

1.

$$\begin{cases} 1.19x_1 + 2.11x_2 - 100x_3 + x_4 = 1.12 \\ 14.2x_1 - 0.112x_2 + 12.2x_3 - x_4 = 3.44 \\ 100x_2 - 99.9x_3 + x_4 = 2.15 \\ 15.3x_1 + 0.110x_2 - 13.1x_3 - x_4 = 4.16 \end{cases}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 15.3 & 0.110 & -13.1 & -1 & 4.16 \\ 14.2 & -0.112 & 12.2 & -1 & 3.44 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 1.19 & 2.11 & -100 & 1 & 1.12 \end{array} \right] \begin{array}{l} \nearrow \frac{14.2}{15.3} \\ \searrow \frac{1.19}{15.3} \end{array}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 15.3 & 0.110 & -13.1 & -1 & 4.16 \\ 0 & -0.2141 & 24.3581 & -0.0719 & -0.4209 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & 2.1014 & -98.0811 & 1.0778 & 0.7964 \end{array} \right] \begin{array}{l} \\ \\ \text{change} \\ \end{array}$$

be (3): $\frac{-0.2141}{100}$ & $\frac{2.1014}{100}$

$$\Rightarrow \left[\begin{array}{cccc|c} 15.3 & 0.110 & -13.1 & -1 & 4.16 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & 0 & 24.144 & -0.070 & -0.416 \\ 0 & 0 & -96.88 & 1.057 & 0.751 \end{array} \right] \begin{array}{l} \\ \\ \text{change} \\ \end{array}$$

be (3): $\frac{24.144}{-96.88}$

$$\Rightarrow \begin{bmatrix} 1.53 & 0.110 & -13.1 & -1 & 4.16 \\ 0 & 100 & -99.9 & 1 & 2.15 \\ 0 & 0 & -96.88 & 1.057 & 0.751 \\ 0 & 0 & 0 & 0.193 & -0.229 \end{bmatrix}$$

Back Substitution

$$x_1 = 0.1767 \quad x_3 = -0.0207$$

$$x_2 = 0.0127 \quad x_4 = -1.1847$$

2.

$$A = \begin{bmatrix} 4 & 1 & -1 & 0 \\ 1 & 3 & -1 & 0 \\ -1 & -1 & 6 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 3 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 6 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2.75 & -0.75 & 0 & -0.25 & 1 & 0 & 0 \\ 0 & -0.75 & 5.75 & 2 & 0.25 & 0 & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 0 & -0.7273 & 0 & 1.0909 & -0.3606 & 0 & 0 \\ 0 & 2.75 & -0.75 & 0 & -0.25 & 1 & 0 & 0 \\ 0 & 0 & 5.5455 & 2 & 0.1818 & 0.2727 & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 0 & 0 & 0.2622 & 1.1147 & -0.3229 & 0.1311 & 0 \\ 0 & 2.75 & 0 & 0.2706 & -0.2254 & 1.0369 & 0.1353 & 0 \\ 0 & 0 & 5.5455 & 2 & 0.1818 & 0.2727 & 1 & 0 \\ 0 & 0 & 0 & 4.2786 & -0.0656 & -0.0983 & -0.3606 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{array}{cccc|cccc} 4 & 0 & 0 & 0 & 1.1187 & -0.3219 & 0.1532 & -0.0613 \\ 0 & 2.175 & 0 & 0 & -0.2213 & 1.0431 & 0.1581 & -0.0632 \\ 0 & 0 & 5.5455 & 0 & 0.2125 & 0.3186 & 1.1686 & -0.4675 \\ 0 & 0 & 0 & 4.2786 & -0.0656 & -0.0983 & -0.3607 & 1 \end{array}$$

$$\Rightarrow \left[\begin{array}{cccc} [1.1187 & -0.3219 & 0.1532 & -0.0613] / 4 \\ [-0.2213 & 1.0431 & 0.1581 & -0.0632] / 2.175 \\ [0.2125 & 0.3186 & 1.1686 & -0.4675] / 5.5455 \\ [-0.0656 & -0.0983 & -0.3607 & 1] / 4.2786 \end{array} \right] = A^{-1}$$

$$A^{-1} \approx \begin{bmatrix} 0.28 & -0.081 & 0.038 & -0.015 \\ -0.081 & 0.38 & 0.058 & -0.023 \\ 0.038 & 0.058 & 0.21 & -0.084 \\ -0.015 & -0.023 & -0.084 & 0.23 \end{bmatrix} \quad \#$$

3.

$$A = \begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} = LU$$

$$LUx = b$$

$$\text{let } Ux = y$$

$$Ly = b$$

$$Ux = y$$

$$b = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\Rightarrow L = \begin{bmatrix} 3 & 0 & 0 & 0 \\ -1 & \frac{3}{8} & 0 & 0 \\ 0 & -1 & \frac{21}{8} & \frac{55}{21} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & \frac{-1}{3} & 0 & 0 \\ 0 & 1 & \frac{-3}{8} & 0 \\ 0 & 0 & 1 & \frac{-8}{21} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Ly = b \Rightarrow y = \begin{bmatrix} \frac{2}{3} \\ \frac{11}{8} \\ \frac{43}{21} \\ \frac{64}{55} \end{bmatrix}$$

$$Ux = y \Rightarrow x = \begin{bmatrix} \frac{889}{792} \approx 1.123 \\ \frac{361}{264} \approx 1.367 \\ \frac{301}{165} \approx 1.824 \\ \frac{64}{55} \approx 1.164 \quad \# \end{bmatrix}$$