Assignment-6

Parameter Estimation

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Q1) Consider a random sample $(x_1, x_2, x_3, \ldots, x_n)$

$\mu = \theta_1$ (mean) & $\sigma^2 = \theta_2$ (Variance)

Likelihood function: $L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-(x_i-\theta_1)^2/2.\theta_2}$

$$\Rightarrow \prod_{i=1}^{n} (\theta_2)^{-1/2} \cdot \prod_{i=1}^{n} (2\pi)^{-1/2} \cdot \prod_{i=1}^{n} e^{-(x_i-\theta_1)^2/2.\theta_2}$$

Taking log on both sides

$$z = \log(L(\theta_1, \theta_2)) = -\frac{n}{2}\log(\theta_2) - \frac{n}{2}\log(2\pi) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

Note: log here is natural log ($\log_e \Rightarrow \ln$)

Differentiate w.r.t θ_1

$$\frac{\partial z}{\partial \theta_1} = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i-\theta_1) = 0 \quad \left\{ \because \frac{\partial z}{\partial \theta_1} = 0 \right\}$$

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$$\Rightarrow \sum_{i=1}^{n} x_i - n\theta_1 = 0.$$

$$\Rightarrow n\theta_1 = \sum_{i=1}^{n} x_i \Rightarrow \theta_1 = \frac{\sum x_i}{n} = \underline{\underline{\bar{x}}}$$

$\therefore \theta_1 = \bar{x}$ is the sample mean.

Differenciate w.r.t θ_2 (Z w.r.t. θ_2)

$$\frac{\partial Z}{\partial \theta_2} = \sum_{i=1}^{n} \left[\frac{-1}{2\theta_2} + \frac{(x_i - \theta_1)^2}{2\theta_2^2} \right]$$

$$\Rightarrow \frac{n}{\cancel{2}\theta_2} = \frac{1}{\cancel{2}\theta_2^{\cancel{2}}} \cdot \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\Rightarrow \theta_2 = \frac{1}{n} \cdot \sum_{i=1}^{n} (x_i - \theta_1)^2 = \underline{\text{Variance}}.$$

$\therefore$ Sol^n: θ_1 is sample mean &
θ_2 is variance.
are Maximum likelihood estimates.

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Q2) Binomial distribution $B(n, \theta)$

$p = \theta; \ q = 1 - p = 1 - \theta$

$$f(x, n, \theta) = {}^nC_x \, \theta^x \, (1-\theta)^{n-x}$$

$$\Rightarrow L(\theta) = \prod_{i=1}^{n} {}^nC_{x_i} \, \theta^{x_i} \, (1-\theta)^{n-x_i}$$

Taking natural log on both sides;

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left[\ln {}^nC_{x_i} + x_i \ln\theta + (n-x_i)\ln(1-\theta) \right]$$

Differenciate ~~[illegible]~~ w.r.t. θ;

$$\frac{d \ln(L(\theta))}{d\theta} \Rightarrow \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right] = 0.$$

$$\Rightarrow \sum_{i=1}^{n} \left[x_i (1-\theta) - (n-x_i)\theta \right] = 0.$$

$$\Rightarrow \theta. \sum_{i=1}^{n} x_i - (n-x_i)\theta = 0$$

$$\Rightarrow \theta \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i . m$$

$$\Rightarrow \theta = \frac{\sum x_i}{n.m}$$

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