}_{\mathbf{\Delta 0}}^{-1}$$

$$\mathbf{Z}_{\mathbf{LDbusrs}} = \mathbf{T}^{-1}\,\mathbf{Z}_{\mathbf{LDbusrph}}\,\mathbf{T}$$

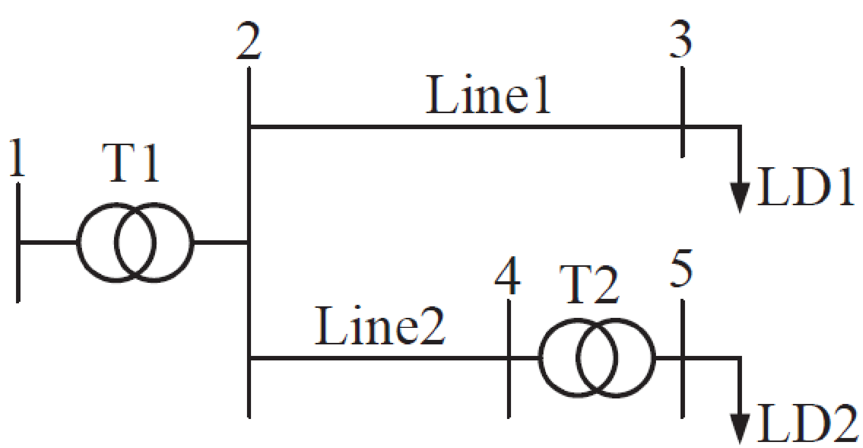
$$\boxed{\mathbf{I}_{\mathbf{\Delta}}(r) = -\left[\mathbf{Z}_{\mathbf{\Delta}}(r,r) + \mathbf{Z}_{\mathbf{LDbusrs}}\right]^{-1}\mathbf{U}_{\mathbf{pre}}(r)}$$

$$\mathbf{U}'_{\mathbf{s}}(r) = -\mathbf{Z}_{\mathbf{LDbusrs}}\,\mathbf{I}_{\mathbf{\Delta}}(r)$$

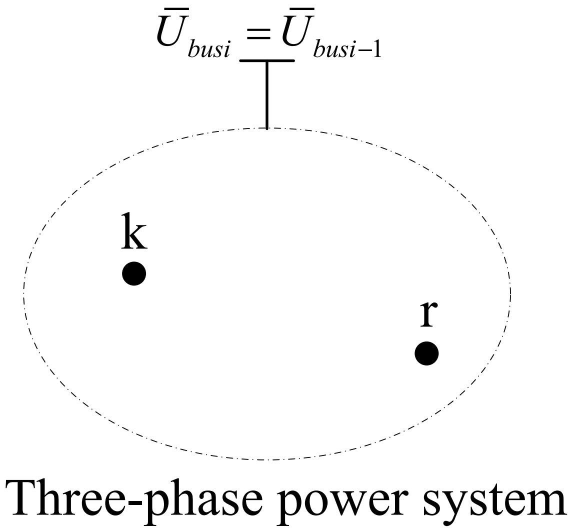
$$\boxed{\begin{aligned} \mathbf{U}'_{\mathbf{1}}(k) &= \mathbf{U}_{\mathbf{pre1}}(k) + \mathbf{Z}_{\mathbf{\Delta 1}}(k,r)\,\mathbf{I}_{\mathbf{\Delta 1}}(r) \\ \mathbf{U}'_{\mathbf{2}}(k) &= \mathbf{Z}_{\mathbf{\Delta 2}}(k,r)\,\mathbf{I}_{\mathbf{\Delta 2}}(r) \\ \mathbf{U}'_{\mathbf{0}}(k) &= \mathbf{Z}_{\mathbf{\Delta 0}}(k,r)\,\mathbf{I}_{\mathbf{\Delta 0}}(r) \end{aligned}}$$

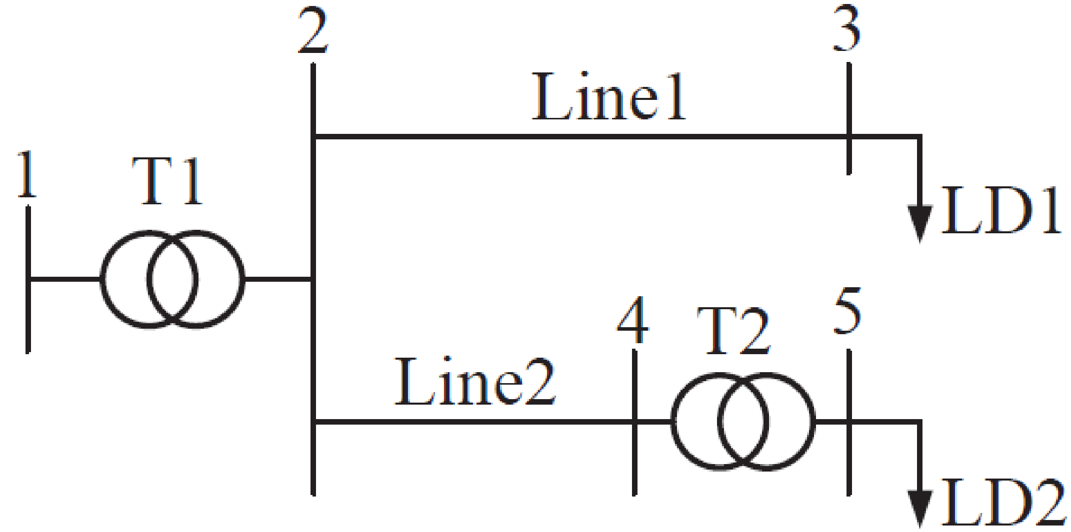
Example 8.6 *Consider again the system described in Example 6.3. The following additional data is also given:*

- *The infinite bus (i.e. bus 1) has and a grounded zero connection point.*
- *Transformer T1 is* Δ*-Y0 connected with Y0 on the 10 kV-side, and* $\overline{Z}_n = 0$.
- *Transformer T2 is Y0-Y0 connected with* $\overline{Z}_n = 0$.
- *The zero-sequence impedances of the lines are 3 times the positive-sequence impedances, and the zero-sequence shunt admittances of the lines are 0.5 times the positive-sequence shunt admittances.*
- *The load LD1 is* Δ*-connected.*
- *The load LD2 is Y0-connected with* $\overline{Z}_n = 0$. *Furthermore, half of the normal load connected to phase a is disconnected while the other phases are loaded as normal,* **i.e. LD2 is an unsymmetrical load**.



Calculate the efficiency of the internal network operating in this unbalanced condition.

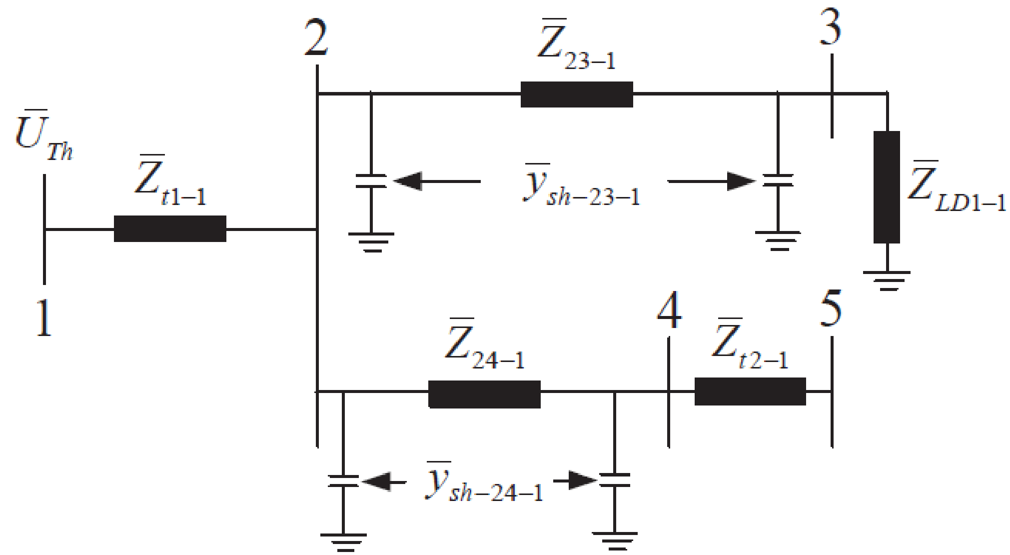




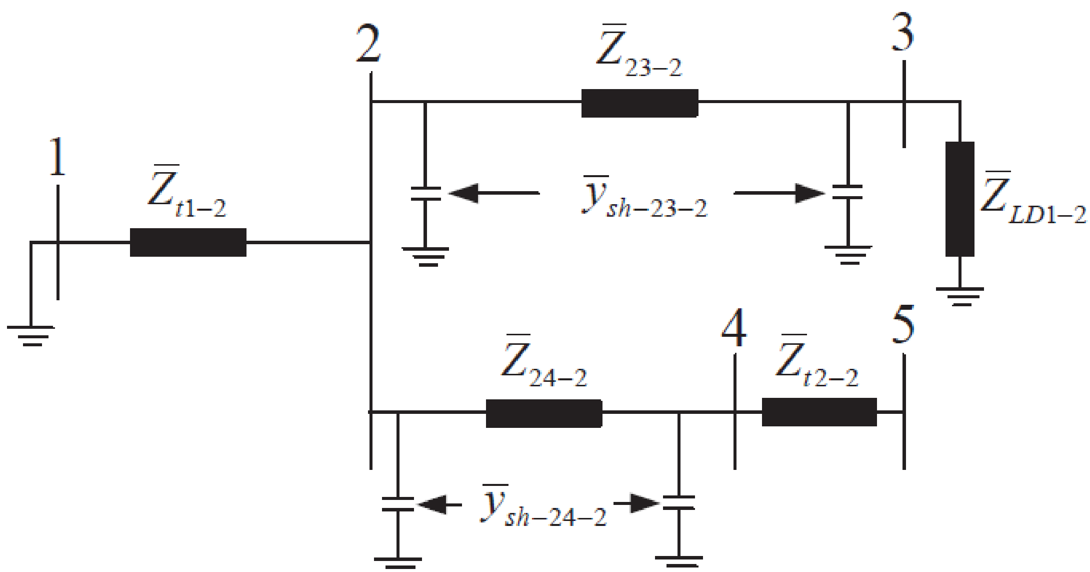
$$\boxed{\mathbf{I}_{\boldsymbol{\Delta}}(r) = -\left[\mathbf{Z}_{\boldsymbol{\Delta}}(r,r) + \mathbf{Z}_{\mathbf{LDbusrs}}\right]^{-1}\mathbf{U}_{\mathbf{pre}}(r)}$$

$$\mathbf{U}'_{\mathbf{s}}(r) \quad = -\mathbf{Z}_{\mathbf{LDbusrs}}\,\mathbf{I}_{\boldsymbol{\Delta}}(r)$$

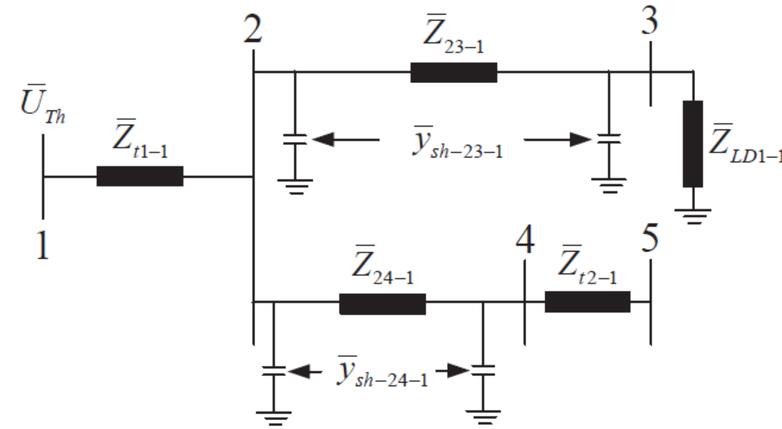
$$\boxed{\begin{aligned} \mathbf{U}'_{\mathbf{1}}(k) &= \mathbf{U}_{\mathbf{pre1}}(k) + \mathbf{Z}_{\boldsymbol{\Delta}\mathbf{1}}(k,r)\,\mathbf{I}_{\boldsymbol{\Delta}\mathbf{1}}(r) \\ \mathbf{U}'_{\mathbf{2}}(k) &= \mathbf{Z}_{\boldsymbol{\Delta}\mathbf{2}}(k,r)\,\mathbf{I}_{\boldsymbol{\Delta}\mathbf{2}}(r) \\ \mathbf{U}'_{\mathbf{0}}(k) &= \mathbf{Z}_{\boldsymbol{\Delta}\mathbf{0}}(k,r)\,\mathbf{I}_{\boldsymbol{\Delta}\mathbf{0}}(r) \end{aligned}}$$



a) Positive-sequence system



b) Negative-sequence system



a) Positive-sequence system

$$\overline{U}_{Th} = \overline{U}_1 = 1\angle 0^\circ$$

$$\overline{Z}_{t1-1} = \overline{Z}_{t1-2} = \overline{Z}_{t1pu} = j\,0.0438$$

$$\overline{Z}_{t2-1} = \overline{Z}_{t2-2} = \overline{Z}_{t2pu} = j\,0.1333$$

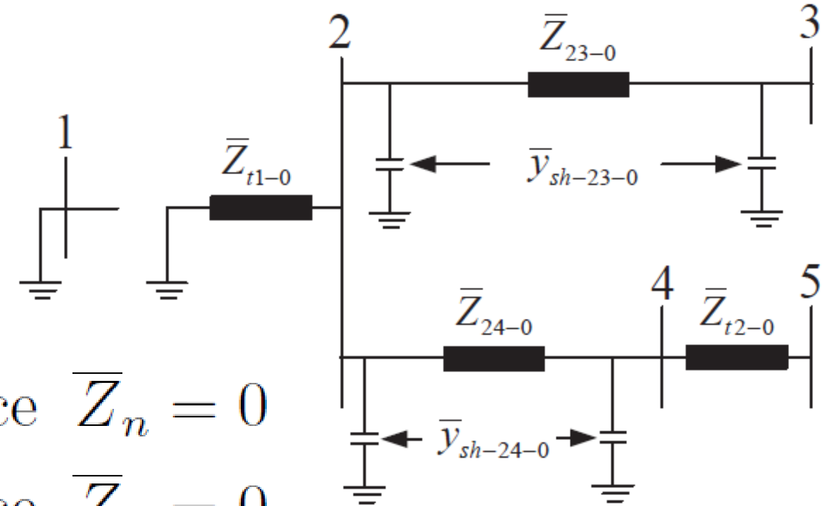
$$\overline{Z}_{23-1} = \overline{Z}_{23-2} = \overline{Z}_{23pu} = 0.0017 + j\,0.003$$

$$\overline{y}_{sh-23-1} = \overline{y}_{sh-23-2} = \overline{y}_{sh-23pu} = \frac{j\,0.0013}{2}$$

$$\overline{Z}_{24-1} = \overline{Z}_{24-2} = \overline{Z}_{24pu} = 0.0009 + j\,0.0015$$

$$\overline{y}_{sh-24-1} = \overline{y}_{sh-24-2} = \overline{y}_{sh-24pu} = \frac{j\,0.00064}{2}$$

$$\overline{Z}_{LD1-1} = \overline{Z}_{LD1-2} = \overline{Z}_{LD1pu} = 0.64 + j\,0.48$$



c) Zero-sequence system

$$\overline{Z}_{t1-0} = \overline{Z}_{t1-1} = j\,0.0438 \quad , \quad \text{since } \overline{Z}_n = 0$$

$$\overline{Z}_{t2-0} = \overline{Z}_{t2-1} = j\,0.1333 \quad , \quad \text{since } \overline{Z}_n = 0$$

$$\overline{Z}_{23-0} = 3\,\overline{Z}_{23-1} = 0.0051 + j\,0.009$$

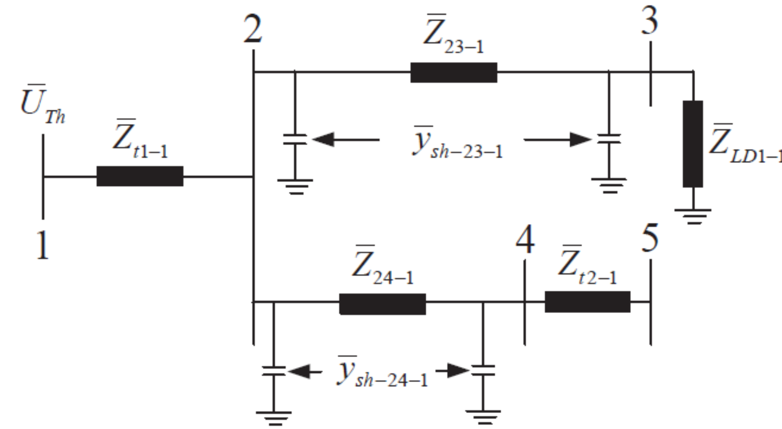
$$\overline{y}_{sh-23-0} = 0.5\,\overline{y}_{sh-23-1} = \frac{j\,0.0013}{4}$$

$$\overline{Z}_{24-0} = 3\,\overline{Z}_{24-1} = 0.0026 + j\,0.0045$$

$$\overline{y}_{sh-24-0} = 0.5\,\overline{y}_{sh-24-1} = \frac{j\,0.00064}{4}$$

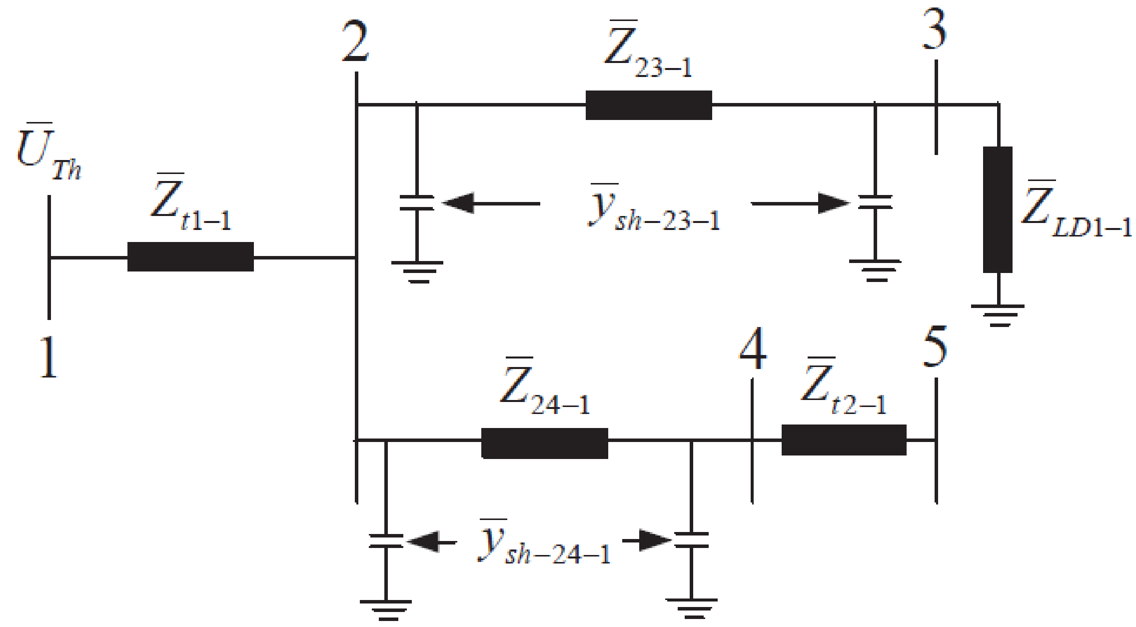
$$\overline{Z}_{LD1-0} = \infty \quad , \quad \text{since } \Delta\text{-connected}$$

$$Y_1 = Y$$



a) Positive-sequence system

$$= \begin{bmatrix} \frac{1}{\overline{Z}_{t1pu}} & -\frac{1}{\overline{Z}_{t1pu}} & 0 & 0 & 0 \\ -\frac{1}{\overline{Z}_{t1pu}} & \overline{Y}_{22} & -\frac{1}{\overline{Z}_{23pu}} & -\frac{1}{\overline{Z}_{24pu}} & 0 \\ 0 & -\frac{1}{\overline{Z}_{23pu}} & \overline{Y}_{33} & 0 & 0 \\ 0 & -\frac{1}{\overline{Z}_{24pu}} & 0 & \overline{Y}_{44} & -\frac{1}{\overline{Z}_{t2pu}} \\ 0 & 0 & 0 & -\frac{1}{\overline{Z}_{t2pu}} & \frac{1}{\overline{Z}_{t2pu}} + \cancel{\frac{1}{\overline{Z}_{LD2pu}}} \end{bmatrix}$$

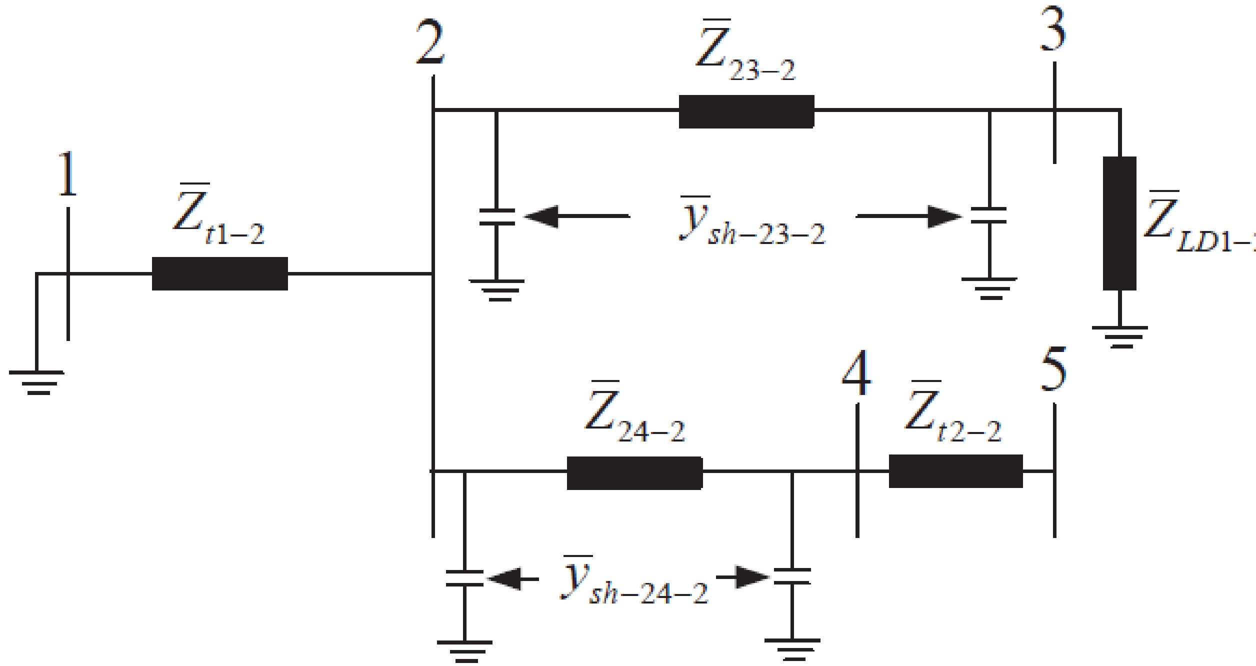


a) Positive-sequence system

$$\mathbf{Z_1} = \mathbf{Y_1^{-1}}$$

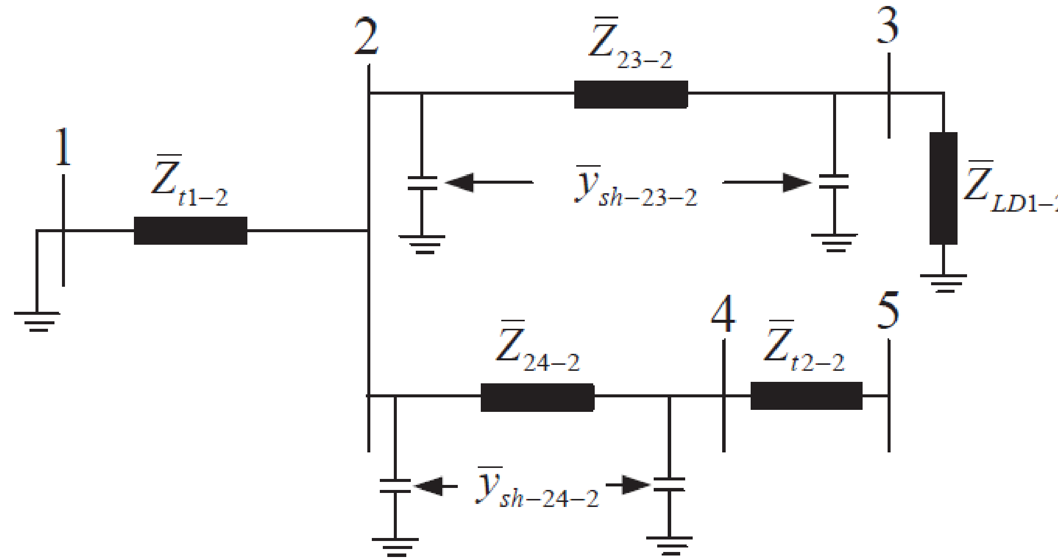
$$\overline{U}_{Thbus5} = \mathbf{Z_1}(5,1)\overline{I}_{bus1-1} =$$

$$= \mathbf{Z_1}(5,1)\,\frac{\overline{U}_{Th}}{\mathbf{Z_1}(1,1)} = 0.968\angle-2.413^\circ$$



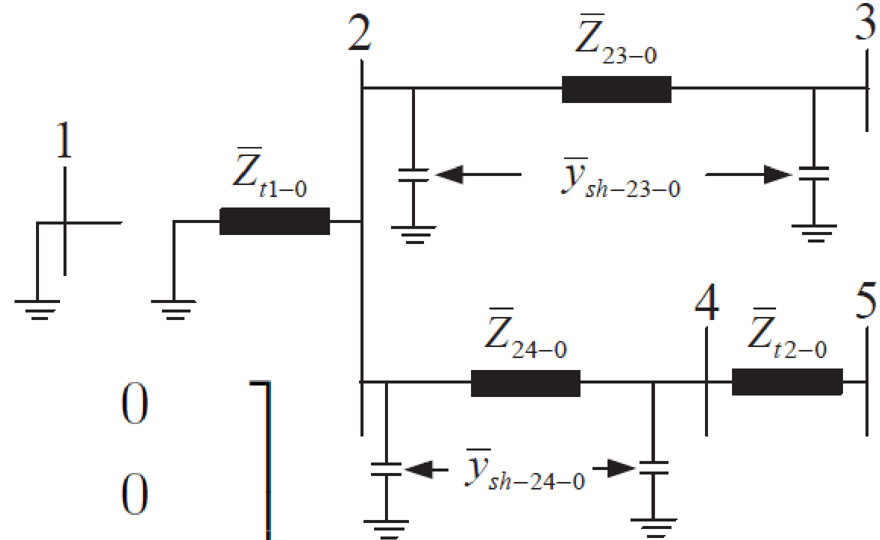
$$\mathbf{Y_{\Delta 2} = Y_{\Delta 1}}$$

$$= \begin{bmatrix} \overline{Y}_{22-1} & -\frac{1}{\overline{Z}_{23-1}} & -\frac{1}{\overline{Z}_{24-1}} & 0 \\ -\frac{1}{\overline{Z}_{23-1}} & \overline{Y}_{33-1} & 0 & 0 \\ -\frac{1}{\overline{Z}_{24-1}} & 0 & \overline{Y}_{44-1} & -\frac{1}{\overline{Z}_{t2-1}} \\ 0 & 0 & -\frac{1}{\overline{Z}_{t2-1}} & \frac{1}{\overline{Z}_{t2-1}} \end{bmatrix}$$



$$\mathbf{Z_{\Delta 1}} = \mathbf{Y_{\Delta 1}^{-1}} = \begin{bmatrix} 0.0018+j0.0423 & 0.0018+j0.0421 & 0.0018+j0.0423 & 0.0018+j0.0423 \\ 0.0018+j0.0421 & 0.0036+j0.0449 & 0.0018+j0.0421 & 0.0018+j0.0421 \\ 0.0018+j0.0423 & 0.0018+j0.0421 & 0.0026+j0.0438 & 0.0026+j0.0438 \\ 0.0018+j0.0423 & 0.0018+j0.0421 & 0.0026+j0.0438 & \boxed{0.0026+j0.1771} \end{bmatrix}$$

$$\overline{Z}_{Thbus5-2} = \overline{Z}_{Thbus5-1} = 0.0026 + j0.1771$$



$$\mathbf{Y_{\Delta 0}} = \begin{bmatrix} \overline{Y}_{22-0} & -\frac{1}{\overline{Z}_{23-0}} & -\frac{1}{\overline{Z}_{24-0}} & 0 \\ -\frac{1}{\overline{Z}_{23-0}} & \overline{Y}_{33-0} & 0 & 0 \\ -\frac{1}{\overline{Z}_{24-0}} & 0 & \overline{Y}_{44-0} & -\frac{1}{\overline{Z}_{t2-0}} \\ 0 & 0 & -\frac{1}{\overline{Z}_{t2-0}} & \frac{1}{\overline{Z}_{t2-0}} \end{bmatrix}$$

$$\mathbf{Z_{\Delta 0}} = \mathbf{Y_{\Delta 0}^{-1}} = \begin{bmatrix} 0.0000+j0.0438 & 0.0000+j0.0438 & 0.0000+j0.0438 & 0.0000+j0.0438 \\ 0.0000+j0.0438 & 0.0051+j0.0528 & 0.0000+j0.0438 & 0.0000+j0.0438 \\ 0.0000+j0.0438 & 0.0000+j0.0438 & 0.0026+j0.0483 & 0.0026+j0.0483 \\ 0.0000+j0.0438 & 0.0000+j0.0438 & 0.0026+j0.0483 & \boxed{0.0026+j0.1816} \end{bmatrix}$$

$$\overline{Z}_{Thbus5-0} = 0.0026 + j0.1816$$

$$\mathbf{U_{pre}}(5) = \begin{bmatrix} \overline{U}_{Thbus5} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.968\angle-2.413^\circ \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{Z_{\Delta}}(5,5) = \begin{bmatrix} \overline{Z}_{Thbus5-1} & 0 & 0 \\ 0 & \overline{Z}_{Thbus5-2} & 0 \\ 0 & 0 & \overline{Z}_{Thbus5-0} \end{bmatrix} =$$

$$= \begin{bmatrix} 0.0026+j0.1771 & 0 & 0 \\ 0 & 0.0026+j0.1771 & 0 \\ 0 & 0 & 0.0026+j0.1816 \end{bmatrix}$$

$$\mathbf{Z_{LDbus5s}} = \mathbf{T}^{-1}\mathbf{Z_{LDbus5p_h}}\mathbf{T} =$$

$$= \mathbf{T}^{-1}\begin{bmatrix} 2\overline{Z}_{LD2pu} & 0 & 0 \\ 0 & \overline{Z}_{LD2pu} & 0 \\ 0 & 0 & \overline{Z}_{LD2pu} \end{bmatrix}\mathbf{T} =$$

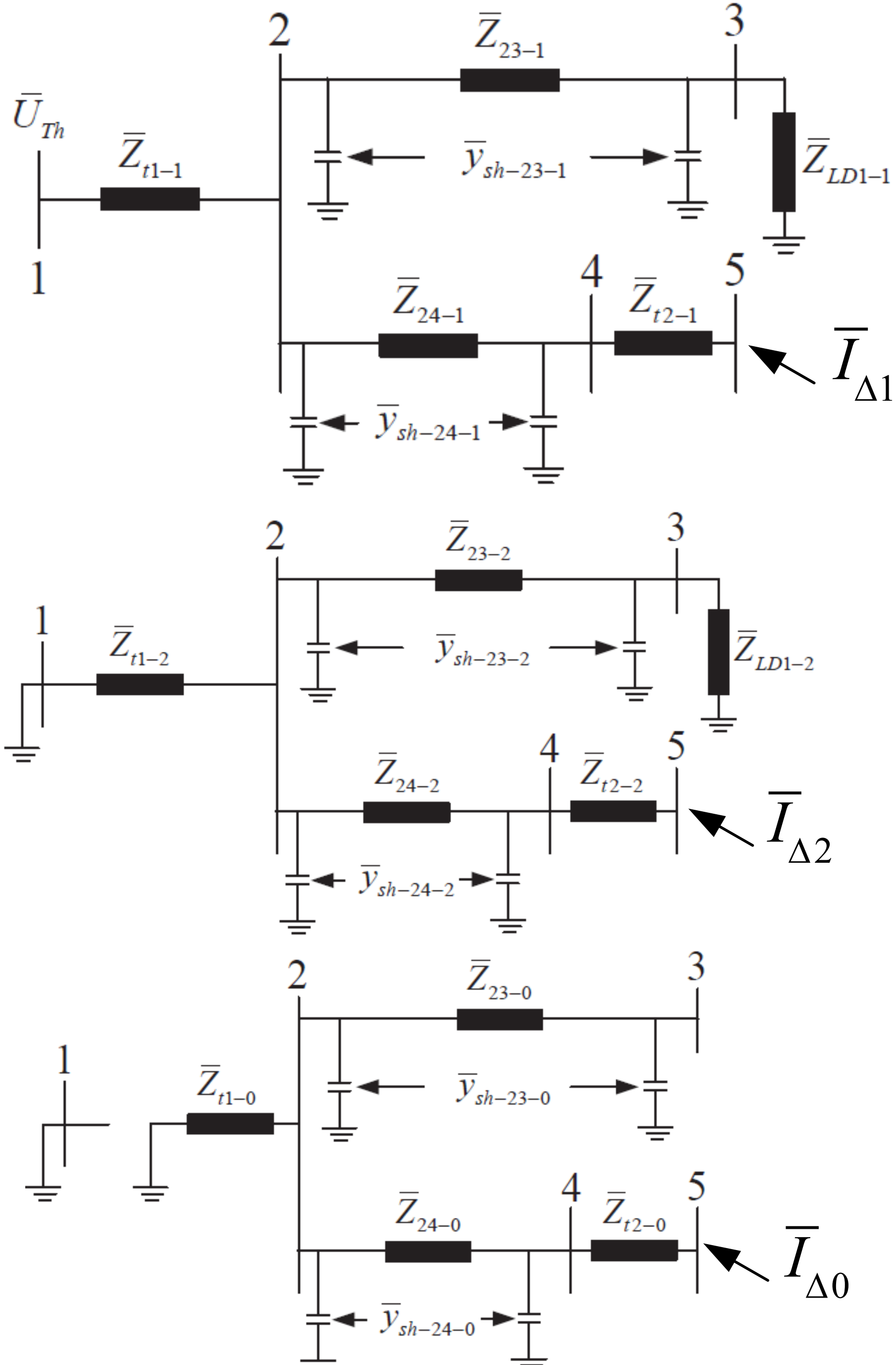
$$= \begin{bmatrix} 3.0083 + j0.9888 & 0.7521 + j0.2472 & 0.7521 + j0.2472 \\ 0.7521 + j0.2472 & 3.0083 + j0.9888 & 0.7521 + j0.2472 \\ 0.7521 + j0.2472 & 0.7521 + j0.2472 & 3.0083 + j0.9888 \end{bmatrix}$$

$$\mathbf{I_\Delta}(5) = -\left[\mathbf{Z_\Delta}(5,5) + \mathbf{Z_{LDbus5s}}\right]^{-1}\mathbf{U}(5) =$$

$$= \begin{bmatrix} 0.3315\angle 155.8442^\circ \\ 0.0653\angle -26.5244^\circ \\ 0.0653\angle -26.6221^\circ \end{bmatrix} = \begin{bmatrix} \mathbf{I_{\Delta 1}}(5) \\ \mathbf{I_{\Delta 2}}(5) \\ \mathbf{I_{\Delta 0}}(5) \end{bmatrix}$$

$$
\begin{aligned}
\mathbf{U'_1}(2) &= \mathbf{Z_1}(2,1)\cdot\overline{I}_{bus1-1}+\mathbf{Z_{\Delta 1}}(2,5)\mathbf{I_{\Delta 1}}(5)=0.9618\angle-3.1760^\circ\\
\mathbf{U'_2}(2) &= \mathbf{Z_{\Delta 2}}(2,5)\mathbf{I_{\Delta 2}}(5)=0.0028\angle 61.0623^\circ\\
\mathbf{U'_0}(2) &= \mathbf{Z_{\Delta 0}}(2,5)\mathbf{I_{\Delta 0}}(5)=0.0029\angle 63.3779^\circ\\
\mathbf{U'_1}(3) &= \mathbf{Z_1}(3,1)\cdot\overline{I}_{bus1-1}+\mathbf{Z_{\Delta 1}}(3,5)\mathbf{I_{\Delta 1}}(5)=0.9581\angle-3.2745^\circ\\
\mathbf{U'_2}(3) &= \mathbf{Z_{\Delta 2}}(3,5)\mathbf{I_{\Delta 2}}(5)=0.0028\angle 60.9638^\circ\\
\mathbf{U'_0}(3) &= \mathbf{Z_{\Delta 0}}(3,5)\mathbf{I_{\Delta 0}}(5)=0.0029\angle 63.3778^\circ\\
\mathbf{U'_1}(4) &= \mathbf{Z_1}(4,1)\cdot\overline{I}_{bus1-1}+\mathbf{Z_{\Delta 1}}(4,5)\mathbf{I_{\Delta 1}}(5)=0.9614\angle-3.1976^\circ\\
\mathbf{U'_2}(4) &= \mathbf{Z_{\Delta 2}}(4,5)\mathbf{I_{\Delta 2}}(5)=0.0029\angle 60.0356^\circ\\
\mathbf{U'_0}(4) &= \mathbf{Z_{\Delta 0}}(4,5)\mathbf{I_{\Delta 0}}(5)=0.0032\angle 60.3527^\circ\\
\mathbf{U'_1}(5) &= \mathbf{Z_1}(5,1)\cdot\overline{I}_{bus1-1}+\mathbf{Z_{\Delta 1}}(5,5)\mathbf{I_{\Delta 1}}(5)=0.9465\angle-5.6970^\circ\\
\mathbf{U'_2}(5) &= \mathbf{Z_{\Delta 2}}(5,5)\mathbf{I_{\Delta 2}}(5)=0.0116\angle 62.6242^\circ\\
\mathbf{U'_0}(5) &= \mathbf{Z_{\Delta 0}}(5,5)\mathbf{I_{\Delta 0}}(5)=0.0119\angle 62.5733^\circ
\end{aligned}
$$

Note that the element numbers given in the above equations are the bus numbers. However, for $\mathbf{Z_\Delta}$ matrices, since the row and column corresponding to the bus connected to the voltage source (in this example bus 1) are removed, $\mathbf{Z_\Delta}(k,r)$ corresponds to the element $\mathbf{Z_\Delta}(k-1,r-1)$, i.e. with $\mathbf{Z_\Delta}(2,5)$ it means the element $\mathbf{Z_\Delta}(1,4)$.

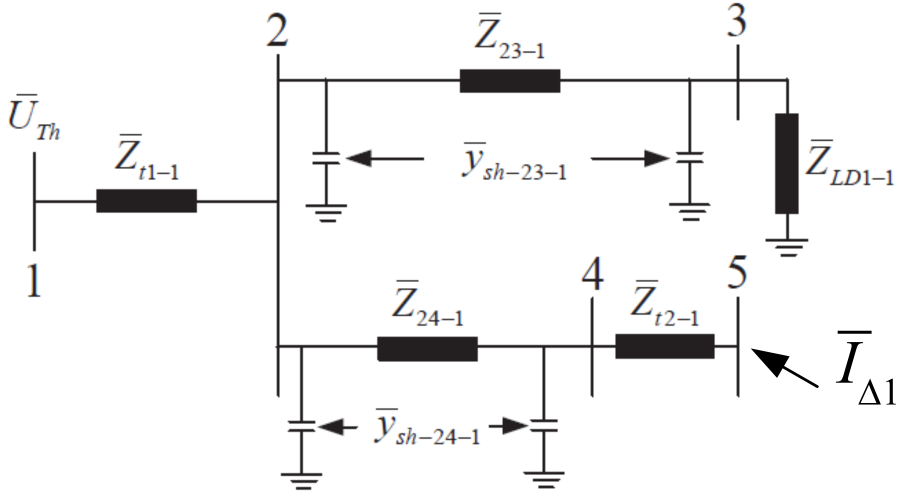


$$\overline{S}_{inj} =$$

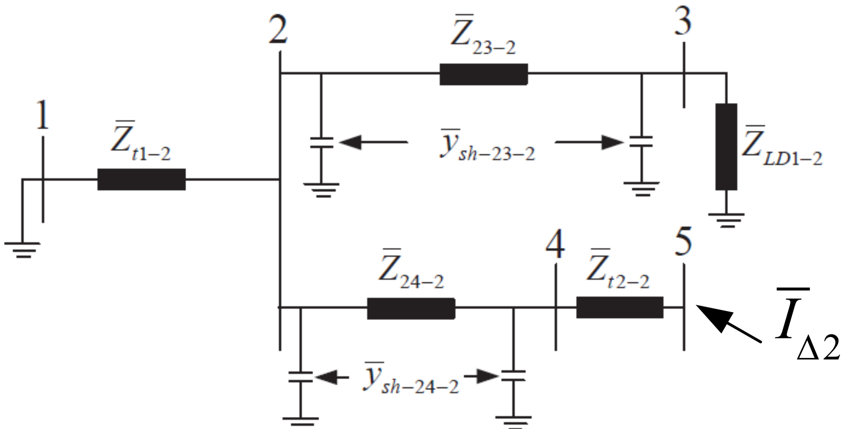
$$\mathbf{U}_{\mathbf{1}}'(2)\left(\frac{\overline{U}_{Th} - \mathbf{U}_{\mathbf{1}}'(2)}{\overline{Z}_{t1-1}}\right)^{*} +$$

$$+\,\mathbf{U}_{\mathbf{2}}'(2)\left(\frac{0 - \mathbf{U}_{\mathbf{2}}'(2)}{\overline{Z}_{t1-2}}\right)^{*} +$$

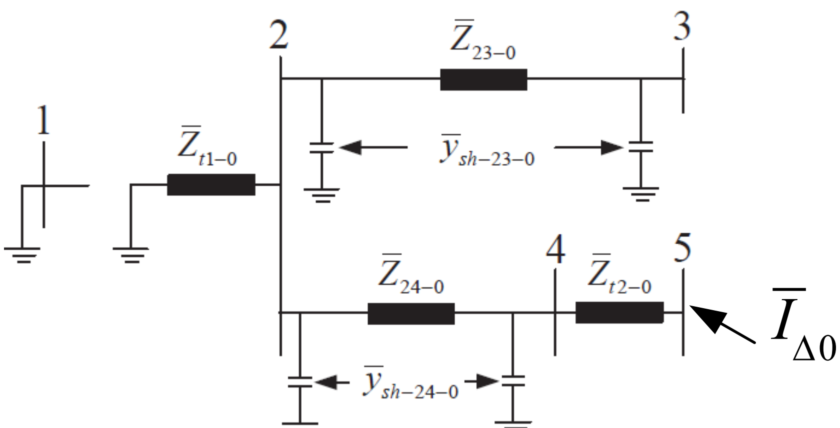
$$+\,\mathbf{U}_{\mathbf{0}}'(2)\left(\frac{0 - \mathbf{U}_{\mathbf{0}}'(2)}{\overline{Z}_{t1-0}}\right)^{*}$$



$$\overline{S}_{LD_{tot}} = \frac{|\mathbf{U}_1'(3)|^2}{\overline{Z}^*_{LD1-1}} + \mathbf{U}_1'(5)[-\mathbf{I}_{\Delta 1}(5)]^*$$



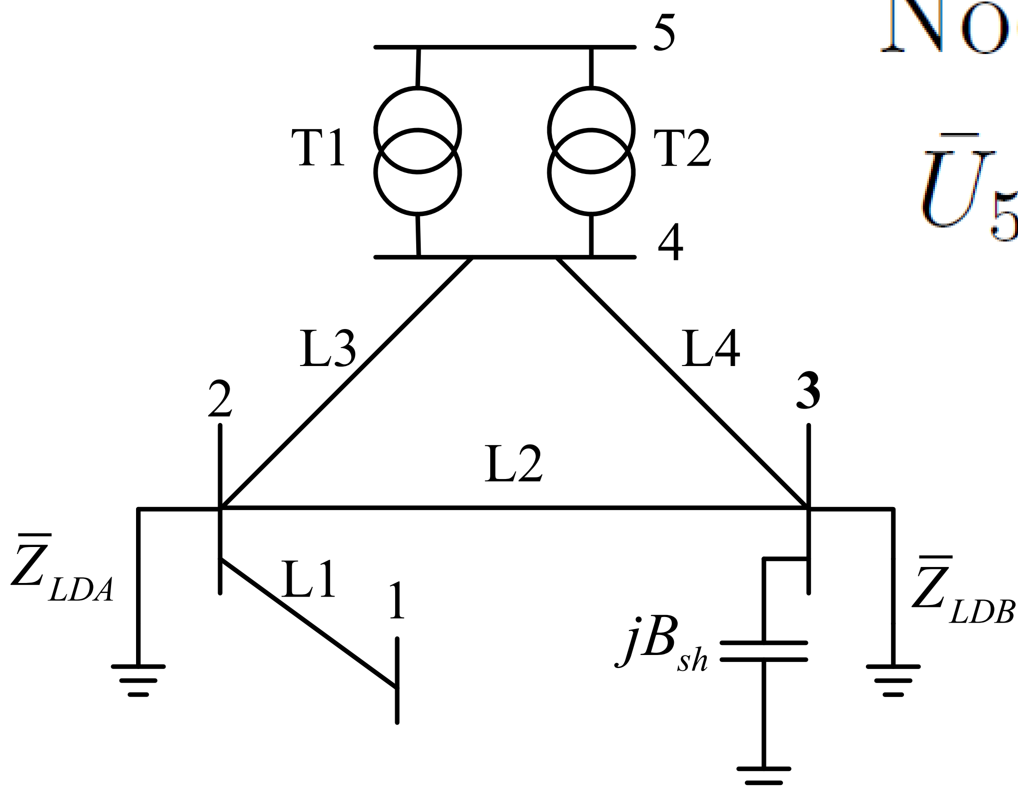
$$+\frac{|\mathbf{U}_2'(3)|^2}{\overline{Z}^*_{LD1-2}} + \mathbf{U}_2'(5)[-\mathbf{I}_{\Delta 2}(5)]^*$$



$$+0 + \mathbf{U}_0'(5)[-\mathbf{I}_{\Delta 0}(5)]^*$$

The efficiency can now be obtained as:

$$\eta = 100 \cdot \frac{Real(\overline{S}_{LD_{tot}})}{Real(\overline{S}_{inj})}\,\% = 99.7911\%$$



Node 5 strong grid

$$\bar{U}_5 = U_{n1} \text{ (kV)}$$

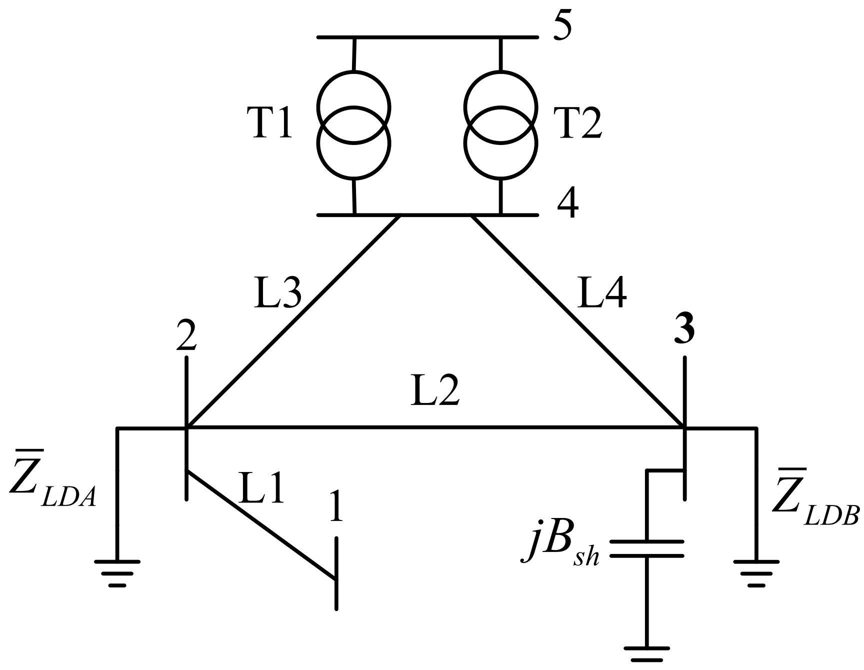
π-equivalents lines

$T1$: U_{n1}/U_{n2} (kV)/(kV), $x_k = \alpha\%$, $S_n = \beta$ (MVA), $Y_0 - Y$

$T2$: U_{n1}/U_{n2} (kV)/(kV), $x_k = \alpha\%$, $S_n = \beta$ (MVA), $Y_0 - Y_0$

LDA, LDB are Δ-connected

B_{sh} is Y-connected

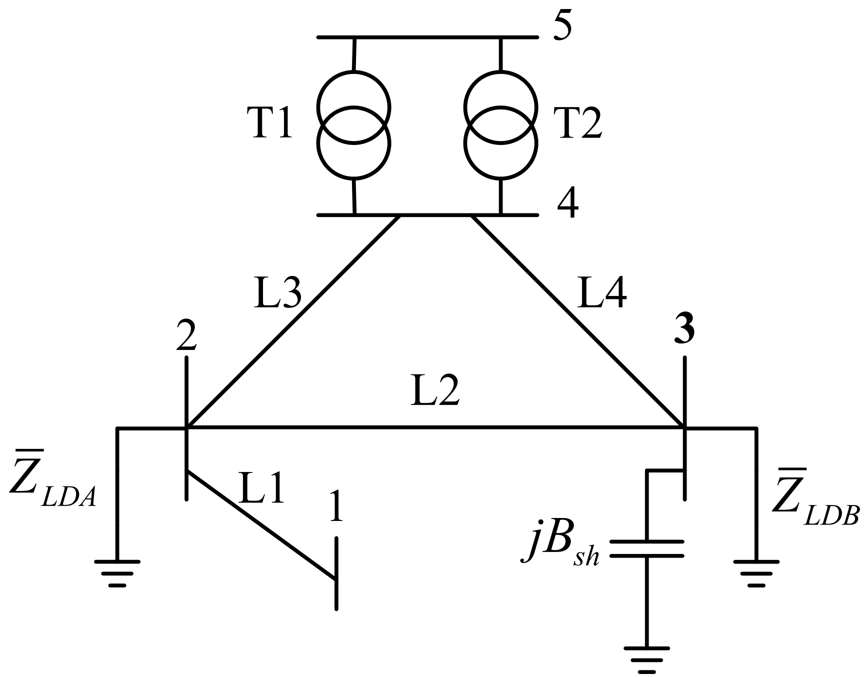


Base power:

S_{base} (MVA)

a) Assume that all positive-, negative- and zero-sequence impedances are known in (p.u). Give the diagonal elements of the admittance matrices

$$Y_1 \quad \text{and} \quad Y_{\Delta 0}$$



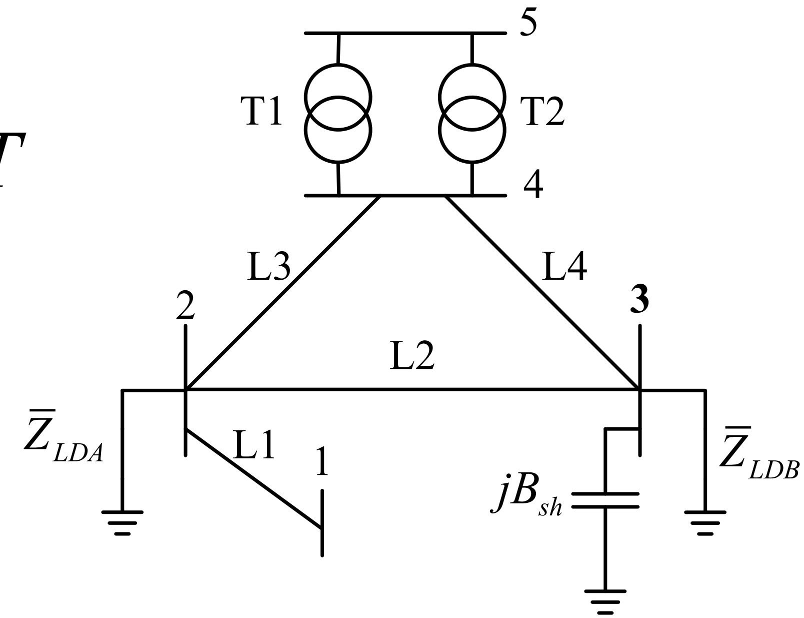
Base power:

S_{base} (MVA)

b) A single line-to-ground fault through a fault impedance $\bar{Z}_f$ occurs at Node 1 on phase a. Calculate the fault current in (kA).

c) Calculate the total active power losses in the system during the line-to-ground fault occurrence.

$$X_{T1} = X_{T2} = X_T$$



$$Y_1(1,1) = \frac{1}{\bar{Z}_{12-1}} + \bar{y}_{sh-12-1}$$

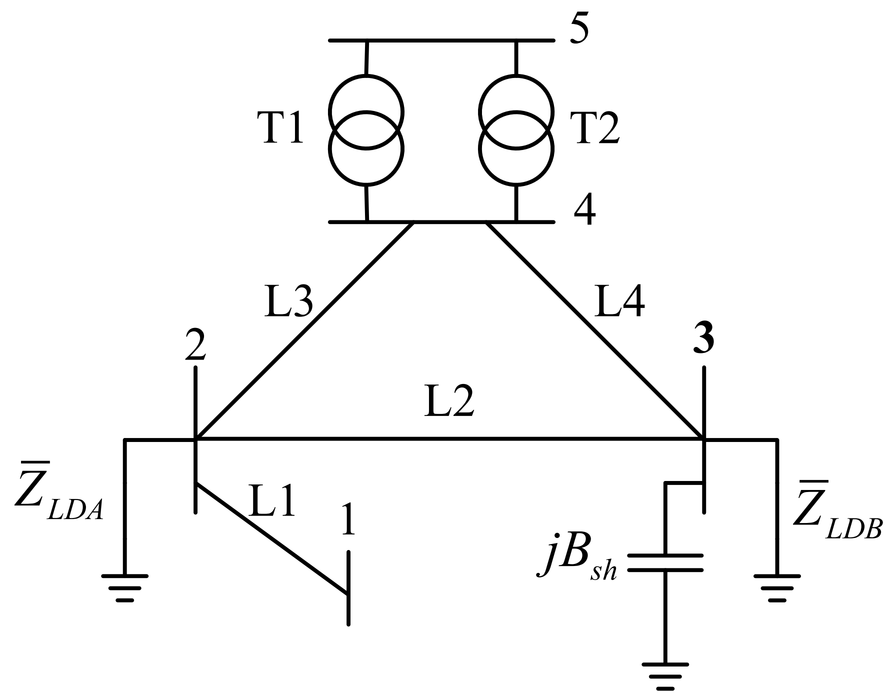
$$Y_1(2,2) = \frac{1}{\bar{Z}_{LDA}} + \frac{1}{\bar{Z}_{21-1}} + \frac{1}{\bar{Z}_{23-1}} + \frac{1}{\bar{Z}_{24-1}} + \bar{y}_{sh-21-1} + \bar{y}_{sh-23-1} + \bar{y}_{sh-24-1}$$

$$Y_1(3,3) = \frac{1}{\bar{Z}_{LDB}} + \frac{1}{\bar{Z}_{32-1}} + \frac{1}{\bar{Z}_{34-1}} + \bar{y}_{sh-32-1} + \bar{y}_{sh-34-1} + jB_{sh}$$

$$Y_1(4,4) = -j\frac{2}{X_T} + \frac{1}{\bar{Z}_{42-1}} + \frac{1}{\bar{Z}_{43-1}} + \bar{y}_{sh-42-1} + \bar{y}_{sh-43-1}$$

$$Y_1(5,5) = -j\frac{2}{X_T}$$

Zero-sequence network

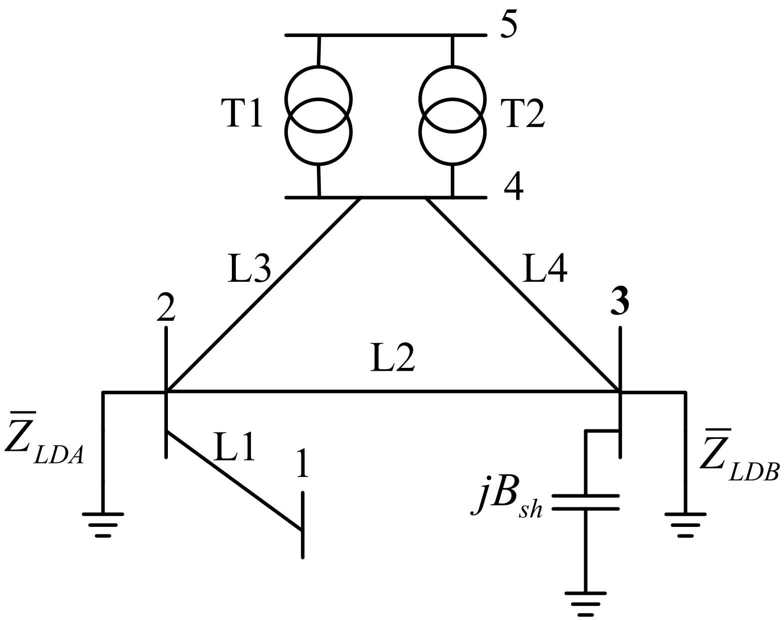


$$Y_{\Delta 0}(1,1)=\frac{1}{\bar{Z}_{12-0}}+\bar{y}_{sh-12-0}$$

$$Y_{\Delta 0}(2,2)=\frac{1}{\bar{Z}_{21-0}}+\frac{1}{\bar{Z}_{23-0}}+\frac{1}{\bar{Z}_{24-0}}+\bar{y}_{sh-21-0}+\bar{y}_{sh-23-0}+\bar{y}_{sh-24-0}$$

$$Y_{\Delta 0}(3,3)=\frac{1}{\bar{Z}_{32-0}}+\frac{1}{\bar{Z}_{34-0}}+\bar{y}_{sh-32-0}+\bar{y}_{sh-34-0}$$

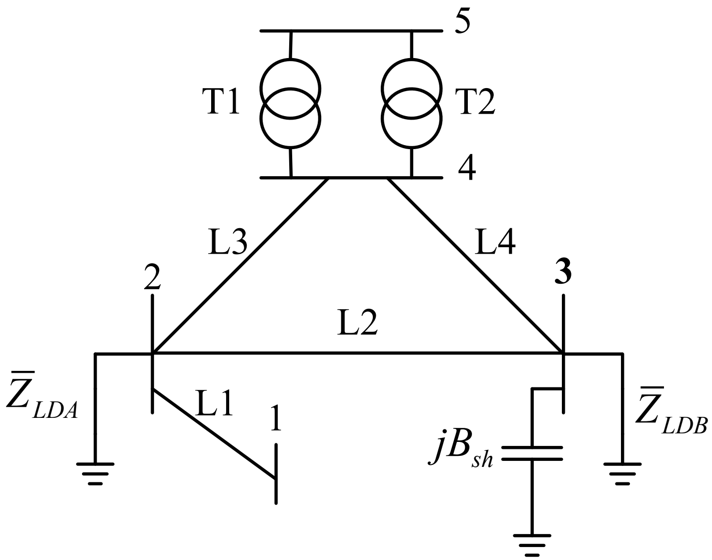
$$Y_{\Delta 0}(4,4)=\text{-j}\frac{1}{X_T}+\frac{1}{\bar{Z}_{42-0}}+\frac{1}{\bar{Z}_{43-0}}+\bar{y}_{sh-42-0}+\bar{y}_{sh-43-0}$$



$$\begin{bmatrix} \bar{I}_{bus1-1} \\ \bar{I}_{bus1-2} \\ \bar{I}_{bus1-0} \end{bmatrix} = T^{-1} \begin{bmatrix} \bar{I}_{fa} \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$\bar{I}_{bus1-1} = \bar{I}_{bus1-2} = \bar{I}_{bus1-0} = \frac{1}{3}\bar{I}_{fa}$$

$$\bar{I}_{fa} = \frac{3\bar{U}_{Thbus1}}{2\bar{Z}_{Thbus1-1} + \bar{Z}_{Thbus1-0} + 3\bar{Z}_f} I_{base_{n2}} \quad \text{(kA)}$$



$$\bar{U}_5 = \frac{U_{n1}}{U_{n1}} \text{ (p.u)}$$

$$\bar{U}_{Thbus1} = \frac{\bar{U}_5}{Z_1(5,5)} Z_1(1,5)$$

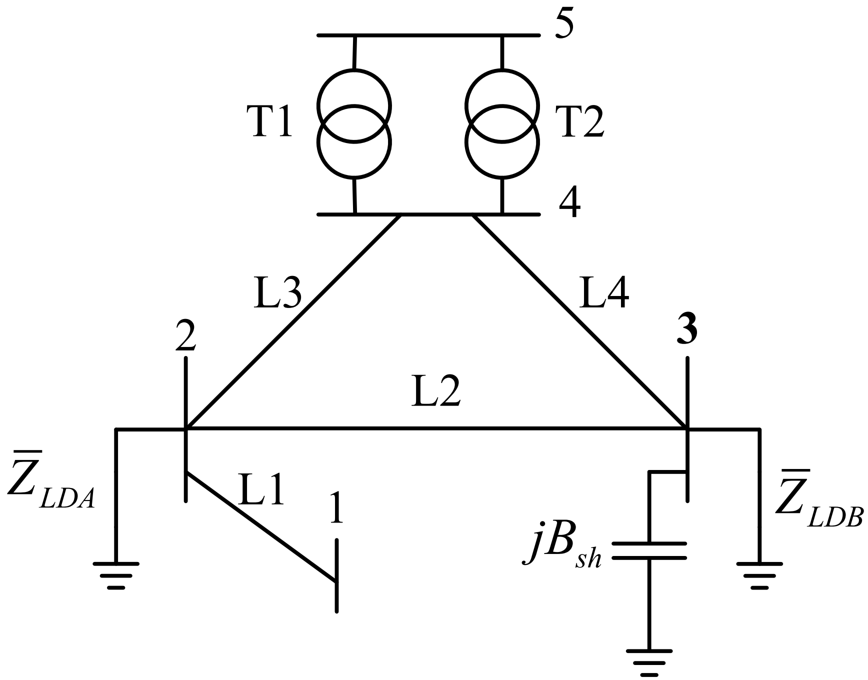
$$Z_1 = Y_1^{-1}$$

$$Y_{\Delta 1} = Y_1(1:4, 1:4)$$

$$Z_{\Delta 1} = Z_{\Delta 2} = Y_{\Delta 1}^{-1}$$

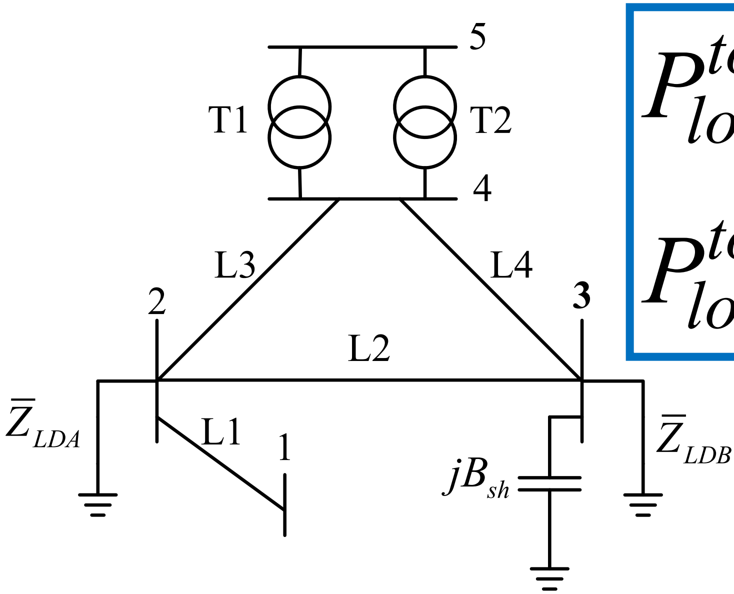
$$Z_{\Delta 0} = Y_{\Delta 0}^{-1}$$

$$\boxed{\begin{aligned} \bar{Z}_{Thbus1-1} &= Z_{\Delta 1}(1,1) \\ \bar{Z}_{Thbus1-0} &= Z_{\Delta 0}(1,1) \end{aligned}}$$



$$I_{base_{n2}} = \frac{S_{base}}{\sqrt{3}U_{n2}}$$

$$\bar{I}_{fa} = \frac{3\bar{U}_{Thbus1}}{2\bar{Z}_{Thbus1-1} + \bar{Z}_{Thbus1-0} + 3\bar{Z}_f} I_{base_{n2}} \quad \mathrm{(kA)}$$



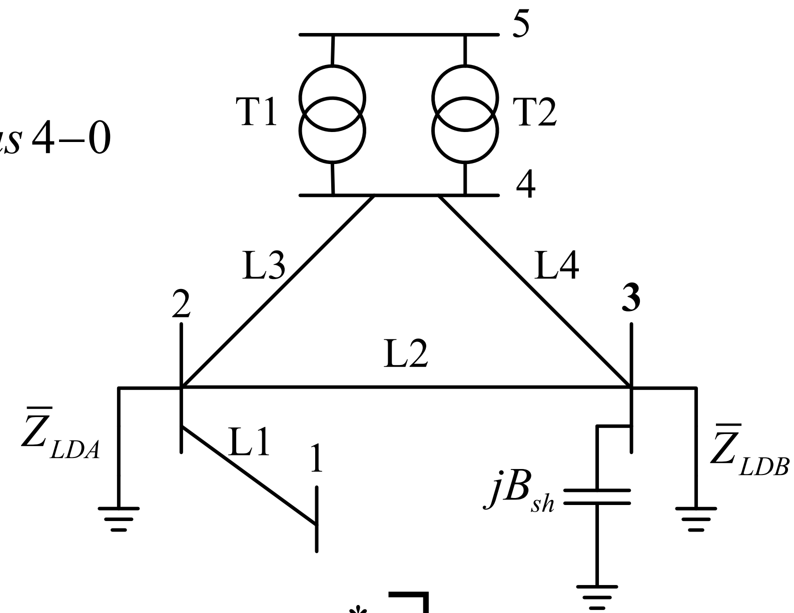
$$P_{losspu}^{tot} = \left(P_{bus4s} - P_{LDAs} - P_{LDBs} \right)$$

$$P_{loss}^{tot} = P_{losspu}^{tot} \, S_{base} \ (MW)$$

$$\begin{aligned}
\mathbf{U}'_{\mathbf{1}}(k) &= \mathbf{U}_{\mathbf{pre1}}(k) + \mathbf{Z}_{\mathbf{\Delta 1}}(k,r)\,\mathbf{I}_{\mathbf{\Delta 1}}(r) \\
\mathbf{U}'_{\mathbf{2}}(k) &= \mathbf{Z}_{\mathbf{\Delta 2}}(k,r)\,\mathbf{I}_{\mathbf{\Delta 2}}(r) \\
\mathbf{U}'_{\mathbf{0}}(k) &= \mathbf{Z}_{\mathbf{\Delta 0}}(k,r)\,\mathbf{I}_{\mathbf{\Delta 0}}(r)
\end{aligned}$$

$$\bar{I}_{bus1-1} = \bar{I}_{bus1-2} = \bar{I}_{bus1-0} = \frac{1}{3}\bar{I}_{fa}$$

$$P_{bus\,4\,s} = P_{bus\,4-1} + P_{bus\,4-2} + P_{bus\,4-0}$$



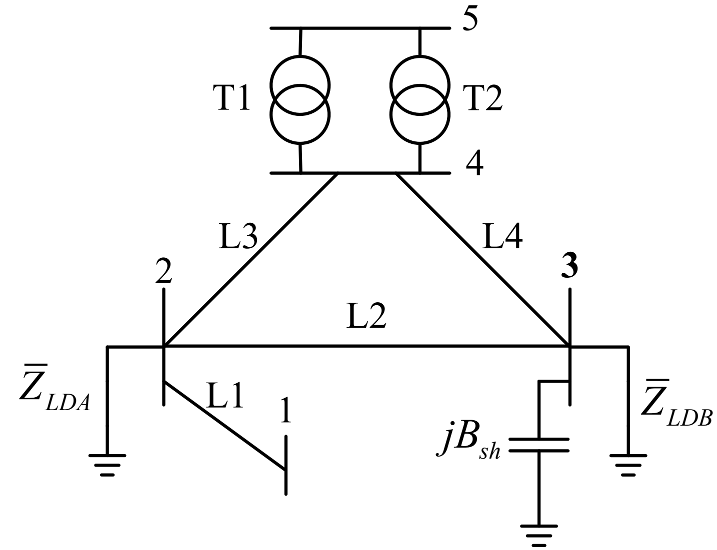
$$P_{bus\,4-1} = \mathrm{Re}\left[\bar{U}'_{bus\,4-1}\left(\frac{\bar{U}_5 - \bar{U}'_{bus\,4-1}}{j2X_T}\right)^*\right]$$

$$P_{bus\,4-2} = ???$$

$$P_{bus\,4-0} = ???$$

$$P_{bus\,4\,s} = P_{bus\,4-1} + 0 + 0$$

$$= \mathrm{Re}\left[\bar{U}'_{bus\,4-1} \left(\frac{\bar{U}_5 - \bar{U}'_{bus\,4-1}}{j2X_T} \right)^* \right]$$

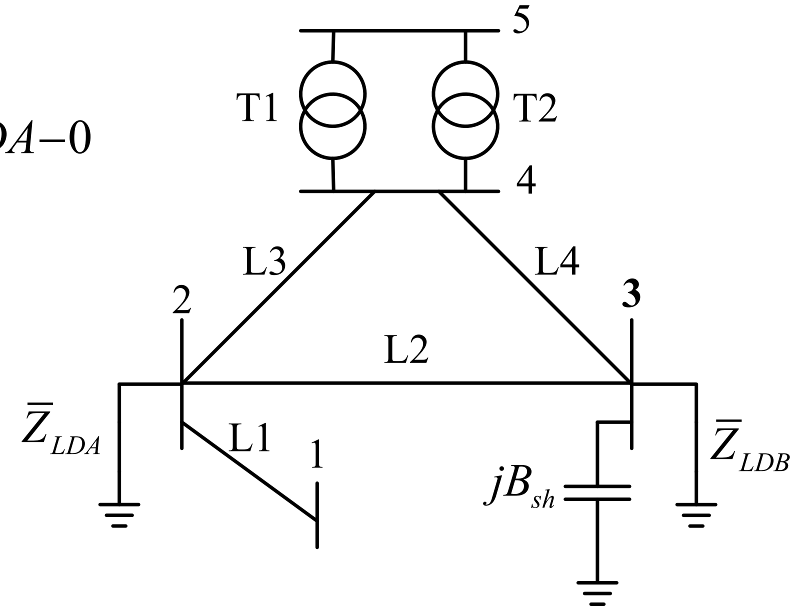


$$\bar{U}'_{Thbus\,4-1} = ???$$

$$\bar{U}'_{bus4-1} = \bar{U}_{Thbus4} - Z_{\Delta 1}(4,1)\,\bar{I}_{bus1-1}$$

$$\bar{U}_{Thbus\,4} = \frac{\bar{U}_5}{\mathrm{Z}_1(5,5)} \mathrm{Z}_1(4,5)$$

$$P_{LDAs} = P_{LDA-1} + P_{LDA-2} + P_{LDA-0}$$



$$P_{LDA-1} = \mathrm{Re}\left[\frac{\left(U'_{bus\,2-1}\right)^2}{\bar{Z}^*_{LDA}}\right]$$

$$P_{LDA-2} = \mathrm{Re}\left[\frac{\left(U'_{bus\,2-2}\right)^2}{\bar{Z}^*_{LDA}}\right]$$

$$P_{LDA-0} = 0$$

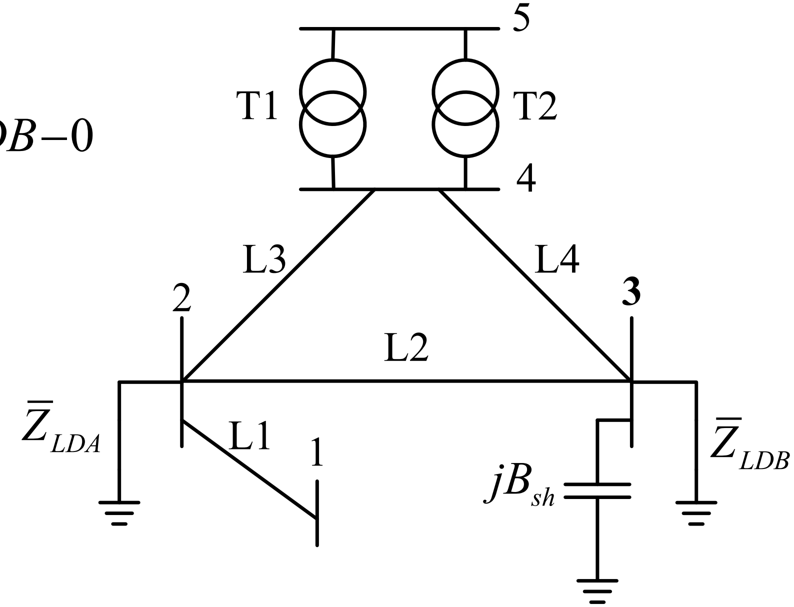
$$P_{LDA-1} = \mathrm{Re}\left[\frac{\left(U'_{bus\,2-1}\right)^2}{\bar{\bar{Z}}^*_{LDA}}\right] \qquad P_{LDA-2} = \mathrm{Re}\left[\frac{\left(U'_{bus\,2-2}\right)^2}{\bar{\bar{Z}}^*_{LDA}}\right]$$

$$\begin{bmatrix} \bar{U}'_{bus2-1} \\ \bar{U}'_{bus2-2} \\ \bar{U}'_{bus2-0} \end{bmatrix} = \begin{bmatrix} \bar{U}_{Thbus2} - Z_{\Delta 1}(2,1)\,\bar{I}_{bus1-1} \\ -Z_{\Delta 1}(2,1)\,\bar{I}_{bus1-2} \\ -Z_{\Delta 0}(2,1)\,\bar{I}_{bus1-0} \end{bmatrix}$$

$$\bar{U}_{Thbus2} = \frac{\bar{U}_5}{Z_1(5,5)} Z_1(2,5)$$

$$\boxed{P_{LDAs} = P_{LDA-1} + P_{LDA-2} + 0}$$

$$P_{LDBs} = P_{LDB-1} + P_{LDB-2} + P_{LDB-0}$$



$$P_{LDB-1} = \mathrm{Re}\left[\frac{\left(U'_{bus\,3-1}\right)^2}{\bar{Z}^*_{LDB}}\right]$$

$$P_{LDB-2} = \mathrm{Re}\left[\frac{\left(U'_{bus\,3-2}\right)^2}{\bar{Z}^*_{LDB}}\right]$$

$$P_{LDA-0} = 0$$

$$P_{LDB-1} = \mathrm{Re}\left[\frac{\left(U'_{bus\,3-1}\right)^2}{\bar{\bar{Z}}^*_{LDB}}\right] \qquad P_{LDB-2} = \mathrm{Re}\left[\frac{\left(U'_{bus\,3-2}\right)^2}{\bar{\bar{Z}}^*_{LDB}}\right]$$

$$\begin{bmatrix} \bar{U}'_{bus3-1} \\ \bar{U}'_{bus3-2} \\ \bar{U}'_{bus3-0} \end{bmatrix} = \begin{bmatrix} \bar{U}_{Thbus3} - Z_{\Delta 1}(3,1)\,\bar{I}_{bus1-1} \\ -Z_{\Delta 1}(3,1)\,\bar{I}_{bus1-2} \\ -Z_{\Delta 0}(3,1)\,\bar{I}_{bus1-0} \end{bmatrix}$$

$$\bar{U}_{Thbus3} = \frac{\bar{\bar{U}}_5}{\mathrm{Z}_1(5,5)}\mathrm{Z}_1(3,5)$$

$$\boxed{P_{LDBs} = P_{LDB-1} + P_{LDB-2} + 0}$$

$$\boxed{P_{loss}^{tot} = \left(P_{bus\,4s} - P_{LDAs} - P_{LDBs} \right) S_{base} \;\; (MW)}$$

$$P_{bus\,4\,s} = \mathrm{Re}\left[\bar{U}'_{bus\,4-1} \left(\frac{\bar{U}_5 - \bar{U}'_{bus\,4-1}}{j2X_T} \right)^* \right] + 0 + 0$$

$$P_{LDAs} = \mathrm{Re}\left[\frac{\left(U'_{bus\,2-1}\right)^2}{\bar{Z}^*_{LDA}} + \frac{\left(U'_{bus\,2-2}\right)^2}{\bar{Z}^*_{LDA}} \right] + 0$$

$$P_{LDBs} = \mathrm{Re}\left[\frac{\left(U'_{bus\,3-1}\right)^2}{\bar{Z}^*_{LDB}} + \frac{\left(U'_{bus\,3-2}\right)^2}{\bar{Z}^*_{LDB}} \right] + 0$$