CS5691 Assignment 1: Regression Models with Polynomial and Gaussian Basis Functions

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Abstract

This report presents polynomial and Gaussian basis function regression models applied to three datasets (univariate, bivariate, and multivariate). Model complexity, regularization, and basis function choices are explored using training, validation, and test datasets. Results are evaluated in terms of ERMS and visualized using curves, surfaces, and scatter plots.

1 dataset1-polynomial regression

1.1 training10

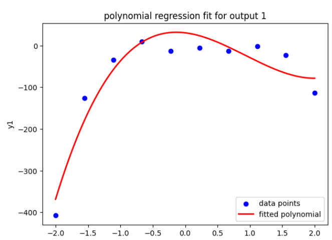

Figure 1: N=10,deg = 3, $\lambda = 0$, $E_{rms} = 35$

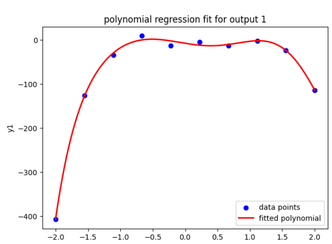

Figure 3: N=10,deg=7,λ=0,$E_{rms} = 5$

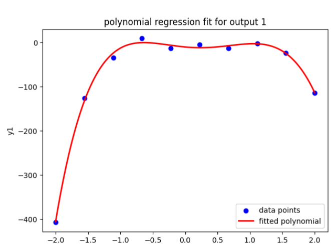

Figure 2: N=10,deg=5,λ=0,$E_{rms} = 6$

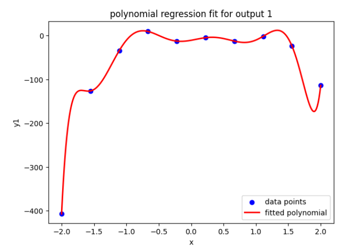

Figure 4: N=10,deg=9,λ=0,$E_{rms} = 0$

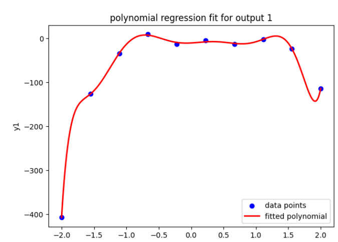

Figure 5: N=10,deg=9,λ=0.001,$E_{rms} = 1.66$

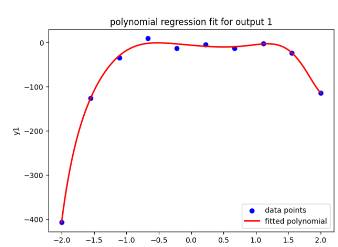

Figure 6: N=10,deg=9,λ=0.1,$E_{rms} = 5.15$

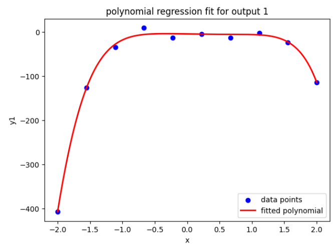

Figure 7: N=10,deg=9,λ=1,E_{rms} = 6.57

1.2 training50

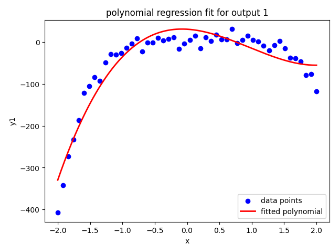

Figure 8: N=50,deg=3,λ=0,E_{rms} = 28.29

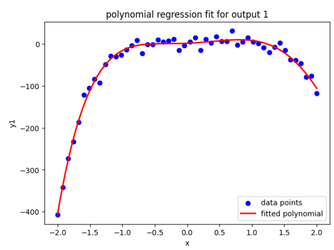

Figure 9: N=50,deg=5,λ=0,E_{rms} = 10.37

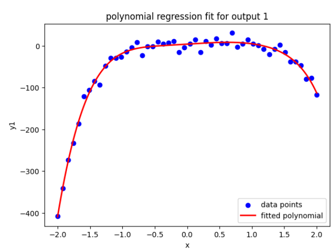

Figure 10: N=50,deg=7,λ=0,E_{rms} = 10.11

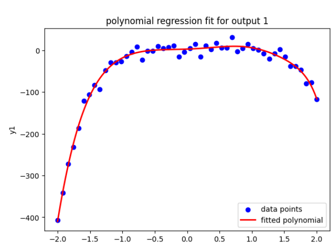

Figure 11: N=50,deg=9,λ=0,E_{rms} = 10.03

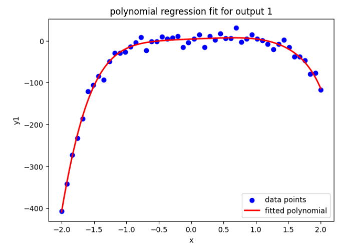

Figure 12: N=50,deg=9,λ=1,E_{rms} = 10.03, no overfitting here, no point using λ and there's practically no change in E_{rms}

1.3 validation

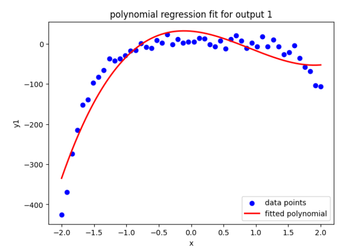

Figure 13: N=50,deg=3,λ=0,E_{rms} = 31.19

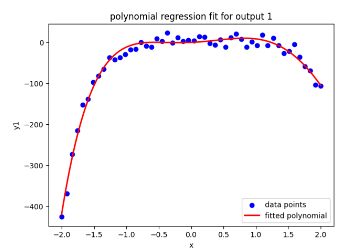

Figure 14: N=50,deg=5,λ=0,E_{rms} = 11.90

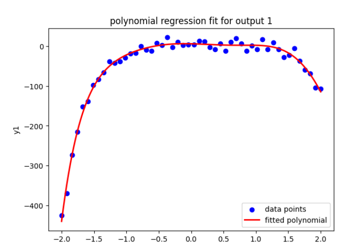

Figure 15: N=50,deg=7,λ=0,E_{rms} = 9.7

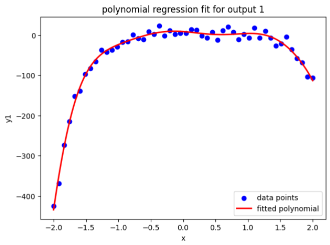

Figure 16: N=50,deg=9,λ=0,E_{rms} = 9.5

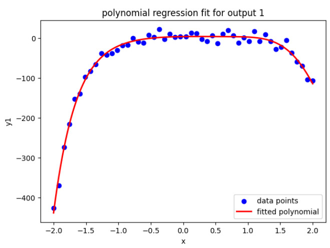

Figure 17: N=50,deg=9,λ=1,E_{rms} = 9.95, here regularization is making the error worse since there is no overfitting, we will not use λ

1.3.1 results

Table 1: E_{rms} values for Dataset 1, Polynomial Regression

Model	Train10	Train50	Val	Test
M=3, $\lambda = 0.001$	34.79	28.30	31.19	32.16
M=3, $\lambda = 0.1$	34.81	28.30	31.19	32.16
M=3, $\lambda = 1$	35.64	28.35	31.25	32.20
M=5, $\lambda = 0.001$	6.02	10.37	11.90	13.37
M=5, $\lambda = 0.1$	6.14	10.37	11.90	13.37
M=5, $\lambda = 1$	8.19	10.42	11.97	13.45
M=7, $\lambda = 0.001$	5.31	10.11	9.78	12.13
M=7, $\lambda = 0.1$	5.83	10.11	9.79	12.13
M=7, $\lambda = 1$	6.55	10.15	9.96	12.17
M=9, $\lambda = 0.001$	1.67	10.04	9.54	11.71
M=9, $\lambda = 0.1$	5.16	10.05	9.67	11.87
M=9, $\lambda = 1$	6.57	10.12	9.95	12.08

we choose the model with M=9 and $\lambda = 0$ as the best model. we now plot its performance on the test data

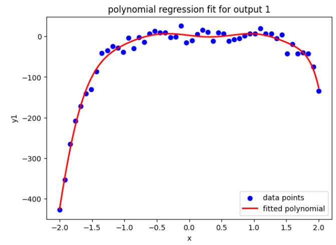

Figure 18: M=9 and $\lambda = 0$ on the test data, $E_{rms} = 11.71$

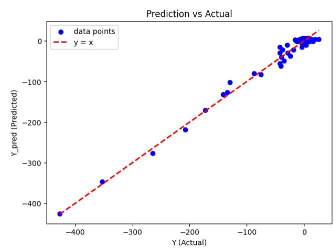

Figure 19: Enter Caption

2 dataset1-gaussian regression

2.1 train10

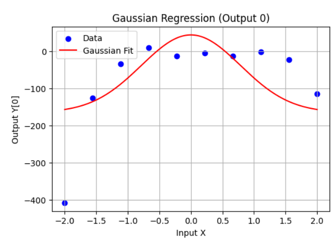

Figure 20: $E_{rms} = 96.12$

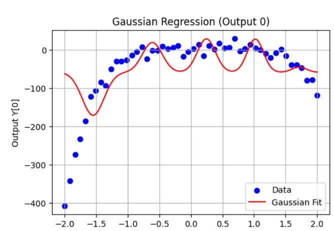

Figure 21: $E_{rms} = 78$

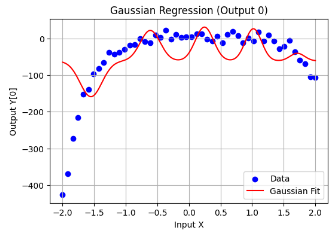

Figure 22: $E_{rms} = 81$

2.2 validation

2.3 test

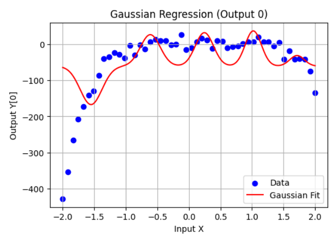

Figure 23: $E_{rms} = 79$, on par with the results from the validation set

3 dataset2-polynomial

3.1 train25

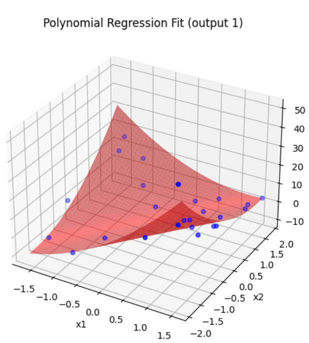

Figure 24: N=25,deg=2,$\lambda = 0$, $E_{rms} = 3.19$

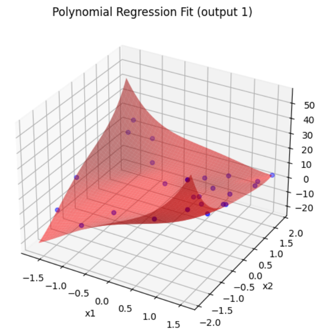

Figure 25: N=25,deg=4,$\lambda = 0$, $E_{rms} = 0.07$

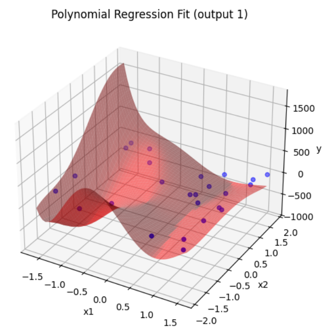

Figure 26: N=25,deg=6,$\lambda = 0$, $E_{rms} = 258.55$

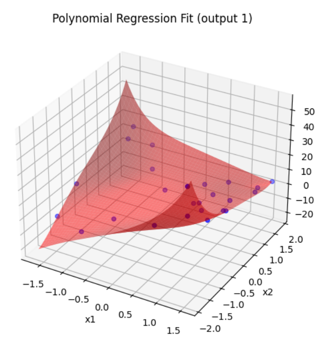

Figure 27: N=50,deg=6,$\lambda = 0.1$, $E_{rms} = 0.11$

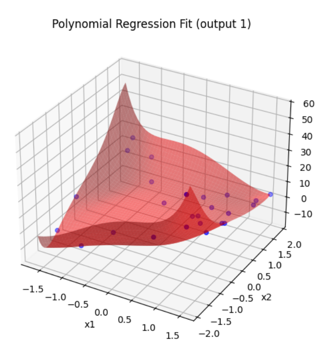

Figure 28: N=50,deg=8,$\lambda = 0.1$, $E_{rms} = 0.08$

3.2 train100

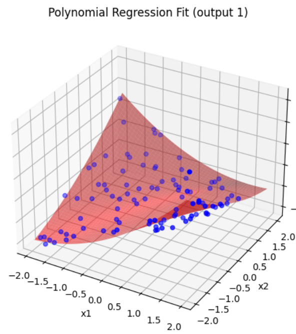

Figure 29: N=100,deg=2,$\lambda = 0$, $E_{rms} = 4.39$

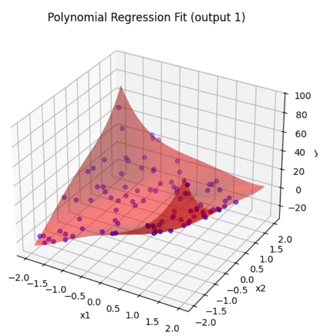

Figure 30: N=100,deg=4,$\lambda = 0$, $E_{rms} = 0.09$

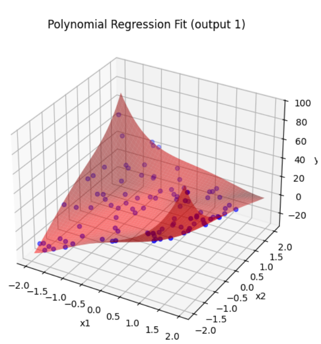

Figure 31: N=100,deg=6,$\lambda = 0$, $E_{rms} = 0.09$

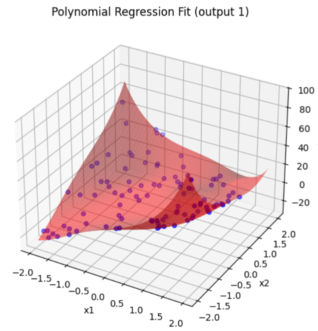

Figure 32: N=100,deg=8,$\lambda = 0$, $E_{rms} = 0.083$

3.3 validation

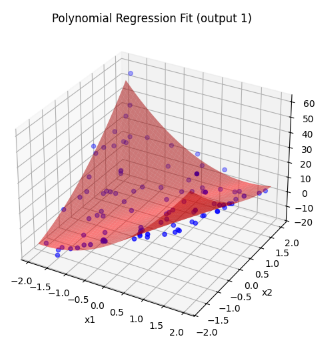

Figure 33: N=100,deg=2,$\lambda = 0$, $E_{rms} = 3.32$

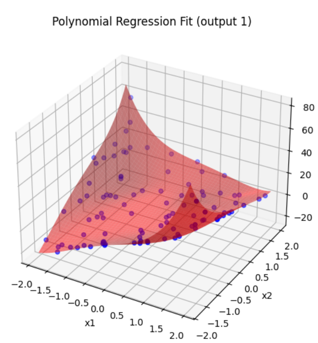

Figure 34: N=100,deg=4,$\lambda = 0$, $E_{rms} = 0.09$

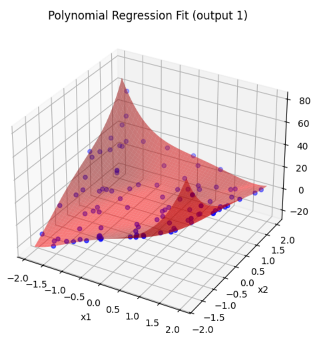

Figure 35: N=100,deg=6,$\lambda = 0$, $E_{rms} = 0.085$

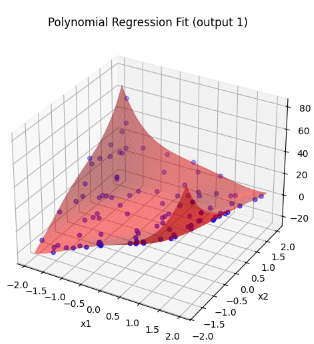

Figure 36: N=100,deg=8,$\lambda = 0$, $E_{rms} = 0.07$

3.4 test

3.4.1 results

Table 2: E_{rms} values for Dataset 2, Polynomial Regression

Model	Train25	Train100	Validation	Test
M=2, $\lambda = 0$	3.19	4.40	3.32	3.91
M=2, $\lambda = 0.001$	3.19	4.40	3.32	3.91
M=2, $\lambda = 0.1$	3.19	4.40	3.32	3.91
M=2, $\lambda = 1$	3.22	4.40	3.32	3.91
M=4, $\lambda = 0$	0.071	0.099	0.094	0.0872
M=4, $\lambda = 0.001$	0.071	0.099	0.094	0.0872
M=4, $\lambda = 0.1$	0.096	0.100	0.095	0.0881
M=4, $\lambda = 1$	0.319	0.149	0.159	0.141
M=6, $\lambda = 0$	258.55	0.093	0.0859	0.0844
M=6, $\lambda = 0.001$	0.020	0.093	0.0859	0.0844
M=6, $\lambda = 0.1$	0.114	0.100	0.0928	0.0896
M=6, $\lambda = 1$	0.317	0.192	0.188	0.186
M=8, $\lambda = 0$	259.77	0.084	0.0716	0.0788
M=8, $\lambda = 0.001$	0.0093	0.0838	0.0716	0.0788
M=8, $\lambda = 0.1$	0.080	0.101	0.0836	0.0936
M=8, $\lambda = 1$	0.266	0.172	0.152	0.169

the model that produced the least E_{rms} in the validation data set is M=8, $\lambda = 0$ and so we will plot the results on this model for the test data

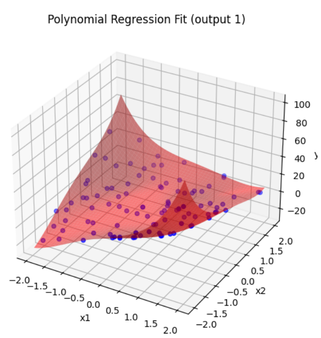

Figure 37: $E_{rms} = 0.07$

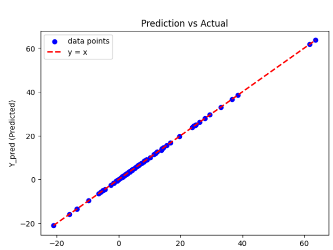

Figure 38: straight line!

4 dataset2-gaussian

4.1 train25

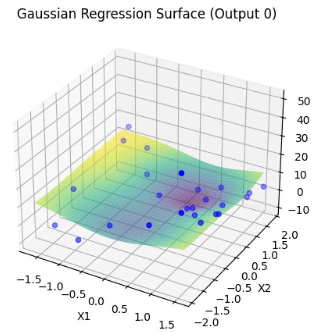

Figure 39: N=25, $E_{rms} = 10.69$

4.2 train100

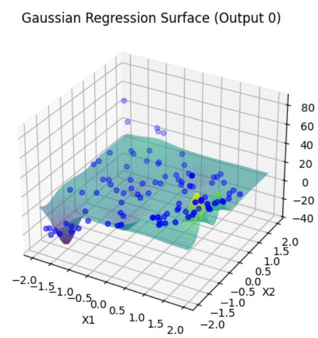

Figure 40: n=100, $E_{rms} = 11.35$

4.3 validation

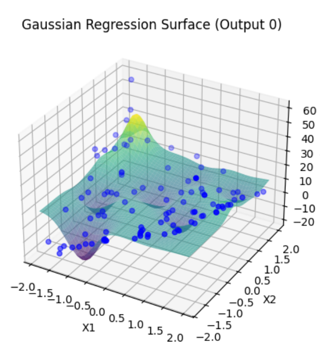

Figure 41: n=100, $E_{rms} = 9.68$

4.4 test

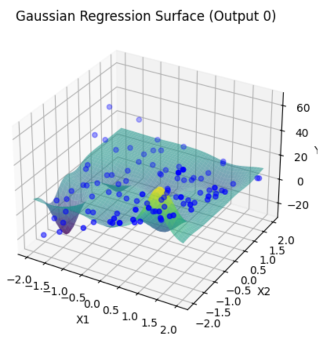

Figure 42: n=100, $E_{rms} = 10$

5 Multiple Parameters - Multiple Output

In this section we will be dealing with the multiple outputs case. The dataset is as follows - There are 3 input features [x1, x2, x3] and 3 output parameters [y1, y2, y3] and a total of 350 training samples. 100 validation samples and 50 test samples. The highlighted part is the best model.

5.1 Polynomial regression -

Here we will be running the polynomial regression to find the curve for the given dataset.

Table 3: ERMS on train, validation, and test data for polynomial ridge regression models. Best models in each block (without/with regularization) are highlighted in bold.

Degree	λ	Train ERMS	Val ERMS	Test ERMS
2	**0**	**0.4054**	**0.4210**	**0.4247**
3	0	0.3979	0.4260	0.4382
4	0	0.3899	0.4396	0.4460
2	1×10^{-6}	0.4054	0.4210	0.4247
2	1×10^{-4}	0.4054	0.4210	0.4247
2	$\mathbf{1 \times 10^{-1}}$	**0.4057**	**0.4206**	**0.4232**
3	1×10^{-6}	0.3979	0.4260	0.4382
3	1×10^{-4}	0.3979	0.4259	0.4381
3	1×10^{-1}	0.4021	0.4218	0.4268
4	1×10^{-6}	0.3899	0.4396	0.4459
4	1×10^{-4}	0.3901	0.4377	0.4442
4	1×10^{-1}	0.3996	0.4237	0.4300

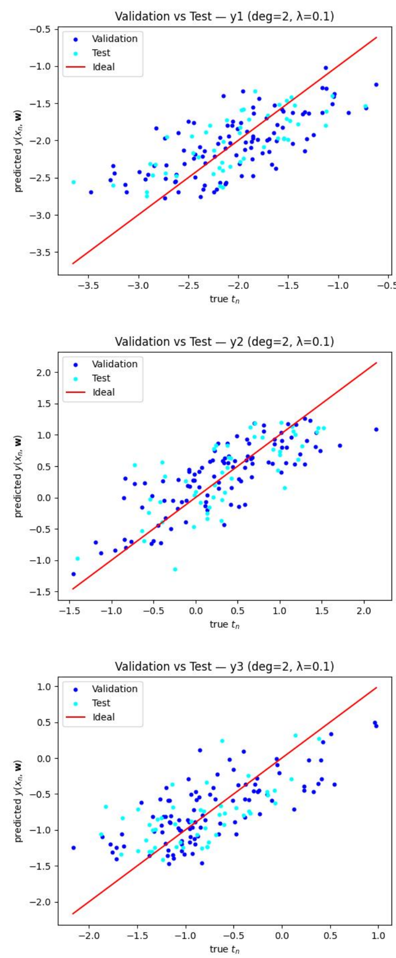

Figure 43: True vs predicted scatter plots (train, val, test).

5.2 Gaussian Basis

Table 4: RBF regression results with different numbers of clusters (k). For each k, the model with the lowest validation ERMS is shown in **bold**.

k	s	λ	Train ERMS	Val ERMS	Test ERMS
$k = 17$					
17	1.0	0.001	0.4057	**0.4224**	0.4238
17	0.7	0.1	0.4095	0.4252	0.4243
17	1.0	0.1	0.4157	0.4253	0.4249
17	0.7	0.001	0.4027	0.4260	0.4272
17	0.7	1.0	0.4168	0.4275	0.4239
17	0.5	0.1	0.4084	0.4291	0.4256
17	0.5	1.0	0.4138	0.4299	0.4247
17	1.0	1.0	0.4226	0.4312	0.4247
17	0.5	0.001	0.3996	0.4318	0.4375
17	0.1	0.1	0.4823	0.5559	0.5142
17	0.1	0.001	0.4821	0.5561	0.5162
17	0.1	1.0	0.4911	0.5633	0.5118
$k = 28$					
28	1.0	0.001	0.4050	**0.4223**	0.4241
28	0.7	0.1	0.4078	0.4242	0.4235
28	1.0	0.1	0.4132	0.4243	0.4245
28	0.7	1.0	0.4143	0.4261	0.4236
28	0.7	0.001	0.4003	0.4265	0.4285
28	0.5	0.1	0.4055	0.4284	0.4257
28	0.5	1.0	0.4116	0.4288	0.4237
28	1.0	1.0	0.4212	0.4291	0.4248
28	0.5	0.001	0.3948	0.4316	0.4388
28	0.1	0.001	0.4393	0.5212	0.4696
28	0.1	0.1	0.4395	0.5219	0.4686
28	0.1	1.0	0.4496	0.5336	0.4725
$k = 35$					
35	1.0	0.001	0.4048	**0.4222**	0.4243
35	0.7	0.1	0.4073	0.4241	0.4236
35	1.0	0.1	0.4120	0.4242	0.4242
35	0.7	1.0	0.4130	0.4260	0.4233
35	0.7	0.001	0.3995	0.4261	0.4300
35	0.5	0.1	0.4043	0.4273	0.4265
35	0.5	1.0	0.4106	0.4285	0.4239
35	1.0	1.0	0.4205	0.4286	0.4247
35	0.5	0.001	0.3943	0.4308	0.4406
35	0.1	0.001	0.4296	0.5052	0.4724
35	0.1	0.1	0.4298	0.5058	0.4704
35	0.1	1.0	0.4406	0.5188	0.4705

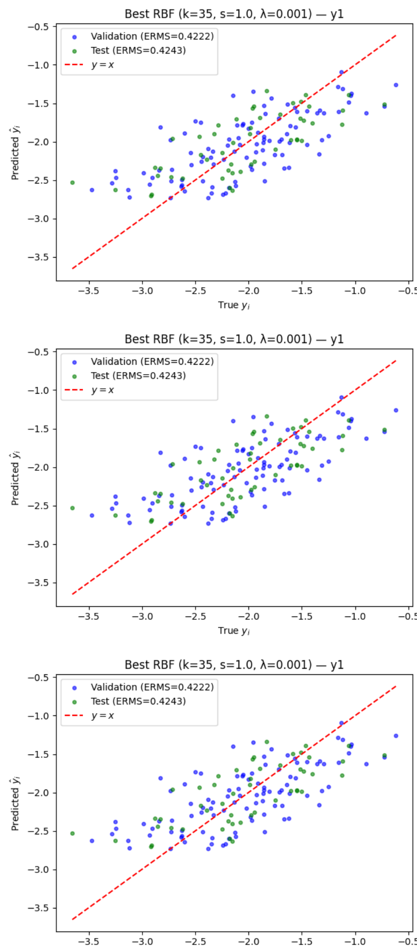

Figure 44: True vs predicted scatter plots (val, test).