

# Recommender Systems

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Enhance customers' satisfaction, so more purchase

# Machine Learning and Business

- Always give good recommendation

3 February, 2012

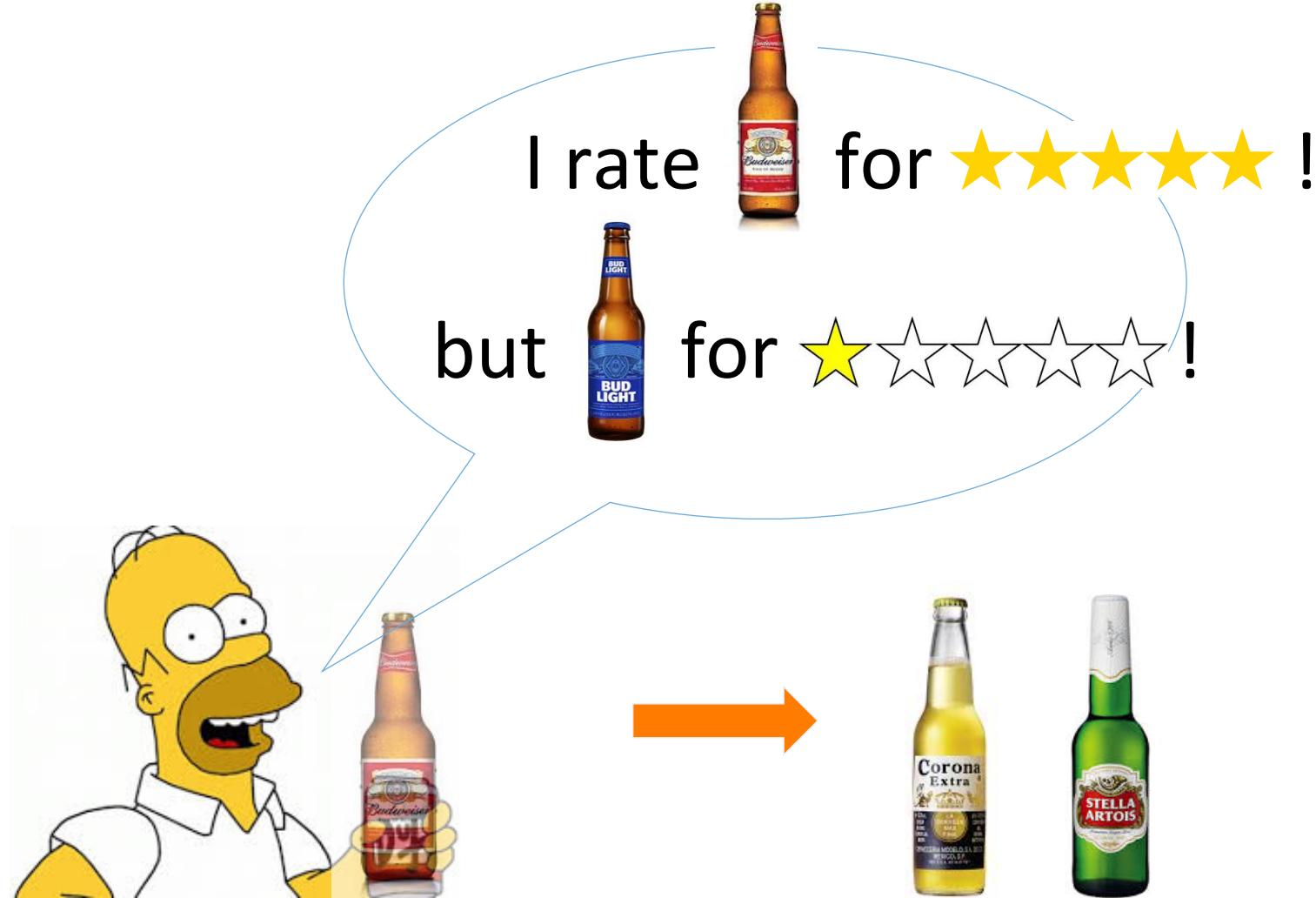


**Recommendations help drive  
27.9% holiday sales growth at  
John Lewis**

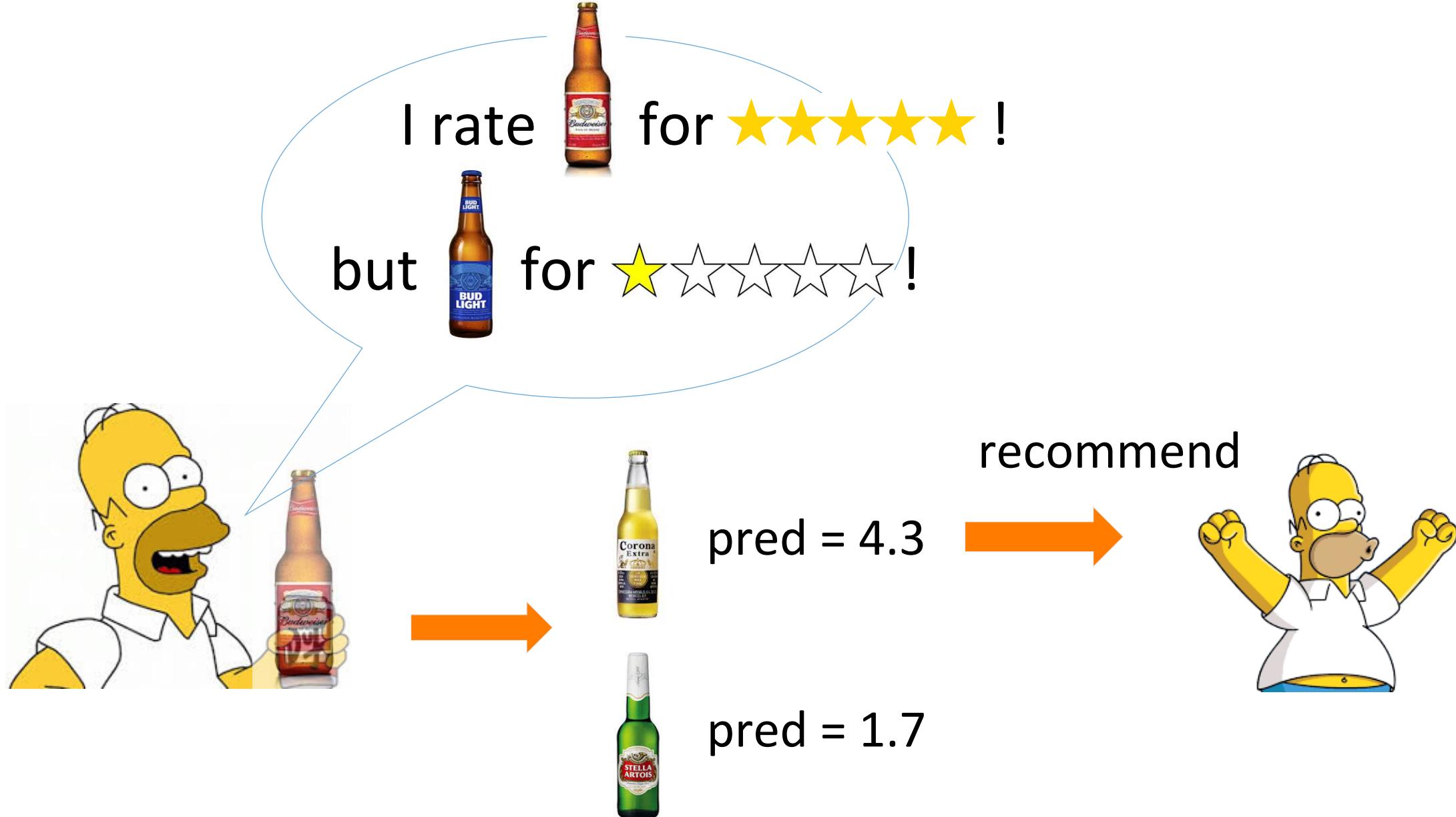


By David Moth @ Econsultancy

- Understand customers' demographic characteristics



Rating = ?



# Personalized Recommendation

Different people have different tastes, so rating.



5



4.5



3



1



1.5

1

3.5

5

# Personalized Recommendation

- Supervised learning, regression problem.
- Central concept: **Similarity**
  - a.) item-item recommendation (Amazon)
  - b.) user-item recommendation

Collaborative filtering (Netflix, Spotify)

# Recommendation Engine UI

When Mr. Simpsons (id=7) is browsing the web:

user_id= 7			
	user_id	beer_id	pred_rating
0	7	146	5
1	7	20	5
2	7	27	5
3	7	31	5
4	7	45	5
5	7	54	5
6	7	57	5
7	7	79	5
8	7	81	5
9	7	106	5

beer: 144			
	beer1	beer2	score
0	144	144	1.000000
1	144	2671	0.352322
2	144	3467	0.281549
3	144	120	0.251629
4	144	129	0.251033
5	144	123	0.249825
6	144	79	0.240634
7	144	32	0.225036
8	144	6959	0.217816
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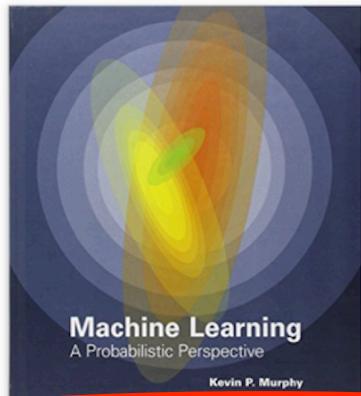
item-item reco

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by Kevin P. Murphy ▾ (Author)

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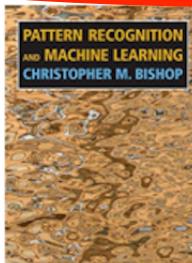
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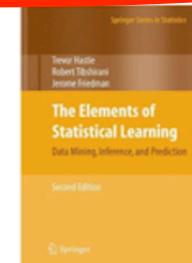
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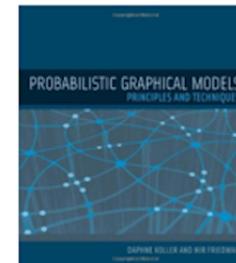


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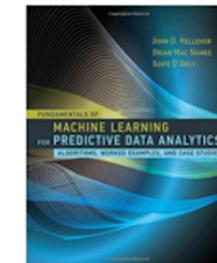


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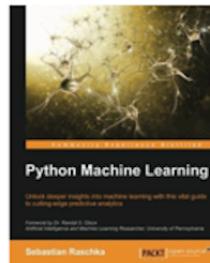
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#1 Best Seller in Data Processing

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# Data Source: BeHoppy (SQL)



# The Beer Content Data

beers.head()

Out[799]:

	id	country	flavor	style	average_rating	number_of_ratings	average_aroma_rating	average_appearance_rating	average_taste_rating
0	5	BEL	Amadeirado	Lager	3.75481	208	3.45238	3.69531	3.61417
1	6	BEL	Doce	Ale	4.22901	131	4.20430	4.22680	4.26804
2	7	MEX	Doce	Lager	3.90435	230	3.39583	3.88194	3.68750
3	8	BRA	Doce	Bohemian Pilsener	3.79474	190	3.16239	3.38136	3.57627
4	9	EUA	Doce	Lager	3.61244	209	3.13492	3.34646	3.36800

Beer id

Beer vectors are represented by features:

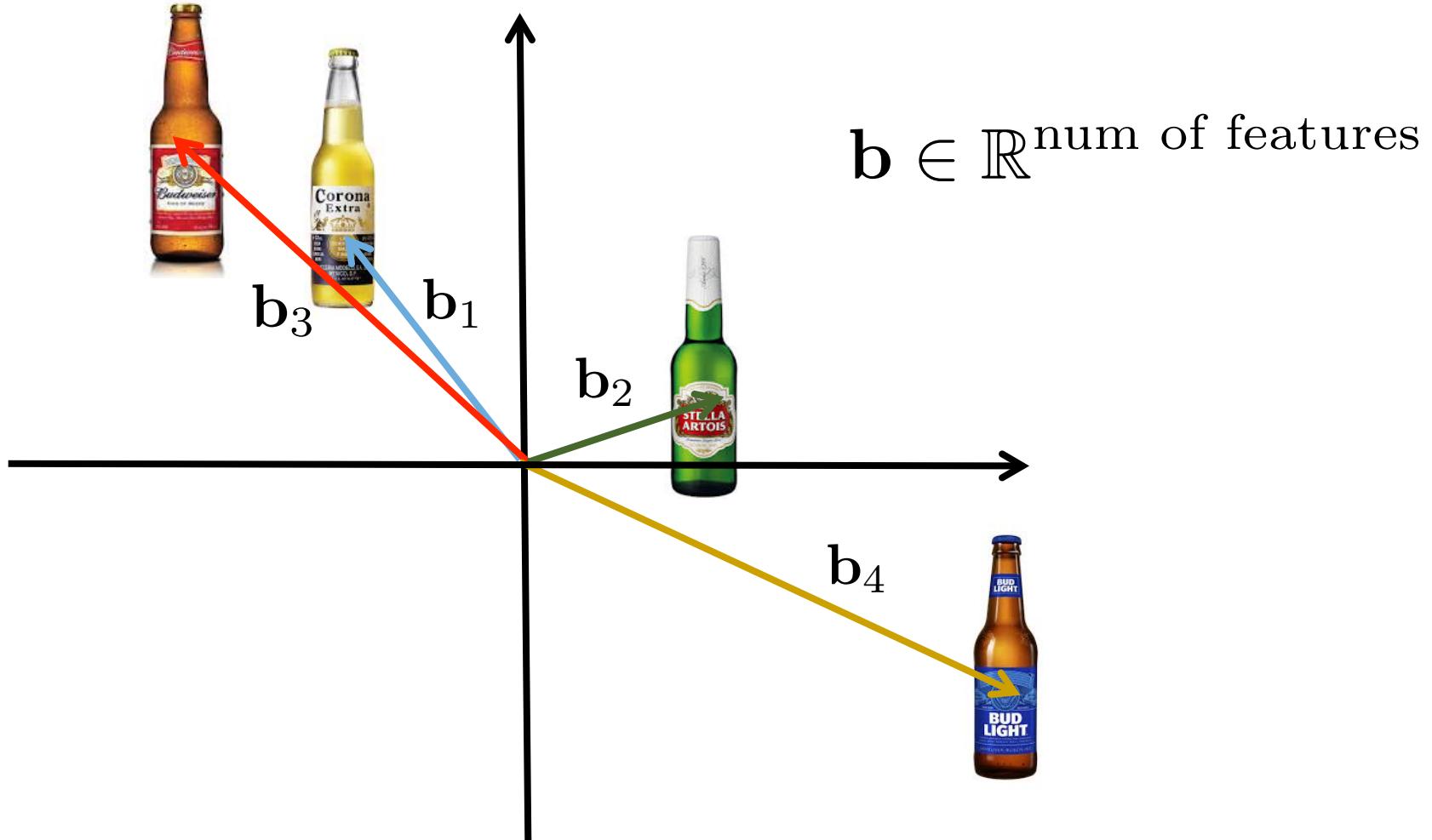


$$\mathbf{b}_1 = (208, 3.45, 3.69, 3.61, 3.44, 0, 0, \dots)$$

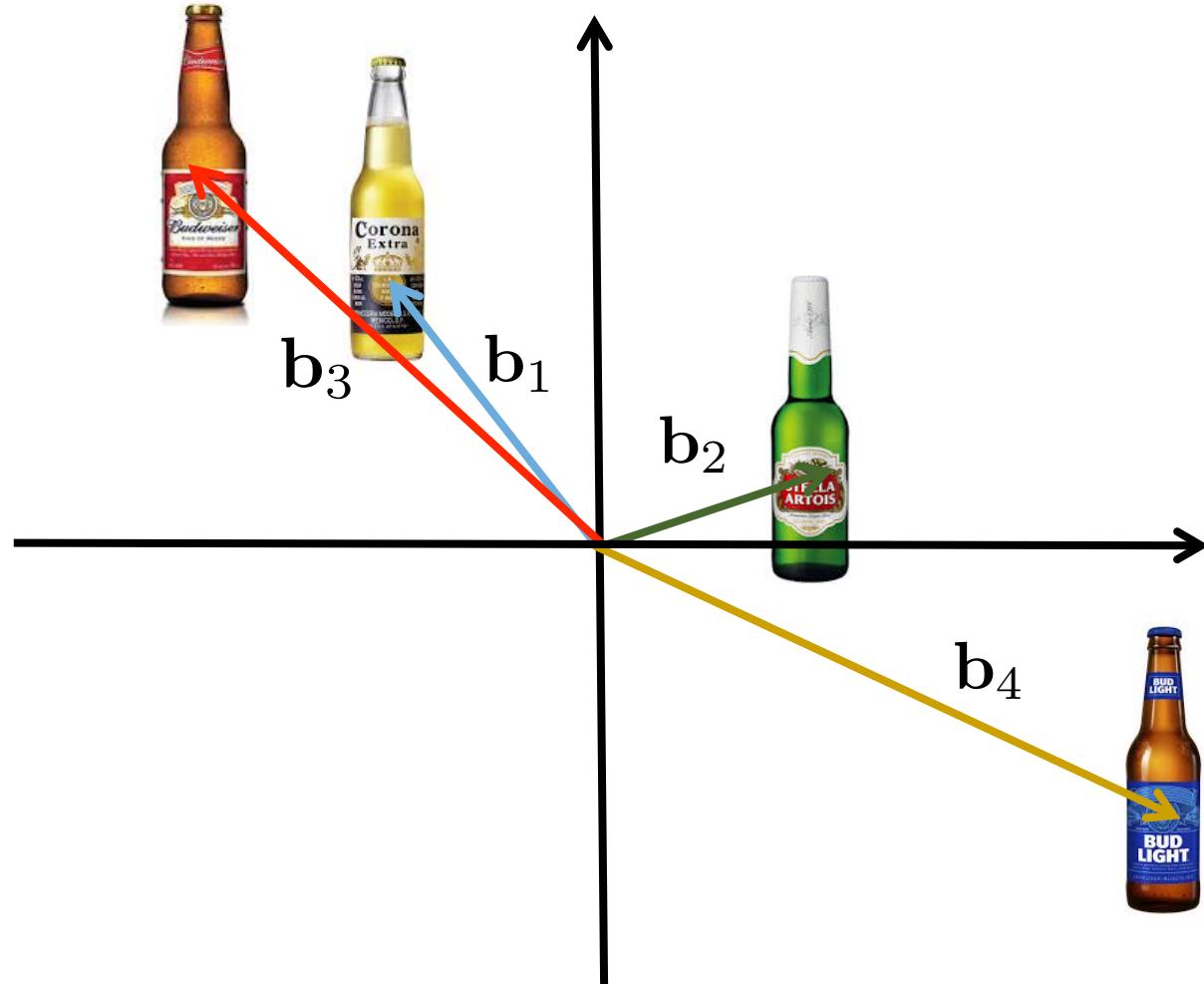
$$\mathbf{b}_2 = (131, 4.2, 4.22, 4.27, 4.1, 0, 0, \dots)$$



# Beer Vector Space



# Cosine Similarity



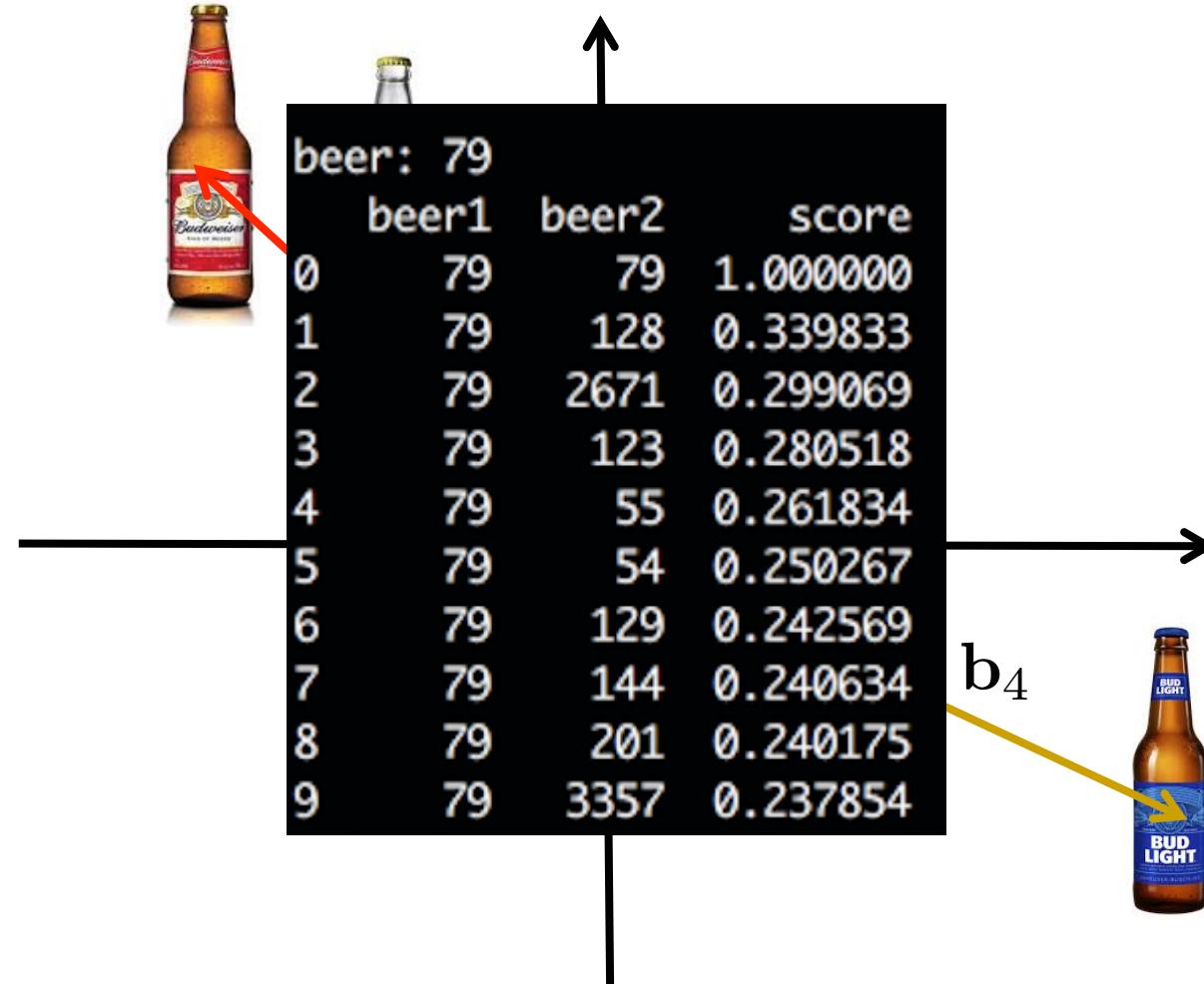
$$s_{A,B} = \frac{\mathbf{b}_A \cdot \mathbf{b}_B}{\|\mathbf{b}_A\| \|\mathbf{b}_B\|}$$

$$s_{31} > s_{32} > s_{34}$$

is more similar  
to      than

The text "is more similar" is positioned above the first two bottles, while "than" is positioned above the last two. All three bottles are enclosed within a red rectangular border.

# Cosine Similarity



$$s_{A,B} = \frac{\mathbf{b}_A \cdot \mathbf{b}_B}{\|\mathbf{b}_A\| \|\mathbf{b}_B\|}$$

$$s_{31} > s_{32} > s_{34}$$



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user-item reco

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# Collaborative Filtering (CF)

- No content features are needed.
- Consider user-item interactions

rating table

	score	user_id	beer_id
0	4	7	6
1	4	48	7
2	4	47	9
3	4	47	10
4	4	50	9



beers

	5	6	7	8	9	10	11
7	5	4	1	5	5	4	
33				1			
34	1	2	5	1	2	2	
42							
45	3	5	2	5	5	4	
47					4	4	
48						4	

users

# Ratings as Features of Beer Vectors

	beers							
	5	6	7	8	9	10	11	12
users	7	5	4	1	5	5	4	
	33		1			1		
	34	1	1	5	1	2	2	
	42		3				3	
	45	4	5	1	5	5	3	1
	47			4	4	4	3	
	48			4	4	1		

$$\mathbf{b}_8 = (1, -, 5, -, 1, -, 4)$$

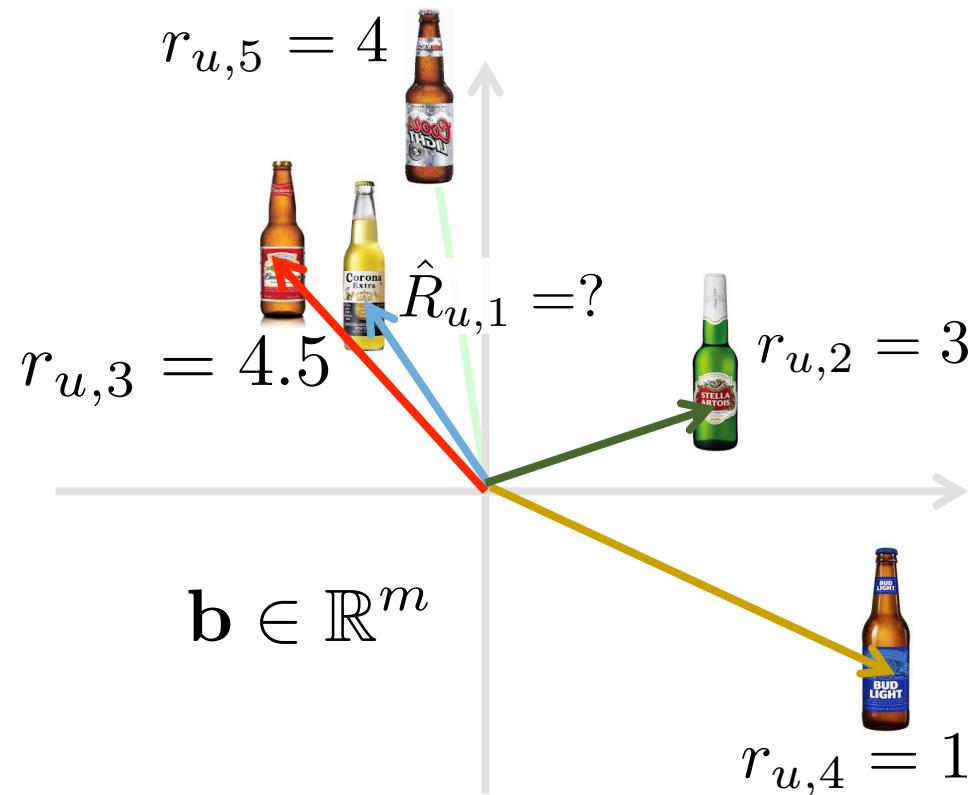
$$\mathbf{b}_9 = (5, -, 1, -, 5, 4, -)$$

$$\mathbf{b}_{10} = (5, 1, 2, -, 5, 4, 1)$$

$$s_{9,10} > s_{9,8}$$

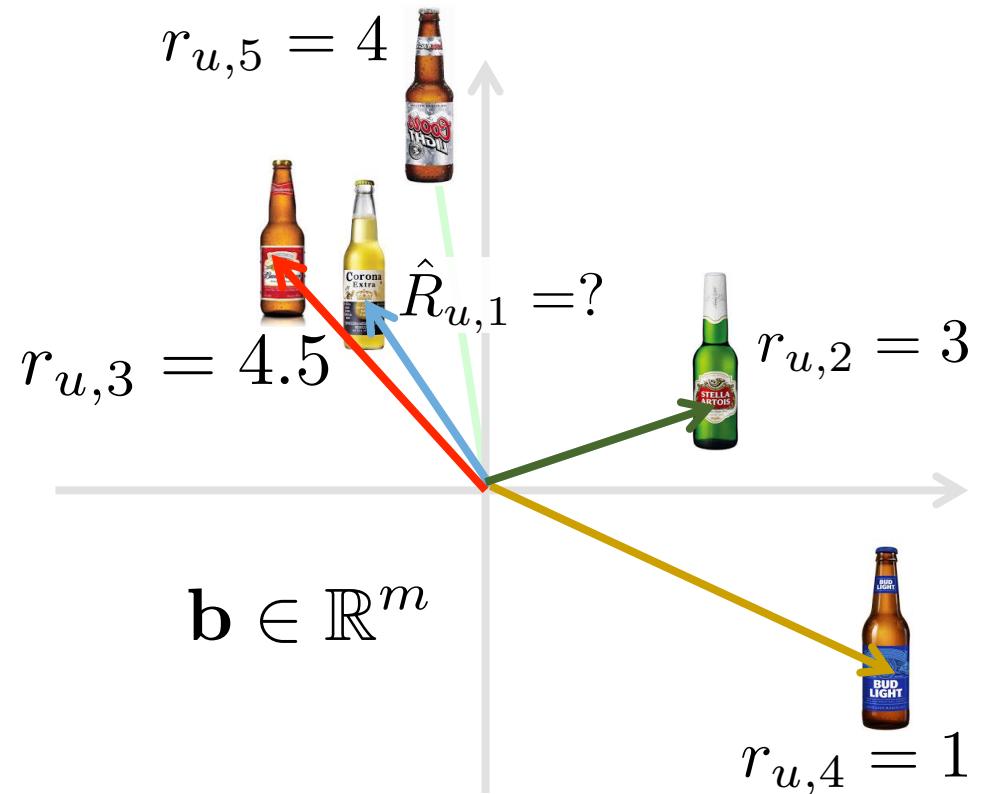
# Neighborhood Model (item-based)

Sarwar, Karypis, Konstan, and Riedl , (2001)



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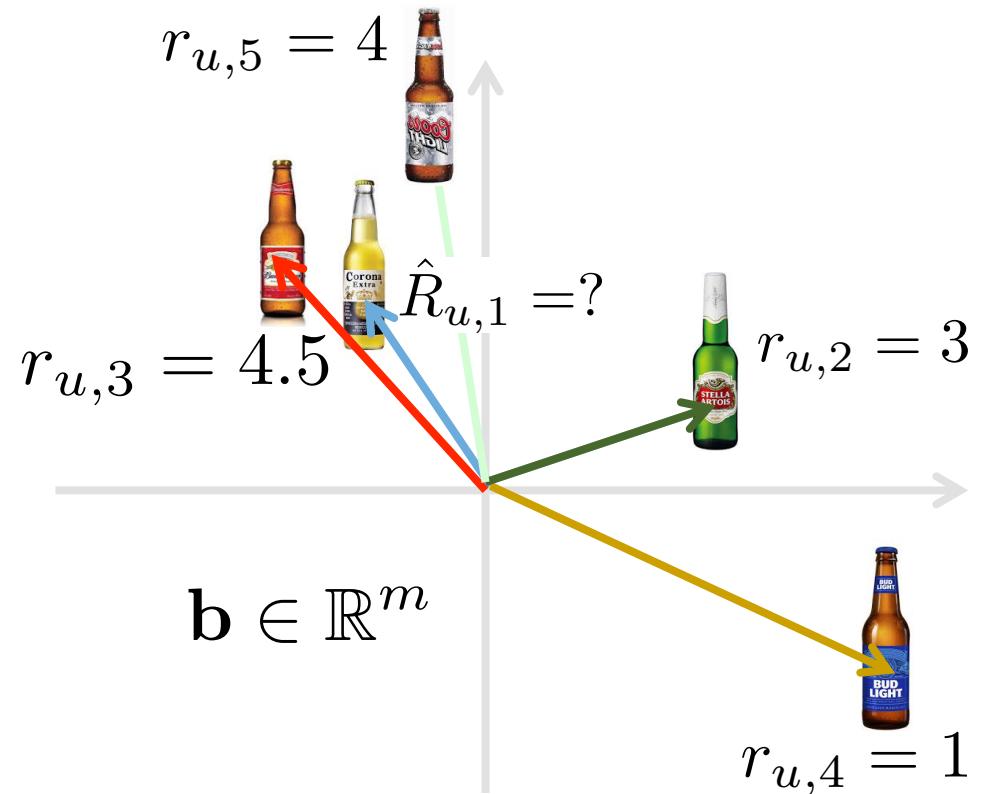


k-nearest-neighbor:

$$\hat{R}_{u,1} = \frac{(4.5 + 4 + 3)}{3}$$

# Neighborhood Model (item-based)

Sarwar, Karypis, Konstan, and Riedl , (2001)



Model = kNN + weight:

$$\hat{R}_{u,i} = \frac{\sum_{j \in S^k} s_{ij} r_{u,j}}{\sum_{j \in S^k} |s_{ij}|}$$
$$= \frac{(0.8 * 4.5 + 0.7 * 4 + 0.2 * 3)}{0.8 + 0.7 + 0.2}$$

# Ratings as Features of Users Vectors

		beers							
		5	6	7	8	9	10	11	12
users	7	5	4		1	5	5	4	
	33				1				
	34	1	1		5	1	2	2	
	42				3				3
	45	4	5		1	5	5	3	1
	47				4	4			3
	48				4	4			
					1				

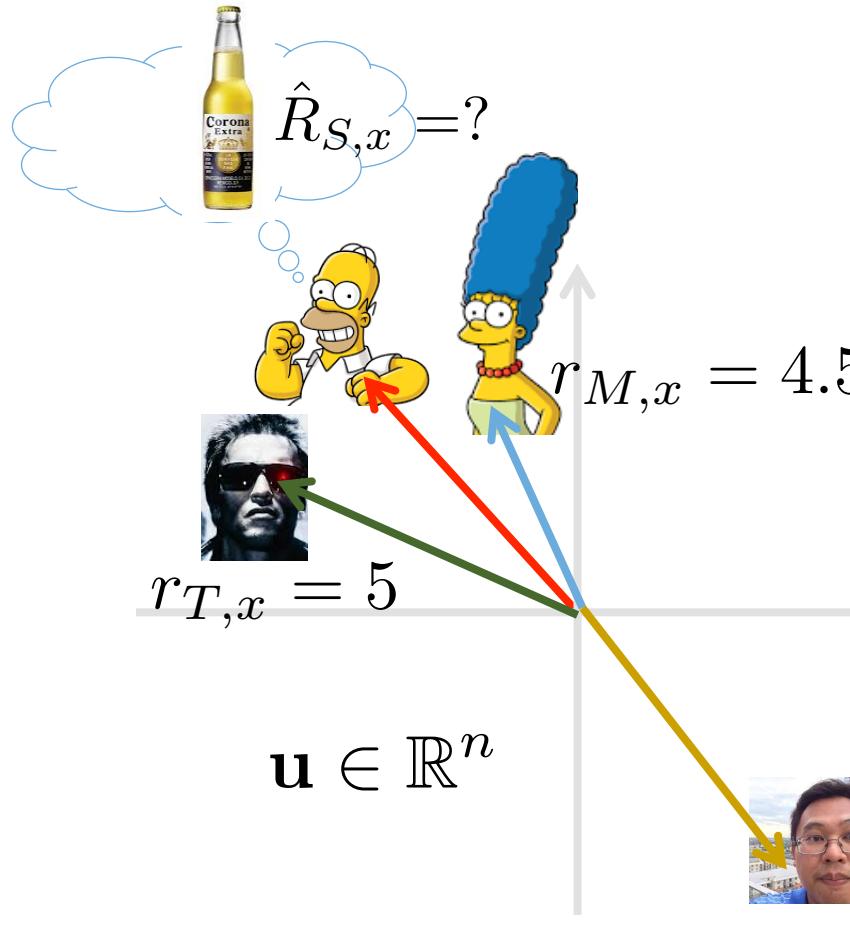
$$\mathbf{u}_7 = (5, 4, -, 1, 5, 5, 4, -)$$

$$\mathbf{u}_{34} = (1, 1, -, 5, 1, 2, 2, -)$$

$$\mathbf{u}_{45} = (4, 5, -, 1, 5, 5, 3, 1)$$

$$s_{45,7} > s_{45,34}$$

# Neighborhood Model (user-based)



Herlocker, Konstan, Borchers and Riedl , (1999)

Find similar users:

$$\hat{R}_{u,x} = \frac{\sum_{v \in S^k} s_{uv} r_{v,x}}{\sum_{v \in S^k} |s_{uv}|}$$

weight

# Metrics

mean absolute error:  $\text{MAE} = \frac{1}{N} \sum_{u,i} |\hat{R}_{u,i} - r_{u,i}|$

a) item-based:

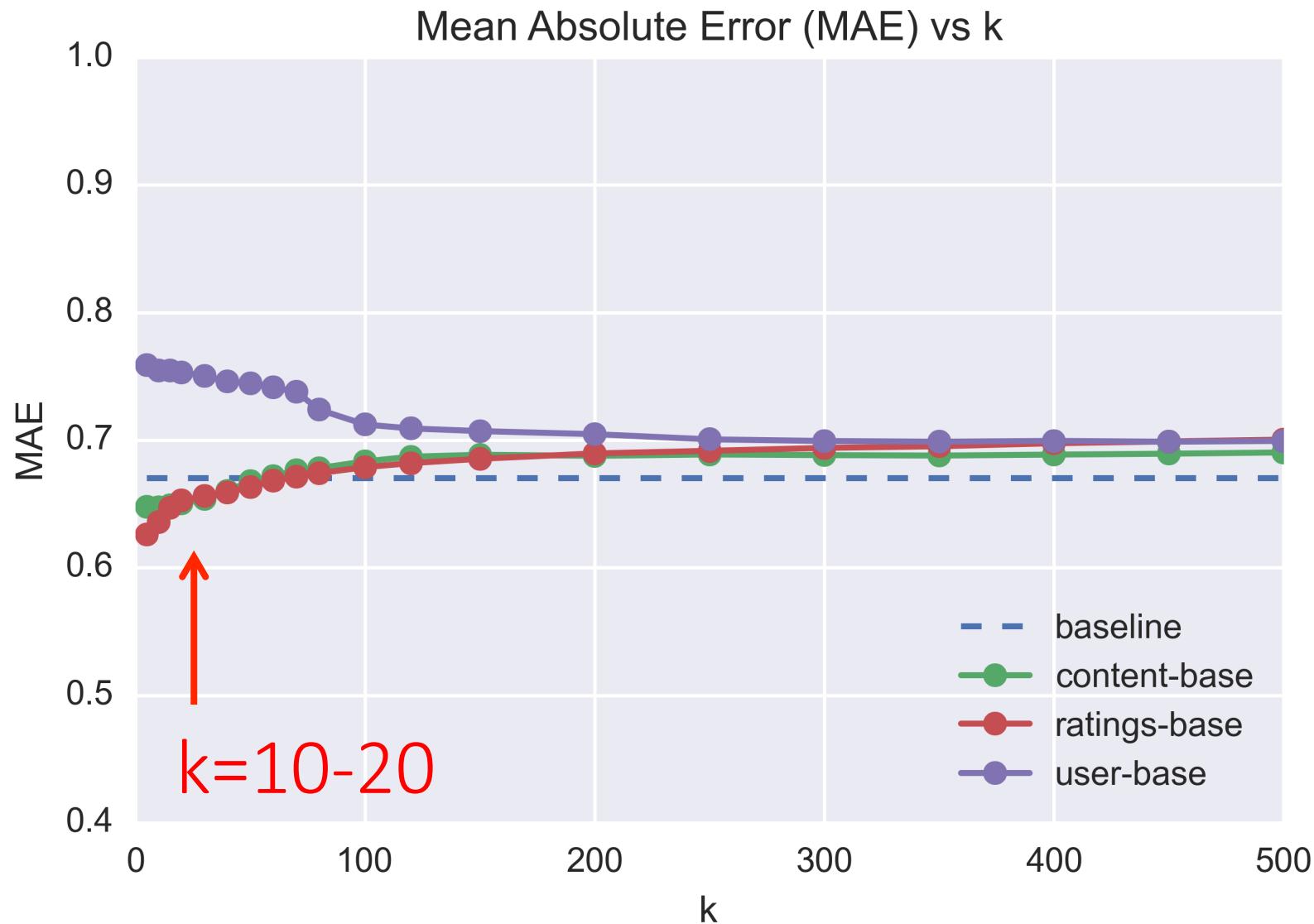
$\mathbf{b}_i$  = beer content and ratings as features

b) user-based:  $\mathbf{u}_u$

c) Baseline:  $\overline{\text{MAE}} = \frac{1}{N} \sum_{u,i} |\bar{R}_i - r_{u,i}|$

[Sarwar, Karypis, Konstan, and Riedl , \(2001\)](#)

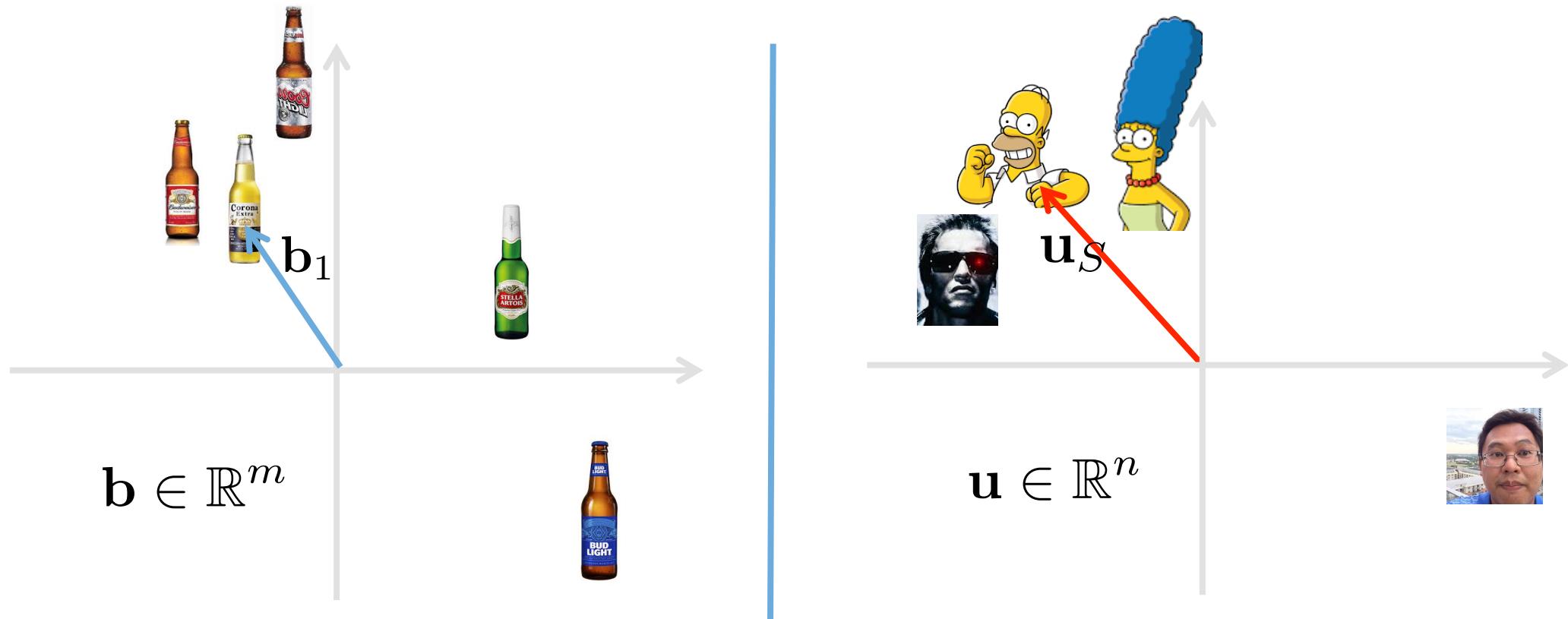
# Model Performance



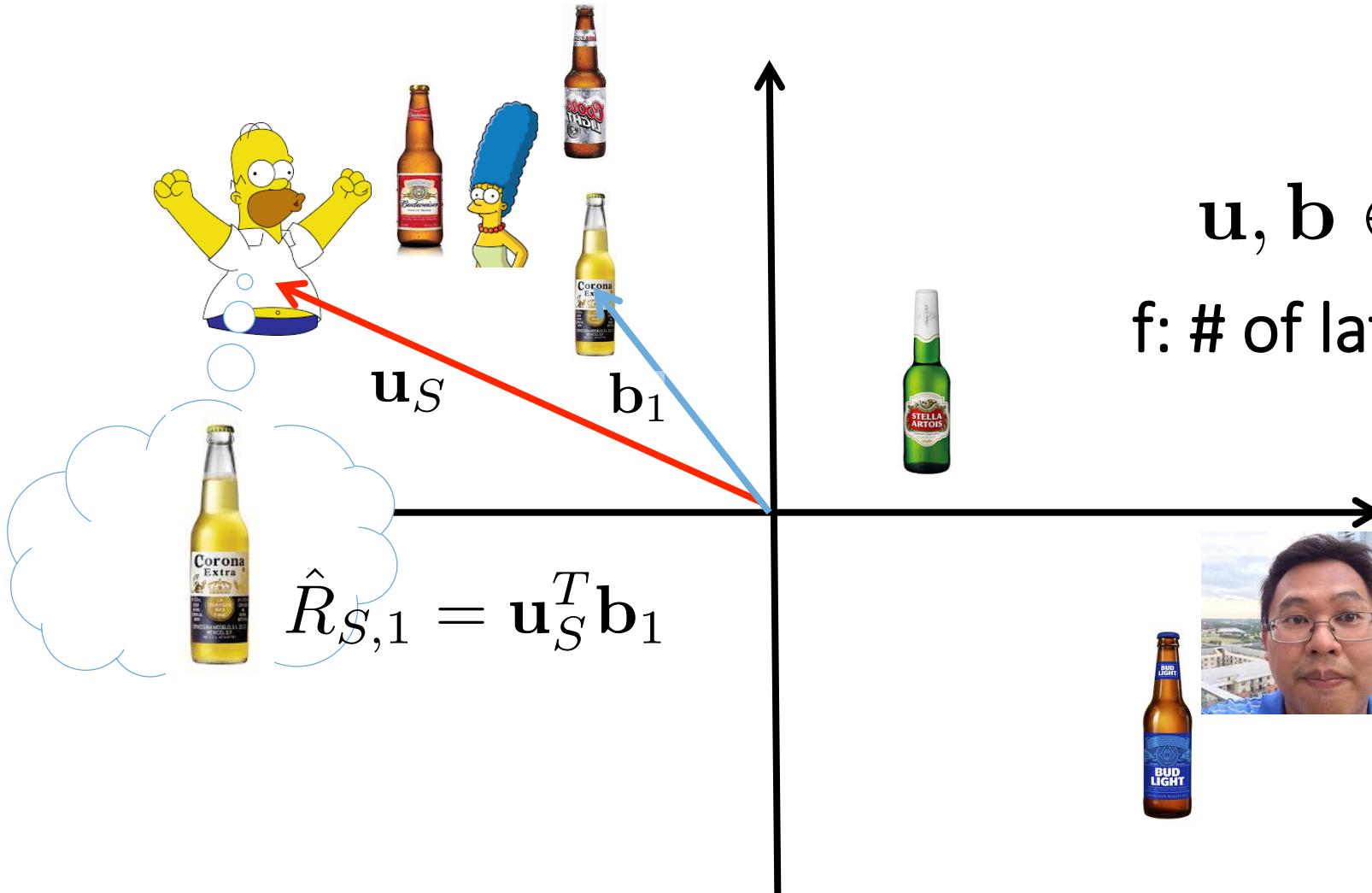
# Disadvantages of the Model

- NOT scalable.
- Not easy to incorporate additional information, e.g. purchase, browse history (implicit data).
- Alternative: latent-factor model!

# Vector Spaces



# Customer-Beer Vector Space



$$\mathbf{u}, \mathbf{b} \in \mathbb{R}^f$$

f: # of latent factors

# Matrix-Factorization for Latent-Factor Model

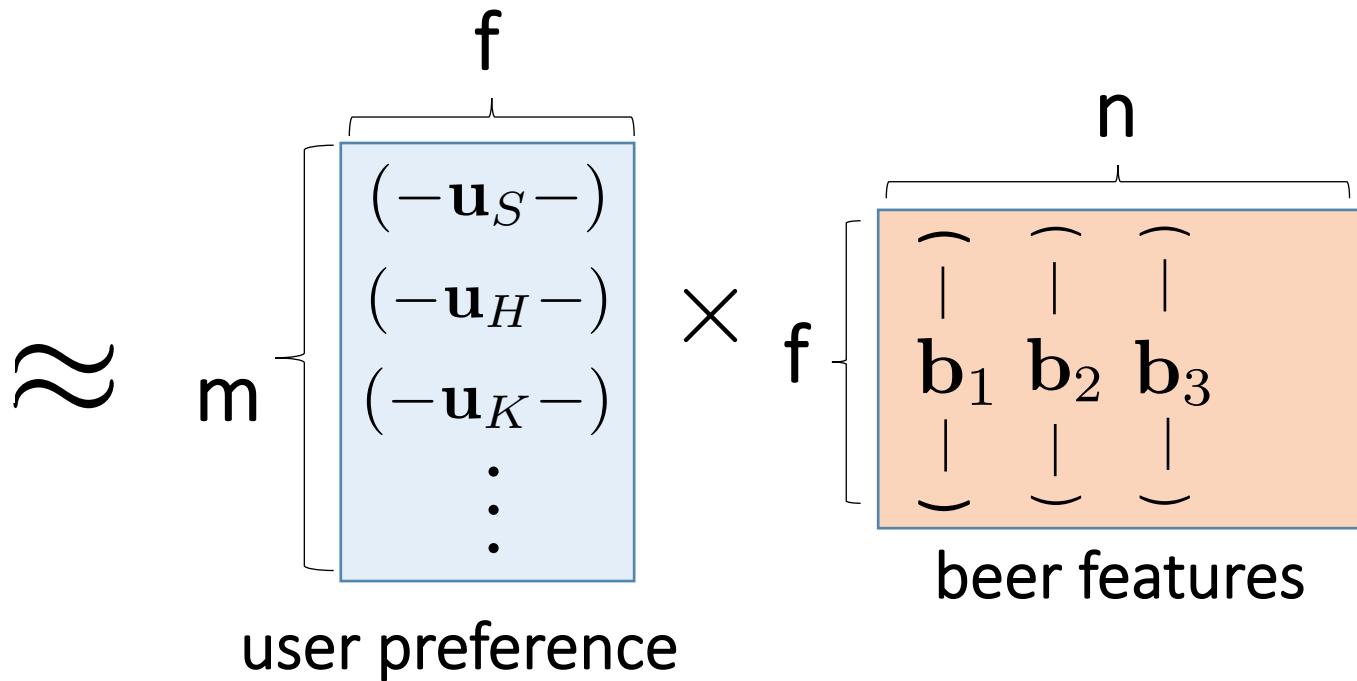
[Computer \(2009\), Koren, Bell and Volinsky](#)

**n beers**

	5	6	7	8	9	10	11
7	5	4	1	5	5	4	
33	1	2	5	1	2	2	
34	3	5	2	5	5	4	
42							
45	3	5	2	5	5	4	
47				4	4		
48					4		

**m users**

**rating matrix**



$$r_{u,i} \simeq \mathbf{u}_u^T \mathbf{b}_i$$

# Matrix-Factorization $\sim$ Linear Regression

$$\min_{\mathbf{u}, \mathbf{b}, \xi} \sum_{(u,i) \text{ if } r_{u,i} \neq 0} \left( r_{u,i} - \underbrace{\mathbf{u}_u^T \mathbf{b}_i}_{\text{user-item interaction}} - \xi_{u,i} \right)^2 + \lambda \left( \underbrace{\sum_u |\mathbf{u}_u|^2}_{u} + \underbrace{\sum_i |\mathbf{b}_i|^2}_{i} \right)$$

bias  
user-item interaction  
regularization

- SVD, not scalable
- Gradient descent, not convex problem
- Alternating least square (ALS)!

# What is the ALS?

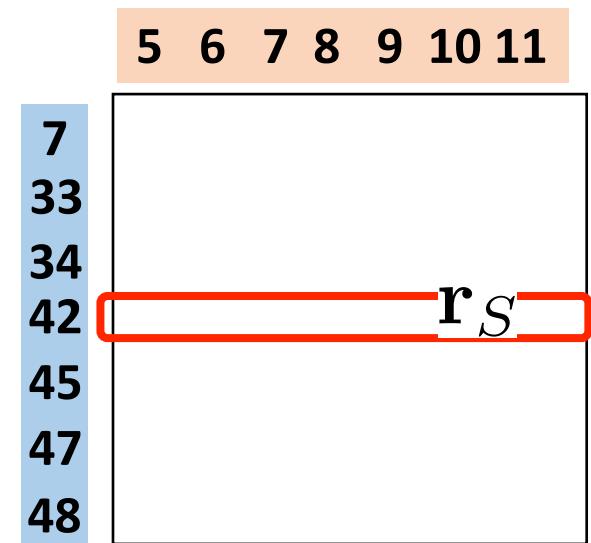
$$\text{Rating matrix} \approx \begin{pmatrix} (-\mathbf{u}_S-) \\ (-\mathbf{u}_H-) \\ (-\mathbf{u}_K-) \\ \vdots \end{pmatrix} \times \begin{pmatrix} \hat{\mathbf{b}}_1 & \hat{\mathbf{b}}_2 & \hat{\mathbf{b}}_3 & \cdots \end{pmatrix} \rightarrow \mathbf{R} = \mathbf{U}\mathbf{B}^T$$

- At each step, fix one variable, and solve minimization:  
fix  $\mathbf{u}$ , solve  $\mathbf{b}$   $\rightarrow$  fix  $\mathbf{b}$ , solve  $\mathbf{u}$   $\rightarrow$  fix  $\mathbf{u}$ , solve  $\mathbf{b}$

# More Detail: Normal Equations

Hu, Koren and Volinsky, 2008

$$\mathbf{u}_S = \begin{pmatrix} u_{S,1} \\ u_{S,2} \\ \vdots \\ u_{S,f} \end{pmatrix} = (\mathbf{B}^T \mathbf{B} + \lambda \mathbf{I})^{-1} \mathbf{B}^T \begin{pmatrix} r_{S,1} \\ r_{S,2} \\ \vdots \\ r_{S,n} \end{pmatrix}$$

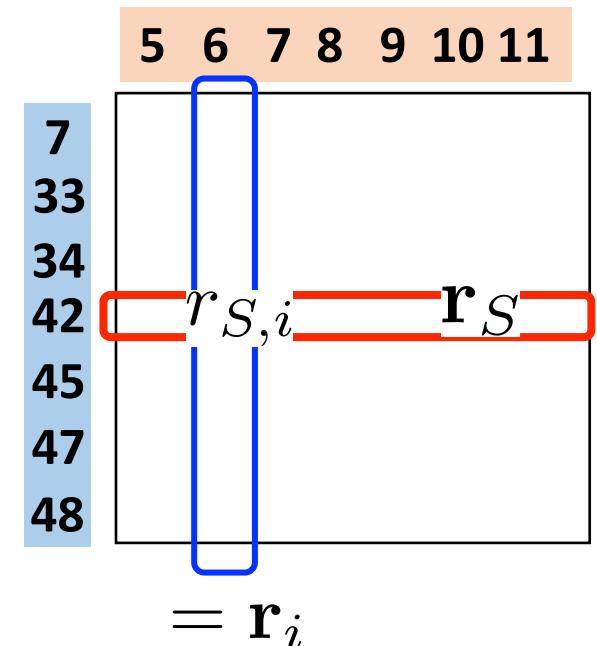


# More Detail: Normal Equations

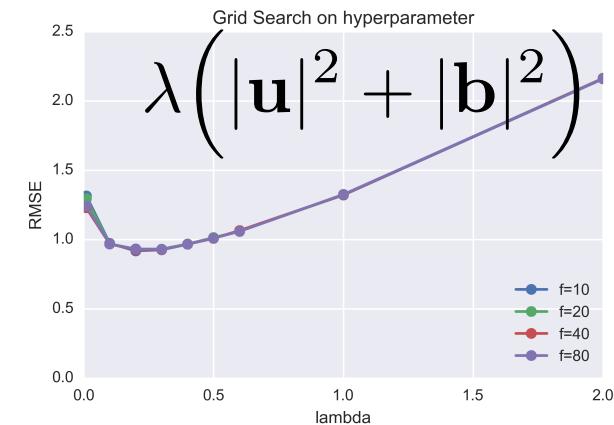
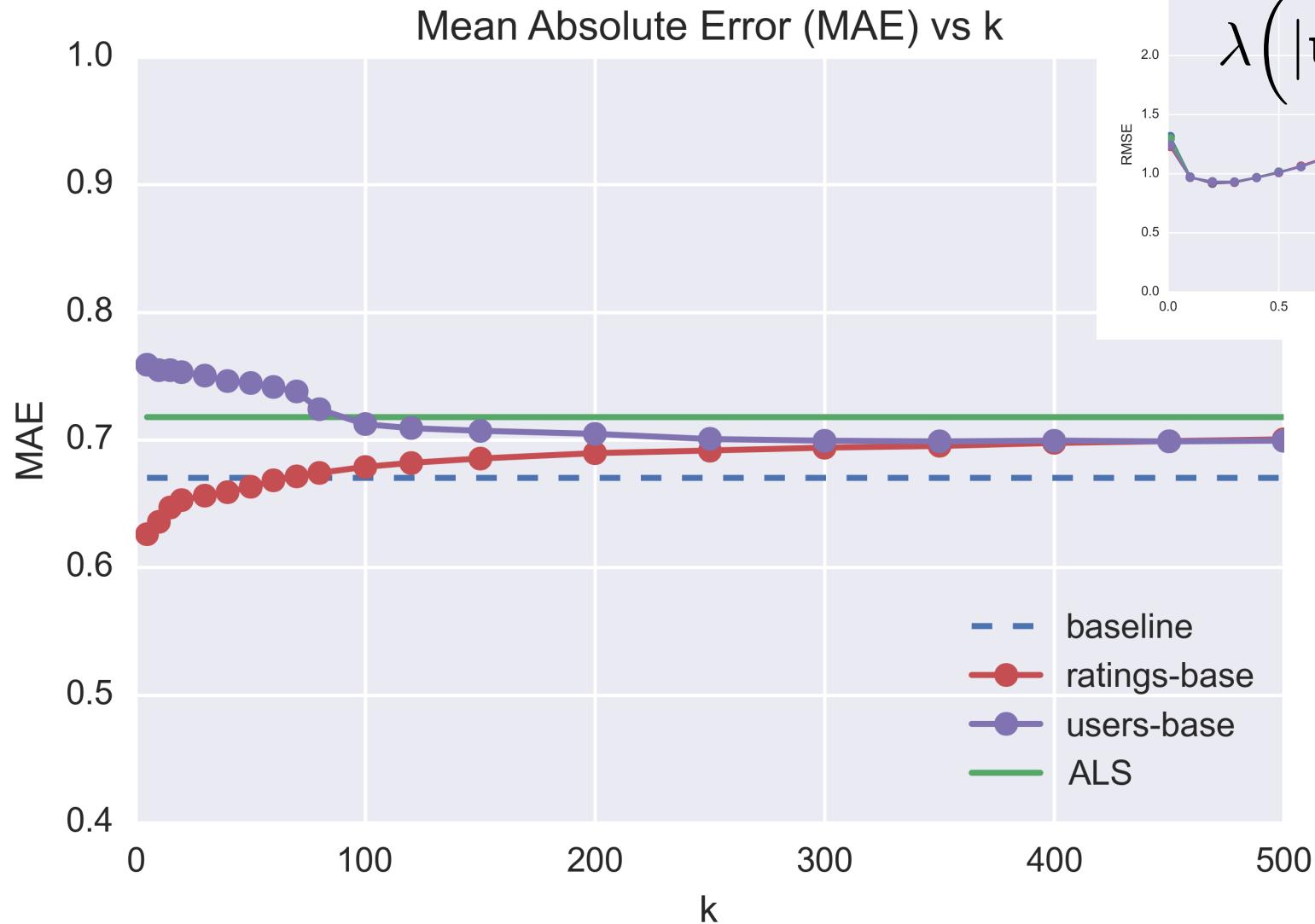
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$$\mathbf{b}_i = \begin{pmatrix} b_{i,1} \\ b_{i,2} \\ \vdots \\ b_{i,f} \end{pmatrix} = (\mathbf{U}^T \mathbf{U} + \lambda \mathbf{I})^{-1} \mathbf{U}^T \begin{pmatrix} r_{1,i} \\ r_{2,i} \\ \vdots \\ r_{m,i} \end{pmatrix}$$



# ALS Model Performance (on Spark)



# Implicit Data: e-Commerce Data

Explicit feedback  
(beHoppy)

	score	user_id	beer_id
0	4	7	6
1	4	48	7
2	4	47	9
3	4	47	10
4	4	50	9

$$r_{u,b} = 1 - 5$$

ratings

Implicit feedback  
(data warehouse)

customer_id	product_id
1	1
1	2
1	1
1	2
1	3
1	191
1	65
2	3
2	3
2	3

$$r_{u,b} \in \mathbb{I}$$

purchase frequency

Hu, Koren and Volinsky, 2008

# CF Using Implicit Data

- Logistic regression + confidence weight

Hu, Koren and Volinsky, 2008

$$c_{u,i} = f(\alpha, r_{u,i})$$

confidence

$$\min_{\mathbf{u}, \mathbf{b}, \xi} \sum_{(u,i)} c_{u,i} \left( p_{u,i} - \underbrace{\mathbf{u}_u^T \mathbf{b}_i}_{\text{user-item interaction}} - \xi_{u,i} \right)^2 + \lambda \left( \underbrace{\sum_u |\mathbf{u}_u|^2 + \sum_i |\mathbf{b}_i|^2}_{\text{regularization}} \right)$$

bias

user-item interaction

regularization

preference  $p_{u,i} = 0/1$  (if  $r_{u,i} > 0$ )

# Implicit Data CF Performance

- Metric: percentile-ranking

$$\overline{rank} = \frac{\sum_{u,i} r_{u,i} * rank_{u,i}}{\sum_{u,i} r_{u,i}}$$

- Random:  $\overline{rank} = 50\%$       Baseline:  $\overline{rank} \sim 29\%$
- CF:  $\overline{rank} \sim 16\%$

# Recommender Pipeline

