REVISION

For Class - 11th

Lecture no. 02

Motion in 2 Dimension

(Part-2).

Rajwant Singh

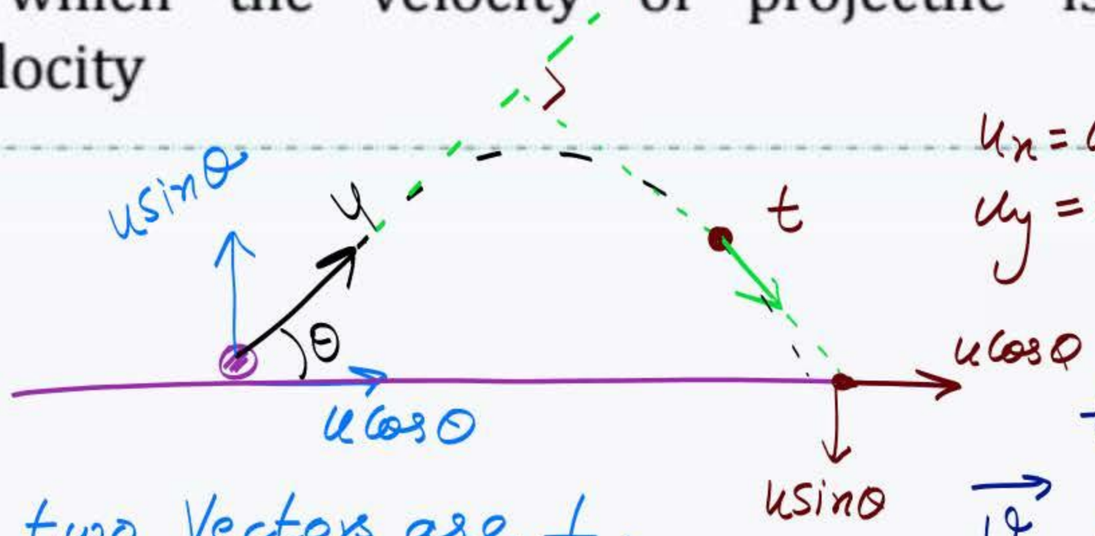
A projectile is thrown upward with velocity u at angle θ. Find the time after projection at which the velocity of projectile is perpendicular to its initial velocity

A (u sin θ)/g

B (u cosec θ)/g Ans

C (u tan θ)/g

D (u cot θ)/g



$u_x = u\cos\theta \quad a_x = 0$

$u_y = u\sin\theta \quad a_y = -g$

$t = ?$

$\vec{v}_t$ is ⊥ to $\vec{v}_i$

When two Vectors are ⊥.

$$\boxed{\vec{A}\cdot\vec{B} = 0}$$

$\vec{u}_i = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$\vec{v} = v_x\hat{i} + v_y\hat{j}$

$\vec{v} = (u_x + a_x t)\hat{i} + (u_y + a_y t)\hat{j}$

$$\boxed{\vec{v} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}}$$

at time t $\vec{u} \cdot \vec{v} = 0$

$$(u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j})(u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}) = 0$$

$$u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta\, gt = 0$$

$$u^2(\cos^2\theta + \sin^2\theta) - ugt\sin\theta = 0$$

$$u^2 = ugt\sin\theta$$

$$\boxed{\frac{u}{g}\operatorname{cosec}\theta = \frac{u}{g\sin\theta} = t}$$

If $\vec{P} + \vec{Q} = \vec{R}$ and $|\vec{P}| = |\vec{Q}| = \sqrt{3}$ and $|\vec{R}| = 3$, then the angle between $\vec{P}$ and $\vec{Q}$ is :

A 0

B $\pi/6$

C $\pi/3$ Ans

D $\pi/2$

$\vec{P} + \vec{Q} = \vec{R}$

when we add $\vec{P}$ & $\vec{Q}$ vectorially there Resultant R.

$|P| = |Q| = \sqrt{3}$

$|R| = 3$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$3 = \sqrt{3 + 3 + 2 \times 3 \cos\theta}$

$\theta =$ ____

$\theta = 60^\circ$ $R = |P|\sqrt{3}$

$= \sqrt{3} \cdot \sqrt{3} = 3$

$\pi/3$

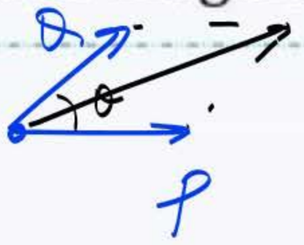
The sum of magnitudes of two forces acting at a point is 16 and magnitude of their resultant is $8\sqrt{3}$. If the resultant is at 90° with the force of smaller magnitude, their magnitudes are

A 3, 13

B 2, 14

C 5, 11

D 4, 12

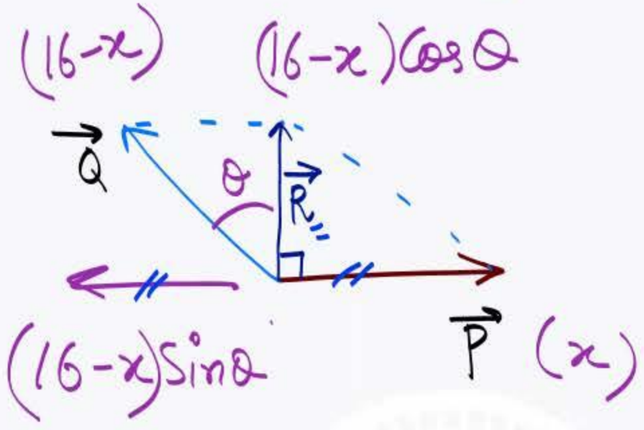


$|P| + |Q| = 16$

$|P| = x$

$|Q| = 16 - x$

$|R| = 8\sqrt{3}$



along X $F_{net} = 0$

along Y $F_{net} = 8\sqrt{3}$

$x = (16 - x)\sin\theta$

$8\sqrt{3} = (16 - x)\cos\theta$

$\sin\theta = \frac{x}{16 - x}$

$\cos\theta = \frac{8\sqrt{3}}{16 - x}$

Sq & adding

$x =$ _____

Two forces, each equal to F, act as shown in figure. Their resultant is

A $F/2$

B F Ans

C $\sqrt{3}\,F$

D $\sqrt{5}\,F$

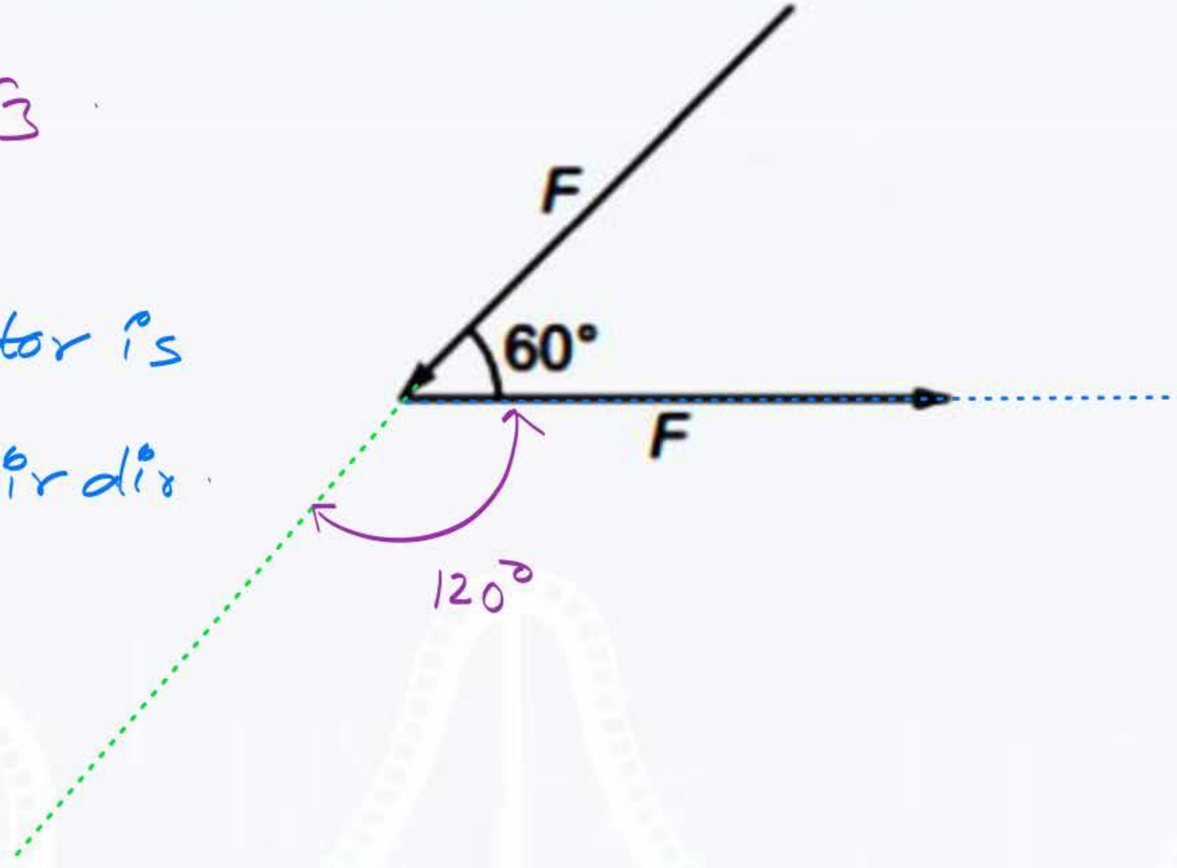
Trick.

$\theta = 60^\circ \quad R = F\sqrt{3}$

Angle between Vector is angle between their dir.

$\theta = 120^\circ \quad R = P$

$R = F$



Resultant of which of the following may be equal to zero ?

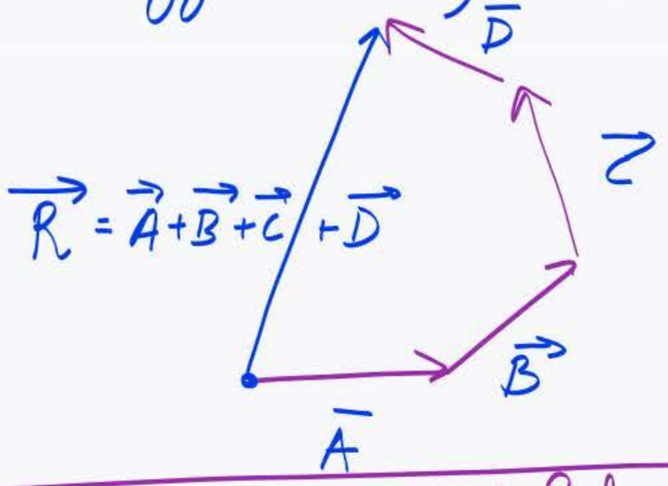
A 10 N, 10 N, 10 N

B 10 N, 10 N, 25 N

C 10 N, 10 N, 35 N

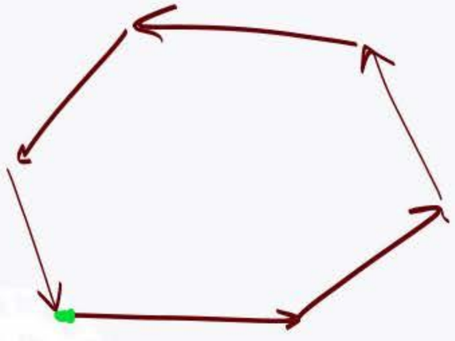
D None of these

Polygon Law of Vectors



$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

$\Rightarrow$ If we have a Closed Polygon



When $R = 0$ we need to Get a Closed Polygon

Symmetric or Asymmetric.

Order Closed polygon

$R = 0$

In our Case Polygon $\rightarrow \Delta$.

Sum of two Sides $\geq$ Third Side.

Resultant of which of the following may be equal to zero ?

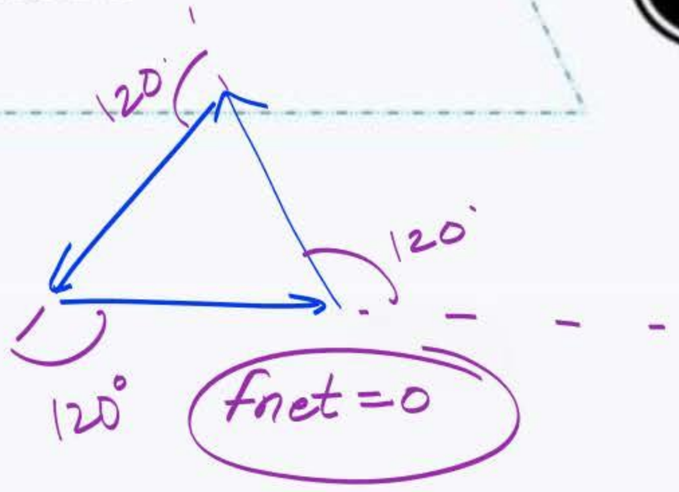
A. 10 N, 10 N, 10 N

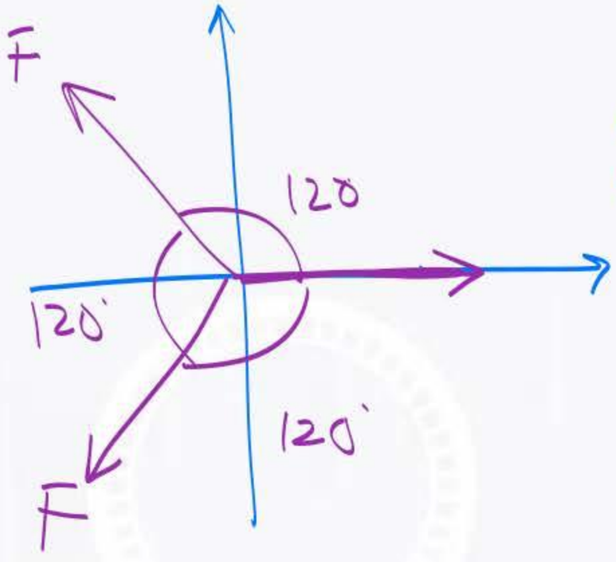
B. 10 N, 10 N, 25 N

C. 10 N, 10 N, 35 N

D. None of these

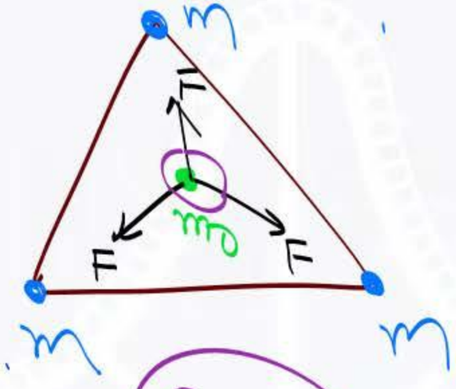
$10 + 10 < 25$ ✗





gravitational

Symmetrical diagram



$F_{net} = 0$

$I_{net} = 0$

$F_{net} = 0$

$\vec{E} = 0$

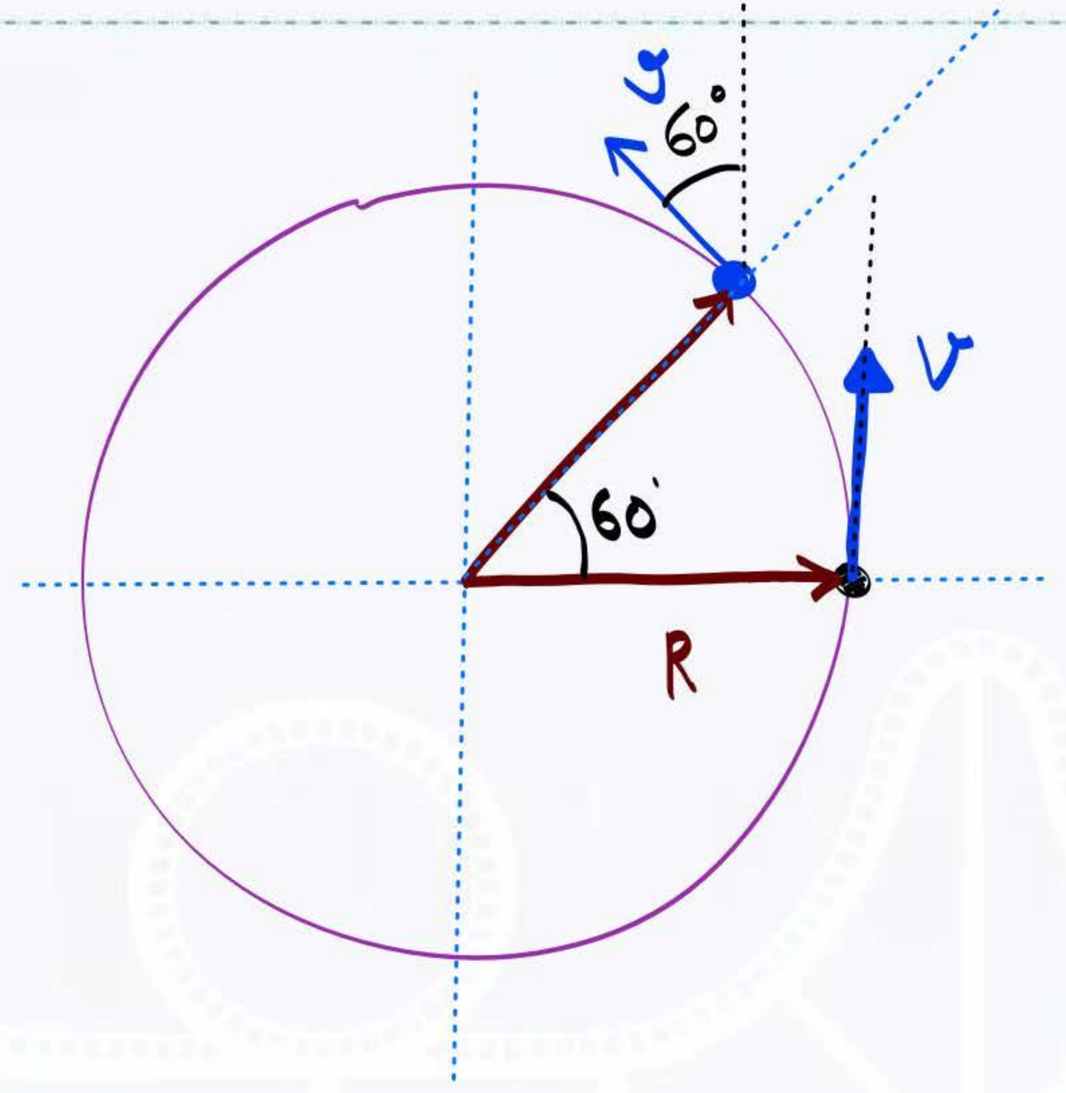
A cyclist is moving on a circular path with constant speed. What is the change in its velocity after it has described an angle of 60° ?

A $v\sqrt{2}$

B v Ans

C $v\sqrt{3}$

D None of these

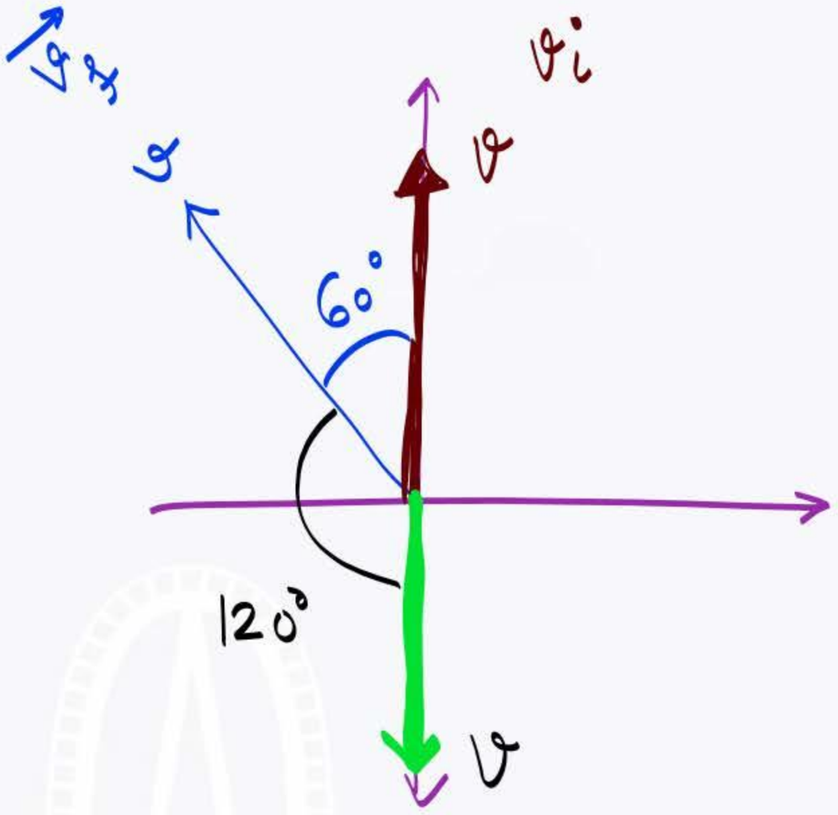


Change in velocity

$\overrightarrow{\Delta v} = \overrightarrow{v_f} - \overrightarrow{v_i}$

Subtraction of Vectors.

Velocity = Vector



$\vec{v}_f - \vec{v}_i$

$\vec{v}_f + (-\vec{v}_i)$

$|A| = |B| = P$

$\theta = 120 \quad R = P.$

$|\text{Change in velocity}| = v.$

Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is : **[JAN 2019]**

A 1 : 2

B 1 : 4

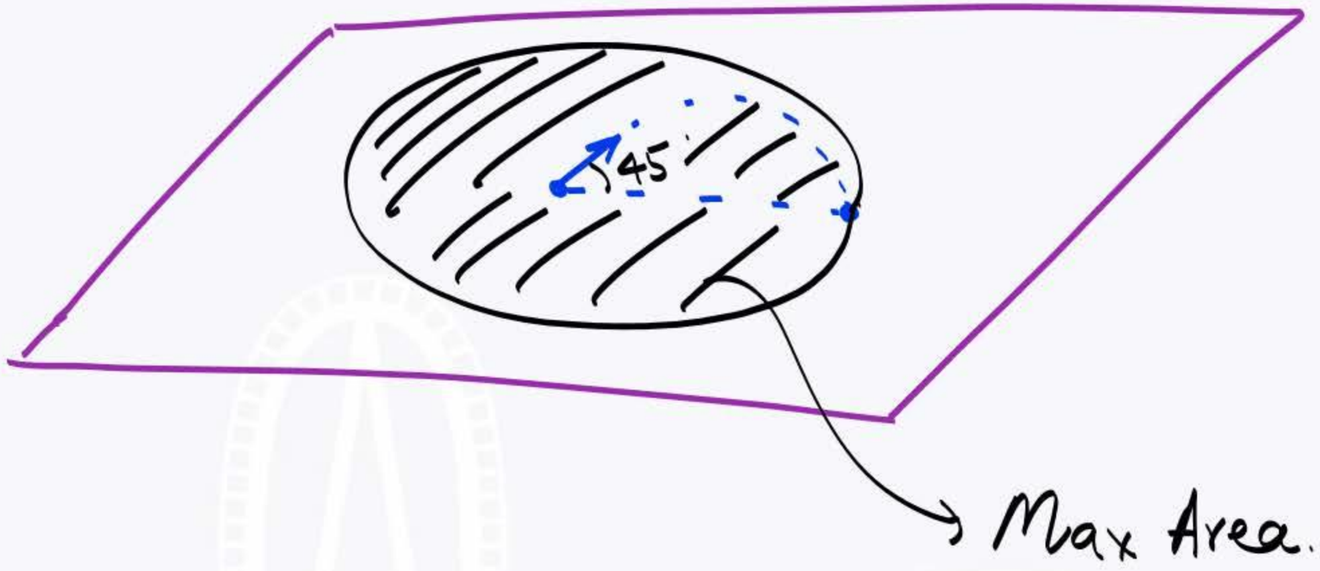
C 1 : 8

D 1 : 16 Ans

$$\frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \frac{1}{16}.$$

Topics to be covered

1. Vectors
2. Projectile Motion
3. Relative Velocity
4. Questions



Range → max

$\theta = 45^\circ$

$R_{max} = \frac{u^2}{g}$

for Max Area. = radius of circle = R_{max}

$\text{Area} = \pi r^2 = \pi R_{max}^2 = \frac{\pi u^4}{g^2}$

A ball is thrown from a point with a speed 'v_0' at an elevation angle of θ. From the same point and at the same instant, a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball ? If yes, what should be the angle of projection θ ? **[2004]**

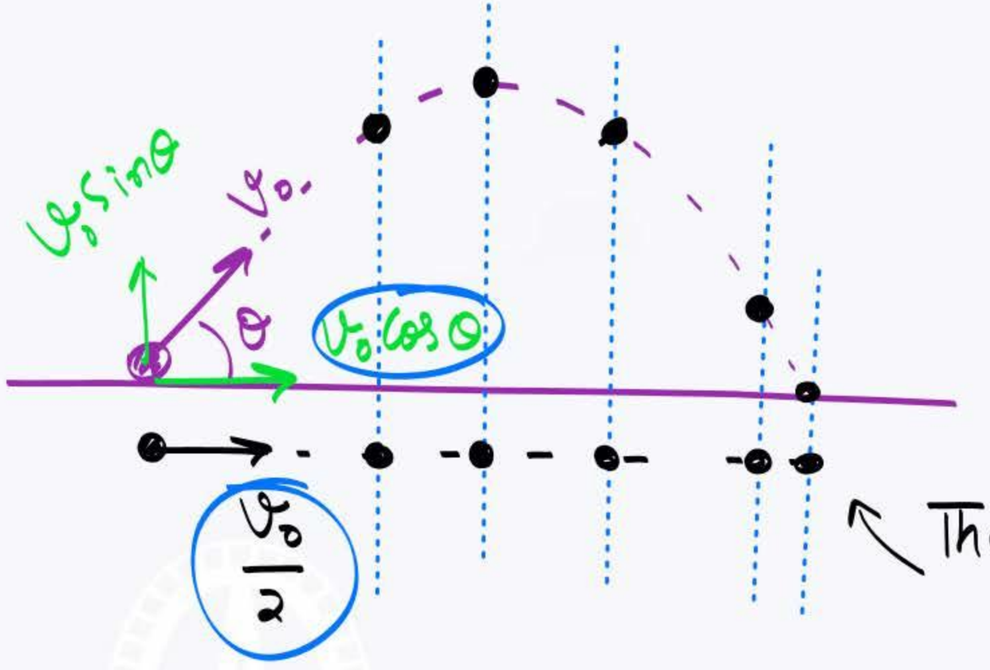
A No

B Yes, 30°

C Yes, 60° Ans

D Yes, 45°

$a_x = 0$



Then he Can Catch.

$$\frac{v_0}{2} = v_0 \cos\theta$$

$$\frac{1}{2} = \cos\theta$$

$$\boxed{\theta = 60^\circ}$$

A particle starts moving from origin with velocity $\vec{u} = 3\hat{i}$ from origin and acceleration $\vec{a} = 6\hat{i} + 4\hat{j}$. Find x- coordinate at the instant when y – coordinate of the particle is 32 m **[JAN 2020]**

A 48

B 60 Ans

C 12

D 24

$t = 0$
$x = 0$
$u = 0$

$u = 3\hat{i}$
$a = 6\hat{i} + 4\hat{j}$.

There is angle between $\vec{v}$ & $\vec{acc}$
(2D Motion)

$y = 32$ $x = ?$

$u_x = 3$ $u_y = 0$
$a_x = 6$ $a_y = 4$

X u_x a_x

$V_x = u_x + a_x t$
$S_x = u_x t + \frac{1}{2} a_x t^2$
$V_x^2 - u_x^2 = 2a_x S_x$

Y (u_y, a_y)

$V_y = u_y + a_y t$
$S_y = u_y t + \frac{1}{2} a_y t^2$
$V_y^2 - u_y^2 = 2a_y S_y$.

Lets find at time $y = 32$?

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$32 = 0 + \frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}} t^2$$

$$16 = t^2$$

$$t = 4s$$

at $t = 4s$

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$= 3 \times 4 + \frac{1}{\cancel{2}} \times 6 \times \overset{2}{\cancel{4}} \times 4$$

$$= 12 + 6 \times 8$$

$$= 12 + 48$$

$$= 60m$$

Alternative

Equation of trajectory.

$x = f(t)$
$y = f(t)$ Time Eliminate $y = f(x)$

Let particle Reach (x, y) at time t

$S_x = u_x t + \frac{1}{2} a_x t^2$

$x = 3t + \frac{1}{2} \times 6t^2$

$\boxed{x = 3t + 3t^2}$

$S_y = u_y t + \frac{1}{2} a_y t^2$

$y = \frac{1}{2} \times 4 \times t^2$

$\boxed{y = 2t^2}$

$t = \frac{y^{1/2}}{\sqrt{2}}$

$y = 2t^2 \quad t^2 = \frac{y}{2}$

$x = 3t + 3t^2$

$x = 3t + 3\left(\frac{y}{2}\right)$

$x = 3\left(\frac{y}{2}\right)^{1/2} + 3\left(\frac{y}{2}\right)$

$y = 32$

$x = ?$

If $\frac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|} = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is :

A 0°

B 45°

C 90° Ans

D 60°

There are two vector $\vec{a}$ & $\vec{b}$

$|\vec{a}+\vec{b}|$

$\Downarrow$

Mag of addition.

$|\vec{a}-\vec{b}|$

Mag of Subtraction.

$$\frac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|} = 1$$

$$\frac{\sqrt{a^2+b^2+2ab\cos\theta}}{\sqrt{a^2+b^2-2ab\cos\theta}} = 1$$

$$\cancel{a^2}+\cancel{b^2}+\cancel{2ab}\cos\theta = \cancel{a^2}+\cancel{b^2}-\cancel{2ab}\cos\theta$$

$$2\cos\theta = 0$$

$$\theta = \pi/2$$

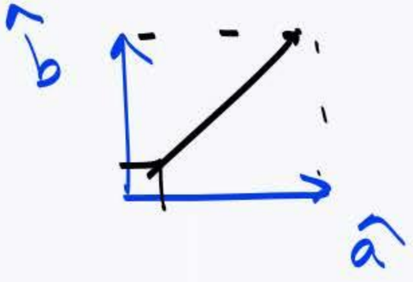
If $|\hat{a} - \hat{b}| = \sqrt{2}$ then calculate the value of $|\hat{a} + \sqrt{3}\hat{b}|$. Ans = 2

$\hat{a}$ = unit vector along $\vec{a}$, direction of $\vec{a}$ $|\hat{a}| = 1$

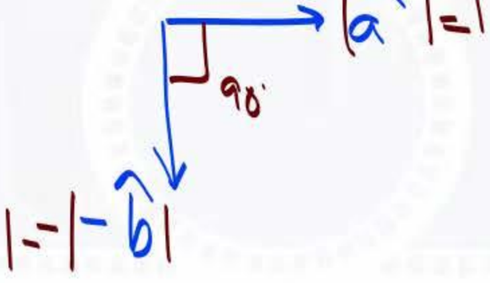
$\hat{b}$ = unit vector along $\vec{b}$, direction of $\vec{b}$. $|\hat{b}| = 1$

$|\hat{a} - \hat{b}| = \sqrt{2}$

Mag of Sub of two unit vectors.



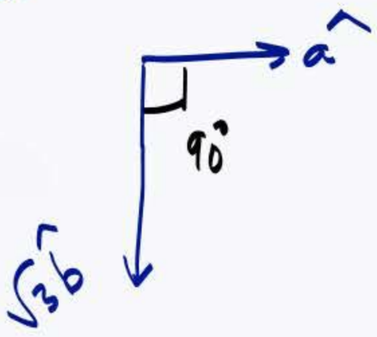
$|\hat{a} + \hat{b}| = \sqrt{2}$



$|\hat{a} - \hat{b}| = \sqrt{2}$

$\hat{a}$ & $\hat{b}$ are ⊥ to Each others.

$|\hat{a} + \sqrt{3}\hat{b}|$ → Scalar multiply dir = same.



$R = \sqrt{A^2 + B^2}$

$= \sqrt{|\hat{a}|^2 + |\sqrt{3}\hat{b}|^2}$

$= \sqrt{1 + 3} = \sqrt{4} = 2$

Ex:- $\vec{F}$

new force $= n\vec{F}$

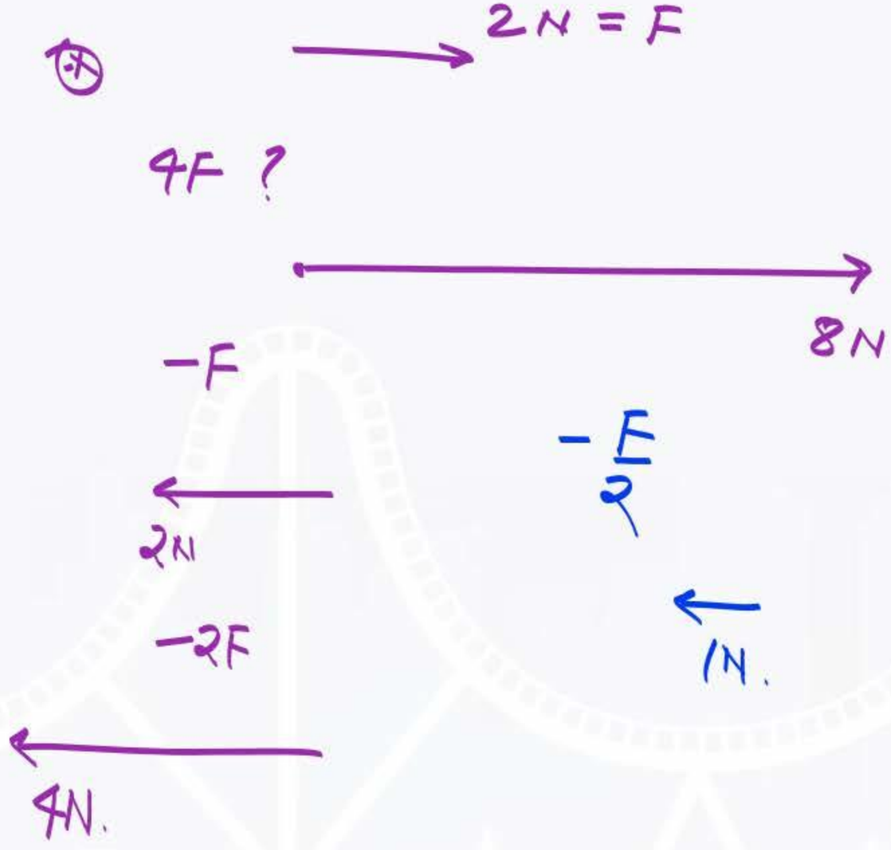
⇓

number (Scalar)

- $n = +ve$ dir same
- $n = -ve$ dir opp.

$|n| > 1$ Magnitude ↑

$|n| < 1$ Magnitude ↓



The displacement of an object along the three axes are given by, $x = 2t^2, y = t^2 - 4t$ and $z = 3t - 5$. The initial velocity of the particle is

A 10 units

B 12 units

C 5 units Ans

D 2 units

3D Motion.

$x = 2t^2$

$y = t^2 - 4t$

$z = 3t - 5$

$v_x = \frac{dx}{dt} = 4t$

$v_y = \frac{dy}{dt} = 2t - 4$

$v_z = \frac{dz}{dt} = 3.$

$t = 0$

$v_x = 0$

$v_y = -4$

$v_z = 3$

$\vec{v}_{net} = 0\hat{i} - 4\hat{j} + 3\hat{k}$

$|v_{net}| = \sqrt{4^2 + 3^2}$

$= 5\ m/s$

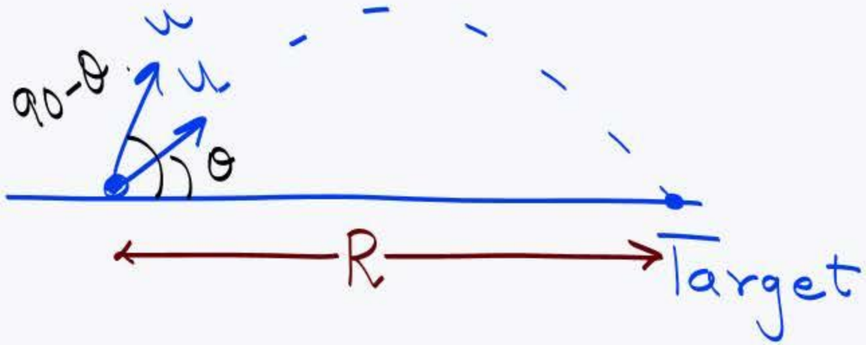
A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time take by it to hit the target in two possible ways, the product t_1t_2 is : **[JAN 2019]**

A $\frac{R}{4g}$

B $\frac{R}{g}$

C $\frac{R}{2g}$

D $\frac{2R}{g}$ Ans



Complementary angle

$R \rightarrow$ fix

$T_\theta = \frac{2u\sin\theta}{g} = t_1$

$T_{90-\theta} = \frac{2u\cos\theta}{g} = t_2$

$t_1t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2}$

$= \frac{2}{g}\left(\frac{2u^2\sin\theta\cos\theta}{g}\right)$ R

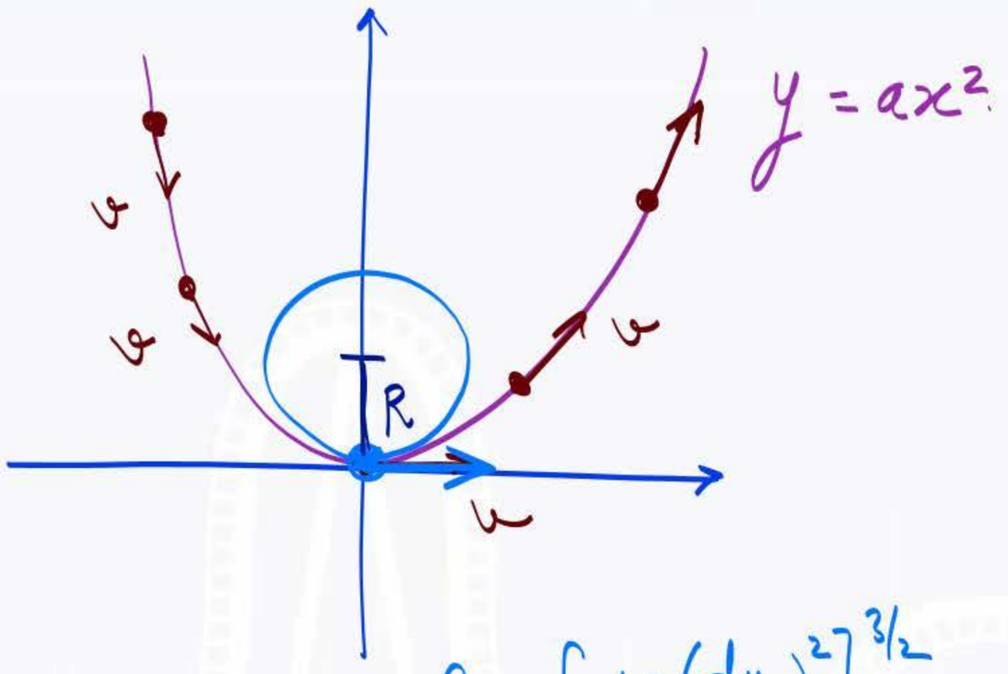
$R = \frac{u^2\sin 2\theta}{g}$

$R = \frac{2u^2\sin\theta\cos\theta}{g}$

$t_1t_2 = \frac{2R}{g}$

A particle moves at uniform speed on a parabiotic trajectory $y = ax^2$ at uniform speed v. Find the acceleration of particle when it passes point $x = 0$ and point $(1, a)$. H.W

$x = 0 \quad a = ?$



$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Since direction of v changing

$\Downarrow$

acceleration

$$a_c = \frac{v^2}{R}$$

$$a_c = \frac{v^2}{R}$$

$$a_c = \frac{v^2}{(1/2a)}$$

$$\boxed{a_{cc} = 2av^2}$$ Ans.

$$y = ax^2$$

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$R = \frac{\left[1 + 0^2\right]^{3/2}}{2a}$$

$$\boxed{R = \frac{1}{2a}}$$ at $x = 0$.

$$y = ax^2$$

$$\frac{dy}{dx} = 2ax$$ at $x = 0$ $\frac{dy}{dx} = 0$

$$\frac{d^2y}{dx^2} = 2a.$$

The area of the parallelogram determined by $\vec{A} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{B} = 12\hat{j} - 2\hat{k}$ is :

A 42

B 56

C 38

D 74

Area of llgm $= |\bar{A} \times \bar{B}|$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & -\hat{j} & k \\ 2 & 1 & -3 \\ 12 & -2 & 0 \end{vmatrix} = \hat{i}(1\times 0 - 6) - \hat{j}(2\times 0 + 36) + \hat{k}(-4-12)$$

$$= -6\hat{i} - 36\hat{j} - 16\hat{k}.$$

$$\text{Area} = |\bar{A} \times \bar{B}| = \sqrt{6^2 + 36^2 + 16^2}$$

The values of x and y for which vectors

$\vec{A} = (6\hat{i} + x\hat{j} - 2\hat{k})$ and $\vec{B} = (5\hat{i} - 6\hat{j} - y\hat{k})$ may be parallel are :

A $x = 0, y = \frac{2}{3}$

B $x = -\frac{36}{5}, y = \frac{5}{3}$ Ans

C $x = -\frac{15}{3}, y = \frac{23}{5}$

D $x = \frac{36}{5}, y = \frac{15}{4}$

$\|^{el}$ Condition $\bar{A} \times \bar{B} = 0$ (Area of $\|gm$) $= 0$

$\vec{A} = 6\hat{i} + x\hat{j} - 2\hat{k}$

$\vec{B} = 5\hat{i} - 6\hat{j} - y\hat{k}$

$\frac{6}{5} = \frac{x}{-6} = \frac{-2}{-y}$

$-\frac{36}{5} = x$

$\frac{\cancel{6}^{3}}{5} = \frac{2}{y}$

$y = 5/3$

The three conterminous edges of a parallelepiped are

$$\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k},\ \vec{b} = 5\hat{j} \text{ and } \vec{c} = -2\hat{i} + \hat{k}.$$

The volume of parallelepiped is :

A 36 cubic unit

B 45 cubic unit

C 40 cubic unit Ans

D 54 cubic unit

Volume = (area of base) h.

Scalar

$(\vec{A} \times \vec{B}) \cdot \vec{C}$.

Volume of 1st pipe.

$\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k}$

$\vec{b} = 5\hat{j}$

$\vec{c} = -2\hat{i} + \hat{k}$

Dot Prod.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & -\hat{j} & k \\ 2 & -6 & 3 \\ 0 & 5 & 0 \end{vmatrix}$$

$$= \hat{i}(-15) - \hat{j}(0) + \hat{k}(10)$$

$$\boxed{\vec{a} \times \vec{b} = -15\hat{i} + 10\hat{k}}$$

Volume = $15 \times 2 + 10$

$= 40$ unit

The unit vector perpendicular to vectors $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{j} - 5\hat{k}$ is :

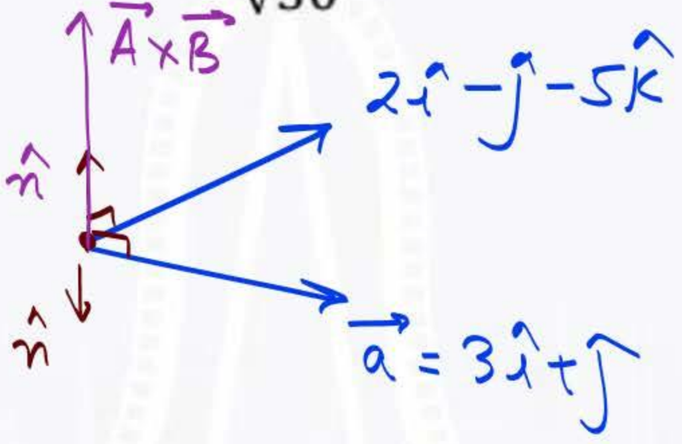
$$\hat{n} = \frac{\vec{C}}{|C|} = \frac{-5\hat{i} + 15\hat{j} - 5\hat{k}}{\sqrt{5^2 + 15^2 + 5^2}}$$

A $\pm \dfrac{(\hat{i} - 3\hat{j} + \hat{k})}{\sqrt{11}}$

B $\pm \dfrac{3\hat{i} + \hat{j}}{\sqrt{11}}$

C $\pm \dfrac{(2\hat{i} - \hat{j} - 5\hat{k})}{\sqrt{30}}$

D none of these



unit vector $\perp$ to A & B = direction of $\vec{A} \times \vec{B}$.

1. $$\begin{vmatrix} \hat{i} & -\hat{j} & k \\ 3 & 1 & 0 \\ 2 & -1 & -5 \end{vmatrix} = \hat{i}(-5-0) - \hat{j}(-15) + \hat{k}(-3-2)$$

$$= -5\hat{i} + 15\hat{j} - 5\hat{k}$$

A force $\vec{F} = (2\hat{i} + 3\hat{j} - \hat{k})$ N is acting on a body at a position $\vec{r} = (6\hat{i} + 3\hat{j} - 2\hat{k})$. The torque about the origin is:

A $(3\hat{i} + 2\hat{j} + 12\hat{k})Nm$ Ans

B $(9\hat{i} + \hat{j} + 7\hat{k})Nm$

C $(\hat{i} + 2\hat{j} + 12\hat{k})Nm$

D $(3\hat{i} + 12\hat{j} + \hat{k})Nm$

Sol:- $\vec{F} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{r} = (6\hat{i} + 3\hat{j} - 2\hat{k})$

$\vec{\tau} = \vec{r} \times \vec{F}$ order is very Imp.

$$= \begin{vmatrix} \hat{i} & -\hat{j} & k \\ 6 & 3 & -2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(-3+6) - \hat{j}(-6+4) + \hat{k}(18-6)$$

$$= 3\hat{i} + 2\hat{j} + 12\hat{k}$$

The coordinates of a moving particle at any time t are given by, $x = 2t^3$ and $y = 3t^3$. Acceleration of the particle is given

A $468\,t$

B $t\sqrt{468}$

C $234\,t^2$

D $t\sqrt{234}$

$x = 2t^3$

$y = 3t^3$

$x = f(t)$

$y = f(t)$

2D Motion

$v_x = \frac{dx}{dt} = 6t^2$ $\quad a_x = \frac{dv_x}{dt} = 12t$

$v_y = \frac{dy}{dt} = 9t^2$ $\quad a_y = \frac{dv_y}{dt} = 18t$

$\vec{a}_{net} = 12t\,\hat{i} + 18t\,\hat{j}$

$|a_{net}| = \sqrt{(12t)^2 + (18t)^2}$

The trajectory of a projectile in vertical plane is $y = ax - bx^2$, where a and b are constants and x and y are respectively horizontal and vertical distances of the projectile from the point of projection. The maximum height attained by the particle and the angle of projection from the horizontal are

H_{max}

θ.

A $\frac{b^2}{4b}, \tan^{-1}(b)$

B $\frac{a^2}{b}, \tan^{-1}(2b)$

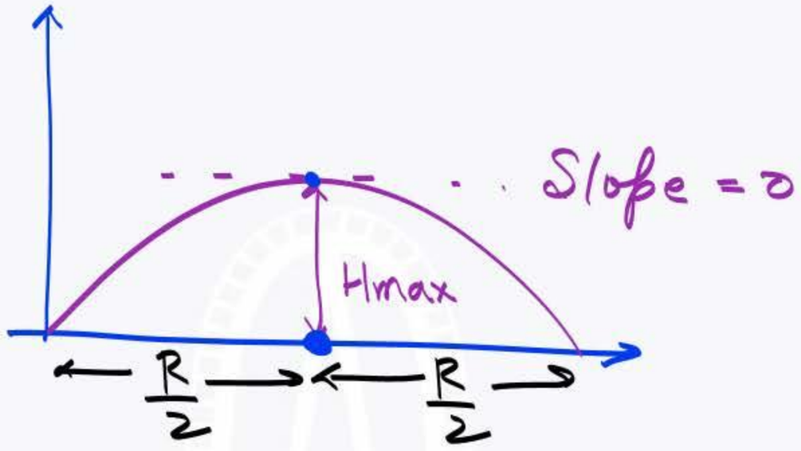
C $\frac{a^2}{4b}, \tan^{-1}(a)$ Ans

D $\frac{2a^2}{b}, \tan^{-1}(a)$

$\boxed{y = ax - bx^2}$

$$y = ax - bx^2$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$



$$\tan\theta = a$$

$$\boxed{\theta = \tan^{-1}(a)}$$

$$y = ax - bx^2$$

$$\frac{dy}{dx} = a - 2bx$$

$$0 = a - 2bx$$

$$x = \frac{a}{2b}$$

$$\frac{\text{Range}}{2}$$

Hmax Slope = 0

$$\boxed{\text{Range} = 2x = \frac{a}{b}}$$

at $x = \frac{a}{2b}$

$y = ?$ ⇒ H_{max}

$$y = ax - bx^2$$

$$y = a\left(\frac{a}{2b}\right) - b\left(\frac{a^2}{4b^2}\right)$$

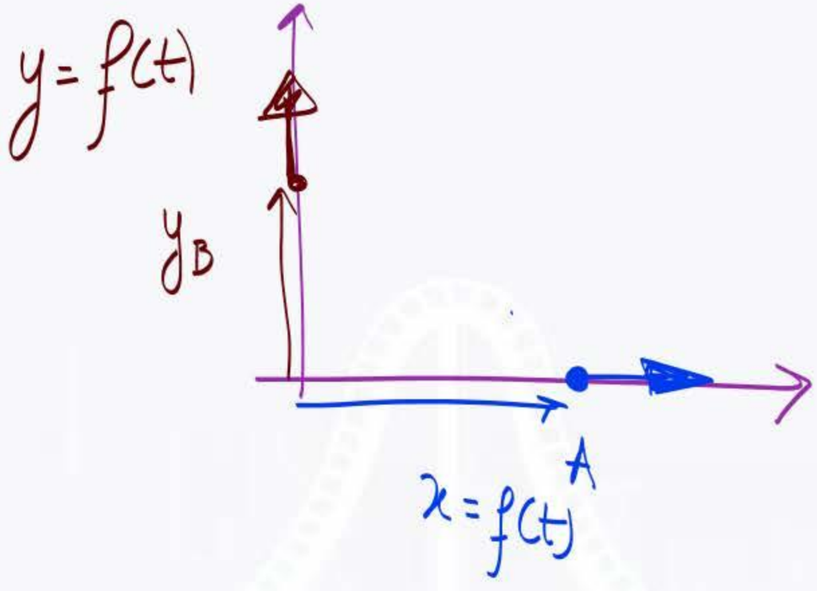
$$y = \frac{a^2}{2b} - \frac{a^2}{4b}$$

$$\boxed{H_{max} = \frac{a^2}{4b}}$$

Position of two particles A and B as a function of time are given by $X_A = -3t^2 + 8t + c$ and $y_B = 10 - 8t^3$. The velocity of B with respect to A at $t = 1$ is $\sqrt{v}$. Find v **(JAN 2020)**

- $x_A = -3t^2 + 8t + c$
- $y_B = 10 - 8t^3$

with time both Particles are moving.



$$v_A = \frac{dx_A}{dt} = (-6t + 8)\hat{i}$$

$$v_B = \frac{dy_B}{dt} = 0 - 24t^2$$

at $t = 1s$ $\quad v_A = (-6+8)\hat{i} = 2\hat{i}$

$v_B = -24\hat{j}$

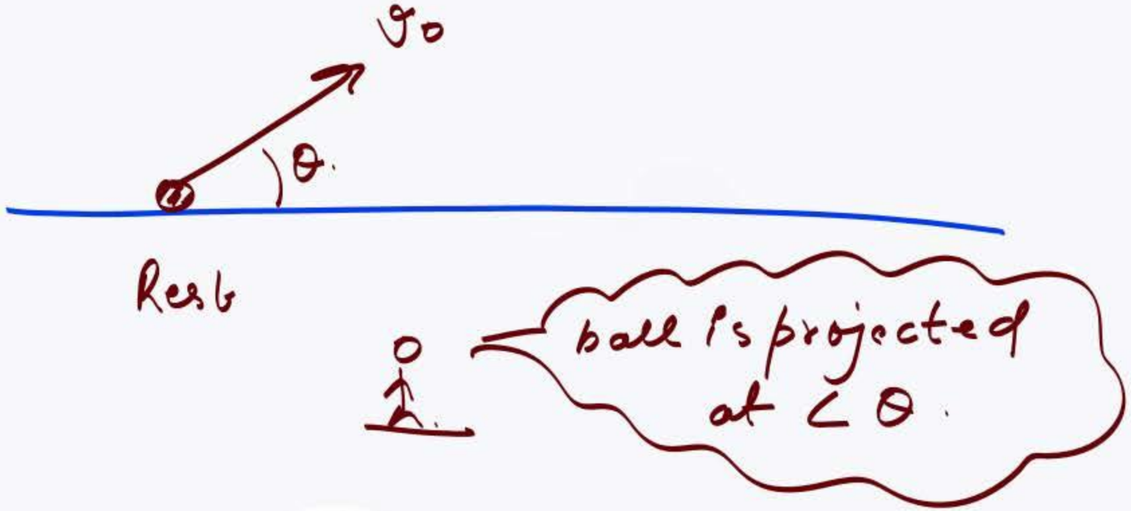
A cricket fielder can throw the cricket ball with a speed v_0. If he throws the ball while running with speed u at an angle θ to the horizontal, what is the effective angle to the horizontal at which the ball is projected in air as seen by a spectator ?

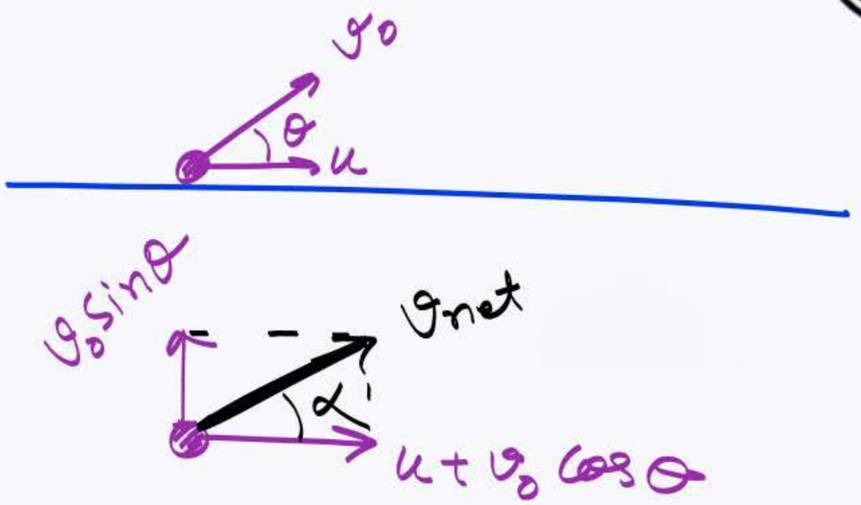
A $\tan^{-1}\left[\frac{v_0\cos\theta}{u+v_0\sin\theta}\right]$

B $\tan^{-1}\left[\frac{v_0\sin\theta}{u+v_0\cos\theta}\right]$ Ans

C $\tan^{-1}\left[\frac{u}{v_0\cos\theta+v_0\sin\theta}\right]$

D $\tan^{-1}\left[\frac{v_0\sin\theta+v_0\cos\theta}{u}\right]$





$$\tan\alpha = \frac{P}{B} = \frac{v_0 \sin\theta}{u + v_0 \cos\theta}$$

$$\alpha = \tan^{-1}\left(\frac{v_0 \sin\theta}{u + v_0 \cos\theta}\right)$$

The initial velocity of a particle of mass 2 kg is $(4\hat{i} + 4\hat{j})\text{ms}^{-1}$. A constant force of $-20\hat{j}$ N is applied on the particle. Initially the particle was at (0, 0). Find the x-coordinate of the point where its y-coordinate is again zero.

A 3.2 m AQ

B 6 m

C 4.8 m

D 1.2 m

Method, done Previously After

$R = \frac{u^2 \sin 2\theta}{g}$

$R = \frac{2 u_x u_y}{g}$

$= \frac{2 \times 4 \times 4}{10} = \frac{16}{5}$

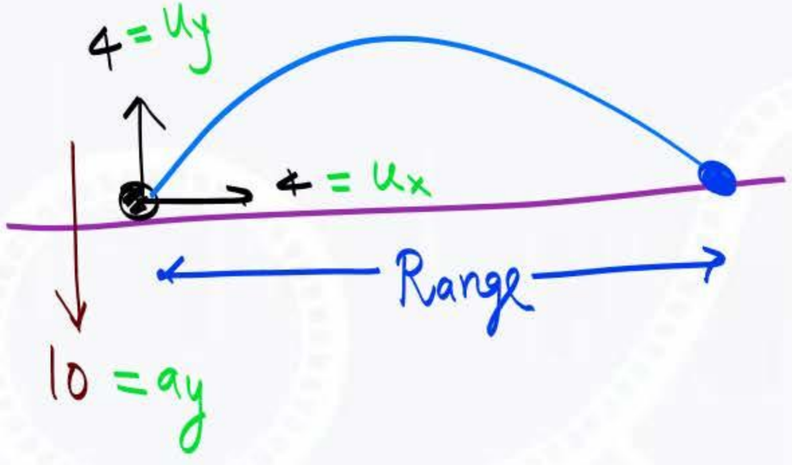
$\vec{u} = 4\hat{i} + 4\hat{j}$

$t = 0$

$x = 0$

$F = -20\hat{j}$

$\frac{F}{m} = a = -10\hat{j}$



The coordinates of a moving particle at any time t are given by $x = ct$ and $y = bt^2$. The speed of the particle is given by

A $2t\sqrt{b^2 - c^2}$

B $\sqrt{4b^2t^2 + c^2}$

Ans

C $2t(b + c)$

D $2t(b - c)$

$x = ct \qquad v_x = c$

$y = bt^2 \qquad v_y = 2bt$

$\vec{v}_{net} = c\hat{i} + 2bt\hat{j}$

$|v_{net}| = \sqrt{c^2 + 4b^2t^2}$

Balls A and B are thrown from two points lying on the same horizontal plane separated by a distance 120 m. Which of the following statement (s) is/are correct ?

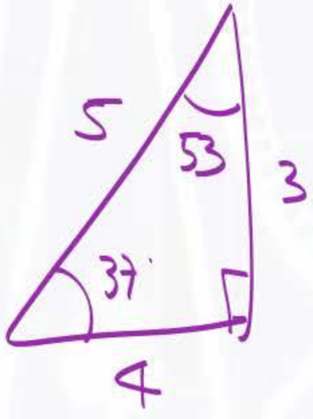
(2 Projectile)

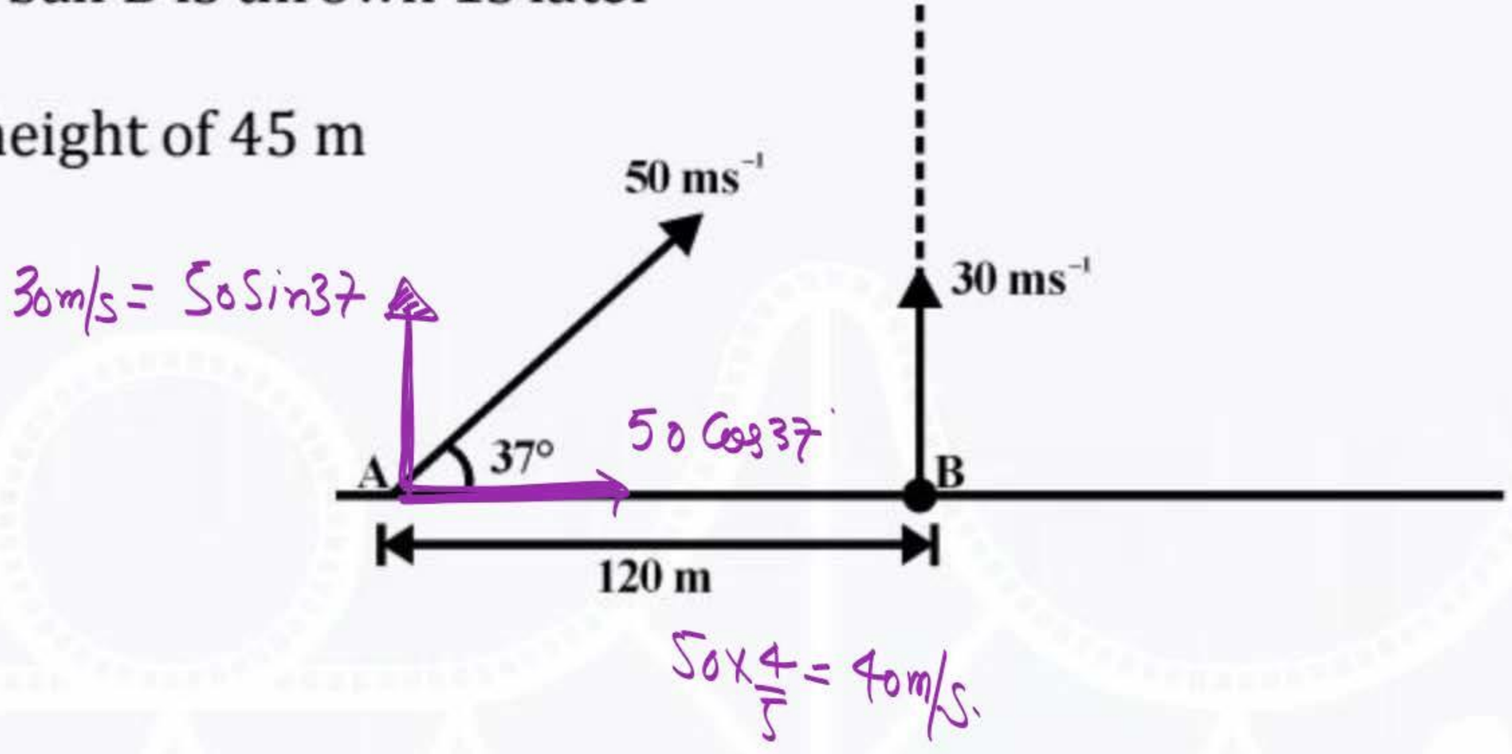
A. The two balls can never meet

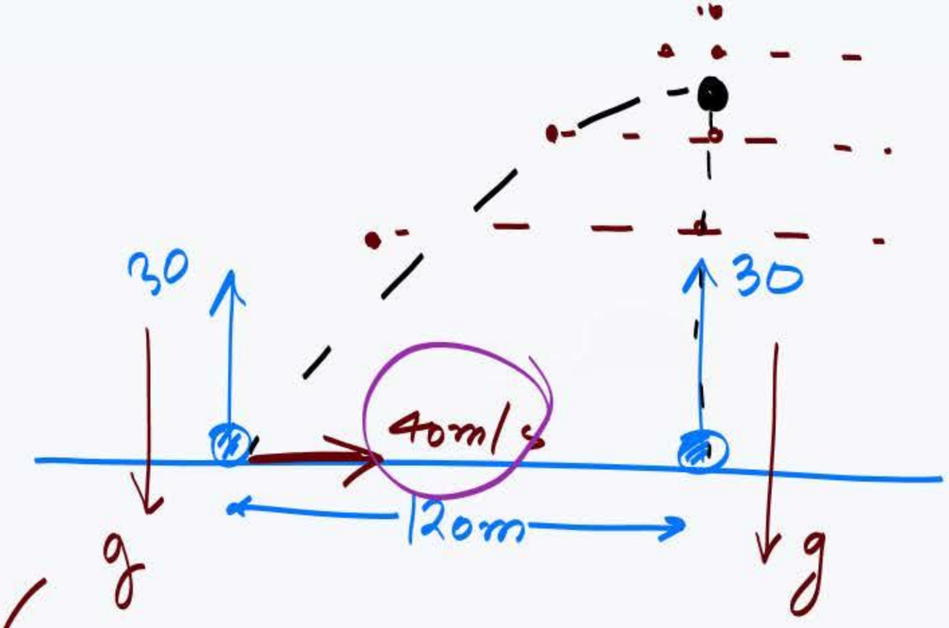
B. The balls can meet if the ball B is thrown 1s later

Ans C. The two balls meet at a height of 45 m

D. None of the above







$a_{rel} = 0$

$u_{rel} = 0$ along Y

40m/s ⇒ Move A towards B.

$$\boxed{t = \frac{120}{40} = 3s}$$ ⇒ Time to Reach Hmax

Time of flight $= \frac{2u_y}{g}$

$$\boxed{T_A = T_B}$$

$$T = \frac{2 \times 30}{10} = 6s$$

$$H_{max} = \frac{u_y^2}{2g} = \frac{\overset{15}{\cancel{30}} \times 3\cancel{0}}{\cancel{2} \times 1\cancel{0}} = 45m$$

When the angle of projection is 75°, a ball falls 10 m shorter of the target. When the angle of projection is 45°, it falls 10 m ahead of the target. Both are projected from the same point with the same speed in the same direction, the distance of the target from the point of projection is

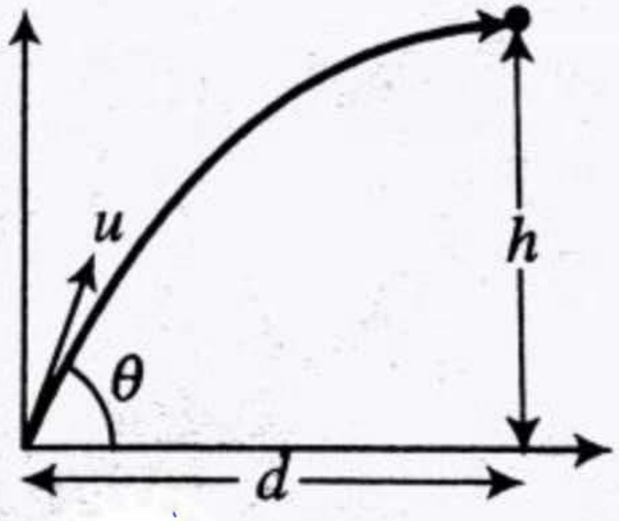
A 15 m

B 30 m

C 45 m

D 10 m

If a stone is to hit at a point which is at a distance d away and at a height h above the point from where the stone starts, then what is the value of initial speed u, if the stone is launched at an angle θ ?



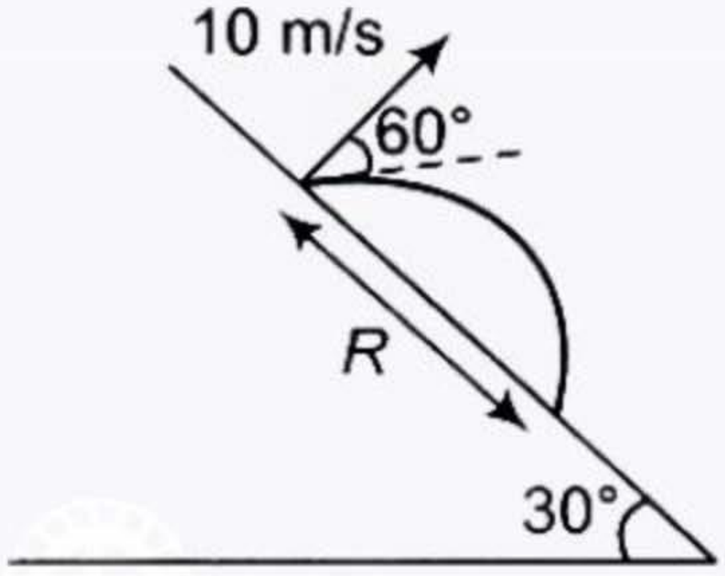
A projectile is launched with a speed of 10 m/s at an angle 60° with the horizontal from a sloping surface of inclination 30°. The range R is. (Take, $g = 10\ m/s^2$)

A 4.9 m

B 13.3 m

C 9.1 m

D 12.6 m



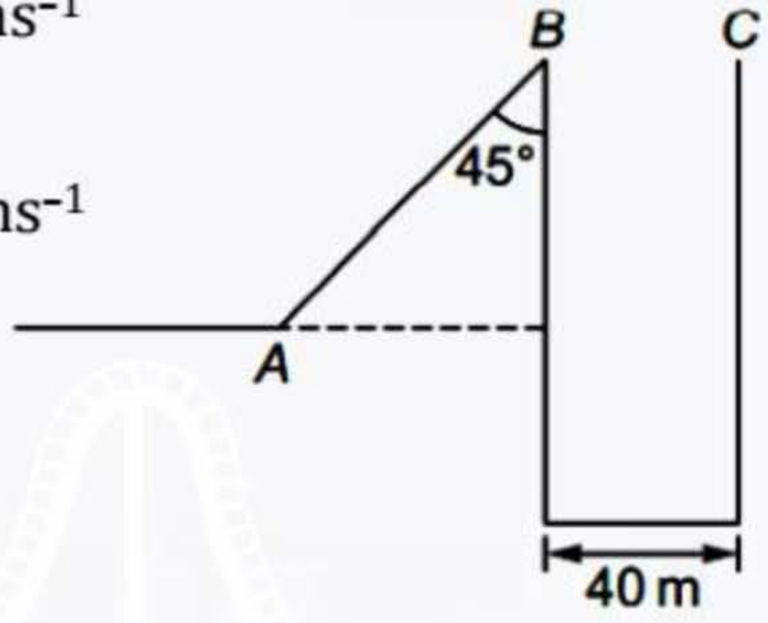
A body is projected up smooth inclined plane with a velocity v_0 from the point A as shown figure. The angle of inclination is 45° and top B of the plane is connected to a well of diameter 40 m. If the body just manages to cross the well,

A $20\ ms^{-1}$

B $20\sqrt{2}\ ms^{-1}$

C $40\ ms^{-1}$

D $40\sqrt{2}\ ms^{-1}$



$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

$$= -24\hat{j} - 2\hat{i}$$

$$|v_{BA}| = \sqrt{24^2 + 2^2}$$

$$= \sqrt{580} = \sqrt{v}$$

$$\boxed{v = 580} \text{ Ans}$$

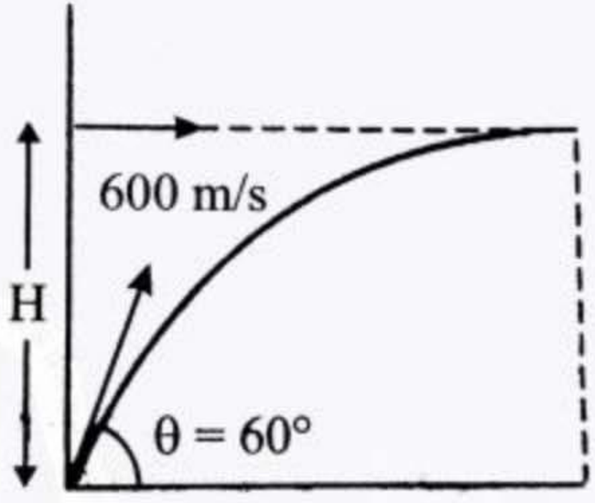
A fighter plane enters inside the enemy territory, at time $t = 0$ with velocity $v_0 = 250\ ms^{-1}$ and moves horizontally with constant acceleration $a = 20\ ms^{-2}$ (see figure). An enemy tank at the border, spot the plane and fire shots at an angle $\theta = 60°$ with the horizontal and with velocity $u = 600\ ms^{-1}$. At what altitude H of the plane it can be hit by the shot ?

A $1500\sqrt{3}$ m

B 125 m

C 1400 m

D 2473 m



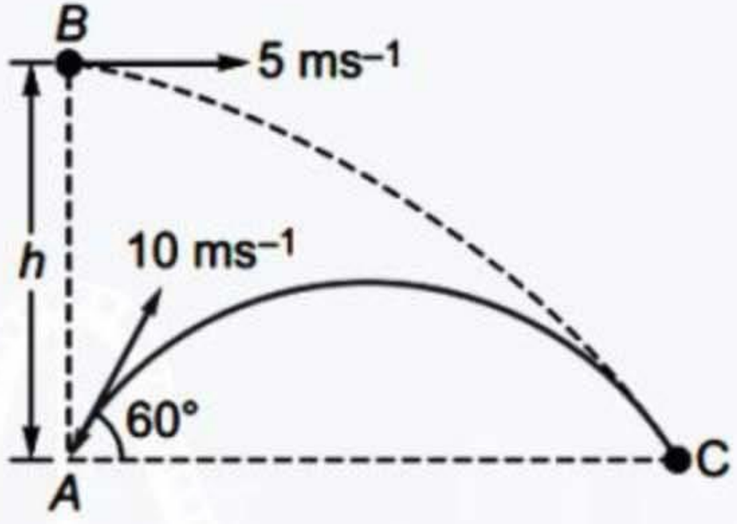
A particle A is projected from the ground with an initial velocity of 10 ms^{-1} at an angle of 60° with horizontal. From what height h should an another particle B be projected horizontal with velocity 5 ms^{-1} so that both the particles collide with velocity 5 ms^{-1} so that both the particles collide on the ground at point C is both are projected simultaneously ? ($g = 10\ ms^{-2}$)

A 10 m

B 30 m

C 15 m

D 25 m



Two boys are standing at the ends A and B of a ground where AB = a. The boy at B starts running in a direction perpendicular to AB, with velocity v_1. The boy at A starts running simultaneously with velocity v and catches the other boy in a time t, then find t.

A particle is moving with a velocity of 10 m/s towards east. After 10 s its velocity changes to 10 m/s towards north. Its average acceleration is:-

A Zero

B $\sqrt{2}$ m/s^2 towards N-W

C $1/\sqrt{2}$ m/s^2 towards N-E

D $1/\sqrt{2}$ m/s^2 towards N-W

When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45°. The value of β is close to : **[SEP 2020]**

A 0.41

B 0.50

C 0.73

D 0.37

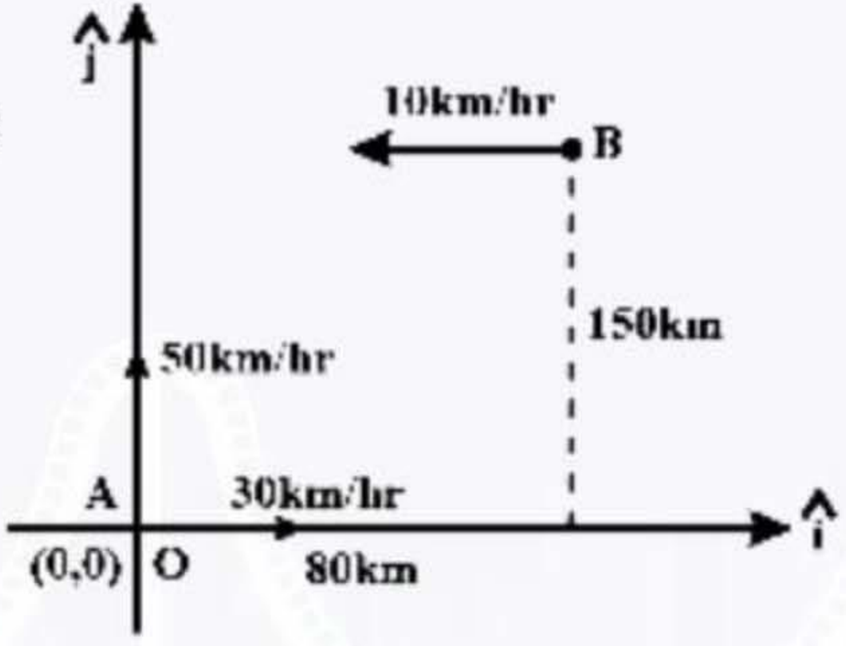
Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$ north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in : **[APRIL 2019]**

A 3.2 hrs

B 2.6 hrs

C 4.2 hrs

D 2.2 hrs



A glass wind screen whose inclination with the vertical can be changed is mounted on a car. The car moves horizontally with a speed of 2m/s. At what angle α with the vertical should the wind screen be placed so that the rain drops falling vertically downwards with velocity 6m/s strike the wind screen perpendicularly?

A $\tan^{-1}(3)$

B $\tan^{-1}(4)$

C $\tan^{-1}(1/3)$

D $\tan^{-1}(1/4)$

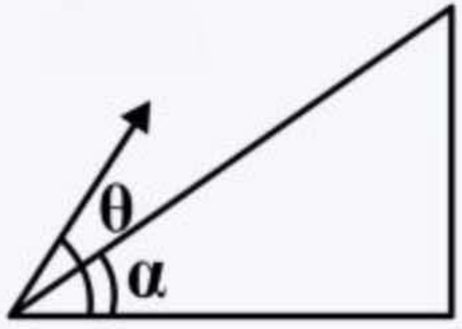
A projectile is fixed at an angle θ with the horizontal, (as shown in the figure), condition under which it lands perpendicular on an inclined plane of inclination α is

A $\sin\alpha = \cos(\theta - \alpha)$

B $\cos\alpha = \sin(\theta - \alpha)$

C $\tan\alpha = \cot(\theta - \alpha)$

D $\cot(\theta - \alpha) = \sin\alpha$



Rain is falling vertically with speed of 35 m/s. A woman rides a bicycle with a speed of 15 m/s in East to West direction. What is the direction is which she should hold her umbrella?

A $\tan^{-1}\left(\frac{3}{7}\right)$ with the vertical towards west

B $\tan^{-1}\left(\frac{4}{7}\right)$ with the vertical towards east

C $\tan^{-1}\left(\frac{5}{7}\right)$ with the vertical towards east

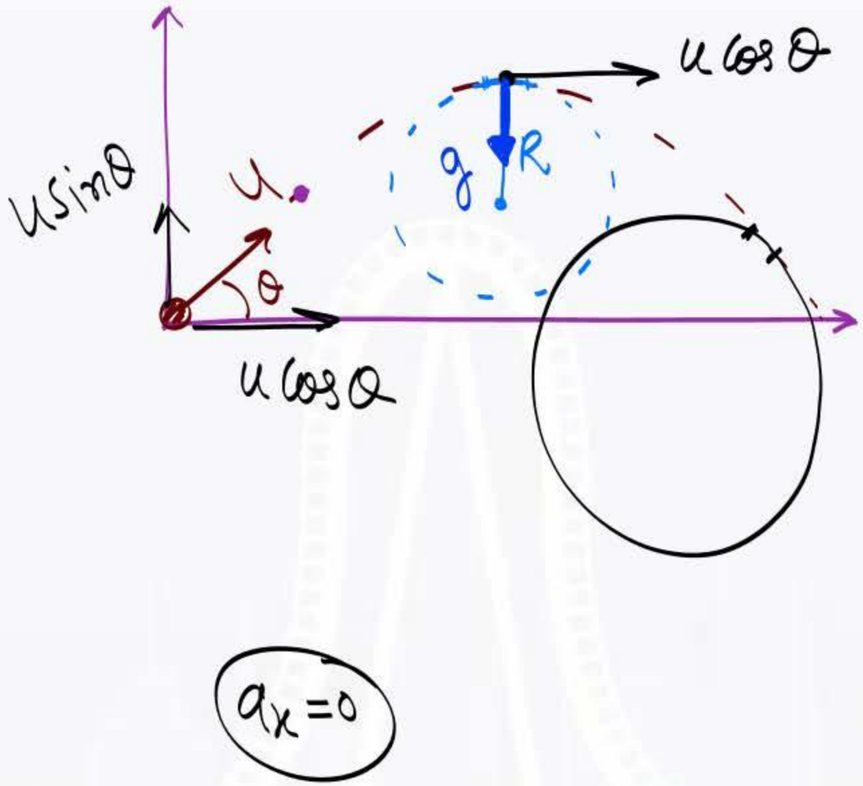
D Towards North downward

A ship a is moving towards the South with a speed 20 km per hour and another ship is moving towards the East with a speed of 20 km per hour. At a certain instant the Ship B is due south of ship A an is at a distance of 10 km from ship A find the shortest distance between the ships and the time after which they are closest to each other

A particle is projected at a speed u at an angle θ with the horizontal. Find the radius of curvature at the highest point of the trajectory of projectile.

(HCV Verma)

Parabolic path



$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

at Each point, Radius of Curvature is diff.

1. draw Circle.
2. Find velocity (Tangential)
3. Find acc along Radius.
4. $a_c = \frac{v^2}{R}$

at top point

$$g = \frac{u^2\cos^2\theta}{R}$$

$$\boxed{R = \frac{u^2\cos^2\theta}{g}}$$

One day in still air, a motor-cyclist riding north at 30 m/s, suddenly the wind starts blowing westward with a velocity 50 m/s, then the apparent velocity with which the motor-cyclist will move, is:

A 58.3 m/s

B 65.4 m/s

C 73.2 m/s

D 53.8 m/s

A motorboat is racing towards north at 25 km/h and the water current in that region is 10 km/h in the direction of 60° east of south. The resultant velocity of the boat is

A 10 km/h

B 20 km/h

C 12 km/h

D 22 km/h

Position of particle as a function of time is given as $\vec{r} = \cos\omega t\hat{i} + \sin\omega t\hat{j}$. Choose correct statement about $\vec{r}$, $\vec{v}$ and $\vec{a}$, where $\vec{v}$ and $\vec{a}$ are velocity and acceleration of particle at time t. **[JAN 2020]**

A $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is towards origin

B $\vec{v}$ and $\vec{a}$ are perpendicular to $\vec{r}$

C $\vec{v}$ is parallel to $\vec{r}$ and $\vec{a}$ parallel to $\vec{r}$

D $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is away from origin.

Train A and train B are running on parallel tracks in the opposite direction with speed of 36 km/hour and 72 km/hour , respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hour. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible) **(SEP 2020)**

A $30.5\ ms^{-1}$

B $29.5\ ms^{-1}$

C $31.5\ ms^{-1}$

D $28.5\ ms^{-1}$

A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when :

(i) they are moving in the same direction, and (ii) in the opposite directions is **(JAN 2019)**

A 5/2

B 25/11

C 3/2

D 11/5

Two particles A and B separated by distance 10m with A ahead of B. A starts from rest with uniform acceleration $2m/s^2$, whereas B is moving with constant velocity of 40m/s. Find time of overtaking

An elevator is accelerating upward at a rate of 6 ft/sec^2 when a bolt from its ceiling falls to the floor of the lift (Distance = 9.5 feet). The time (in seconds) taken by the falling bolt to hit the floor is (take g = 32 ft/ sec^2)

A $\sqrt{2}$

B $\frac{1}{\sqrt{2}}$

C $2\sqrt{2}$

D $\frac{1}{2\sqrt{2}}$

Two bodies are held separated by 9.8 m vertically one above the other. They are released simultaneously to fall freely under gravity. After 2 s the relative distance between them is

A 4.9 m

B 19.6 m

C 9.8 m

D 392 m

Two balls are thrown simultaneously, (A) vertically upwards with a speed of 20 m/s from the ground and (B) vertically downwards from a height of 40 m with the same speed and along the same line of motion. At which point will the balls collide? (take $g = 10\ m/s^2$)

A 15 m above from the ground

B 15 m below from the top of the tower

C 20 m above from the ground

D 20 m below from the top of the tower

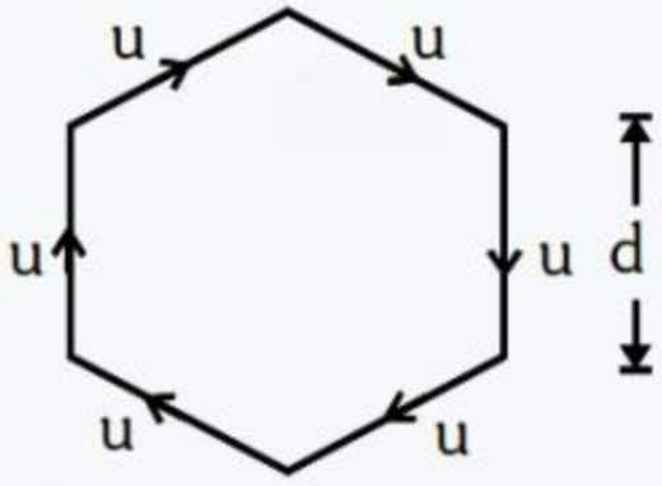
Six persons of same mass travel with same speed u along a regular hexagon of side 'd' such that each one always faces the other. After how what will they meet each other ?

A $\frac{d}{u}$

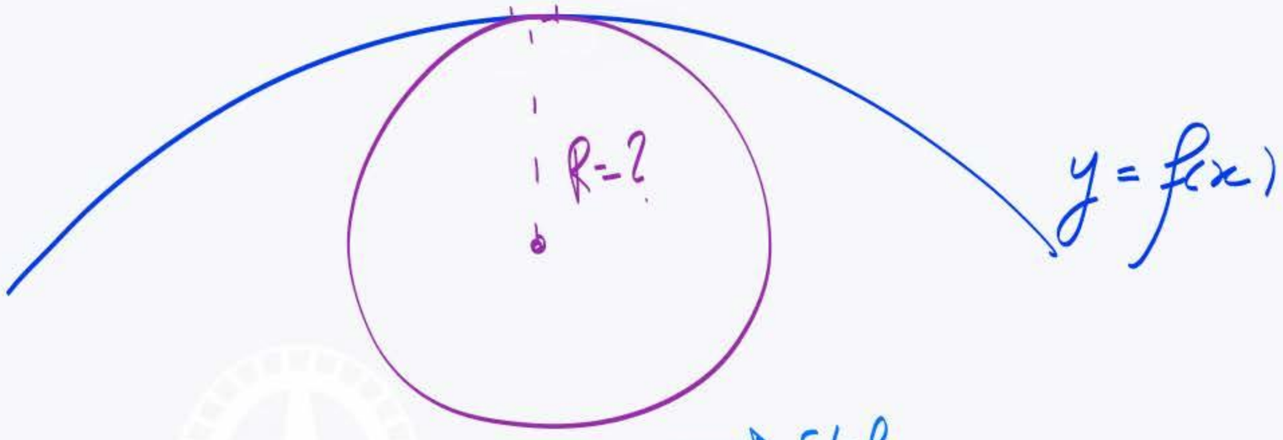
B $\frac{2d}{3u}$

C $\frac{2d}{u}$

D $d\sqrt{3}u$



Ans.



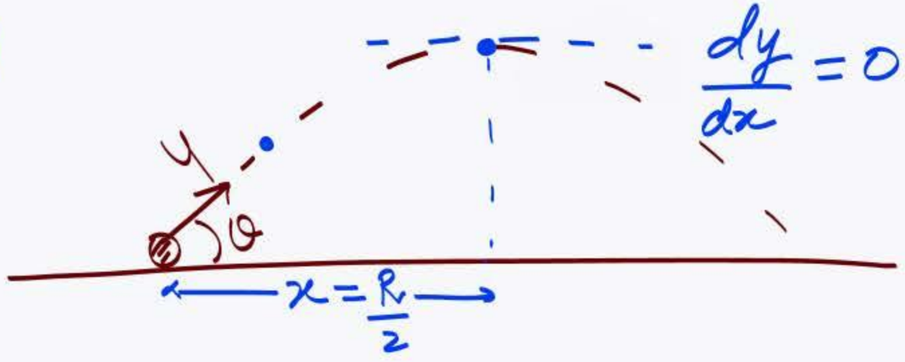
$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Slope.

Point

Point.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$



$$\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \tan\theta - \frac{gx}{u^2\cos^2\theta}$$

$$\frac{d^2y}{dx^2} = -\frac{g}{u^2\cos^2\theta}$$

$$\left|\frac{d^2y}{dx^2}\right| = \frac{g}{u^2\cos^2\theta}$$

Two cars A and B start moving from the same point with same speed v = 5 km/minute. Car A moves towards North and car B is moving towards East. What is the relative velocity of B with respect to A?

A $5\sqrt{2}$ km/min towards South-East

B $5\sqrt{2}$ km/min towards North-West

C $5\sqrt{2}$ km/min towards South-West

D $5\sqrt{2}$ km/min towards North-East

A man wishes to swim across a river 0.5 km wide. If he can swim at the rate of 2 km/h in still water and the river flows at the rate of 1 km/h. The angle made by the direction (w.r.t. the flow of the river) along which he should swim so as to reach a point exactly opposite his starting point, should be:

A 60°

B 120°

C 145°

D 90°

A boat-mat can row a boat to make it move with a speed of 10 km/h in still water. River flows steadily at the rate of 5 km/h. and the width of the river is 2 km. If the boat man cross the river along the minimum distance of approach then time elapsed in rowing the boat will be:

A $\frac{2\sqrt{3}}{5}$ h

B $\frac{2}{5\sqrt{3}}$ h

C $\frac{3\sqrt{2}}{5}$ h

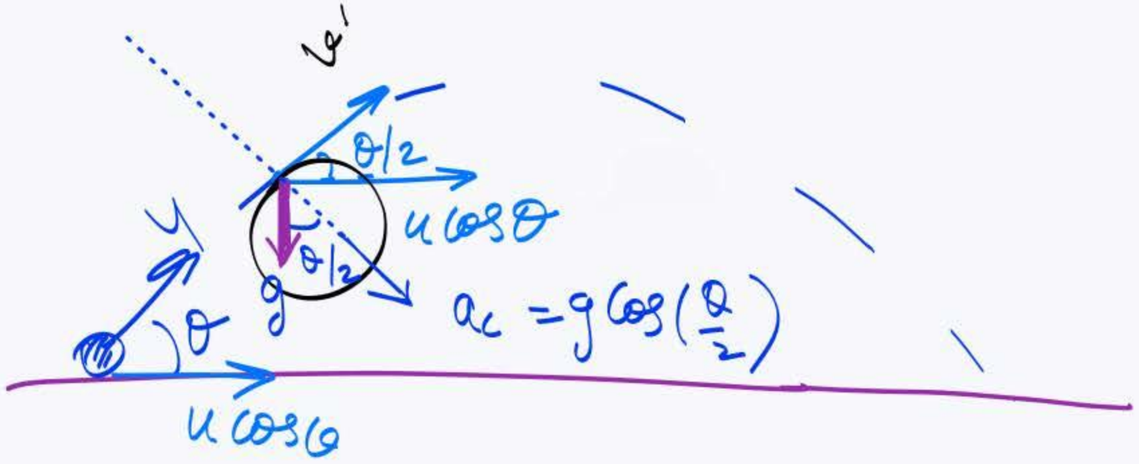
D $\frac{5\sqrt{2}}{3}$ h

THANK YOU!

$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+0^2\right]^{3/2}}{\frac{g}{u^2\cos^2\theta}} = \frac{u^2\cos^2\theta}{g}$$

$$\boxed{R = \frac{u^2\cos^2\theta}{g}}$$

HCV Circular



$$v'\cos\left(\frac{\theta}{2}\right) = u\cos\theta$$

$$v' = \frac{u\cos\theta}{\cos\left(\frac{\theta}{2}\right)}$$

Find R of Curvature at point where angle is $\theta/2$

$$a_c = \frac{v_T^2}{R}$$

$$R = \frac{u^2\cos^2\theta}{\cos^2(\theta/2)\, g\cos\left(\frac{\theta}{2}\right)}$$