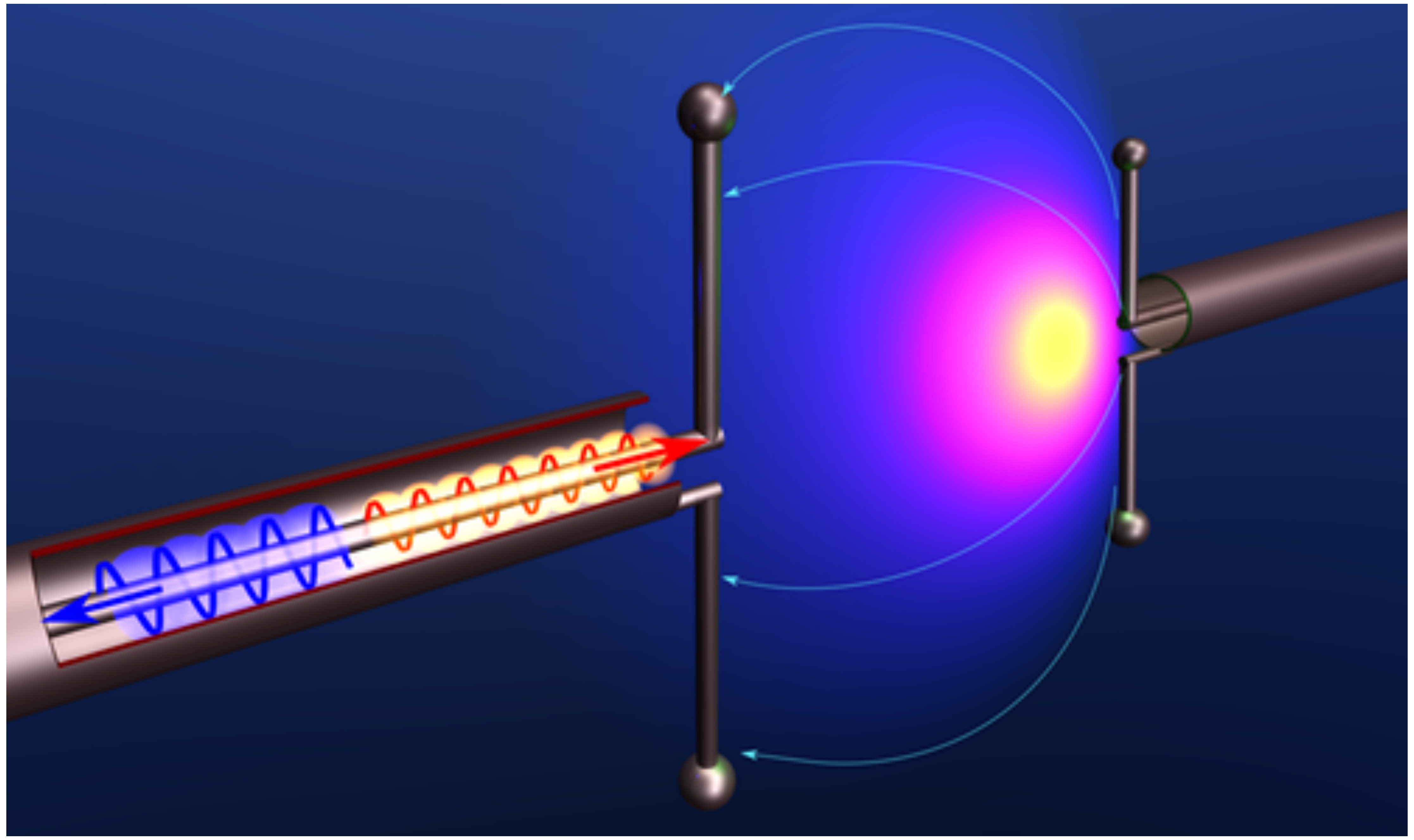
Hyperfréquence (Radiofrequency)



In this scheme for wireless transmission, a transmitting antenna (right) radiates radio-frequency waves to the receiving antenna, which sends power (blue wave) through a coaxial cable to the receiving circuit.

Part 2

Dr. Lana Damaj

Part 2.1: outlines

- Introduction
- Common types of transmission lines
- Lumped- element model of transmission lines
- Transmission line parameters
- Transmission line equation
- Wave propagation on a transmission line
- Voltage reflection coefficient
- Standing wave

Introduction

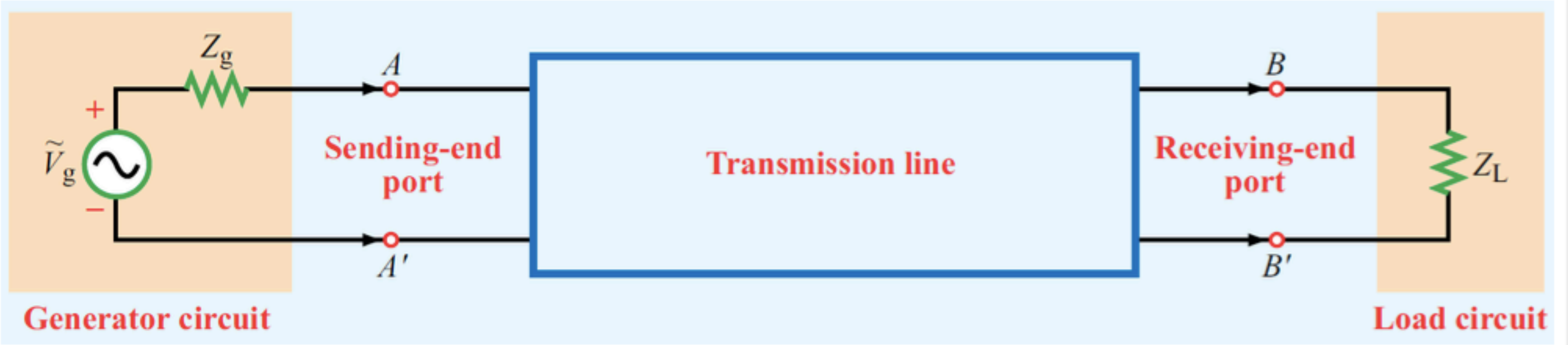
- We draw circuit diagrams with “lumped”components: ideal R’s, C’s, L’s, transistors, etc., connected by lines that represent zero-length wires.
- But all real wires, if not much shorter than the shortest relevant wavelength, are themselves complicated circuit elements; the current (and voltage) is not the same everywhere along such a wire, even if the wire has no resistance.
- On the other hand, when interconnections are made with transmission lines we can accurately predict circuit behavior.

Introduction

- Transmission lines in communication carry telephone signals, computer data in LANs, TV signals in cable TV systems, and signals from a transmitter to an antenna or from an antenna to a receiver.
- Transmission lines in RF systems are also circuits (not only a connection like in electric systems)
- Their electrical characteristics are critical and must be matched to the equipment for successful communication to take place.

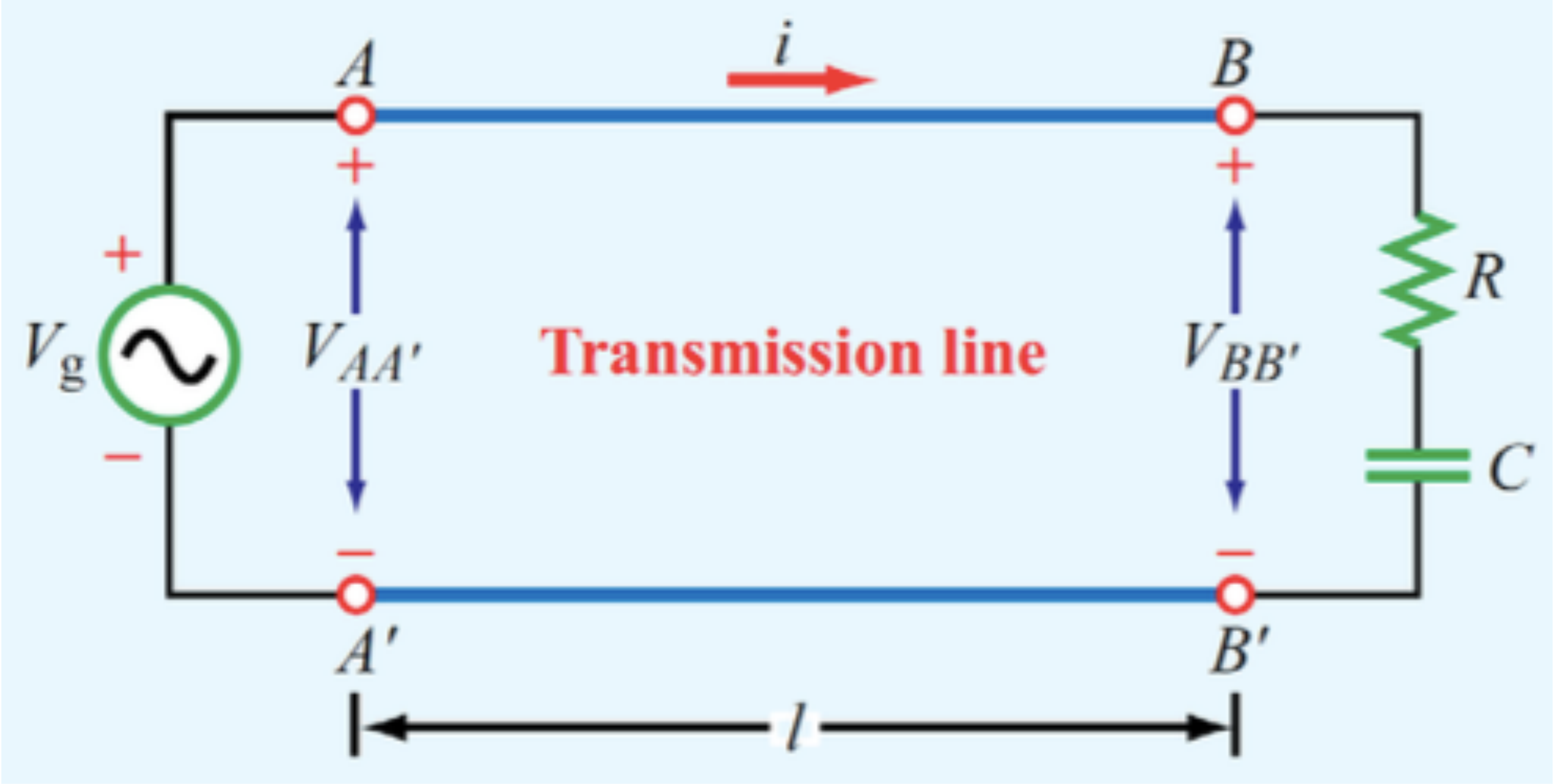
Introduction

- Transmission line: a bridge between circuit theory and electromagnetic theory.
- By modeling transmission lines in the form of equivalent circuits, we can use Kirchhoff's voltage and current laws to develop wave equations whose solutions provide an understanding of wave propagation, standing waves, and power transfer.
- Fundamentally, a transmission line is a two-port network, with each port consisting of two terminals. One of the ports, the line's sending end, is connected to a source (also called the generator). The other port, the line's receiving end, is connected to a load.



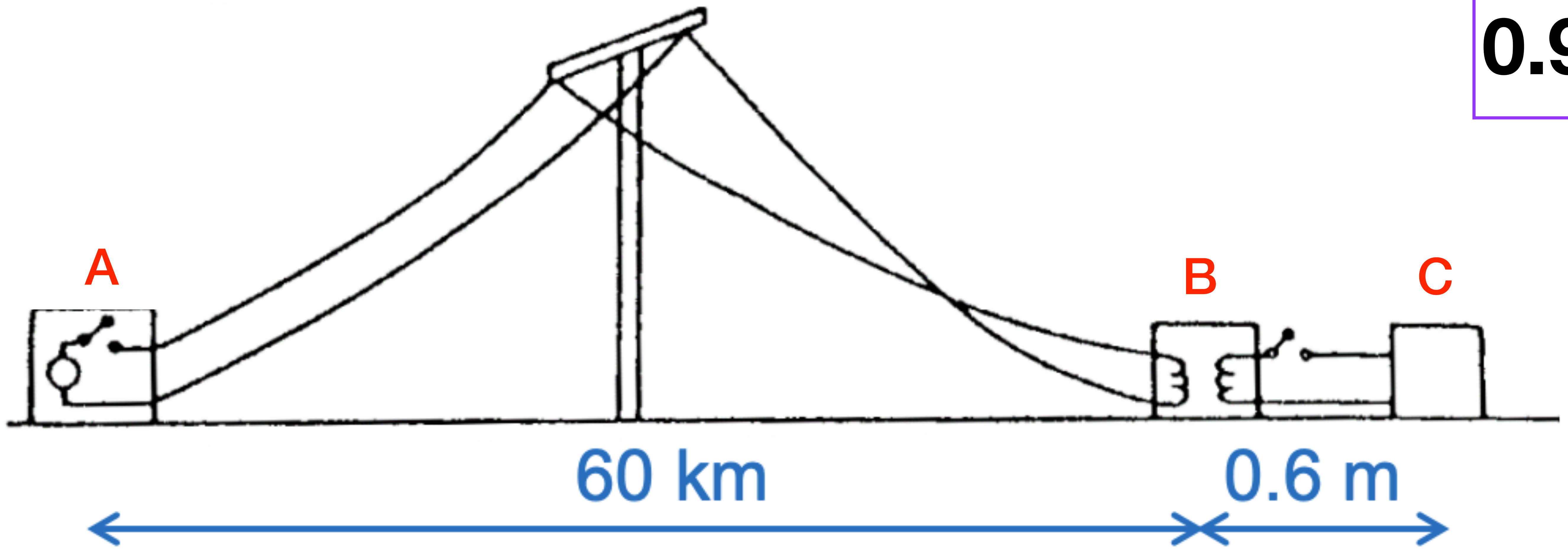
Effect of the wavelength

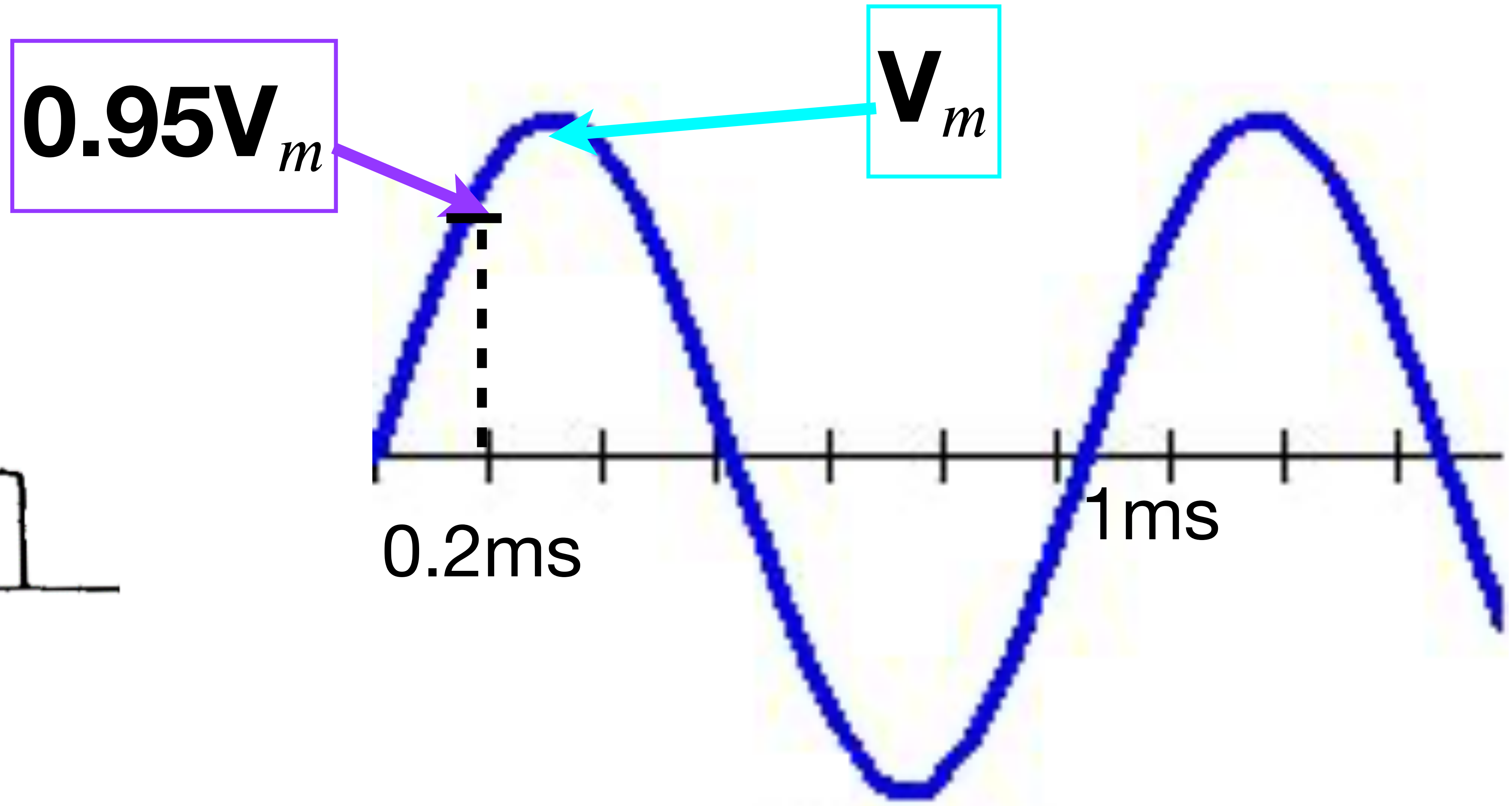
- **Is the pair of wires between terminals AA' and terminals BB' a transmission line? If so, under what set of circumstances should we explicitly treat the pair of wires as a transmission line, as opposed to ignoring their presence?**
  - **The factors that determine whether or not we should treat the wires as a transmission line are governed by the length of the line l and the frequency f of the signal provided by the generator.**
  - **When l/λ is very small, transmission-line effects may be ignored**
  - **When $l/\lambda \gtrsim 0.01$, it may be necessary to account not only for the phase shift due to the time delay, but also for the presence of reflected signals (this principal will be detailed later) that may have been bounced back by the load toward the generator.**



Effect of the wavelength: example

- Assume that $V = V_m \sin 2\pi ft$ with $f = 1$ kHz when A is closed. The corresponding wavelength is 300 km and is comparable to the distance between A and B. This voltage signal will not appear instantaneously at point B, as a certain time is required for the signal to travel from points A to B⇒transmission line



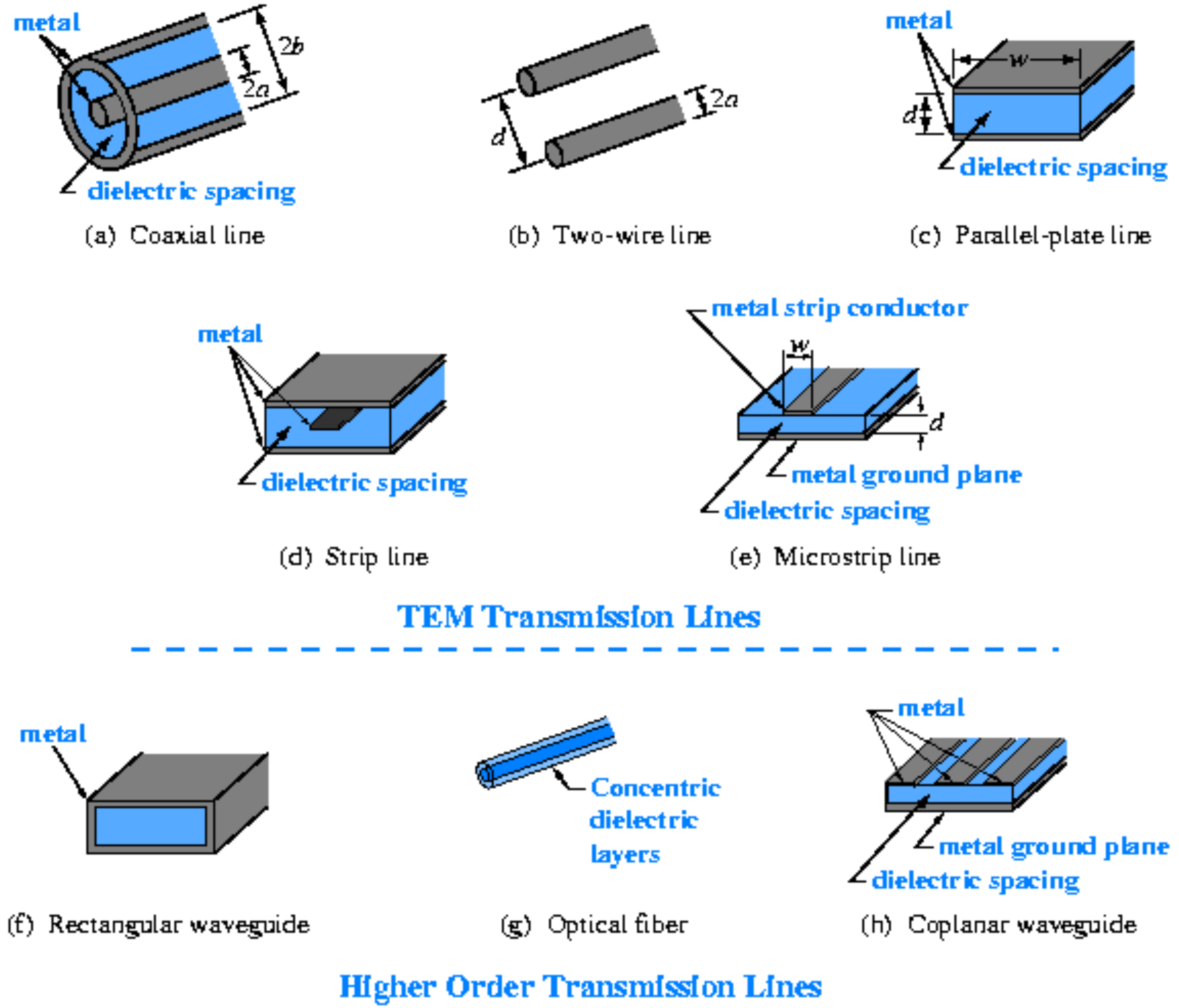


- How about B-C? Because the distance between B and C is small, the voltage at point B will appear almost instantaneously at point C.→electric circuit

Transmission lines: definition

- Transmission line is a specialized cable or other structure designed to carry alternating current of radio frequency
- This lines are consider to be impedance matching circuits design to deliver power, for exemple, from transmitter to antenna and maximum signal from antenna to the receiver

Common types of transmission lines



Common types of transmission lines

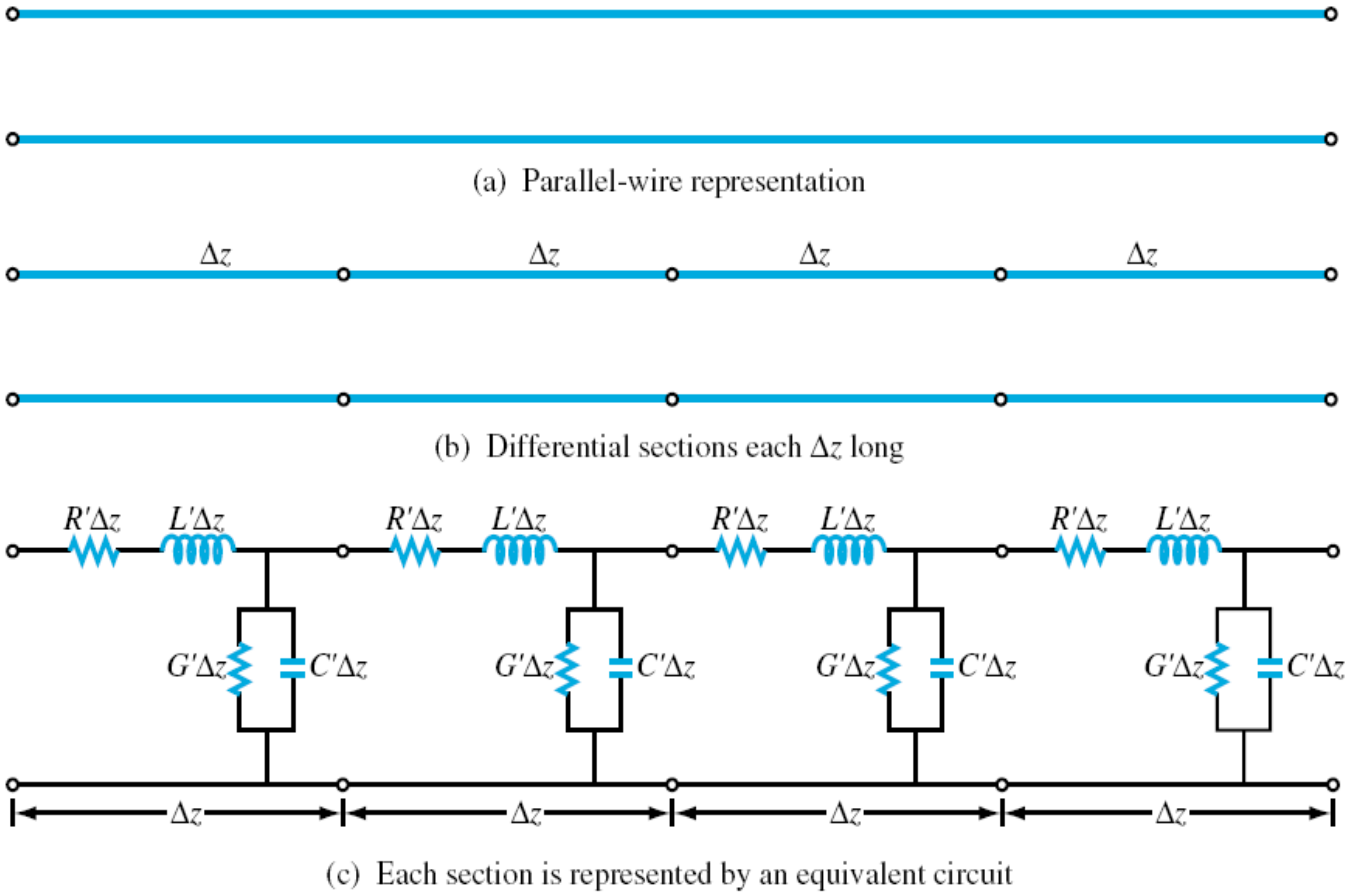
- Transmission lines may be classified into two types:
  - a) Transverse electromagnetic (TEM) transmission lines – waves propagating along these lines having electric and magnetic field that are entirely transverse to the direction of propagation (ex: coaxial cable, two-wire line, microstrip line, …)
  - b) Higher order transmission lines – waves propagating along these lines have at least one significant field component in the direction of propagation (ex: waveguides, fiber optics,…)

Lumped- element model of transmission lines

- **A transmission line is represented by a parallel-wire configuration regardless of the specific shape of the line, i.e coaxial line, two-wire line or any TEM line.**
- **Lumped element circuit model consists of four basic elements called 'the transmission line parameters' : R' , L' , G' , C'**

Lumped-element transmission line parameters:

– R' : combined resistance of both conductors per unit length, in Ω/m
– L' : the combined inductance of both conductors per unit length, in H/m
– G' : the conductance of the insulation medium per unit length, in S/m
– C' : the capacitance of the two conductors per unit length, in F/m



(c) Each section is represented by an equivalent circuit

Transmission line parameters

Parameter	Coaxial	Two Wire	Parallel Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{R_s}{\pi a}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]$	$\frac{\mu d}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]}$	$\frac{\sigma w}{d}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]}$	$\frac{\varepsilon w}{d}$	F/m

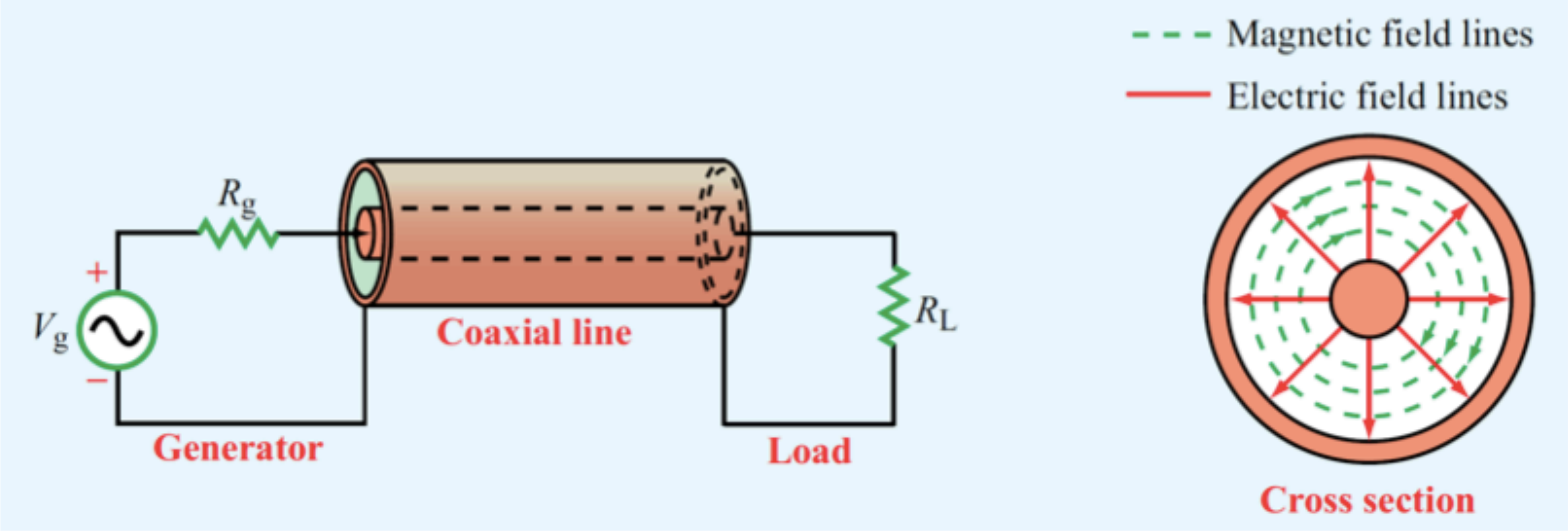
$$R_S = \sqrt{\pi f \mu_c/\sigma_c}$$

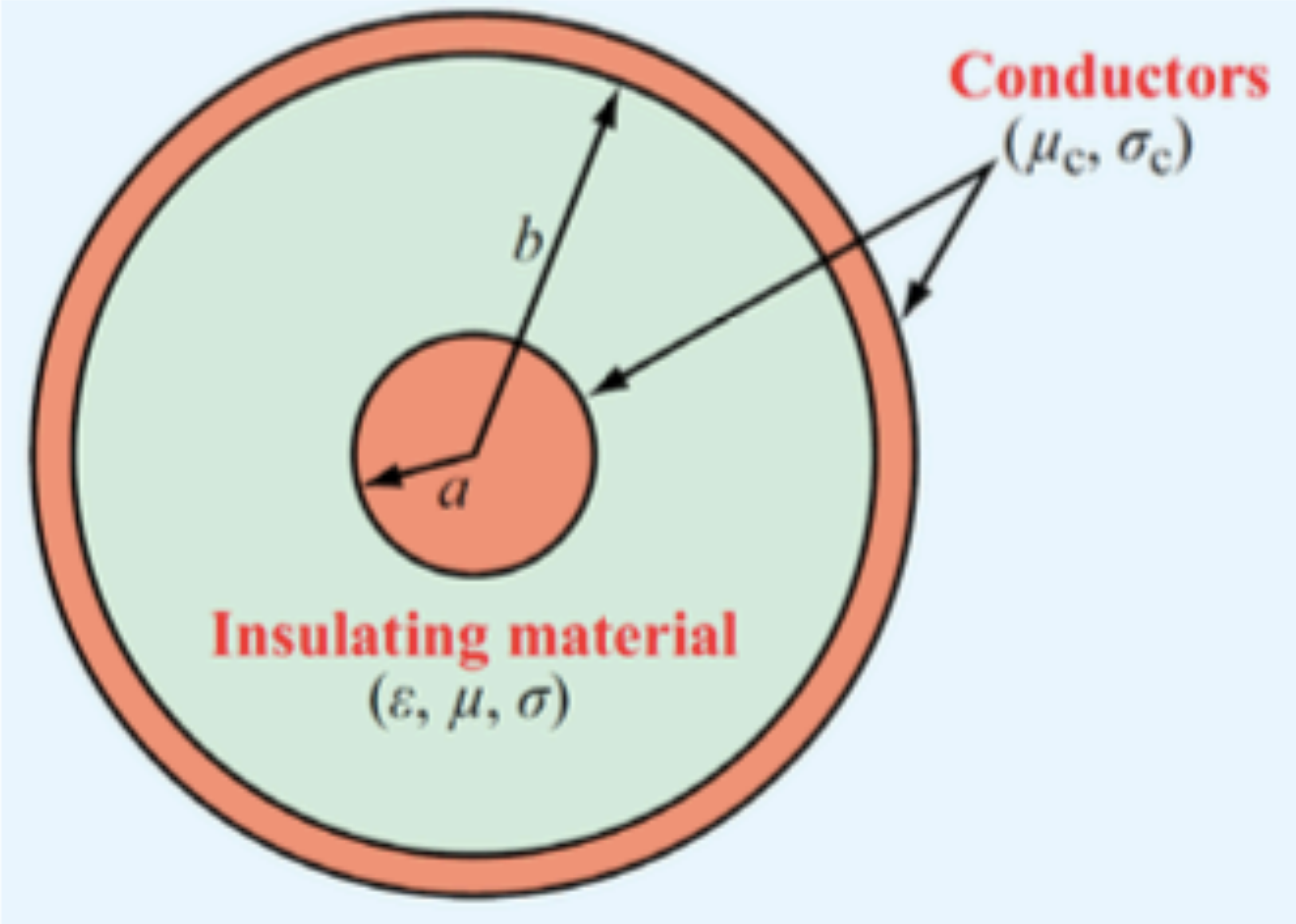
μ, σ, ε are the permeability, the conductivity, and the permittivity of the insulating material between conductors

μ_c, σ_c, are the permeability and the conductivity of the conductors

a, b, w and d are the dimensions of the lines represented in slide 9

Coaxial transmission line





$$R' = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right) \quad \text{where } R_s = \sqrt{\frac{\pi f \mu_c}{\sigma_c}}$$

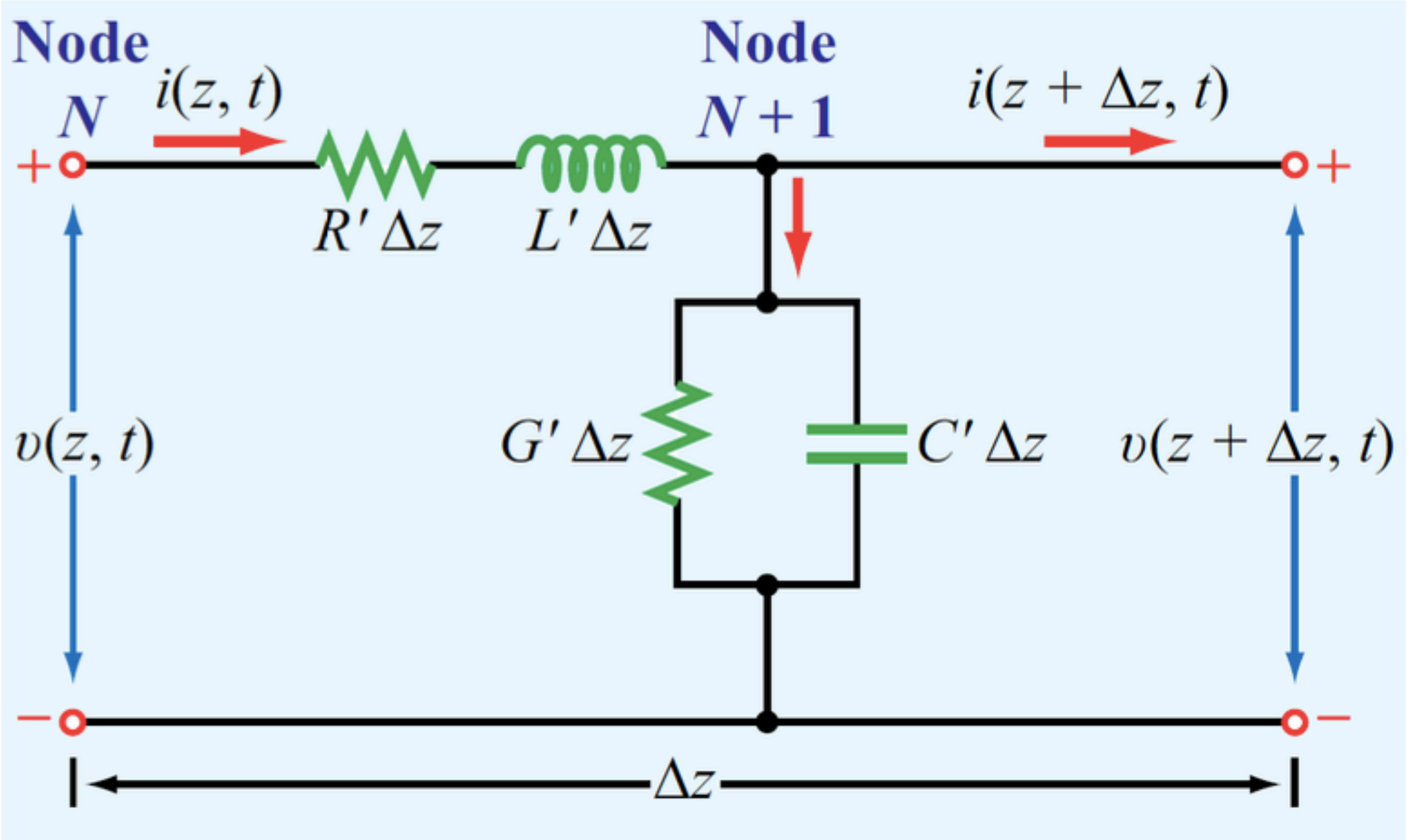
Surface resistance

- For all TEM lines,

$$L'C' = \mu\epsilon \qquad \frac{G'}{C'} = \frac{\sigma}{\epsilon}$$

Transmission line equation

- Equivalent circuit of a two-conductor transmission line of differential length Δz.



- Based on Kirchhoff’s voltage law

$$v(z,t) - R'\Delta z\, i(z,t) - L'\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

- Based on Kirchhoff’s current law
- $$i(z,t) - G'\Delta z\, v(z+\Delta z,t) - C'\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

Transmission line equation

- **By rearranging KVL and KCL equations, we obtain two first-order differential equations known as telegrapher's equations.**

$$-\frac{\partial v(z,t)}{\partial z} = R'\, i(z,t) + L'\,\frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial i(z,t)}{\partial z} = G'\, v(z,t) + C'\,\frac{\partial v(z,t)}{\partial t}$$

- **Phasor notation for sinusoidal signals**

$$v(z,t) = Re[\tilde{V}(z)e^{j\omega t}] \qquad i(z,t) = Re[\tilde{I}(z)e^{j\omega t}]$$

- **Telegrapher's equation in phasor form**

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\tilde{I}(z)$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\tilde{V}(z)$$

Wave propagation on a transmission line

- The two first-order coupled equations can be combined to give two second-order uncoupled wave equations, one for $\tilde{V}(z)$ and another for $\tilde{I}(z)$.

$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0$$

$$\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\tilde{I}(z) = 0$$

where $\boxed{\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}}$

Propagation constant

- Complex propagation constant γ consists of a real part α, called the attenuation constant of the line with units of Np/m, and an imaginary part β, called the phase constant of the line with units of rad/m.

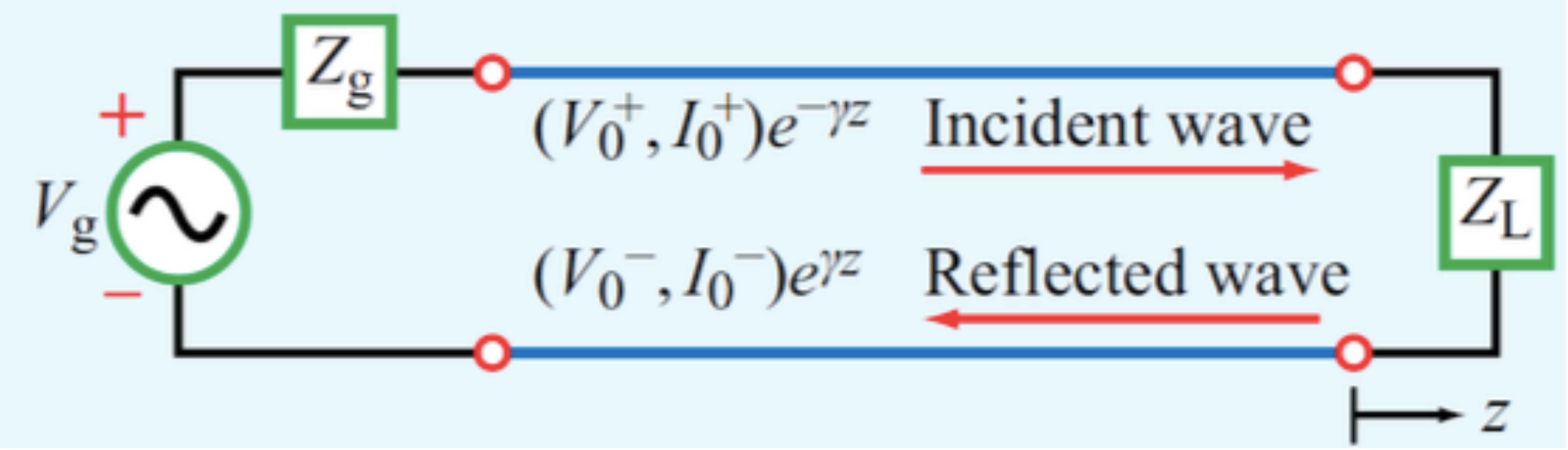
$$\gamma = \alpha + j\beta$$

Neper unit: $L_{Np} = \ln(x_1/x_2)$

1 Np = $20\log_{10}e$ dB ~ 8.686 dB

- The wave equations have traveling wave solutions of the following form:

$$\boxed{\begin{aligned}\tilde{V}(z) &= V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \\ \tilde{I}(z) &= I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}\end{aligned}}$$



Wave propagation on a transmission line

- **The voltage-current relationship**

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z} = -\frac{1}{(R' + j\omega L')}\frac{d\tilde{V}(z)}{dz} = \frac{\gamma}{(R' + j\omega L')}[V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}]$$

$$= \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

where $\frac{V_0^+}{I_0^+} = Z_0 = -\frac{V_0^-}{I_0^-}$ Characteristic impedance

- **Characteristic impedance of the line**

$$\boxed{Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}}$$

Unit: Ohm

- **It should be noted that Z_0 is equal to the ratio of the voltage amplitude to the current amplitude for each of the traveling waves individually (with an additional minus sign in the case of the $-z$ propagating wave), but it is not equal to the ratio of the total voltage $\tilde{V}(z)$ to the total current $\tilde{I}(z)$, unless one of the two waves is absent.**

Lossless transmission line

- In many practical situations, the transmission line can be designed to exhibit low ohmic losses by selecting conductors with very high conductivities ($R' \approx 0$) and dielectric materials with negligible conductivities ($G' \approx 0$).

- Propagation constant for lossless line.

$$\gamma = \alpha + j\beta \approx \sqrt{(j\omega L')(j\omega C')} = j\omega\sqrt{L'C'}$$

$$\boxed{\alpha = 0, \qquad \beta = \omega\sqrt{L'C'}}$$

- Characteristic impedance of lossless line

$$\boxed{Z_0 \approx \sqrt{\frac{j\omega L'}{j\omega C'}} = \sqrt{\frac{L'}{C'}}}$$

- Phase velocity

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu\epsilon}} \approx \frac{1}{\sqrt{\mu_0\epsilon_r\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0}}\frac{1}{\sqrt{\epsilon_r}} = \frac{c}{\sqrt{\epsilon_r}}$$

- Guide wavelength

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{L'C'}} = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} \approx \frac{1}{f\sqrt{\mu_0\epsilon_r\epsilon_0}} = \frac{c}{f}\frac{1}{\sqrt{\epsilon_r}} = \frac{\lambda_0}{\sqrt{\epsilon_r}}$$

Parameters of transmission lines

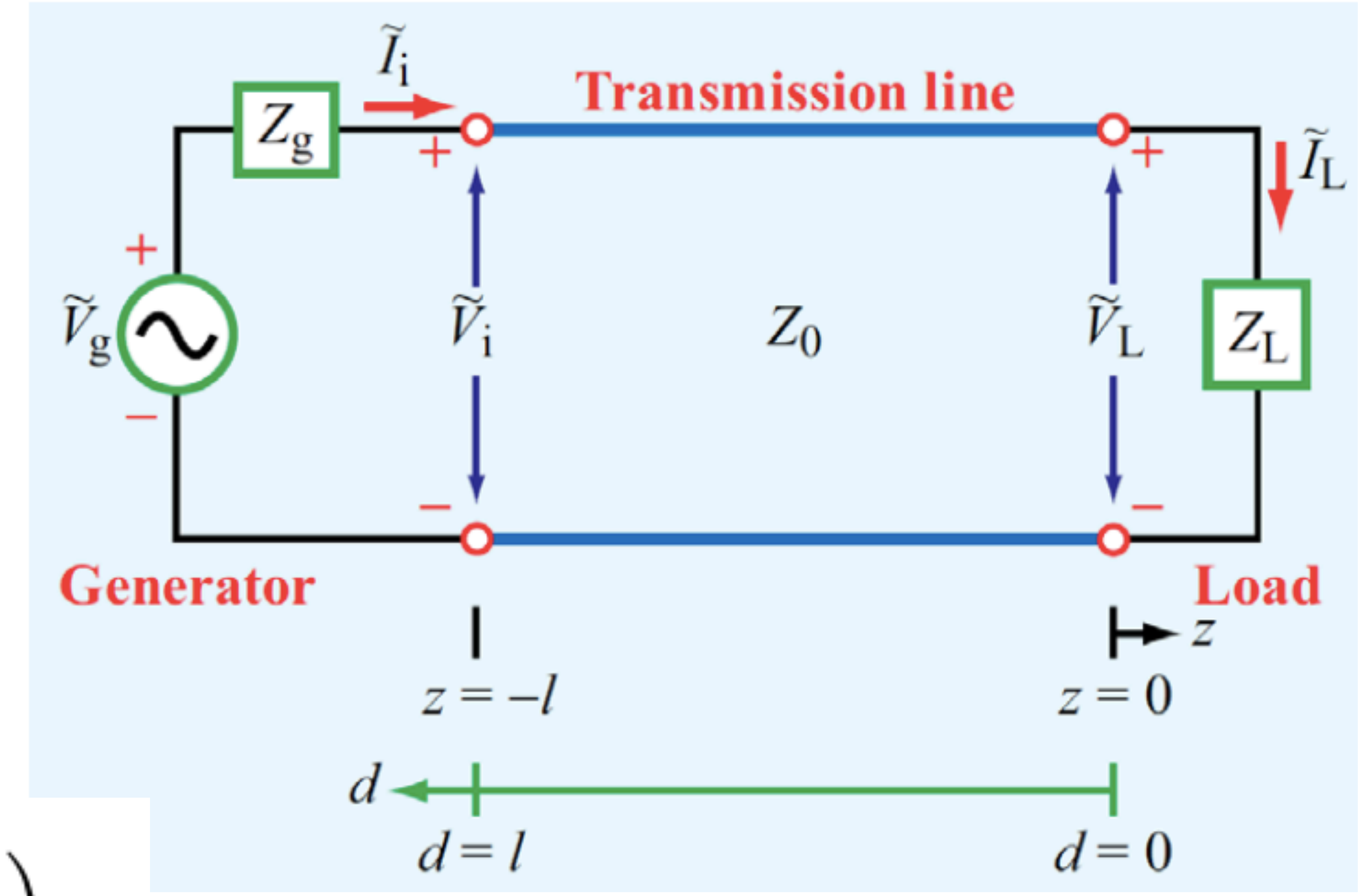
	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\dfrac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless ($R' = G' = 0$)	$\alpha = 0,\ \beta = \omega\sqrt{\epsilon_r}/c$	$u_p = c/\sqrt{\epsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\epsilon_r}/c$	$u_p = c/\sqrt{\epsilon_r}$	$Z_0 = \left(60/\sqrt{\epsilon_r}\right)\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\epsilon_r}/c$	$u_p = c/\sqrt{\epsilon_r}$	$Z_0 = \left(120/\sqrt{\epsilon_r}\right) \cdot \ln\left[(D/d) + \sqrt{(D/d)^2 - 1}\right]$ $Z_0 \approx \left(120/\sqrt{\epsilon_r}\right)\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\epsilon_r}/c$	$u_p = c/\sqrt{\epsilon_r}$	$Z_0 = \left(120\pi/\sqrt{\epsilon_r}\right)(h/w)$

Voltage reflection coefficient

- With $\gamma = j\beta$ for the lossless line, the total voltage and current become

$$\tilde{V}(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z}$$



- At the load ($z = 0$)

$$\tilde{V}_L = \tilde{V}(z=0) = V_0^+ + V_0^-$$

$$\tilde{I}_L = \tilde{I}(z=0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

- The load impedance

$$Z_L = \frac{\tilde{V}_L}{\tilde{I}_L} = \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right) Z_0 \quad \Rightarrow \quad V_0^- = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) V_0^+$$

- Voltage reflection coefficient

- $$\boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}}$$

$$\Gamma = -\frac{I_0^-}{I_0^+}$$

$z_L = Z_L / Z_0$

Normalized load impedance

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Voltage reflection coefficient

- Z_0 of a lossless line is a real number, but Z_L is in general a complex quantity. Hence, in general Γ also is complex and given by

$$\Gamma = |\Gamma| e^{-j\theta_r}$$

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

Load	$\lvert\Gamma\rvert$	θ_r
Z_0 $Z_L = (r + jx)Z_0$	$\left[\frac{(r-1)^2 + x^2}{(r+1)^2 + x^2}\right]^{1/2}$	$\tan^{-1}\left(\frac{x}{r-1}\right) - \tan^{-1}\left(\frac{x}{r+1}\right)$
Z_0 Z_0	0 (no reflection)	irrelevant
Z_0 (short)	1	$\pm 180°$ (phase opposition)
Z_0 (open)	1	0 (in-phase)
Z_0 $jX = j\omega L$	1	$\pm 180° - 2\tan^{-1} x$
Z_0 $jX = \frac{-j}{\omega C}$	1	$\pm 180° + 2\tan^{-1} x$

- Matched load($Z_L = Z_0$) →$\Gamma = 0$ and $V_0^- = 0$
- Open load($Z_L = \infty$) →$\Gamma = 1$ and $V_0^- = V_0^+$
- Short load($Z_L = 0$) →$\Gamma = -1$ and $V_0^- = -V_0^+$
-

Example 1

- We are interested in the distribution of voltage on a coaxial cable of length L = 5 m and characterized by a signal propagation velocity v = 200 000 m / s:
- Find τ delay introduced by the cable section
- If e (t) = 5 cos (ωt), find the voltage v (t) at the position x as a function of ω, x and λ
- If the charge has now an impedance R=150 Ω, find the amplitude of the reflected wave, the power of the incident, reflected and transmitted wave at the load.
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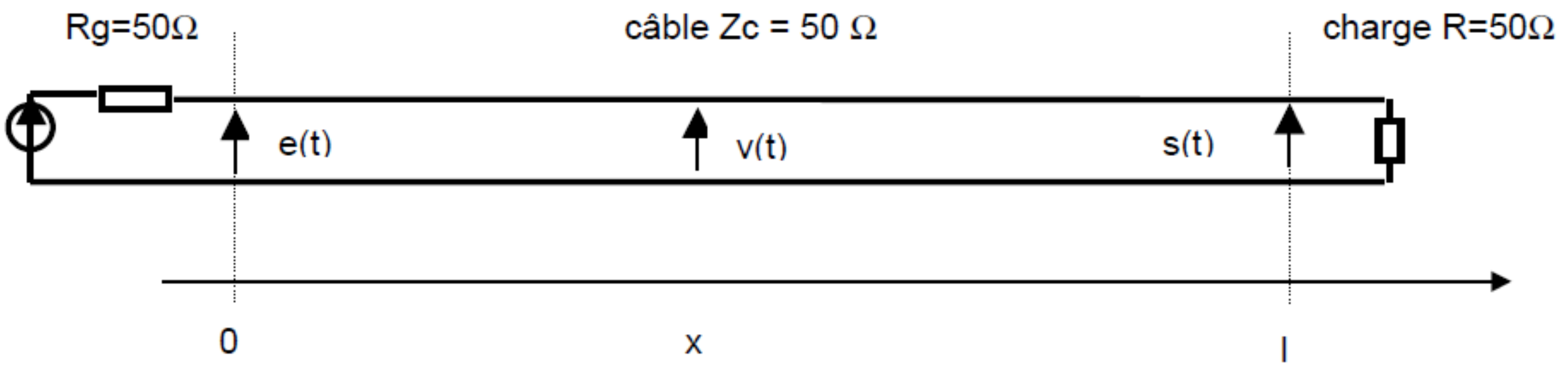
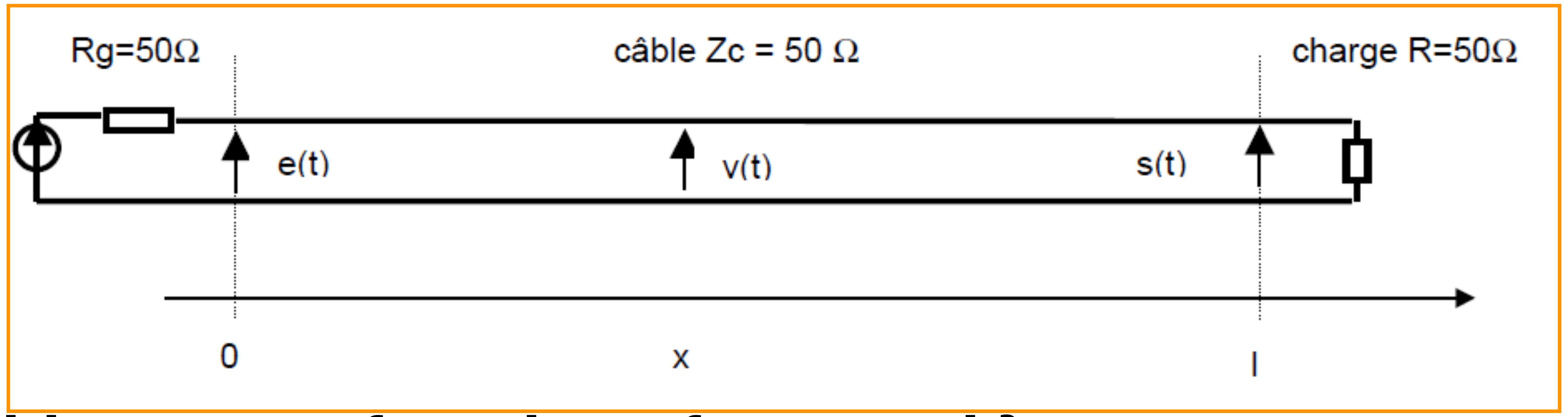


Example 1: Solution



- Find τ delay introduced by the cable section
  - $v = \frac{l}{\tau} \Longrightarrow \tau = \frac{l}{v} = 2.5 \times 10^{-5}s \longrightarrow 25\mu s$
- If e (t) = 5 cos (ωt), find the voltage v (t) at the position x as a function of ω, x and λ
  - $V(t) = 5cos(\omega(t - t_1)) = 5cos(\omega(t - \frac{x}{v})) = 5cos(\omega t - 2\pi f \frac{x}{v})$
  - $V(t) = 5cos(\omega t - \frac{2\pi}{\lambda}x)$
- If the charge has now an impedance R=150 Ω, find the amplitude of the reflected wave, the power of the incident, reflected and transmitted wave at the load.
  - $\Gamma = \frac{R - Z_c}{R + Z_c} = 0.5 = \frac{V_{refl}}{V_{inc}} \Longrightarrow V_{refl} = 0.5 \times 5 = 2.5V$
  - $P_{inc} = \frac{1}{2}\frac{V_{inc}^2}{R} = 0.083W$, $P_{refl} = \frac{1}{2}\frac{V_{refl}^2}{R} = |\Gamma|^2 P_{inc} = 0.0208W$, $P_{trans} = P_{inc} - P_{refl} = (1 - |\Gamma|^2)P_{inc} = 0.06225W$

Example 2

- We propose to study the properties of an RG58C/U coaxial cable whose characteristics are as follows:

1. Calculate the relative permittivity ε_r of the insulation used in this cable.
2. Calculate its linear inductance L and check the value of its characteristic impedance Zc.
3. Deduce the value of the propagation speed of the signal on the cable.
4. A section of l=10m of this cable is used to transport a signal of frequency f= 400 MHz. Knowing that the voltage at the cable input is written: e(t) = 5cos(ωt), give the expression for the voltage s(t) at the cable output.

- The inner conductor is made up of 19 strands and has a diameter of d1 = 0.8 mm
- The metal braid has a diameter d2 = 2.95 mm
- attenuation is 34 dB for 100m at 400 MHz and 20 dB/100m at 100 MHz
- The expression of the linear capacitance and inductance of a coaxial cable:

$$C=\frac{2\pi\varepsilon_0\varepsilon_r}{Ln(\frac{r_2}{r_1})} \qquad L=\frac{\mu_0}{2\pi}Ln(\frac{r_2}{r_1})$$

- The permittivity of the vacuum :

$$\varepsilon_0=\frac{1}{36\pi10^9}$$

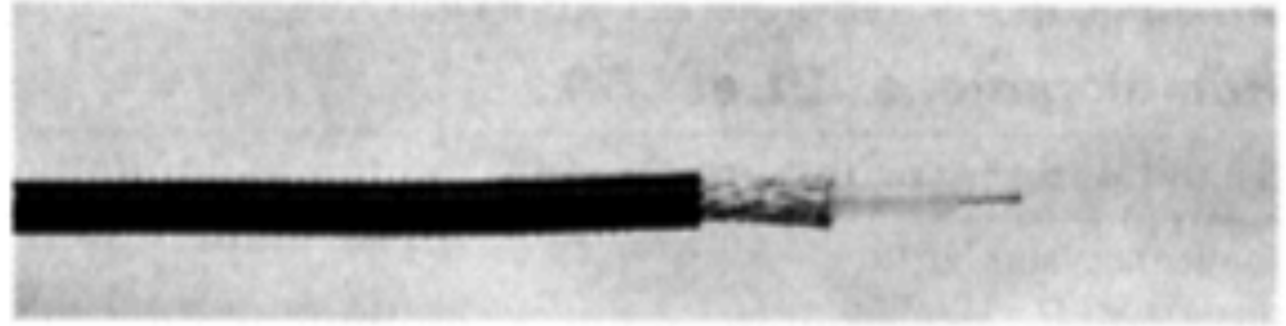
- The magnetic permeability of the vacuum :

$$\mu_0=4\pi10^{-7}$$

Type RG

RG 58C/U

Alcatel Câble - Filotex®



- ▶ Le câble RG 58C/U est composé d'une âme en cuivre étamé de 19 x 0,18 mm, d'un diélectrique en polyéthylène plein de diamètre extérieur 2,95 mm.
- ▶ La tresse en cuivre étamé est recouverte d'une gaine en PVC noir de diamètre 4,95 mm.

Homologation: MIL-C 17D

Spécifications techniques

Impédance: 50 Ω ±2 Ω
Capacité: 100 pF/m
Tension maximale: 1,4 kV
Affaiblissement: 34 dB/100 m à 400 MHz
Température d'utilisation: -40°C à +85°C

réf. Alcatel Câble	code commande	prix de l'U.D.V. 1-4	5-9	10+
U.D.V. = 1 bobine de 100m				
RG 58C/U	388-338	382,55	325,17	286,91
U.D.V. = 1 bobine de 305m				
RG 58C/U	252-5129	1165,05	990,29	873,79

Example 2

1. **Calculate the relative permittivity ε_r of the insulation used in this cable.**

$$C = 100pF/m,\ \epsilon_r = \frac{C \times ln(\frac{r_2}{r_1})}{2\pi\epsilon_0} = 2.34$$

2. **Calculate its linear inductance L and check the value of its characteristic impedance Zc.**

$$L = \frac{\mu_0}{2\pi} ln(\frac{r_2}{r_1}) = 2.6 \times 10^-7 H = 260nH,\ Z_c = \sqrt{\frac{L}{C}} = 50.99\Omega$$

3. **Deduce the value of the propagation speed of the signal on the cable.**

$$speed = \frac{C_0}{\sqrt{\epsilon_r}} = 1.96 \times 10^8 m/s$$

4. **A section of l=10m of this cable is used to transport a signal of frequency f= 100 MHz. Knowing that the voltage at the cable input is written: e(t) = 5cos(ωt) V, give the expression for the voltage s(t) at the cable output.**

The signal s(t) is attenuated and delayed.

$s(t) = S_{max} cos(\omega(t - t_0))$, **where** $S_{max} = \frac{E_{max}}{10^{att/20}} = 3.38V$

(att=34dB*10m/100m=3.4 dB at 400 MHz)

and $t_0 = \frac{l}{speed} = 5.1 \times 10^{-8} s = 51ns,$

so $s(t) = 3.38cos(\omega t - 10.2\pi) = 3.38cos(\omega t - 0.2\pi)V$

- The inner conductor is made up of 19 strands and has a diameter of d1 = 0.8 mm
- The metal braid has a diameter d2 = 2.95 mm
- attenuation is 34 dB for 100m at 400 MHz and 20 dB/100m at 100 MHz
- The expression of the linear capacitance and inductance of a coaxial cable:

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{Ln(\frac{r_2}{r_1})} \qquad L = \frac{\mu_0}{2\pi} Ln(\frac{r_2}{r_1})$$

- The permittivity of the vacuum :

$$\varepsilon_0 = \frac{1}{36\pi 10^9}$$

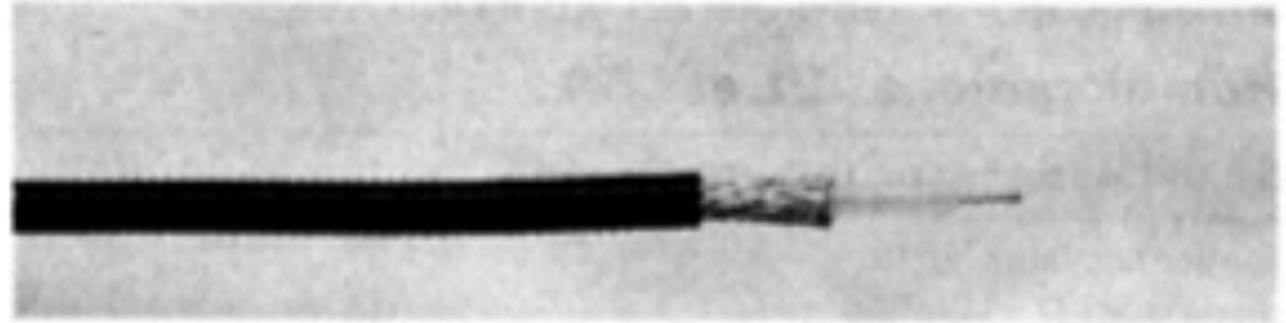
- The magnetic permeability of the vacuum :

$$\mu_0 = 4\pi 10^{-7}$$

Type RG

RG 58C/U

Alcatel Câble - Filotex®



- Le câble RG 58C/U est composé d'une âme en cuivre étamé de 19 x 0,18 mm, d'un diélectrique en polyéthylène plein de diamètre extérieur 2,95 mm.
- La tresse en cuivre étamé est recouverte d'une gaine en PVC noir de diamètre 4,95 mm.

Homologation: MIL-C 17D

Spécifications techniques

Impédance: 50 Ω ± 2 Ω
Capacité: 100 pF/m
Tension maximale: 1,4 kV
Affaiblissement: 34 dB/100 m à 400 MHz
Température d'utilisation: -40°C à +85°C

U.D.V. = 1 bobine de 100m				
réf.	**code Alcatel Câble commande**	**prix de l'U.D.V. 1-4**	**5-9**	**10+**
RG 58C/U	**388-338**	**382,55**	**325,17**	**286,91**
U.D.V. = 1 bobine de 305m				
RG 58C/U	**252-5129**	**1165,05**	**990,29**	**873,79**

Voltage standing wave ratio

- Using $V^- = \Gamma V^+$, the total voltage and current become

$$\tilde{V}(z) = V_0^+(e^{-j\beta z} + \Gamma e^{+j\beta z})$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma e^{+j\beta z})$$

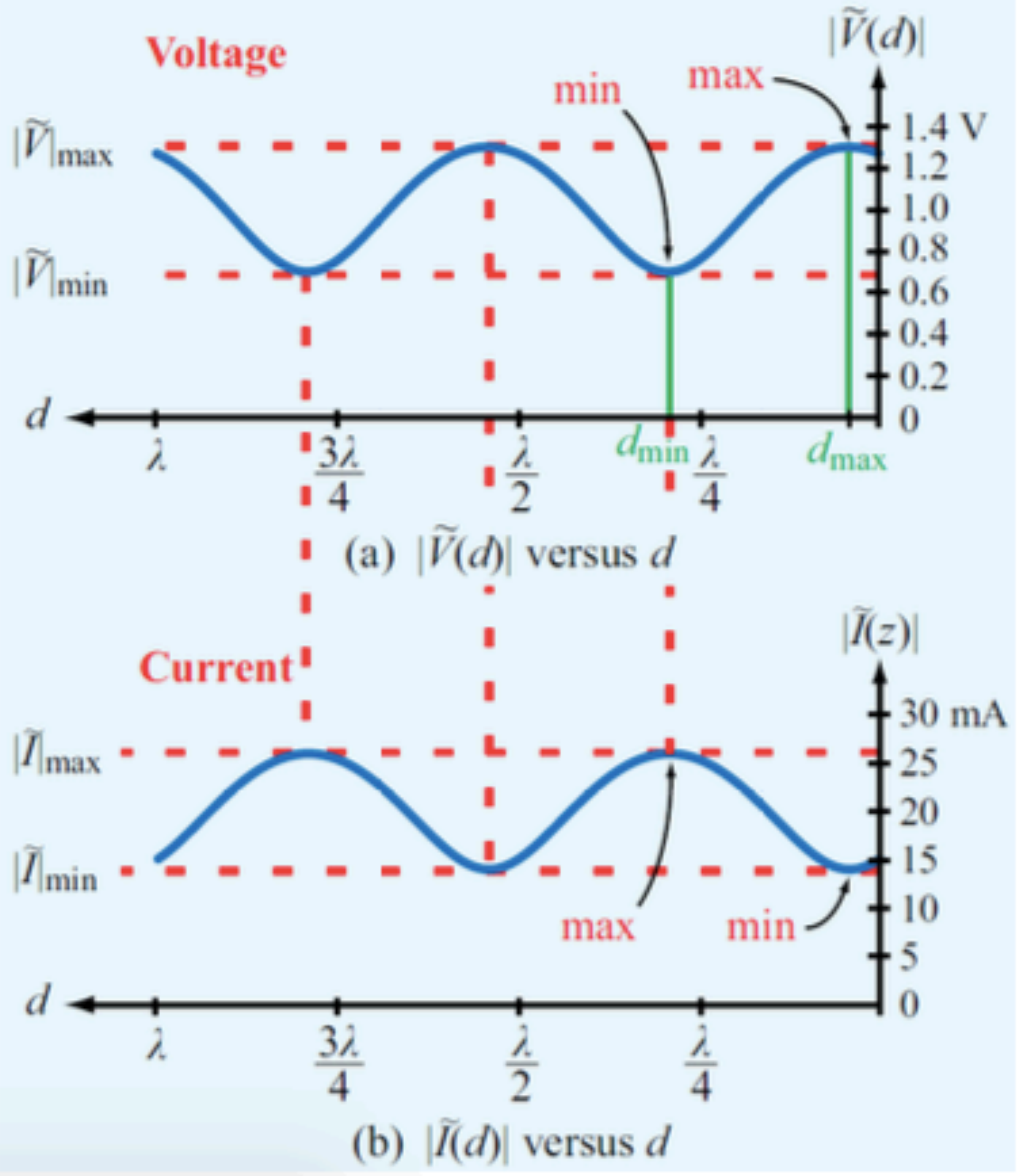
- Replacing z with $-d$, Magnitude of $\tilde{V}(d)$ and $\tilde{I}(d)$

$$|\tilde{V}(d)| = |V_0^+|[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_r)]^{1/2}$$

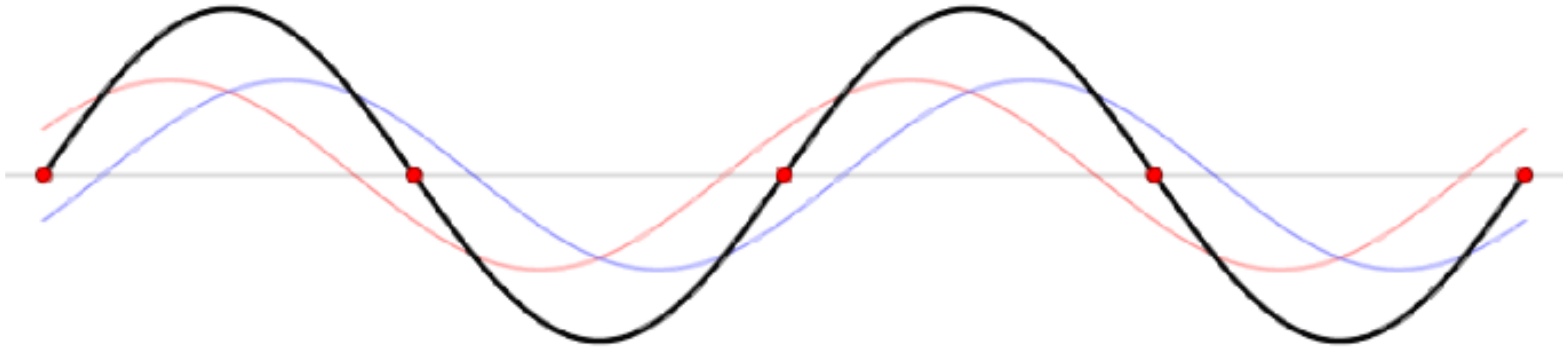
$$|\tilde{I}(d)| = \frac{|V_0^+|}{Z_0}[1 + |\Gamma|^2 - 2|\Gamma|\cos(2\beta d - \theta_r)]^{1/2}$$

- The sinusoidal patterns are called standing waves and are caused by the interference of the two traveling waves.
  - Maximum values when $2\beta d - \theta_r = 2n\pi$
  - Minimum values when $2\beta d - \theta_r = (2n+1)\pi$
- With no reflected wave present, there are no interference and no standing waves.

Standing-wave pattern for (a) $|\tilde{V}(d)|$ and (b) $|\tilde{I}(d)|$ for a lossless transmission line of characteristic impedance $Z_0 = 50$ Ω, terminated in a load with a reflection coefficient $\Gamma = 0.3e^{j30°}$. The magnitude of the incident wave $|V_0^+| = 1$ V. The standing-wave ratio is $S = |\tilde{V}|_{max}/|\tilde{V}|_{min} = 1.3/0.7 = 1.86$.

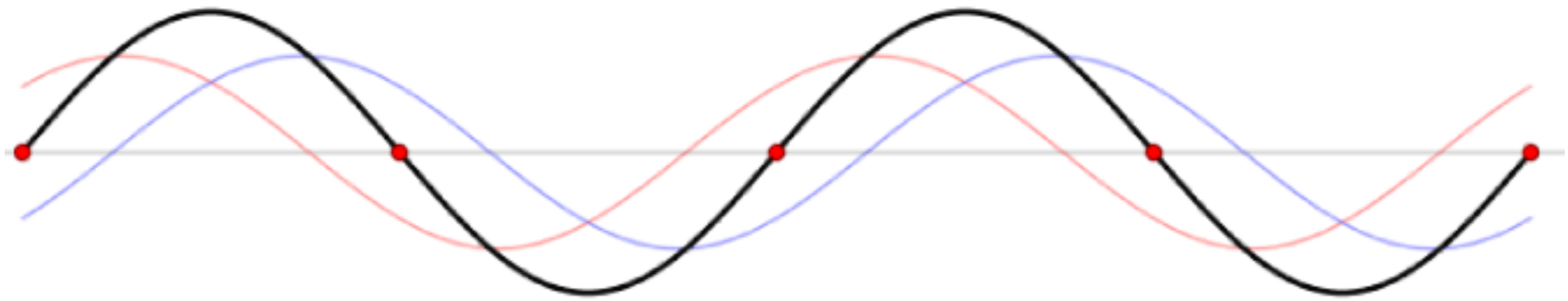


Standing wave



A **standing wave** is the phenomenon resulting from the simultaneous propagation in different directions of several waves of the same frequency in the same physical environment, which form a figure elements of which are fixed in time.

Standing wave



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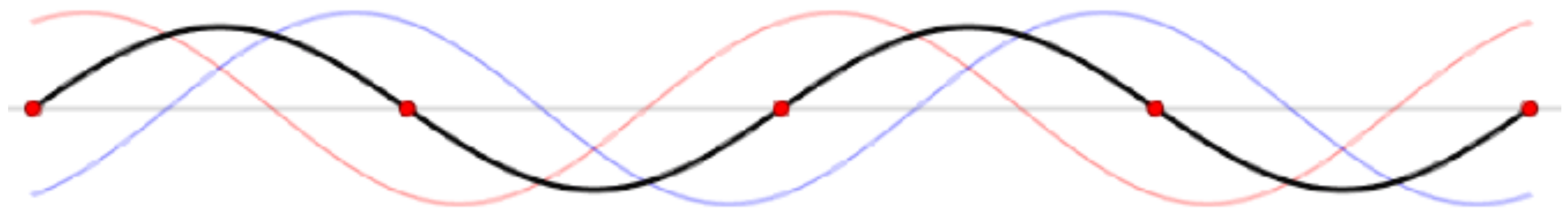
Standing wave



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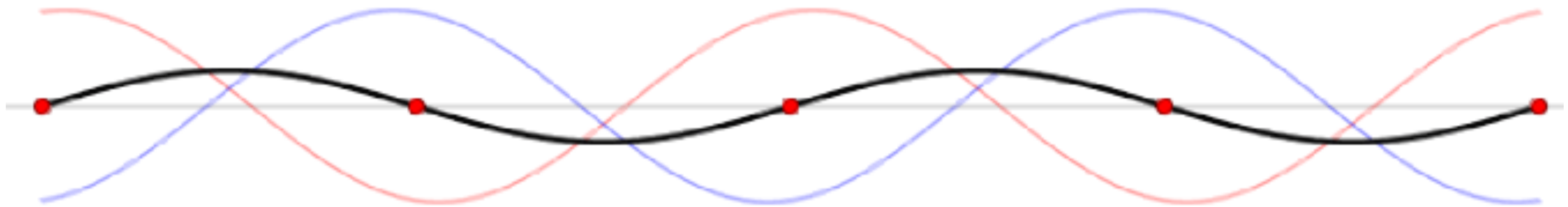

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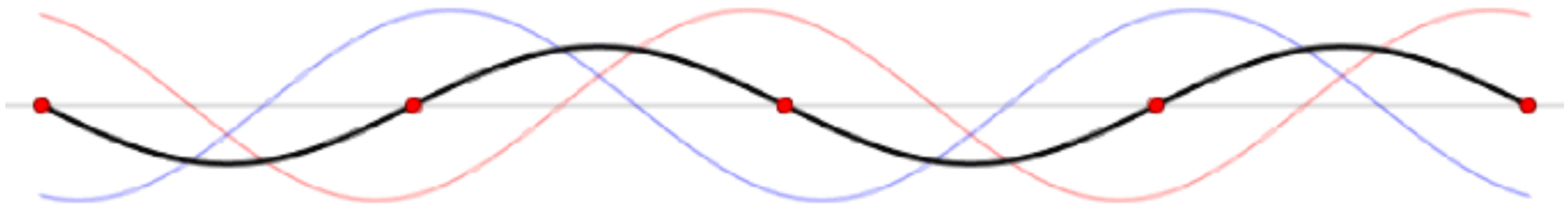
Standing wave



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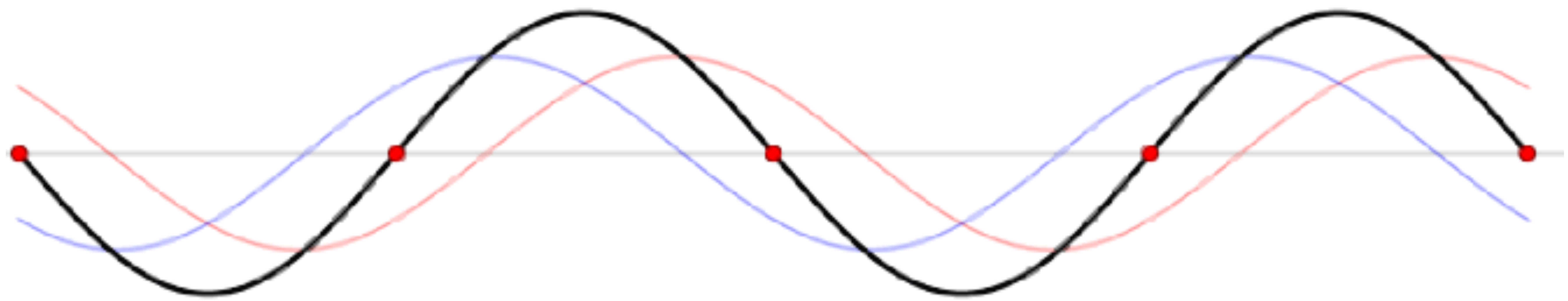
Standing wave



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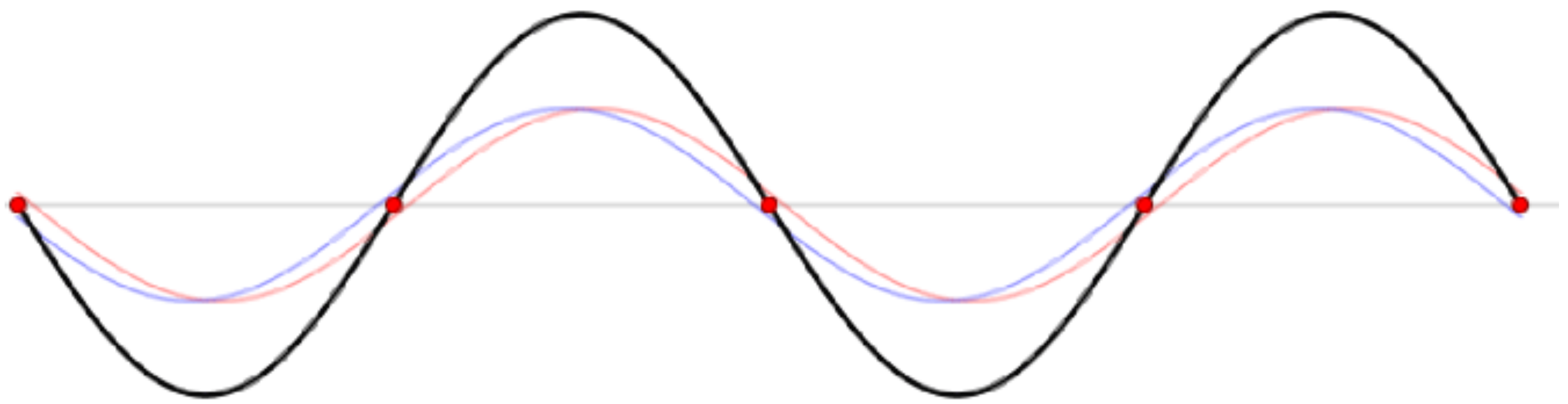
Standing wave



A **standing wave** is the phenomenon resulting from the simultaneous propagation in different directions of several waves of the same frequency in the same physical environment, which form a figure elements of which are fixed in time.

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Voltage standing wave ratio (VSWR)

$$|\tilde{V}(d)| = |V_0^+|[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_r)]^{1/2}$$

$$|\tilde{I}(d)| = \frac{|V_0^+|}{Z_0}[1 + |\Gamma|^2 - 2|\Gamma|\cos(2\beta d - \theta_r)]^{1/2}$$

- **Maximum and minimum values of $|\tilde{V}(d)|$**

$$|\tilde{V}|_{max} = |\tilde{V}(d_{max})| = |V_0^+|[1 + |\Gamma|] \quad \text{at } 2\beta d_{max} - \theta_r = 2n\pi$$

$$|\tilde{V}|_{min} = |\tilde{V}(d_{min})| = |V_0^+|[1 - |\Gamma|] \quad \text{at } 2\beta d_{min} - \theta_r = (2n+1)\pi$$

- **VSWR:** $\boxed{S = \frac{|\tilde{V}|_{max}}{|\tilde{V}|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}}$

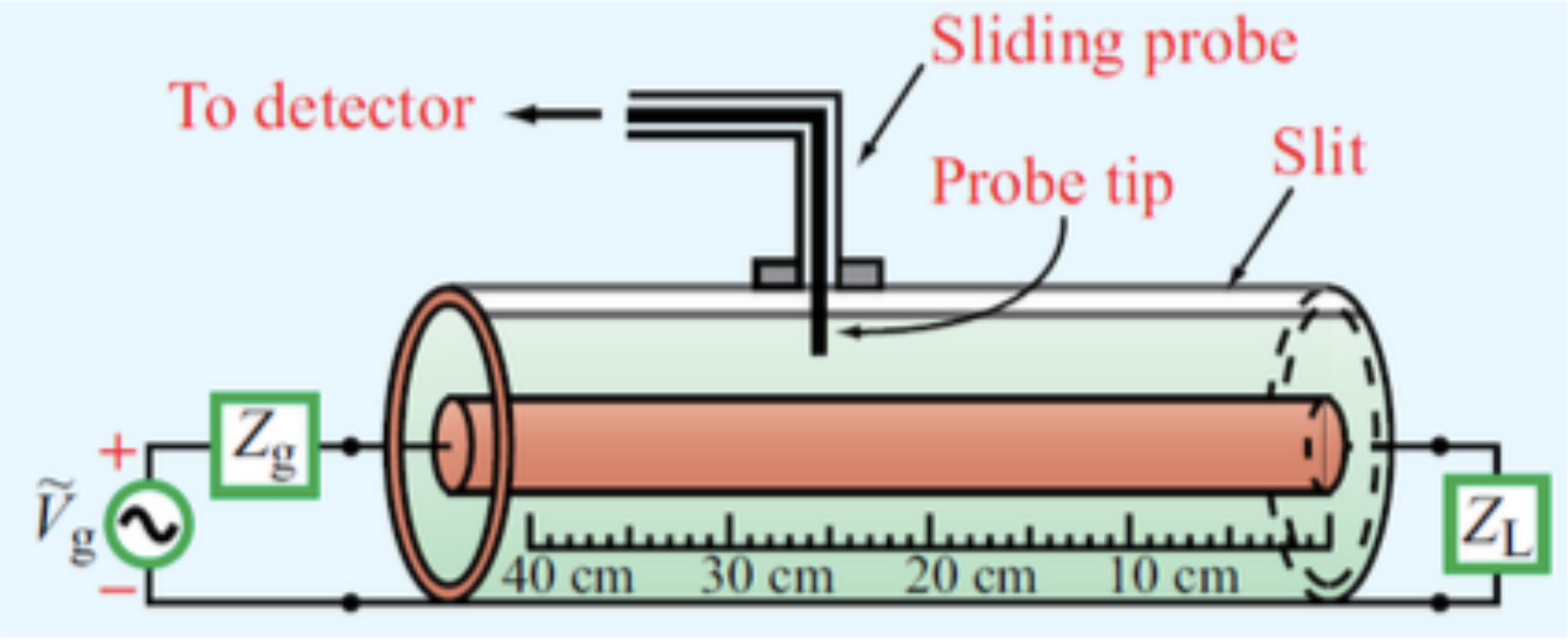
A measure of the mismatch between the load and the transmission line

For a matched load with $\Gamma = 0$, $S = 1$, and for a line with $|\Gamma| = 1$, $S = \infty$.

Measuring Z_L using slotted-line probe

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} \qquad \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Measure ↑ (S) — Known ↑ (Z_0)



Wave impedance of lossless lines

- **Since the voltage and current magnitudes are oscillatory with position along the line and in phase opposition with each other, the wave impedance $Z(d)$ must vary with position also.**

$$Z(d) = \frac{\tilde{V}(d)}{\tilde{I}(d)} = \frac{V_0^+(e^{+j\beta d} + \Gamma e^{-j\beta d})}{V_0^+(e^{+j\beta d} - \Gamma e^{-j\beta d})} Z_0 = Z_0 \left[\frac{1 + \Gamma e^{-j2\beta d}}{1 - \Gamma e^{-j2\beta d}}\right] = Z_0 \left[\frac{1 + \Gamma_d}{1 - \Gamma_d}\right]$$

- **Phase-shifted voltage reflection coefficient**

$$\Gamma_d = \Gamma e^{-j2\beta d} = |\Gamma| e^{j\theta_r} e^{-j2\beta d} = |\Gamma| e^{j(\theta_r - 2\beta d)}$$

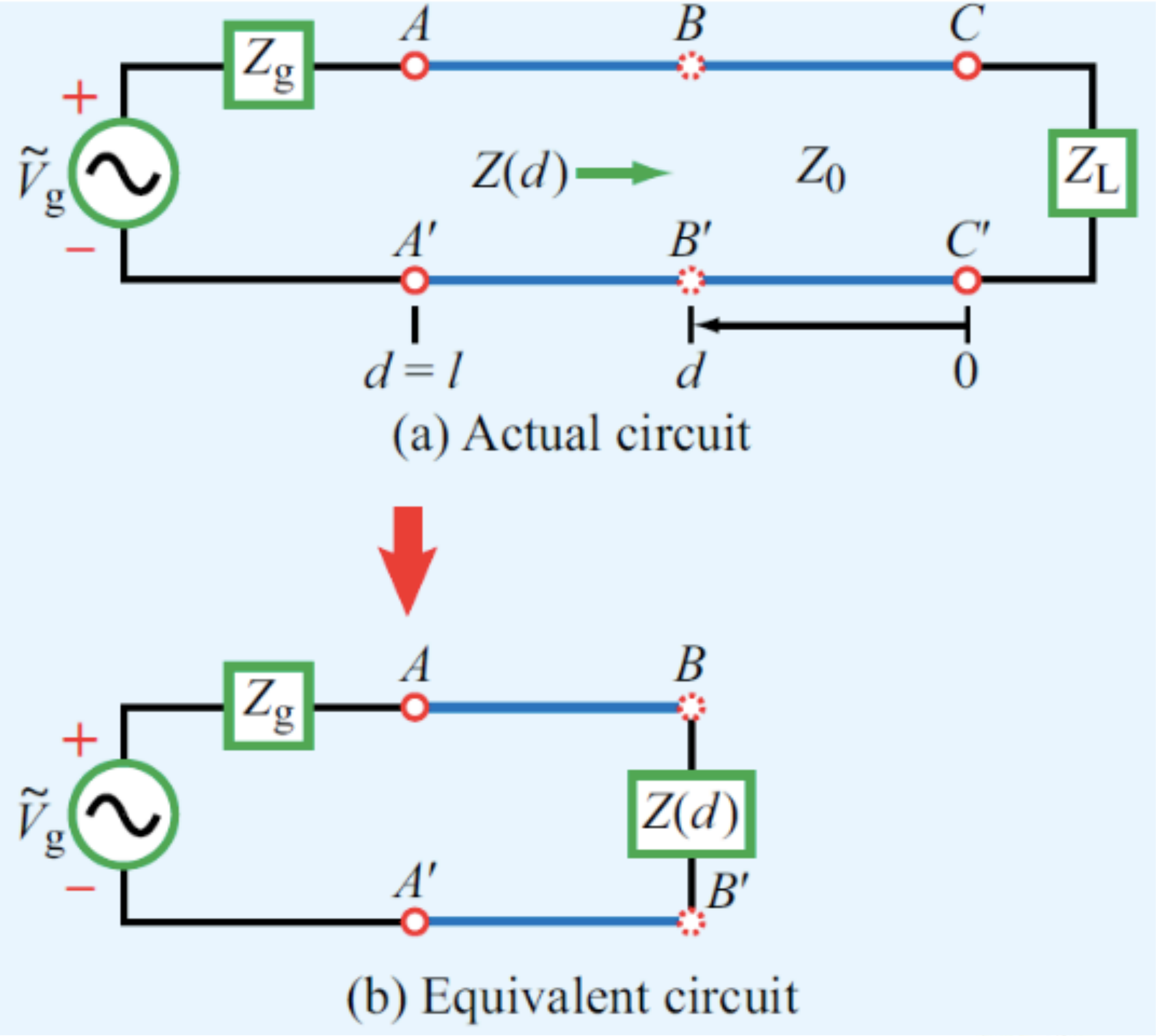
  - Γ_d has the same magnitude as Γ, but its phase is shifted by $2\beta d$ relative to that of Γ.

- **$Z(d)$ is the ratio of the total voltage (incident and reflected-wave voltages) to the total current at any point d on the line, in contrast with the characteristic impedance of the line Z_0, which relates the voltage and current of each of the two waves individually ($Z = V^+/I^+ = -V^-/I^-$).**

Input impedance

- In the actual circuit (a), at terminals BB' at an arbitrary location d on the line, $Z(d)$ is the wave impedance of the line when "looking" to the right (i.e., towards the load).

- Application of the equivalence principle allows us to replace the segment to the right of terminals BB' with a lumped impedance of value $Z(d)$.



Input impedance

- **Of particular interest in many transmission-line problems is the input impedance at the source end of the line, at $d = l$, which is given by:**

$$Z_{in} = Z(l) = Z_0 \left[\frac{1+\Gamma_l}{1-\Gamma_l}\right]$$

Where, $\Gamma_l = \Gamma e^{-j2\beta l} = |\Gamma| e^{j(\theta_r - j2\beta l)}$

Using $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

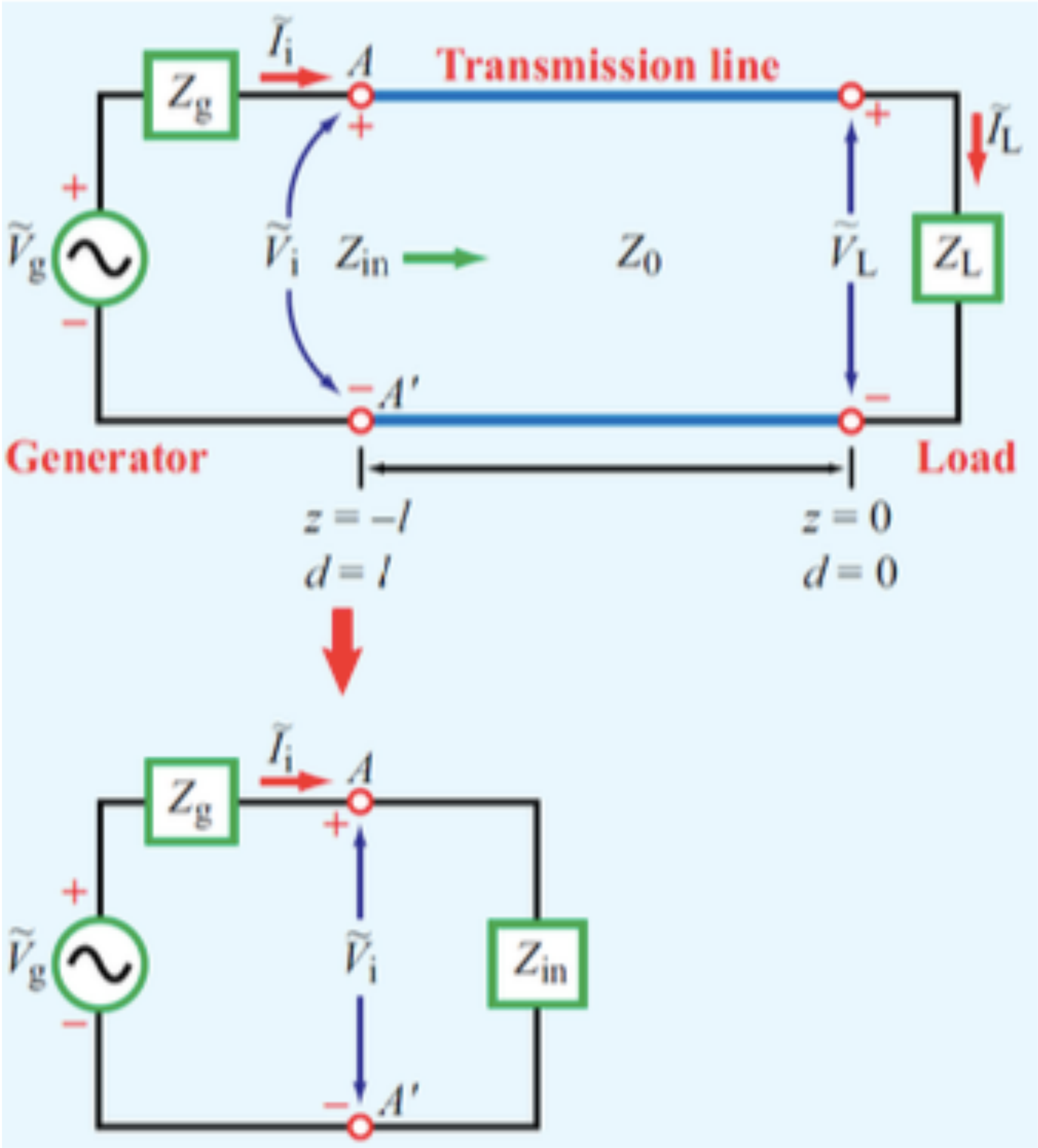
We obtain the input impedance

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]}$$

Final solution is

$$\tilde{V}_i = \tilde{I}_i Z_{in} = \frac{Z_{in}}{Z_g + Z_{in}} \tilde{V}_g \qquad = \tilde{V}(-l)$$

- $$\boxed{V_0^+ = \left(\frac{Z_{in}}{Z_g + Z_{in}} \tilde{V}_g\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right)}$$



Input impedance of short-circuited line

Since $Z_L = 0$, the input impedance is

$$Z_{in}^{sc} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] = jZ_0 \tan\beta l$$

Purely reactive

$$\tilde{V}(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{+j\beta z})$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} (e^{-j\beta z} - \Gamma e^{+j\beta z})$$

- $Z_L = 0 \rightarrow \Gamma = -1$

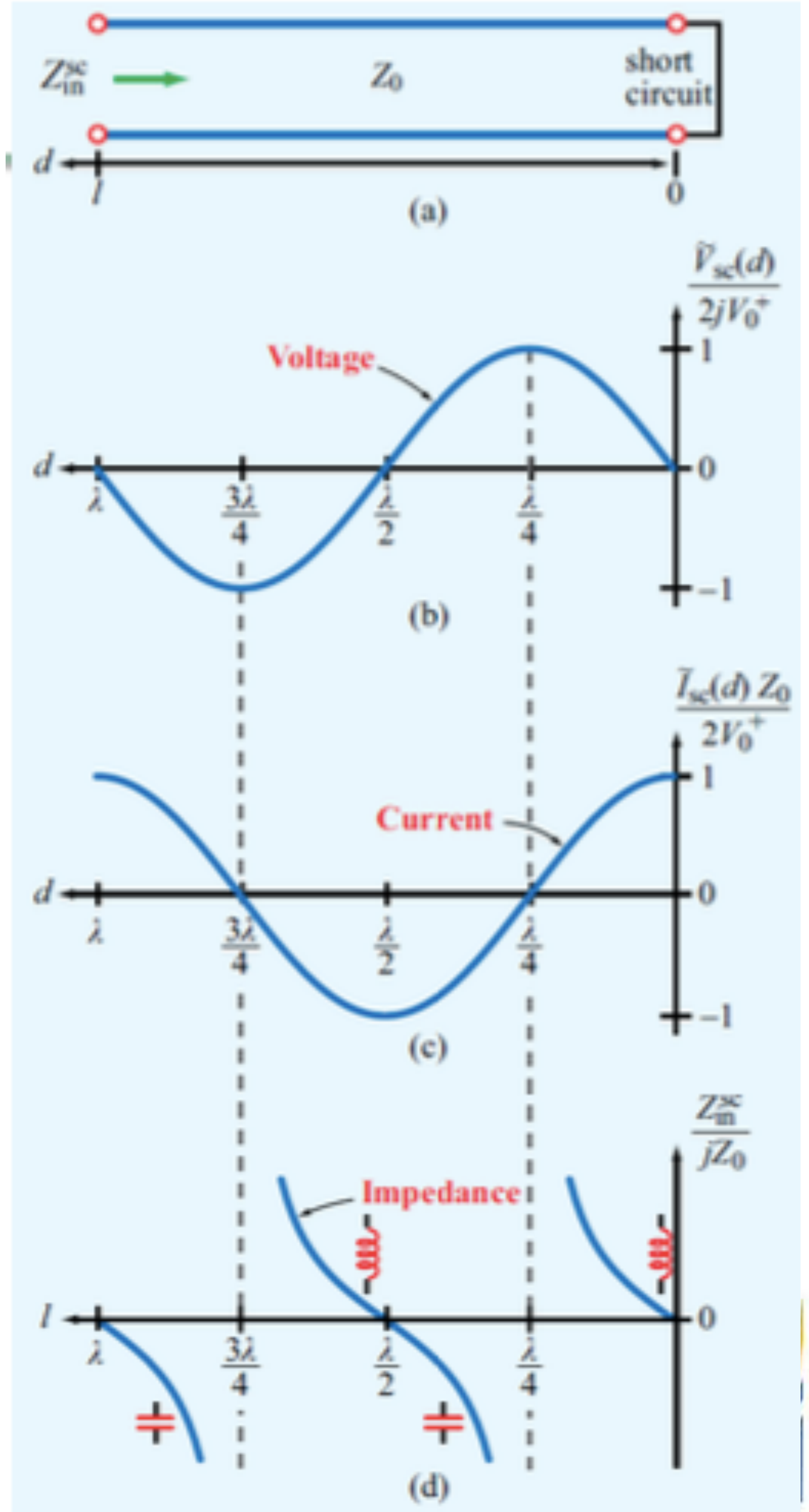
If $\tan\beta l \geq 0$, the line appears inductive to the source, acting like an equivalent inductor L_{eq}

$$L_{eq} = \frac{Z_0 \tan\beta l}{\omega}$$

If $\tan\beta l \leq 0$, the input impedance is capacitive, in which case the line acts like an equivalent capacitor with capacitance C_{eq}

$$C_{eq} = -\frac{1}{\omega Z_0 \tan\beta l}$$

Through proper choice of the length of a short-circuited line, we can make them into equivalent capacitors and inductors of any desired reactance.



Input impedance of open-circuited line

Since $Z_L = \infty$, the input impedance of an open-circuited line is

$$Z_{in}^{oc} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] = -jZ_0 \cot\beta l$$

A network analyzer is a radio-frequency (RF) instrument capable of measuring the impedance of any load connected to its input terminal.

The combination of the two measurements (Z_{in}^{sc} and Z_{in}^{oc}) can be used to determine the characteristic impedance of the line Z_0 and its phase constant β.

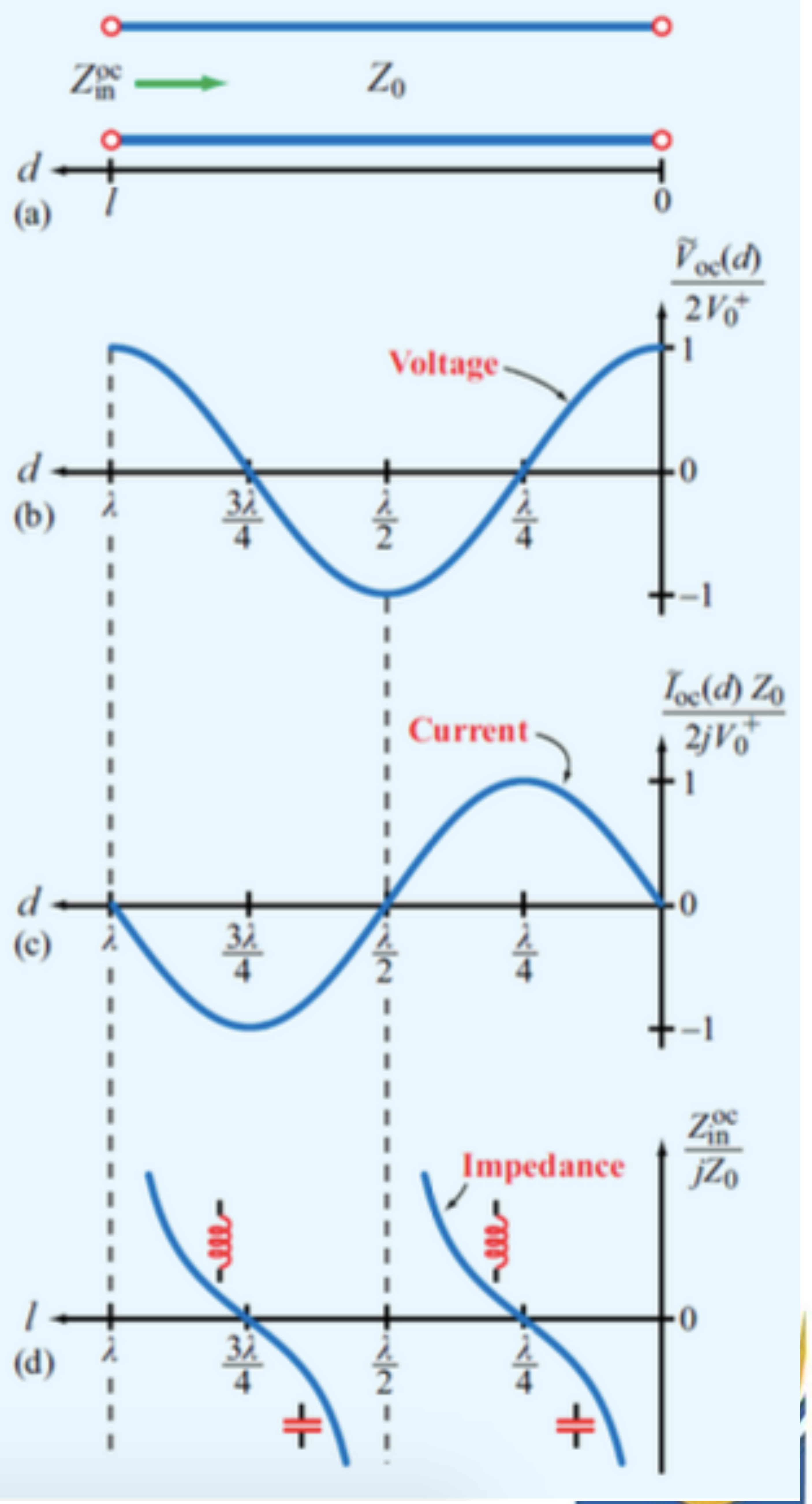
$$Z_0 = \sqrt{Z_{in}^{sc} Z_{in}^{oc}}$$

$$\tan\beta l = \sqrt{-\frac{Z_{in}^{sc}}{Z_{in}^{oc}}}$$

$$\tilde{V}(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{+j\beta z})$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} (e^{-j\beta z} - \Gamma e^{+j\beta z})$$

- $Z_L = \infty \rightarrow \Gamma = 1$



Problem 1

1. Distributed Parameters Model

RG-223/U coax has an inner conductor radius $a = 0.47$ mm and inner radius of the outer conductor $b = 1.435$ mm. The conductor is copper, and polyethylene is the dielectric. Calculate the distributed parameters at 800 MHz.

for copper: $\sigma_{Cu} = 5.8x10^7 \dfrac{S}{m}$

for polyethylene: $\varepsilon_r = 2.26, \sigma = 10^{-16} \dfrac{S}{m}$

Parameter	Coaxial
R'	$\frac{R_S}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$
L'	$\frac{\mu}{2\pi}\ln(b/a)$
G'	$\frac{2\pi\sigma}{\ln(b/a)}$
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$

$$R_S = \sqrt{\pi f \mu_c / \sigma_c}$$

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Problem 1: Solution

$$R' = \frac{1}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)\sqrt{\frac{\pi f \mu}{\sigma_c}}$$

$$= \frac{1}{2\pi}\left(\frac{1}{0.47x10^{-3}}+\frac{1}{1.435x10^{-3}}\right)\sqrt{\frac{\pi\left(800x10^{6}\right)\left(4\pi x10^{-7}\right)}{\left(5.8x10^{7}\right)}} = 3.32\frac{\Omega}{m}$$

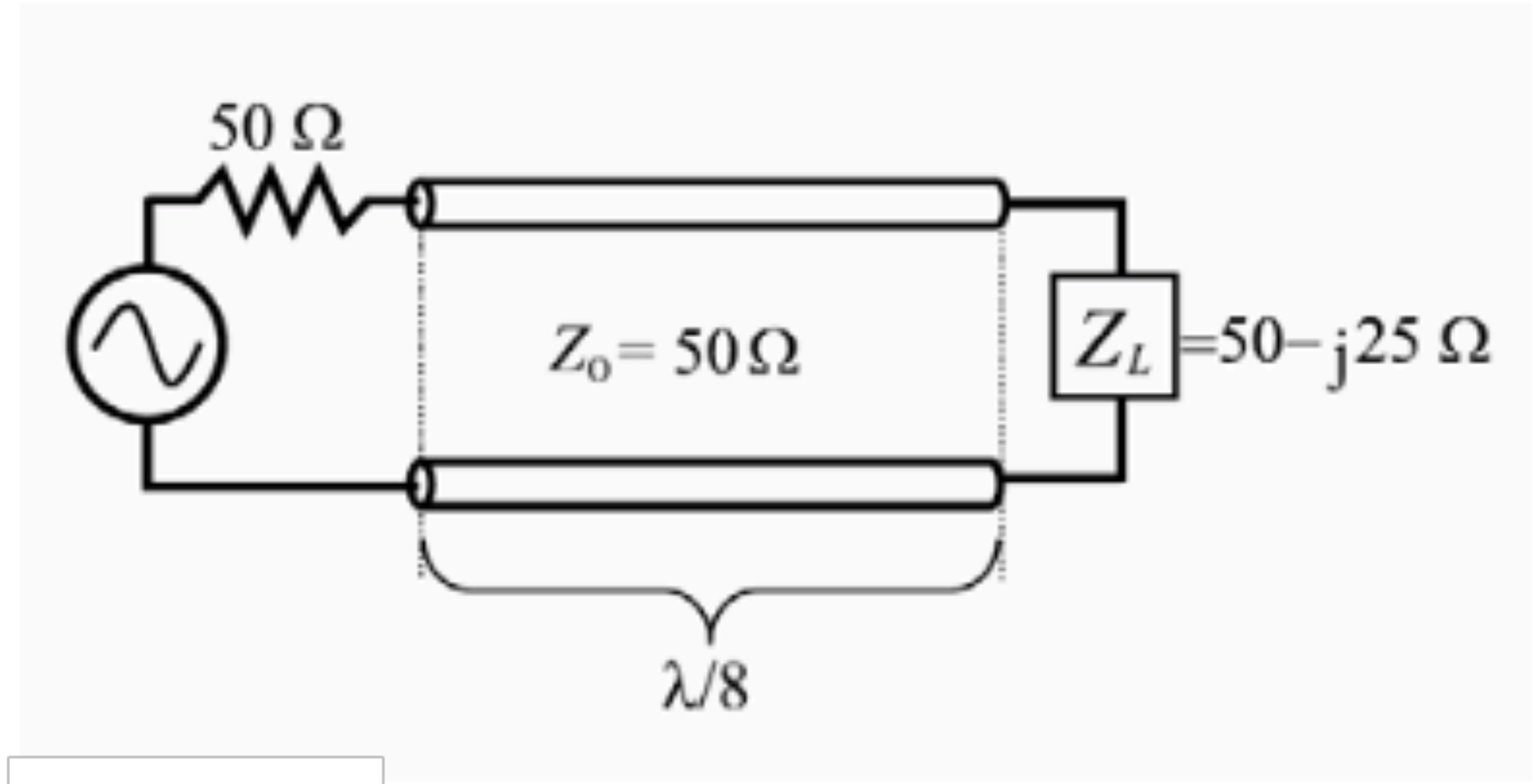
$$L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{4\pi x10^{-7}}{2\pi}\ln\left(\frac{1.435}{0.47}\right) = 223\frac{nH}{m}$$

$$G' = \frac{2\pi\sigma}{\ln\left(b/a\right)} = \frac{2\pi\left(10^{-16}\right)}{\ln\left(1.435/0.47\right)} = 560x10^{-18}\frac{S}{m \approx 0}$$

$$C' = \frac{2\pi\varepsilon}{\ln\left(b/a\right)} = \frac{2\pi\left(2.26\right)\left(8.854x10^{-12}\right)}{\ln\left(1.435/0.47\right)} = 112\frac{pF}{m}$$

Problem 2

A source with 50 Ω source impedance drives a 50 Ω T-Line that is 1/8 of a wavelength long, terminated in a load $Z_L = 50 - j25\ \Omega$. Calculate Γ_L, *VSWR*, and the input impedance seen by the source.



Problem 2: Solution

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{50 - j25 - 50}{50 - j25 + 50} = 0.242 e^{-j76^\circ}$$

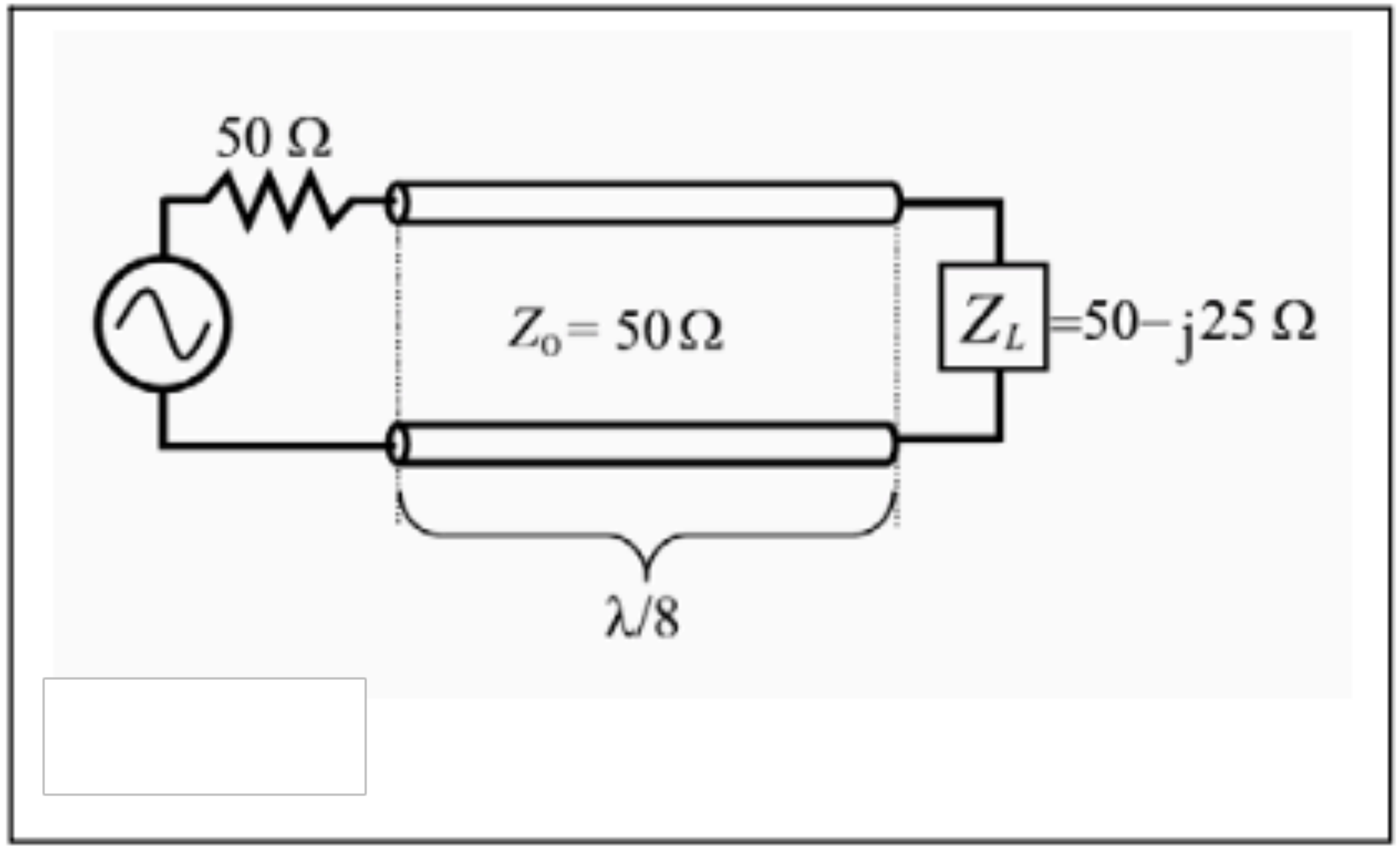
$$VSWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = 1.64$$

$$\beta \ell = \frac{2\pi}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4}, \quad \tan\left(\frac{\pi}{4}\right) = 1$$

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)}$$

$$= 50 \frac{50 - j25 + j50}{50 + j50 + 25}$$

$$= 30.8 - j3.8\ \Omega$$



Problem 3

The reflection coefficient at the load for a 50 Ω line is measured as $\Gamma_L = 0.516e^{j8.2°}$ at $f = 1$ GHz. Find the equivalent circuit for Z_L.

Problem 3: Solution

Rearranging $\Gamma_L = \dfrac{Z_L - Z_o}{Z_L + Z_o}$, we find $Z_L = Z_o \dfrac{1+\Gamma_L}{1-\Gamma_L} = 150 + j30\ \Omega$.

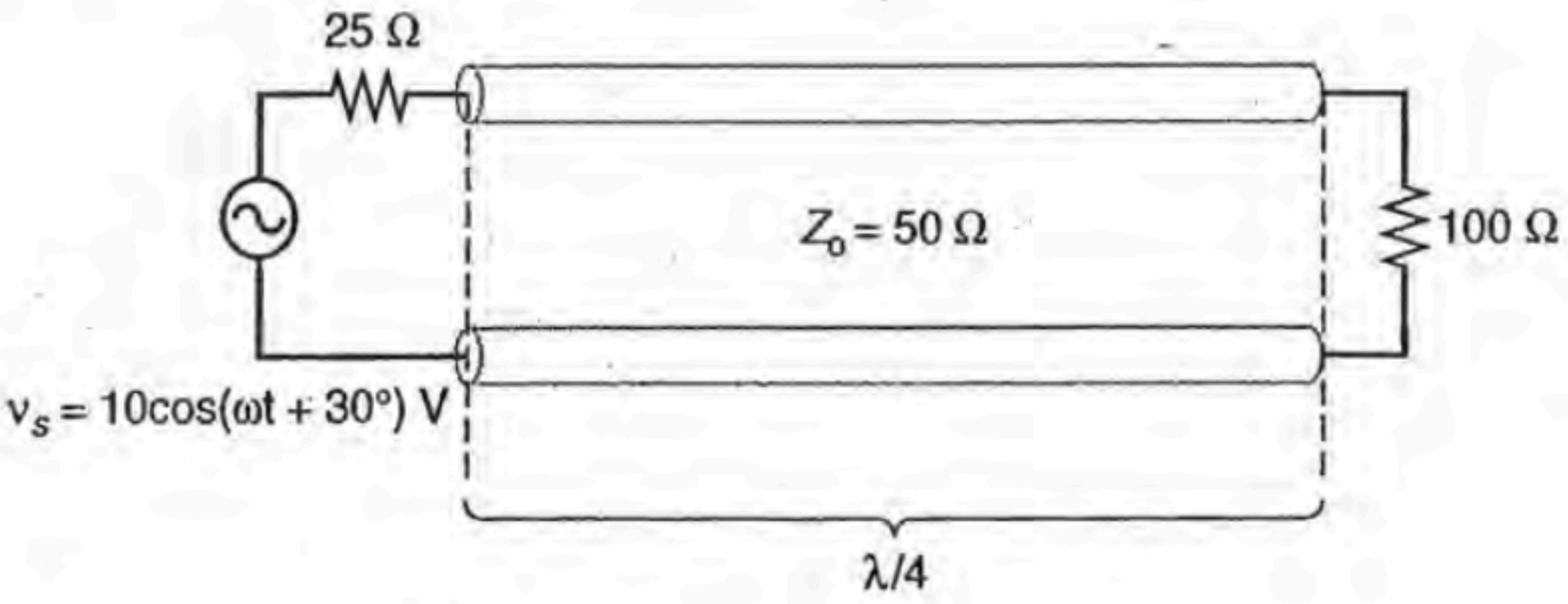
This is a resistor in series with an inductor. The inductor is found by considering

$j\omega L = j30$, or $L = \dfrac{30}{2\pi\left(1x10^9\right)} = 4.8nH$,

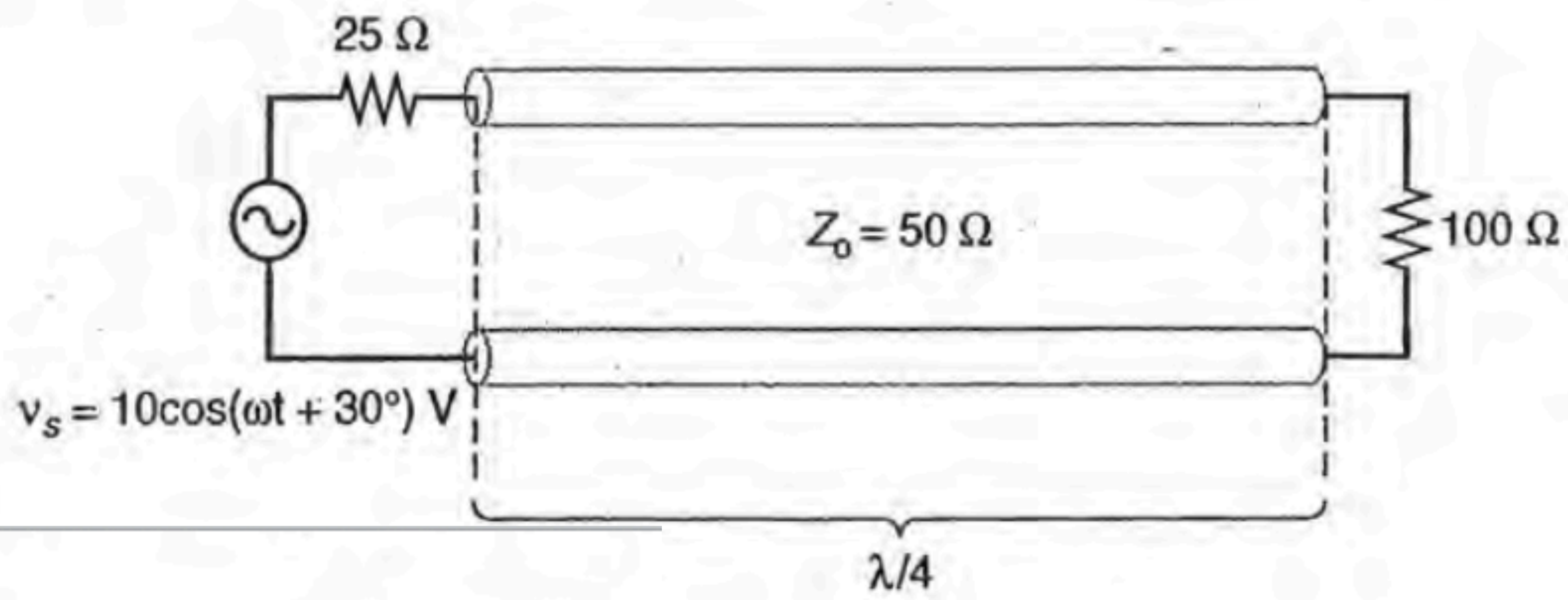
So the load is a 150 Ω resistor in series with a 4.8 nH inductor.

Problem 4

- Consider the lossless T-line circuit of Figure below. We want to find the voltage across the 100-Ω load.



Problem 4: Solution



To begin, the source voltage is converted to its phasor,

$$V_{SS} = 10e^{j30°}V$$

To find V_L, we need Γ_L and V_o^+. We have $V_L = V_s(z=0) = V_o^+(1+\Gamma_L)$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{100\ \Omega - 50\ \Omega}{100\ \Omega + 50\ \Omega} = \frac{1}{3}$$

Finding V_o^+ requires that we know Z_{in}, which we can find for a lossless T-line,

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta\ell)}{Z_o + jZ_L \tan(\beta\ell)}$$

Since in this example

$$\beta\ell = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

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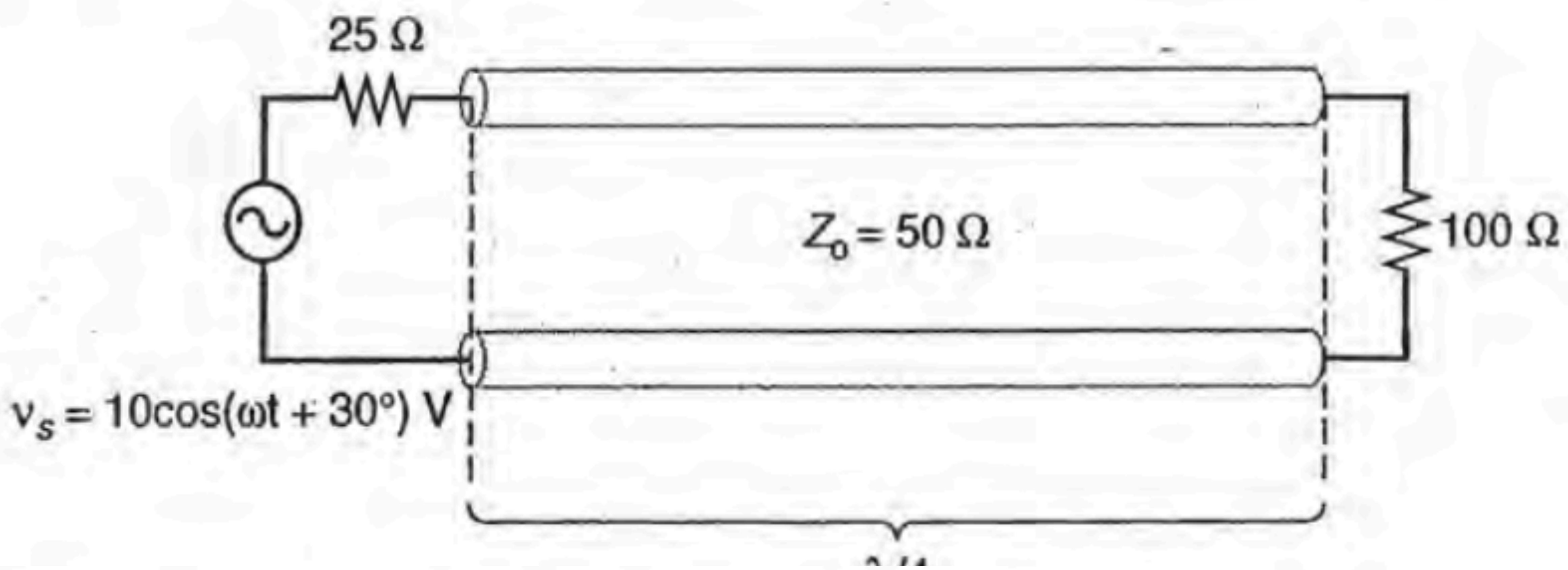
Problem 4: Solution



The equation for Z_{in} reduces to

$$\tan\beta\ell = \tan\frac{\pi}{2} = \infty \qquad Z_{in} = \frac{Z_o^2}{Z_L} = 25\ \Omega$$

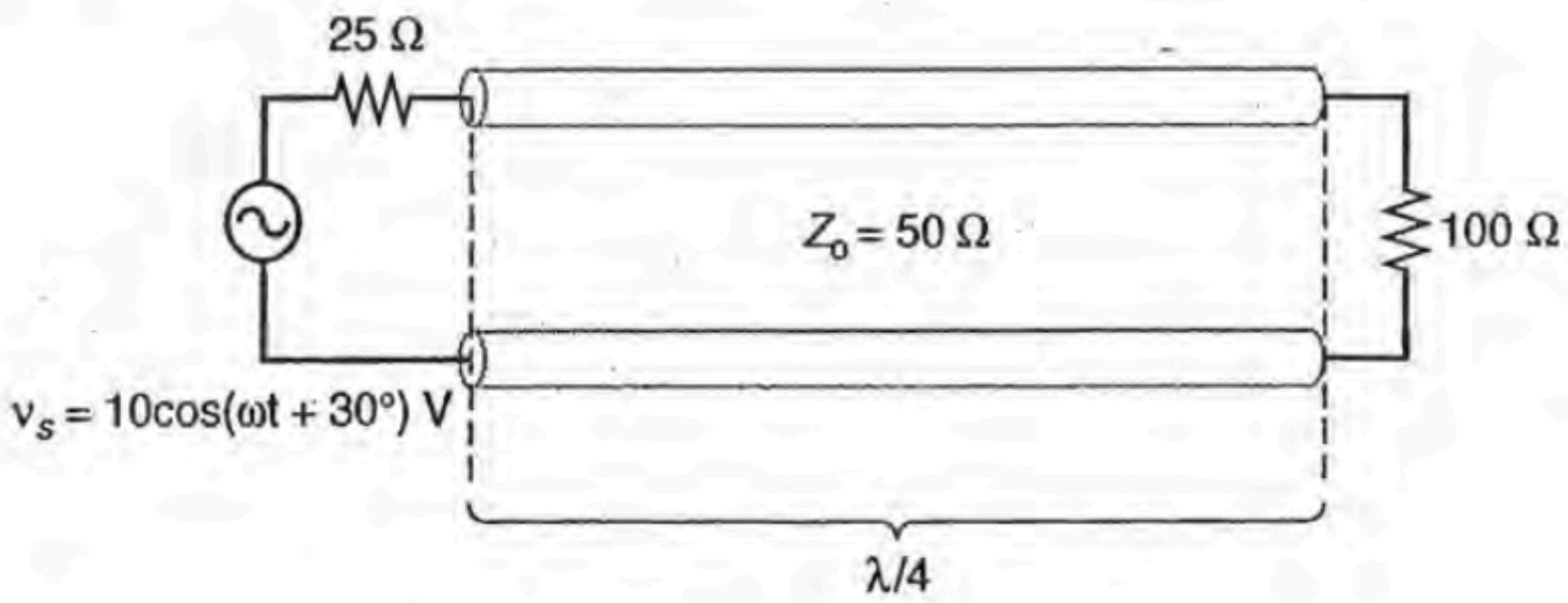
Now we have for the input voltage

$$V_{in} = V_{ss}\frac{Z_{in}}{Z_s + Z_{in}} = 10e^{j30°}\frac{25}{25+25} = 5e^{j30°}\ \text{V}$$

To evaluate for V_o^+ we also need $e^{\gamma\ell}$ and $e^{-\gamma\ell}$. Euler's equation can be used to convert $e^{\gamma\ell}$ and $e^{-\gamma\ell}$ into complex numbers in rectangular coordinates if so desired. For the lossless case, $\gamma = j\beta$ and $\gamma\ell = j\beta\ell = j\pi/2$. So

$$V_o^+ = \frac{V_{in}}{e^{+\gamma\ell} + \Gamma_L e^{-\gamma\ell}} = \frac{5e^{j30°}}{e^{j90°} + e^{-j90°}/3} = 7.5e^{-j60°}\ \text{V}$$

Problem 4: Solution



Finally,

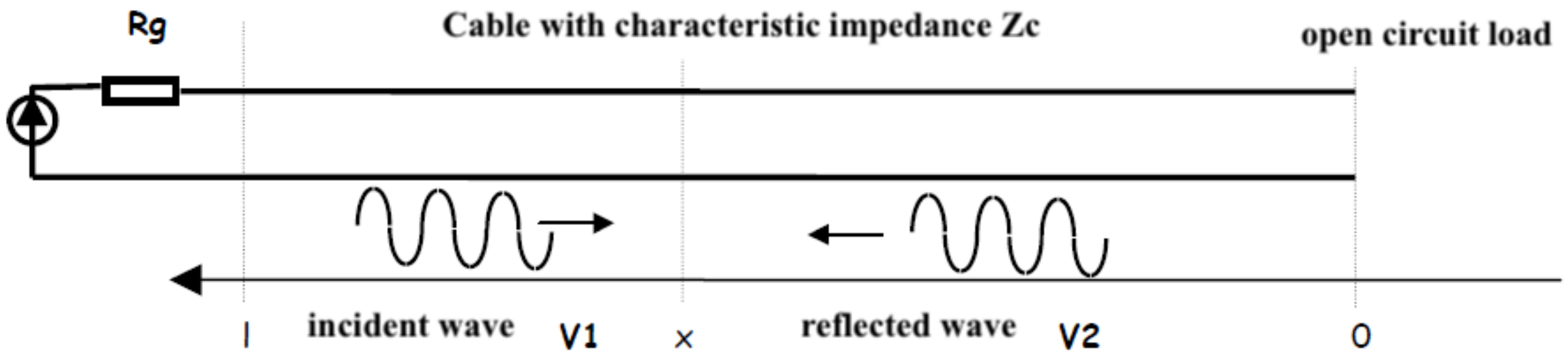
$$V_L = 7.5e^{-j60°}\left(1+\tfrac{1}{3}\right) = 10e^{-j60°}\ \text{V}$$

Converting this phasor to its instantaneous form gives us the voltage across the load,

$$v_L = 10\cos(\omega t - 60°)\text{V}$$

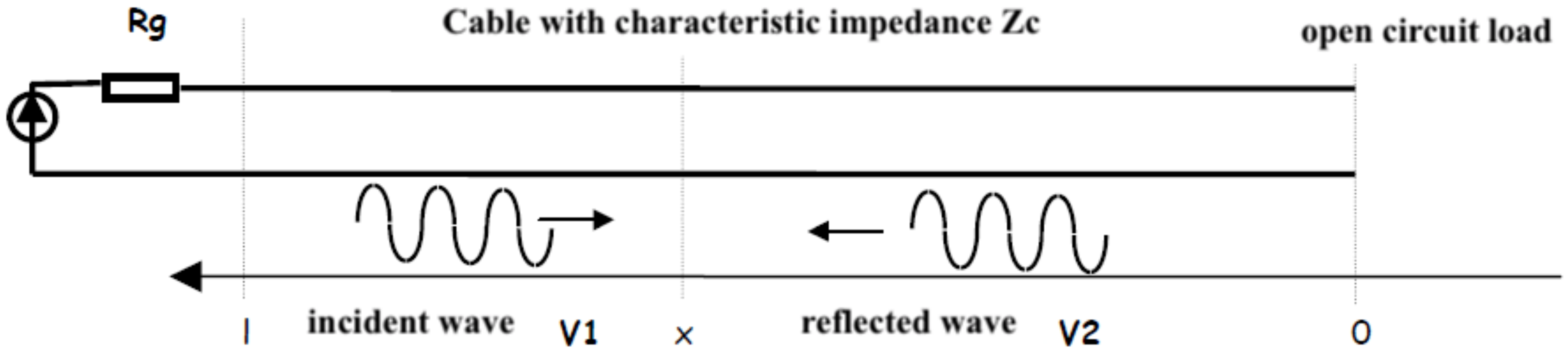
Problem 5:

- We consider the following circuit consisting of a section of transmission line of length l = 1 m ended with an open circuit and powered by a sinusoidal voltage.



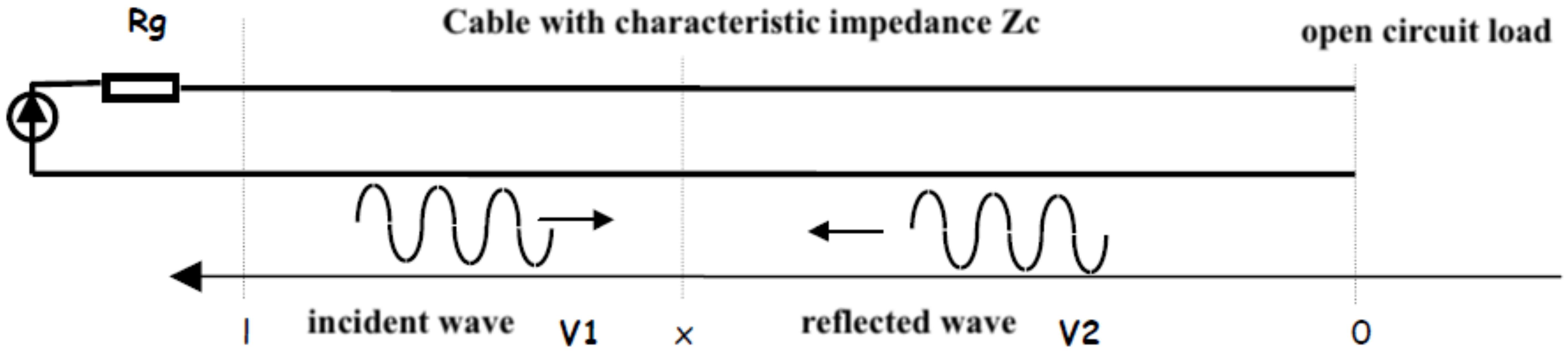
- Voltage and current at a point of abscissa x are then written:
- $v(x,t) = V_1\cos(\omega t + kx) + V_2\cos(\omega t - kx)$
- $i(x,t) = (V_1/Z_c)\cos(\omega t + kx) - (V_2/Z_c)\cos(\omega t - kx)$
- Where $k = \omega/v$ and the speed of propagation on this cable is v = 200000 km/s.
- 1- Find the reflection coefficient. In this case, simplify the expressions for v(x,t) and i(x,t).
- 2-Deduce the modulus Ze of the line input impedance as a function of Zc, f, l and v.

Problem 5: Solution



- **1)** $\Gamma_L = \dfrac{Z_L - Z_c}{Z_L + Z_c} = 1$ **since** $Z_L = \infty$ **and we have:** $\Gamma_L = \dfrac{V_2}{V_1}$ **so** $V_2 = V_1$**. To simplify the equations of v(x,t) and i(x,t) we get:** $v(x,t) = V_1[cos(\omega t - kx) + cos(\omega t + kx)] = 2V_1 cos(kx)cos(\omega t)$ **and** $i(x,t) = \dfrac{V_1}{Z_c}[cos(\omega t + kx) - cos(\omega t - kx)] = -2\dfrac{V_1}{Z_c}sin(kx)sin(\omega t)$

Problem 5: Solution



- **2)**

$$|Z_e| = \frac{V_e}{I_e} = \frac{V(x=l)}{I(x=l)} = \frac{2V_1 cos(kl)}{2\frac{V_1}{Z_c}sin(kl)} = Z_c cot(kl) = Z_c cot(\frac{\omega}{v}l) = Z_c cot(2\pi\frac{f}{v}l)$$