REVISION

For Class - 11^{th}

Lecture no. 02

Motion in 2 Dimension

(Part-2).

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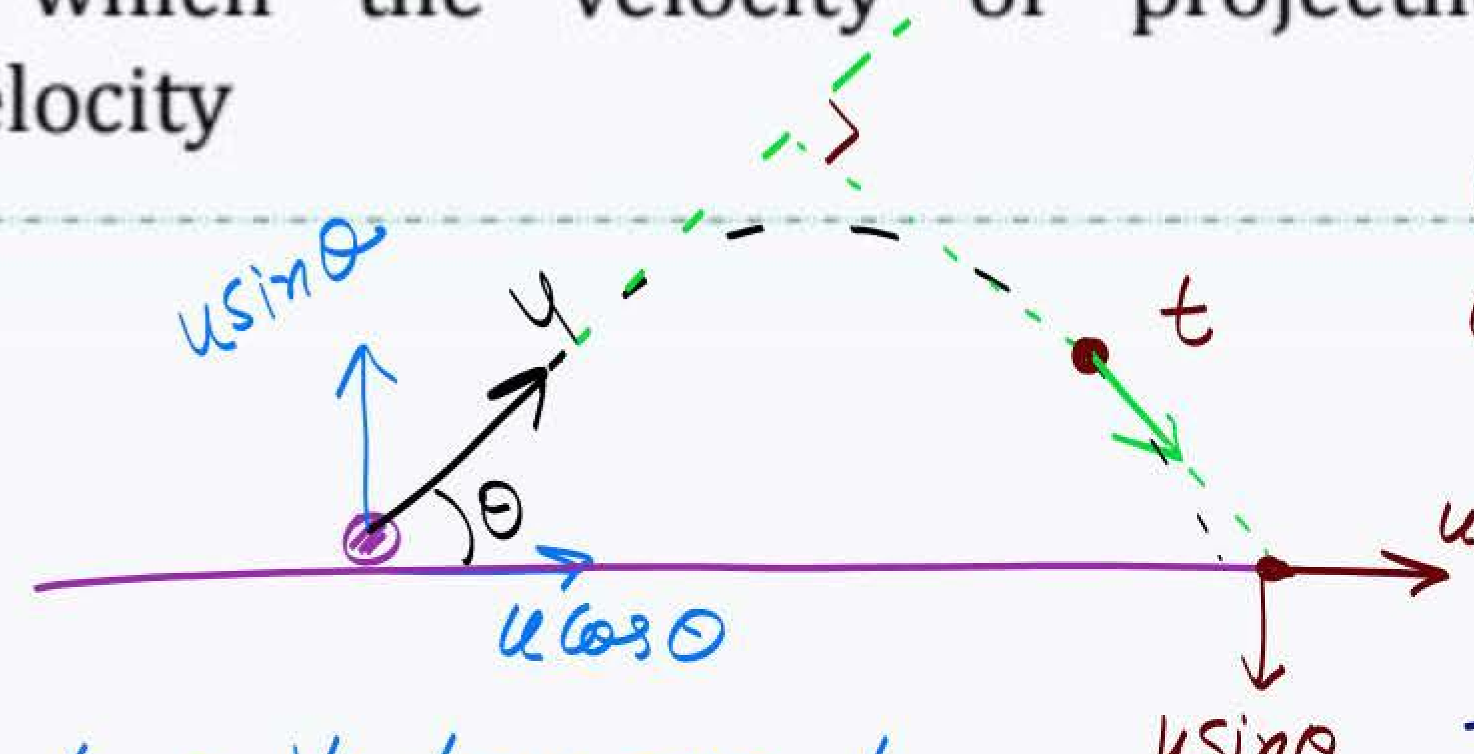
A projectile is thrown upward with velocity u at angle θ. Find the time after projection at which the velocity of projectile is perpendicular to its initial velocity

A (u sin θ)/g

B (u cosec θ)/g Ans

C (u tan θ)/g

D (u cot θ)/g



$u_x = u\cos\theta \quad a_x = 0$

$u_y = u\sin\theta \quad a_y = -g$

$t = ?$

When two Vectors are $\perp$.

$\boxed{\vec{A}\cdot\vec{B} = 0}$

$\vec{v}_t$ is $\perp$ to $\vec{v}_i$

$\vec{u}_i = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$\vec{v} = v_x\hat{i} + v_y\hat{j}$

$\vec{v} = (u_x + a_x t)\hat{i} + (u_y + a_y t)\hat{j}$

$\boxed{\vec{v} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}}$

at time t $\vec{u} \cdot \vec{v} = 0$

$$(u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j})(u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}) = 0$$

$$u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta\, gt = 0$$

$$u^2(\cos^2\theta + \sin^2\theta) - ugt\sin\theta = 0$$

$$u^2 = ugt\sin\theta$$

$$\boxed{\frac{u}{g}\operatorname{cosec}\theta = \frac{u}{g\sin\theta} = t}$$

If $\vec{P} + \vec{Q} = \vec{R}$ and $|\vec{P}| = |\vec{Q}| = \sqrt{3}$ and $|\vec{R}| = 3$, then the angle between $\vec{P}$ and $\vec{Q}$ is :

A 0

B $\pi/6$

C $\pi/3$ Ans

D $\pi/2$

$\vec{P} + \vec{Q} = \vec{R}$

When we add $\vec{P}$ & $\vec{Q}$ vectorially there Resultant R.

$|P| = |Q| = \sqrt{3}$

$|R| = 3$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$3 = \sqrt{3 + 3 + 2\times 3\cos\theta}$$

$\theta =$ ____ .

$\theta = 60^\circ$ $R = |P|\sqrt{3}$

$\pi/3$ $= \sqrt{3}\cdot\sqrt{3} = 3$

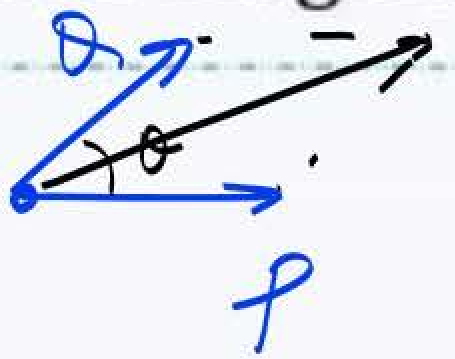
The sum of magnitudes of two forces acting at a point is 16 and magnitude of their resultant is $8\sqrt{3}$. If the resultant is at 90° with the force of smaller magnitude, their magnitudes are

A 3, 13

B 2, 14

C 5, 11

D 4, 12

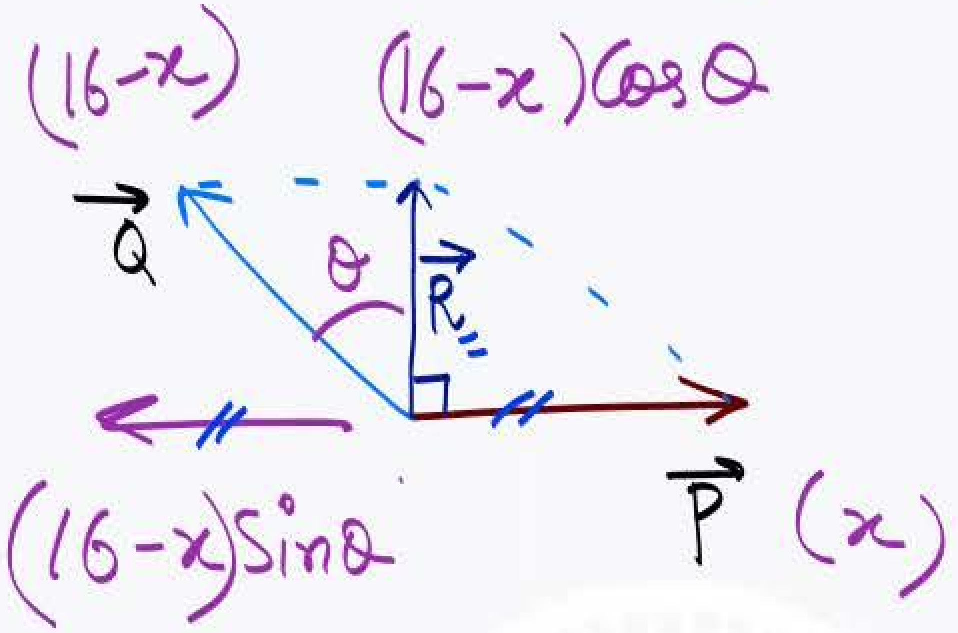


$|P| + |Q| = 16$

$|P| = x$

$|Q| = 16 - x$

$|R| = 8\sqrt{3}$



$\boxed{\text{along } x \;\; F_{net} = 0}$

along y $F_{net} = 8\sqrt{3}$

$x = (16 - x)\sin\theta$

$8\sqrt{3} = (16 - x)\cos\theta$

$\sin\theta = \frac{x}{16 - x}$

$\cos\theta = \frac{8\sqrt{3}}{16 - x}$

Sq & adding

$x =$ ______

Two forces, each equal to F, act as shown in figure. Their resultant is

Trick

$\theta = 60^\circ \quad R = F\sqrt{3}$

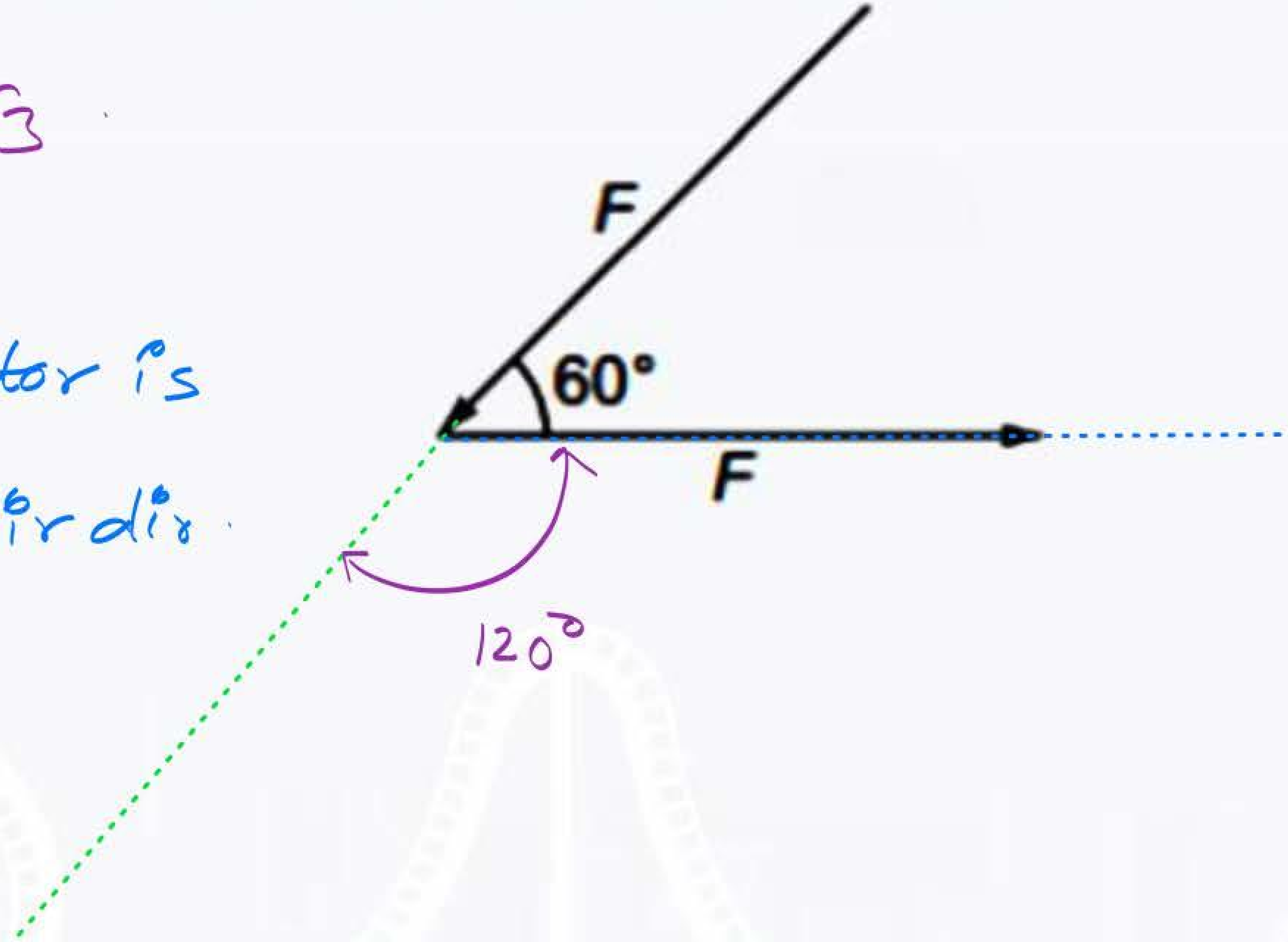
A $F/2$

B F Ans

C $\sqrt{3}\,F$

D $\sqrt{5}\,F$

Angle between Vector is angle between their dir.



$\theta = 120^\circ \quad R = F$

$R = F$

Resultant of which of the following may be equal to zero ?

A. 10 N, 10 N, 10 N

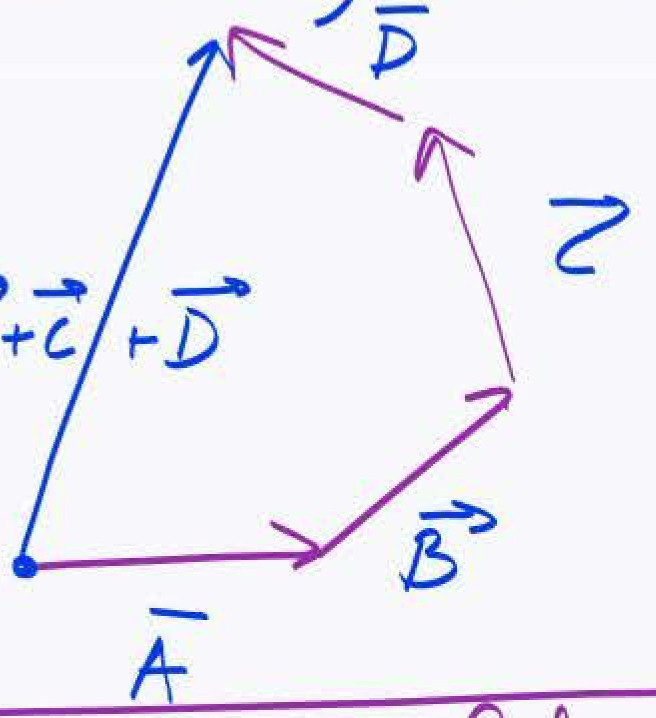
B. 10 N, 10 N, 25 N

C. 10 N, 10 N, 35 N

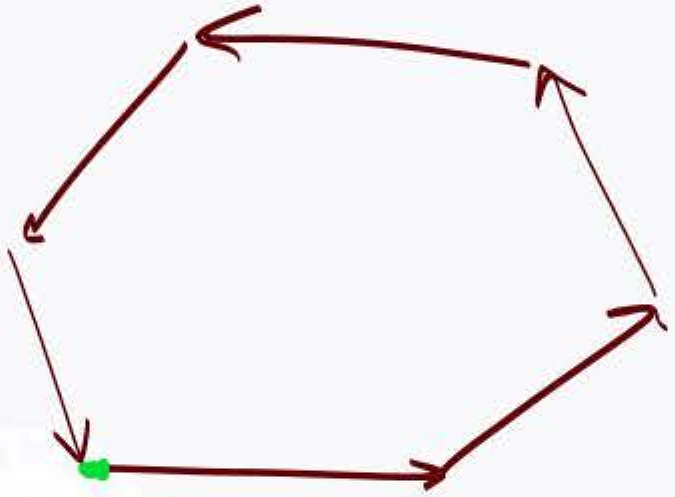
D. None of these

Polygon Law of Vectors

$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$



$\Rightarrow$ If we have a Closed Polygon



When $R = 0$ we need to Get a Closed Polygon

Symmetric or Asymmetric Order Closed polygon

$R = 0$

In our Case Polygon $\rightarrow \Delta$.

Sum of two Sides $\geq$ Third Side.

Resultant of which of the following may be equal to zero ?

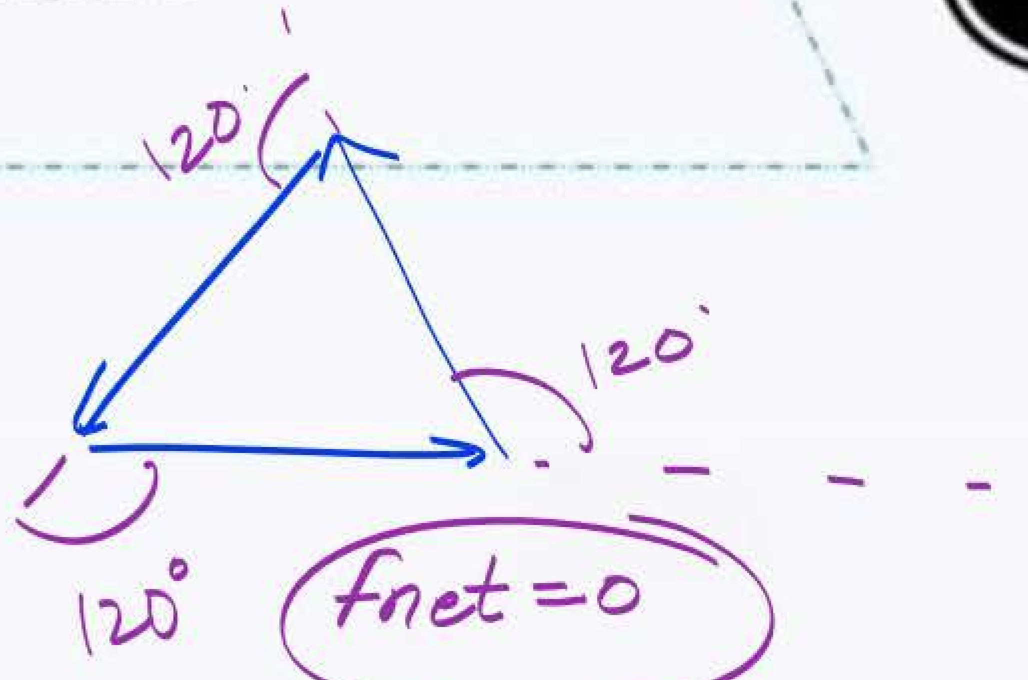
A ~~10 N, 10 N, 10 N~~

B 10 N, 10 N, 25 N

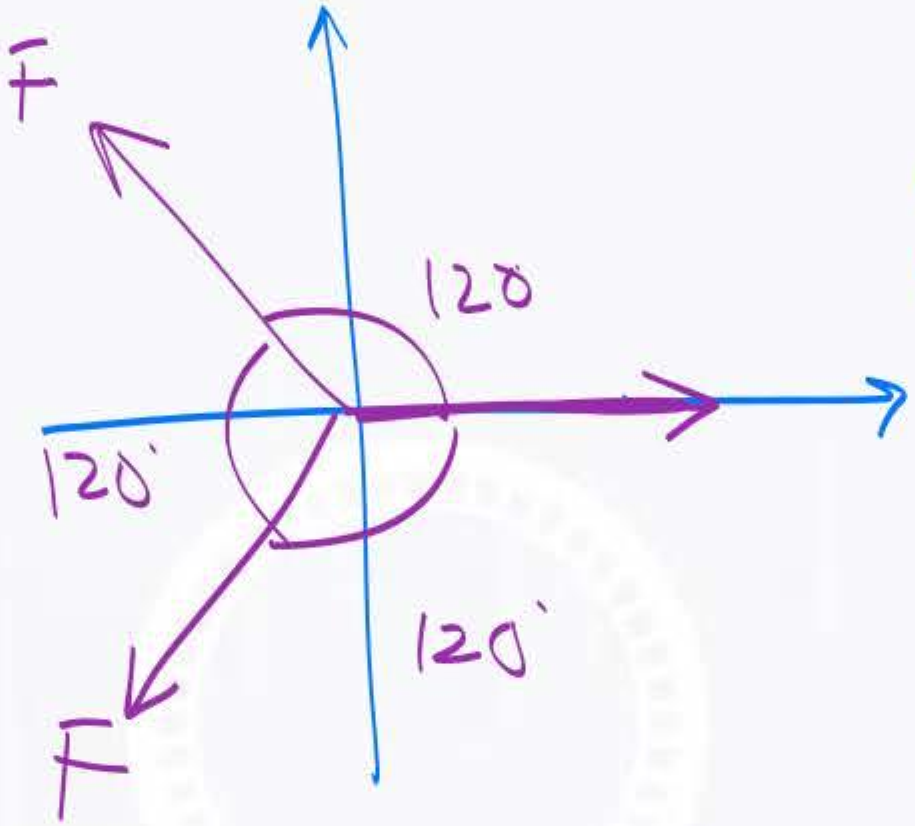
$10+10<25$ ✗

C 10 N, 10 N, 35 N

D None of these

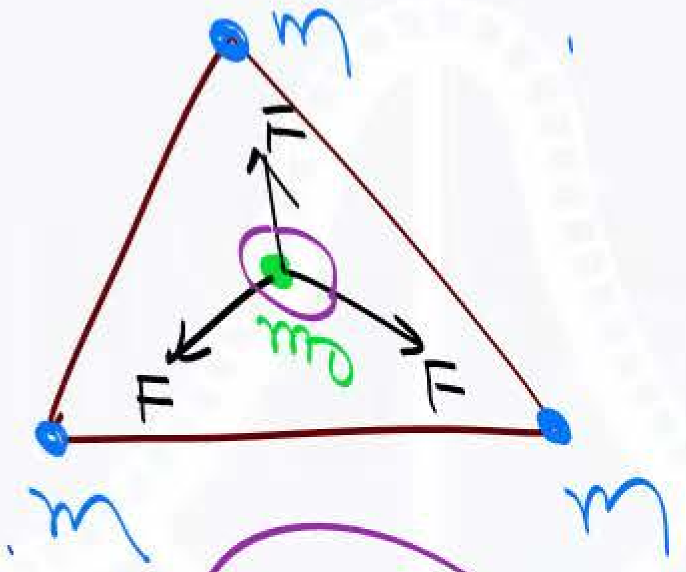


$F_{net}=0$



Gravitational

Symmetrical diagram



$F_{net}=0$

$F_{net}=0$

$I_{net}=0$

$F_{net}=0$

$\vec{E}=0$

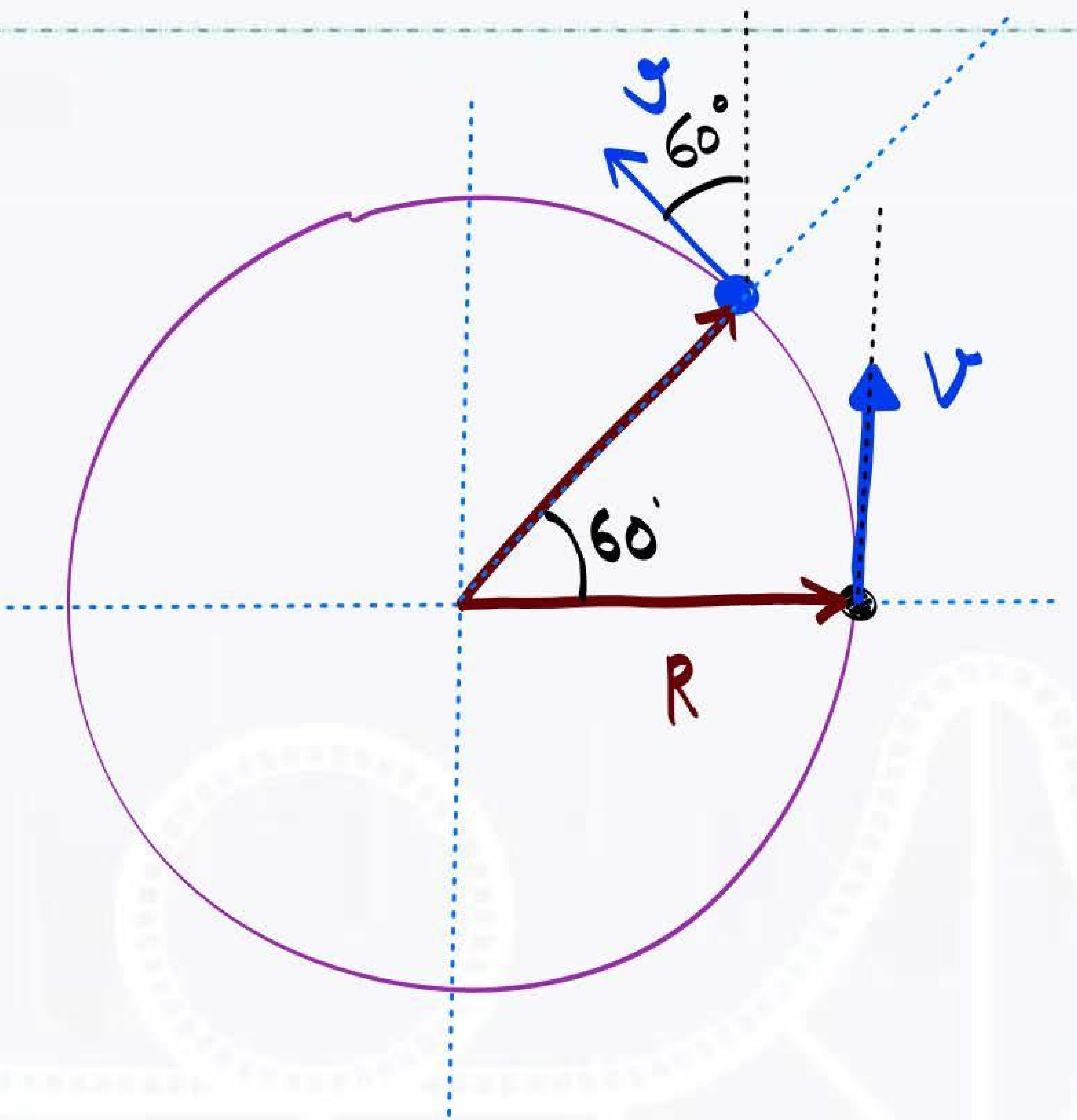
A cyclist is moving on a circular path with constant speed. What is the change in its velocity after it has described an angle of 60° ?

A $v\sqrt{2}$

B v Ans

C $v\sqrt{3}$

D None of these

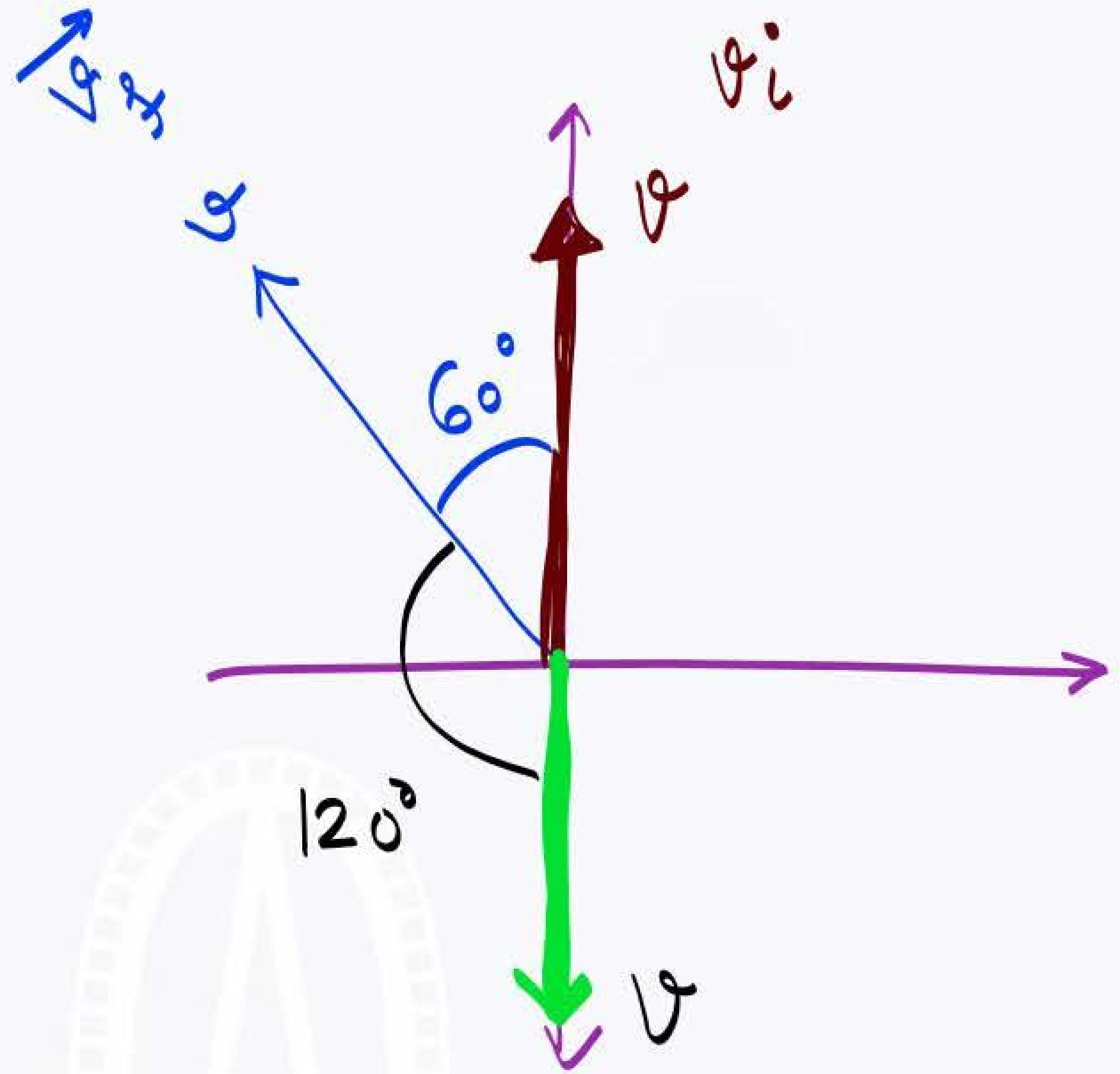


Change in velocity

$\vec{\Delta v} = \vec{v_f} - \vec{v_i}$

Subtraction of Vectors.

Velocity = Vector



$\vec{v_f} - \vec{v_i}$

$\vec{v_f} + (-\vec{v_i})$

$|A| = |B| = P$

$\theta = 120$ $R = P.$ $|\text{Change in velocity}| = v.$

Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is : **[JAN 2019]**

A 1 : 2

B 1 : 4

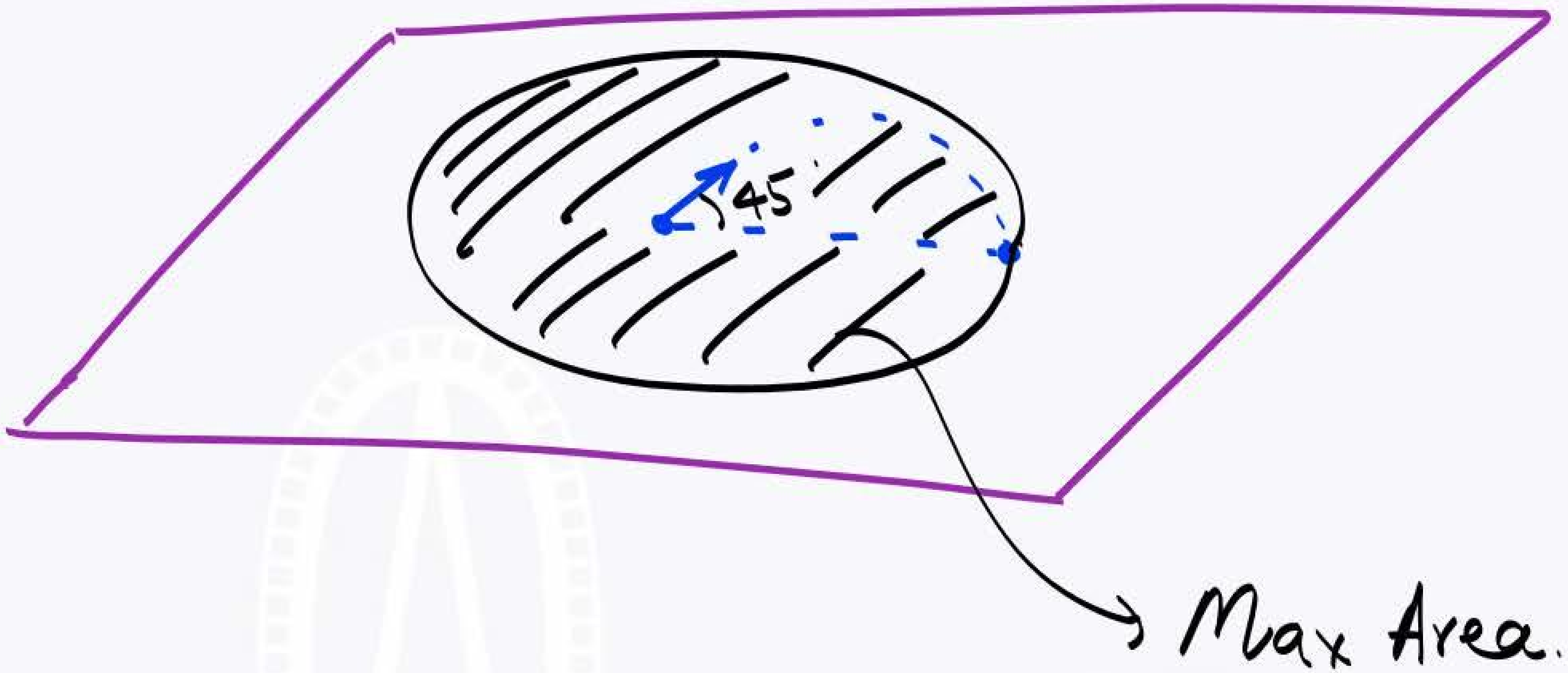
C 1 : 8

D 1 : 16 Ans

$$\frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \frac{1}{16}$$

Topics to be covered

1. Vectors
2. Projectile Motion
3. Relative Velocity
4. Questions



Range $\rightarrow$ max

$\theta = 45^\circ$

$$R_{max} = \frac{u^2}{g}$$

for Max Area. = radius of circle = R_{max}

$$\text{Area} = \pi r^2 = \pi R_{max}^2 = \frac{\pi u^4}{g^2}$$

A ball is thrown from a point with a speed 'v_0' at an elevation angle of θ. From the same point and at the same instant, a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball ? If yes, what should be the angle of projection θ ? **[2004]**

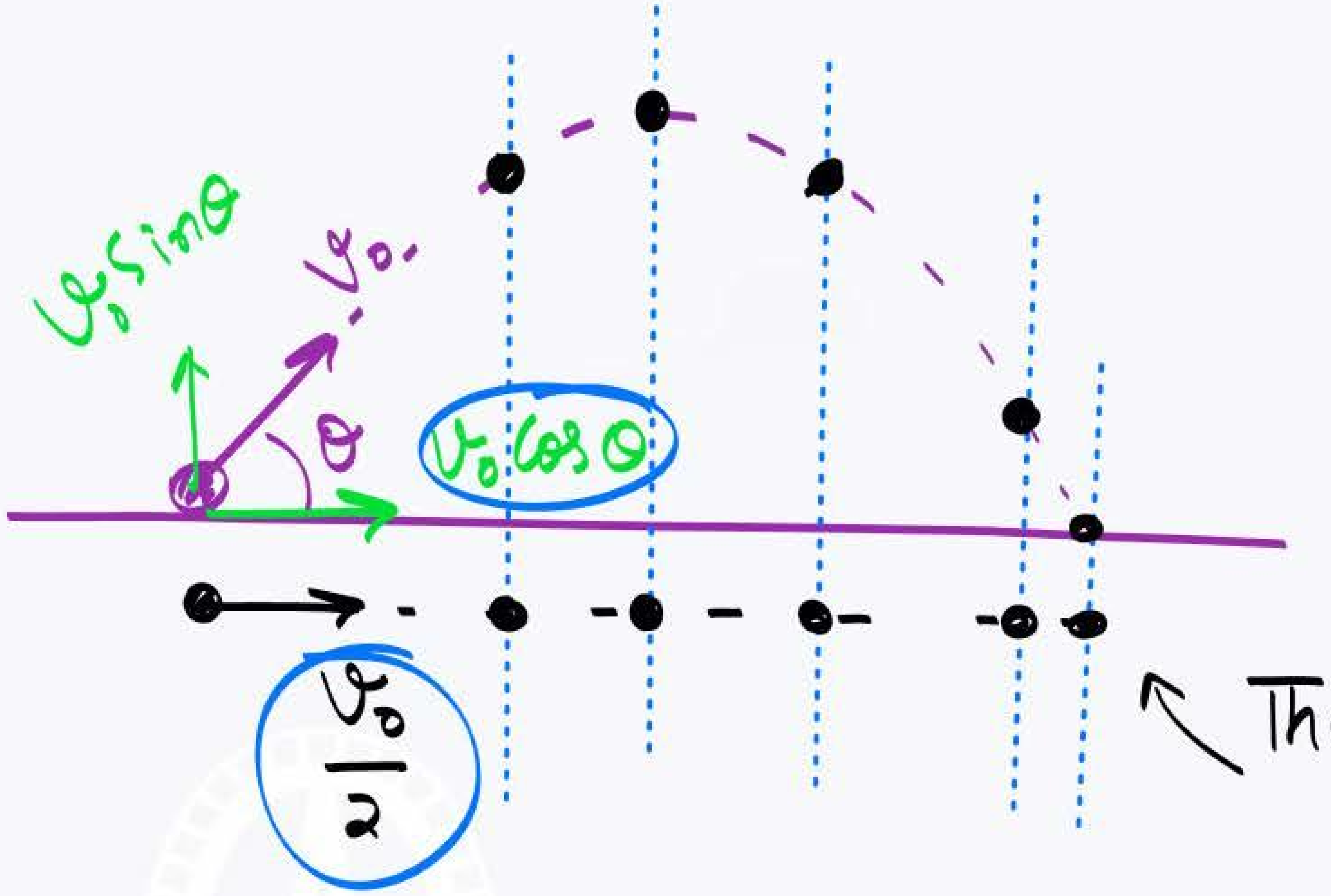
A No

B Yes, 30°

C Yes, 60° Ans

D Yes, 45°

$a_x = 0$



Then he can catch.

$$\frac{v_0}{2} = v_0 \cos\theta.$$

$$\frac{1}{2} = \cos\theta$$

$$\boxed{\theta = 60^\circ}$$

A particle starts moving from origin with velocity $\vec{u} = 3\hat{\imath}$ from origin and acceleration $\vec{a} = 6\hat{\imath} + 4\hat{\jmath}$. Find x- coordinate at the instant when y – coordinate of the particle is 32 m **[JAN 2020]**

A 48

~~B~~ 60 Ans

C 12

D 24

$t = 0$
$x = 0$
$u = 0$

$u = 3\hat{i}$
$a = 6\hat{i} + 4\hat{j}$

There is angle between $\vec{v}$ & $\vec{acc}$
(2D Motion)

$y = 32 \quad x = ?$

$u_x = 3 \quad u_y = 0$
$a_x = 6 \quad a_y = 4$

X $u_x\ a_x$ Y (u_y, a_y)

$v_x = u_x + a_x t \qquad v_y = u_y + a_y t$

$S_x = u_x t + \frac{1}{2} a_x t^2 \qquad S_y = u_y t + \frac{1}{2} a_y t^2$

$v_x^2 - u_x^2 = 2 a_x S_x \qquad v_y^2 - u_y^2 = 2 a_y S_y$

Lets find at time $y = 32$?

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$32 = 0 + \frac{1}{\cancel{2}} \times \overset{2}{4}\, t^2$$

$$16 = t^2$$

$$t = 4s$$

at $t = 4s$

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$= 3 \times 4 + \frac{1}{\cancel{2}} \times 6 \times \overset{2}{\cancel{4}} \times 4$$

$$= 12 + 6 \times 8$$

$$= 12 + 48$$

$$= 60m$$

Alternative

Equation of trajectory.

$x = f(t)$
$y = f(t)$ — Time Eliminate — $y = f(x)$

Let particle Reach (x, y) at time t

$S_x = u_x t + \frac{1}{2} a_x t^2$

$x = 3t + \frac{1}{2} \times 6t^2$

$\boxed{x = 3t + 3t^2}$

$S_y = u_y t + \frac{1}{2} a_y t^2$

$y = \frac{1}{2} \times 4 \times t^2$

$\boxed{y = 2t^2}$

$t = \frac{y^{1/2}}{\sqrt{2}}$

$y = 2t^2 \quad t^2 = \frac{y}{2}$

$x = 3t + 3t^2$

$x = 3t + 3\left(\frac{y}{2}\right)$

$x = 3\left(\frac{y}{2}\right)^{1/2} + 3\left(\frac{y}{2}\right)$

$y = 32$

$x = ?$

If $\frac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|} = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is :

A $0°$

B $45°$

C $90°$ Ans

D $60°$

There are two vector $\vec{a}$ & $\vec{b}$

$|\vec{a}+\vec{b}|$

$\Downarrow$

Mag of addition.

$|\vec{a}-\vec{b}|$

Mag of Subtraction.

$$\frac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|} = 1$$

$$\frac{\sqrt{a^2+b^2+2ab\cos\theta}}{\sqrt{a^2+b^2-2ab\cos\theta}} = 1$$

$$\cancel{a^2}+\cancel{b^2}+2\cancel{ab}\cos\theta = \cancel{a^2}+\cancel{b^2}-\cancel{2ab}\cos\theta$$

$$2\cos\theta = 0 \qquad \theta = \pi/2$$

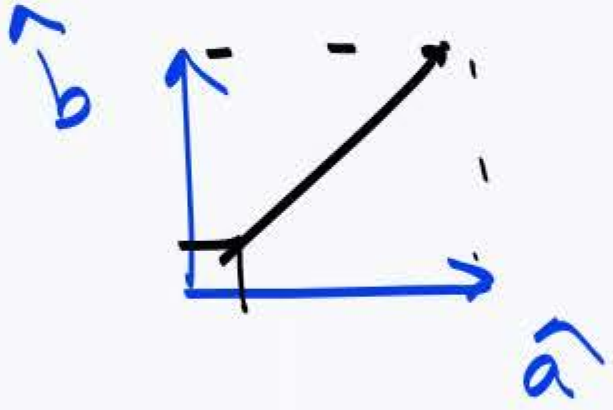
If $|\hat{a} - \hat{b}| = \sqrt{2}$ then calculate the value of $|\hat{a} + \sqrt{3}\hat{b}|$. Ans = 2

$\hat{a}$ = unit vector along $\vec{a}$, direction of $\vec{a}$ $\quad |\hat{a}| = 1$

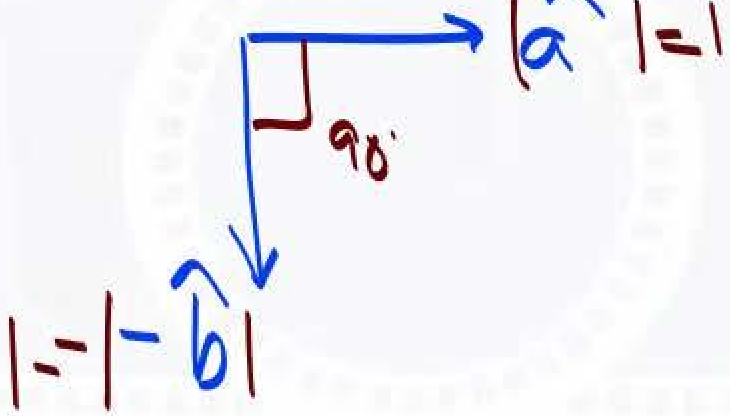
$\hat{b}$ = unit vector along $\vec{b}$, direction of $\vec{b}$. $\quad |\hat{b}| = 1$

$|\hat{a} - \hat{b}| = \sqrt{2}$

Mag of Sub of two unit vectors.



$|\hat{a} + \hat{b}| = \sqrt{2}$

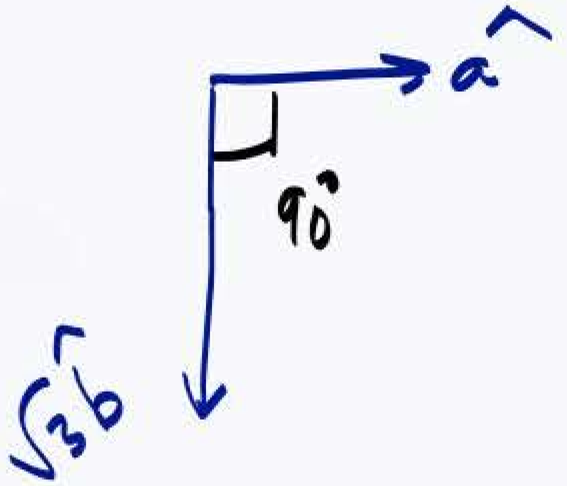


$|\hat{a} - \hat{b}| = \sqrt{2}$

$\hat{a}$ & $\hat{b}$ are $\perp$ to Each others.

$|\hat{a} + \sqrt{3}\hat{b}|$

→ Scalar multiply
dir = same.



$$R = \sqrt{A^2 + B^2}$$
$$= \sqrt{|\hat{a}|^2 + |\sqrt{3}\hat{b}|^2}$$
$$= \sqrt{1 + 3} = \sqrt{4} = 2$$

Ex:- $\vec{F}$

new force $= n\vec{F}$

⇓

number (Scalar)

‖

$n = +ve$
dir Same

$n = -ve$
dir opp.

$|n| > 1$ Magnitude ↑

$|n| < 1$ Magnitude ↓

$2N = F$

$4F$?

$8N$

$-F$

$2N$

$-2F$

$4N.$

$-\frac{F}{2}$

$1N.$

The displacement of an object along the three axes are given by, $x = 2t^2$, $y = t^2 - 4t$ and $z = 3t - 5$. The initial velocity of the particle is

A 10 units

B 12 units

C 5 units Ans

D 2 units

3D Motion.

$x = 2t^2$

$y = t^2 - 4t$

$z = 3t - 5$

$v_x = \frac{dx}{dt} = 4t$

$v_y = \frac{dy}{dt} = 2t - 4$

$v_z = \frac{dz}{dt} = 3$

$t = 0$

$v_x = 0$

$v_y = -4$

$v_z = 3$

$\vec{v}_{net} = 0\hat{i} - 4\hat{j} + 3\hat{k}$

$|v_{net}| = \sqrt{4^2 + 3^2}$

$= 5 \text{ m/s}$

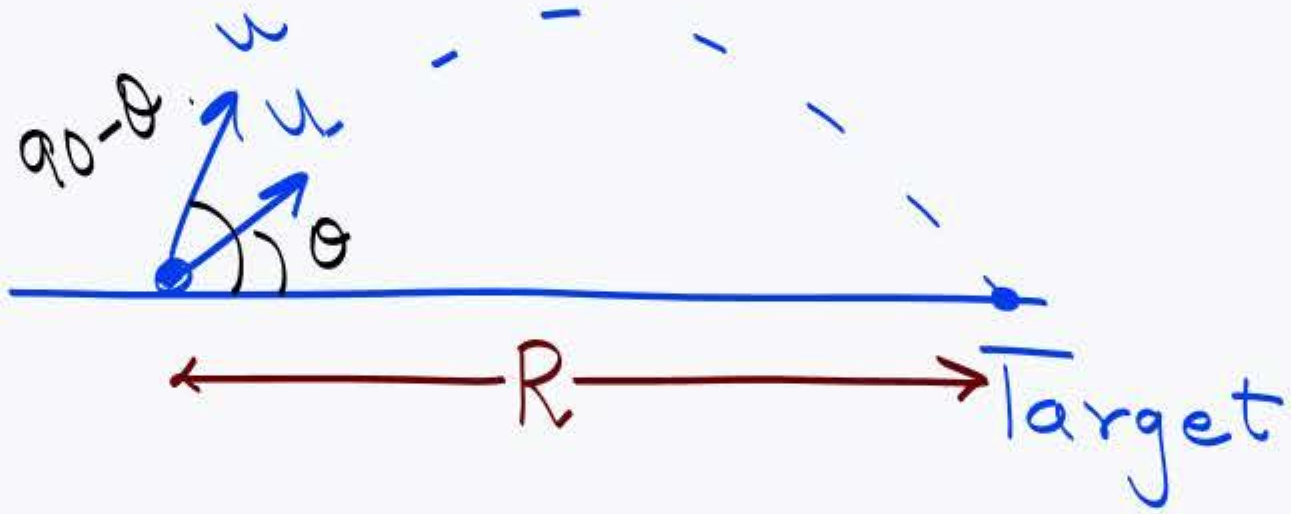
A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time take by it to hit the target in two possible ways, the product $t_1 t_2$ is : **[JAN 2019]**

A $\frac{R}{4g}$

B $\frac{R}{g}$

C $\frac{R}{2g}$

D $\frac{2R}{g}$ Ans



Complementary angle

$\boxed{R \rightarrow \text{fix}}$

$$T_\theta = \frac{2u\sin\theta}{g} = t_1$$

$$T_{90-\theta} = \frac{2u\cos\theta}{g} = t_2$$

$$t_1 t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2}$$

$$= \frac{2}{g}\underbrace{\left(\frac{2u^2\sin\theta\cos\theta}{g}\right)}_{R}$$

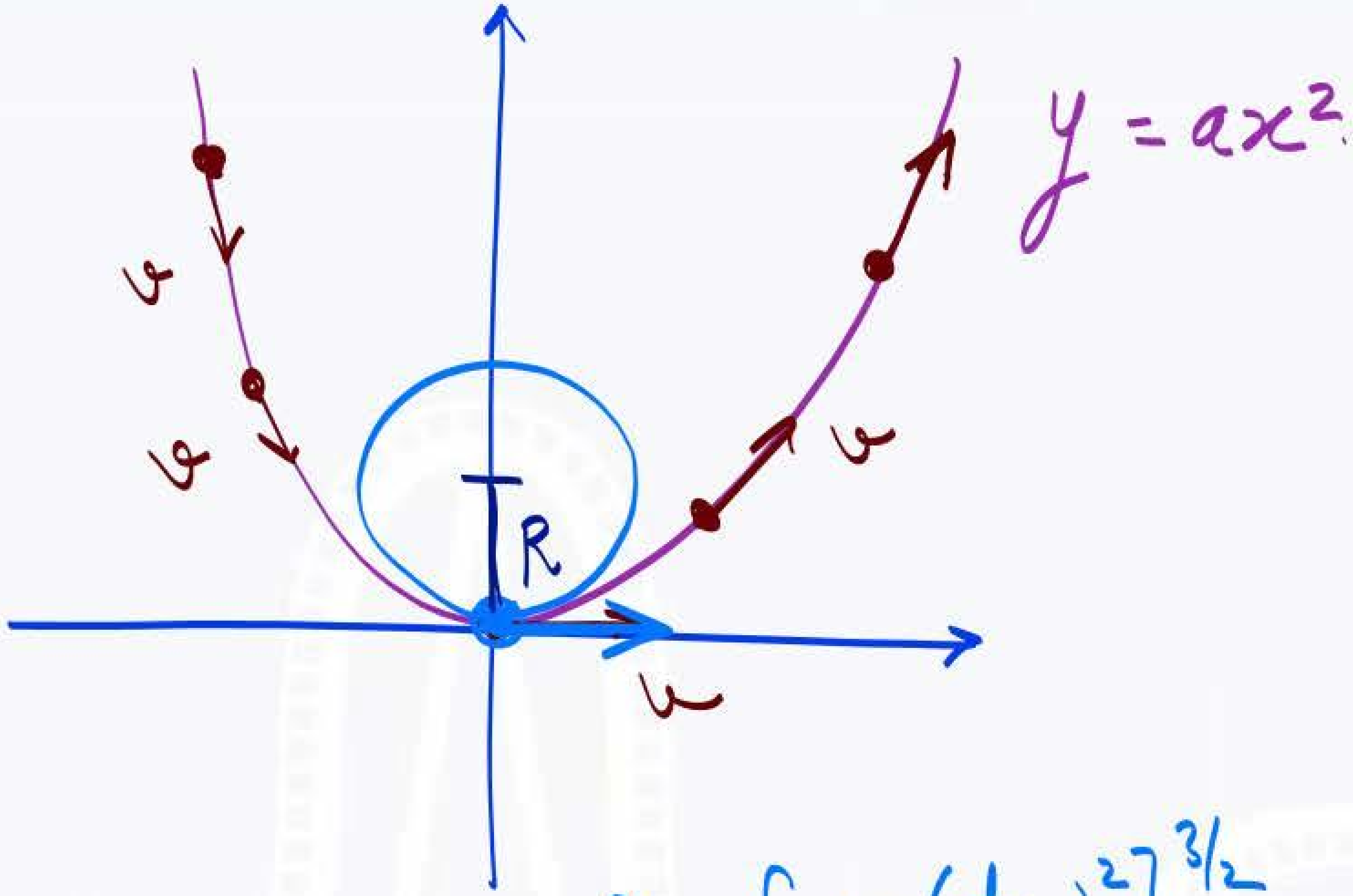
$$\boxed{t_1 t_2 = \frac{2R}{g}}$$

$$R = \frac{u^2\sin 2\theta}{g}$$

$$R = \frac{2u^2\sin\theta\cos\theta}{g}$$

A particle moves at uniform speed on a parabiotic trajectory $y = ax^2$ at uniform speed v. Find the acceleration of particle when it passes point $x = 0$ and point $(1, a)$. H.W

$x = 0 \quad a = ?$



Since direction of v changing → acceleration

$$a_c = \frac{v^2}{R}$$

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$a_c = \frac{v^2}{R}$$

$$a_c = \frac{v^2}{(1/2a)}$$

$$\boxed{a_{cc} = 2av^2} \text{ Ans.}$$

$$y = ax^2$$

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$y = ax^2$$

$$\frac{dy}{dx} = 2ax \qquad \text{at } x = 0 \quad \frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 2a.$$

$$R = \frac{\left[1 + 0^2\right]^{3/2}}{2a}$$

$$\boxed{R = \frac{1}{2a}} \quad \text{at } x = 0.$$

The area of the parallelogram determined by $\vec{A} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{B} = 12\hat{j} - 2\hat{k}$ is :

A. 42

B. 56

C. 38

D. 74

Area of ||gm = $|\bar{A} \times \bar{B}|$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 12 & -2 & 0 \end{vmatrix} = \hat{i}(1\times0 - 6) - \hat{j}(2\times0 + 36) + \hat{k}(-4-12)$$

$$= -6\hat{i} - 36\hat{j} - 16\hat{k}$$

$$\text{Area} = |\bar{A} \times \bar{B}| = \sqrt{6^2 + 36^2 + 16^2}$$

The values of x and y for which vectors $\vec{A} = (6\hat{i} + x\hat{j} - 2\hat{k})$ and $\vec{B} = (5\hat{i} - 6\hat{j} - y\hat{k})$ may be parallel are :

A. $x = 0, y = \frac{2}{3}$

B. $x = -\frac{36}{5}, y = \frac{5}{3}$ Ans

C. $x = -\frac{15}{3}, y = \frac{23}{5}$

D. $x = \frac{36}{5}, y = \frac{15}{4}$

|| Condition $\bar{A} \times \bar{B} = 0$ (Area of ||gm) = 0

$\vec{A} = 6\hat{i} + x\hat{j} - 2\hat{k}$

$\vec{B} = 5\hat{i} - 6\hat{j} - y\hat{k}$

$\frac{6}{5} = \frac{x}{-6} = \frac{-2}{-y}$

$-\frac{36}{5} = x$

$\frac{\cancel{6}^{3}}{5} = \frac{\cancel{2}}{y}$

$y = 5/3$

The three conterminous edges of a parallelepiped are

$\vec{a} = 2\hat{\imath} - 6\hat{\jmath} + 3\hat{k}$, $\vec{b} = 5\hat{\jmath}$ and $\vec{c} = -2\hat{\imath} + \hat{k}$.

The volume of parallelepiped is :

A 36 cubic unit

B 45 cubic unit

C 40 cubic unit Ans

D 54 cubic unit

Volume = (area of base) h.

Scalar

$(\vec{A} \times \vec{B}) \cdot \vec{C}$

$\Downarrow$

Volume of ||^d pipe.

$\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k}$

$\vec{b} = 5\hat{j}$

$\vec{c} = -2\hat{i} + \hat{k}$

Dot Prod.

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & -\hat{j} & k \\ 2 & -6 & 3 \\ 0 & 5 & 0 \end{vmatrix}$$

$$= \hat{i}(-15) - \hat{j}(0) + \hat{k}(10)$$

$$\boxed{\vec{a} \times \vec{b} = -15\hat{i} + 10\hat{k}}$$

Volume $= 15 \times 2 + 10$

$= 40$ unit

The unit vector perpendicular to vectors

$\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{j} - 5\hat{k}$ is :

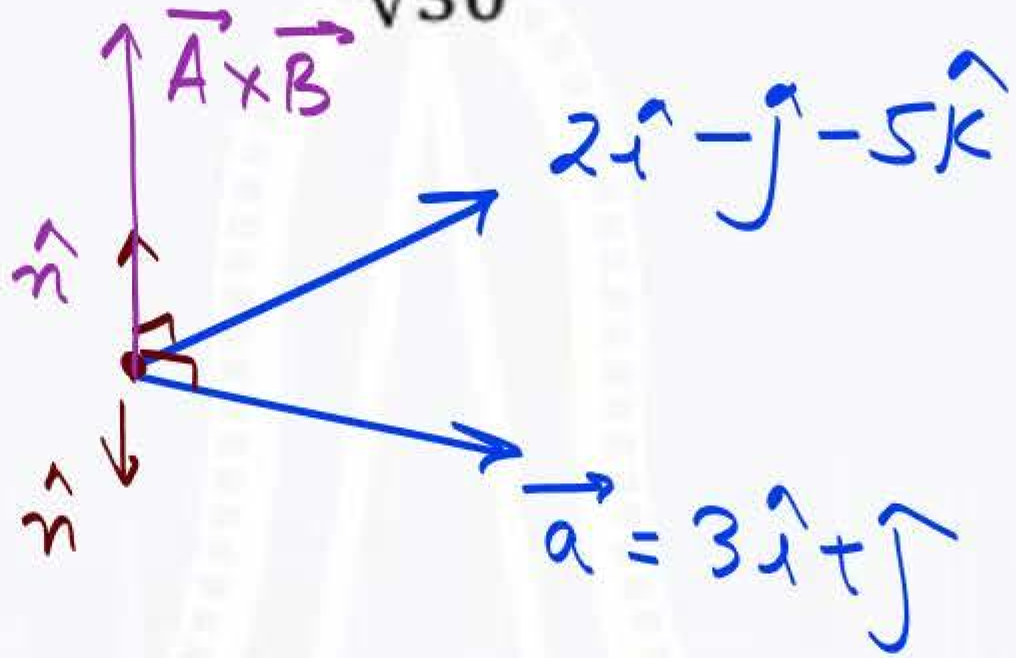
$$\hat{n} = \frac{\vec{C}}{|C|} = \frac{-5\hat{i} + 15\hat{j} - 5\hat{k}}{\sqrt{5^2 + 15^2 + 5^2}}$$

A $\pm \dfrac{(\hat{i} - 3\hat{j} + \hat{k})}{\sqrt{11}}$

B $\pm \dfrac{3\hat{i} + \hat{j}}{\sqrt{11}}$

C $\pm \dfrac{(2\hat{i} - \hat{j} - 5\hat{k})}{\sqrt{30}}$

D none of these



unit vector $\perp$ to A & B = direction of $\overline{A \times B}$.

1. $\begin{vmatrix} \hat{i} & -\hat{j} & k \\ 3 & 1 & 0 \\ 2 & -1 & -5 \end{vmatrix} = \hat{i}(-5-0) - \hat{j}(-15) + \hat{k}(-3-2)$

$= -5\hat{i} + 15\hat{j} - 5\hat{k}$

A force $\vec{F} = (2\hat{\imath} + 3\hat{\jmath} - \hat{k})$ N is acting on a body at a position $\vec{r} = (6\hat{\imath} + 3\hat{\jmath} - 2\hat{k})$. The torque about the origin is:

A $(3\hat{\imath} + 2\hat{\jmath} + 12\hat{k})Nm$ Ans

B $(9\hat{\imath} + \hat{\jmath} + 7\hat{k})Nm$

C $(\hat{\imath} + 2\hat{\jmath} + 12\hat{k})Nm$

D $(3\hat{\imath} + 12\hat{\jmath} + \hat{k})Nm$

Sol:- $\vec{F} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{r} = (6\hat{i} + 3\hat{j} - 2\hat{k})$

$\vec{\tau} = \vec{r} \times \vec{F}$ order is very Imp.

$$= \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 6 & 3 & -2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(-3+6) - \hat{j}(-6+4) + \hat{k}(18-6)$$

$$= 3\hat{i} + 2\hat{j} + 12\hat{k}$$

The coordinates of a moving particle at any time t are given by, $x = 2t^3$ and $y = 3t^3$. Acceleration of the particle is given

A. $468\,t$

B. $t\sqrt{468}$

C. $234\,t^2$

D. $t\sqrt{234}$

$x = 2t^3$ $\quad$ $x = f(t)$

$y = 3t^3$ $\quad$ $y = f(t)$

2D Motion

$v_x = \frac{dx}{dt} = 6t^2$ $\quad$ $a_x = \frac{dv_x}{dt} = 12t$

$v_y = \frac{dy}{dt} = 9t^2$ $\quad$ $a_y = \frac{dv_y}{dt} = 18t$

$\vec{a}_{net} = 12t\,\hat{i} + 18t\,\hat{j}$

$|a_{net}| = \sqrt{(12t)^2 + (18t)^2}$

The trajectory of a projectile in vertical plane is $y = ax - bx^2$, where a and b are constants and x and y are respectively horizontal and vertical distances of the projectile from the point of projection. The maximum height attained by the particle and the angle of projection from the horizontal are

H_{max}

θ.

A $\frac{b^2}{4b}, \tan^{-1}(b)$

B $\frac{a^2}{b}, \tan^{-1}(2b)$

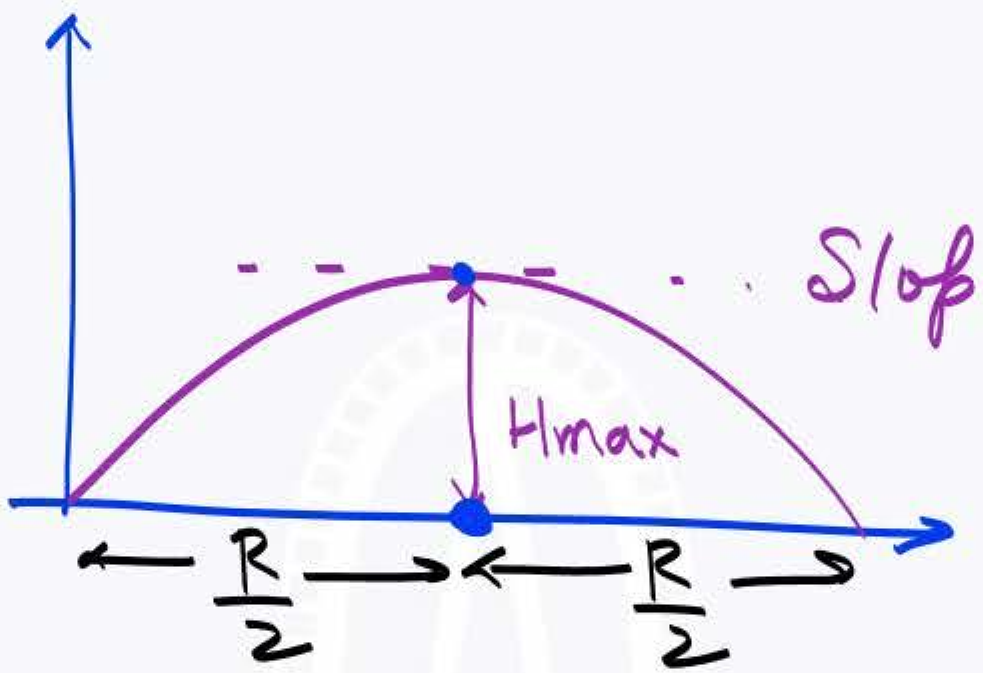
C $\frac{a^2}{4b}, \tan^{-1}(a)$ Ans

D $\frac{2a^2}{b}, \tan^{-1}(a)$

$$\boxed{y = ax - bx^2}$$

$$y = ax - bx^2$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$



$\tan\theta = a$

$$\boxed{\theta = \tan^{-1}(a)}$$

$y = ax - bx^2$

Hmax   Slope = 0

$\frac{dy}{dx} = a - 2bx$

$0 = a - 2bx$

$x = \frac{a}{2b} \rightarrow \frac{\text{Range}}{2}$

$$\boxed{\text{Range} = 2x = \frac{a}{b}}$$

at $x = \frac{a}{2b}$

$y = ? \Rightarrow H_{max}$

$y = ax - bx^2$

$y = a\left(\frac{a}{2b}\right) - b\left(\frac{a^2}{4b^2}\right)$

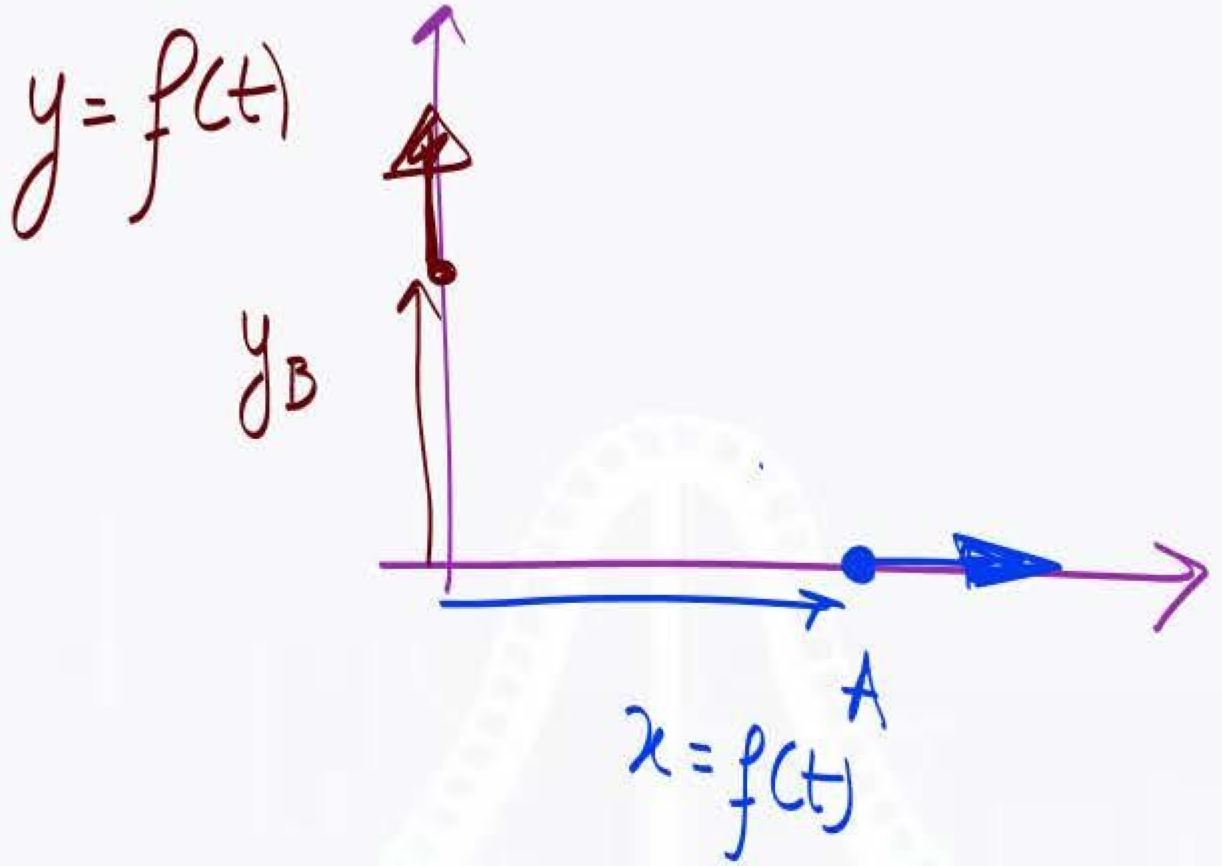
$y = \frac{a^2}{2b} - \frac{a^2}{4b}$

$$\boxed{H_{max} = \frac{a^2}{4b}}$$

Position of two particles A and B as a function of time are given by $X_A = -3t^2 + 8t + c$ and $y_B = 10 - 8t^3$. The velocity of B with respect to A at $t = 1$ is $\sqrt{v}$. Find v **(JAN 2020)**

- $x_A = -3t^2 + 8t + c$
- $y_B = 10 - 8t^3$

with time both Particles are moving



$v_A = \frac{dx_A}{dt} = (-6t + 8)\hat{i}$

$v_B = \frac{dy_B}{dt} = 0 - 24t^2$

at $t = 1s$ $\quad v_A = (-6+8)\hat{i} = 2\hat{i}$

$v_B = -24\hat{j}$

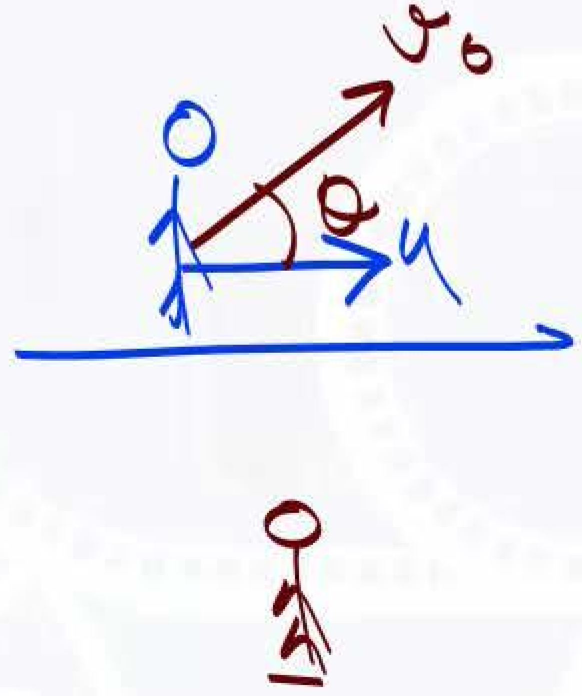
A cricket fielder can throw the cricket ball with a speed v_0. If he throws the ball while running with speed u at an angle θ to the horizontal, what is the effective angle to the horizontal at which the ball is projected in air as seen by a spectator ?

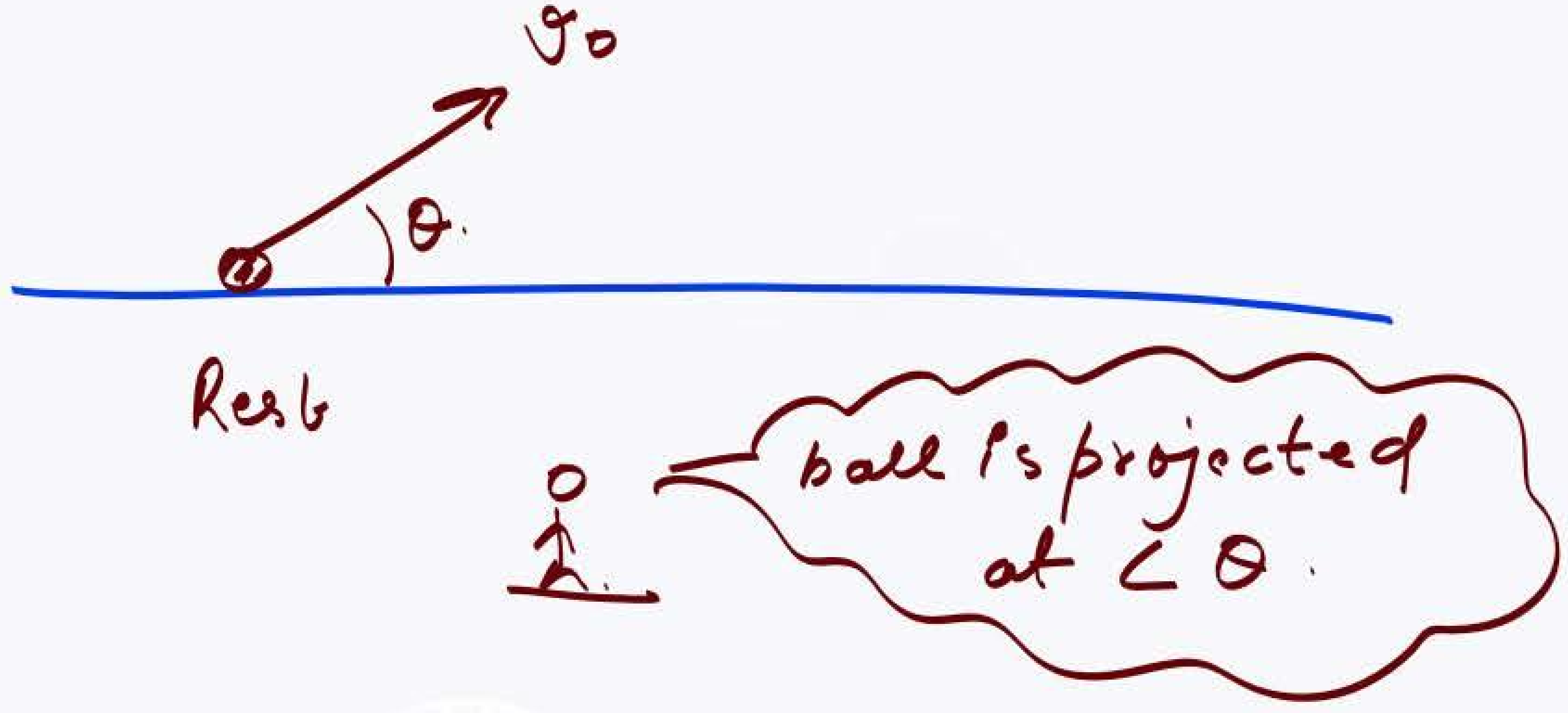
A. $\tan^{-1}\left[\frac{v_0\cos\theta}{u+v_0\sin\theta}\right]$

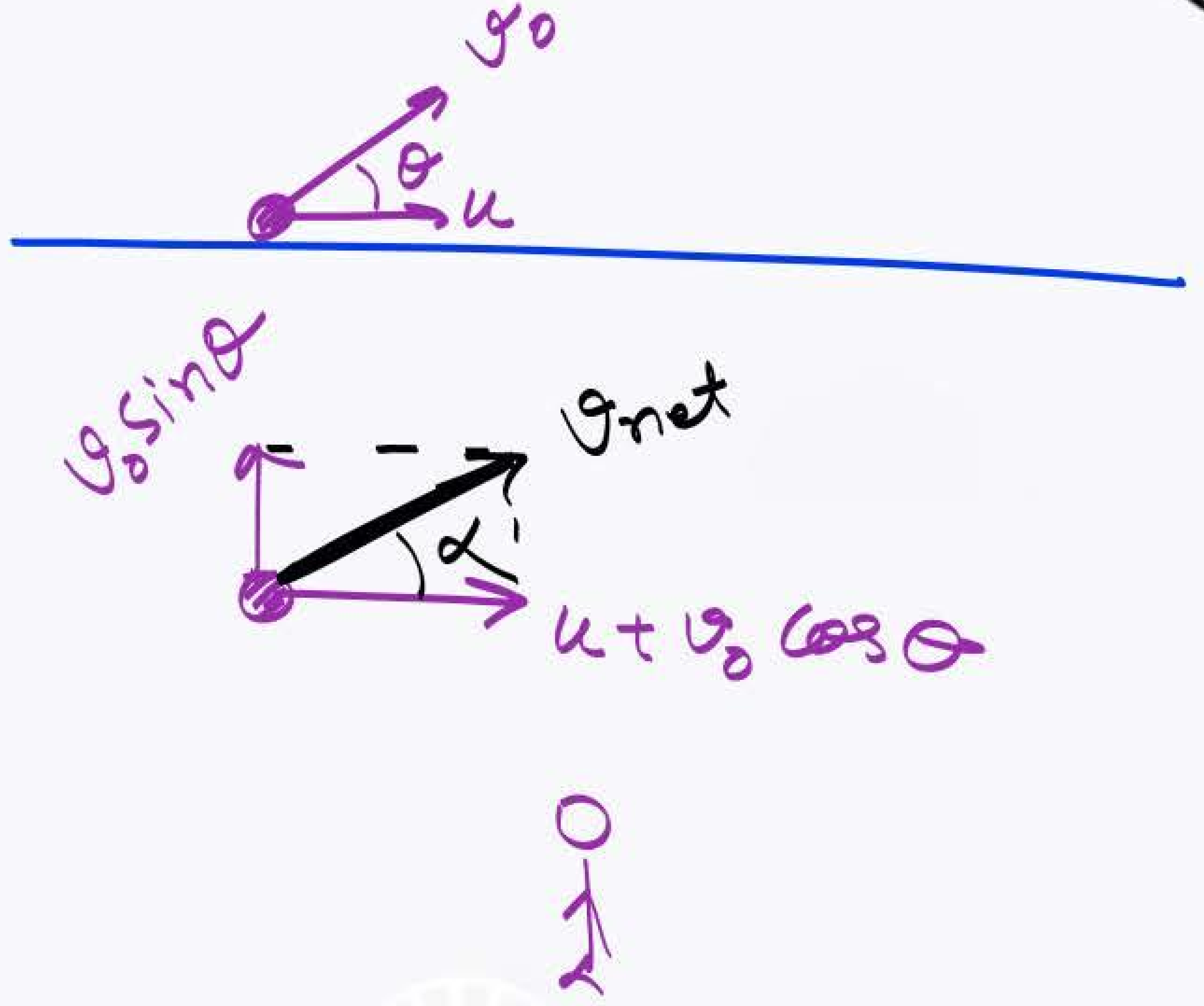
B. $\tan^{-1}\left[\frac{v_0\sin\theta}{u+v_0\cos\theta}\right]$ Ans

C. $\tan^{-1}\left[\frac{u}{v_0\cos\theta+v_0\sin\theta}\right]$

D. $\tan^{-1}\left[\frac{v_0\sin\theta+v_0\cos\theta}{u}\right]$







$$\tan\alpha = \frac{P}{B} = \frac{v_0 \sin\theta}{u + v_0 \cos\theta}$$

$$\alpha = \tan^{-1}\left(\frac{v_0 \sin\theta}{u + v_0 \cos\theta}\right)$$

The initial velocity of a particle of mass 2 kg is $(4\hat{\imath} + 4\hat{\jmath})\text{ms}^{-1}$. A constant force of $-20\hat{\jmath}$ N is applied on the particle. Initially the particle was at (0, 0). Find the x-coordinate of the point where its y-coordinate is again zero.

A. 3.2 m Ans

B. 6 m

C. 4.8 m

D. 1.2 m

Method 1 done Previously

Alter

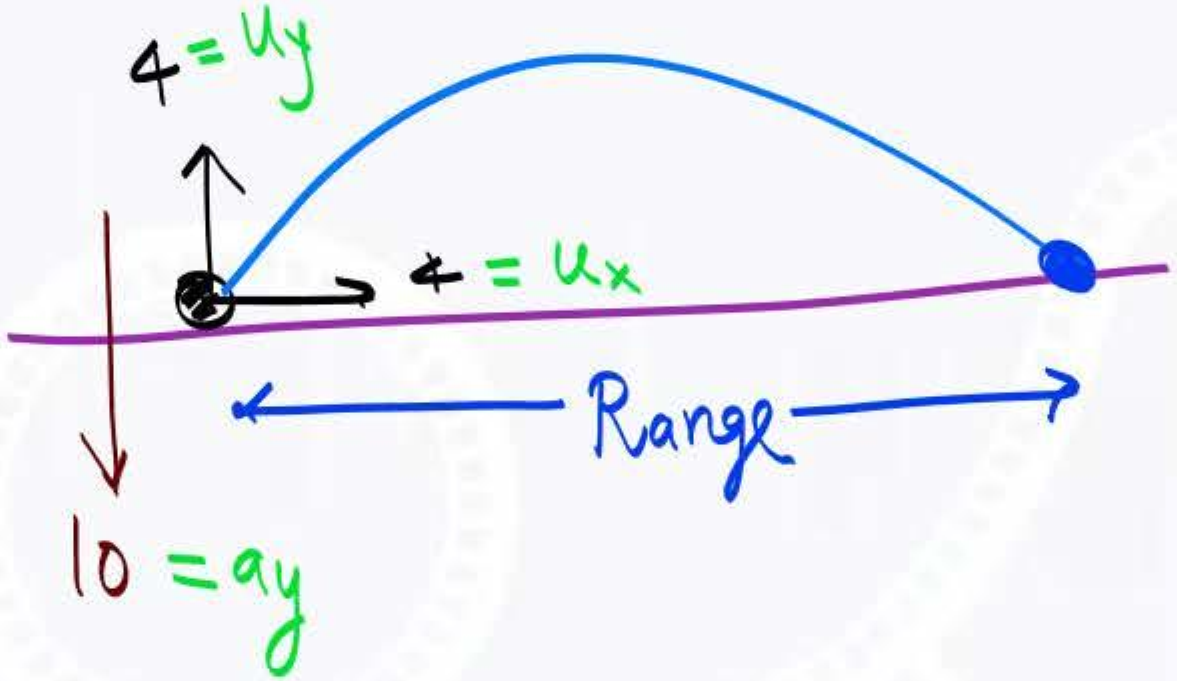
$\vec{u} = 4\hat{i} + 4\hat{j}$

$t = 0$

$x = 0$

$F = -20\hat{j}$

$\frac{F}{m} = a = -10\hat{j}$



$R = \frac{u^2 \sin 2\theta}{g}$

$R = \frac{2 u_x u_y}{g}$

$= \frac{\cancel{2} \times 4 \times 4}{\cancel{10}\,5} = \frac{16}{5}$

$=$

The coordinates of a moving particle at any time t are given by $x = ct$ and $y = bt^2$. The speed of the particle is given by

A $2t\sqrt{b^2 - c^2}$

B $\sqrt{4b^2t^2 + c^2}$ Ans

C $2t(b + c)$

D $2t(b - c)$

$x = ct \qquad v_x = c$

$y = bt^2 \qquad v_y = 2bt$

$$\vec{v}_{net} = c\hat{i} + 2bt\hat{j}$$

$$|v_{net}| = \sqrt{c^2 + 4b^2t^2}$$

(2 Projectile)

Balls A and B are thrown from two points lying on the same horizontal plane separated by a distance 120 m. Which of the following statement (s) is/are correct ?

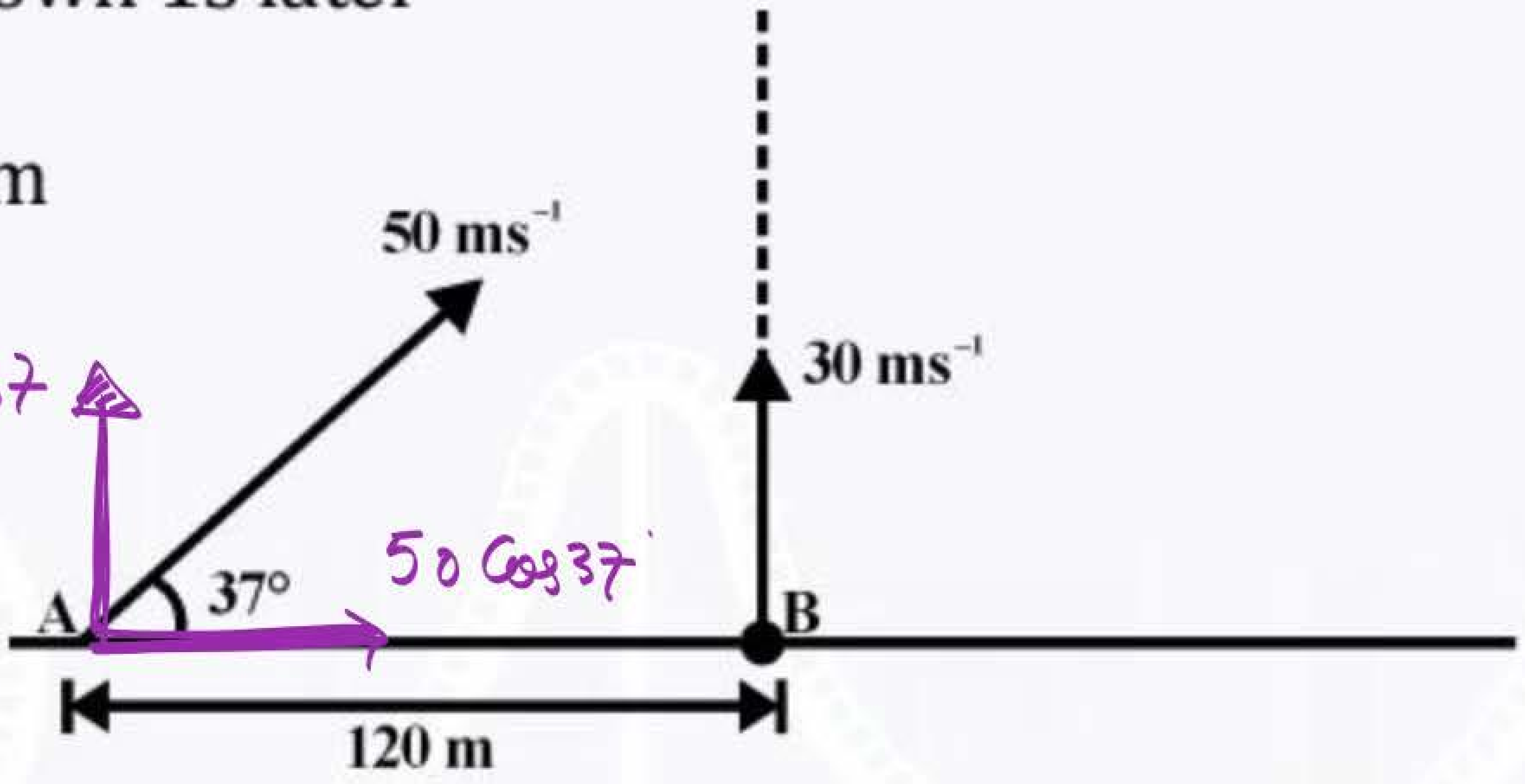
A. ~~The two balls can never meet~~

B. ~~The balls can meet if the ball B is thrown 1s later~~

Ans C. The two balls meet at a height of 45 m

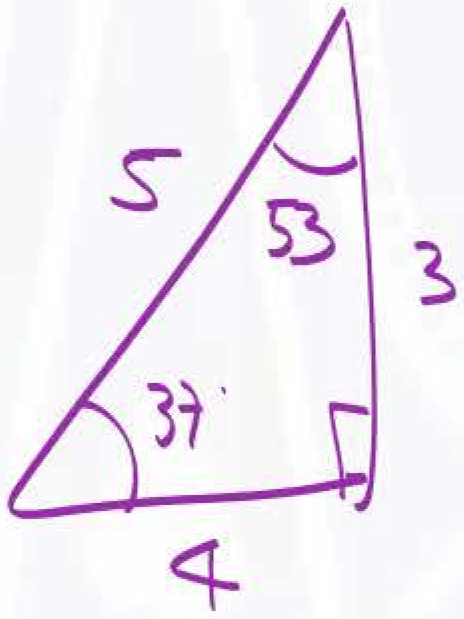
D. None of the above

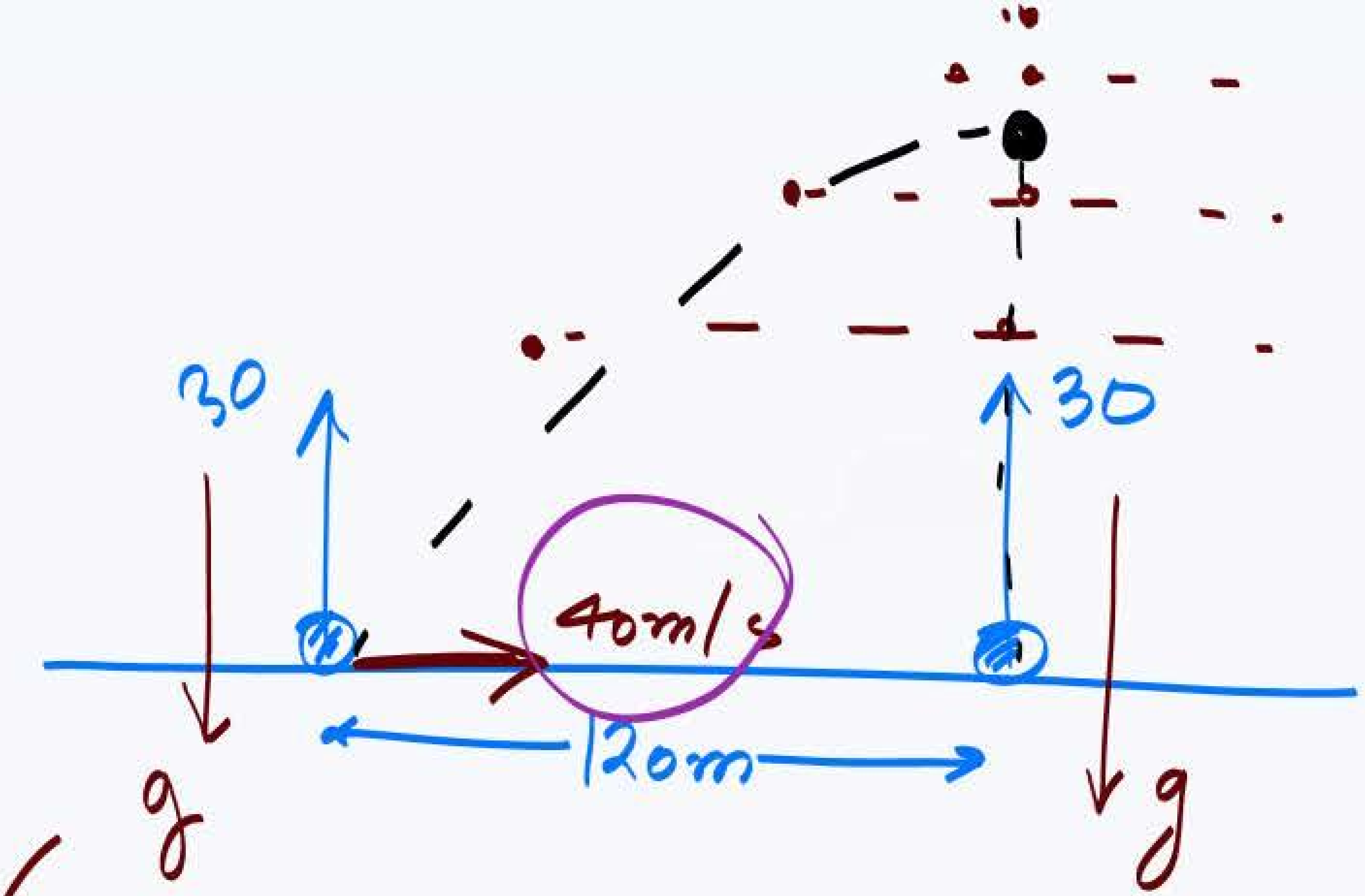
$30 m/s = 50 \sin 37$



$50 \cos 37$

$50 \times \frac{4}{5} = 40 m/s$





$a_{rel} = 0$

$u_{rel} = 0$ along Y

40m/s $\Rightarrow$ Move A towards B.

$$\boxed{t = \frac{120}{40} = 3\,s} \Rightarrow \text{Time to Reach } H_{max}$$

$$\text{Time of flight} = \frac{2u_y}{g}$$

$$\boxed{T_A = T_B} \quad T = \frac{2\times 30}{10} = 6\,s$$

$$H_{max} = \frac{u_y^2}{2g} = \frac{\overset{15}{\cancel{30}}\times 3\cancel{0}}{\cancel{2}\times 1\cancel{0}} = 45\,m$$

When the angle of projection is 75°, a ball falls 10 m shorter of the target. When the angle of projection is 45°, it falls 10 m ahead of the target. Both are projected from the same point with the same speed in the same direction, the distance of the target from the point of projection is

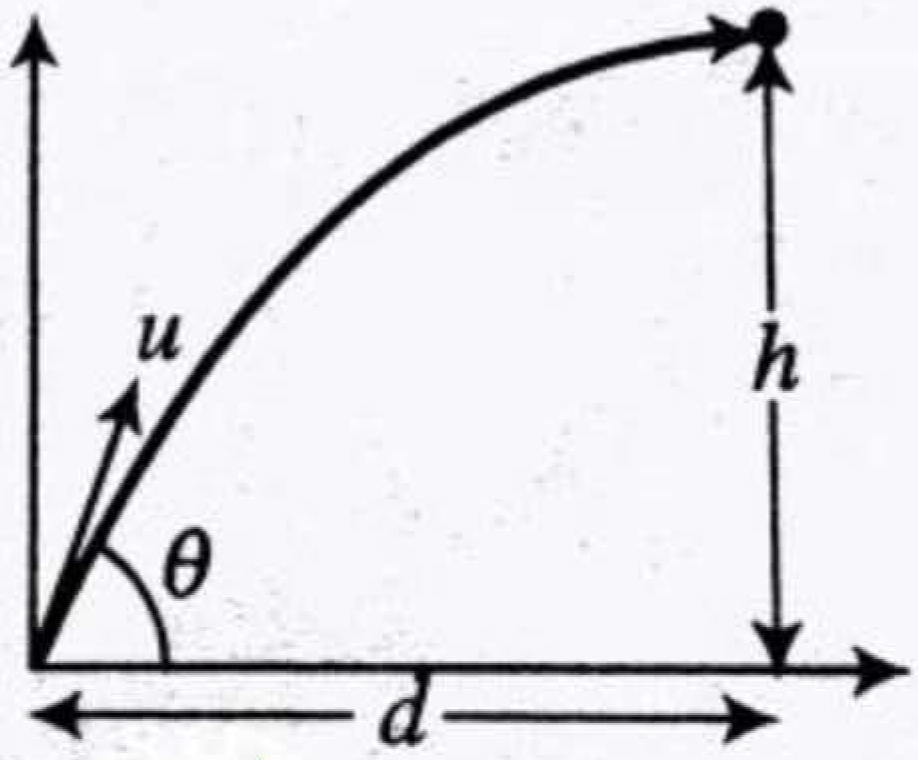
A. 15 m

B. 30 m

C. 45 m

D. 10 m

If a stone is to hit at a point which is at a distance d away and at a height h above the point from where the stone starts, then what is the value of initial speed u, if the stone is launched at an angle θ ?



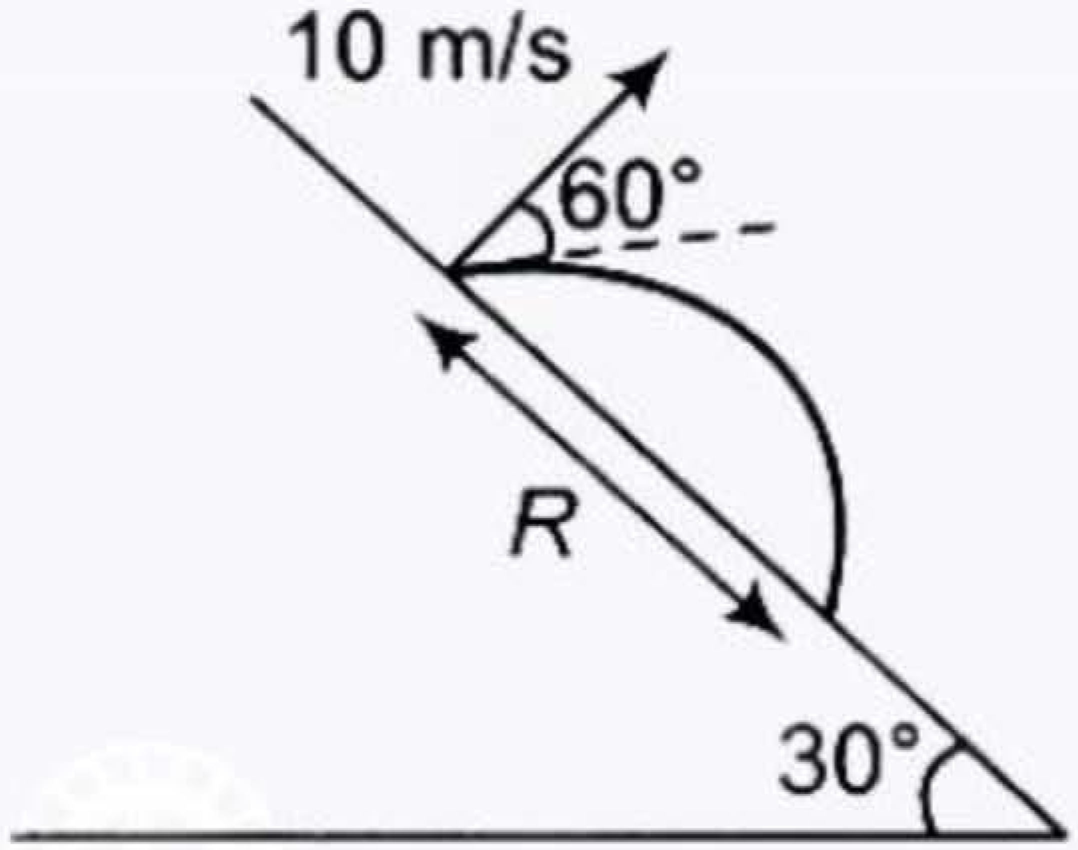
A projectile is launched with a speed of 10 m/s at an angle $60°$ with the horizontal from a sloping surface of inclination $30°$. The range R is. (Take, $g = 10\ m/s^2$)

A 4.9 m

B 13.3 m

C 9.1 m

D 12.6 m



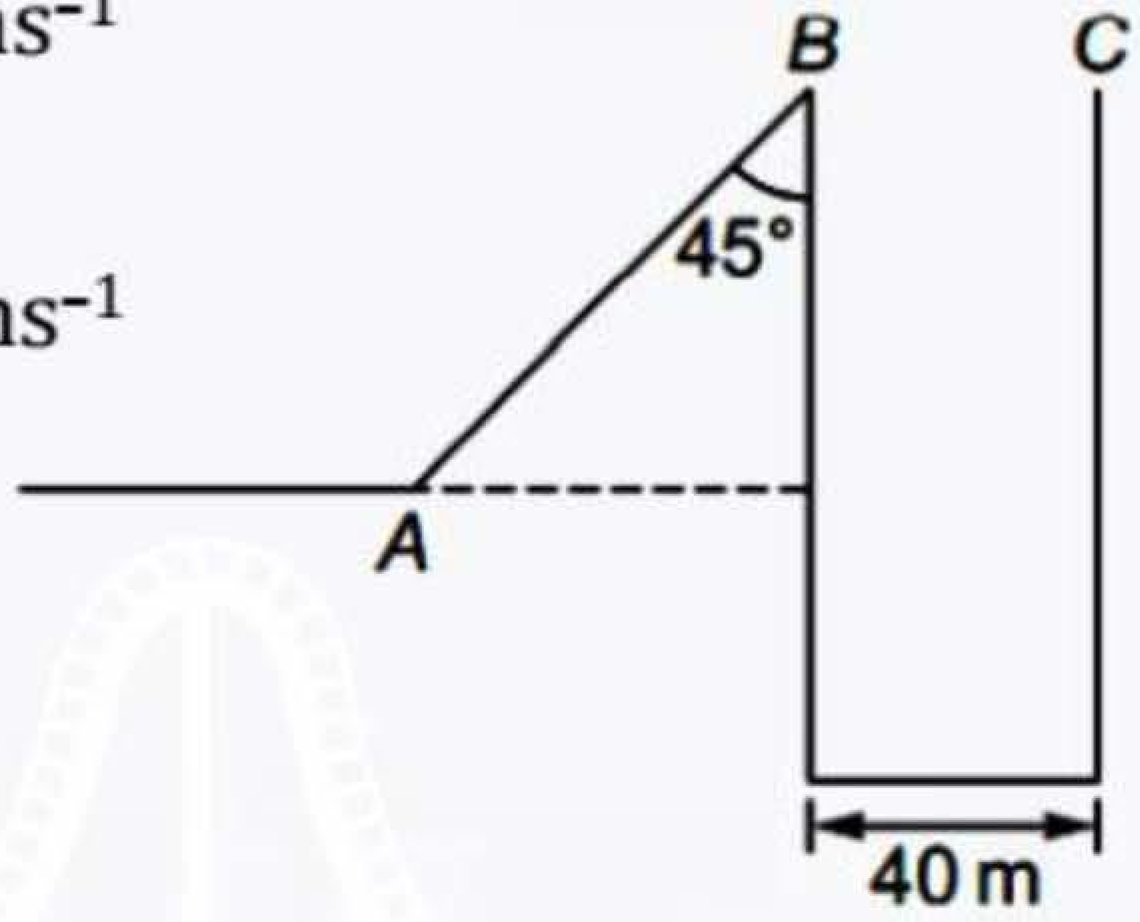
A body is projected up smooth inclined plane with a velocity v_0 from the point A as shown figure. The angle of inclination is 45° and top B of the plane is connected to a well of diameter 40 m. If the body just manages to cross the well,

A. $20\ ms^{-1}$

B. $20\sqrt{2}\ ms^{-1}$

C. $40\ ms^{-1}$

D. $40\sqrt{2}\ ms^{-1}$



$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

$$= -24\hat{j} - 2\hat{i}$$

$$|v_{BA}| = \sqrt{24^2 + 2^2}$$

$$= \sqrt{580} = \sqrt{v}$$

$$\boxed{v = 580}$$ Ans

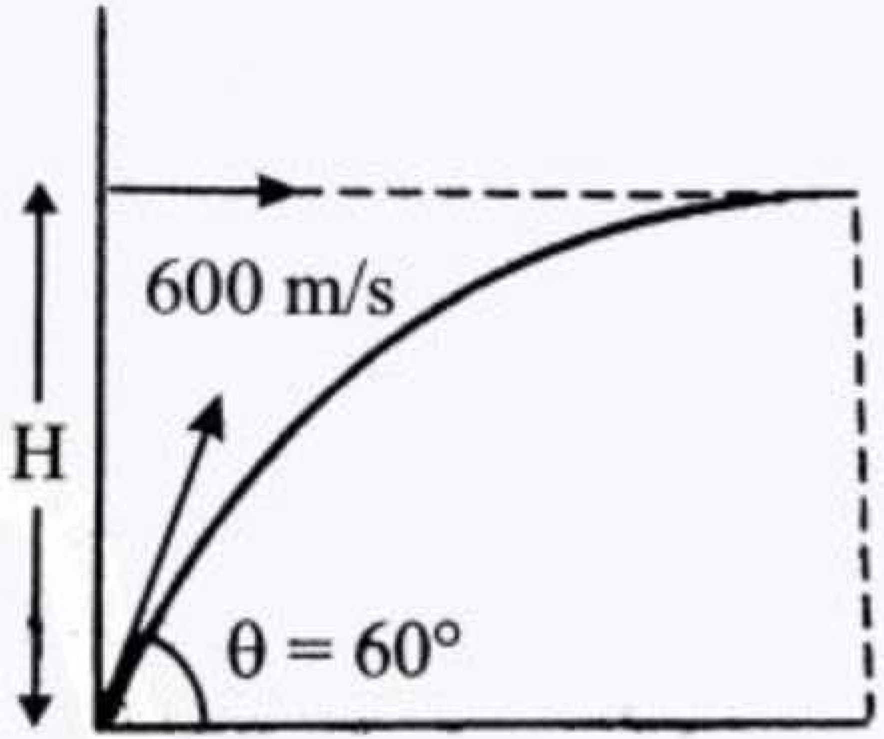
A fighter plane enters inside the enemy territory, at time $t = 0$ with velocity $v_0 = 250\ ms^{-1}$ and moves horizontally with constant acceleration $a = 20\ ms^{-2}$ (see figure). An enemy tank at the border, spot the plane and fire shots at an angle $\theta = 60°$ with the horizontal and with velocity $u = 600\ ms^{-1}$. At what altitude H of the plane it can be hit by the shot ?

A $1500\sqrt{3}$ m

B 125 m

C 1400 m

D 2473 m



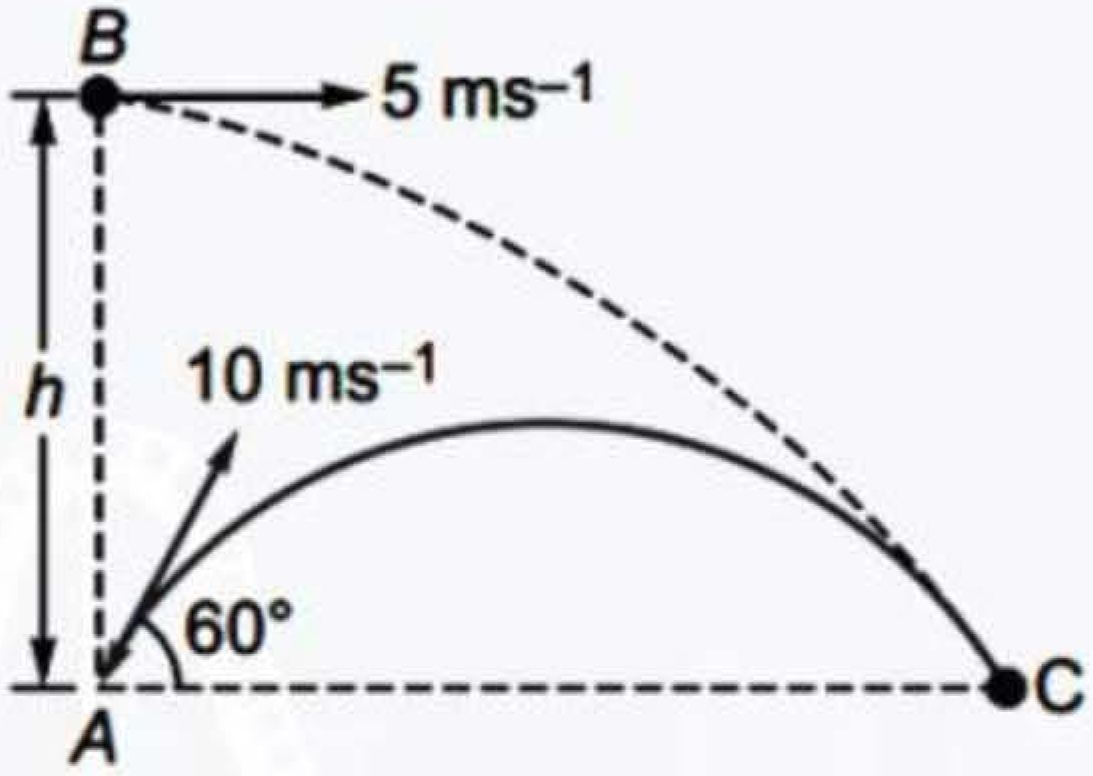
A particle A is projected from the ground with an initial velocity of 10 ms^{-1} at an angle of 60° with horizontal. From what height h should an another particle B be projected horizontal with velocity 5 ms^{-1} so that both the particles collide with velocity 5 ms^{-1} so that both the particles collide on the ground at point C is both are projected simultaneously ? ($g = 10\ ms^{-2}$)

A 10 m

B 30 m

C 15 m

D 25 m



Two boys are standing at the ends A and B of a ground where AB = a. The boy at B starts running in a direction perpendicular to AB, with velocity v_1. The boy at A starts running simultaneously with velocity v and catches the other boy in a time t, then find t.

A particle is moving with a velocity of 10 m/s towards east. After 10 s its velocity changes to 10 m/s towards north. Its average acceleration is:-

A. Zero

B. $\sqrt{2}$ m/s^2 towards N-W

C. $1/\sqrt{2}$ m/s^2 towards N-E

D. $1/\sqrt{2}$ m/s^2 towards N-W

When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops coming at an angle $60°$ from the horizontal. On further increasing the speed of the car to $(1 + \beta)v$, this angle changes to $45°$. The value of β is close to :

[SEP 2020]

A 0.41

B 0.50

C 0.73

D 0.37

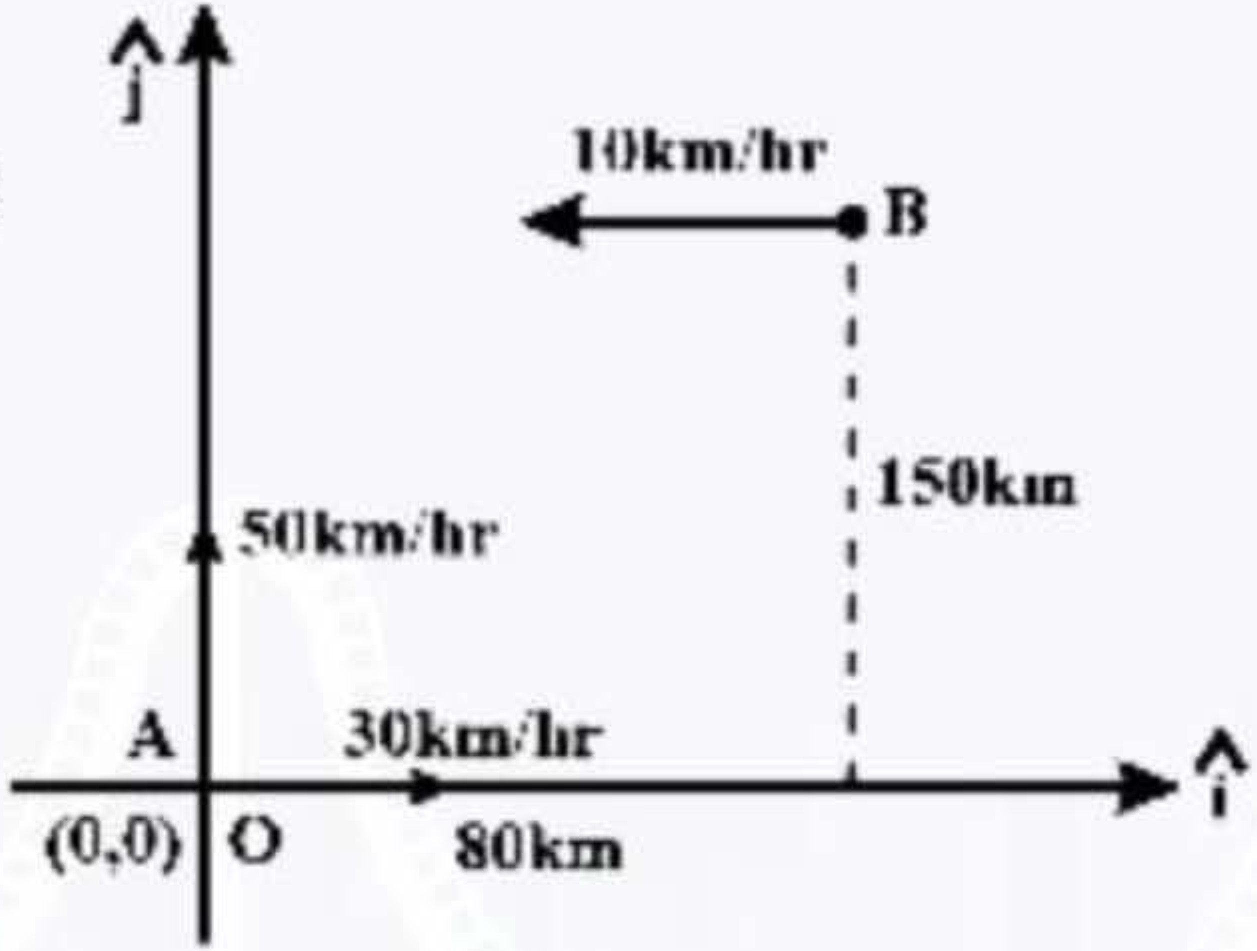
Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{\imath} + 50\hat{\jmath}$ km/hr where $\hat{\imath}$ points east and $\hat{\jmath}$ north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in : **[APRIL 2019]**

A. 3.2 hrs

B. 2.6 hrs

C. 4.2 hrs

D. 2.2 hrs



A glass wind screen whose inclination with the vertical can be changed is mounted on a car. The car moves horizontally with a speed of 2m/s. At what angle α with the vertical should the wind screen be placed so that the rain drops falling vertically downwards with velocity 6m/s strike the wind screen perpendicularly?

A $\tan^{-1}(3)$

B $\tan^{-1}(4)$

C $\tan^{-1}(1/3)$

D $\tan^{-1}(1/4)$

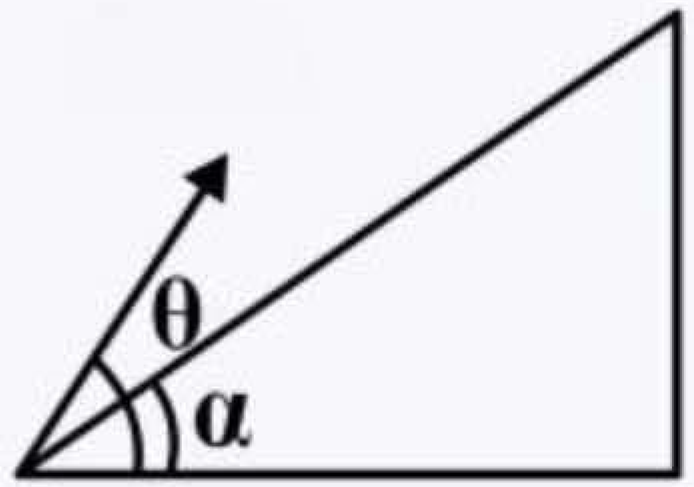
A projectile is fixed at an angle θ with the horizontal, (as shown in the figure), condition under which it lands perpendicular on an inclined plane of inclination α is

A $\sin\alpha = \cos(\theta - \alpha)$

B $\cos\alpha = \sin(\theta - \alpha)$

C $\tan\alpha = \cot(\theta - \alpha)$

D $\cot(\theta - \alpha) = \sin\alpha$



Rain is falling vertically with speed of 35 m/s. A woman rides a bicycle with a speed of 15 m/s in East to West direction. What is the direction is which she should hold her umbrella?

A. $\tan^{-1}\left(\frac{3}{7}\right)$ with the vertical towards west

B. $\tan^{-1}\left(\frac{4}{7}\right)$ with the vertical towards east

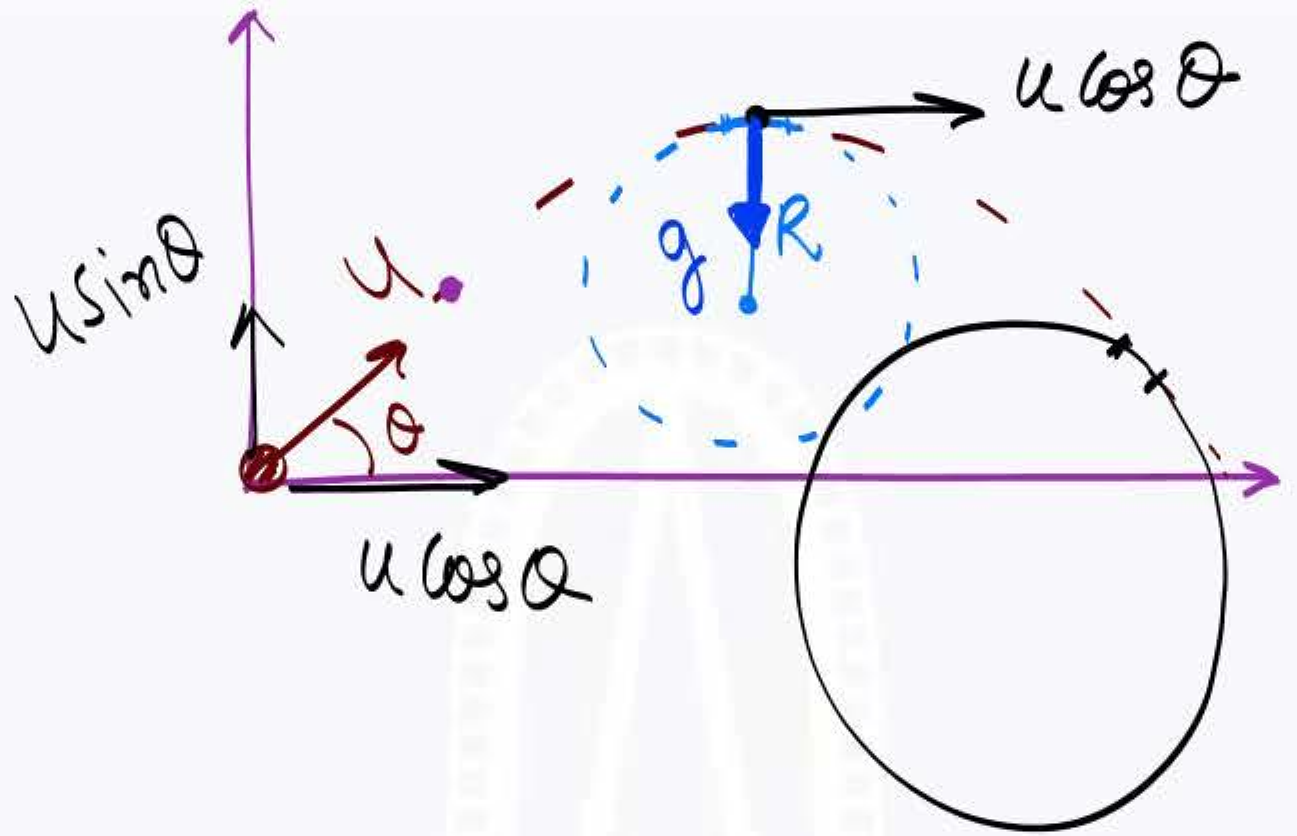
C. $\tan^{-1}\left(\frac{5}{7}\right)$ with the vertical towards east

D. Towards North downward

A ship a is moving towards the South with a speed 20 km per hour and another ship is moving towards the East with a speed of 20 km per hour. At a certain instant the Ship B is due south of ship A an is at a distance of 10 km from ship A find the shortest distance between the ships and the time after which they are closest to each other

A particle is projected at a speed u at an angle θ with the horizontal. Find the radius of curvature at the highest point of the trajectory of projectile.

(HCV Verma)



$a_x = 0$

Parabolic path

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

at Each point, Radius of Curvature is diff.

1. draw Circle.
2. Find velocity (Tangential)
3. find acc along Radius.
4. $a_c = \frac{v^2}{R}$

at top point

$$g = \frac{u^2\cos^2\theta}{R}$$

$$\boxed{R = \frac{u^2\cos^2\theta}{g}}$$

One day in still air, a motor-cyclist riding north at 30 m/s, suddenly the wind starts blowing westward with a velocity 50 m/s, then the apparent velocity with which the motor-cyclist will move, is:

A. 58.3 m/s

B. 65.4 m/s

C. 73.2 m/s

D. 53.8 m/s

A motorboat is racing towards north at 25 km/h and the water current in that region is 10 km/h in the direction of 60° east of south. The resultant velocity of the boat is

A 10 km/h

B 20 km/h

C 12 km/h

D 22 km/h

Position of particle as a function of time is given as $\vec{r} = \cos\omega t\hat{i} + \sin\omega t\hat{j}$. Choose correct statement about $\vec{r}$, $\vec{v}$ and $\vec{a}$, where $\vec{v}$ and $\vec{a}$ are velocity and acceleration of particle at time t. **[JAN 2020]**

A $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is towards origin

B $\vec{v}$ and $\vec{a}$ are perpendicular to $\vec{r}$

C $\vec{v}$ is parallel to $\vec{r}$ and $\vec{a}$ parallel to $\vec{r}$

D $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is away from origin.

Train A and train B are running on parallel tracks in the opposite direction with speed of 36 km/hour and 72 km/hour , respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hour. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible) **(SEP 2020)**

A. $30.5\ ms^{-1}$

B. $29.5\ ms^{-1}$

C. $31.5\ ms^{-1}$

D. $28.5\ ms^{-1}$

A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when :

(i) they are moving in the same direction, and (ii) in the opposite directions is **(JAN 2019)**

A 5/2

B 25/11

C 3/2

D 11/5

Two particles A and B separated by distance 10m with A ahead of B. A starts from rest with uniform acceleration $2 m/s^2$, whereas B is moving with constant velocity of $40 m/s$. Find time of overtaking

An elevator is accelerating upward at a rate of 6 ft/sec^2 when a bolt from its ceiling falls to the floor of the lift (Distance = 9.5 feet). The time (in seconds) taken by the falling bolt to hit the floor is (take $g = 32\ ft/sec^2$)

A $\sqrt{2}$

B $\frac{1}{\sqrt{2}}$

C $2\sqrt{2}$

D $\frac{1}{2\sqrt{2}}$

Two bodies are held separated by 9.8 m vertically one above the other. They are released simultaneously to fall freely under gravity. After 2 s the relative distance between them is

A 4.9 m

B 19.6 m

C 9.8 m

D 392 m

Two balls are thrown simultaneously, (A) vertically upwards with a speed of 20 m/s from the ground and (B) vertically downwards from a height of 40 m with the same speed and along the same line of motion. At which point will the balls collide? (take $g = 10\ m/s^2$)

A. 15 m above from the ground

B. 15 m below from the top of the tower

C. 20 m above from the ground

D. 20 m below from the top of the tower

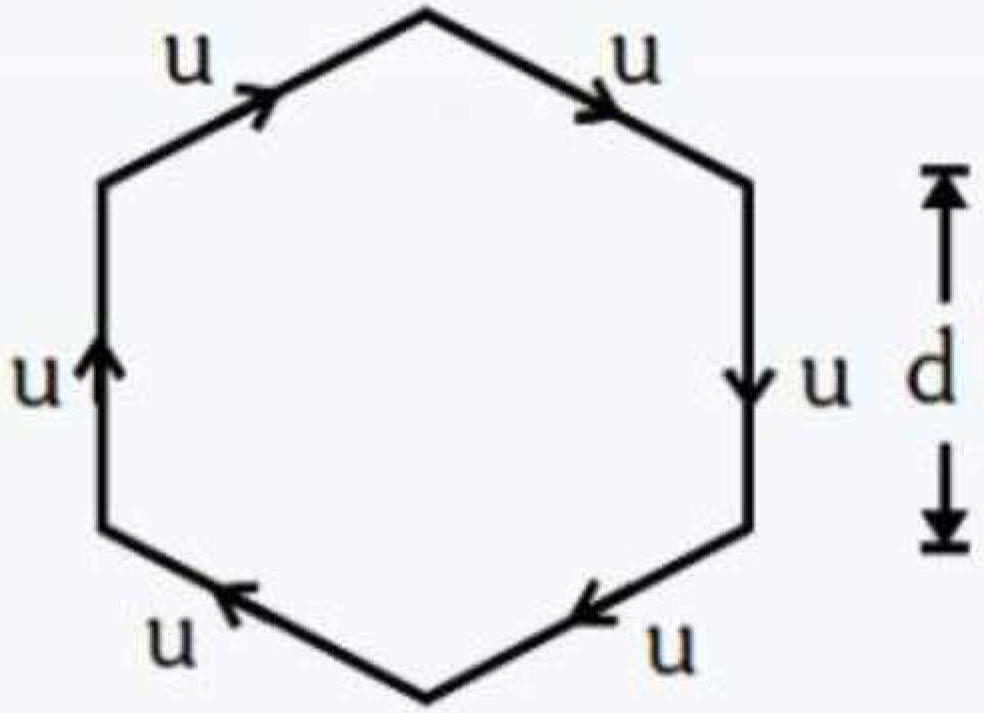
Six persons of same mass travel with same speed u along a regular hexagon of side 'd' such that each one always faces the other. After how what will they meet each other ?

A $\frac{d}{u}$

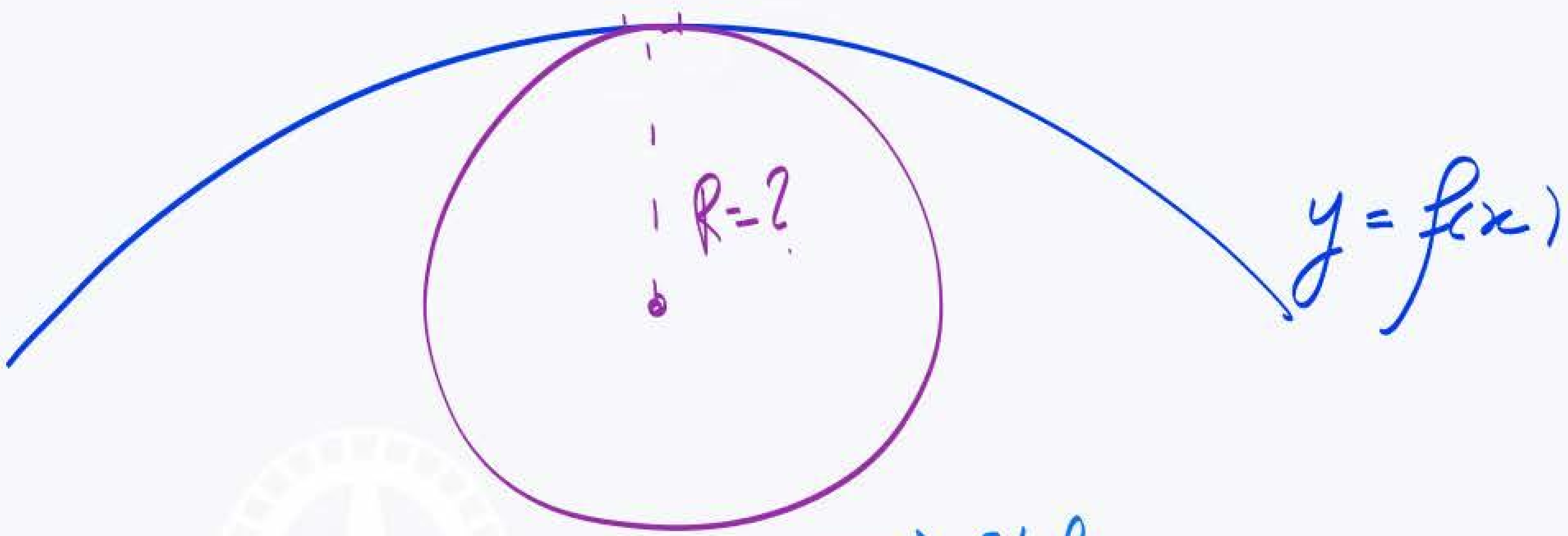
B $\frac{2d}{3u}$

C $\frac{2d}{u}$

D $d\sqrt{3}u$

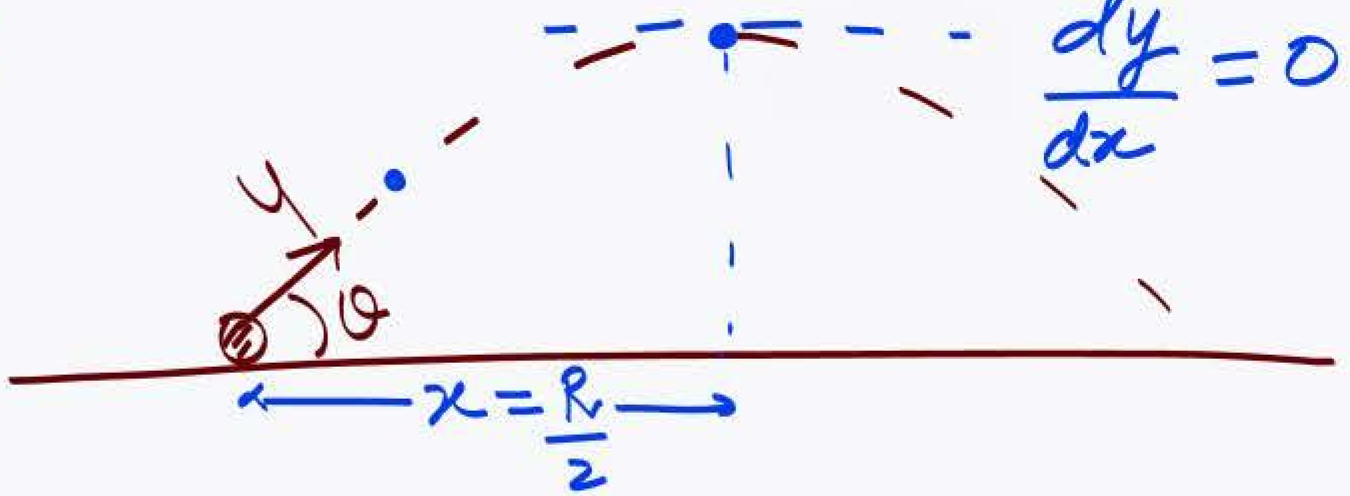


Alter



$$R = \frac{\left[1 + \left(\underbrace{\frac{dy}{dx}}_{\text{Slope}}\right)^2\right]^{3/2}_{\text{Point}}}{\left|\frac{d^2y}{dx^2}\right|_{\text{Point}}}$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$



$$\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \tan\theta - \frac{gx}{u^2\cos^2\theta}$$

$$\frac{d^2y}{dx^2} = -\frac{g}{u^2\cos^2\theta}$$

$$\left|\frac{d^2y}{dx^2}\right| = \frac{g}{u^2\cos^2\theta}$$

Two cars A and B start moving from the same point with same speed $v = 5$ km/minute. Car A moves towards North and car B is moving towards East. What is the relative velocity of B with respect to A?

A. $5\sqrt{2}$ km/min towards South-East

B. $5\sqrt{2}$ km/min towards North-West

C. $5\sqrt{2}$ km/min towards South-West

D. $5\sqrt{2}$ km/min towards North-East

A man wishes to swim across a river 0.5 km wide. If he can swim at the rate of 2 km/h in still water and the river flows at the rate of 1 km/h. The angle made by the direction (w.r.t. the flow of the river) along which he should swim so as to reach a point exactly opposite his starting point, should be:

A. $60°$

B. $120°$

C. $145°$

D. $90°$

A boat-mat can row a boat to make it move with a speed of 10 km/h in still water. River flows steadily at the rate of 5 km/h. and the width of the river is 2 km. If the boat man cross the river along the minimum distance of approach then time elapsed in rowing the boat will be:

A $\frac{2\sqrt{3}}{5}$ h

B $\frac{2}{5\sqrt{3}}$ h

C $\frac{3\sqrt{2}}{5}$ h

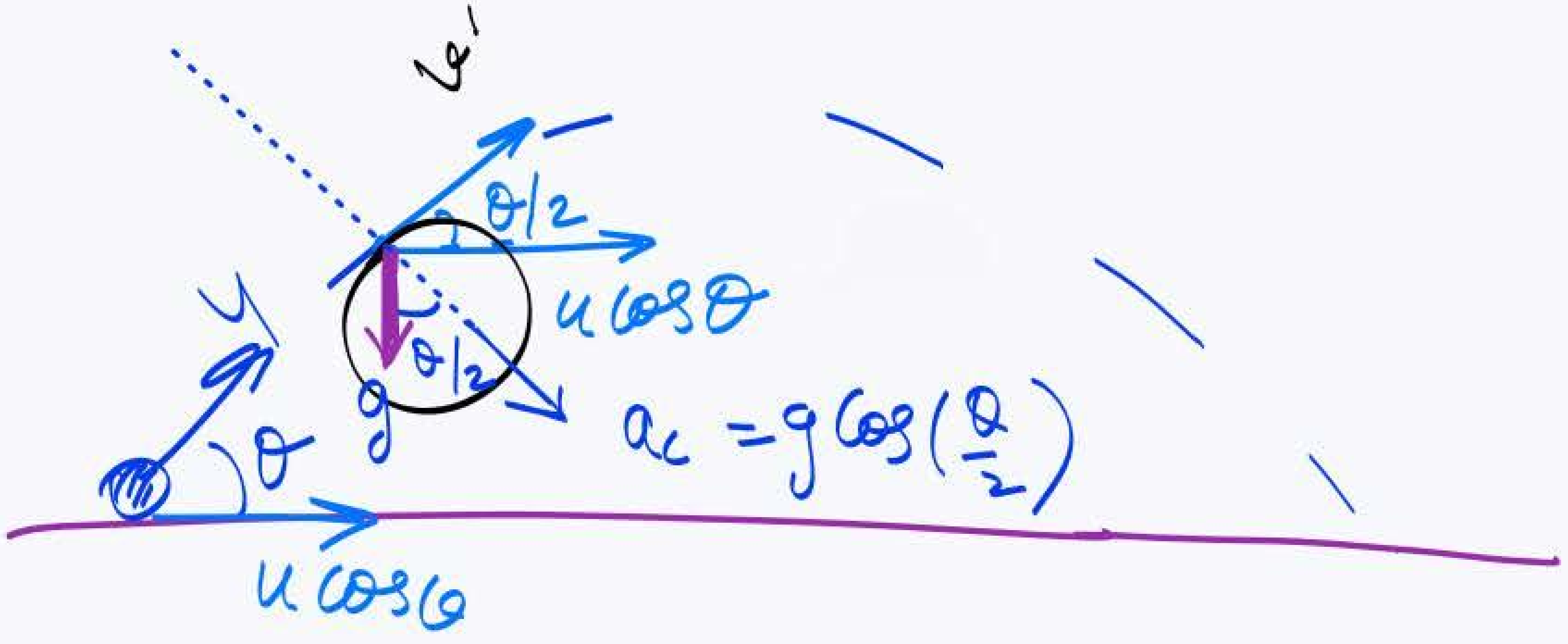
D $\frac{5\sqrt{2}}{3}$ h

THANK YOU!

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left(1+0^2\right)^{3/2}}{\frac{g}{u^2\cos^2\theta}} = \frac{u^2\cos^2\theta}{g}$$

$$\boxed{R = \frac{u^2\cos^2\theta}{g}}$$

HCV Circular



$$v'\cos\left(\frac{\theta}{2}\right) = u\cos\theta$$

$$v' = \frac{u\cos\theta}{\cos\left(\frac{\theta}{2}\right)}$$

Find R of Curvature at point where angle is $\theta/2$

$$a_c = \frac{v_T^2}{R}$$

$$R = \frac{u^2\cos^2\theta}{\cos^2(\theta/2)\, g\cos\left(\frac{\theta}{2}\right)}$$