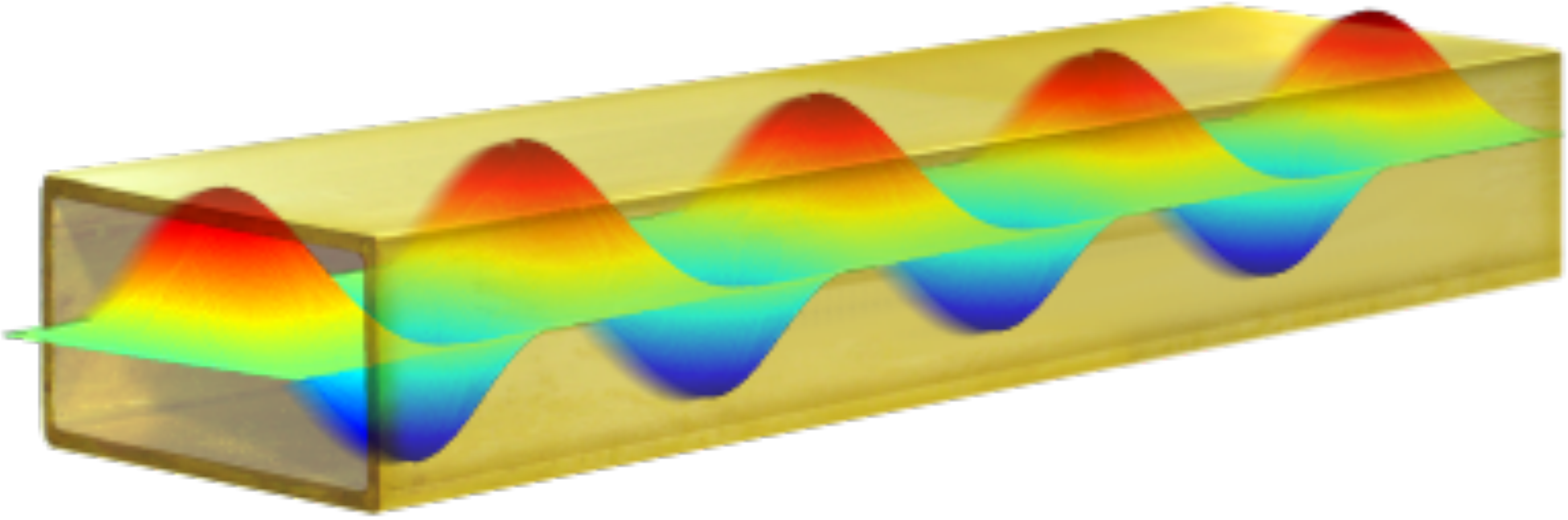
Hyperfréquence (Radiofrequency)



Three-dimensional view of the electric field for the TE_{10}– mode in a rectangular waveguide

Part 2

Dr. Lana Damaj

Part 2.3: outlines

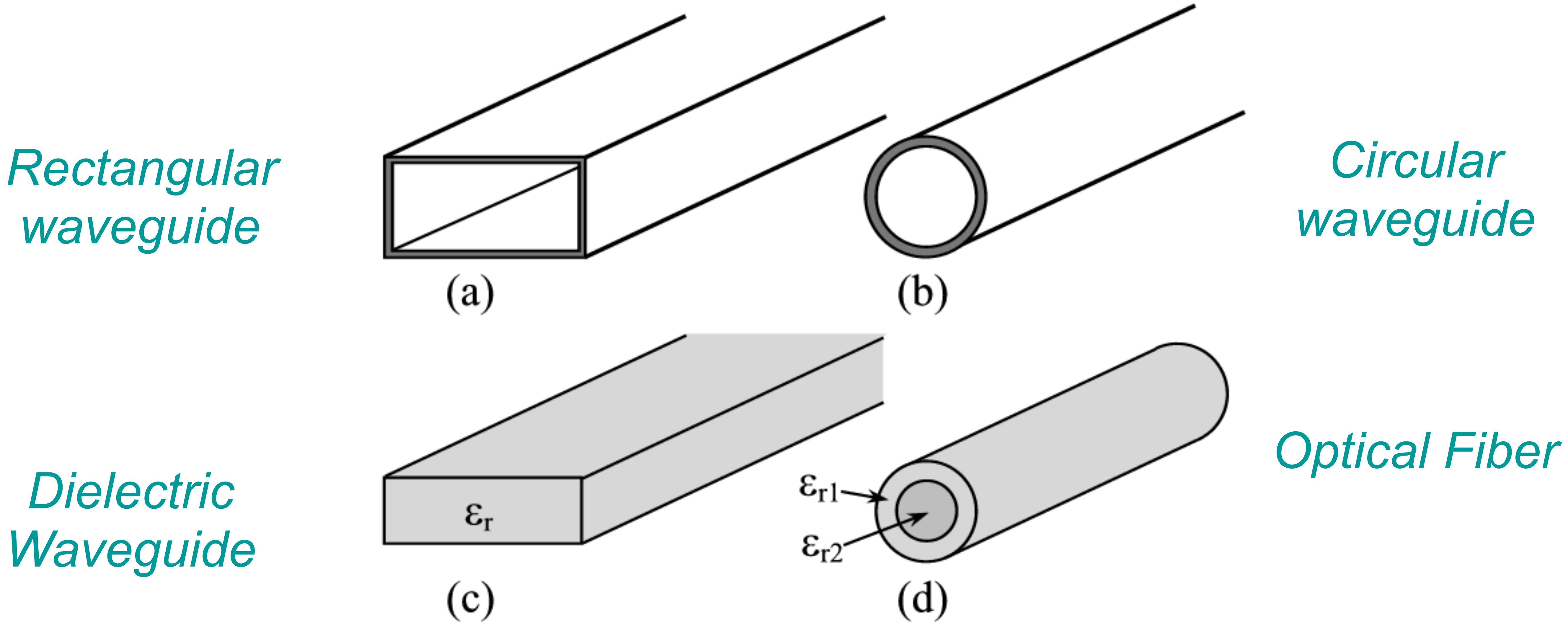
- **Waveguide:**
  - **rectangular waveguide fundamentals**
  - **Waveguide field propagation: TE and TM modes**
  - **Wave propagation**

Introduction

- In the previous chapter, we saw how a pair of conductors was used to guide electromagnetic wave propagation.
- This propagation was via the TEM mode, meaning both the electric and magnetic field components were transverse, or perpendicular, to the direction of propagation.
- In this chapter we investigate wave-guiding structures that support propagation in non- TEM modes, namely in the TE and TM modes.

Introduction

- The generic term waveguide can mean any structure that supports propagation of a wave.
- Although T-lines are technically a subset of waveguides, in general usage the term waveguide refers to constructs that only support non-TEM mode propagation.
- Such constructs share an important criteria: They are unable to support wave propagation below a certain frequency, termed the cut off frequency.



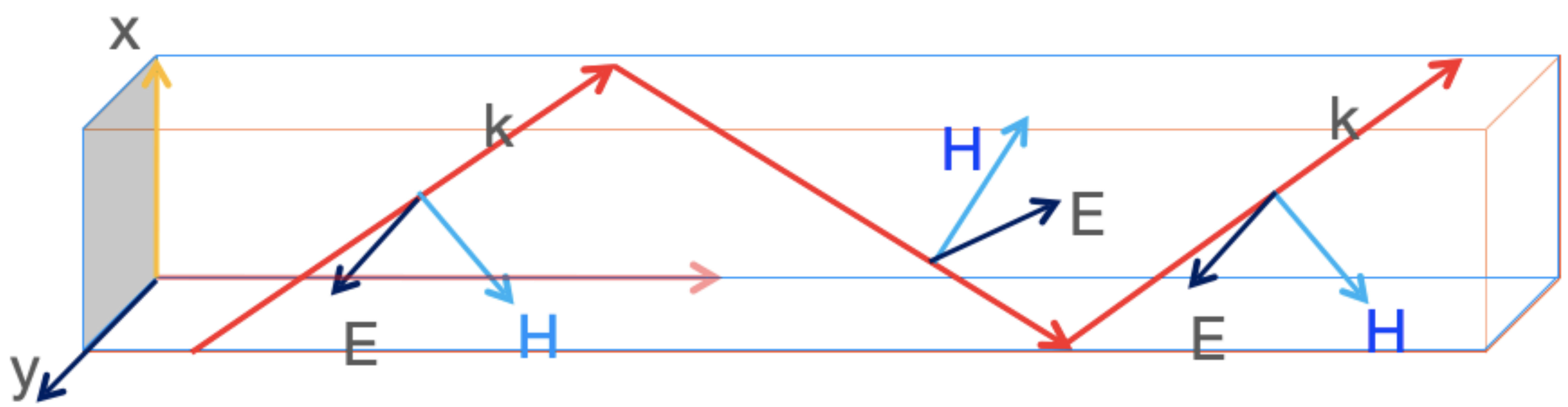
TE and TM modes (review)

TE mode

- The vector of the electric field is perpendicular to the plane of incidence (xoz), E//oy and it keeps this position during guidance but it changes the sign keeping the direct direction (k,E,H)

TM mode

- The electric field vector is in the plane of incidence and the magnetic field vector is perpendicular to it.

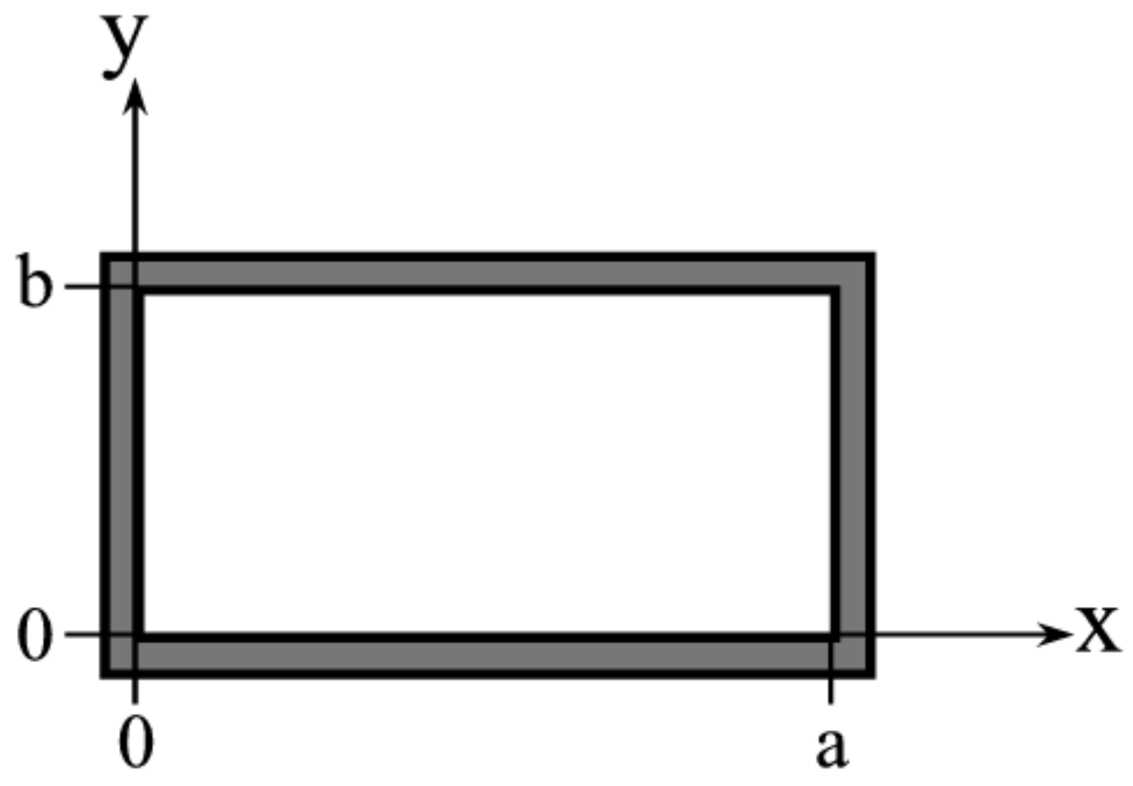


TE mode

Rectangular waveguide

- **Let us consider a rectangular waveguide with interior dimensions are a and b,**
- **Waveguide can support TE and TM modes.**
  - **In TE modes, the electric field is transverse to the direction of propagation.**
  - **In TM modes, the magnetic field that is transverse and an electric field component is in the propagation direction.**
- **The order of the mode refers to the field configuration in the guide, and is given by m and n integer subscripts, TE_{mn} and TM_{mn}.**
  - **The m subscript corresponds to the number of half-wave variations of the field in the x direction, and**
  - **The n subscript is the number of half-wave variations in the y direction.**

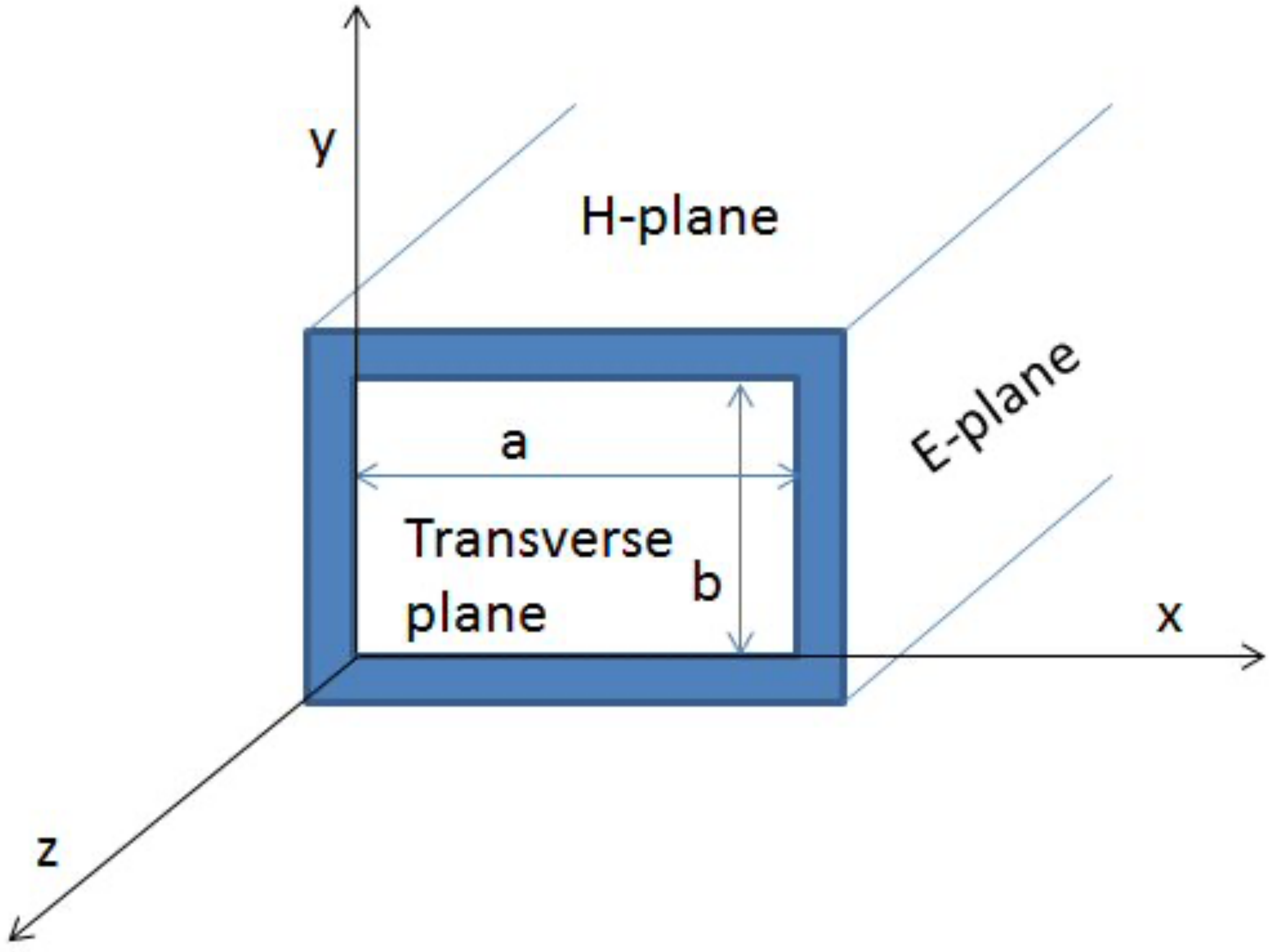
Rectangular Waveguide



Why is TEM not supported by rectangular guide?

- The TEM mode requires at least a pair of conductors to propagate and is therefore not supported by hollow guide like rectangular waveguide.
- To see why this is so, let us suppose that a hollow guide does support the TEM mode. By definition, the magnetic field must be entirely in the transverse plane, and from Gauss's law for magnetic fields, $\nabla \cdot \mathbf{B} = 0$, these field lines must form closed loops. Now, by Ampere's circuital law,

$$\oint \mathbf{H} \cdot d\mathbf{L} = \int \mathbf{J}_c \cdot d\mathbf{S} + \frac{\partial}{\partial t} \int \mathbf{D} \cdot d\mathbf{S}$$



- Since no conductive element can be enclosed in the hollow waveguide, the conduction current J_c term must be zero. The displacement-current term requires a component of D, and therefore E, in the direction of propagation, that is, normal to the transverse plane. But for TEM mode propagation, the E must be entirely transverse. Therefore, the TEM mode can- not be supported by hollow waveguide.

Rectangular waveguide

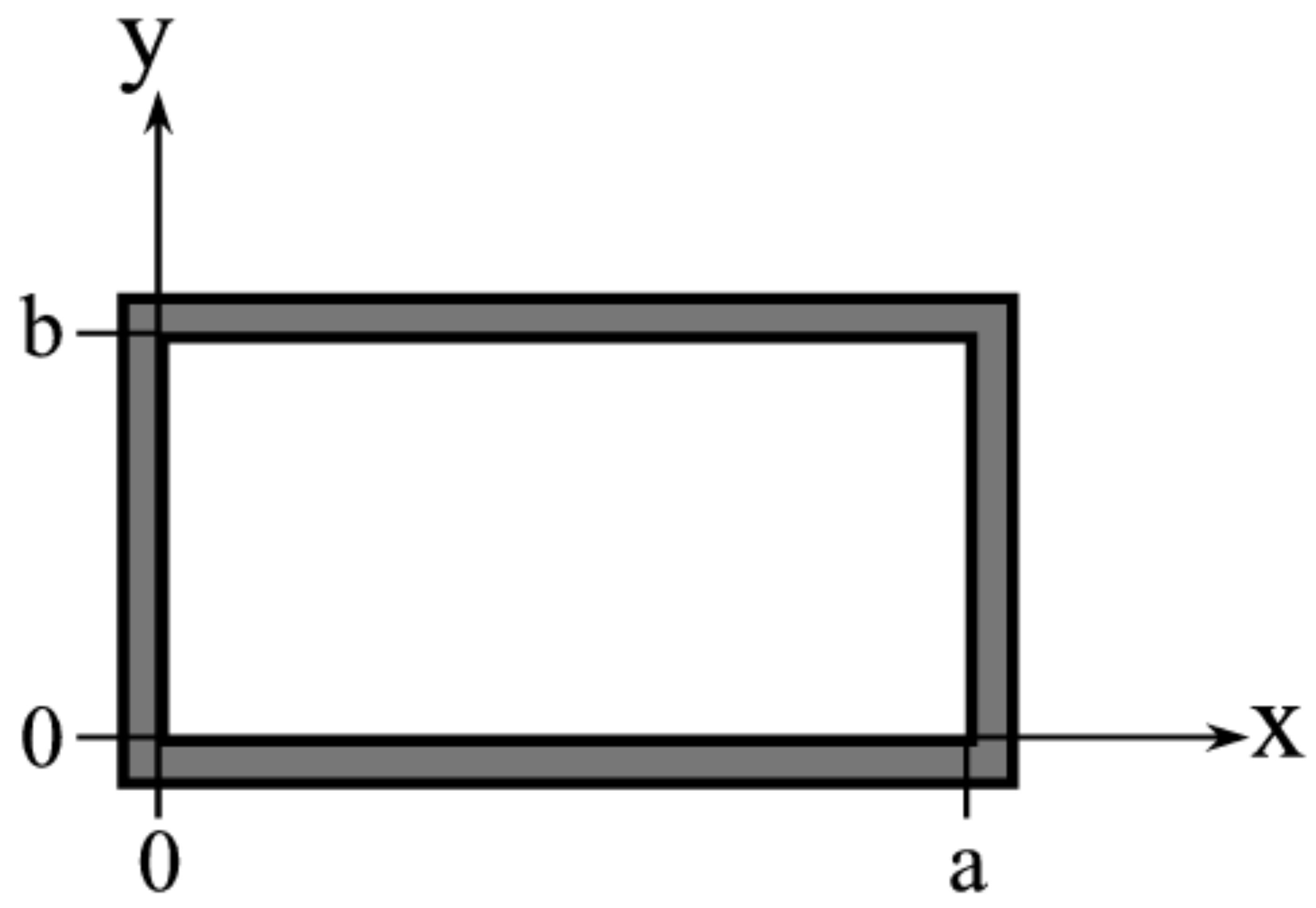
- **A particular mode is only supported above its cutoff frequency. The cutoff frequency is given by**

$$f_{c_{mn}} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{c}{2\sqrt{\mu_r\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$u = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_o\mu_r\varepsilon_o\varepsilon_r}} = \frac{1}{\sqrt{\mu_o\varepsilon_o}}\frac{1}{\sqrt{\mu_r\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

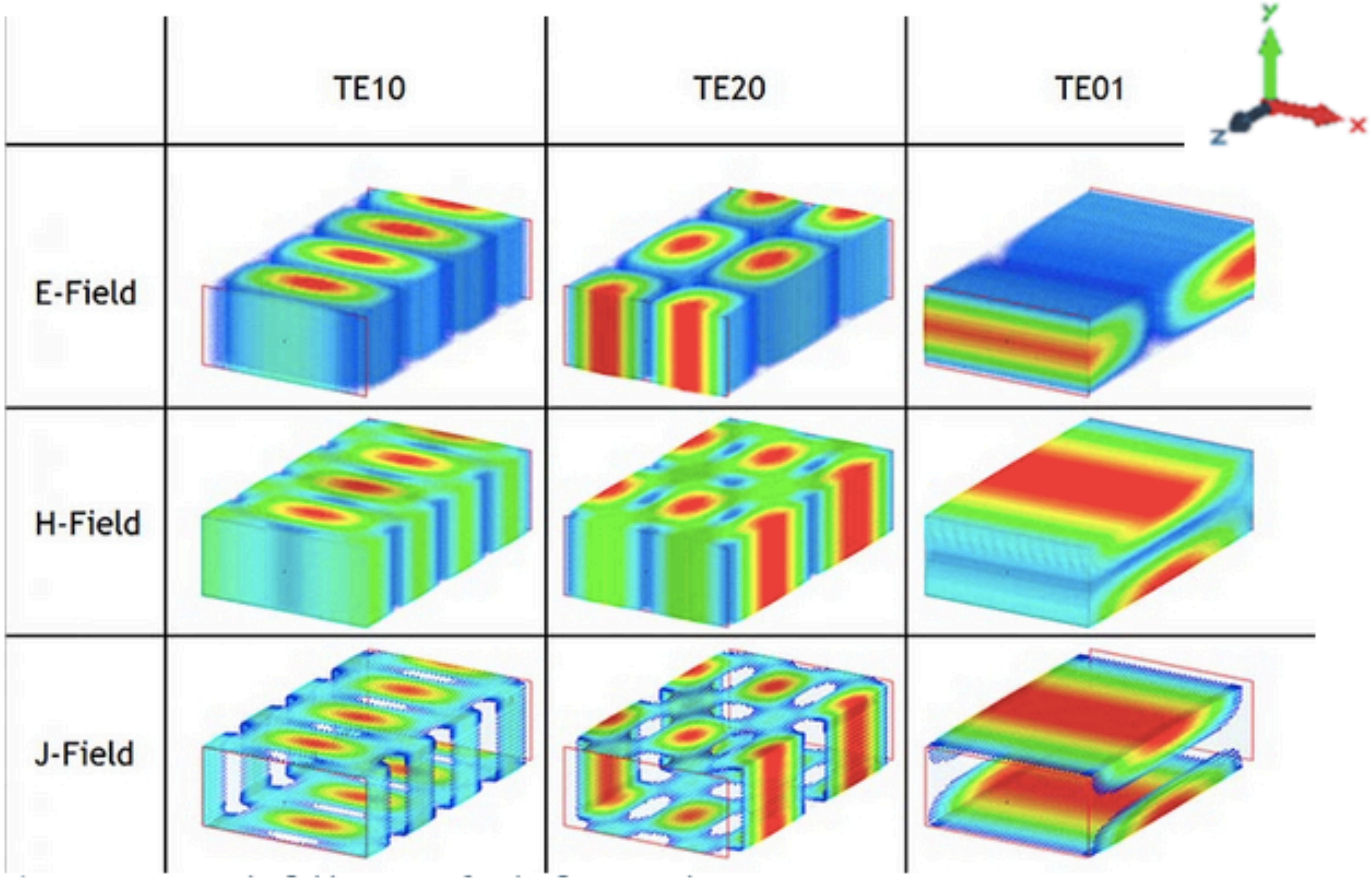
where $c = 3\times10^8$ m/s

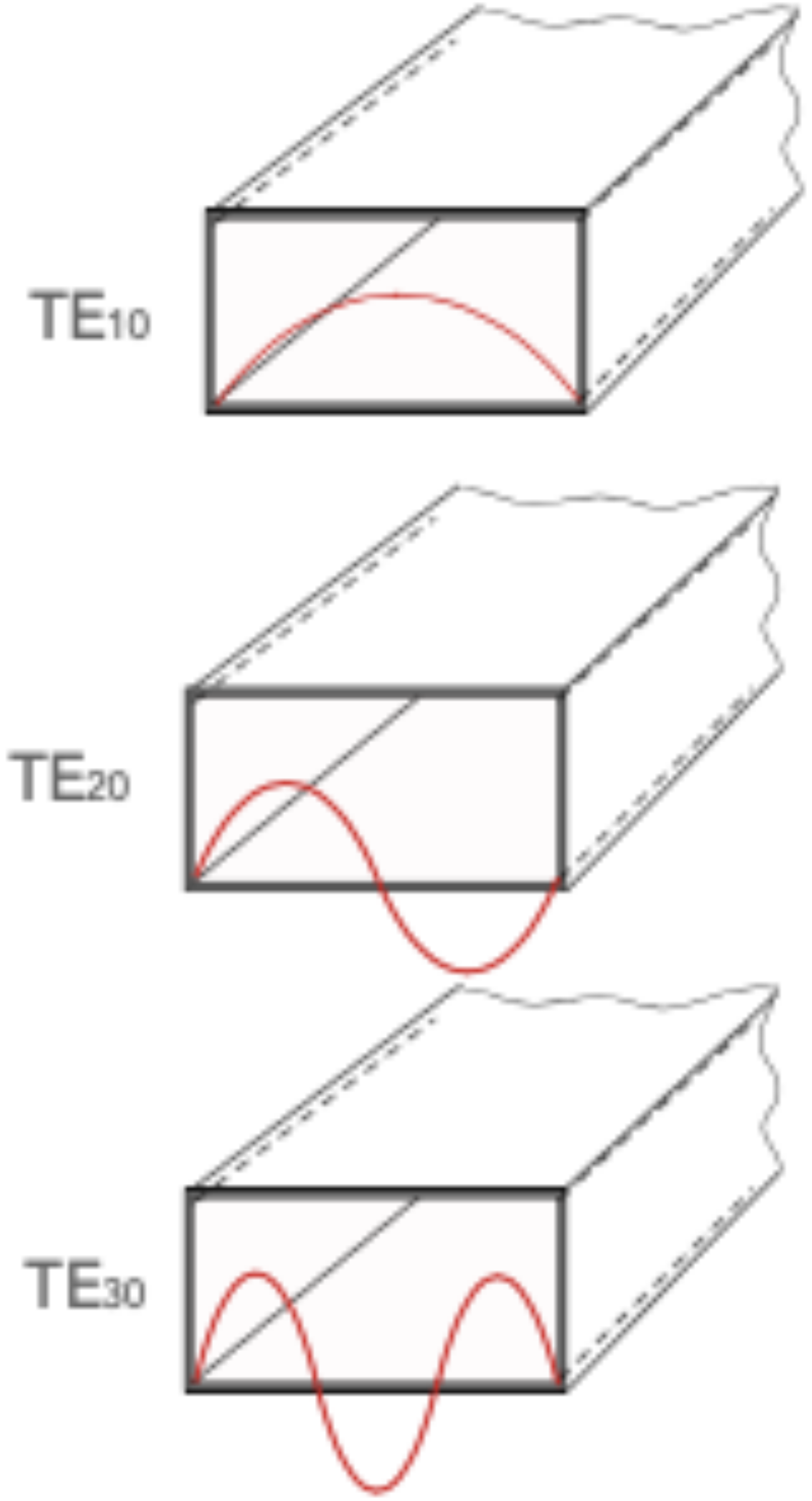
Rectangular Waveguide



Rectangular waveguide

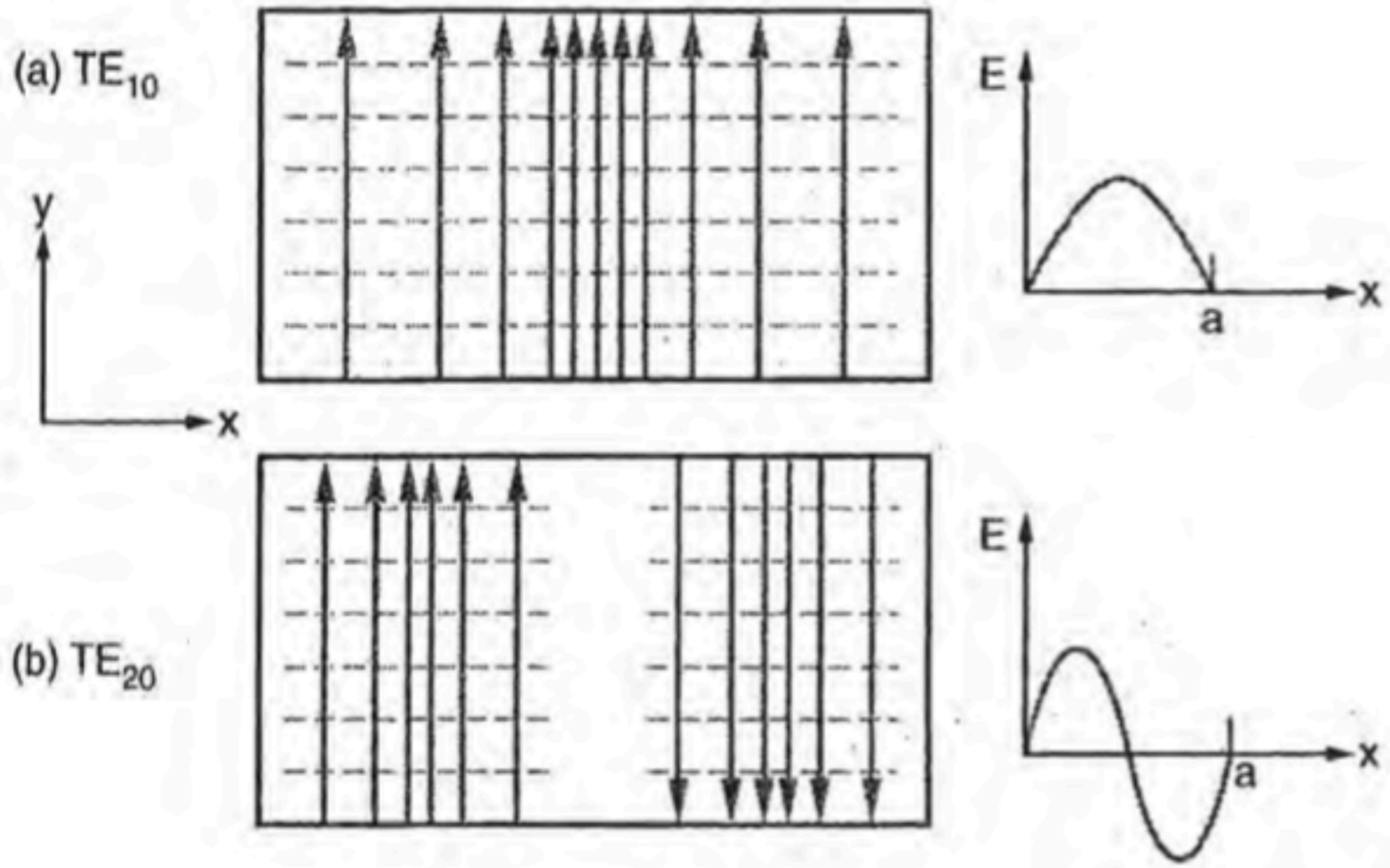
- **Mode of propagation**





Rectangular waveguide

- **Mode of propagation**

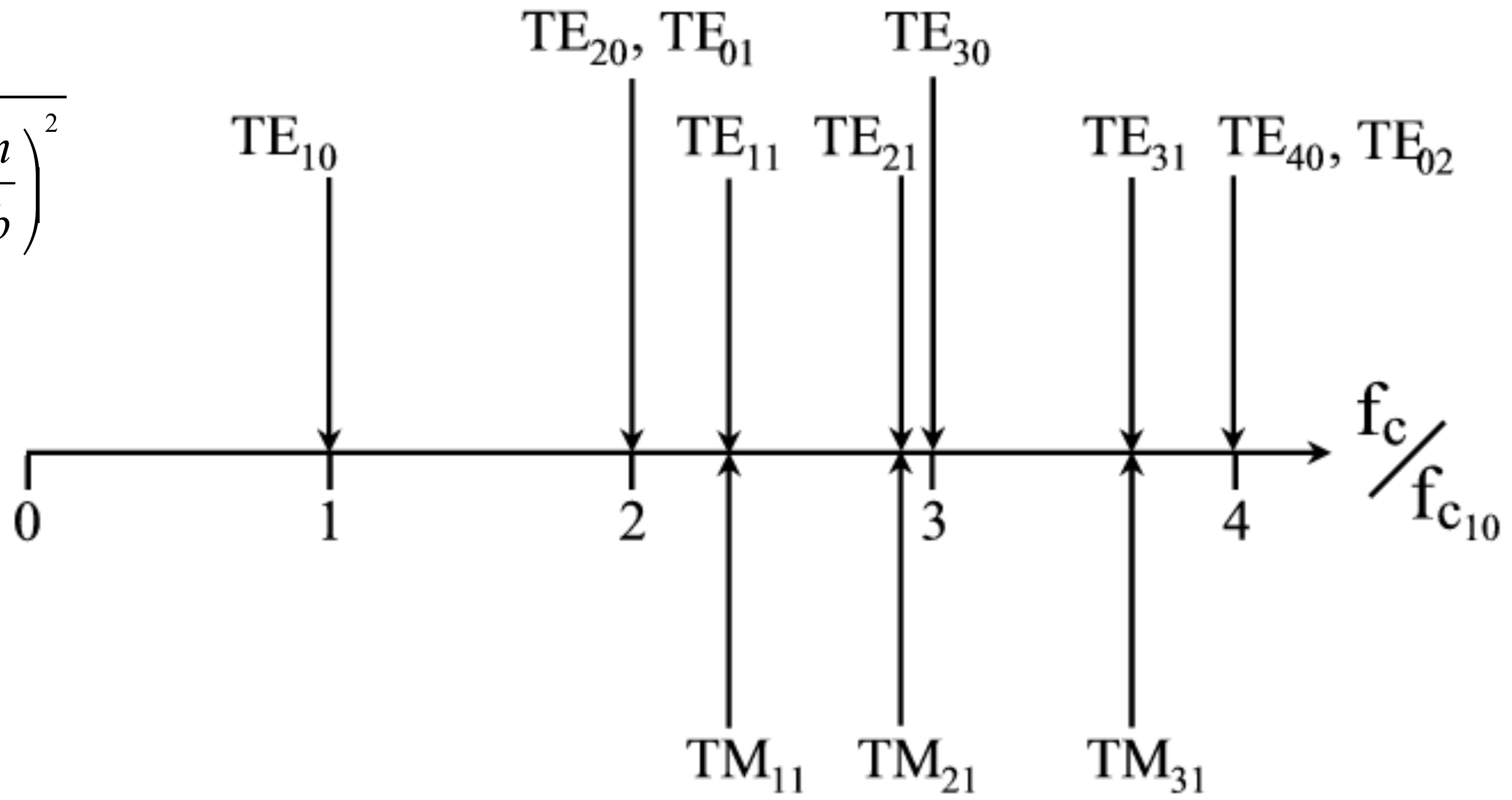


The field patterns and associated field intensities in a cross section of rectangular waveguide for (a) TE_{10} and (b) TE_{20}. Solid lines indicate electric field; dashed lines are the magnetic field.

Rectangular waveguide

- For conventional rectangular waveguide filled with air, where a = 2b, the dominant or lowest order mode is TE_{10} with a cutoff frequency $f_{c,10} = c/2a$.
- The relative cutoff frequencies for the first 12 modes of this waveguide are shown in the figure below
- For instance, owing to this particular wave- guide's condition a = 2b, the TE_{20} and TE_{01} modes have the same cutoff frequency. Also, the TE11 and TM11 modes share a cutoff frequency.
- Notice also that there are no modes where both m and n are zero and also no TM modes with either m or n equal to zero.

$$f_{c_{mn}} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{c}{2\sqrt{\mu_r\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$



Rectangular waveguide

- **Some Standard Rectangular Waveguide**

Waveguide Designation	a (in)	b (in)	t (in)	f_{c10} (GHz)	freq range (GHz)
WR975	9.750	4.875	.125	.605	.75 – 1.12
WR650	6.500	3.250	.080	.908	1.12 – 1.70
WR430	4.300	2.150	.080	1.375	1.70 – 2.60
WR284	2.84	1.34	.080	2.08	2.60 – 3.95
WR187	1.872	.872	.064	3.16	3.95 – 5.85
WR137	1.372	.622	.064	4.29	5.85 – 8.20
WR90	.900	.450	.050	6.56	8.2 – 12.4
WR62	.622	.311	.040	9.49	12.4 - 18

Example 1

- **Let us calculate the cutoff frequency for the first four modes of WR284 waveguide. From the table, the guide dimensions are a = 2.840 inches and b = 1.340 inches. Converting to metric units we have a = 7.214 cm and b = 3.404 cm.**
-

$$f_{c_{mn}} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{c}{2\sqrt{\mu_r\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

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Example 1

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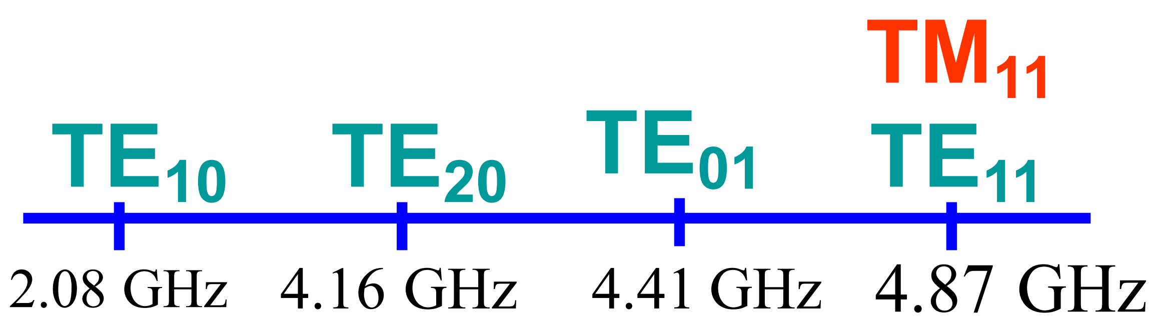
- $$f_{c_{mn}} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \text{where } c = 3\times10^8 \text{ m/s}$$

TE$_{10}$: $$f_{c10} = \frac{c}{2a} = \frac{3x10^8 \, m/s}{2(7.214cm)} \frac{100cm}{1m} = 2.08 \text{ GHz}$$

TE$_{01}$: $$f_{c01} = \frac{c}{2b} = \frac{3x10^8 \, m/s}{2(3.404cm)} \frac{100cm}{1m} = 4.41 \text{ GHz}$$

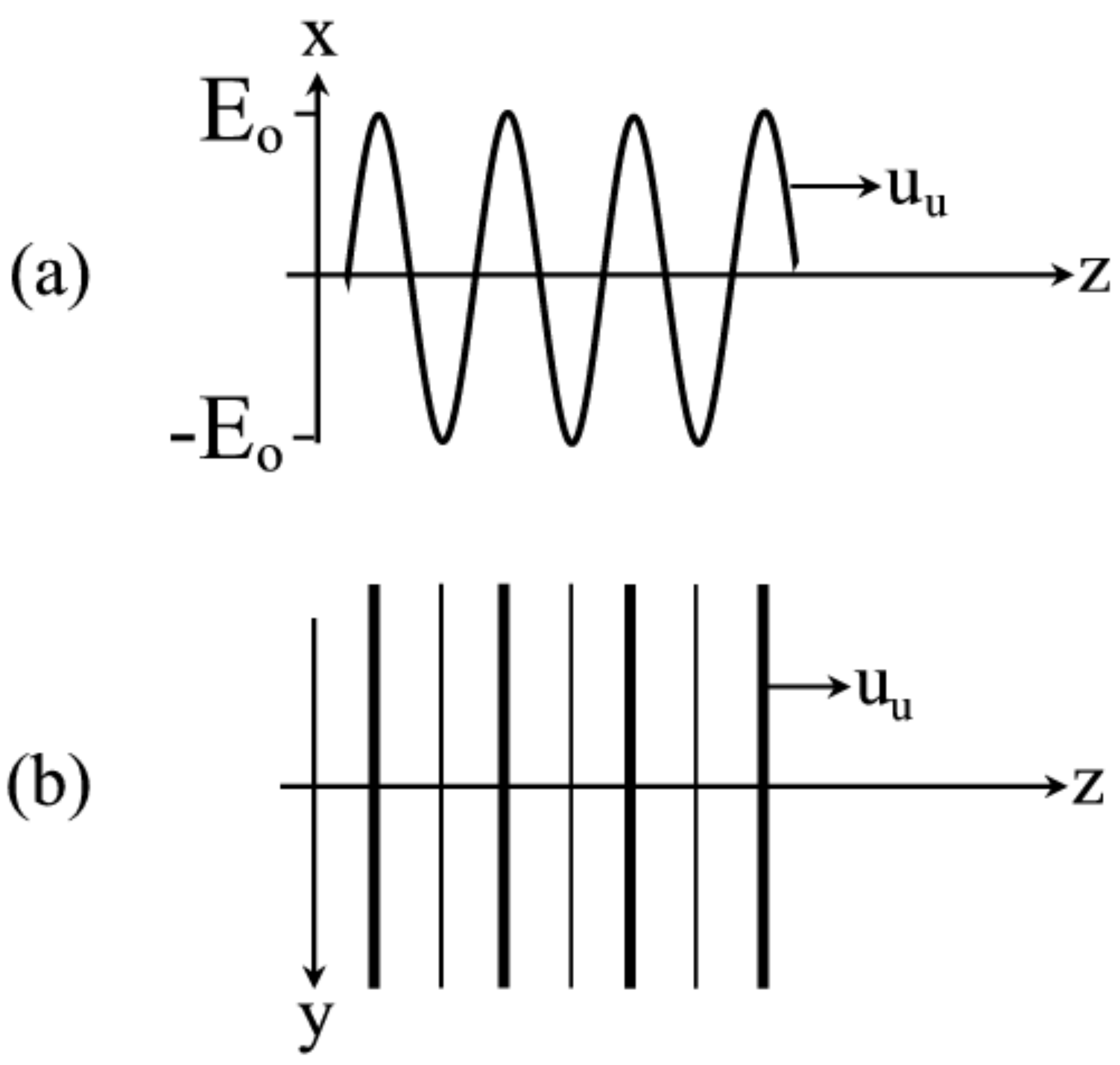
TE$_{20}$: $$f_{c20} = \frac{c}{a} = 4.16 \text{ GHz}$$

TE$_{11}$: $$f_{c11} = \frac{3x10^8 \, m/s}{2}\sqrt{\left(\frac{1}{7.214cm}\right)^2 + \left(\frac{1}{3.404cm}\right)^2} \frac{100cm}{1m} = 4.87 \text{ GHz}$$



Rectangular Waveguide - Wave Propagation

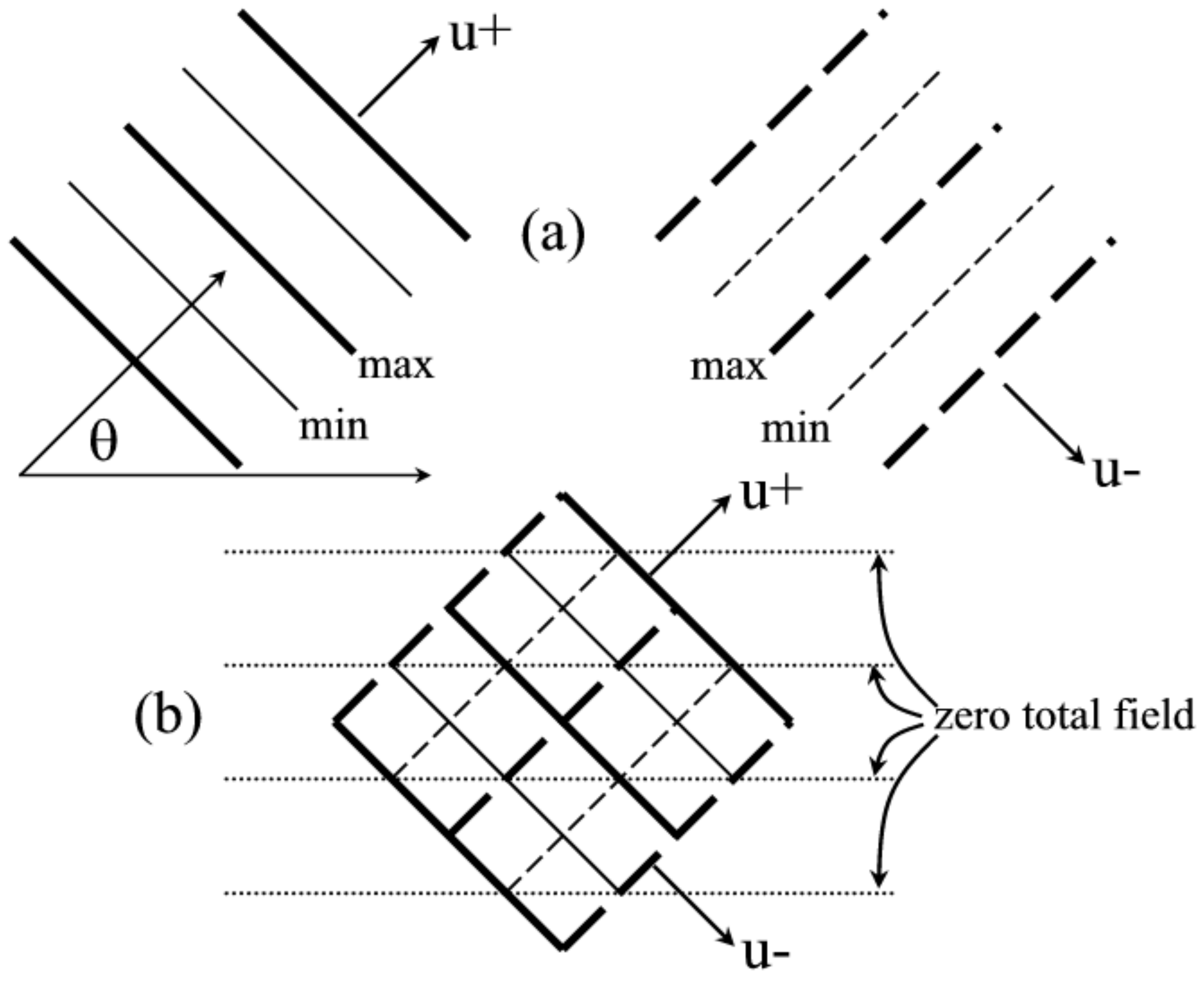
- We can achieve a qualitative understanding of wave propagation in waveguide by considering the wave to be a superposition of a pair of TEM waves.
- Let us consider a TEM wave propagating in the z direction. Figure shows the wave fronts; bold lines indicating constant phase at the maximum value of the field ($+E_o$), and lighter lines indicating constant phase at the minimum value ($-E_o$).
- The waves propagate at a velocity u_u, where the u subscript indicates media unbounded by guide walls. In air, $u_u = c$.



Rectangular Waveguide - Wave Propagation

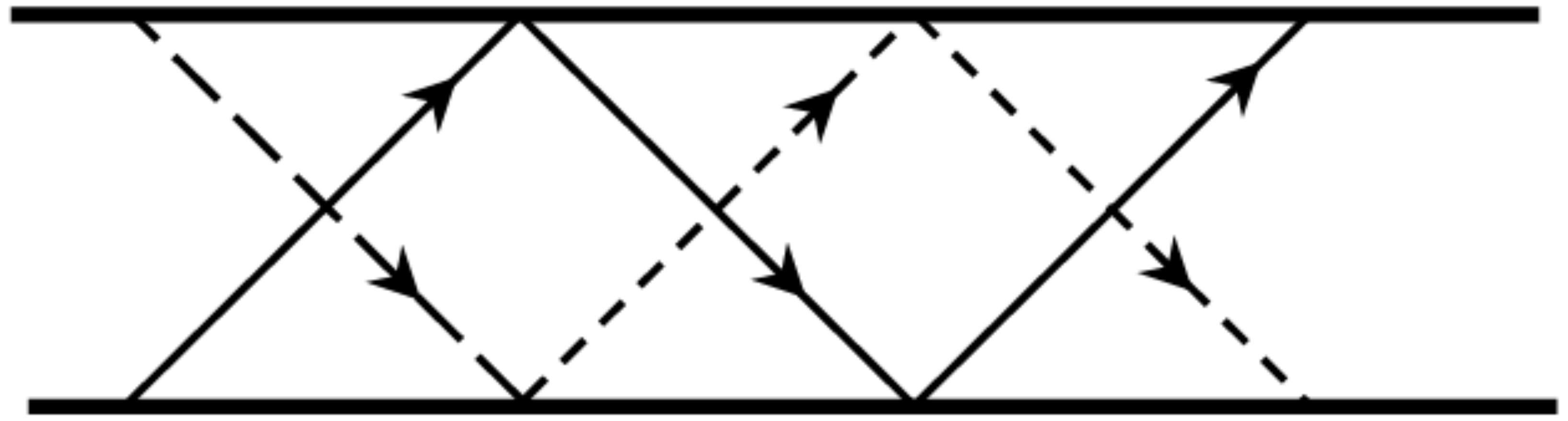
- **Now consider a pair of identical TEM waves, labeled as u+ and u- in Figure (a). The u+ wave is propagating at an angle +θ to the z axis, while the u- wave propagates at an angle –θ.**

- **These waves are combined in Figure (b). Notice that horizontal lines can be drawn on the superposed waves that correspond to zero field. Along these lines the u+ wave is always 180° out of phase with the u- wave.**

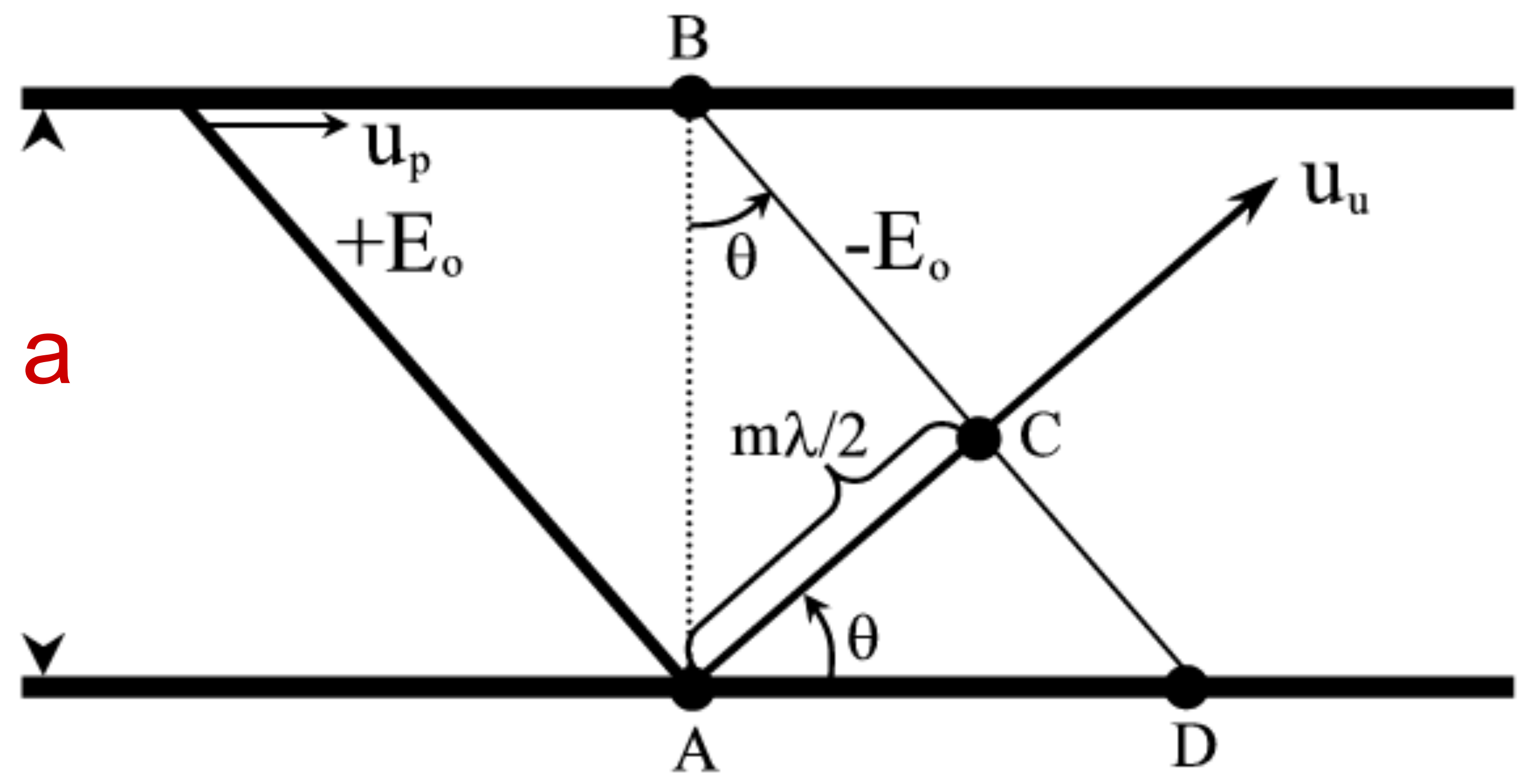


Rectangular Waveguide - Wave Propagation

- **Since we know E = 0 on a perfect conductor, we can replace the horizontal lines of zero field with perfect conducting walls. Now, u+ and u- are reflected off the walls as they propagate along the guide.**
- **The distance separating adjacent zero-field lines in Figure (b), or separating the conducting walls in Figure (a), is given as the dimension a in Figure (b).**
- **The distance *a* is determined by the angle θ and by the distance between wavefront peaks, or the wavelength λ. For a given wave velocity u_u, the frequency is $f = u_u/\lambda$.**
- **If we fix the wall separation at a, and change the frequency, we must then also change the angle θ if we are to maintain a propagating wave. Figure (b) shows wave fronts for the u+ wave.**
- **The edge of a $+E_o$ wave front (point A) will line up with the edge of a $-E_o$ front (point B), and the two fronts must be $\lambda/2$ apart for the m = 1 mode.**



(a)



(b)

Rectangular Waveguide - Wave Propagation

For any value of m, we can write by simple trigonometry

$$\sin\theta = \frac{m\lambda/2}{a} \quad \Rightarrow \quad \lambda = \frac{2a}{m}\sin\theta = \frac{u_u}{f}$$

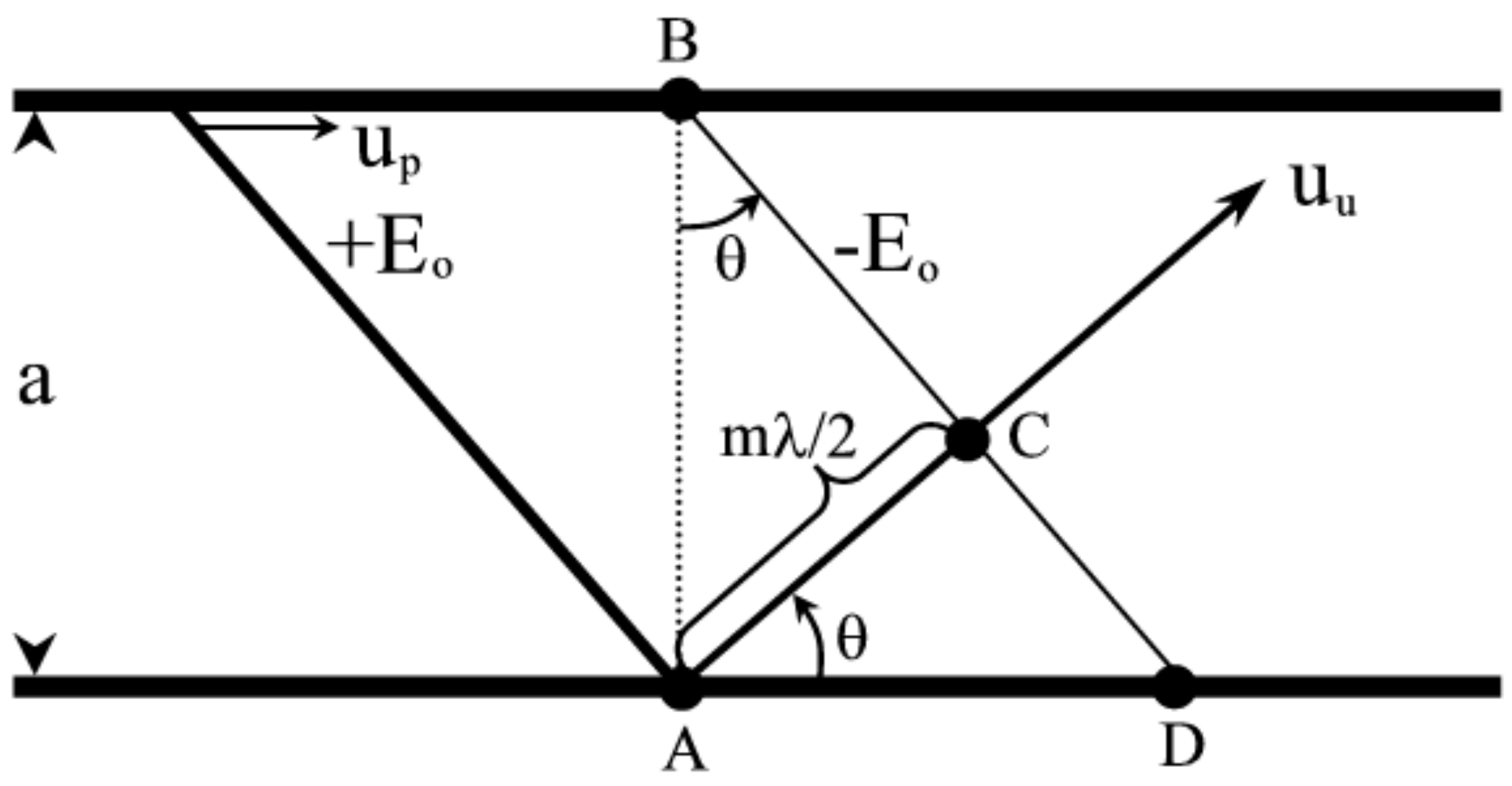
The waveguide can support propagation as long as the wavelength is smaller than a critical value, λ_c, that occurs at $\theta = 90°$, or

$$\lambda_c = \frac{2a}{m} = \frac{u_u}{f_c}$$

Where f_c is the cutoff frequency for the propagating mode.

We can relate the angle θ to the operating frequency and the cutoff frequency by

$$\sin\theta = \frac{\lambda}{\lambda_c} = \frac{f_c}{f}$$



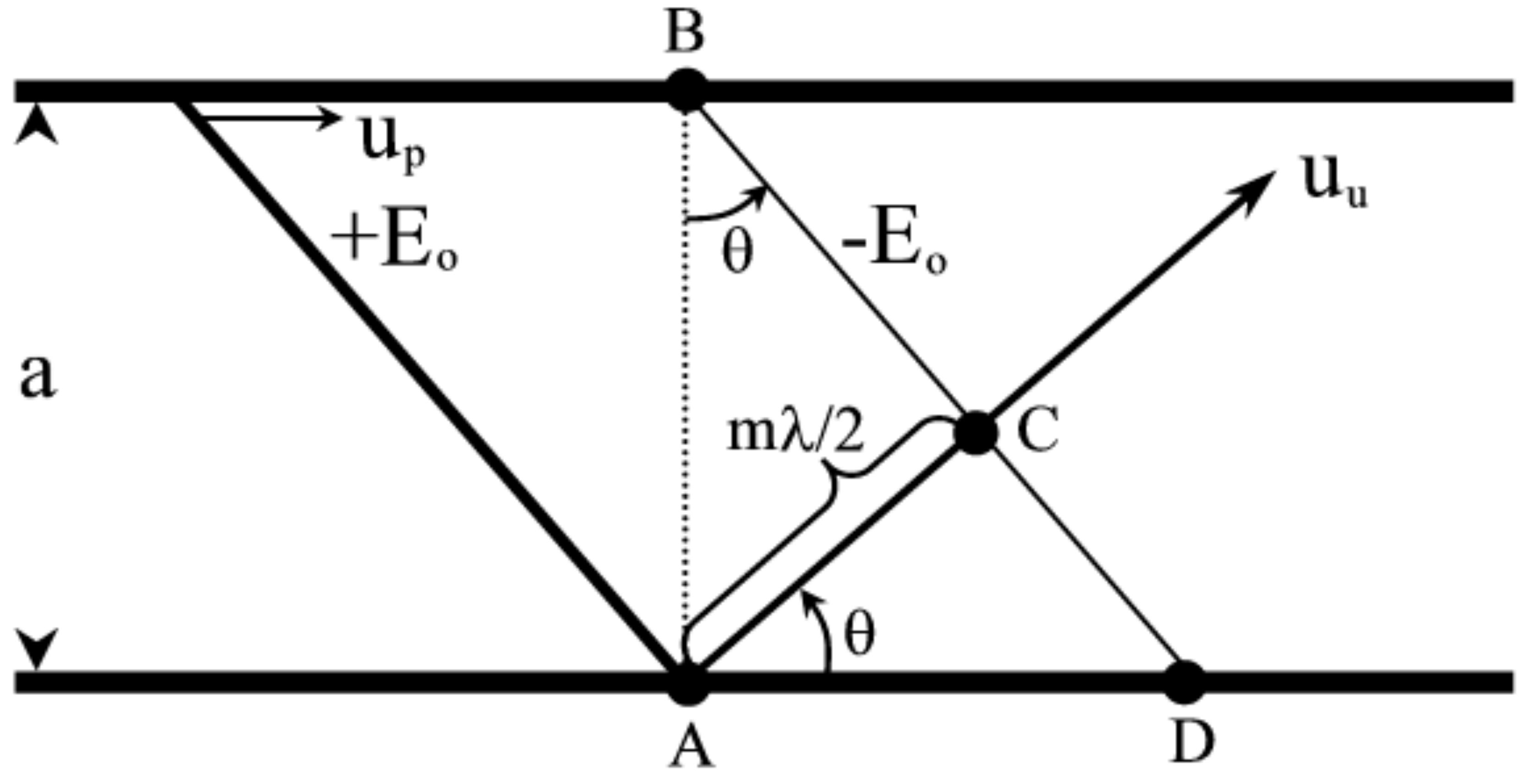
Rectangular Waveguide - Wave Propagation

The time t_{AC} it takes for the wavefront to move from A to C (a distance l_{AC}) is

$$t_{AC} = \frac{\text{Distance from A to C}}{\text{Wavefront Velocity}} = \frac{l_{AC}}{u_u} = \frac{m\lambda/2}{u_u}$$

A constant phase point moves along the wall from A to D. Calling this phase velocity u_p, and given the distance l_{AD} is

$$l_{AD} = \frac{m\lambda/2}{\cos\theta}$$



Then the time t_{AD} to travel from A to D is

$$t_{AD} = \frac{l_{AD}}{u_p} = \frac{m\lambda/2}{\cos\theta\ u_p}$$

Since the times t_{AD} and t_{AC} must be equal, we have

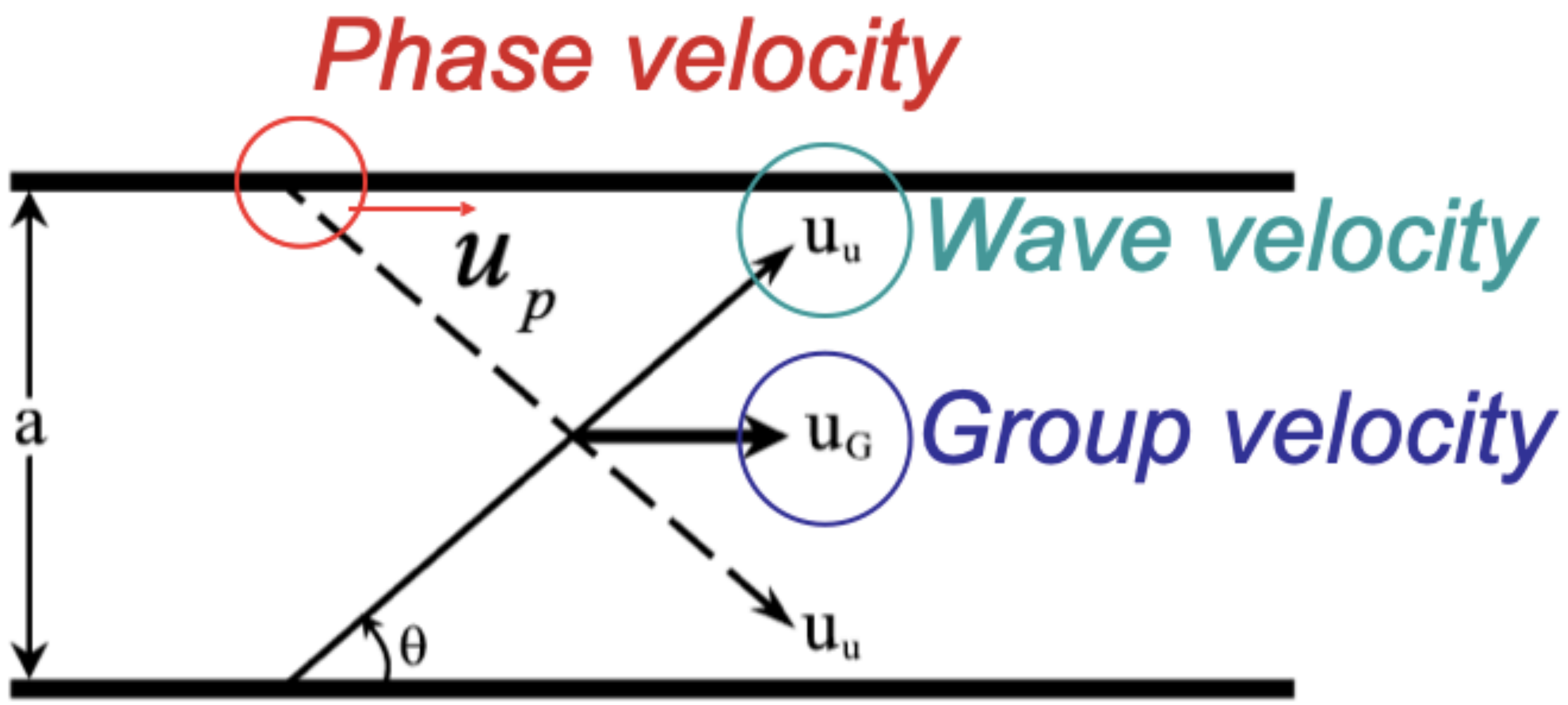
$$u_p = \frac{u_u}{\cos\theta}$$

Rectangular Waveguide - Wave Propagation

The *Wave velocity* is given by

$$u_u = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_o\mu_r\varepsilon_o\varepsilon_r}} = \frac{1}{\sqrt{\mu_o\varepsilon_o}}\frac{1}{\sqrt{\mu_r\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

where $c = 3\times10^8$ m/s

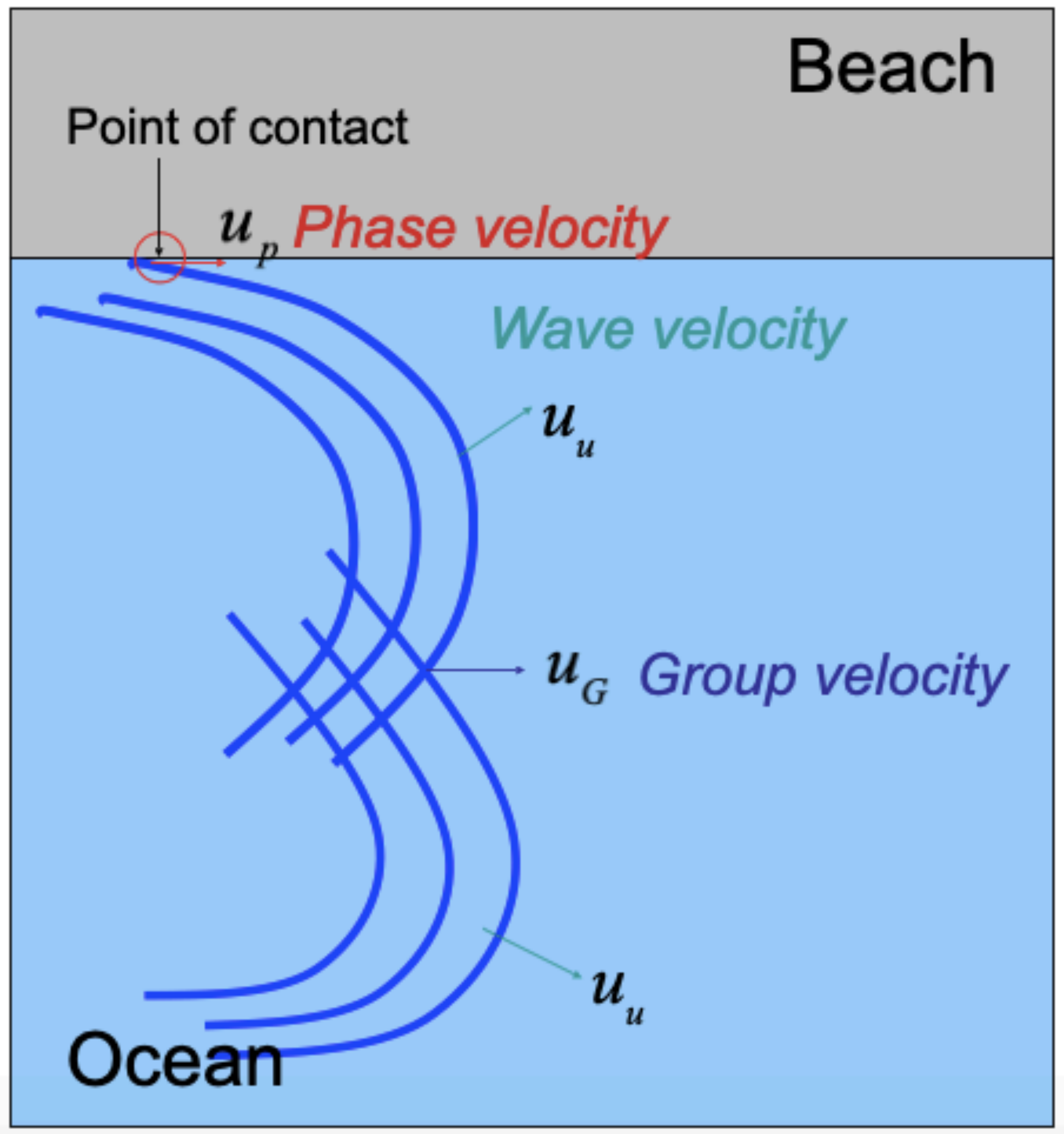


The *Phase velocity* is given by

$$u_p = \frac{u_u}{\cos\theta} \quad \xrightarrow{\text{using}} \quad u_p = \frac{u_u}{\sqrt{1-\left(f_c/f\right)^2}}$$

$$\cos\theta = \sqrt{\cos^2\theta} = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(f_c/f\right)^2}$$

Analogy!



The *Group velocity* is given by

$$u_G = u_u\cos\theta$$

$$u_G = u_u\sqrt{1-\left(f_c/f\right)^2}$$

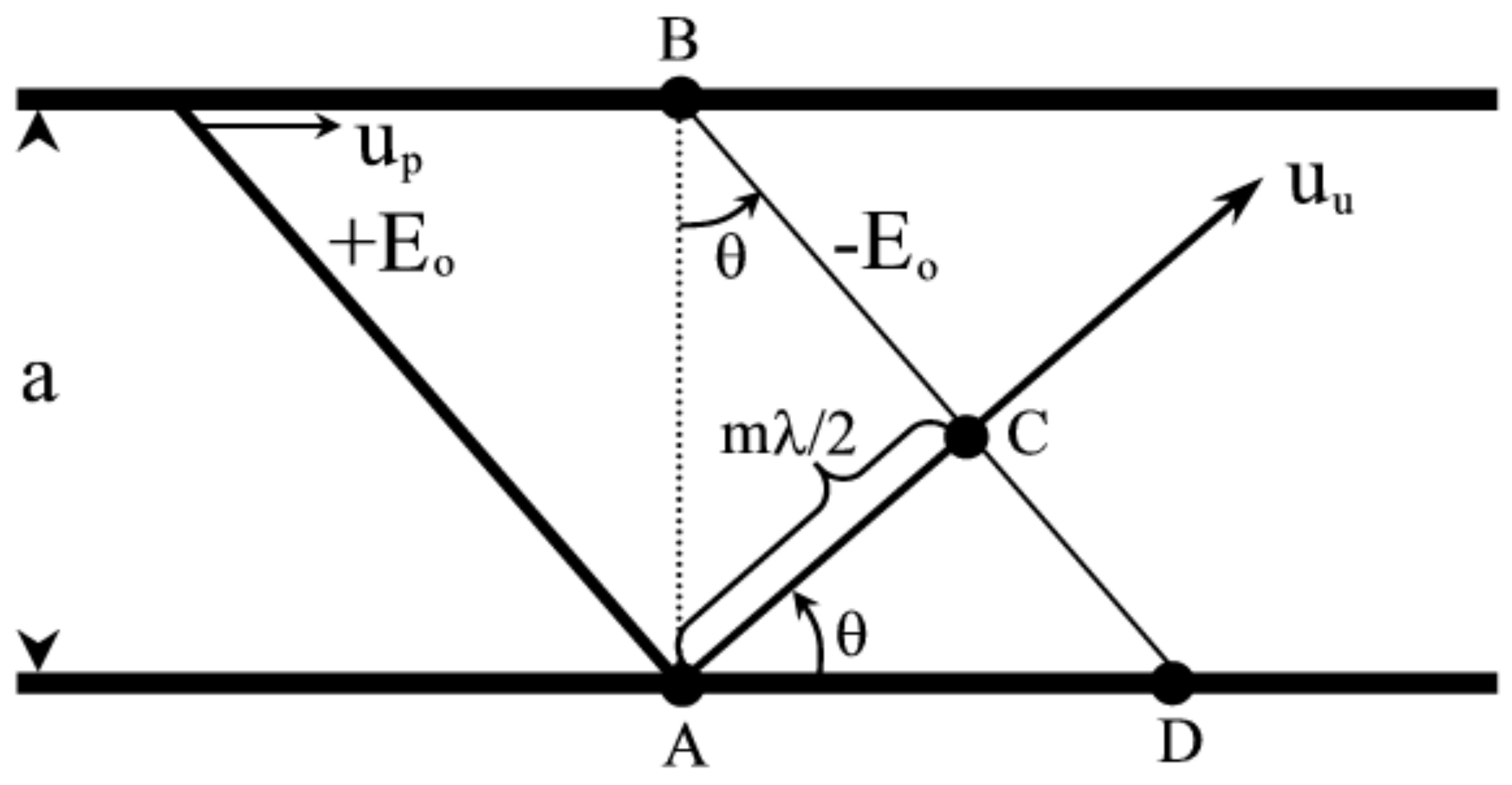
Rectangular Waveguide - Wave Propagation

The **phase constant** is given by

$$\beta = \beta_u \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

The **guide wavelength** is given by

$$\lambda = \frac{\lambda_u}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$



The ratio of the transverse electric field to the transverse magnetic field for a propagating mode at a particular frequency is the ***waveguide impedance***.

For a TE mode, the wave impedance is

$$\boxed{Z_{mn}^{TE} = \frac{\eta_u}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}},$$

For a TM mode, the wave impedance is

$$\boxed{Z_{mn}^{TM} = \eta_u \sqrt{1 - \left(\frac{f_c}{f}\right)^2}}.$$

$$\eta_u = \sqrt{\frac{\mu}{\epsilon}}$$

$\eta_u = 120\pi\Omega$ in free space

Example 2

Consider WR975 is filled with polyethylene. Find (a) u_u, (b) u_p and (c) u_G at 600 MHz.

From Table for WR975 we have $a = 9.75$ in and $b = 4.875$ in. Then

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$$fc_{10} = \frac{c}{2\sqrt{\varepsilon_r}a} = \frac{3x10^8\ m/s}{2\sqrt{2.26}} \frac{1}{9.75in}\left(\frac{1in}{0.0254m}\right) = 403MHz$$

$$F = \sqrt{1-\left(\frac{fc}{f}\right)^2} = \sqrt{1-\left(\frac{403}{600}\right)^2} = 0.741$$

Now,

$$u_U = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3x10^8}{\sqrt{2.26}} = 2x10^8\,\frac{m}{s}$$

$$u_P = \frac{u_U}{F} = 2.7x10^8\,\frac{m}{s}$$

$$u_G = u_U F = 1.48x10^8\,\frac{m}{s}$$

Example 3

Let's determine the TE mode looking into a 20 cm long section of shorted WR90 waveguide operating at 10 GHz.

From the Waveguide Table, a = 0.9 inch (or) 2.286 cm and b = 0.450 inch (or) 1.143 cm.

Example 3

Let's determine the TE mode looking into a 20 cm long section of shorted WR90 waveguide operating at 10 GHz.

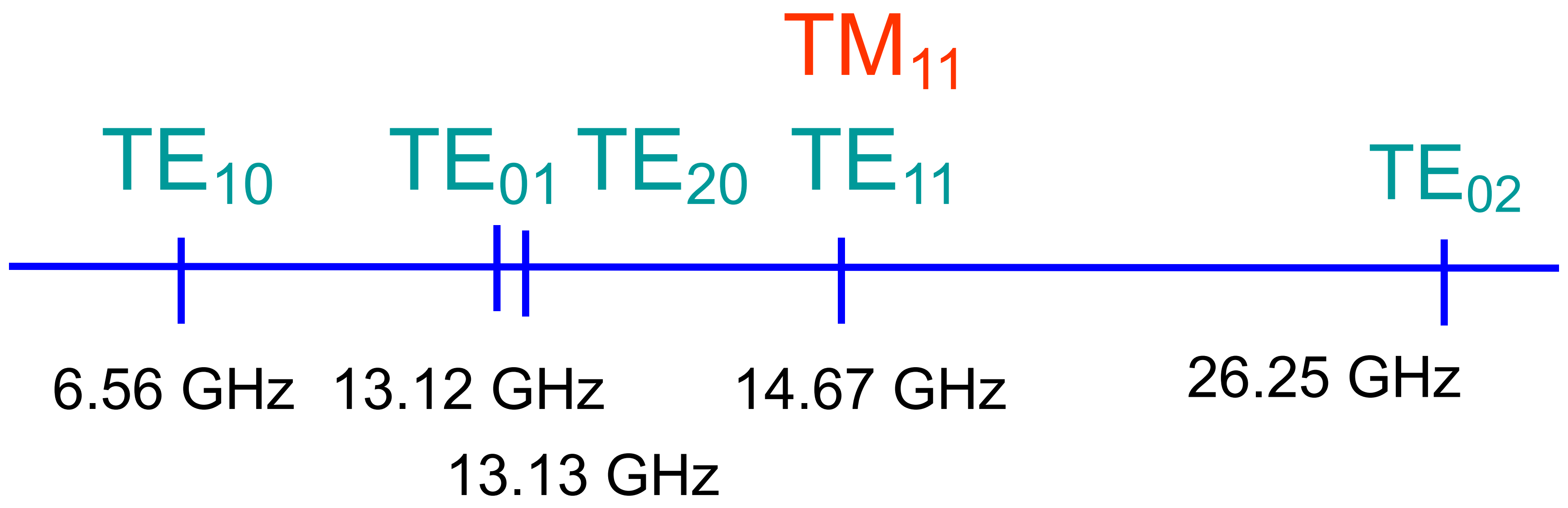
From the Waveguide Table, a = 0.9 inch (or) 2.286 cm and b = 0.450 inch (or) 1.143 cm.

$$f_{c_{mn}} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Mode	Cutoff Frequency
TE_{10}	6.56 GHz
TE_{01}	13.12 GHz
TE_{11}	14.67 GHz
TE_{20}	13.13 GHz
TE_{02}	26.25 GHz

Rearrange

Mode	Cutoff Frequency
TE_{10}	6.56 GHz
TE_{01}	13.12 GHz
TE_{20}	13.13 GHz
TE_{11}	14.67 GHz
TE_{02}	26.25 GHz



At 10 GHz, only the TE_{10} mode is supported!