

1

```
u(0.00, 0.00) = 1.0000 u(0.00, 0.31) = 0.9511 u(0.00, 0.63) = 0.8090 u(0.00, 0.94) = 0.5878 u(0.00, 1.26) = 0.3090 u(0.00, 1.57) = 0.0000
u(0.31, 0.00) = 0.9511 u(0.31, 0.31) = -2.5208 u(0.31, 0.63) = 8.4904 u(0.31, 0.94) = -26.6388 u(0.31, 1.26) = 88.0999 u(0.31, 1.57) = -0.9511
u(0.63, 0.00) = 0.8090 u(0.63, 0.31) = -3.7736 u(0.63, 0.63) = 16.3808 u(0.63, 0.94) = -65.2751 u(0.63, 1.26) = 254.6504 u(0.63, 1.57) = -0.8090
u(0.94, 0.00) = 0.5878 u(0.94, 0.31) = -3.6647 u(0.94, 0.63) = 20.0417 u(0.94, 0.94) = -96.1531 u(0.94, 1.26) = 435.6331 u(0.94, 1.57) = -0.5878
u(1.26, 0.00) = 0.3090 u(1.26, 0.31) = -2.6920 u(1.26, 0.63) = 18.8377 u(1.26, 0.94) = -107.6668 u(1.26, 1.26) = 562.0103 u(1.26, 1.57) = -0.3090
u(1.57, 0.00) = 0.0000 u(1.57, 0.31) = -1.1910 u(1.57, 0.63) = 13.5198 u(1.57, 0.94) = -96.4033 u(1.57, 1.26) = 588.4865 u(1.57, 1.57) = -0.0000
u(1.88, 0.00) = -0.3090 u(1.88, 0.31) = 0.5742 u(1.88, 0.63) = 5.3043 u(1.88, 0.94) = -64.5276 u(1.88, 1.26) = 499.4016 u(1.88, 1.57) = 0.3090
u(2.20, 0.00) = -0.5878 u(2.20, 0.31) = 2.3751 u(2.20, 0.63) = -4.4740 u(2.20, 0.94) = -17.3715 u(2.20, 1.26) = 303.9931 u(2.20, 1.57) = 0.5878
u(2.51, 0.00) = -0.8090 u(2.51, 0.31) = 4.0108 u(2.51, 0.63) = -14.5076 u(2.51, 0.94) = 38.2377 u(2.51, 1.26) = 28.6993 u(2.51, 1.57) = 0.8090
u(2.83, 0.00) = -0.9511 u(2.83, 0.31) = 4.8362 u(2.83, 0.63) = -22.4944 u(2.83, 0.94) = 91.2641 u(2.83, 1.26) = -278.0640 u(2.83, 1.57) = 0.9511
u(3.14, 0.00) = 1.0000 u(3.14, 0.31) = 0.9511 u(3.14, 0.63) = 0.8090 u(3.14, 0.94) = 0.5878 u(3.14, 1.26) = 0.0000 u(3.14, 1.57) = 0.0000
```

2

a

```
前向差分法結果：
r/t 0.50 0.60 0.70 0.80 0.90 1.00 1.10
0.0 0.000000e+00 2.000000e+01 4.000000e+01 6.000000e+01 8.000000e+01 1.000000e+02 1.000000e+02
0.5 1.538462e+01 4.366667e+02 3.971429e+02 3.725000e+02 3.577778e+02 -2.275000e+03 1.200000e+02
1.0 3.358974e+02 -5.318742e+04 1.684345e+03 1.305040e+03 -3.452846e+05 6.247111e+05 1.400000e+02
1.5 -4.091340e+04 1.351024e+07 -6.418159e+06 -4.598568e+07 1.685571e+08 -1.965372e+08 1.600000e+02
2.0 1.039250e+07 -4.237864e+09 -2.992521e+09 3.308479e+10 -7.333158e+10 6.895395e+10 1.800000e+02
2.5 -3.259896e+09 6.511818e+11 4.684242e+12 -1.832815e+13 3.126350e+13 -2.587766e+13 2.000000e+02
3.0 5.009091e+11 4.718067e+14 -3.545456e+15 9.264827e+15 -1.336276e+16 1.015608e+16 2.200000e+02
3.5 3.629282e+14 -5.975362e+17 2.178407e+18 -4.497167e+18 5.761130e+18 -4.115691e+18 2.400000e+02
4.0 -4.596433e+17 4.438207e+20 -1.214079e+21 2.140227e+21 -2.508480e+21 1.708941e+21 2.600000e+02
4.5 3.414005e+20 -2.749706e+23 6.404582e+23 -1.008349e+24 1.102762e+24 -7.234084e+23 2.800000e+02
5.0 -2.115158e+23 1.552355e+26 -3.264370e+26 4.725932e+26 -4.890787e+26 3.110817e+26 3.000000e+02
5.5 1.194119e+26 -8.288289e+28 1.625950e+29 -2.208858e+29 2.186184e+29 -1.355374e+29 3.200000e+02
6.0 -6.375607e+28 4.266959e+31 -7.968940e+31 1.030899e+32 -9.839618e+31 5.970975e+31 3.400000e+02
6.5 3.282276e+31 -2.142331e+34 3.860207e+34 -4.807623e+34 4.454936e+34 -2.655228e+34 3.600000e+02
7.0 -1.647947e+34 1.056553e+37 -1.853733e+37 2.241137e+37 -2.027188e+37 1.190175e+37 3.800000e+02
7.5 8.127329e+36 -5.142968e+39 8.843674e+39 -1.044513e+40 9.263856e+39 -5.370822e+39 4.000000e+02
8.0 -3.956129e+39 2.479111e+42 -4.197928e+42 4.867562e+42 -4.248456e+42 2.437418e+42 4.200000e+02
8.5 1.907009e+42 -1.186221e+45 1.984944e+45 -2.268216e+45 1.954112e+45 -1.111421e+45 4.400000e+02
9.0 -9.124781e+44 5.643821e+47 -9.357164e+47 1.056927e+48 -9.009953e+47 5.087947e+47 4.600000e+02
9.5 4.341401e+47 -2.673473e+50 4.400547e+50 -4.924925e+50 4.162565e+50 -2.336831e+50 4.800000e+02
10.0 -2.056518e+50 1.262100e+53 -2.065638e+53 2.294836e+53 -1.926225e+53 1.076175e+53 5.000000e+02
```

b

```
後向差分法結果：
r/t 0.50 0.60 0.70 0.80 0.90 1.00 1.10
0.0 0.000000e+00 2.000000e+01 4.000000e+01 6.000000e+01 8.000000e+01 1.000000e+02 1.000000e+02
0.5 1.538462e+01 -1.580345e+00 -1.263517e+00 -1.005976e+00 -7.879593e-01 9.860558e+00 1.200000e+02
1.0 -1.215650e+00 -7.921856e+00 4.059837e-03 8.76319e-04 -5.498257e+00 -5.251566e+01 1.400000e+02
1.5 -6.093735e+00 -7.369093e+00 3.666945e+00 2.908290e+00 2.210750e+01 -1.231216e+01 1.600000e+02
2.0 -5.668533e+00 -6.565607e+00 5.522873e+00 -1.050156e+01 8.546155e+01 -2.172461e+02 1.800000e+02
2.5 -5.050467e+00 -6.957518e+00 1.416247e+01 -5.830066e+01 2.046017e+02 -3.518021e+02 2.000000e+02
3.0 -5.351937e+00 -1.229272e+01 4.848794e+01 -1.731745e+02 4.169565e+02 -5.531824e+02 2.200000e+02
3.5 -9.455936e+00 -3.600918e+01 1.465750e+02 -4.164383e+02 7.892018e+02 -8.654876e+02 2.400000e+02
4.0 -2.769937e+01 -1.107709e+02 3.854301e+02 -9.024647e+02 1.440035e+03 -1.364363e+03 2.600000e+02
4.5 -8.520842e+01 -3.060688e+02 9.181916e+02 -1.844386e+03 2.581713e+03 -2.181611e+03 2.800000e+02
5.0 -2.354376e+02 -7.625429e+02 2.043856e+03 -3.639144e+03 4.596021e+03 -3.549453e+03 3.000000e+02
5.5 -5.865714e+02 -1.757742e+03 4.338258e+03 -7.025293e+03 8.173518e+03 -5.880738e+03 3.200000e+02
6.0 -1.352109e+03 -3.830495e+03 8.899657e+03 -1.337563e+04 1.456916e+04 -9.915022e+03 3.400000e+02
6.5 -2.946535e+03 -8.014689e+03 1.780798e+04 -2.523975e+04 2.607234e+04 -1.698559e+04 3.600000e+02
7.0 -6.165146e+03 -1.627714e+04 3.498169e+04 -4.734974e+04 4.687253e+04 -2.950886e+04 3.800000e+02
7.5 -1.252087e+04 -3.233563e+04 6.777370e+04 -8.848390e+04 8.465512e+04 -5.188331e+04 4.000000e+02
8.0 -2.487356e+04 -6.318425e+04 1.299401e+05 -1.649201e+05 1.535465e+05 -9.214346e+04 4.200000e+02
8.5 -4.860327e+04 -1.219331e+05 2.471568e+05 -3.068334e+05 2.795527e+05 -1.650071e+05 4.400000e+02
9.0 -9.379469e+04 -2.330886e+05 4.672633e+05 -5.701473e+05 5.106091e+05 -2.974956e+05 4.600000e+02
9.5 -1.792989e+05 -4.423632e+05 8.792710e+05 -1.058485e+06 9.351587e+05 -5.393092e+05 4.800000e+02
10.0 -3.402794e+05 -8.348862e+05 1.648622e+06 -1.963825e+06 1.716509e+06 -9.819909e+05 5.000000e+02
```

C

```
Crank-Nicolson 結果：
r/t 0.50 0.60 0.70 0.80 0.90 1.00 1.10
0.0 0.000000e+00 2.000000e+01 4.000000e+01 6.000000e+01 8.000000e+01 1.000000e+02 1.000000e+02
0.5 2.849714e+01 3.704628e+01 4.710895e+01 5.851197e+01 7.109025e+01 8.468941e+01 1.200000e+02
1.0 5.107990e+01 6.640387e+01 8.279862e+01 1.003647e+02 1.192475e+02 1.396249e+02 1.400000e+02
1.5 4.735688e+01 6.156394e+01 7.657036e+01 9.204776e+01 1.076281e+02 1.228993e+02 1.600000e+02
2.0 5.941908e+01 7.724480e+01 9.702942e+01 1.190598e+02 1.437558e+02 1.716776e+02 1.800000e+02
2.5 6.395431e+01 8.314061e+01 1.030701e+02 1.231358e+02 1.425765e+02 1.604301e+02 2.000000e+02
3.0 6.970116e+01 9.061151e+01 1.138409e+02 1.399242e+02 1.696937e+02 2.043476e+02 2.200000e+02
3.5 7.889712e+01 1.025663e+02 1.273370e+02 1.523123e+02 1.762451e+02 1.974023e+02 2.400000e+02
4.0 8.137900e+01 1.057927e+02 1.325691e+02 1.624629e+02 1.967719e+02 2.375207e+02 2.600000e+02
4.5 9.267539e+01 1.204780e+02 1.499727e+02 1.800315e+02 2.089028e+02 2.339235e+02 2.800000e+02
5.0 9.401596e+01 1.222208e+02 1.526718e+02 1.862612e+02 2.247415e+02 2.710961e+02 3.000000e+02
5.5 1.056760e+02 1.373788e+02 1.714637e+02 2.066704e+02 2.407787e+02 2.700876e+02 3.200000e+02
6.0 1.072713e+02 1.394527e+02 1.737147e+02 2.109781e+02 2.533928e+02 3.049868e+02 3.400000e+02
6.5 1.181971e+02 1.536563e+02 1.921953e+02 2.325359e+02 2.720637e+02 3.059748e+02 3.600000e+02
7.0 1.208858e+02 1.571516e+02 1.953582e+02 2.363388e+02 2.825528e+02 3.391193e+02 3.800000e+02
7.5 1.304624e+02 1.696011e+02 2.124650e+02 2.578728e+02 3.029140e+02 3.416521e+02 4.000000e+02
8.0 1.346682e+02 1.750687e+02 2.173431e+02 2.621265e+02 3.120813e+02 3.734329e+02 4.200000e+02
8.5 1.426340e+02 1.854242e+02 2.324969e+02 2.828717e+02 3.334548e+02 3.771743e+02 4.400000e+02
9.0 1.484826e+02 1.930273e+02 2.394775e+02 2.881744e+02 3.418672e+02 4.078784e+02 4.600000e+02
9.5 1.548243e+02 2.012716e+02 2.524540e+02 3.076778e+02 3.637841e+02 4.125852e+02 4.800000e+02
10.0 1.622372e+02 2.109084e+02 2.616244e+02 3.143573e+02 3.718244e+02 4.424168e+02 5.000000e+02
```

3

```
r/θ 0.00 0.35 0.70 1.05
0.5 0.0000 50.0000 50.0000 0.0000
0.6 0.0000 62.8146 62.8146 0.0000
0.7 0.0000 73.8167 73.8167 0.0000
0.8 0.0000 83.4907 83.4907 0.0000
0.9 0.0000 92.1478 92.1478 0.0000
1.0 0.0000 100.0000 100.0000 0.0000
```

4

```
Crank-Nicolson 結果：
x/t 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
0.0 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
0.1 0.8090 1.0238 0.6440 0.9168 0.5759 0.8755 0.5533 0.8663 0.5539 0.8740 0.5667
0.2 0.3090 0.8476 0.1582 0.7496 0.0997 0.7204 0.0921 0.7290 0.1127 0.7583 0.1485
0.3 -0.3090 0.3476 -0.3056 0.3644 -0.2757 0.4058 -0.2247 0.4644 -0.1603 0.5331 -0.0886
0.4 -0.8090 -0.2852 -0.6298 -0.1267 -0.4853 0.0084 -0.3567 0.1322 -0.2367 0.2489 -0.1232
0.5 -1.0000 -0.8090 -0.7044 -0.5504 -0.4733 -0.3404 -0.2801 -0.1609 -0.1122 -0.0030 0.0367
0.6 -0.8090 -1.0238 -0.4970 -0.7388 -0.2344 -0.4962 -0.0101 -0.2885 0.1826 -0.1095 0.3492
0.7 -0.3090 -0.8476 -0.0565 -0.5902 0.1926 -0.3551 0.4116 -0.1523 0.5988 0.0204 0.7585
0.8 0.3090 -0.3476 0.5651 -0.1080 0.7845 0.0915 0.9659 0.2567 1.1166 0.3945 1.2429
0.9 0.8090 0.7852 1.0663 0.9737 1.2154 1.0982 1.3230 1.1932 1.4082 1.2703 1.4783
1.0 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000 2.0000
```