



Hybrid 4D cardiovascular modeling based on patient-specific clinical images for real-time PCI surgery simulation



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ABSTRACT

In this paper, we proposed a hybrid 4D cardiovascular modeling method based on the clinical CTA and MRI to simulate the motion of the heart, as well as the patient-specific vessel structure attached to the heart model, in order to support the personalized Percutaneous Coronary Intervention (PCI) simulation which is used to train surgeons of skills and to help planning surgery. To obtain patient-specific vessel structure, a coarse segmentation with the centerlines extraction subsequently is applied to the computed tomography (CT) scans and the vessels along the centerlines is modeled using a lofted 2D segmentation method. The vessels are then combined with a template heart model to construct a cardiovascular system. For the cardiac motion, we estimate the ventricles motion from 4D Magnetic Resonance Imaging (MRI) sequences to drive the whole heart motion. And the position-based method coupling with a mass-spring model constructed with elastic spheres is used to simulate the cardiac motion cycle stably in the interactive PCI simulator. With our method, a personalized highly realistic beating motion of a whole heart is able to be created and applied to our patient-specific PCI surgery simulation system.

1. Introduction

Cardiovascular disease (CVD) remains the leading cause of mortality worldwide. Within CVD, coronary artery disease (CAD; also called coronary heart disease, CHD) in which a plaque builds up inside the coronary arteries is a serious health problem. Percutaneous Coronary Intervention (PCI), which is a medical procedure using a balloon to open a blockage in a coronary artery and usually involving placement of stents to improve the blood flow to the heart, is an effective treatment for CHD. The PCI procedures are minimally invasive and complex that need to be performed by experienced and highly skilled cardiologists. Moreover, PCI surgery tends to cause many potential life-threatening complications, it is very dangerous for new surgeons to directly conduct surgery on human patients. Meanwhile, with the continued developing in simulation, visualization, and haptic interaction, virtual reality (VR) based simulation plays a more and more significant role in clinical training and surgery rehearsing of higher risk clinical situations. For the skills training and surgery planning of PCI procedures, some PCI simulators are developed. However, most of the simulators can only deal with certain pre-settled data and scenarios, which cannot accommodate patient-specific surgery planning and rehearsing. Therefore, a virtual dynamic

patient-specific cardiovascular environment modeling method is necessary for a PCI simulator to satisfy various therapy requirements as the vessel structures and pathology positions is usually different among patients.

There are many works dedicated to image segmentation and 3D modeling, such as [1] which uses a particle filtering method for semi-automatic muscle tissue segmentation and [2] which generates 3D mechanism models containing not only geometric shapes but also internal motion structures. In this paper, we advocate a hybrid 4D cardiovascular modeling method based on the clinical images to obtain patient-specific dynamic cardiovascular system for simulation of PCI surgery. First, the patient-specific vessel model is extracted from CT data by using a level-set method for a coarse segmentation and lofting based on the centerlines to obtain a fine model subsequently. And then a template heart model is combined with the extracted vessel model to construct a patient-specific cardiovascular system. Next, we estimate the bi-ventricular motion of the heart from cine-MRI data containing a cardiac cycle to drive the motion of the whole heart. Finally, we apply position-based method to the heart model which is modeled as a mass-spring system with elastic spheres for the heart motion simulation in the PCI simulator [3]. The framework of the proposed method is illustrated as in Fig. 1.

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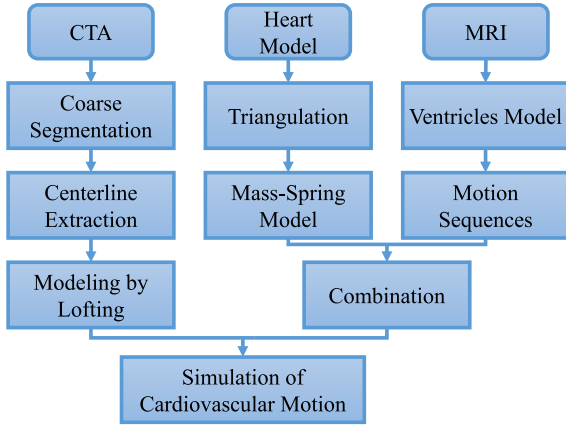


Fig. 1. The framework of the proposed method.

2. Patient-specific vessel modeling

Due to the versatility of the vessel structures and the pathology positions, it is important to extract personalized vessels from patients' clinical data to support guidance and training of PCI procedure for different cases. We use the pre-operative coronary Computed Tomography Angiogram (CTA), which is usually used to help detecting coronary pathology before an operation, of a specified patient, to obtain patient-specific geometric model of the heart vessels. The pre-processing procedure of coronary CTA is described as those in Fig. 2. Given a sequence of clinical CTA slices, the data is represented as 3D volume data, and coarse segmentations by the level-set method, centerlines extraction, radius estimation and lofting to generate a fine model are applied in sequence. First, we apply the level-set method to the clinical CT data to obtain an initial coarse segmentation. The level-set method, as is implicit, can change topology and deform far from their initial conditions without re-parameterization and is a powerful numerical technique with great potential for 3D medical image segmentation. To improve the computational efficiency of the level-set method, we employ the Work-Efficient GPU level-set segmentation algorithm [4]. In this coarse segmentation stage, user-specified seeds are placed at region-of-interest (ROI) in the images to initialize the level set field which is updated at all active elements according to the equation as follows,

$$\Phi(\mathbf{x}, t) = \Phi(\mathbf{x}, t - \Delta t) + \Delta t F(\mathbf{x}, t) |\nabla \Phi(\mathbf{x}, t - \Delta t)|, \quad (1)$$

where $F(\mathbf{x}, t)$ is a speed function, \mathbf{x} is a coordinate in the image, t is the current time and scalar field $\Phi(\mathbf{x}, t) = 0$ represents the 3D level set surface. Here we adopt the speed function proposed in [5], which determines the motion speed based on the local mean surface curvature and the local intensity of the image. The active computational domain is determined according to both the temporal and spatial derivatives of the level set field to define the minimal set of active coordinates at current time. And the level-set method evolves until there are no active coordinates which implies that the segmentation has globally converged. Although the GPU level-set method is efficient, it cannot separate the coronary vessels from the tissues/organs around. The result of this step is a little rough and need to be clipped using geometry processing tools. Then we extract the centerline for the segmented volumetric vessels, and employ cubic B-splines to fit the vascular branches. Finally, a 3D coronary vessels model could be created using a lofted 2D segmentation method [6]. For each vessel branch, a set of 3D points are sampled along the vessel centerline and a set of 2D cross sections corresponding to these points are determined. Then we apply a 2D level-set method to these cross sections to estimate the radius of each branch at corresponding positions. Within the 2D level-set method, a contour is initialized by a sample point of the centerline, and grows in the directions of changing intensity values to find the location of sharpest change. The 2D moving level set front is governed by

$$\Phi_t = -v|\nabla \Phi| - \nabla g \nabla \Phi, \quad (2)$$

where Φ_t represents the front, v is the velocity normal to the front, $g = \frac{1}{1+\nabla I p}$ is an edge detection function intended to stop the evolving curve on object boundary as mentioned in [7], ∇I is intensity gradient of a 2D cross section and p is a constant controlling the sensitivity of gradient-based edge detection. The 2D segmentations or contours along the centerline can be stitched together to construct a solid 3D vessel model.

3. Construction of cardiovascular model

In order to reconstruct a complete cardiovascular model including a whole heart model and patient-specific coronary vessels attached to it, a generic heart model is used to combine with the 3D vessels model constructed previously. As the template heart model is insufficient to match the coronary vessels model sometimes, the volumetric shape registration governed deformable model [8], of which the generic template tetrahedral mesh is deformed towards the underlying geometry of the voxel-represented heart cells in CTA scans via global registration and elastic registration, is resorted to reconstruct a patient-specific heart model which is suitable for the vessel model. As both the vessels model

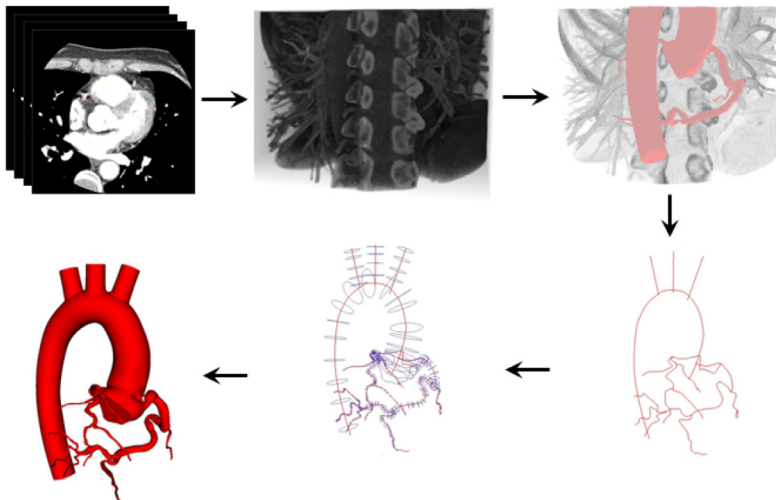


Fig. 2. Overview of the CTA data pre-processing procedure to extract personalized vessels.

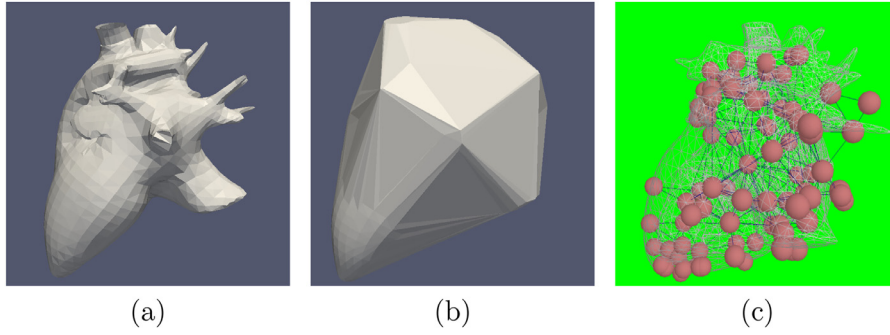


Fig. 3. Heart modeling. (a) The reconstructed surface model of the whole heart. (b) The tetrahedral model of the heart. (c) Heart model constructed with elastic spheres.

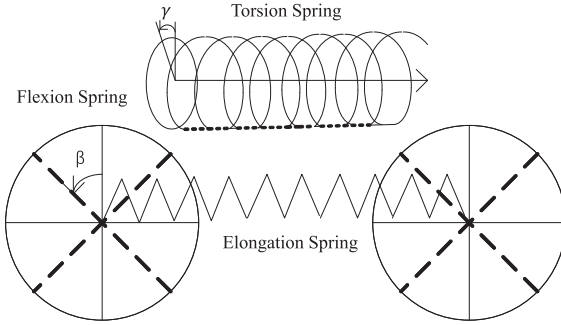


Fig. 4. Three dimensional springs used to connected the spheres. There are three different types of springs governing three kind of deformations, such as twisting, bending and tensiling.

and the heart model contain a segment of aorta, we select this common part from both models to bring the vessels and heart model into the same coordinate system via the iterate closest point (ICP) algorithm [9] and generate a cardiovascular system model which is ready to be used in the PCI simulator.

4. Dynamic cardiovascular system simulation

Actually, the heart of a patient keeps beating all the time during a clinical PCI surgery. In order to improve the reality of the PCI surgery simulation, we develop a dynamic cardiovascular simulation system to recover and simulate the highly realistic beating motion of the whole heart as well as the coronary vessels attached to it by integrating Position Based Dynamics (PBD) [10,11] and mass-spring models.

For the surface triangular model of the heart, we build a six degree-of-freedom mass-spring model which is a simple physical model for real-time simulation. As the difficulty for mass-spring model to preserve volume, we adopt the method proposed by Conti et al. [12] to construct

the model with elastic spheres placing along the medial axis transform of the object. Given the reconstructed 3D cardio-vascular model, we use TetGen [13] to generate a tetrahedral model from the surface model of the heart based on Delaunay tetrahedralization firstly. Then, elastic spheres are placed at the vertices of each tetrahedron to character the volumetric properties and each pair of these spheres are connected together with three dimensional springs which control elongation, flexion and torsion, as illustrated in Figs. 3 and 4. The movement of spheres leads to the deformation of the surface model. And the filling-spheres model of the heart decouples local deformation and global deformation, namely, forces applied generate a local deformation first and then propagate to the global model, which guarantees the system's stability. Moreover, sphere packings are proved to be efficient when doing collision detection [14], and therefore are suitable for real-time application. Interactions between different objects can be computed directly from the deformable model using collision spheres which are placed at each node of the skeleton.

To simulate realistic beating motion, we then estimate the ventricles motion from 4D Magnetic Resonance Imaging (MRI) sequences consisting of a complete cycle of heart beating motion. The estimated ventricles motion sequences are used to drive the whole heart model beating in a visually plausible way. For the motion estimating, the bi-ventricular geometry of the heart is extracted from the first frame of the cardiac MRI sequence corresponding to the end-diastolic of the cardiac cycle and a registration algorithm is applied to compute deformation fields [15]. The nonlinear image registration based on the diffeomorphic demons [16] is used to register the first frame of the cardiac sequence to the following frames to find deformation fields between frames. And therefore, a smooth cycle motion of the heart bi-ventricular model is then obtained by interpolating linearly between frames based on the deformation fields. The extracted bi-ventricular geometry model is illustrated in Fig. 5.

We then build a relationship between the whole heart model and the ventricular model, so that the kinematic motion modeling is imple-

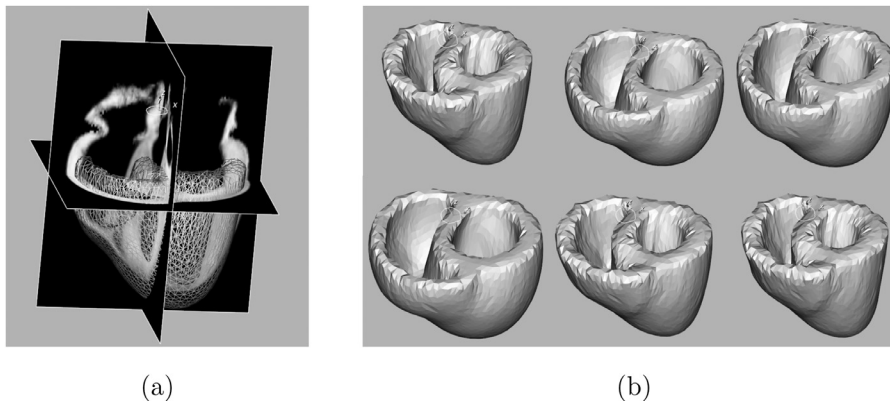


Fig. 5. The geometry of the ventricles extracted from MRI sequence. (a) The geometry of the ventricles coupling with the corresponding MRI images frame. (b) The ventricles geometry models at six different times extracted from the MRI sequence.

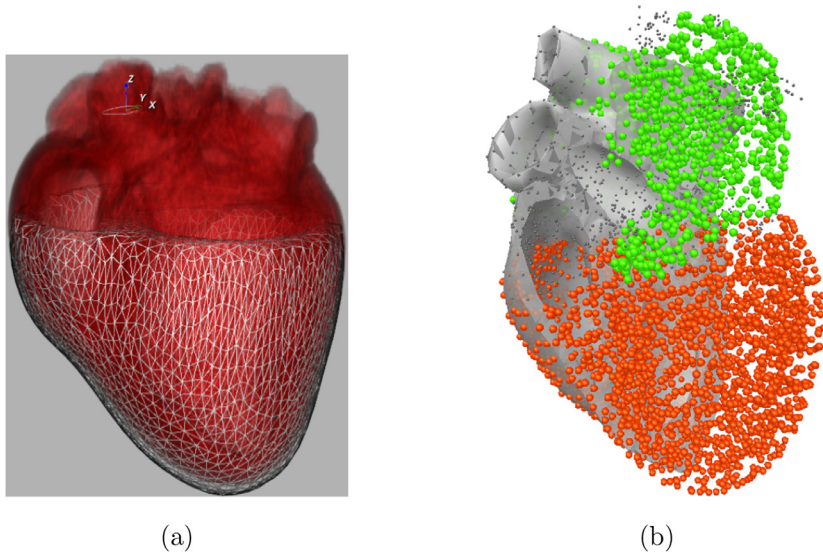


Fig. 6. Relations between the ventricles and the whole heart model. (a) The geometry of the ventricles extracted from the first frame of the cardiac MRI sequence coupling with the whole heart model. (b) The whole heart geometry illustrating the relationship between ventricles and the whole heart with the particles in red corresponding to the ventricles. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

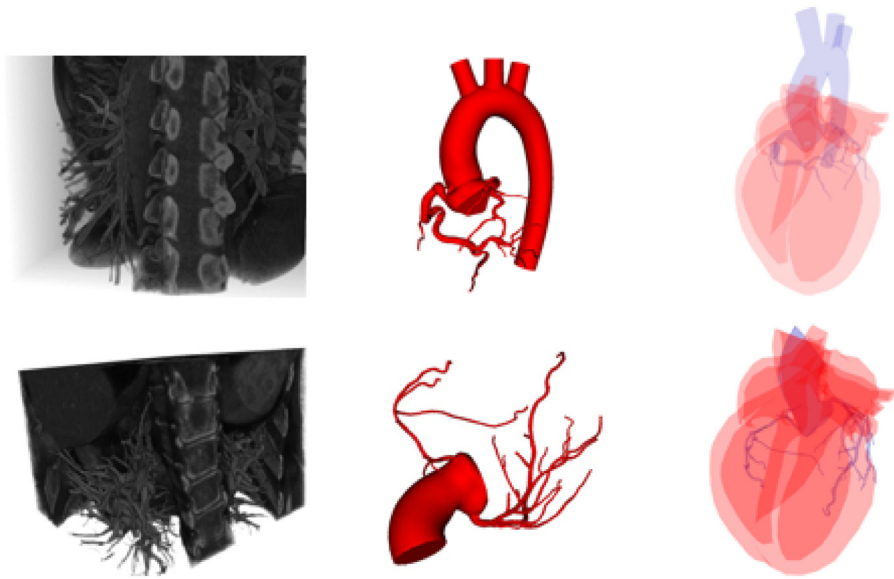


Fig. 7. The result of vessel modeling.

mented driven by the ventricular motion. As illustrated in Fig. 6, the ventricle geometry and the whole heart model of which the motion of the particles in red are driven directly by the estimated ventricle motion while the particles in green are simulated based on the PBD method and the mass-spring model is described.

In the force based approach for simulation of dynamic systems, internal and external forces are accumulated from which accelerations are computed based on Newton's second law of motion. To compute the accelerations, there are some challenges, such as how to determine the mass of particles for mass-spring system or mass of spheres for filling-spheres method and how to avoid the overshooting problem of explicit integration. Position Based Dynamics proposed by Müller et al. [10] and Bender et al. [11] avoid these problems and is of great simplicity and robustness thus becomes a popular method for simulating deformable bodies in computer graphics and interactive applications. Instead of computing velocities and positions based on forces, position-based methods work directly on the positions which provides a high level of control. And PBD method is proved to be stable even when simple and fast explicit time integration schemes are used. Moreover, constraints and penetration problems in collision detection are able to be solved using projected Gauss-Seidel algorithm. With the position-based method, the

computation of contact forces and the simulation of the soft-body deformation are decoupled, so that the computation of virtual force does not dependent on the result of deformation which ensures the stability of the simulation system. Therefore, we apply this method to the dynamic simulation of the cardiovascular system consisting of coronary artery and heart model. The objects to be simulated with PBD method are generally represented by a set of particles and a set of constraints. We construct the heart model with spheres placed at each node of the skeleton as described previously. Each sphere is assigned three attributes: mass, position and velocity. The parameters including the position, velocity and mass of each particle of the model are initialized in a pre-processing step and some constraints are generated. The constraints are of the following forms,

$$C_i(\mathbf{x} + \Delta\mathbf{x}) = 0, i = 1, \dots, n. \quad (3)$$

$$C_j(\mathbf{x} + \Delta\mathbf{x}) \geq 0, j = 1, \dots, n. \quad (4)$$

where \mathbf{x} is the position of each particle. The constraints C are linearized in the neighborhood of the current solution using

$$C_i(\mathbf{x} + \Delta\mathbf{x}) \approx C_i(\mathbf{x}) + \nabla C_i(\mathbf{x})\Delta\mathbf{x} = 0. \quad (5)$$

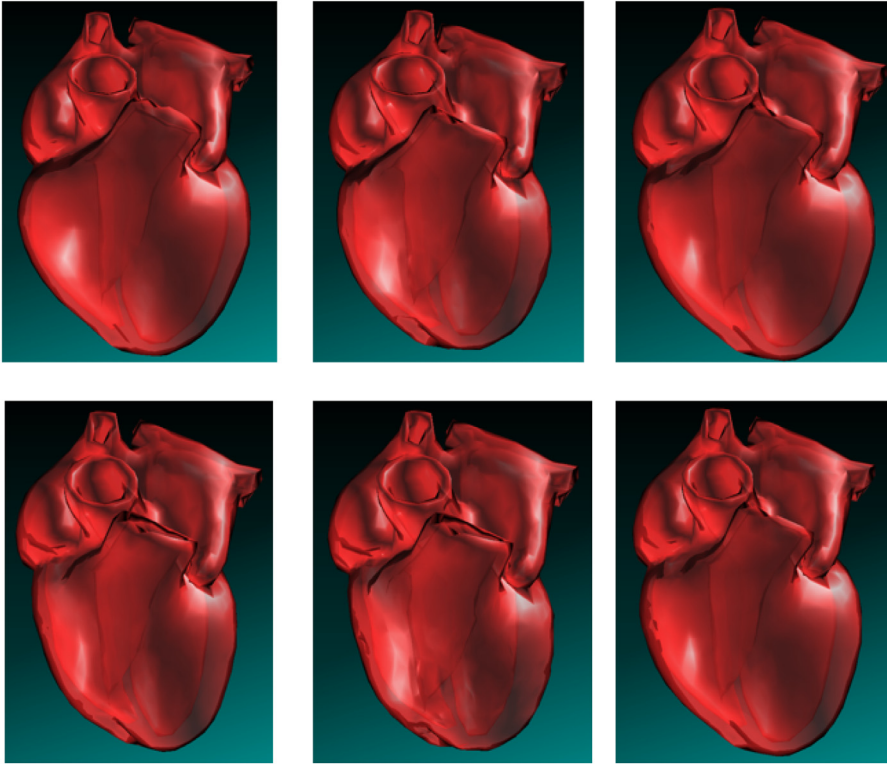


Fig. 8. The simulation of the heart motion at six different times.

and the non-linear Gauss–Seidel iteration is typically used to solve each constraint equation. $\Delta \mathbf{x}$ in Eq. (5) is restricted along the constraint gradient, a Lagrange multiplier has been used such that the correction

$$\Delta \mathbf{x} = \lambda \mathbf{M}^{-1} \nabla C_i(\mathbf{x}) \quad (6)$$

solves Eq. (5), where $\mathbf{M} = \text{diag}\{m_1, \dots, m_n\}$ is the mass matrix. Then positions are updated after each constraint is processed. After iterations a new velocity is computed according to the total constraint delta $\Delta \mathbf{v} = \Delta \mathbf{x} / \Delta t$. Different constraints can be used, such as distance constraint and bending constraint. During the simulation, the velocities of the particles in the whole heart model corresponding to the ventricular model are set using the deformation fields estimated previously and the velocities of the other particles are computed through explicit integration under the effect of spring forces generated by the three-dimensional springs due to the displacement of their neighboring particles and then new positions of next time step are predicted. These predicted positions are corrected to satisfy the presetting constraints. Due to the time requirement of iterating over the spheres and updating the state, the movement of the model barycenter will lag behind as time integration. We use the shape matching method [17] which can directly be integrated in the position-based dynamics algorithm as a form of constraint projection to modify the global deformation of the model. For the whole heart modeled with springs and elastic spheres, two set of positions are used, the initial positions \mathbf{x}^0 and the set of predicted positions \mathbf{x} . The aim of the shape matching method is to find the corresponding rotation matrix \mathbf{R} and the translational vectors \mathbf{t} and \mathbf{t}_0 by minimizing

$$\sum_i w_i (\mathbf{R}(\mathbf{x}_i^0 - \mathbf{t}_0) + \mathbf{t} - \mathbf{x}_i)^2, \quad (7)$$

where w_i are the weights of the sphere nodes. The mass centers of the initial shape and the actual shape can be defined as:

$$\mathbf{t}_0 = \mathbf{x}_{cm}^0 = \frac{\sum_i m_i \mathbf{x}_i^0}{\sum_i m_i}, \quad \mathbf{t} = \mathbf{x}_{cm}^i = \frac{\sum_i m_i \mathbf{x}_i}{\sum_i m_i}, \quad (8)$$

where the m_i is the mass of point \mathbf{x}_i . To further find the optimal linear transformation \mathbf{A} , the derivatives with respect to all the coefficients of

\mathbf{A} are set to be zero, which yields

$$\mathbf{A} = \left(\sum_i m_i \mathbf{p}_i \mathbf{q}_i^T \right) \left(\sum_i m_i \mathbf{q}_i \mathbf{q}_i^T \right)^{-1} = \mathbf{A}_{pq} \mathbf{A}_{qq}, \quad (9)$$

and $\mathbf{q}_i = \mathbf{x}_i^0 - \mathbf{x}_{cm}^0$ and $\mathbf{p}_i = \mathbf{x}_i - \mathbf{x}_{cm}$ are the relative locations with respect to their mass centers. The second term \mathbf{A}_{qq} is a symmetric matrix that involves scaling without rotation, and $\mathbf{A}_{pq} = \mathbf{R}\mathbf{S}$ controls the optimal rotation, where $\mathbf{S} = \sqrt{\mathbf{A}_{pq}^T \mathbf{A}_{pq}}$ is the symmetric part of the polar decomposition and \mathbf{R} is the rotational part. Thus, the rotation matrix becomes $\mathbf{R} = \mathbf{A}_{pq} \mathbf{S}^{-1}$. Finally, the target position can be calculated as

$$\mathbf{g}_i = \mathbf{R}(\mathbf{x}_i^0 - \mathbf{x}_{cm}^0) + \mathbf{x}_{cm}. \quad (10)$$

5. Result

We implement our method on a PC with an NVIDIA GeForce GTX 970 GPU, Intel Core I7 CPU using C/C++ and CUDA. Two CTA datasets are used to extract the coronary vessels model. CTA scans were acquired using a dual-source CT scanner. Voxel sizes of the reconstructed volumes are approximately $0.77 \times 0.77 \times 0.50 \text{ mm}^3$. We apply the patient-specific vessel modeling method on the clinical CTA scans of two different patients to extract the coronary artery models and combine these coronary models with a template heart model to construct complete patient-specific cardiovascular models as illustrated in Fig. 7. We evaluate the performance of cardiovascular modeling procedure on these two CTA data for each step in details, including the coarse segmentation of the CTA volume data, centerline extraction and modeling the fine vessel models by lofting, as shown in Table 1. This procedure is semi-automatic, with only a few user interactions, a patient-specific coronary model can be reconstructed. The total time of the vessel modeling procedure in addition with user interactions is about fifteen minutes.

And the motion simulation of the heart driven by ventricles is depicted in Fig. 8. It takes only a few tens of milliseconds to simulate an entire circle of the heart beating motion in the PCI simulator. These models then can be used in a patient-specific PCI simulator where a wire or a

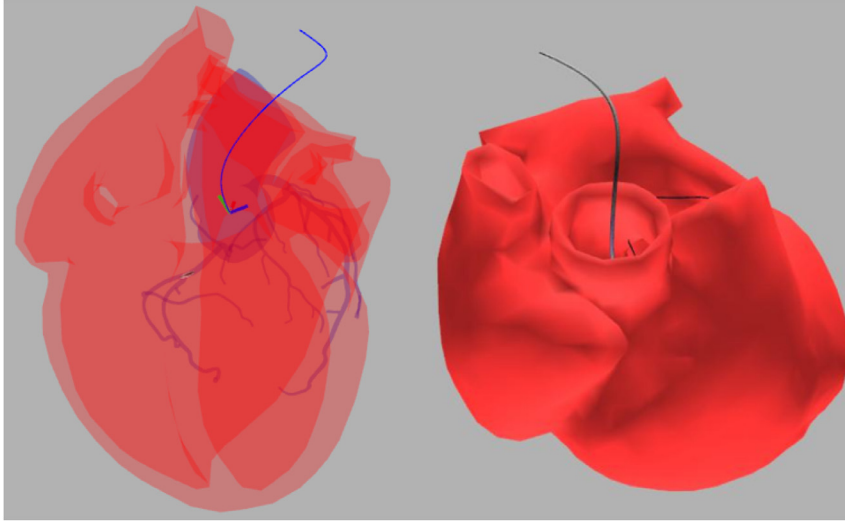


Fig. 9. The cardiovascular model with a wire inserted.

Table 1
Statistics of cardiovascular modeling procedure.

CTA	Patient1	Patient2
Volume resolution	512 × 512 × 224	512 × 512 × 276
Model triangular faces	205,804	221,382
Coarse segmentation	56s	75s
Clipping	42s	61s
Centerline extraction	24s	30s
Estimate cross-section radius	11s	13s
Lofting	8s	9s
Total	141s	188s

Table 2
Statistics of PCI simulation operation.

Operation	Inserting	Rotation	Turning
Model triangular faces	221,382	221,382	221,382
Simulation frame rate	31FPS	38FPS	30FPS

balloon is inserted into the vessels of the heart for training and planning of PCI procedures as illustrated in Fig. 9. We also simulate three different types of operation using our cardiovascular modeling equipped PCI simulator, including inserting a wire into the vessels, rotating the wire and turning the wire into branches of vessels, and the performance evaluations of these PCI simulation operations are shown in Table 2, which indicates great clinical potential of our modeling method because of its high efficiency.

6. Conclusion

In this paper, we have introduced a hybrid 4D cardiovascular modeling method based on the clinical images to construct patient-specific dynamic cardiovascular models and simulate the heart motion to support PCI simulation in order to train cardiologists of skills and perform preoperative planning for the PCI surgery procedures. We use a level-set method to obtain a coarse segmentation and further get a fine surface model of the vessel structure by lofting the extracted centerlines. And then we combine the vessels model with a template heart model of which the motion is driven by ventricles motion extracted from 4D MRI sequence. A personalized highly realistic beating motion of a whole heart is generated and applied to the patient-specific PCI surgery simulation system.

Acknowledgments

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