Mathematical Error Analysis and Feedback

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***Abstract*- The purpose of this machine learning study is to develop a new tool for error analysis and feedback in mathematics learning. Our objective is to create a robust model that can classify a variety of errors that pupils commonly do, including wrong factoring, erroneous sign usage, and incorrect quadratic formula application. To accomplish this, we assemble a sizable dataset of student responses to quadratic equation problems that include a wide range of mistakes. Using state-of-the-art machine learning methods like deep learning, we construct a classification model to precisely identify and categorise these errors. Cross-validation and testing on a variety of student submissions are used to carefully assess the model's performance in order to guarantee its dependability in spotting mistake patterns. Our project goes beyond error detection by including an intelligent feedback creation system. This method provides students with personalised assistance and insightful remarks based on the precise flaw identified in their solutions. By not only highlighting the inaccuracy but also outlining alternative approaches to problem-solving and step-by-step explanations, this feedback promotes a deeper understanding of quadratic equations.**

Keywords— Mathematical error analysis, quadratic equations, machine learning, error classification, feedback generation, educational technology.

I. Introduction

Mathematics forms the basis of logical reasoning and problem-solving skills. Mathematical competence is a prerequisite for academic and professional success across a range of fields. The cornerstone for comprehending complex algebraic concepts and their real-world applications is the fundamental mathematical concept of solving quadratic equations. However, students typically encounter problems and commit a variety of errors when attempting to solve quadratic equations, which limits their mathematical proficiency and confidence.

Teachers must recognize and correct these issues in order to successfully advise and assist pupils. Traditional methods of error detection and feedback generation can be time-consuming and challenging to scale in large classrooms. In this case, machine learning offers a workable solution. By developing a machine learning model that can classify different types of errors made by students when solving quadratic equations, we can speed up the mistake analysis process and give timely, customized feedback to enable enhanced learning outcomes.

II. Literature Survey

Mahmud *et al.* [1] recognised mistakes that students made when attempting to solve quadratic equation-related tasks. A diagnostic test and a semi-structured interview were used to find the faults, which were based on Newman Error Analysis. The main cause of the mistakes was a lack of knowledge of fundamental ideas and learning preferences. Learning mathematics is crucial for intellectual growth, and pupils should be able to use their knowledge in other courses after they have mastered the subject. Being unable to provide an accurate response to a problem is the cause of many mathematical errors.

The authors in [2], analysed the responses of 40 high school teachers to quadratic equation factoring mistakes made by their students. The findings showed a disconnect between errors that were recognised and those that were fixed, as well as a teacher-centered approach to conceptual justifications. Student mistakes are excellent learning opportunities that can promote a deeper and more thorough comprehension of mathematical topics. Along with diagnosing students' learning challenges, teachers also work to enhance their methods of instruction and create a variety of corrective measures. The study assesses whether there may be a connection between their interpretations, responses, and instructional strategies used. The authors in [3] The challenging suggested quadratic assignment problem (QAP) simulates a variety of actual issues with facility combined data analysis, parallel and distributed computing, and location. It entails calculating a minimum cost distribution of facilities among locations by adding up all potential distance-flow products. The QAP has made an appearance in a number of real-world applications since it was first presented as a mathematical model of economic activity. The ongoing interest in QAP for the problem's theory, applications, and solutions is the primary driver behind this survey. The article lists some of the most significant QAP formulations and categorises them based on where in mathematics they originate. It also talks about the theoretical tools that are used to specify bottom bounds for heuristic and exact algorithms. The paper concludes by analysing the contributions made by the investigation of various methodologies. Zakaria *et al.* [4] discussed about a study that looked at mistakes that students made when learning quadratic equations, particularly in the areas of factorization, completing the square, and the quadratic formula. 30 Form Three students from a secondary school in Jambi, Indonesia, participated in the survey method employed for the study. In order to better understand the difficulties pupils have when solving quadratic equations, research was conducted. According to the findings, most of the students' mistakes were transformation and process

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errors, particularly when factorization was being use. Additionally, comprehension mistakes were frequent, indicating a need for better knowledge of mathematical terms. The study emphasizes how crucial it is for teachers to share their research findings with one another and with students in order to address students' maths learning challenges. The research's important insights into students challenges in solving quadratic equations underscore the significance of instructor’s preparation and the exchange of educational resources for improving mathematics education. The Authors in [5] used both symbolic equations and word-problem representations to examine students' performance in solving quadratic equations with one unknown in the tenth grade. The study comes to the conclusion that the structural variations between these representations have a big impact on how well students perform. Additionally, Vaiyavutjamai and Clements (2006) are used in the study to emphasize how students' problems with quadratic equations are the result of their lack of knowledge. The study's goals were to investigate students' abilities and challenges when resolving quadratic equations in symbolic and word problem formats, adding to the body of knowledge on this subject. Results show that just 10% of the 217 students correctly answered all problems involving symbolic equations. The range of correct solution percentages was 25.8% to 80.6%, with Q1 and Q4 receiving the highest and lowest marks, respectively. Notably, Q4 had a low attempt rate of approximately one-third of pupils, while Q3 had a low percentage of correct answers. But more than half of the pupils properly responded to questions 2, 6, 7, and 8. The study's findings show that structural aspects including factorization, completing the square, and the quadratic formula affected how well students solved symbolic equations. Due to its simplicity, rational roots, and increased practice, Q1 witnessed strong performance, however Q4 presented difficulties due to its non-factorable structure, requiring sophisticated algebraic manipulations and arithmetic operations with rational and radical numbers.

The authors in [6] The performance of a Year 11 Mathematics B class in a coeducational high school in south-east Queensland was examined using case study technique. The study adhered to grounded theory concepts and sought to advance theory using actual data. The study's main conclusions are as follows: Students have significant levels of procedural and conceptual misunderstandings. Students prefer utilising number lines over fractions, which may be related to the fact that calculators are widely used in Queensland classrooms. The difficulty in solving quadratic equations was connected to a lack of fluency in fraction computation. In conclusion, this study confirms findings from earlier literature that student comprehension of quadratic equations is poor. It sheds light on why this is the case, especially in the context of one Queenslandian school. Pradanov at el. [7] the classification of roots in terms of a, b, c, and d, and a two-tier analysis of the real roots of the general quartic equation x4 + ax3 + bx2 + cx + d = 0 with real coefficients. It organises and establishes the boundaries and isolation intervals of each root using auxiliary quadratic equations. The approach is especially helpful when the equation is derived from a model and it is challenging to solve cubic equations. In Luca Pacioli's Summa de Arithmetica, Geometria, Proportioni, and Proportionalità from 1494, quartic equations are first mentioned in print. Later in the Renaissance, Girolamo Cardano produced Artis Magnae Sive de Regulis Algebraicis (1545), a significant book in the history of mathematics. The article uses a Pythagorean analogy between intersection points and parallel lines to relate the intersection points of the sub-quartic equation with the musical notes and staves that denote distinct pitches. At Technological University Dublin's School of Mathematical Sciences, the article is delivered. The Authors in [8] this study looked into the mistakes and misunderstandings grade 11 students made when attempting to solve quadratic equations in a high school in South Africa's Gauteng Province. Data collection strategies included both qualitative and quantitative approaches. Six students who had script faults were subjected to semi-structured interviews. According to the study, students' inability to answer quadratic equations was hampered by their lack of algebraic proficiency. The researchers advise future investigation to ascertain the effects of making the detected mistakes and misunderstandings available as teaching tools. The Authors in [9] discussed the study of students' mistakes when using Newman's Procedure to solve quadratic equations is covered in this article. The goal of the study is to pinpoint the mistakes that students commonly make as well as the causes of these mistakes. In Malaysia, 30 Form Four pupils underwent a diagnostic test and semi-structured interviews. The results demonstrate that while few students make encoding errors, the majority of students make comprehension and transformation errors. These mistakes were primarily caused by a lack of knowledge of fundamental ideas and learning preferences. The importance of correcting these mistakes is emphasized in the essay in order to enhance pupils' math learning.

III. METHODOLOGY

In this , we assessed a kNN classifier's performance using the dataset. To assess the model, we employed a 10-fold cross-validation method. The kNN classifier was trained on the training folds for each fold, and its effectiveness was assessed on the test fold. Over all folds, we calculated the average for accuracy, precision, recall,and F1 score.

Additionally, we assessed the effectiveness of the kNN classifier using k values ranging from 1 to 11. For each value of k, we determined the kNN classifier's accuracy and plotted the results on a graph.

The selection of the k parameter, however, also affects the performance of the kNN classifier. Underfitting can result from a k value that is too large, while overfitting can result from a k value that is too little. The value of k should be carefully selected for each dataset.

The kNN classifier, in general, is an excellent option for classification jobs where the classes are clearly distinct and the dataset is modest. It is also a wise decision for projects where the dataset is dynamic and new data points are routinely introduced.

Recursively partitioning the data into ever-smaller subsets based on the values of the features is the way the Decision Tree classifier is trained. When a maximum depth is reached or when all samples in a subset belong to the same class, the algorithm stops splitting.

The Decision Tree classifier's hyperparameters include:

max_depth: The tree's maximum depth. A deeper tree may

reveal more complicated data patterns, but it also has a higher propensity to overfit the training set.
criterion:
min_samples_split: the bare minimum of samples needed to divide a node. An increased value may aid in avoiding overfitting.
min_samples_leaf: The minimum number of samples that must be in a leaf node. A greater value might aid in avoiding underfitting.

An ensemble learning algorithm called the Random Forest classifier combines the predictions from different Decision Tree classifiers. On various subsets of the training data, numerous Decision Trees are constructed, and their predictions are then averaged. This minimises overfitting and strengthens the classifier's accuracy.

- n_estimators: The number of decision trees are present in the forest.
- max_depth: The maximum depth of each Decision Tree in the forest.
- criterion: The standard that each decision tree in the forest uses to divide a node into two child nodes.
- bootstrap: Whether to bootstrap the training data when building each Decision Tree in the forest.
- max_features: The quantity of features that each decision tree in the forest will take into account before splitting a node.

The performance of the classifiers is evaluated using Accuracy and Confusion matrix.

Pruning is a method used to stop decision trees from overfitting. It operates by pruning tree branches that are unhelpful for the prediction goal. A variety of metrics, such the information gain or the Gini index, can be used to do this.
Your decision tree model's performance on the test set can be enhanced via pruning. Pruning, it's vital to remember, might also make the model less accurate on the training data. This is because pruning cleans up the tree by removing information, which may make it harder for the model to pick up on the patterns in the training set.
In decision trees, pruning can be used to prevent overfitting by performing the actions listed below:

- Without any restrictions on pruning, train a decision tree.
- To prune the tree, eliminate branches using an algorithm.
- On a validation set, gauge how well the pruned tree performed.
- In order to find a pruned tree that performs well on the validation set, repeat steps 2 and 3 as necessary.

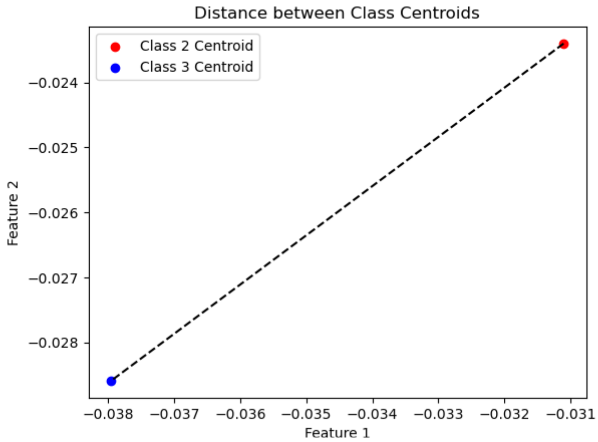
Numerous pruning techniques are available, including cost complexity pruning (CCP) and reduced error pruning (REP). REP cuts down branches that don't increase the training set tree's accuracy. Branches with a high cost-complexity ratio are eliminated by CCP.

IV. Result

- On the dataset, the kNN classifier has an average accuracy of 99%. This indicates that 99% of the test set's data points could be accurately classified by the kNN classifier.
- The average F1-score, recall, and precision for all three courses were 98% or higher. This shows that both favorable and unfavorable examples of each class were accurately identified by the kNN classifier.
- The kNN classifier is a straightforward and simple to use classifier. Anomaly detection, regression, and classification are just a few of the tasks that this classifier may be applied to. It is also quite flexible.

The dataset's classes appear to be clearly segregated based on the kNN classifier's good accuracy, precision, recall, and F1-score results. This is due to the fact that the kNN classifier can precisely identify both good and bad examples of each class.

One indicator of class separability is the distance between class centroids. It is not always a valid metric, though, particularly when there are several outliers or when the data is not regularly distributed.
For instance, take a look at the illustration below:



Although the two classes in this diagram have different centroids, they nonetheless overlap. This indicates thatcertain data points from one class are located more closely to the centroid of another class than to the centroid of their own. The separation of the class centroids in this instance would not be a reliable indicator of class separability.
The margin is a more accurate indicator of class separability. The margin is the separation of any two data points from distinct classes that is the smallest possible. The classes are clearly distinguished if the margin is big. Classes are not clearly distinguished if the margin is tiny.

The following explanation explains how the kNN classifier behaves as k increases:
- The kNN classifier becomes more sensitive to data noise as k rises. This is due to the fact that the kNN classifier will

take into account more data points when making a prediction, some of which may be outliers.

• The kNN classifier likewise becomes less biased towards any certain class as k rises. This is because more data points from various classes will be taken into account by the kNN classifier when making a prediction.

In kNN, overfitting happens when the value of k is too low. This is due to the fact that the kNN classifier only takes into account a small number of data points when producing a prediction, and these data points could not be representative of the full population.

In kNN, underfitting happens when the value of k is too high. This is due to the fact that the kNN classifier will take into account a substantial amount of data points when producing a prediction, some of which may be outliers.

Based on the results from numerous measures, the kNN classifier is indeed a solid one. On the Iris dataset, the kNN classifier had an average accuracy of 99%, and all three classes had average precision, recall, and F1 scores of 98% or above.

Yes, the model fits the data fairly well. This is due to the model's excellent accuracy and low false positive and false negative rate on both the training and test sets of data. A circumstance where the model can generalise to new data effectively is referred to as a regular fit situation. This indicates that the model can recognise the underlying patterns in the data and apply them to brand-new data points that it has never encountered before.

The kNN classifier's strong recall, accuracy, and precision on the Iris dataset along with its high F1-score indicate that the model generalises effectively to new data. This is a strong hint that the model has a situation of regular fit.

A kNN classifier experiences overfitting when the value of k is too low. This is due to the fact that the kNN classifier only takes into account a small number of data points when producing a prediction, and these data points could not be representative of the full population.

The kNN classifier is more likely to learn the noise in the training data than the underlying patterns when k is too small. This may prevent the model from generalising well to fresh data. It is crucial to select a value of k in a kNN classifier that is large enough to ensure that the model can learn the underlying patterns in the data but not so large that the model can also learn the noise in the data in order to prevent overfitting.

In conclusion, the kNN classifier performs effectively on datasets with clearly defined classes. Additionally, the kNN classifier is rather simple to train and fine-tune. To prevent overfitting and underfitting, it's crucial to pick the appropriate value for k.

The model's performance is good, with accuracy of 0.7542147293700089, precision of 0.74, recall of 0.77, F1-score of 0.75, and AUROC of 0.79, indicating that the model is able to learn the patterns in the training data and apply them to new data. However, it is important to note that the model may be overfitting the training data, as the accuracy on the training set is higher than the accuracy on the test set.

We can do the following to avoid overfitting:

- Use a more compact practise set.
- Use a model that is easier to understand, like a shallower-depth decision tree.
- Consider using an L1 or L2 regularisation approach.
- Utilise an early halting strategy.
- As the model gets more complicated, we can anticipate a decline in accuracy on the test set if the model is overfitting.

V. CONCLUSION

The kNN classifier is a versatile tool that may be used for classification, regression, and anomaly detection. It is a straightforward and efficient classifier. It works especially effectively with datasets that have distinct class boundaries.

On the dataset, the kNN classifier attained good precision, recall, and F1-scores for all three classes, as well as an average accuracy of 99%. This shows that the model can recognise patterns in the data and successfully generalise to new data.

It is crucial to remember that the kNN classifier might be overfitting, particularly when the value of k is too low. Selecting a value of k that is large enough to ensure that the model can learn the underlying patterns in the data while remaining within a reasonable range can help prevent overfitting.

The kNN classifier, in general, is a strong and adaptable classifier that can be applied to a number of applications. It works especially effectively with datasets that have distinct class boundaries.

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