**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Answer**: We have a normal distribution with = 45 and = 8.0. Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have X ≤ 50 so the question is to find Pr(X > 50).

Pr(X > 50) = 1 - Pr(X ≤ 50).

Z = (X - mu)/sigma = (X - 45)/8.0

Thus the question can be answered by using the normal table to find

Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Answer**:   
Mean = 38

SD = 6

Z score = (Value - Mean)/SD

Z score for 44 = (44 - 38)/6 = 1 => 84.13 %

=> People above 44 age = 100 - 84.13 = 15.87% ≈ 63    out of 400

Z score for 38 = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44 age = 84.13 - 50 = 34.13 % ≈ 137 out of 400

Hence more employees at the processing centre are older than 44 than between 38 and 44. is F**ALSE**

Z score for 30 = (30 - 38)/6 = -1.33 = 9.15 %   ≈ 36 out of 400

Hence A training program for employees under the age of 30 at the centre would be expected to attract about 36 employees - **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Answer**: The difference between https://tex.z-dn.net/?f=2%20X_1 and https://tex.z-dn.net/?f=X_1%20%2B%20X_2 ishttps://tex.z-dn.net/?f=N(%200%2C6%20%5Csigma%5E2).

Given https://tex.z-dn.net/?f=X_1%20%5Csim%20N(%5Cmu%2C%20%5Csigma%5E2)~~%5Cmbox%7Band%7D~~%20X_2%20%5Csim%20N(%5Cmu%2C%20%5Csigma%5E2)  are two independent identically distributed random variables.

The **Normal distribution** is defined by two parameters, the **mean**,https://tex.z-dn.net/?f=%5Cmu, and the **variance**, https://tex.z-dn.net/?f=%5Csigma%5E%7B2%7D .

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Answer**: Option C is correct

Two values symmetric about mean for the given standard normal distribution are [22, 78]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Answer**:

Given that:

$1 = Rs. 45

https://tex.z-dn.net/?f=Profit_1%20%5Csim%20N(5%2C%203%5E2)%20%5C%5C%20Profit_2%20%5Csim%20N(7%2C%204%5E2)

Thus,

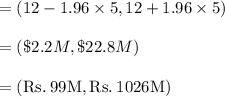
Company's profit:

https://tex.z-dn.net/?f=P%20%5Csim%20N(%205%2B7%2C%203%5E2%20%2B%204%5E2)%20%3D%20N(12%2C%205%5E2)

**A):**

95% of the probability lies between 1.96 standard deviations of the mean.

Thus range is:



**B):** Fifth percentile is calculated as:

https://tex.z-dn.net/?f=P(Z%20%5Cleq%20%5Cdfrac%7Bp-12%7D%7B5%7D)%20%3D%200.05

From p values of z score table, we get:

https://tex.z-dn.net/?f=%5Cdfrac%7Bp-12%7D%7B5%7D%20%3D%20-1.644%5C%5Cp%20%3D%2012%20-%208.22%20%3D%203.78%5C%5C

Thus at $3.78M dollars, or Rs. 170.1M amount, 5th percentile of profit lies.

Or 5th percentile of profit is Rs. 170.1 Million.

**C):** Loss is when profit < 0

Thus: p < 0

The first division of company, thus have larger probability of making a loss in a given year.