

REVISION

For Class - 11th



Lecture no. 02

Motion in 2 Dimension

(Part-2).



Physics Wallah

Rajwant Singh

Topics to be covered

- 1 Vectors
- 2 Projectile Motion
- 3 Relative Velocity
- 4 Questions *

A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product $t_1 t_2$ is:

[JAN 2019]

A

$$\frac{R}{4g}$$

B

$$\frac{R}{g}$$

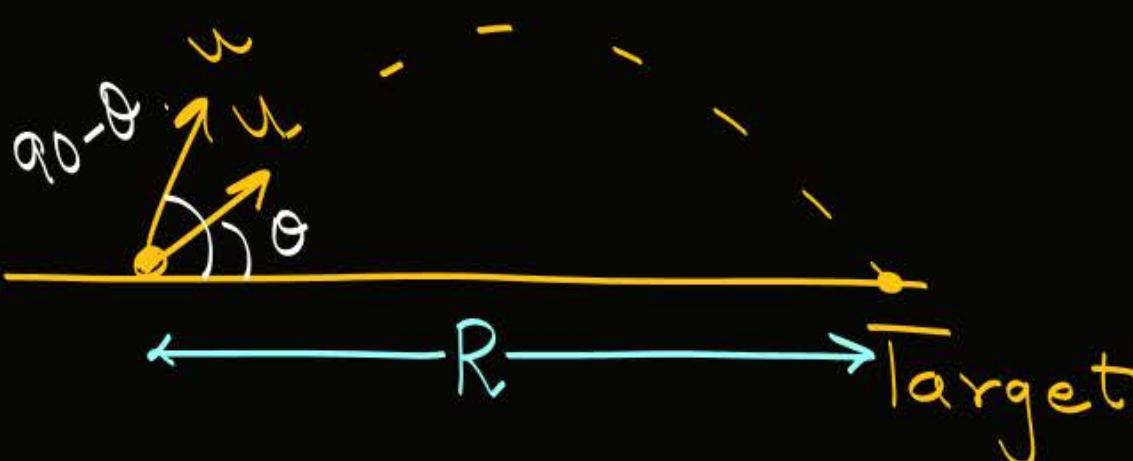
C

$$\frac{R}{2g}$$

D

$$\frac{2R}{g}$$

Answ



Complementary angle

$$R \rightarrow f_{ix}$$

$$T_\theta = \frac{2u \sin \theta}{g} = t_1$$

$$T_{90-\theta} = \frac{2u \cos \theta}{g} = t_2$$

$$t_1 t_2 = \frac{4u^2 \sin \theta \cos \theta}{g^2} \cdot R = u^2 \frac{\sin 2\theta}{g} \cdot R$$

$$= \frac{2(2u^2 \sin \theta \cos \theta)}{g \cdot g} \cdot R = \frac{2u^2 \sin 2\theta}{g} \cdot R$$

$$t_1 t_2 = \frac{2R}{g}$$



Position of two particles A and B as a function of time are given by $x_A = -3t^2 + 8t + c$ and $y_B = 10 - 8t^3$. The velocity of B with respect to A at $t = 1$ is \underline{v} . Find v (JAN 2020)

$$\bullet x_A = -3t^2 + 8t + c$$

with time both particles are moving

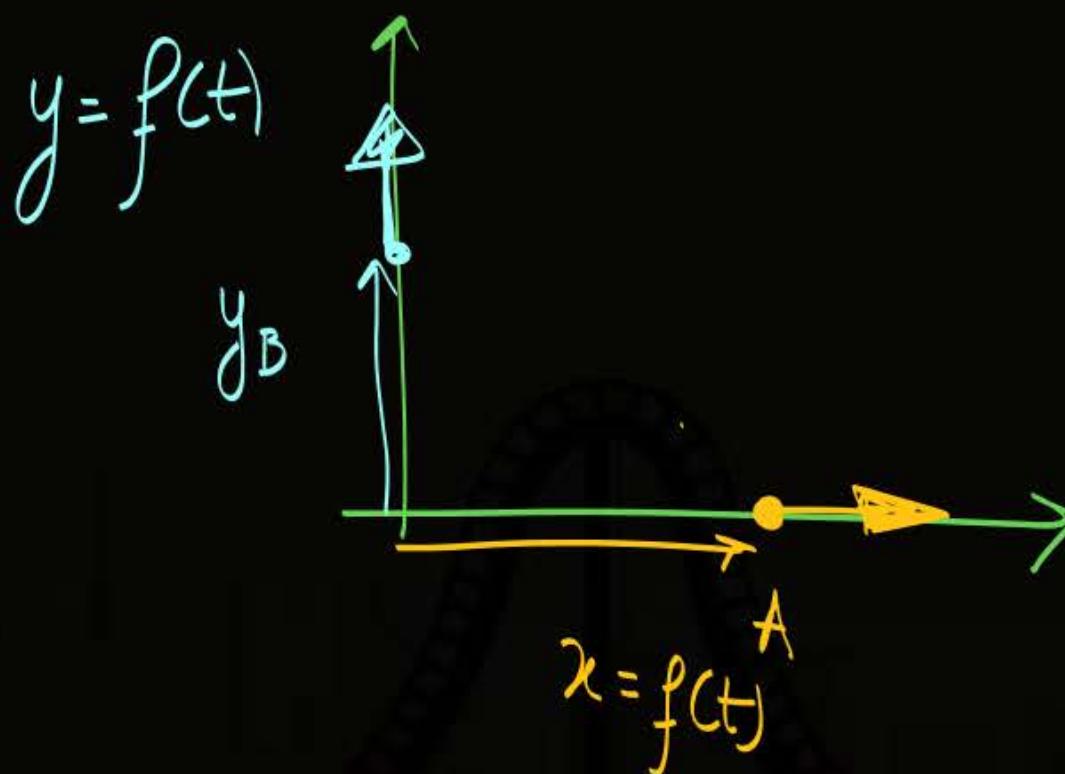
$$\bullet y_B = 10 - 8t^3$$

$$v_A = \frac{dx_A}{dt} = (-6t + 8)\hat{i}$$

$$v_B = \frac{dy_B}{dt} = 0 - 24t^2\hat{j}$$

$$\text{at } t=1s \quad v_A = (-6+8)\hat{i} = 2\hat{i}$$

$$v_B = -24\hat{j}$$



$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

$$= -24\hat{j} - 2\hat{i}$$

$$|v_{BA}| = \sqrt{24^2 + 2^2}$$

$$= \sqrt{580} = \sqrt{58}$$

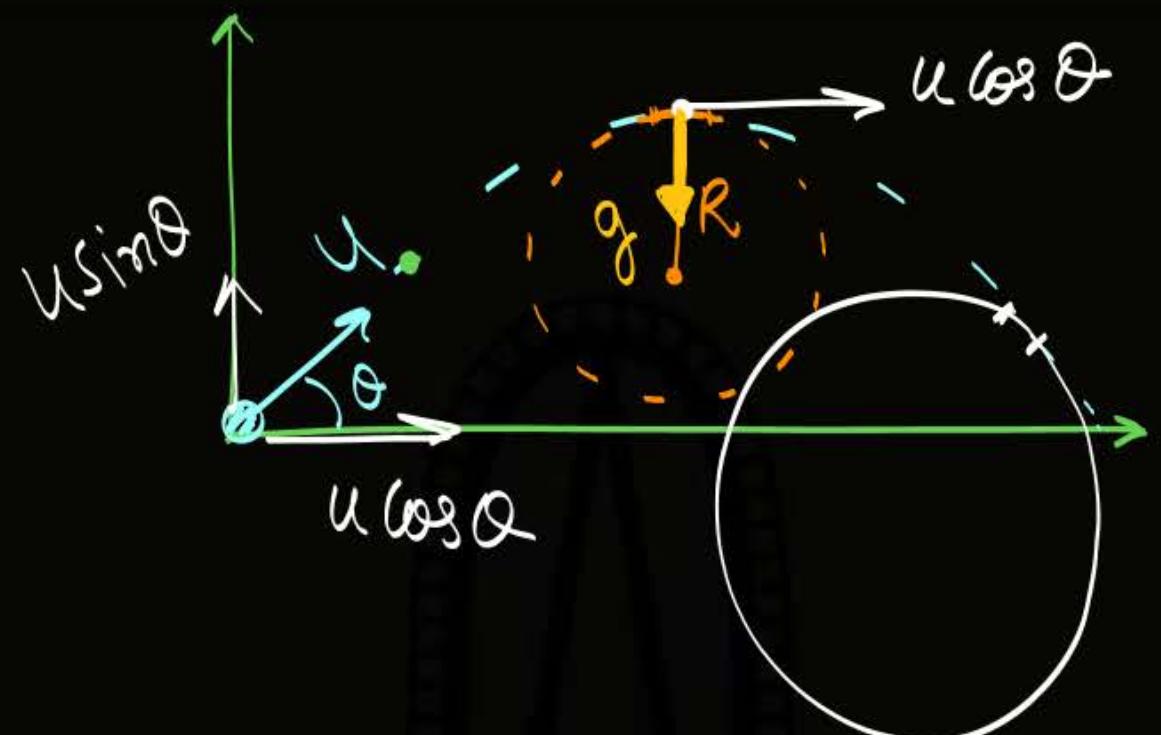
$$v = 580 \text{ m/s}$$



A particle is projected at a speed u at an angle θ with the horizontal.
Find the radius of curvature at the highest point of the trajectory of projectile.

P
W
(HCV Verma)

Parabolic path



$$a_x = 0$$

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

at Each point, Radius of Curvature is diff.

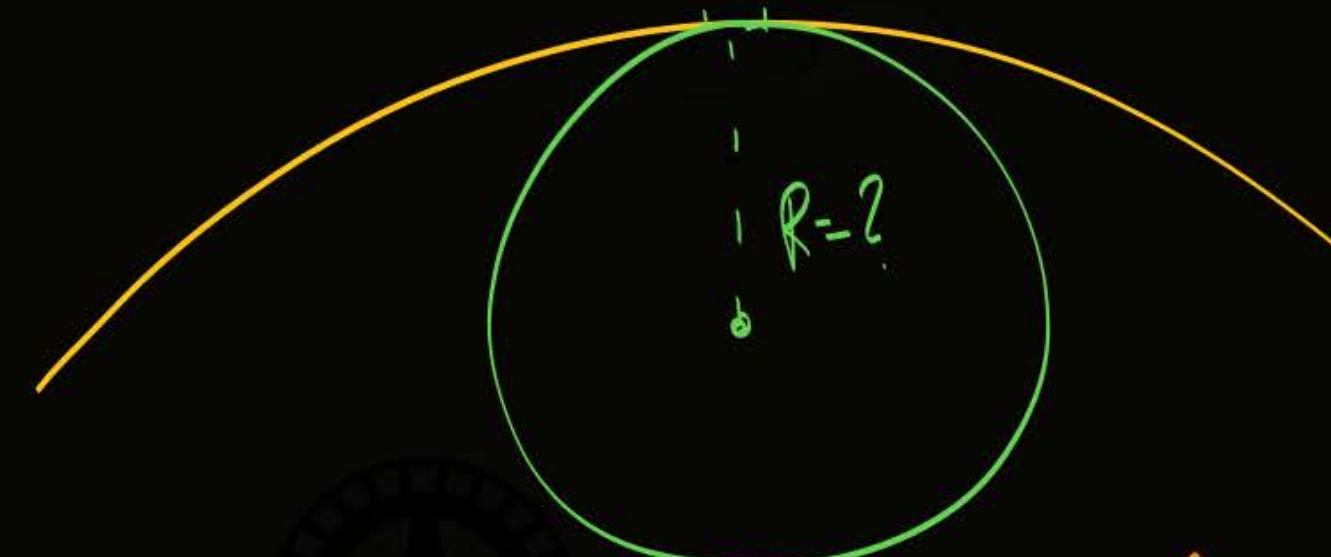
1. draw Circle.
2. Find velocity (Tangential)
3. find acc along radius.
4. $R_c = \frac{v^2}{a_r}$

$$a_r = \frac{u^2 \cos^2 \theta}{R}$$

$$R = \frac{u^2 \cos^2 \theta}{g}$$

at top point

Ansatz

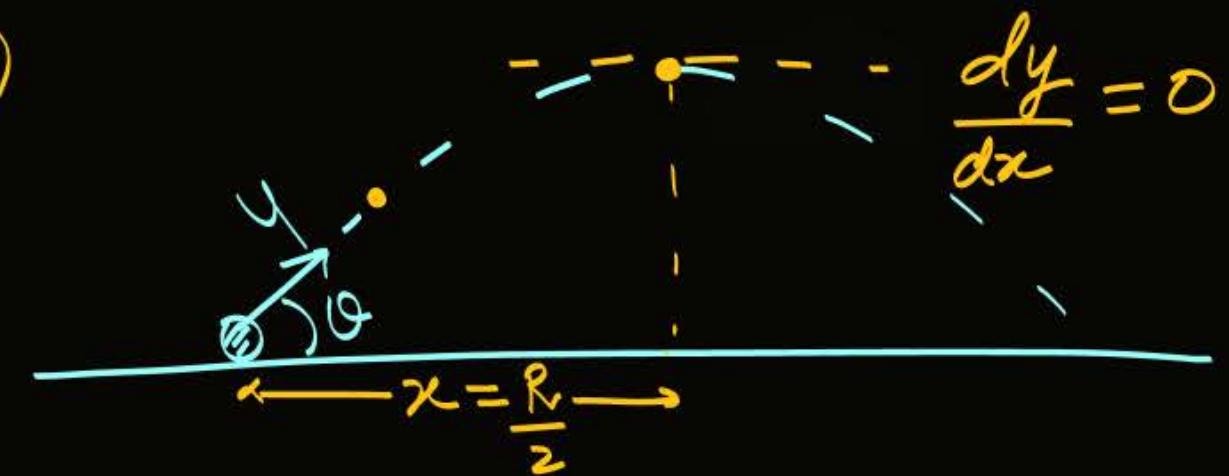


$$y = f(x)$$

$$R = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\left| \frac{d^2y}{dx^2} \right|}$$

Point

$$\dagger \quad y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$



$$\frac{dy}{dx} = \tan \theta - \frac{gx}{u^2 \cos^2 \theta}$$

$$\frac{d^2y}{dx^2} = -\frac{g}{u^2 \cos^2 \theta}$$

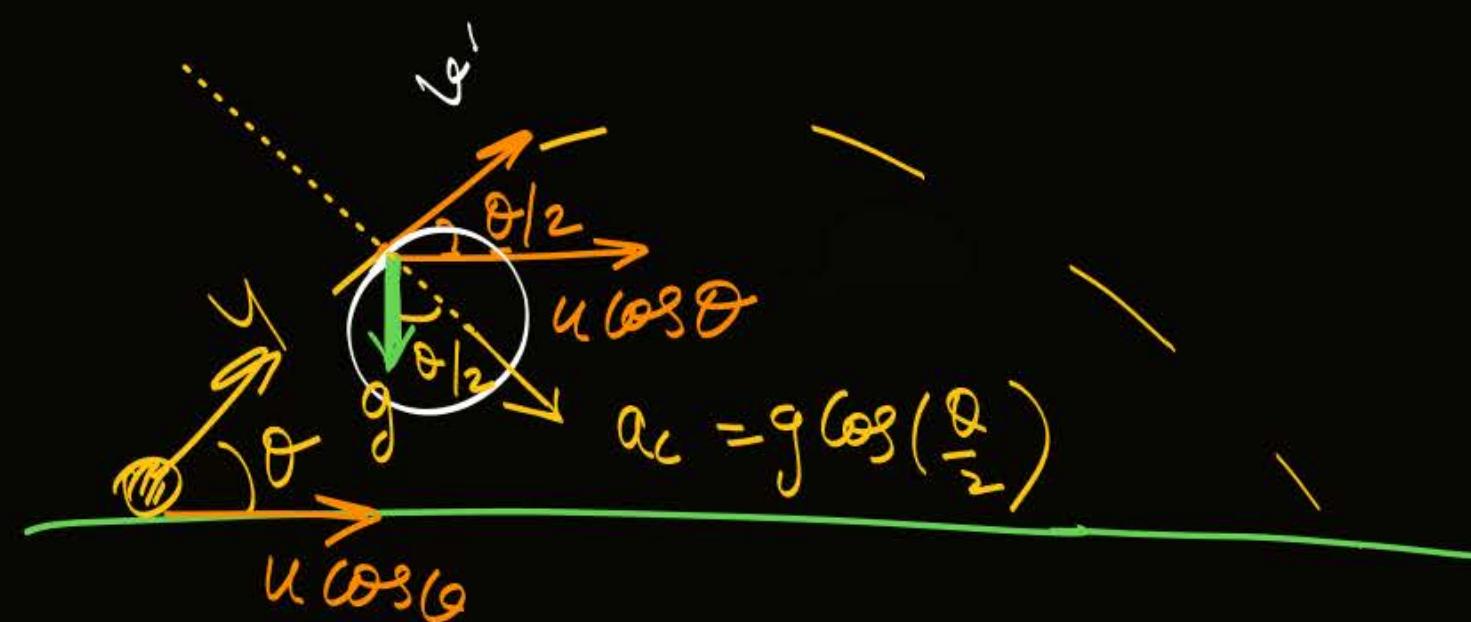
$$\left| \frac{d^2y}{dx^2} \right| = \frac{g}{u^2 \cos^2 \theta}$$

$$R = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\left| \frac{d^2y}{dx^2} \right|} = \frac{\left(1 + \sigma^2 \right)^{3/2}}{\frac{g}{u^2 \cos^2 Q}} = \frac{u^2 \cos^2 Q}{g}$$

$$\boxed{R = \frac{u^2 \cos^2 Q}{g}}$$

P
W

H/T Circular



$$v' \cos\left(\frac{Q}{2}\right) = u \cos \theta$$

$$v' = \frac{u \cos Q}{\cos\left(\frac{Q}{2}\right)}$$

Find R of Curvature at point where angle is $\theta/2$

$$a_c = \frac{v_T^2}{R}$$

$$R = \frac{u^2 \cos^2 Q}{\cos^2(\theta/2) g \cos(\frac{Q}{2})}$$



A projectile is thrown upward with velocity u at angle θ . Find the time after projection at which the velocity of projectile is perpendicular to its initial velocity

A $(u \sin \theta)/g$

B $(u \operatorname{cosec} \theta)/g$ Ans

C $(u \tan \theta)/g$

D $(u \cot \theta)/g$



$$u_x = u \cos \theta \quad a_x = 0$$

$$u_y = u \sin \theta \quad a_y = -g$$

$$t = ?$$

\vec{v}_t is \perp to \vec{v}_i

When two Vectors are \perp .

$$\boxed{\vec{A} \cdot \vec{B} = 0}$$

$$\vec{v}_i = u \cos \theta \hat{i} + u \sin \theta \hat{j}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

$$\vec{v} = (u_x + a_x t) \hat{i} + (u_y + a_y t) \hat{j}$$

$$\boxed{\vec{v} = u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}}$$

at time $t \quad \vec{a} \cdot \vec{v} = 0$

$$(\underline{u \cos \theta \hat{i}} + \underline{u \sin \theta \hat{j}}) (\underline{u \cos \theta \hat{i}} + \underline{(u \sin \theta - gt) \hat{j}}) = 0$$

$$u^2 \cos^2 \theta + u^2 \sin^2 \theta - u \sin \theta g t = 0$$

$$u^2 (\cos^2 \theta + \sin^2 \theta) - u g t \sin \theta = 0$$

$$u^2 = u g t \sin \theta$$

$$\boxed{\frac{u \cos \theta}{g} = \frac{u}{g \sin \theta} = t}$$



If $\vec{P} + \vec{Q} = \vec{R}$ and $|\vec{P}| = |\vec{Q}| = \sqrt{3}$ and $|\vec{R}| = 3$, then the angle between \vec{P} and \vec{Q} is:



A 0

B $\pi/6$

C $\pi/3$ Ans

D $\pi/2$

$\vec{P} + \vec{Q} = \vec{R}$ ↗
when we add \vec{P} & \vec{Q} vectorially
there resultant R .

$$|P| = |Q| = \sqrt{3}$$

$$|R| = 3$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos Q}$$

$$3 = \sqrt{3+3+2 \times 3 \cos Q}$$

$$\cos Q = \underline{\hspace{2cm}}$$

⊗ $Q = 60^\circ$ $R = |P| \sqrt{3}$

$\sqrt{3} \cdot \sqrt{3} = 3$



The sum of magnitudes of two forces acting at a point is 16 and magnitude of their resultant is $8\sqrt{3}$. If the resultant is at 90° with the force of smaller magnitude, their magnitudes are

\star

A 3, 13

B 2, 14

C 5, 11

D 4, 12

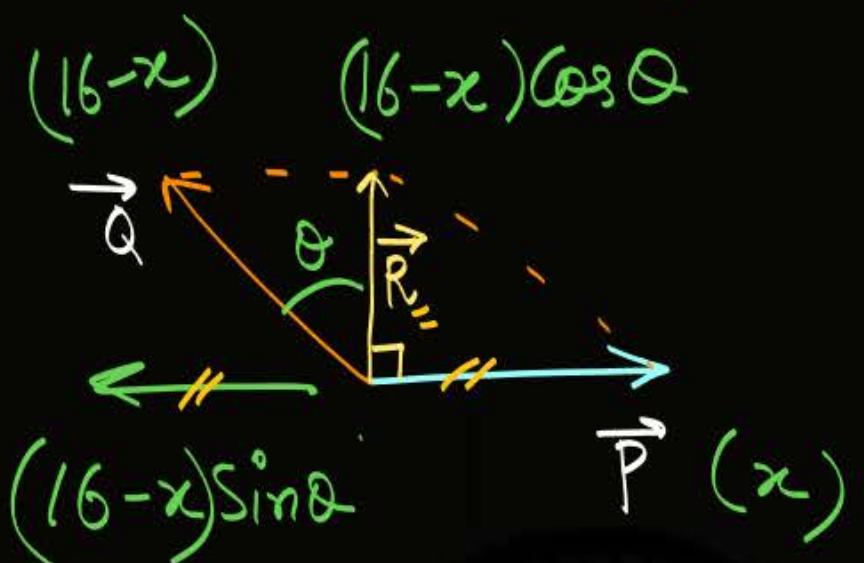


$$|P| + |Q| = 16$$

$$|P| = x$$

$$|Q| = 16 - x$$

$$|R| = 8\sqrt{3}$$



along X $f_{net} = 0$

along Y $f_{net} = 8\sqrt{3}$

$$x = (16 - x) \sin \theta$$

$$8\sqrt{3} = (16 - x) \cos \theta$$

$$\sin \theta = \frac{x}{16 - x}$$

$$\cos \theta = \frac{8\sqrt{3}}{16 - x}$$

Eq of adding

$$x = \underline{\hspace{2cm}}$$



Two forces, each equal to F , act as shown in figure. Their resultant is



A $F/2$

B F Ans

C $\sqrt{3} F$

D $\sqrt{5} F$

Top

$\theta = 60^\circ$ $R = F\sqrt{3}$

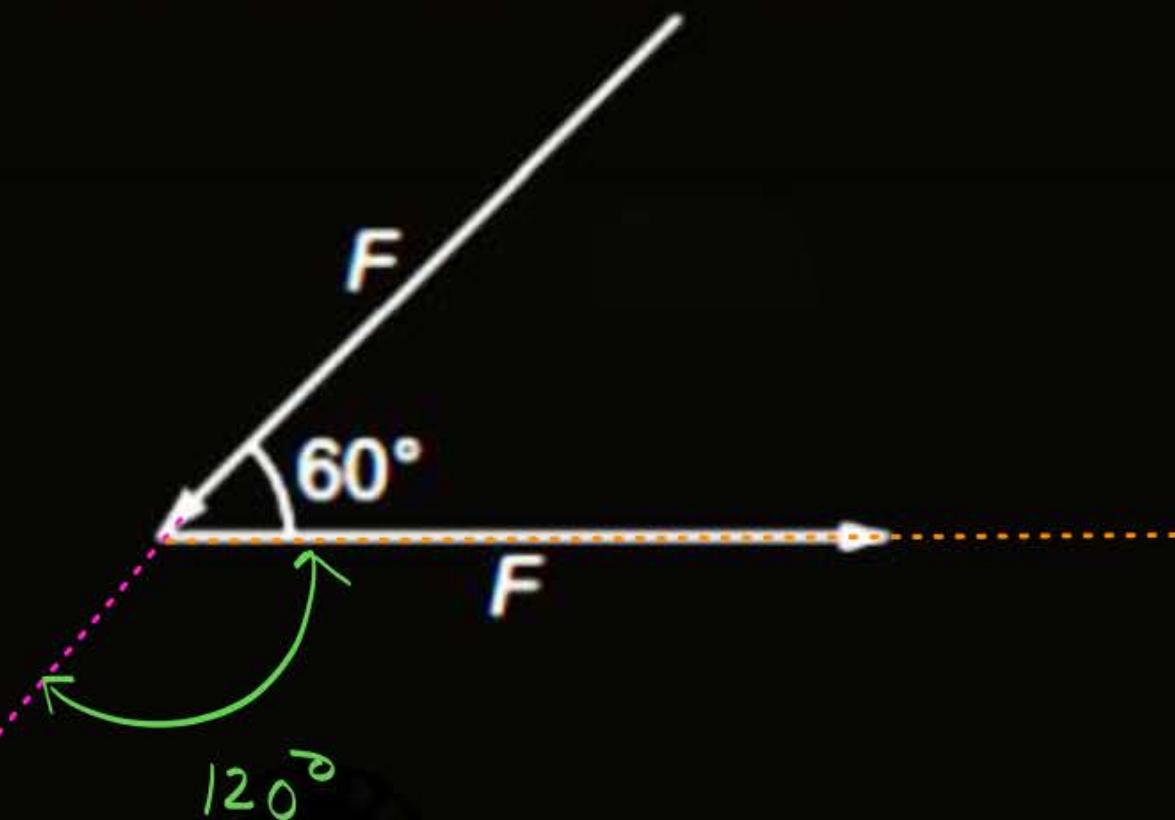
Angle between Vector is

angle between their dir

$\theta = 120^\circ$

$R = \sqrt{F^2 + F^2 - 2F \cdot F \cos 120^\circ}$

$R = F$

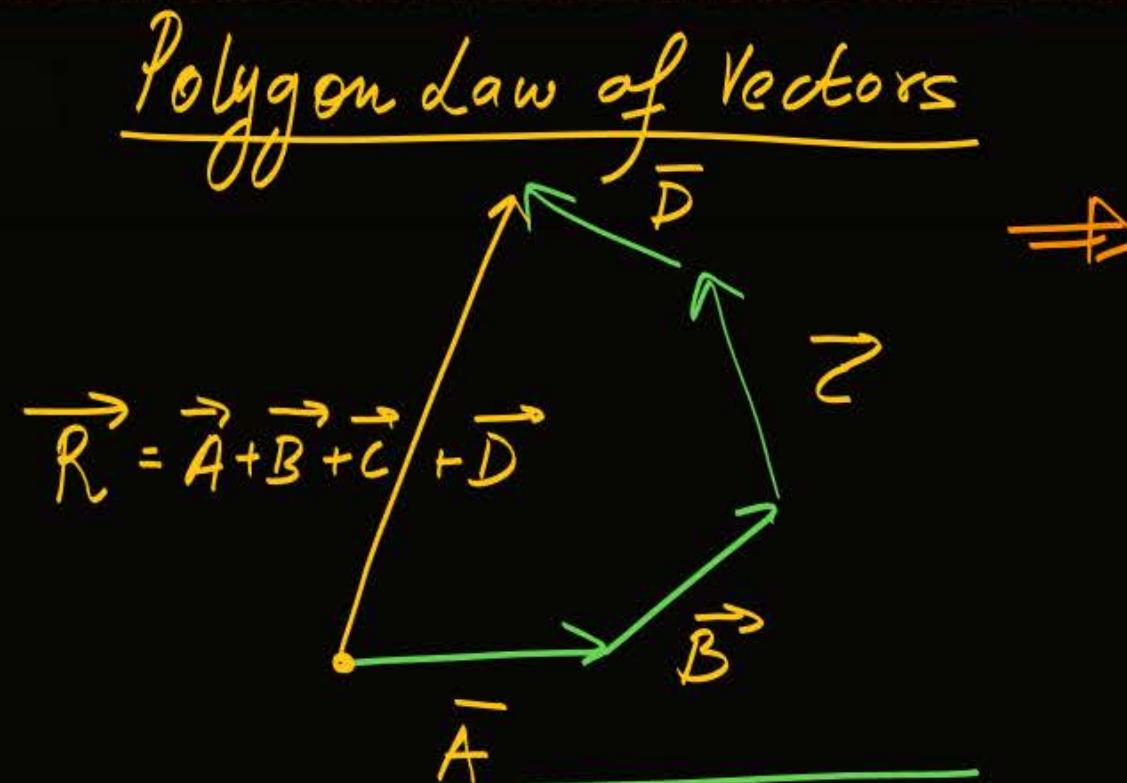




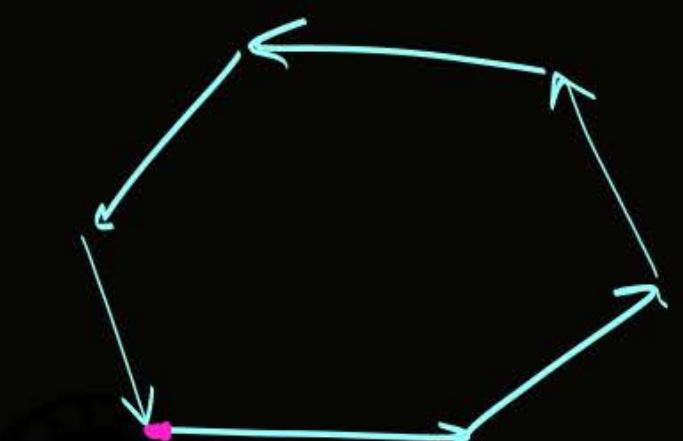
Resultant of which of the following may be equal to zero ?

- A 10 N, 10 N, 10 N
- B 10 N, 10 N, 25 N
- C 10 N, 10 N, 35 N
- D None of these

When $R=0$ we need to get a closed Polygon



If we have a Closed Polygon



Symmetric or Asymmetric
Ordered Closed polygon

$$R=0$$

In our Case Polygon $\rightarrow \Delta$.

Sum of two Sides \geq Third Side.



Resultant of which of the following may be equal to zero ?



A

10 N, 10 N, 10 N

B

10 N, 10 N, 25 N

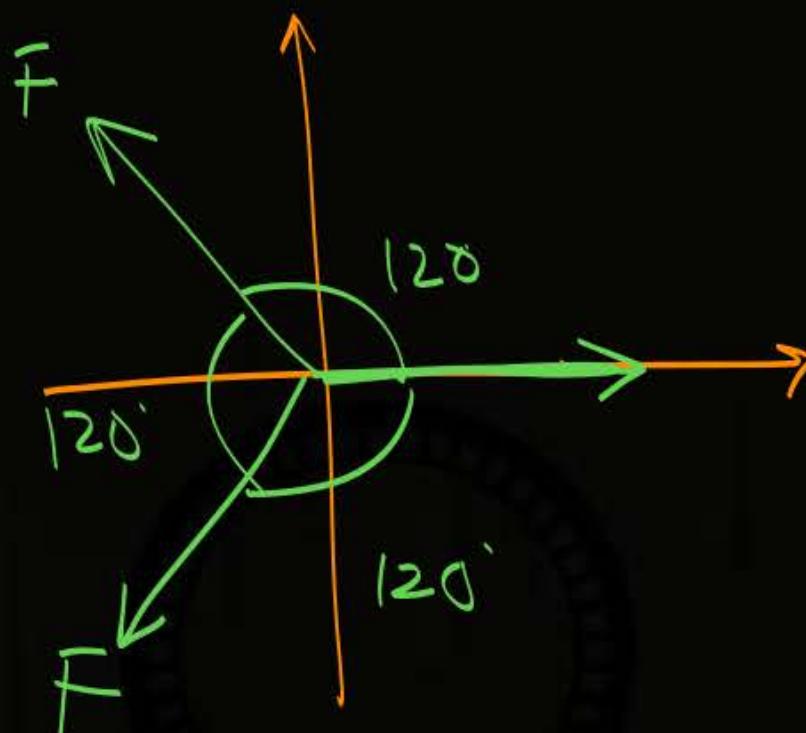
$$10 + 10 < 25 \quad \text{X}$$

C

10 N, 10 N, 35 N

D

None of these



$$10 + 10 < 25 \quad \text{X}$$

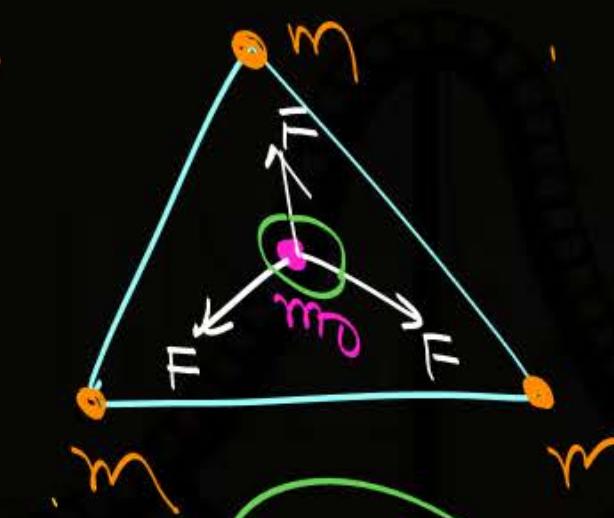
gravitational

Symmetrical diagram

$$\vec{f}_{\text{net}} = 0$$

$$\vec{I}_{\text{net}} = 0$$

$$\vec{F}_{\text{net}} = 0$$

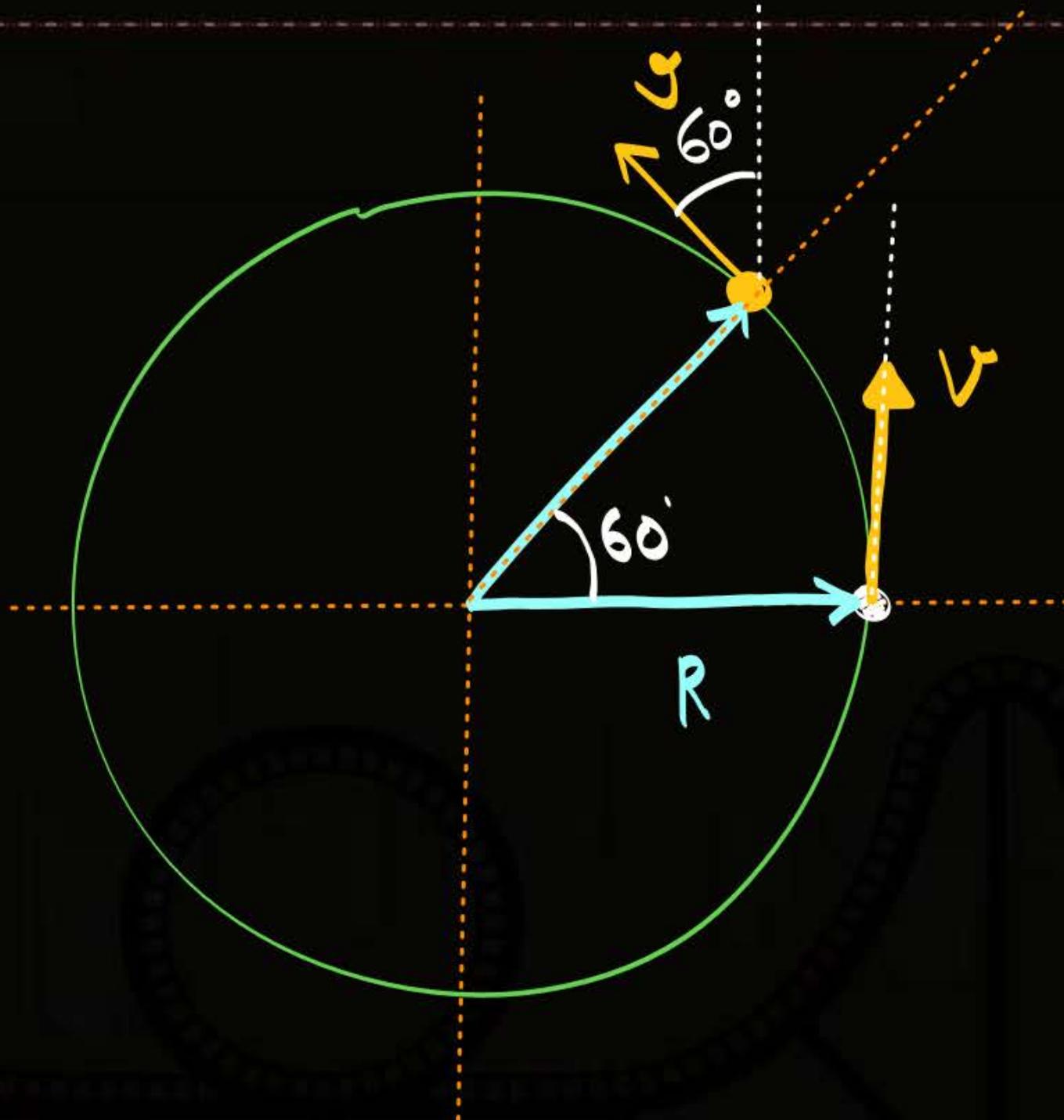


$$\vec{f}_{\text{net}} = 0$$

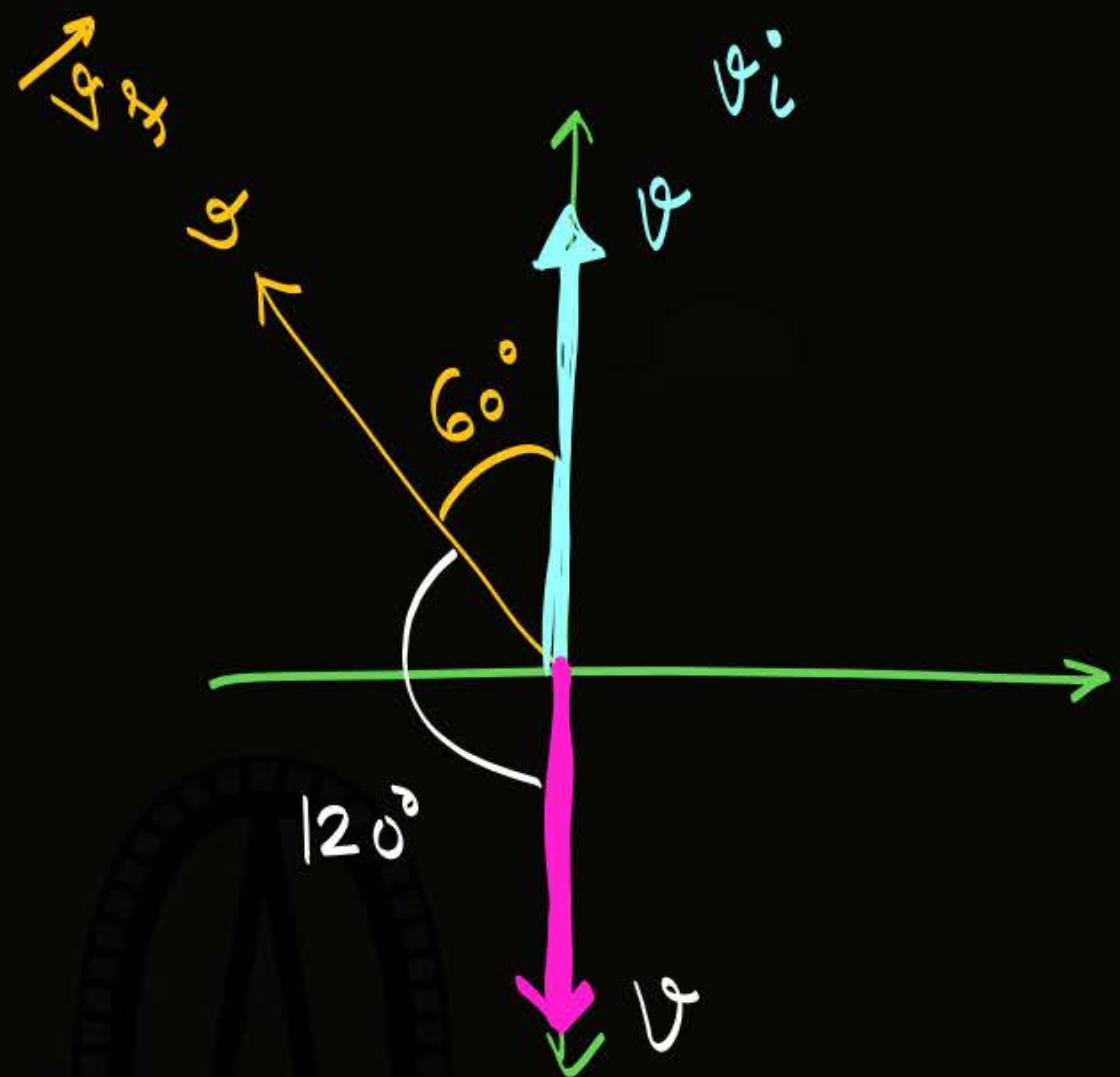


A cyclist is moving on a circular path with constant speed. What is the change in its velocity after it has described an angle of 60° ?

- A $v\sqrt{2}$
- B v Ans
- C $v\sqrt{3}$
- D None of these



Change in velocity
 $\vec{\Delta v} = \vec{v}_f - \vec{v}_i$
Subtraction of Vectors
Velocity = Vector



$$\vec{v}_f - \vec{v}_i$$

$$\vec{v}_f + (-\vec{v}_i)$$

$$|A| = |B| = P$$

$$Q = 120 \quad R = P.$$

$$|\text{Change in velocity}| = v.$$



Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is : [JAN 2019]

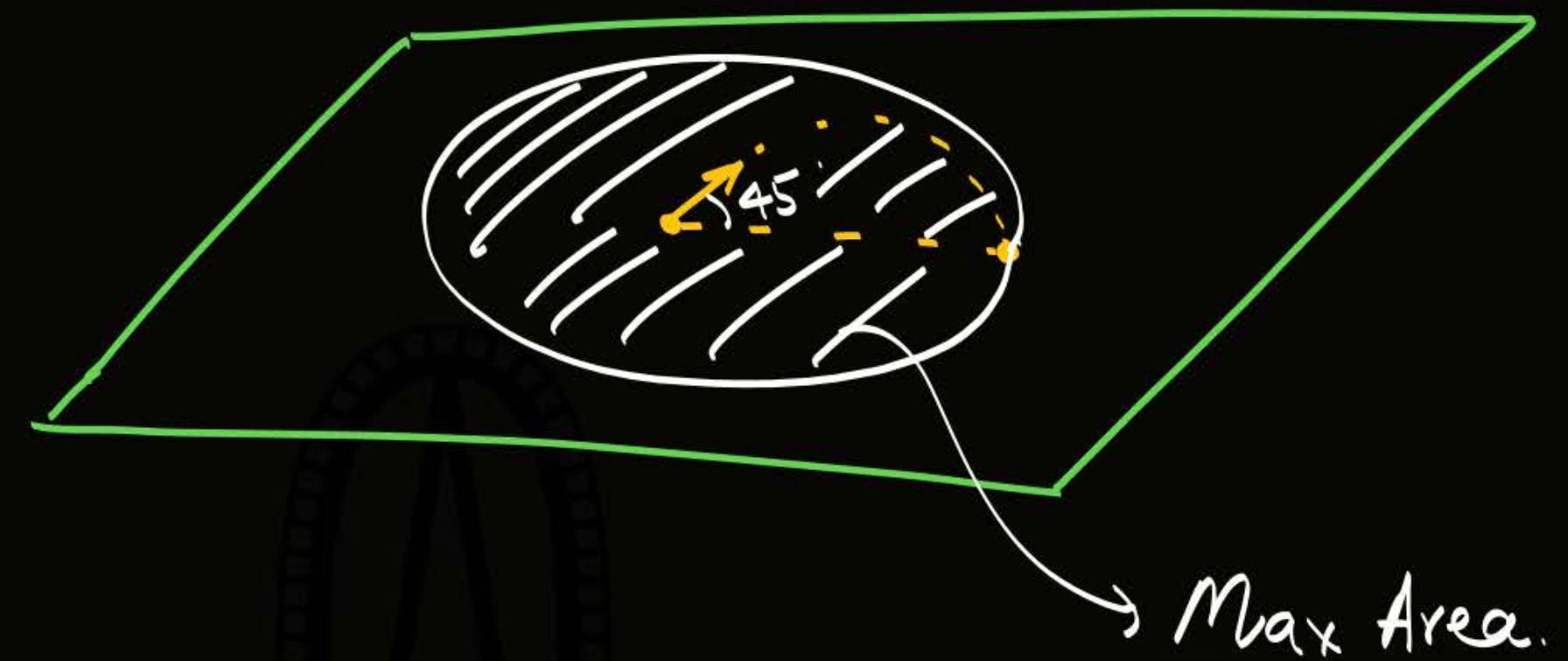
A 1 : 2

C 1 : 8

B 1 : 4

D 1 : 16 Ans

$$\frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \frac{1}{16}$$



Max Area.

for
Max Area.

$R_{\text{range}} \rightarrow \max$

$$\theta = 45^\circ$$

$$R_{\max} = \frac{u^2}{g}$$

= Radius of Circle = R_{\max}

$$\text{Area} = \pi r^2 = \pi R_{\max}^2 = \frac{\pi u^4}{g^2}$$



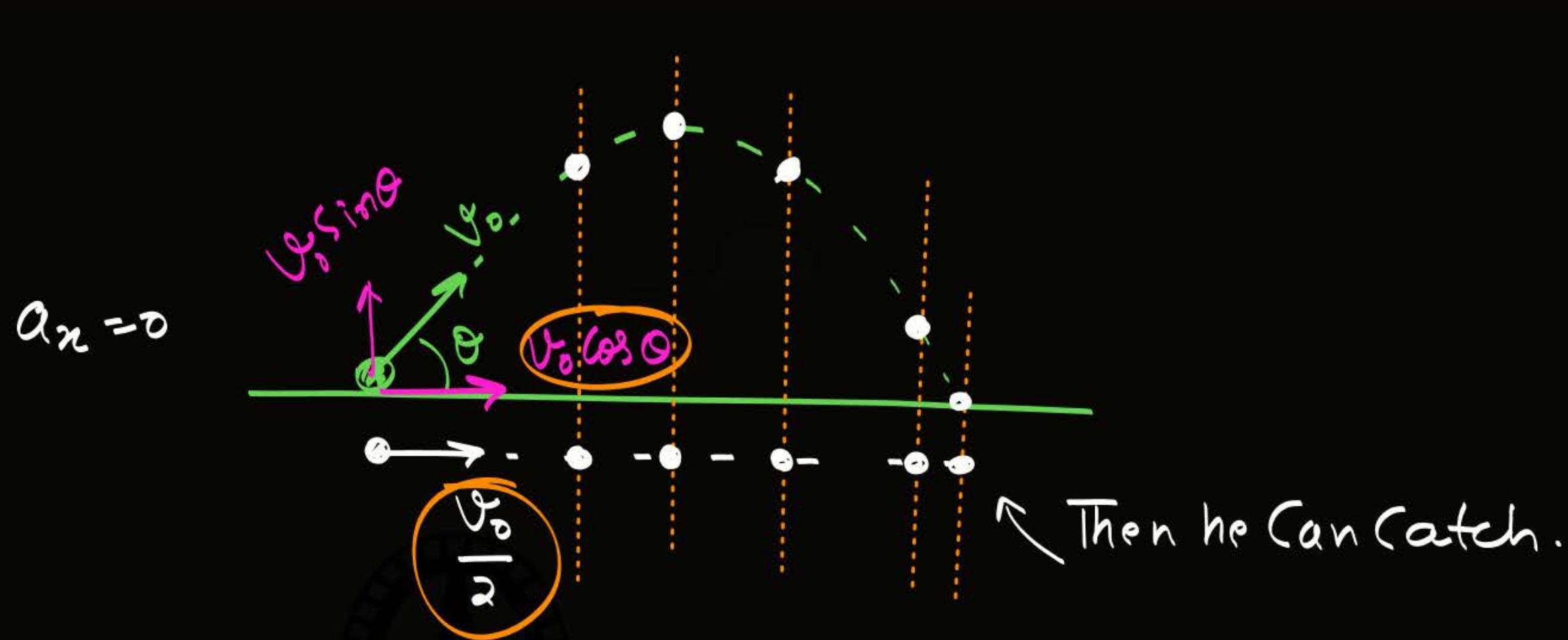
A ball is thrown from a point with a speed ' v_0 ' at an elevation angle of θ . From the same point and at the same instant, a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball ? If yes, what should be the angle of projection θ ? [2004]

A No

B Yes, 30°

C Yes, 60° Ane

D Yes, 45°



Then he can catch.

$$\frac{v_0}{2} = v_0 \cos \theta .$$

$$\frac{1}{2} = \cos \theta$$

$$\boxed{\theta = 60^\circ}$$



A particle starts moving from origin with velocity $\vec{u} = 3\hat{i}$ from origin and acceleration $\vec{a} = 6\hat{i} + 4\hat{j}$. Find x- coordinate at the instant when y - coordinate of the particle is 32 m [JAN 2020]

A 48

C 12

$$t=0$$

$$\alpha=0$$

$$u=0$$

$$u = 3\hat{i}$$

$$a = 6\hat{i} + 4\hat{j}$$

$$y = 32 \quad x=?$$

$$u_x = 3 \quad u_y = 0$$

$$a_x = 6 \quad a_y = 4$$

B 60 Ans

D 24

} There is angle between \vec{v} & \vec{a}
(2D Motion)

$$X \quad u_x \ a_x$$

$$v_x = u_x + a_x t$$

$$s_x = u_x t + \frac{1}{2} a_x t^2$$

$$v_x^2 - u_x^2 = 2 a_x s_x$$

$$Y \quad (u_y, a_y)$$

$$v_y = u_y + a_y t$$

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$$v_y^2 - u_y^2 = 2 a_y s_y$$

lets find at time $y = 32$?

at $t = 4s$

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$32 = 0 + \frac{1}{2} \times 4 t^2$$

$$16 = t^2$$

$$t = 4s$$

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$= 3 \times 4 + \frac{1}{2} \times 6 \times 4^2$$

$$= 12 + 48$$

$$= 60m$$

$$= 60m$$

Alternative

Equation of Trajectory

$$x = f(t)$$

$$y = g(t)$$

Time Eliminate

$$y = f(x)$$

Let particle reach (x, y) at time t

$$s_x = u_x t + \frac{1}{2} a_x t^2$$

$$x = 3t + \frac{1}{2} \times 6t^2$$

$$\boxed{x = 3t + 3t^2}$$

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$$y = \frac{1}{2} \times 4xt^2$$

$$\boxed{y = 2t^2}$$

$$t = \frac{y^{\frac{1}{2}}}{\sqrt{2}}$$

$$y = 2t^2 \quad t^2 = \frac{y}{2}$$

$$x = 3t + 3t^2$$

$$x = 3t + 3\left(\frac{y}{2}\right)$$

$$x = 3\left(\frac{y}{2}\right)^2 + 3\left(\frac{y}{2}\right)$$

$$y = 32$$

$$x = ?$$



If $\frac{|\vec{a} + \vec{b}|}{|\vec{a} - \vec{b}|} = 1$, then the angle between \vec{a} and \vec{b} is :

P
W

A

A 0°

C 90° *Ans*

B 45°

D 60°

There are two vector \vec{a} & \vec{b}

$$|\vec{a} + \vec{b}|$$

mag of addition.

$$|\vec{a} - \vec{b}|$$

mag of subtraction.

$$\frac{|\vec{a} + \vec{b}|}{|\vec{a} - \vec{b}|} = 1$$

$$\frac{\sqrt{a^2 + b^2 + 2ab \cos \theta}}{\sqrt{a^2 + b^2 - 2ab \cos \theta}} = 1$$

$$a^2 + b^2 + 2ab \cos \theta = a^2 + b^2 - 2ab \cos \theta$$

$$2ab \cos \theta = 0$$

$$\theta = 90^\circ$$



If $|\hat{a} - \hat{b}| = \sqrt{2}$ then calculate the value of $|\hat{a} + \sqrt{3}\hat{b}|$.

Ans = 2

P
W

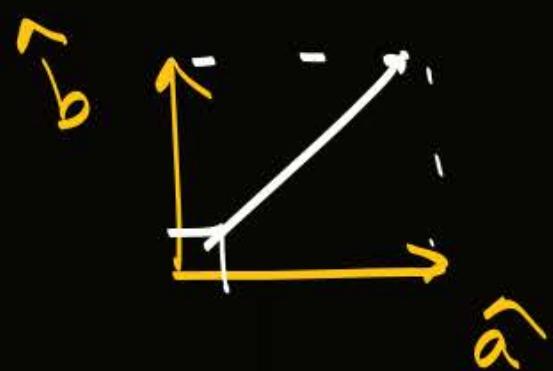
\hat{a} = unit vector along \vec{a} , direction of \vec{a} $|\hat{a}| = 1$

\hat{b} = unit vector along \vec{b} , direction of \vec{b} . $|\hat{b}| = 1$

$$|\hat{a} - \hat{b}| = \sqrt{2}$$

Mag of Sub of

two unit vectors.



$$|\hat{a} + \hat{b}| = \sqrt{2}$$

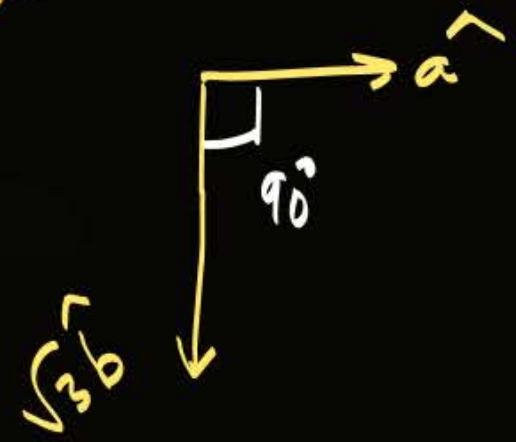
\hat{a} & \hat{b} are \perp to each others.

$$|\hat{a}| = 1 \quad |\hat{a} - \hat{b}| = \sqrt{2}$$

$|-| = |\hat{b}|$

$$|\vec{a} + \sqrt{3}\vec{b}|$$

Scalar multiply
dir= same



Ex:- \vec{F}

$$\text{new force} = n\vec{F}$$

↓
number (Scalar)

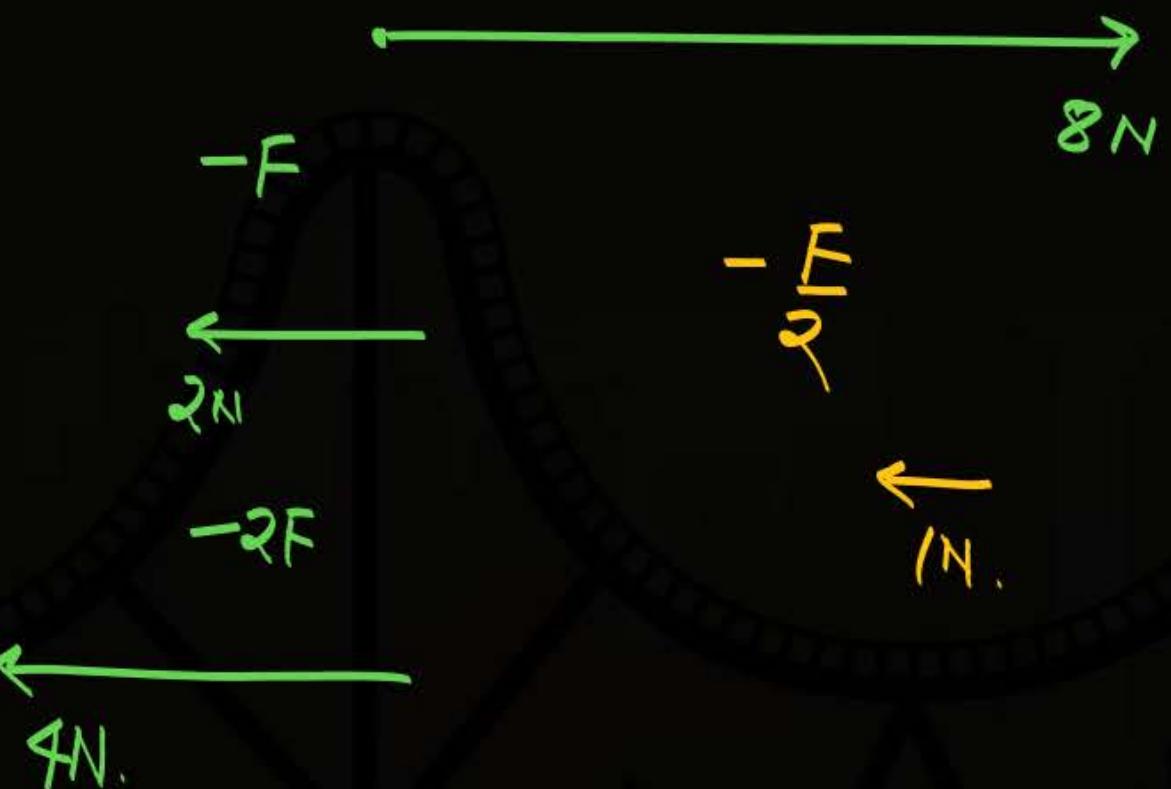
||
 ↓
 $n = +ve$
 $n = -ve$
 dir opp.

$|n| > 1$ Magnitude ↑
 $|n| < 1$ Magnitude ↓

$$\begin{aligned}
 R &= \sqrt{A^2 + B^2} \\
 &= \sqrt{|\vec{a}|^2 + |\sqrt{3}\vec{b}|^2} \\
 &= \sqrt{1 + 3} = \sqrt{4} = 2
 \end{aligned}$$

⊗ $2N = F$

4F ?





The displacement of an object along the three axes are given by, $x = 2t^2$, $y = t^2 - 4t$ and $z = 3t - 5$. The initial velocity of the particle is

A 10 units

B 12 units

C 5 units Ans

D 2 units

$$x = 2t^2$$

$$y = t^2 - 4t$$

$$z = 3t - 5$$

3D Motion

$$v_x = \frac{dx}{dt} = 4t$$

$$v_y = \frac{dy}{dt} = 2t - 4$$

$$v_z = \frac{dz}{dt} = 3$$

$$t = 0$$

$$v_x = 0$$

$$v_y = -4$$

$$v_z = 3$$

$$\vec{v}_{\text{net}} = 0\hat{i} - 4\hat{j} + 3\hat{k}$$

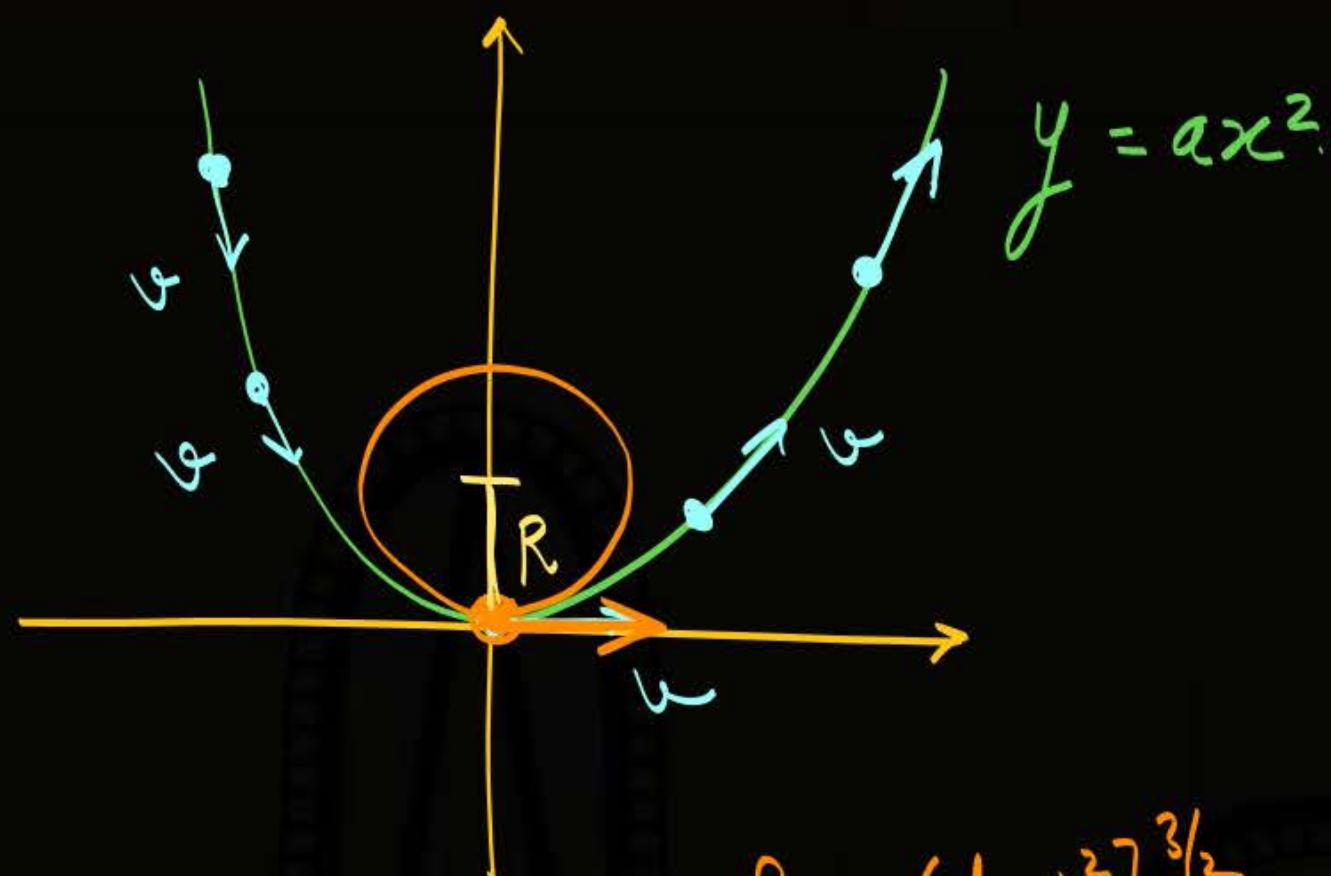
$$|v_{\text{net}}| = \sqrt{4^2 + 3^2}$$

$$= 5 \text{ m/s}$$



P
W

A particle moves at uniform speed on a parabolic trajectory $y = ax^2$ at uniform speed v . Find the acceleration of particle when it passes point $x = 0$ and point $(1, a)$. H.W.



$$R = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\left| \frac{d^2y}{dx^2} \right|}$$

$$x=0 \quad a=?$$

Since direction of v changing
so
acceleration

$$a_c = \frac{v^2}{R}$$

$$a_c = \frac{v^2}{R}$$

$$a_c = \frac{v^2}{(2a)}$$

$a_{cc} = 2av^2$

 Ans.

$$y = ax^2$$

$$R = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}^{3/2}$$

$\left| \frac{d^2y}{dx^2} \right|$

$$y = ax^2$$

$$\frac{dy}{dx} = 2ax \quad \text{at } x=0 \quad \frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 2a$$

$$R = \frac{\sqrt{1 + 0^2}}{2a}^{3/2}$$

$$R = \frac{1}{2a} \quad \text{at } x=0$$



The area of the parallelogram determined by

$$\vec{A} = 2\hat{i} + \hat{j} - 3\hat{k} \text{ and } \vec{B} = 12\hat{j} - 2\hat{k}$$

A 42

B 56

C 38

D 74

$$\text{Area of llgm} = |\vec{A} \times \vec{B}|$$

$$\begin{aligned}\vec{A} \times \vec{B} &= \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 12 & -2 & 0 \end{vmatrix} = \hat{i}(1 \times 0 - 6) - \hat{j}(2 \times 0 + 36) + \hat{k}(-4 - 12) \\ &= -6\hat{i} - 36\hat{j} - 16\hat{k}.\end{aligned}$$

$$\text{Area} = |\vec{A} \times \vec{B}| = \sqrt{6^2 + 36^2 + 16^2}$$



The values of x and y for which vectors

$\vec{A} = (6\hat{i} + x\hat{j} - 2\hat{k})$ and $\vec{B} = (5\hat{i} - 6\hat{j} - y\hat{k})$ may be parallel are :

A $x = 0, y = \frac{2}{3}$

C $x = -\frac{15}{3}, y = \frac{23}{5}$

B ~~$x = -\frac{36}{5}, y = \frac{5}{3}$~~ Anse

D $x = \frac{36}{5}, y = \frac{15}{4}$

1st Condition $\vec{A} \times \vec{B} = 0$ (Area of ||gm) = 0

$$\vec{A} = 6\hat{i} + x\hat{j} - 2\hat{k}$$

$$\vec{B} = 5\hat{i} - 6\hat{j} - y\hat{k}$$

$$\frac{6}{5} = \frac{x}{-6} = \frac{-2}{-y}$$

$\frac{-36}{5} = x$

$$\frac{3}{5} = \frac{2}{y}$$

$$y = \frac{5}{3}$$



The three conterminous edges of a parallelepiped are

$$\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k}, \vec{b} = 5\hat{j} \text{ and } \vec{c} = -2\hat{i} + \hat{k}.$$

The volume of parallelepiped is :

A 36 cubic unit

B 45 cubic unit

C 40 cubic unit *Ans*

D 54 cubic unit

$$\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k}$$

$$\vec{b} = 5\hat{j}$$

$$\vec{c} = -2\hat{i} + \hat{k}$$

Dot Prod.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{-j} & \hat{k} \\ 2 & -6 & 3 \\ 0 & 5 & 0 \end{vmatrix}$$

$$= \hat{i}(-15) - \hat{j}(0) + \hat{k}(10)$$

$$\boxed{\vec{a} \times \vec{b} = -15\hat{i} + 10\hat{k}}$$

PW
Volume = (area of base)h
Scalar

$$(\vec{A} \times \vec{B}) \cdot \vec{C}$$



Volume of 1st pipe

$$\text{Volume} = 15 \times 2 + 10$$

$$= 40 \text{ unit}$$

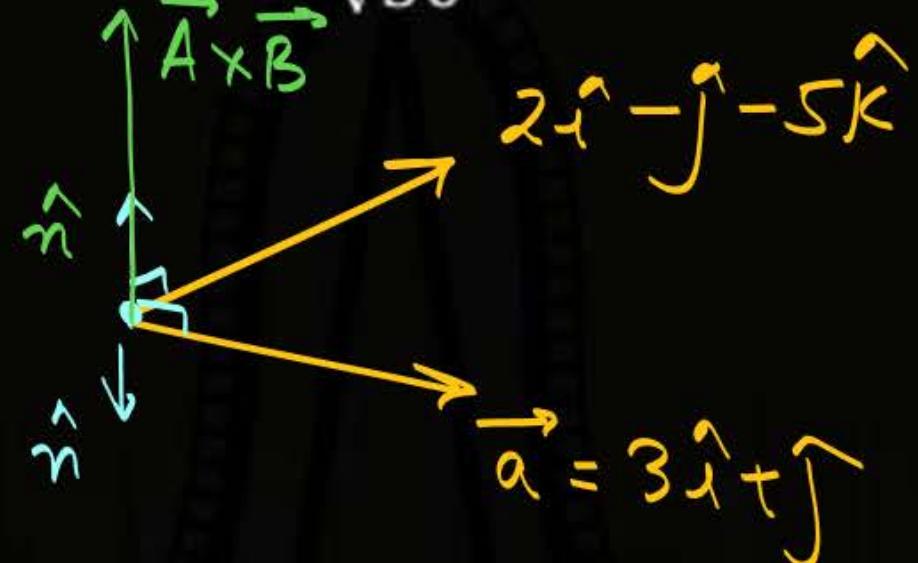


The unit vector perpendicular to vectors

$$\vec{a} = 3\hat{i} + \hat{j} \text{ and } \vec{b} = 2\hat{i} - \hat{j} - 5\hat{k}$$

A $\pm \frac{(\hat{i} - 3\hat{j} + \hat{k})}{\sqrt{11}}$

C $\pm \frac{(2\hat{i} - \hat{j} - 5\hat{k})}{\sqrt{30}}$



B $\pm \frac{3\hat{i} + \hat{j}}{\sqrt{11}}$

D none of these

unit vector \perp to $\vec{A} \times \vec{B}$ = direction of

$$1. \quad \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 3 & 1 & 0 \\ 2 & -1 & -5 \end{vmatrix} = \hat{i}(-5 - 0) - \hat{j}(15) + \hat{k}(-3 - 2)$$

$$= -5\hat{i} + 15\hat{j} - 5\hat{k},$$





A force $\vec{F} = (2\hat{i} + 3\hat{j} - \hat{k})$ N is acting on a body at a position $\vec{r} = (6\hat{i} + 3\hat{j} - 2\hat{k})$. The torque about the origin is:

A $(3\hat{i} + 2\hat{j} + 12\hat{k})\text{Nm}$ *Ans*

B $(9\hat{i} + \hat{j} + 7\hat{k})\text{Nm}$

C $(\hat{i} + 2\hat{j} + 12\hat{k})\text{Nm}$

D $(3\hat{i} + 12\hat{j} + \hat{k})\text{Nm}$

Sol:- $\vec{F} = 2\hat{i} + 3\hat{j} - \hat{k}$

$$\vec{r} = (6\hat{i} + 3\hat{j} - 2\hat{k})$$

$\vec{\tau} = \vec{r} \times \vec{F}$ *order is very Imp.*

$$\begin{aligned}
 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 3 & -2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(-3+6) - \hat{j}(-6+4) + \hat{k}(18- \\
 &\quad = 3\hat{i} + 2\hat{j} + 12\hat{k}
 \end{aligned}$$



The coordinates of a moving particle at any time t are given by, $x = 2t^3$ and $y = 3t^3$. Acceleration of the particle is given

A $468t$

B $t\sqrt{468}$

C $234t^2$

D $t\sqrt{234}$

$$x = 2t^3$$

$$x = f(t)$$

$$y = 3t^3$$

$$y = f(t)$$

2D Motion

$$v_x = \frac{dx}{dt} = 6t^2$$

$$a_x = \frac{dv_x}{dt} = 12t$$

$$v_y = \frac{dy}{dt} = 9t^2$$

$$a_y = \frac{dv_y}{dt} = 18t$$

$$\vec{a}_{net} = 12t\hat{i} + 18t\hat{j}$$

$$|a_{net}| = \sqrt{(12t)^2 + (18t)^2}$$

The trajectory of a projectile in vertical plane is $y = ax - bx^2$, where a and b are constants and x and y are respectively horizontal and vertical distances of the projectile from the point of projection. The maximum height attained by the particle and the angle of projection from the horizontal are θ .

A  $\frac{b^2}{4b}, \tan^{-1}(b)$

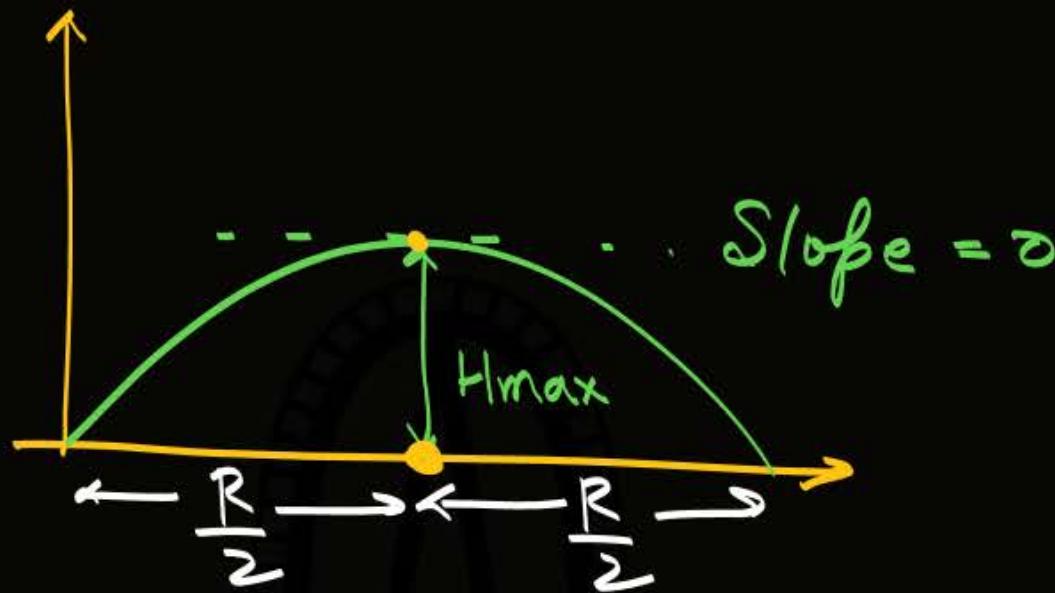
B  $\frac{a^2}{b}, \tan^{-1}(2b)$

C  $\frac{a^2}{4b}, \tan^{-1}(a)$ *(Ans)*

D  $\frac{2a^2}{b}, \tan^{-1}(a)$

$$y = ax - bx^2$$

$$y = ax - bx^2$$



$$y = ax - bx^2$$

$$\frac{dy}{dx} = a - 2bx$$

$$0 = a - 2bx$$

$$x = \frac{a}{2b}$$

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$\tan \theta = a$$

$$\theta = \tan^{-1}(a)$$

$$H_{\max} \text{ Slope} = 0,$$

$$\text{Range} = 2x = \frac{a}{b}$$

$$\text{at } x = \frac{a}{2b}$$

$$y = ? \Rightarrow H_{\max}$$

$$y = ax - bx^2$$

$$y = a\left(\frac{a}{2b}\right) - b\left(\frac{a^2}{4b^2}\right)$$

$$y = \frac{a^2}{2b} - \frac{a^2}{4b}$$

$$H_{\max} = \frac{a^2}{4b}$$



A cricket fielder can throw the cricket ball with a speed v_0 . If he throws the ball while running with speed u at an angle θ to the horizontal, what is the effective angle to the horizontal at which the ball is projected in air as seen by a spectator ?



$$A \quad \tan^{-1} \left[\frac{v_0 \cos \theta}{u + v_0 \sin \theta} \right]$$



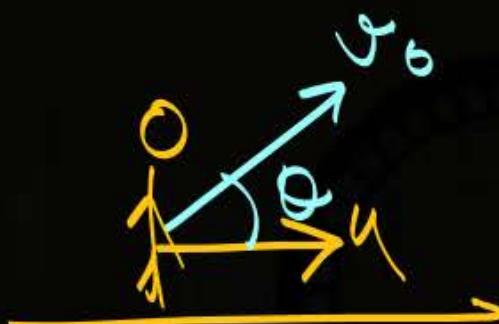
$$B \quad \tan^{-1} \left[\frac{v_0 \sin \theta}{u + v_0 \cos \theta} \right] \quad \text{Ans}$$

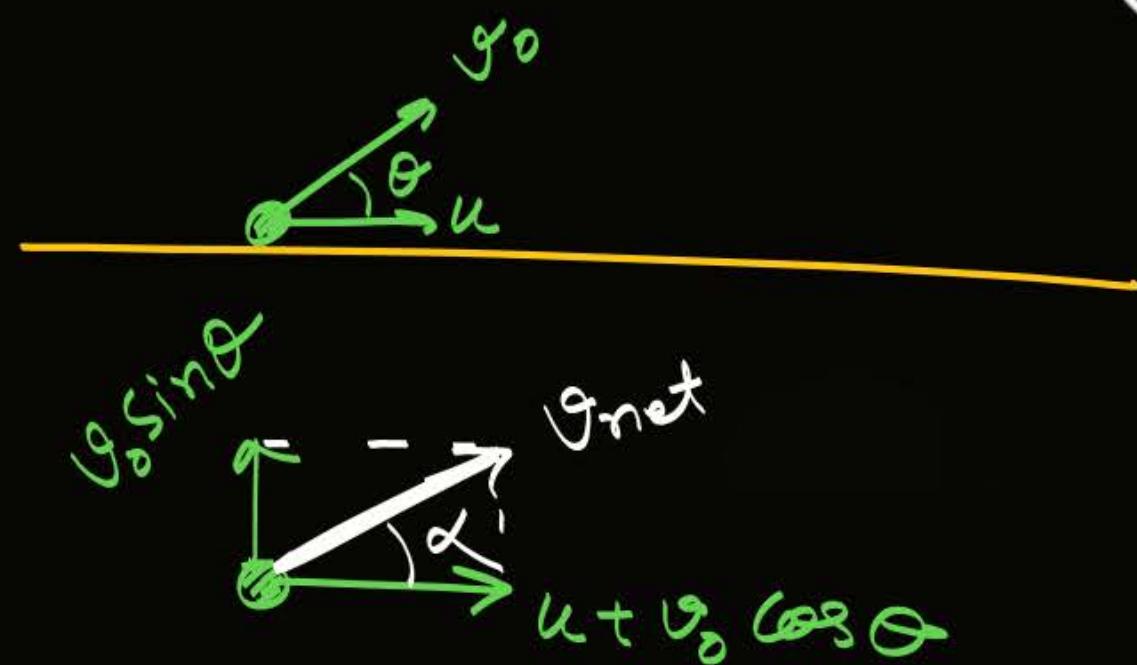
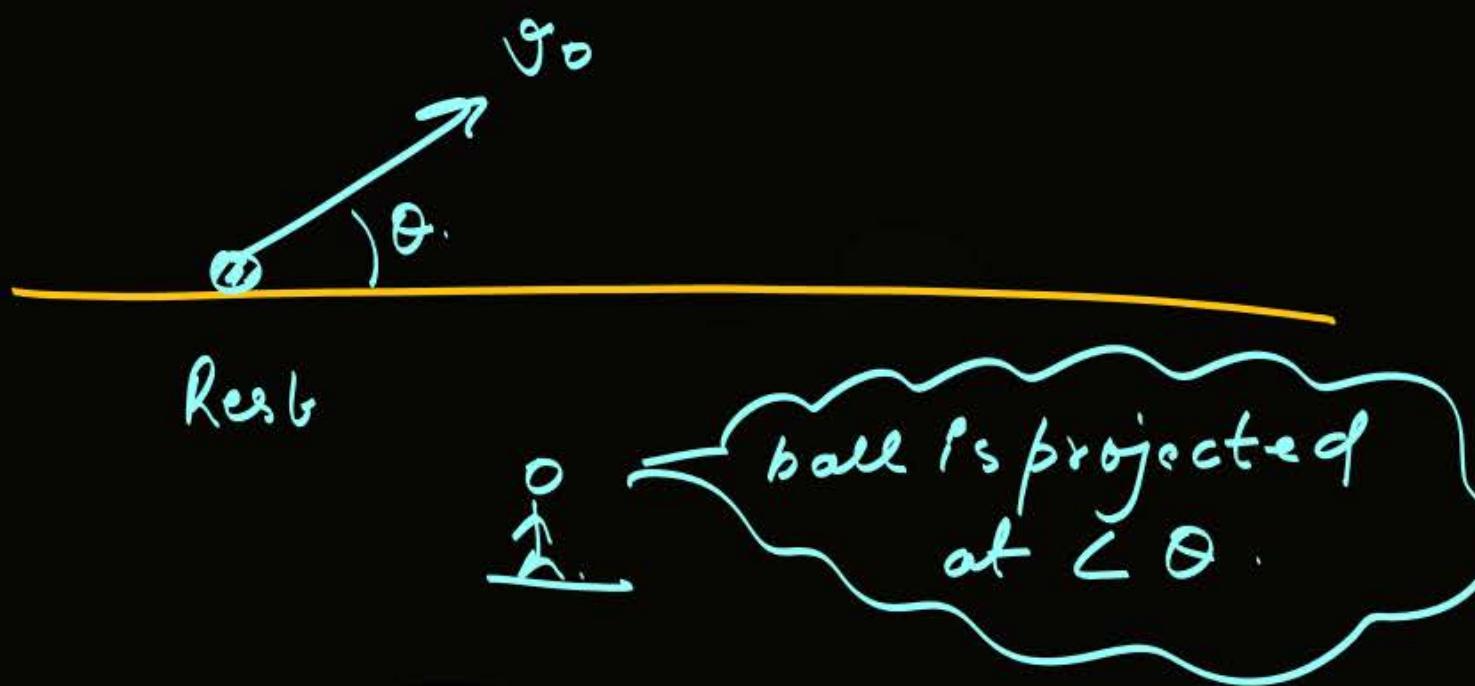


$$C \quad \tan^{-1} \left[\frac{u}{v_0 \cos \theta + v_0 \sin \theta} \right]$$



$$D \quad \tan^{-1} \left[\frac{v_0 \sin \theta + v_0 \cos \theta}{u} \right]$$





$$\tan \alpha = \frac{P}{B} = \frac{v_0 \sin \theta}{u + v_0 \cos \theta}$$

$$\alpha = \tan^{-1} \left(\frac{v_0 \sin \theta}{u + v_0 \cos \theta} \right)$$



The initial velocity of a particle of mass 2 kg is $(4\hat{i} + 4\hat{j}) \text{ ms}^{-1}$. A constant force of $-20\hat{j}$ N is applied on the particle. Initially the particle was at $(0, 0)$. Find the x-coordinate of the point where its y-coordinate is again zero.

A 3.2 m AQ

C 4.8 m

$$\vec{u} = 4\hat{i} + 4\hat{j}$$

$$t = 0$$

$$x = 0$$

$$F = -20\hat{j}$$

$$\frac{F}{m} = a = -10\hat{j}$$

Method 1
done previously

B 6 m

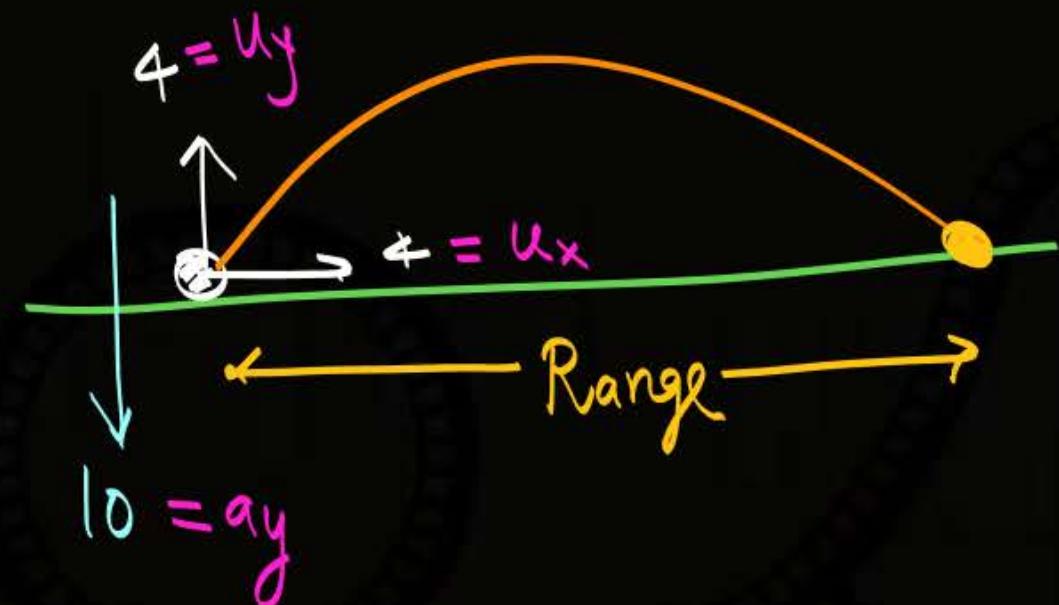
D 1.2 m

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R = \frac{2u_x u_y}{g}$$

$$= \frac{\cancel{2} \times 4 \times 4}{10 \cancel{5}} = \frac{16}{5} \text{ m}$$

=





The coordinates of a moving particle at any time t are given by $x = ct$ and $y = bt^2$. The speed of the particle is given by

A $2t\sqrt{b^2 - c^2}$

Ans B $\sqrt{4b^2t^2 + c^2}$

C $2t(b + c)$

D $2t(b - c)$

$$x = ct \quad v_x = c$$

$$y = bt^2 \quad v_y = 2bt$$

$$\vec{v}_{net} = c\hat{i} + 2bt\hat{j}$$

$$|v_{net}| = \sqrt{c^2 + 4b^2t^2}$$





Balls A and B are thrown from two points lying on the same horizontal plane separated by a distance 120 m. Which of the following statement (s) is/are correct?

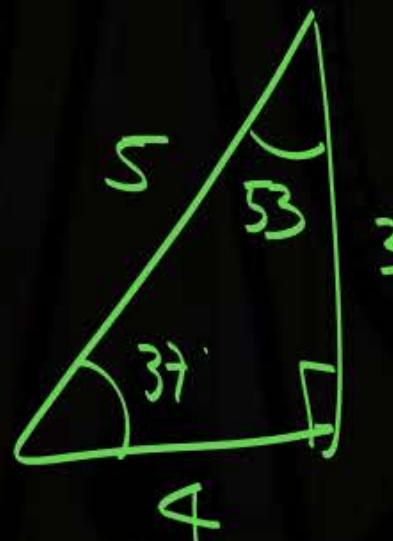
(Q Projectile)

A The two balls can never meet

B The balls can meet if the ball B is thrown 1s later

C Ans The two balls meet at a height of 45 m

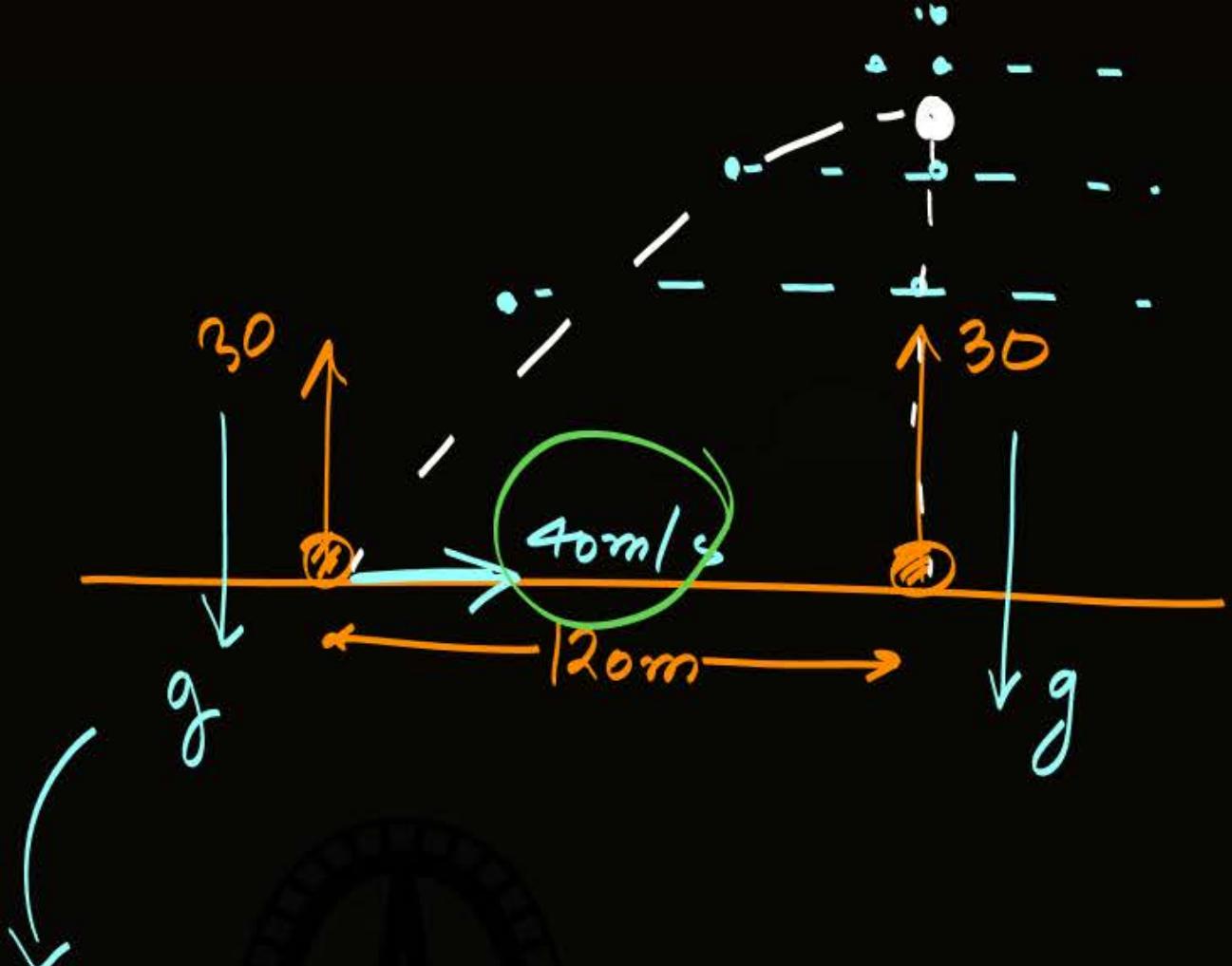
D None of the above



$$30 \text{ m/s} = 50 \sin 37^\circ$$



$$50 \times \frac{4}{5} = 40 \text{ m/s}$$



$$a_{rel} = 0$$

$u_{rel} = 0$ along Y

40 m/s \Rightarrow Move A towards B.

$$\text{Time of flight} = \frac{2u_y}{g}$$

$$T_A = T_B$$

$$T = \frac{2 \times 30}{10} = 6s$$

$$t = \frac{120}{40} = 3s \Rightarrow \text{Time to reach}$$

$$H_{\max} = \frac{u_y^2}{2g} = \frac{30 \times 30}{2 \times 10} = 45m$$



When the angle of projection is 75° , a ball falls 10 m shorter of the target. When the angle of projection is 45° , it falls 10 m ahead of the target. Both are projected from the same point with the same speed in the same direction, the distance of the target from the point of projection is

A 15 m

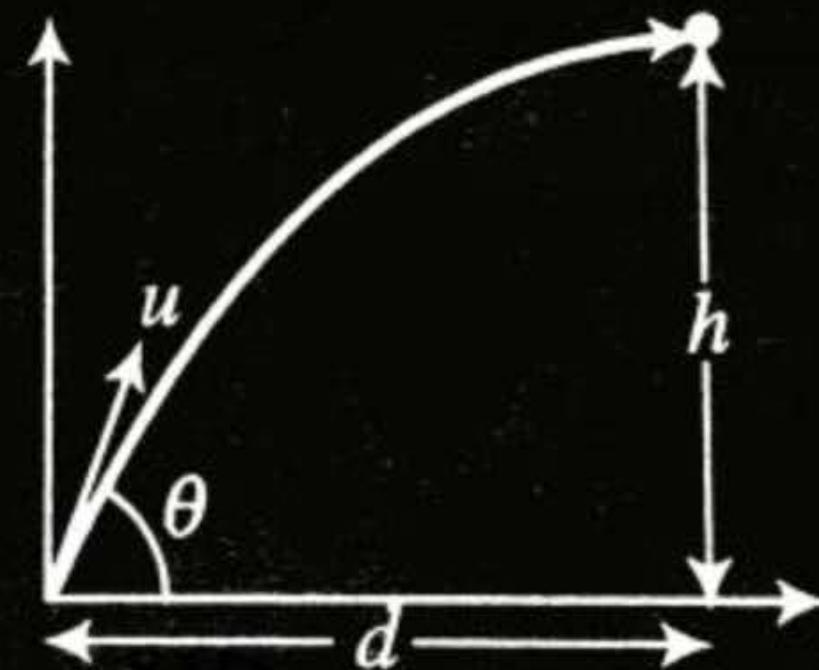
B 30 m

C 45 m

D 10 m



If a stone is to hit at a point which is at a distance d away and at a height h above the point from where the stone starts, then what is the value of initial speed u , if the stone is launched at an angle θ ?



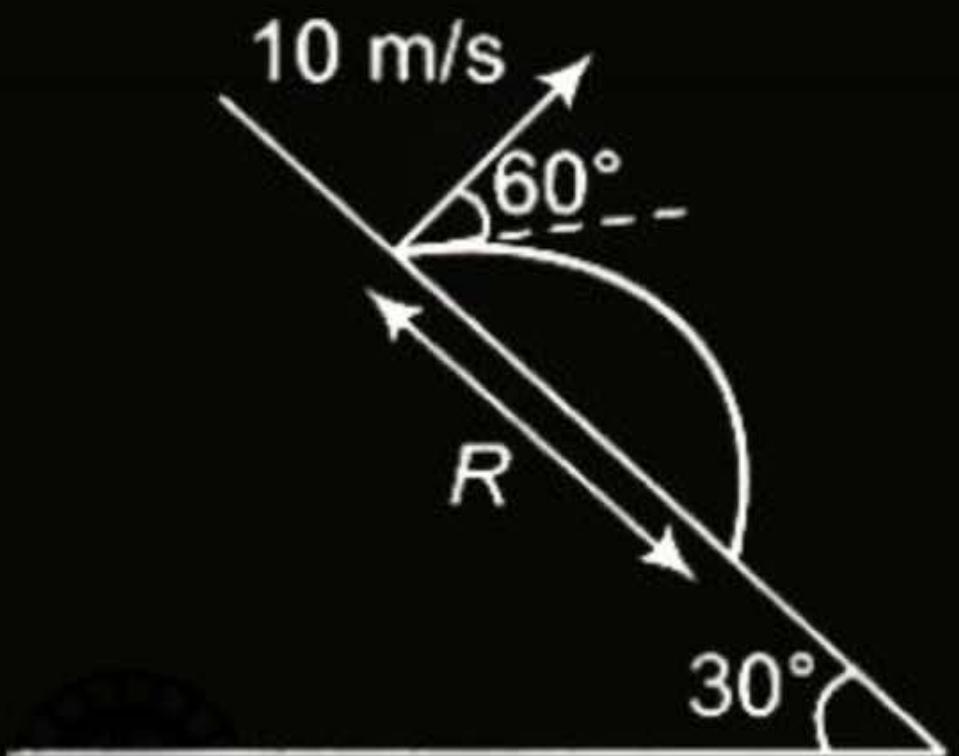
A projectile is launched with a speed of 10 m/s at an angle 60° with the horizontal from a sloping surface of inclination 30° . The range R is. (Take, $g = 10 \text{ m/s}^2$)

A  4.9 m

B  13.3 m

C  9.1 m

D  12.6 m



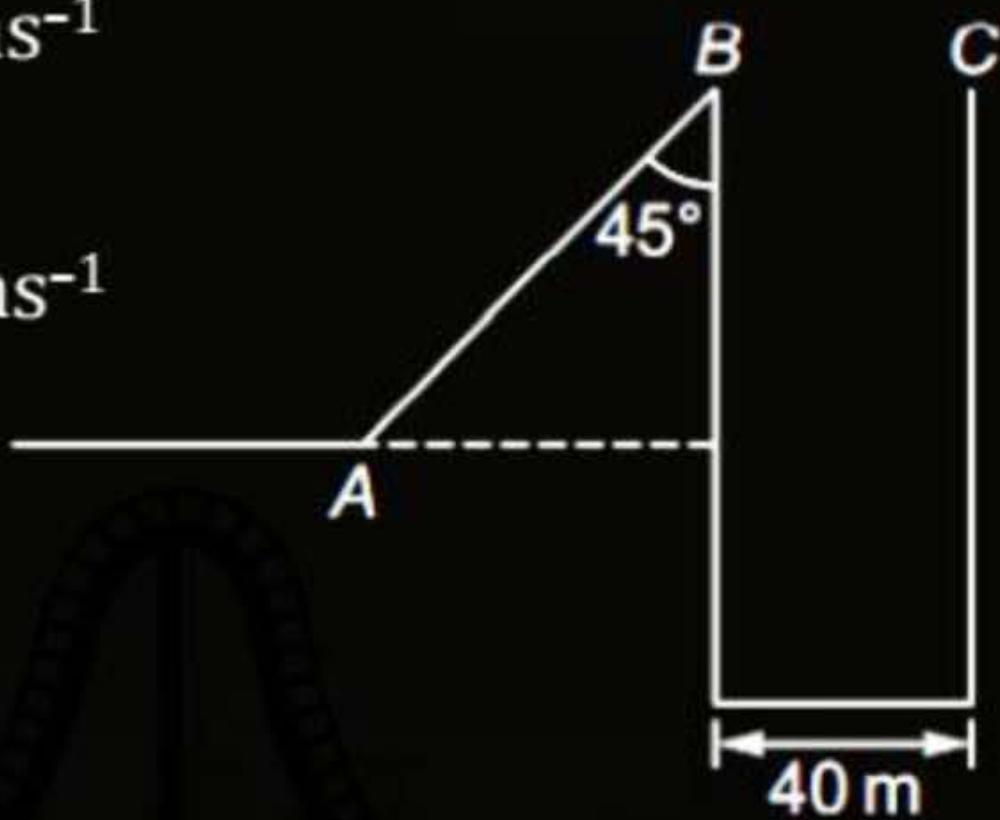
A body is projected up smooth inclined plane with a velocity v_0 from the point A as shown figure. The angle of inclination is 45° and top B of the plane is connected to a well of diameter 40 m. If the body just manages to cross the well,

A 20 ms^{-1}

C 40 ms^{-1}

B $20\sqrt{2} \text{ ms}^{-1}$

D $40\sqrt{2} \text{ ms}^{-1}$





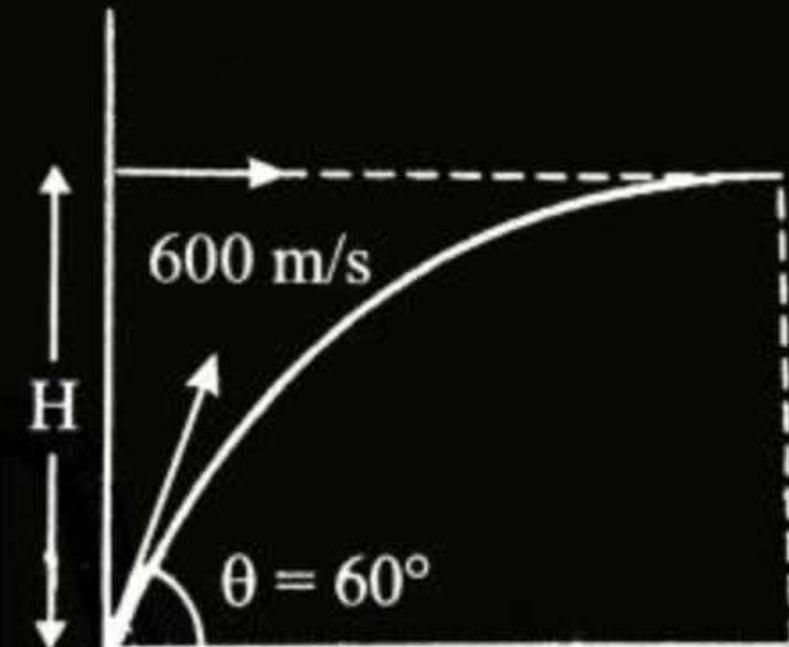
A fighter plane enters inside the enemy territory, at time $t = 0$ with velocity $v_0 = 250 \text{ ms}^{-1}$ and moves horizontally with constant acceleration $a = 20 \text{ ms}^{-2}$ (see figure). An enemy tank at the border, spot the plane and fire shots at an angle $\theta = 60^\circ$ with the horizontal and with velocity $u = 600 \text{ ms}^{-1}$. At what altitude H of the plane it can be hit by the shot?

A $1500\sqrt{3} \text{ m}$

C 1400 m

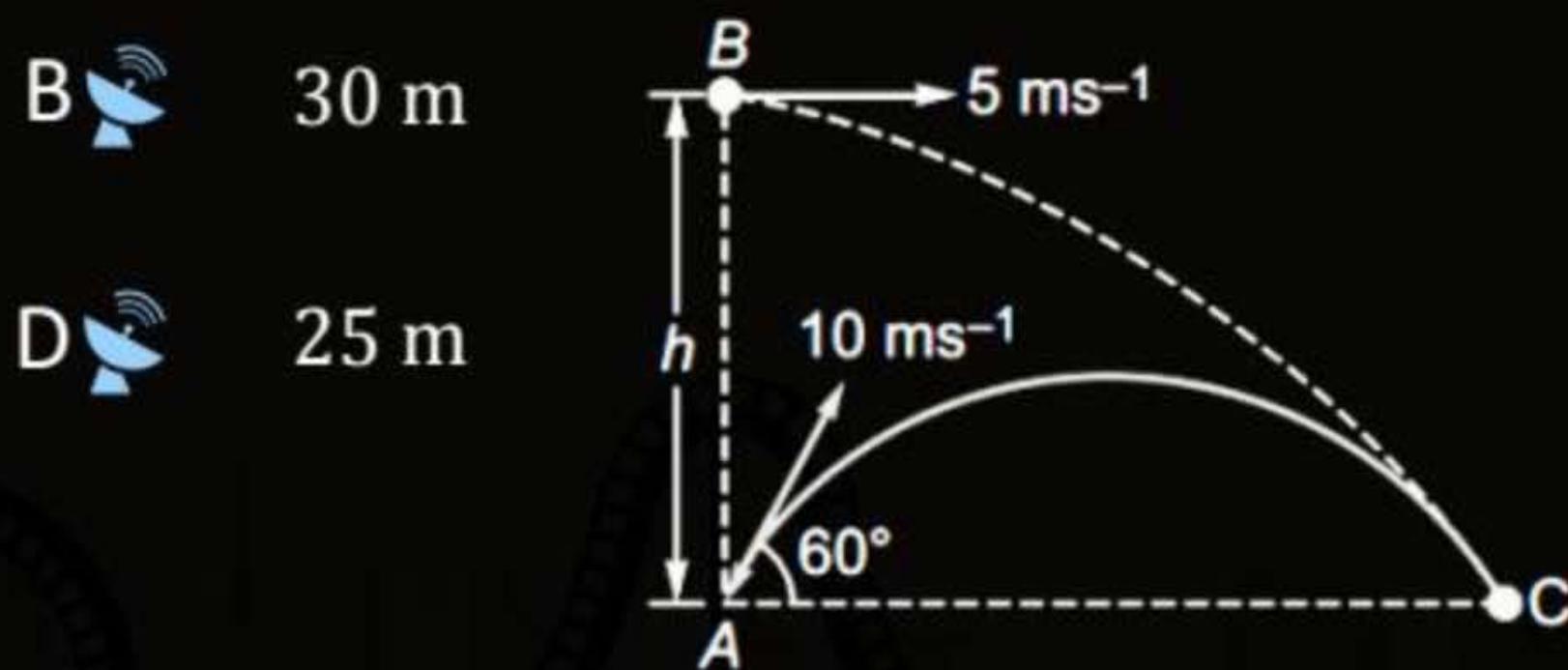
B 125 m

D 2473 m





A particle A is projected from the ground with an initial velocity of 10 ms^{-1} at an angle of 60° with horizontal. From what height h should another particle B be projected horizontal with velocity 5 ms^{-1} so that both the particles collide with velocity 5 ms^{-1} so that both the particles collide on the ground at point C is both are projected simultaneously? ($g = 10 \text{ ms}^{-2}$)



Two boys are standing at the ends A and B of a ground where $AB = a$. The boy at B starts running in a direction perpendicular to AB, with velocity v_1 . The boy at A starts running simultaneously with velocity v and catches the other boy in a time t , then find t .



A particle is moving with a velocity of 10 m/s towards east. After 10 s its velocity changes to 10 m/s towards north. Its average acceleration is:-

- A Zero
- B $\sqrt{2}$ m/s² towards N-W
- C $1/\sqrt{2}$ m/s² towards N-E
- D $1/\sqrt{2}$ m/s² towards N-W





When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v , he sees that rain drops coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45° . The value of β is close to :

[SEP 2020]

A 0.41

B 0.50

C 0.73

D 0.37



Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where \hat{i} points east and \hat{j} north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in :

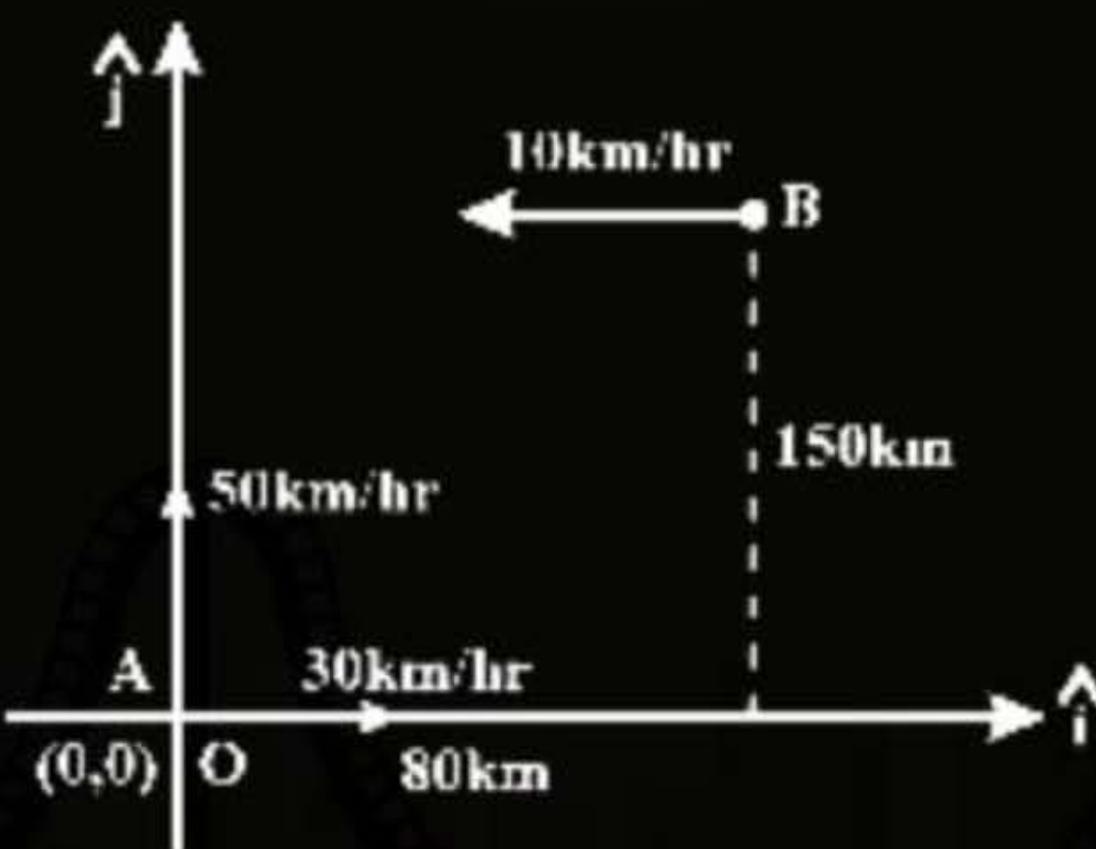
[APRIL 2019]

A 3.2 hrs

C 4.2 hrs

B 2.6 hrs

D 2.2 hrs





A glass wind screen whose inclination with the vertical can be changed is mounted on a car. The car moves horizontally with a speed of 2m/s. At what angle α with the vertical should the wind screen be placed so that the rain drops falling vertically downwards with velocity 6m/s strike the wind screen perpendicularly?

A $\tan^{-1}(3)$

B $\tan^{-1}(4)$

C $\tan^{-1}(1/3)$

D $\tan^{-1}(1/4)$



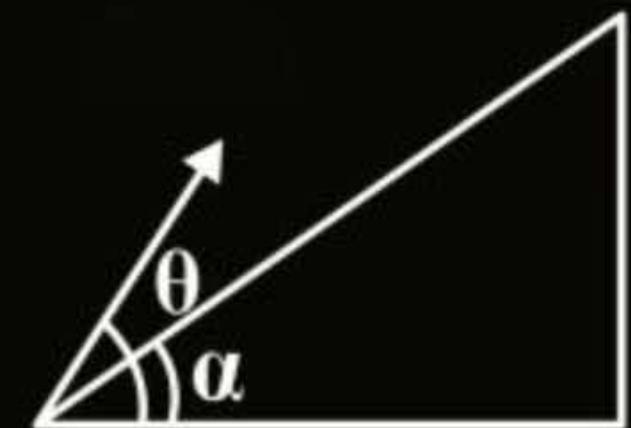
A projectile is fixed at an angle θ with the horizontal, (as shown in the figure), condition under which it lands perpendicular on an inclined plane of inclination α is

A $\sin \alpha = \cos (\theta - \alpha)$

B $\cos \alpha = \sin (\theta - \alpha)$

C $\tan \alpha = \cot (\theta - \alpha)$

D $\cot (\theta - \alpha) = \sin \alpha$





Rain is falling vertically with speed of 35 m/s. A woman rides a bicycle with a speed of 15 m/s in East to West direction. What is the direction in which she should hold her umbrella?



- A $\tan^{-1} \left(\frac{3}{7} \right)$ with the vertical towards west
- B $\tan^{-1} \left(\frac{4}{7} \right)$ with the vertical towards east
- C $\tan^{-1} \left(\frac{5}{7} \right)$ with the vertical towards east
- D Towards North downward



A ship A is moving towards the South with a speed 20 km per hour and another ship is moving towards the East with a speed of 20 km per hour. At a certain instant the Ship B is due south of ship A and is at a distance of 10 km from ship A find the shortest distance between the ships and the time after which they are closest to each other



One day in still air, a motor-cyclist riding north at 30 m/s, suddenly the wind starts blowing westward with a velocity 50 m/s, then the apparent velocity with which the motor-cyclist will move, is:

A 58.3 m/s

B 65.4 m/s

C 73.2 m/s

D 53.8 m/s



A motorboat is racing towards north at 25 km/h and the water current in that region is 10 km/h in the direction of 60° east of south. The resultant velocity of the boat is

A 10 km/h

B 20 km/h

C 12 km/h

D 22 km/h





Position of particle as a function of time is given as $\vec{r} = \cos\omega t\hat{i} + \sin\omega t\hat{j}$. Choose correct statement about \vec{r} , \vec{v} and \vec{a} , where \vec{v} and \vec{a} are velocity and acceleration of particle at time t.

[JAN 2020]

- A \vec{v} is perpendicular to \vec{r} and \vec{a} is towards origin
- B \vec{v} and \vec{a} are perpendicular to \vec{r}
- C \vec{v} is parallel to \vec{r} and \vec{a} parallel to \vec{r}
- D \vec{v} is perpendicular to \vec{r} and \vec{a} is away from origin.





Train A and train B are running on parallel tracks in the opposite direction with speed of 36 km/hour and 72 km/hour , respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hour. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible)

(SEP 2020)

A 30.5 ms^{-1}

B 29.5 ms^{-1}

C 31.5 ms^{-1}

D 28.5 ms^{-1}



A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when :
(i) they are moving in the same direction, and (ii) in the opposite directions is

(JAN 2019)

A  5/2

B  25/11

C  3/2

D  11/5



Two particles A and B separated by distance 10m with A ahead of B. A starts from rest with uniform acceleration 2m/s^2 , whereas B is moving with constant velocity of 40m/s . Find time of overtaking





An elevator is accelerating upward at a rate of 6 ft/sec^2 when a bolt from its ceiling falls to the floor of the lift (Distance = 9.5 feet). The time (in seconds) taken by the falling bolt to hit the floor is (take $g = 32 \text{ ft/sec}^2$)

A $\sqrt{2}$

B $\frac{1}{\sqrt{2}}$

C $2\sqrt{2}$

D $\frac{1}{2\sqrt{2}}$





Two bodies are held separated by 9.8 m vertically one above the other. They are released simultaneously to fall freely under gravity. After 2 s the relative distance between them is

A 4.9 m

B 19.6 m

C 9.8 m

D 392 m



Two balls are thrown simultaneously, (A) vertically upwards with a speed of 20 m/s from the ground and (B) vertically downwards from a height of 40 m with the same speed and along the same line of motion. At which point will the balls collide? (take $g = 10 \text{ m/s}^2$)



- A 15 m above from the ground
- B 15 m below from the top of the tower
- C 20 m above from the ground
- D 20 m below from the top of the tower



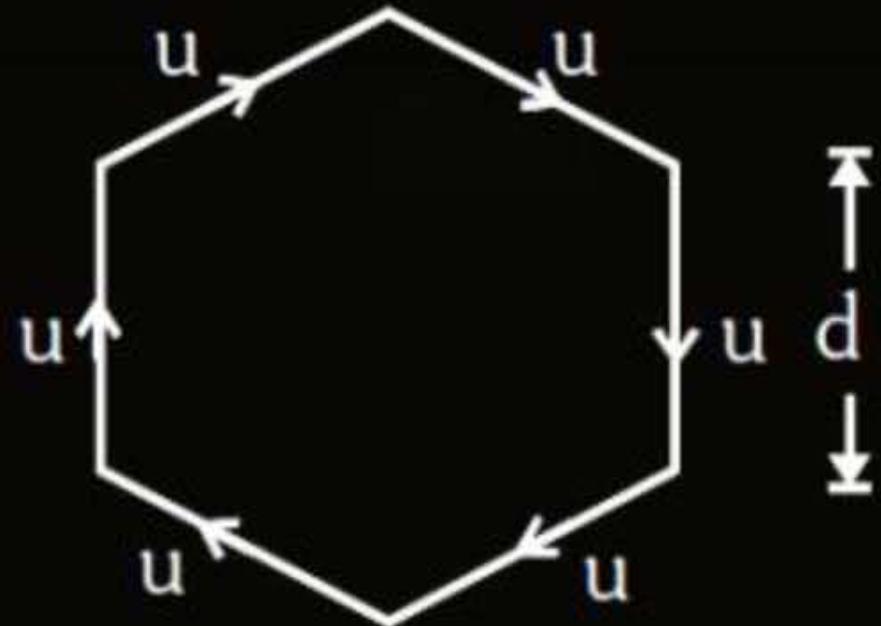
Six persons of same mass travel with same speed u along a regular hexagon of side ' d ' such that each one always faces the other. After how much time will they meet each other?

A $\frac{d}{u}$

C $\frac{2d}{u}$

B $\frac{2d}{3u}$

D $d\sqrt{3}u$





Two cars A and B start moving from the same point with same speed $v = 5 \text{ km/minute}$. Car A moves towards North and car B is moving towards East. What is the relative velocity of B with respect to A?



- A $5\sqrt{2} \text{ km/min}$ towards South-East
- B $5\sqrt{2} \text{ km/min}$ towards North-West
- C $5\sqrt{2} \text{ km/min}$ towards South-West
- D $5\sqrt{2} \text{ km/min}$ towards North-East



A man wishes to swim across a river 0.5 km wide. If he can swim at the rate of 2 km/h in still water and the river flows at the rate of 1 km/h. The angle made by the direction (w.r.t. the flow of the river) along which he should swim so as to reach a point exactly opposite his starting point, should be:

A 60°

B 120°

C 145°

D 90°



A boat-man can row a boat to make it move with a speed of 10 km/h in still water. River flows steadily at the rate of 5 km/h. and the width of the river is 2 km. If the boat man cross the river along the minimum distance of approach then time elapsed in rowing the boat will be:

A

$$\frac{2\sqrt{3}}{5} \text{ h}$$

B

$$\frac{2}{5\sqrt{3}} \text{ h}$$

C

$$\frac{3\sqrt{2}}{5} \text{ h}$$

D

$$\frac{5\sqrt{2}}{3} \text{ h}$$



THANK
YOU!