Function Approximation (Regression)

Evaluation

$\hat{y} = f(x)$

$f(x) = w^T \text{Basis}$

$\phi = [x_1, x_2, \ldots, x_d]$

Example:

$\hat{y} = \frac{1}{2}x_1 - x_2 + \ldots + x_d$

$w = \text{vec of weights}$

Extrinsic: G

$\hat{y} = f(G(x)) \quad y = G(x)$

$G = [x_1, \ldots, x_{784}]$

y: w.r.t. $\quad$ Change this

$\text{Model}_{\text{Extrinsic}} = \text{Model}_{\text{Intrinsic}} \circ G$

$\hat{y}_1 = \frac{1}{2}x_1 - x_2 + \ldots$

$\hat{y}_2 = \frac{1}{2}x_1 - x_2 + \ldots$

$\hat{y}_3 = \frac{1}{2}x_1 - x_2 + \ldots$

$\hat{y}_4 = \frac{1}{2}x_1 - x_2 + \ldots$

$\hat{y}_5 = \frac{1}{2}x_1 - x_2 + \ldots$

$\hat{y}_6 = \frac{1}{2}x_1 - x_2 + \ldots$

Hyperplane $w_1 x_1 + w_2 x_2 + \ldots + w_{784} x_{784}$

Linear regression.

$y = w_1 x_1 + w_2 x_2 + \ldots + w_{784} x_{784}$

Binary model for regression:

$y \in$ discrete values to predict

$\hat{y} \in$ continuously valued $\mathbb{R}$

Hindsight with MNIST:

$y \in$ continuously valued $\mathbb{R}$ or $\mathbb{Z}$

Polynomial curve fitting:

$x \in \mathbb{R} \quad y \in \mathbb{R}(x)$

Polynomial regression degree M

$\hat{y} = w_0 + w_1 x + w_2 x^2$

$\hat{y} = w_0 + w_1 x + w_2 x^2 + \ldots + w_M x^M$

Polynomial regression

$x \in \mathbb{R} \quad y : \mathbb{R}(x)$

$x \in \mathbb{R} \quad y : \mathbb{R}(x, x^2)$

Note: $\hat{y} = w_0 + w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4 + w_5 x^5$

Not: $\hat{y} = w_0 + w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4 + w_5 x^5 + w_6 x^6$

$\text{SE} = (\hat{y} - y)^2$ $\quad$ Squared error

$y = 0.5 \quad \hat{y} = 0.1$

$\text{Minimize}_{\text{Model}} \left((0.5 - 0.1)^2 \right)$

$\hat{y} = \frac{\text{Model}}{\text{Model}}$

$\hat{y} = \text{Model}$

$y = w_0 + w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4 + w_5 x^5 + w_6 x^6 + w_7 x^7 + w_8 x^8 + w_9 x^9$

$\hat{y} = w_0 + w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4 + w_5 x^5 + w_6 x^6 + w_7 x^7 + w_8 x^8 + w_9 x^9 + w_{10} x^{10}$

$\hat{y} = \{w_0, w_1, \ldots, w_{10}\}$

$\hat{y}(x) = \{x, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$

$\hat{y} = \{w_0, w_1, \ldots, w_{10}\}$

Training model:

$\text{Model}_{\text{Intrinsic}} = \text{Model}_{\text{Extrinsic}} \circ G$

$\hat{y} = \text{Model}_{\text{Intrinsic}}(x)$